NEW KEYNESIAN PHILLIPS CURVES:
A REASSESSMENT BASED ON EURO-AREA DATA

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Abstract:
In this paper, we re-examine the empirical success of New Keynesian Phillips Curves using euro-area data. The nature of our re-evaluation relies on the actual empirical underpinnings of such estimates: we find existing estimates un-robust and – given that key deep parameters are generally calibrated rather than estimated – potentially at odds with the data. We re-estimate with a well-specified optimising supply-side (which attempts to treat non-stationarity in factor income shares and mark-ups) and this allows us to derive directly estimates of technology parameters and real marginal costs. Our resulting estimates of the euro-area New Keynesian Phillips curves are robust, provide reasonable estimates for fixed-price durations and discount rates and embody plausible dynamic properties.

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1. Introduction

New Keynesian Phillips curves (NKPC) have become an increasingly popular model for analysing and accounting for inflation persistence. They differ from traditional forms by replacing measures of cyclical pressures (e.g., output gaps) with real marginal cost indicators and further assume that prices are set optimally subject to adjustment constraints. Enthusiasm for the New (over the Old) might be thought to stem from at least four sources. First, whilst old style Phillips curves fit the data robustly, their recent forecasting performance appears to have weakened (e.g., Anderson and Wascher, 2000).\(^1\) This might reflect such things as measurement errors in output gap estimates (Orphanides et al., 2000); new economy effects or, the failure of backward-looking expectations to capture improved policy credibility. Second, many have argued that inflation persistence is better matched by real marginal cost indicators than cyclical measures such as traditionally-defined output gaps (see the discussions in Galí and Gertler, 1999, Neiss and Nelson, 2002).\(^2\) Third, in using real marginal costs as the fundamental determinant of inflation, they potentially provide a richer description of and justification for the inflationary process than output gaps (given their decomposition into wage and productivity components). Finally, being based on clear micro-foundations, they provide a more theoretically satisfying model of inflation dynamics.

Conventional Phillips curves are of the form, \(\pi_t = \sum_{i=1}^{l} \alpha_i \pi_{t-i} + \eta \tilde{y}_t\) with the priors \(\sum \alpha_i = 1\) and \(\eta > 0\). Where \(\pi_t\) is the contemporaneous inflation rate and \(\tilde{y}_t\) is, typically, an output gap. Thus, a positive output gap increases inflation subject to there being no long-run trade off. The NKPC, by contrast, posits: \(\pi_t = \delta E_t(\pi_{t+1}) + \lambda x_t\) where \(x\) is the assumed fundamental: \(\pi_t = \lambda \sum_{j=0}^{\infty} \delta^j x_{t+j}\).

Inflation becomes a jump process and thus any sluggishness in inflation derives directly from that in the fundament -- which may be either the output gap or the log deviation of real marginal costs from steady state. Under certain proportionality conditions, both measures are perfectly correlated. That allows the interpretation that real marginal costs may in fact capture the “true” output gap better than conventional de-trended measures.

Although the literature has mostly focused on US data, recently NKPCs (as here) have also been estimated for the euro-area (e.g. Amato and Gerlach, 2000, Bårdsen et al. (2002), Galí et al. 2001). Indeed the latter claim that the New Phillips curves fits euro-area data better than the US

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\(^1\) See also Gordon (1998).

\(^2\) GG (1999) suggest that in recent US data, output gaps and marginal costs have moved in different directions.
with a lesser role for backward-looking expectations in the euro area. In spite of the recent success, however, common criticisms of the NKPC approach include: whether they capture actual inflation persistence (Fuhrer, 1997, Fuhrer and Moore, 1995), the plausibility of their implied dynamics (Ball, 1994, Mankiw, 2001) and their estimation methodology. Notably, on this last point, Rudd and Whelan (2001) suggest that hybrid model suffers low power against the backward-looking one. Also Bårdsen et al. (2002) argue that, as statistical, model both the pure and hybrid NKPC are inadequate, and the significance of the forward term in the hybrid model of Gali et al. (2001) in the euro area is therefore misleading. Likewise they show that, applying the encompassing principle also to UK and Norwegian inflation, leads to clear rejection of the NKPC.

