Consumption-led Growth

Markus Brunnermeier  Pierre-Olivier Gourinchas  Oleg Itskhoki
markus@princeton.edu  pog@berkeley.edu  itskhoki@princeton.edu

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Introduction
• Gourinchas and Jeanne (2013): the capital allocation puzzle

Average productivity growth and capital inflows between 1980 and 2000 for 68 non-OECD countries.
Motivation I

- Gourinchas and Jeanne (2013): **the capital allocation puzzle**

![Average productivity growth and capital inflows between 1980 and 2000 for 68 non-OECD countries.](image_url)

- In this paper, we swap the axes of this plot: can international capital flows alter productivity growth trajectories?
1. What is the relationship between openness and growth?
   - trade openness
   - financial openness
Motivation II

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   - trade openness
   - financial openness

2. Is it possible to borrow like Argentina or Spain and grow like China?
   (i) What is wrong with Spanish-style (consumption-led) growth?
   (ii) What is special about Chinese-style (export-led) growth?
1. What is the relationship between openness and growth?
   - trade openness
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   (ii) What is special about Chinese-style (export-led) growth?

- A model of endogenous convergence growth
  - to open the blackbox of productivity evolution under different openness regimes
  - a “neoclassical” (DRS) environment with endogenous innovation decisions by entrepreneurs
  - emphasis on the feedback from international borrowing into the pace and composition (T vs NT) of convergence
Figure 1: CA imbalances in the Euro Zone
Figure 1: Sectoral reallocation in the Euro Zone (Piton, 2017)
Main Insights

- Openness has two effects (on incentives for innovation):
  (i) change in relative market size
  (ii) increase in foreign competition and domestic cost of production, a price effect
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- Trade deficits (a) unambiguously favor non-tradable sector and (b) tend to reduce pace of innovation
  - reduced-form relationship between $NX$ and sectoral growth
  - furthermore, $NX/Y$ is a sufficient statistic
  - trade surpluses promote GDP growth
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- Laissez-faire productivity growth is in general suboptimal
  - capital controls may improve upon market allocation

Learning-by-doing and Dutch disease

Trade and growth

Financial flows and growth:

Trade and growth with Frechet distributions and beyond
Model Setup
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- Real small open economy in continuous time
  - exogenous world interest rate $r^*$ in terms of world good

- Two sector economy:
  - $\gamma$ tradable (exportable) and
  - $1 - \gamma$ non-tradable (non-exportable)

and symmetric in all other respects
Model Setup

- Real small open economy in continuous time
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  - $\gamma$ tradable (exportable) and
  - $1 - \gamma$ non-tradable (non-exportable)
  and symmetric in all other respects

- Rest of the world (ROW) in steady state:
  \[ W^* = A_T^* = A_N^* = A^* \quad \text{and} \quad P_F^* = P_N^* = P^* = 1 \]

- We study convergence growth trajectories:
  \[ A_T(0), A_N(0) < \bar{A} \leq A^* \]

- Growth results from new product creation by profit-maximizing entrepreneurs
Households

- Representative household:

\[
\max \{C(t), L(t)\} \int_0^\infty e^{-\vartheta t} U(t) dt, \quad U = \frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\varphi} L^{1+\varphi}
\]

s.t. \[\dot{B} = r^* B + WL + \Pi - PC \]
\[= GDP \]
\[= Y\]
Households

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\max_{\{C(t), L(t)\}} \int_0^\infty e^{-\theta t} U(t) dt, \quad U = \frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\varphi} \frac{1}{1+\varphi} L^{1+\varphi}
\]

s.t. \( \dot{B} = r^* B + WL + \Pi - PC \) \( \stackrel{=}{GDP} \) \( \stackrel{=}{Y} \)

- Static market clearing (goods and labor):

\[
WL = Y + NX, \quad C^\sigma L^\varphi = W/P
\]
Demand

- Two sectors:

\[ Y = PC = \gamma P_T C_T + (1 - \gamma) P_N C_N \]

where

\[ C = C_T^{\gamma} C_N^{1-\gamma} \quad \text{and} \quad C_T = \left[ \kappa \frac{1}{\rho} C_F^{\frac{\rho-1}{\rho}} + (1 - \kappa) \frac{1}{\rho} C_H^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \ \rho > 1 \]