Our paper takes a more fundamental perspective. From an empirical point of view, we consider the specification and estimation of a realistic, data-consistent but theoretically well-founded supply-side, to determine technology parameters and real marginal costs, as key. Indeed it is perhaps surprising that, in a literature that emphasises both clear micro foundations, properly-measured production costs and long-run inflation determinants, the derivation of the supply side has been by passed quite lightly – being largely calibrated (thus neglecting issues of data compatibility) and based on either short-run production functions omitting the capital stock or where capital is determined outside the optimisation framework. For instance, on the issue of data compatibility, a well-documented stylised fact in many European countries is the “hump-shaped” labour income share in GDP; after increasing strongly in the 1970s, the share of labour income share in the euro area decreased continuously in the two subsequent decades. Developments in the United States, however, remain broadly in line with the stable labour income share (and, in turn, in line with the common Cobb-Douglas prior) (e.g. Blanchard (1997) and Caballero and Hammour (1998)).

This apparent data incompatibility of euro-area data with the underlying theoretical assumption is routinely ignored. One consequence of doing so, however, is that cointegration between the fundamant and the dependent price variable is not examined or likely to hold. In that sense, therefore, our re-evaluation takes this underlying data-compatibility in estimating the NKPCs for the euro area quite seriously. Clearly, if NKPCs are to give accurate view of inflation dynamics, the approach should be kept as close as possible to the data; otherwise whatever results derived (through, say, calibration) risk being spurious and un-representative. Our theoretical framework contains a multi-sector model of imperfect competition, where the two-factor production function with capital and labour as inputs is otherwise same across firms except for the

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3 For instance, well-known problems with GMM estimation such as those related to orthogonality conditions and instrument choice (e.g., Fuhrer et al., 1995).
sectorally-differentiated scale and technological progress parameters of the production function. By allowing price and income elasticity to differ across sectors, the aggregation of the firm-level conditions of profit maximisation implies a time-varying aggregated mark-up, the development of which reflects changes in the production shares of sectors with the high or low mark-ups and/or with a fast or slow speed of technological progress. Furthermore, the assumption of non-isoelastic demand curves implies time-variant sectoral mark-ups, as competing foreign prices also affect the pricing behaviour. This offers an avenue for explaining the development of the labour income share in the euro area – even with Cobb-Douglas technology.

Paying attention to the underlying supply side and relaxing a number of over-simplifying assumptions typically found in the literature, allows us not only a better identification of the fundament (marginal costs) but also a more robust verdict on NKPCs. Furthermore, an important implication of abandoning overly simplifying assumptions, in favour of more general optimising framework is that although the formal estimated equations doesn’t change, the interpretation changes from inflation equation to that of as dynamic price one.

The paper is organised as follows: Sections 2 reviews the Calvo staggered price model and leads to the specification of the NKPCs and, in particular, discusses some Gali et al. (2001) estimates for the euro area. Section 3 highlights problems win the conventional approach and 4 discusses our favoured supply-side system. The next section shows our estimates of both the supply side parameters and the NKPC themselves. Section 5 presents estimation results and test results based on the estimation of the reduced form present value specification. Section 6 concludes.

2. The Calvo staggered price model

A cornerstone of the New Keynesian Phillips Curve literature is price staggering. A simple way to capture this is to assume that in any given period each firm has a fixed probability $1-\theta$ that it may adjust its price during that period and, hence, a probability $\theta$ that it will keep its price unchanged, Calvo (1983). The expected time between price adjustments is thus $\frac{1}{1-\theta}$. Because these adjustment opportunities occur randomly, the interval between price changes for an individual firm will be a random variable. The aggregate price level can then be presented as the weighted sum of the prices of the firms which do not change their prices (equalling the previous period price level) and the price $p_t^i$ selected by firms that are able to change price at time t. Denoting by lowercase letters the logarithm of variables, the aggregate price level can be expressed as:
Following Galí and Gertler (1999) we assume that in each sector \( j \) a fraction \( 1 - \omega \) of the firms are 'forward looking' and set prices optimally given the constraint on timing of adjustments and using all the available information in order to forecast future marginal costs:

\[
p_t^* = \theta p_{t-1} + (1 - \theta) p_t^v
\]  

(1)

The rest of the firms are 'backward looking' and follow a rule of thumb in their price setting. Backward looking firms follow the price setting of their competitors and are unable to identify whether any competitor is backward- or forward-looking. Depending on whether there is or is not a lag in the information set available for the backward looking firms the following two backward looking price-setting rules are specified. If the latest information of the backward looking firms is dated on \( t-1 \), (a situation we refer to as information lag) as assumed by Galí and Gertler (1999), then the backward looking rule is:

\[
p_t^b = p_{t-1}^* + \pi_{t-1}
\]  

(3a)

In (3a) the lagged inflation proxies current inflation \( \pi_t \). As this kind of constraint on the available information seems unconventionally constrained, we assume alternatively that no information lag exist and backward looking firms set their prices following the rule (no information lag):

\[
p_t^b = p_{t-1}^* + \pi_t
\]  

(3b)

Alternative pricing rules (3a) and (3b) can also be interpreted in terms of Mankiw(2001a)’s “sticky-information” model; the main difference to Mankiw's (2001a) formulation is Mankiw assumes that in each period only a fraction firms updates information. In our formulation all firms, which reset their prices, update their information with, at most, one period information lag.