- Aggregators of individual varieties:

\[ C_H = \left[ \frac{1}{\gamma} \int_0^{\Lambda_T} C_H(i) \frac{\rho-1}{\rho} \, di \right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad C_N = \left[ \frac{1}{1 - \gamma} \int_0^{\Lambda_N} C_N(i) \frac{\rho-1}{\rho} \, di \right]^{\frac{\rho}{\rho-1}} \]
Exports and Imports

- Tradable expenditure:

\[ \gamma P_T C_T = \int_0^{\Lambda_T} P_H(i) C_H(i) \, di + \gamma P_F C_F \]

- Aggregate imports:

\[ X^* = \gamma P_F C_F = \gamma \kappa \left( \frac{P_F}{P_T} \right)^{1-\rho} Y, \quad P_F = \tau P^*_F = \tau \]

- Aggregate exports:

\[ X = \gamma P_H^* C_H^* = \gamma \kappa (\tau P_H)^{1-\rho} Y^* \]

- Net exports:

\[ NX = X - X^* = \gamma \kappa \tau^{1-\rho} \left[ P_H^{1-\rho} Y^* - P_T^{\rho-1} Y \right] \]
Technology and Revenues

- Technology of product $i \in [0, \Lambda_J]$ in sector $J \in \{T, N\}$:

  $Y_J(i) = A_J(i)L_J(i)$
Technology and Revenues

• Technology of product $i \in [0, \Lambda_J]$ in sector $J \in \{T, N\}$:

$$Y_J(i) = A_J(i)L_J(i)$$

• Marginal cost pricing if technology is non-excludable:

$$P_H = W / A_T \quad \text{where} \quad A_T = \left[ \frac{1}{\gamma} \int_0^{\Lambda_T} A_T(i)^{\rho-1} \, \text{d}i \right]^{\frac{1}{\rho-1}}$$
Technology and Revenues

- Technology of product \( i \in [0, \Lambda_J] \) in sector \( J \in \{ T, N \} \):
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  Y_J(i) = A_J(i)L_J(i)
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  \]

- Revenues:
  \[
  R_N(i) = P_N(i)C_N(i) = \left( \frac{P_N(i)}{P_N} \right)^{1-\rho} R_N,
  \]
  \[
  R_T(i) = P_H(i)C_H(i) + P_H^*(i)C_H^*(i) = \left( \frac{P_H(i)}{P_H} \right)^{1-\rho} R_T
  \]

  where \( R_N = Y \) and
  \[
  R_T = (1 - \kappa) \left( \frac{P_H}{P_T} \right)^{1-\rho} Y + \kappa(\tau P_H)^{1-\rho} Y^* = Y \left[ 1 + \frac{NX}{\gamma Y} \right]
  \]
Technology Draws

• An entrepreneur has \( n \gg 1 \) possible ideas (projects):

\[
Z_{J(\ell)}(\ell) \overset{iid}{\sim} \text{Frechet}(z, \theta), \quad \ell = 1..n, \quad \theta > \rho - 1
\]

• A fraction \( \gamma \) of ideas are tradable, \( J(\ell) = T \)
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- An entrepreneur can adopt only one project

- The technology is privately owned for one period
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- Period profits:
  \[ \Pi_T(\ell) = \frac{1}{\rho} \left( \frac{\rho}{\rho - 1} \frac{W}{Z_T(\ell)} \frac{1}{P_H} \right)^{1-\rho} R_T \]
  \[ \Pi_N(\ell) = \frac{1}{\rho} \left( \frac{\rho}{\rho - 1} \frac{W}{Z_N(\ell)} \frac{1}{P_N} \right)^{1-\rho} R_N \]
Technology Draws

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  $$\Pi_N(\ell) = \frac{1}{\rho} \left( \frac{\rho}{\rho - 1} \frac{W}{Z_N(\ell)} \frac{1}{P_N} \right)^{1-\rho}$$
  
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Technology Adoption

- Project choice:
  \[
  \hat{\ell} = \arg \max_{\ell=1..n} \Pi_{J(\ell)}(\ell)
  \]
  and we define \((\hat{Z}_T, \hat{Z}_N, \hat{Z})\) and \((\hat{\Pi}_T, \hat{\Pi}_N, \hat{\Pi})\)
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- **Lemma 1** (i) *The probability to adopt a tradable project:*
  \[
  \pi_T \equiv \mathbb{P}\{\hat{\Pi}_T \geq \hat{\Pi}_N\} = \frac{\gamma \cdot \chi^{\rho-1}}{\gamma \cdot \chi^{\rho-1} + 1 - \gamma}, \quad \chi \equiv \left( \frac{P_H}{P_N} \right)^{\rho-1} \frac{R_T}{R_N}.
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Technology Adoption

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Technology Adoption

- **Project choice:**

  \[
  \hat{\ell} = \arg \max_{\ell=1 \ldots n} \Pi_J(\ell)
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- **Lemma 1**  (i) The probability to adopt a tradable project:

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  \]

  (ii) The productivity conditional on adoption:

  \[
  \mathbb{E}\left\{\hat{Z}_T^{\rho-1} \mid \hat{\Pi}_T \geq \hat{\Pi}_N\right\} = \left(\frac{\pi_T}{\gamma}\right)^{\nu-1} A^{*\rho-1},
  \]

  where \(A^* \equiv \mathbb{E}\hat{Z} = (nz)^{1/\theta} \Gamma(\nu)^{1/\rho-1}\) and \(\nu \equiv 1 - \frac{\rho-1}{\theta} \in (0, 1)\).
• $\lambda$ is the innovation rate and $\delta$ is the rate at which technologies become obsolete:

$$\dot{\Lambda}_T = \lambda \pi_T - \delta \Lambda_T$$

• Assume $\lambda$ is country-specific and $\lambda \leq \delta$
Productivity Dynamics

- \( \lambda \) is the innovation rate and \( \delta \) is the rate at which technologies become obsolete:
  \[
  \dot{\Lambda}_T = \lambda \pi_T - \delta \Lambda_T
  \]

- Assume \( \lambda \) is country-specific and \( \lambda \leq \delta \)

- **Lemma 2** *The sectoral productivity dynamics is given by:*
  \[
  \frac{\dot{A}_T}{A_T} = \frac{\delta}{\rho - 1} \left[ \left( \frac{\bar{A}}{A_T} \right)^{\rho-1} \left( \frac{\pi_T}{\gamma} \right)^\nu - 1 \right]
  \]
  where \( \bar{A} \equiv A^* \left( \frac{\lambda}{\delta} \right)^{\frac{1}{\rho-1}} \).
\[ \frac{\dot{A}_T(t)}{A_T(t)} = \frac{1}{\rho - 1} \left[ \lambda \left( \frac{A^*}{A_T(t)} \right)^{\rho - 1} \left( \frac{\pi_T(t)}{\gamma} \right)^\nu - \delta \right], \]

\[ \frac{\pi_T(t)}{1 - \pi_T(t)} = \frac{\gamma}{1 - \gamma} \chi(t)^{\rho - 1}, \]

\[ \chi = \left( \frac{P_H}{P_N} \right)^{\rho - 1} \frac{R_T}{R_N} = \left( \frac{A_N}{A_T} \right)^{\rho - 1} \left[ 1 + \frac{NX}{\gamma Y} \right], \]

\[ B(0) + \int_0^\infty e^{-rt} NX(t) = 0. \]
Closed Economy
Closed Economy, $\kappa \equiv 0$

- In closed economy $R_T = R_N = Y$, and therefore:

$$\chi = \left(\frac{P_H}{P_N}\right)^{\rho-1} = \left(\frac{A_N}{A_T}\right)^{\rho-1}$$

- The project choice is, thus:

$$\left\{\frac{\pi_T(t)}{1 - \pi_T(t)} = \frac{\gamma}{1 - \gamma} \left(\frac{A_N(t)}{A_T(t)}\right)^{\theta}\right.$$
Closed Economy, $\kappa \equiv 0$

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- The project choice is, thus:
  \[ \frac{\pi_T(t)}{1 - \pi_T(t)} = \gamma \left( \frac{A_N(t)}{A_T(t)} \right)^{\theta} \]

- Proposition 1  (i) Starting from $A_T(0) = A_N(0)$, equilibrium project choice in the closed economy is $\pi_T(t) \equiv \gamma$,
  \[ A_T(t) = \left[ e^{-\delta t} A_T(0)^{\rho^{-1}} + (1 - e^{-\delta t}) \bar{A}^{\rho^{-1}} \right]^{\frac{1}{\rho^{-1}}} \text{ and } \bar{\Lambda}_T = \gamma \frac{\lambda}{\delta}. \]

  (ii) Equilibrium allocation $C = w \frac{1+\varphi}{\sigma+\varphi}, \ L = w \frac{1-\sigma}{\sigma+\varphi}, \ w = A.$

  (iii) Efficiency: 


Balanced Trade
Balanced Trade

- Consider open economy with $\kappa > 0$ and $\tau \geq 1$

- **Lemma 3** (i) The relative revenue shifter is given by:

$$\frac{R_T}{R_N} = (1 - \kappa) \left( \frac{P_H}{P_T} \right)^{1-\rho} + \kappa (\tau P_H)^{1-\rho} \frac{Y^*}{Y} = 1 + \frac{NX}{\gamma Y}.$$

(ii) Under balanced trade, $\chi = (A_N/A_T)^{\rho-1}$, and hence $\pi_T(t)$ and $(A_T(t), A_N(t))$ follow the same path as in autarky.
Balanced Trade

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(ii) *Under balanced trade, $\chi = (A_N/A_T)^{\rho-1}$, and hence $\pi_T(t)$ and $(A_T(t), A_N(t))$ follow the same path as in autarky.*

- Equilibrium allocation is nonetheless different from autarkic:

$$w = C = A \cdot \left( \frac{1}{\tau^{2\rho-1}} \frac{A^*}{A_T} \right)^{\kappa \gamma \frac{1}{1+(2-\kappa)(\rho-1)}}$$
Balanced Trade

- Consider open economy with $\kappa > 0$ and $\tau \geq 1$

- **Lemma 3**  
  (i) The relative revenue shifter is given by:

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- Equilibrium allocation is nonetheless different from autarkic:

  $$ w = c = A \cdot \left( \frac{1}{\tau^{2\rho-1}} \frac{A^*}{A_T} \right)^{\frac{\kappa \gamma}{1+(2-\kappa)(\rho-1)}} $$

- Laisser-faire productivity dynamics is suboptimal. The planner would choose $\pi_T(t) < \gamma$ for all $t \geq 0$.  

Open Economy
Financial Openness

- With open current account:

\[
\frac{\pi_T}{1 - \pi_T} = \frac{\gamma}{1 - \gamma} \left( \frac{A_N}{A_T} \right)^\theta \left[ 1 + \frac{NX}{\gamma Y} \right]^{\frac{\theta}{\rho - 1}}
\]

- Lemma 4

\[NX(t) < 0 \text{ and } A_T(t) \geq A_N(t) \Rightarrow \dot{A_T}(t) < \dot{A_N}(t)\]

- Proposition 5

In st. st. with \(NX = -r^* \bar{B} > 0\):

\[\bar{A}_T > \bar{A}_N > \bar{A}_T\]

- Proposition 6

Starting from \(A_T(0) = A_N(0) < \bar{A}\), there exist two cut-off/unique:

1. \(NX(t) < 0\) for \(t \in [0, t_1)\) and \(NX(t) > 0\) for \(t > t_1\), and

2. \(A_T(t) < A_N(t)\) for \(t \in (0, t_2)\) and \(A_T(t) > A_N(t)\) for \(t > t_2\).

At \(t = t_2\), \(A_T(t) = A_N(t) = A(t) < A(t)\).
Financial Openness

- With open current account:
  \[
  \frac{\pi_T}{1 - \pi_T} = \frac{\gamma}{1 - \gamma} \left( \frac{A_N}{A_T} \right)^\theta \left[ 1 + \frac{N\bar{X}}{\gamma Y} \right]^{\frac{\theta}{\rho - 1}}
  \]

- **Lemma 4** \(NX(t) < 0 \text{ and } A_T(t) \geq A_N(t) \Rightarrow \frac{\dot{A}_T}{\dot{A}_N} < 1\).

- **Proposition 5** In st. st. with \(\overline{N\bar{X}} = -r^*\bar{B} > 0\): \(\bar{A}_T > \bar{A} > \bar{A}_N\).

- **Proposition 6** Starting from \(A_T(0) = A_N(0) < \bar{A}\), there exist two cutoffs \(0 < t_1 < t_2 < \infty\):
  - \(NX(t) < 0 \text{ for } t \in [0, t_1) \text{ and } NX(t) > 0 \text{ for } t > t_1\), and
  - \(A_T(t) < A_N(t) \text{ for } t \in (0, t_2) \text{ and } A_T(t) > A_N(t) \text{ for } t > t_2\).

At \(t = t_2\), \(A_T(t) = A_N(t) = A(t) < A^a(t)\).
Figure 2: Productivity convergence in closed and open economies
Impact of Openness

- Two effects of openness:
  1. Relative size of the market: $Y/Y^*$
  2. Competition: $P_T/P_H < 1$

$$1 + \frac{NX}{\gamma Y} = \left( \frac{P_H}{P_T} \right)^{1-\rho} \cdot \left[ (1 - \kappa) + \kappa \left( \frac{\tau}{P_H} \right)^{1-\rho} \frac{P_H^{1-\rho} Y^*}{P_T^{\rho-1} Y} \right]$$

$\gamma = 0.3$
Endogenous Innovation
Endogenous Innovation Rate

- Entrepreneurship decision as in Lucas (1978) if $\mathbb{E}\hat{\Pi} \geq \phi W$:

$$\lambda = \varphi \left( \mathbb{E}\hat{\Pi} / W \right) \quad \text{and} \quad \mathbb{E}\hat{\Pi} / W = \frac{\phi R N / W}{A_{\theta}^{\rho - 1}} \mathbb{E}_{\max} \left\{ \chi \hat{Z}^\rho_{\hat{T}}, \hat{Z}^\rho_{N} \right\}$$

- Lemma 5

$$\mathbb{E}\hat{\Pi} / W = \varphi \cdot \left( \frac{A^*}{A} \cdot \frac{A}{\hat{A}_{\theta}} \right)^{\rho - 1} \cdot \Psi \left( 1 + \frac{N X}{Y} \right)$$
Endogenous Innovation Rate

- Entrepreneurship decision as in Lucas (1978) if $\hat{E} \hat{\Pi} \geq \phi W$:

$$\lambda = \Phi \left( \frac{\hat{E} \hat{\Pi}}{W} \right) \quad \text{and} \quad \frac{\hat{E} \hat{\Pi}}{W} = \frac{\rho R_N/W}{A_N^{\rho-1}} \max \left\{ \chi \hat{Z}_T^{\rho-1}, \hat{Z}_N^{\rho-1} \right\}$$

- Lemma 5 

$$\frac{\hat{E} \hat{\Pi}}{W} = \rho \cdot \left( \frac{A^*}{A} \cdot \frac{A}{\hat{A}_\theta} \right)^{\rho-1} \cdot \psi \left( 1 + \frac{NX}{Y} \right)$$

- Proposition 8  

(i) $\lambda$ is increasing in $A^*/A$ and in $A/\hat{A}_\theta \geq 1$.

(ii) $\lambda$ increases with trade openness iff $\sigma < 1$ and $\phi < \infty$.

(iii) When $\sigma = 1$, $\Psi \approx 1 + \left[ \left( \frac{A_N}{A_T} \right)^{1-\gamma} - \frac{\phi}{1+\phi} \right] \frac{NX}{Y}$, and $\lambda$ increases with $NX$ when $A_N \geq A_T$. 

Endogenous Innovation Rate

- Entrepreneurship decision as in Lucas (1978) if $E \hat{\Pi} > \phi W$:
  \[
  \lambda = \Phi \left( \frac{E \hat{\Pi}}{W} \right) \quad \text{and} \quad \frac{E \hat{\Pi}}{W} = \frac{\varrho R_N / W}{A_\theta^{\rho-1}} \max \left\{ \chi \hat{Z}^\rho_T, \hat{Z}^\rho_N \right\}
  \]

- Lemma 5
  \[
  \frac{E \hat{\Pi}}{W} = \varrho \cdot \left( \frac{A^*}{A} \cdot \frac{A}{\hat{A}_\theta} \right)^{\rho-1} \cdot \psi \left( 1 + \frac{NX}{Y} \right)
  \]

- Proposition 8 (i) $\lambda$ is increasing in $A^*/A$ and in $A/\hat{A}_\theta \geq 1$.
  (ii) $\lambda$ increases with trade openness iff $\sigma < 1$ and $\varphi < \infty$.
  (iii) When $\sigma = 1$, \( \Psi \approx 1 + \left[ \left( \frac{A_N}{A_T} \right)^{1-\gamma} - \frac{\varphi}{1+\varphi} \right] \frac{NX}{Y}, \)
  and $\lambda$ increases with $NX$ when $A_N \geq A_T$.

- Endogenous non-tradable tilt reinforces the negative effect of trade deficits on innovation rate

- Induced $NX > 0$ with policy if the goal is max growth rate
Empirical Implications
Empirical Implications

- Reduced-form relationship between $NX$ and sectoral growth:

$$\frac{\dot{A}_T(t)}{A_T(t)} - \frac{\dot{A}_N(t)}{A_N(t)} = g_0 \left[ - (\rho - 1) (1 + \mu) \log \frac{A_T(t)}{A_N(t)} + \frac{\mu}{\gamma} \frac{NX(t)}{Y(0)} \right],$$

with $g_0 \equiv \frac{\delta}{\rho - 1} \left( \frac{\lambda}{\delta} \frac{A^*_T}{A_0} \right)^{\rho - 1}$, which is also the base growth rate

- holds whether $NX \neq 0$ are market outcomes or policy-induced
- i.e., applies equally for $NX < 0$ in Spain and $NX > 0$ in China

- $NX/Y$ is a sufficient statistic for the feedback effect from equilibrium allocation to sectoral productivity growth
Preliminary empirical results

- KLEMS panel of sector-country productivity growth
  (17 OECD countries, 33 ~3-digit sectors, 2001–2007 change)
- Empirical specification:
  \[ \Delta \log A_{ks} = d_k + d_s + b \cdot \log A_{ks}^0 + c \cdot \Lambda_s \cdot nx_k + \varepsilon_{ks} \]
  - \( \Delta \log A_{ks} \) is productivity growth in sector \( s \), country \( k \)
  - \( \Lambda_s \) is median sector-level *home share* across countries
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  \]

<table>
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<tr>
<th>Dep. var:</th>
<th>VA/L (1)</th>
<th>RVA/L (2)</th>
<th>KLEMS (3)</th>
<th>VA/L (4)</th>
<th>RVA/L (5)</th>
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<td>\Delta \log A_{ks}</td>
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<tr>
<td>\Lambda_s \cdot n x_k</td>
<td>-0.36*** (0.10)</td>
<td>-0.41** (0.15)</td>
<td>0.07 (0.20)</td>
<td>-0.20 (0.14)</td>
<td>-0.00 (0.14)</td>
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<td>\log A_{ks}^0</td>
<td>-4.75** (1.76)</td>
<td>-4.43*** (0.98)</td>
<td>-0.74 (0.72)</td>
<td>-2.17** (0.73)</td>
<td>-3.40*** (0.56)</td>
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<td>(R^2)</td>
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- 6% trade deficit reduces relative sectoral productivity growth by 1% across tradability quartiles (25th–75th)
Unit Labor Costs

- Two ULC measures: $w/A$ and $W/A_T$
  - move together holding $\tau$ constant

- Autarky (assume $\sigma = 1$):
  \[ w^a(t) = C^a(t) = A(t) \]

- Balanced trade:
  \[ w^b(t) = C^b(t) = A(t) \left( \frac{A^*}{A_T(t)} \right)^{\frac{\kappa \gamma}{1+(2-\kappa)(\rho-1)}} > A(t) \]

- Open financial account:
  \[ w^b(0) < w(0) < C(0) \]

- ULC increase on impact and gradually fall along the convergence path
Applications
1. Physical capital and financial frictions

2. Misallocation and growth policy

3. Rollover crisis
   - Sudden stop in capital flows during transition triggers a reversal in trade deficits and a recession in non-tradable sector
   - Rapid take off in tradable productivity growth, provided labor market can flexibly adjust by a sharp decline in wages
Conclusion
Conclusion

• Standard endogenous growth forces have a robust implication for the relationship between trade deficits and:
  1. non-tradable tilt of innovation
  2. overall lower speed of convergence growth

• Countries that borrow along the convergence growth trajectory are likely to experience asymmetric and slower convergence
  – lagging tradable productivity
  – high unit labor costs and depressed innovation rate
  – particularly vulnerable to rollover crisis along such trajectories

• Countries that are concerned with GDP growth rather than welfare might find it optimal to subsidize exports
Appendix
Price Indexes

- **Average sectoral prices:**
  \[
  P_H = \left[ \frac{1}{\gamma} \int_0^{A_T} P_H(i)^{1-\rho} \, di \right]^{\frac{1}{1-\rho}} \quad \text{and} \quad P_N = \left[ \frac{1}{1-\gamma} \int_0^{A_N} P_N(i)^{1-\rho} \, di \right]^{\frac{1}{1-\rho}}
  \]

- **Aggregate price indexes:**
  \[
  P = P_T^\gamma P_N^{1-\gamma} \quad \text{where} \quad P_T = \left[ \kappa P_F^{1-\rho} + (1 - \kappa) P_H^{1-\rho} \right]^{\frac{1}{1-\rho}}
  \]

- **Equilibrium sectoral prices:**
  \[
  P_H = \frac{W}{A_T}, \quad P_N = \frac{W}{A_N} \quad \text{and} \quad P_F = \tau
  \]

- **Real wage rate:**
  \[
  w = \frac{W}{P} = \frac{W}{P} = A \left[ 1 - \kappa + \kappa \left( \frac{W}{\tau A_T} \right)^{\rho-1} \right]^{\frac{\gamma}{\rho-1}}, \quad A \equiv A_T^\gamma A_N^{1-\gamma}
  \]
Solution for NX

- Equilibrium system:

\[ C = w^{\frac{1+\varphi}{\sigma+\varphi}} \left[ 1 + \frac{NX}{Y} \right]^{-\frac{\varphi}{\sigma+\varphi}} \text{ where } w = A \left( \frac{W}{\tau A_T} \right)^{\kappa \gamma} \]

and

\[ \frac{NX}{Y} = \frac{\gamma \kappa}{\left( \frac{W}{\tau A_T} \right)^{\rho - \kappa \gamma}} \left[ \tau^{1-2\rho} \frac{A^{\frac{1+\varphi}{\sigma+\varphi}}}{C} \frac{A}{A_T} - \left( \frac{W}{\tau A_T} \right)^{(1-\kappa \gamma)+(2-\kappa)(\rho-1)} \right] \]
Efficiency in Closed Economy

- **Proposition** (i) If $A_T(0) = A_N(0)$, then $\pi^*_T(t) = \gamma$ and $A_T(t) = A_N(t)$ for all $t$ maximizes $A(t)$ for all $t$. (ii) If $A_N(t) > A_T(t)$ at some $t$, then $\pi^*_T(t) \in (\gamma, \pi_T(t))$, and laissez-faire dynamics with $\pi_T(t)$ is suboptimal.

- Optimal policy satisfies (for $J \in \{T, N\}$):

$$
\left(\frac{\pi^*_T}{1 - \pi^*_T} \frac{1 - \gamma}{\gamma}\right)^{1-\nu} = \frac{\xi_T}{\xi_N} \left(\frac{A_N}{A_T}\right)^{\rho-1},
$$

where $b_J(t)\xi_T(t) - \dot{\xi}_J(t) = a_J(t)$,

and $a_J(t) \equiv \left(\frac{A_J(t)}{A(t)}\right)^{\eta-1} A(t)^\zeta$, $b_J(t) \equiv \vartheta + \delta \left(\frac{\bar{A}}{A_J(t)}\right)^{\rho-1} \left(\frac{\pi_J(t)}{\gamma_J}\right)^\nu$

- $b_J(t)$ plays the role of discount rate and $a_J(t)$ is the flow benefit

- $\xi_T/\xi_N = R_T/R_N$ in the limit of $\vartheta \to \infty$ (perfect impatience)

  Otherwise, $\xi_T/\xi_T \in (1, R_T/R_N)$

- Patents with finite time-varying duration can decentralize $\pi^*_T(t)$
Comparison with Learning-by-Doing

- General learning-by-doing formulation:

\[
Y_T(t) = F(A_T(t), L_T(t)), \\
\dot{A}_T(t) = G(A_T(t), A_N(t), L_T(t), L_N(t))
\]
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\]

- Mapping of the baseline model into learning-by-doing:

\[
F(A_T, L_T) = AL,
\]
\[
G(A_T, A_N, L_T, L_N) = \tilde{G}(A_T, \pi_T(A_T, A_N, L_T, L_N)),
\]
\[
\tilde{G}(A_T, \pi_T) = \frac{\delta}{\rho - 1} \left[ \left( \frac{\bar{A}}{A_T} \right)^{\rho - 1} \left( \frac{\pi_T}{\gamma} \right)^{\nu} - 1 \right],
\]
\[
\frac{\pi_T}{1 - \pi_T} \frac{1 - \gamma}{\gamma} = \left( \frac{A_N}{A_T} \right)^{\theta} \left( \frac{R_T}{R_N} \right)^{\frac{\theta}{\rho - 1}} \quad \text{and} \quad \frac{R_T}{R_N} = \frac{L_T}{L_N}
\]