Following Rotemberg (1987), we assume that forward looking firms set their prices to minimise a quadratic loss function that depends on the difference between the reset price over the period it
is expected to remain fixed and the optimal price in the absence of restrictions in price setting. Hence, the representative firm \( j \) minimises

\[
\frac{1}{2} \sum_{t=0}^{\infty} (\delta \theta)^t \mathbb{E}_t \left( p_{jt} - p_{j,t+1}^* \right)^2
\]

(4)

Where \( \delta \) is (as before) a subjective discount factor and \( p_{j,t+1}^* \) is the optimal price, i.e. the latter is the profit-maximising price for firm in the absence of any restrictions or costs associated with price adjustment. It equals the mark-up over nominal marginal costs, i.e. in logarithms \( p_{j,t+1}^* = mcn_{j,t+1}^f + \mu \), where \( mcn_{j,t+1}^f \) is the marginal costs of the forward-looking firm \( j \) and \( \mu \geq 0 \) is the mark-up. The minimisation of (4) implies the following relation for the optimal reset price of firm \( j \),

\[
p_{jt} = (1 - \delta \theta) \sum_{t=0}^{\infty} (\delta \theta)^t (mcn_{j,t+1}^f + \mu)
\]

(5)

If, in addition, firms are assumed to share common production technology and labour markets are homogenous, then marginal costs are same for all firms adjusting the price at \( t \), i.e. \( mcn_{j,t+1}^f = mcn_{t+1}^f \) \( \forall j \), and, hence, also the optimal reset price is the same, \( p_{jt} = p_t^f \) \( \forall j \), for all forward-looking firms adjusting their price at \( t \). Accordingly, at the aggregate, we can write:

\[
p_t^f = (1 - \delta \theta) \sum_{t=0}^{\infty} (\delta \theta)^t (mcn_{t+1}^f + \mu)
\]

(6)

Equations (1)-(2), (3a) and (6), when written in terms of aggregate inflation (where \( \pi_t = p_t - p_{t-1} \)), imply,

\[
\pi_t = \gamma_t^f E_t(\pi_{t+1}) + \gamma_0^f \pi_{t-1} + \lambda_0 (mcn_t^f + \mu - p_t)
\]

(7)

where \( \gamma_t^f = \delta \theta \phi^{-1} \), \( \gamma_0^f = \omega \phi^{-1} \), \( \lambda_0 = (1 - \omega)(1 - \theta)(1 - \delta \theta) \phi^{-1} \) and \( \phi = \theta + \omega[1 - \theta(1 - \delta)] \).

\footnote{Thus, the fundamental should comprise the discounted value of the mark-up over marginal costs instead of merely marginal costs itself (c.f., Galí et al. (2001), equation A.3, p 1268).}
The adoption of the backward looking rule (3b) results in:

\[ \pi_t = \gamma'_b E_t(\pi_{t+1}) + \gamma''_b \pi_{t-1} + \lambda_t (mcn_t^f + \mu - p_t) \]  

(8)

where \( \gamma'_b = \delta \theta (1 - \omega (1 - \theta)) \zeta^{-1}, \gamma''_b = \omega \theta \zeta^{-1}, \lambda_t = (1 - \omega) (1 - \theta) (1 - \delta \theta) \zeta^{-1} \) and \( \zeta = \theta (1 + \delta \theta \omega) \).

An interesting feature of equations (7) and (8) is that, although the assumed underlying pricing rules of backward looking firms differ, they are, in fact, observationally equivalent. It can be shown that that between the parameter values of \( \omega \) associated with the rule (3a) (denote by \( \omega_0 \)) and the rule (3b) (denote by \( \omega_1 \)) the following relation prevails: \( \omega_1 = \frac{\omega_0}{\theta + \omega_0 (1 - \theta)} \). By substituting this relation to (8) we end up with (7) or vice versa and, hence, for the composite parameters \( \gamma'_b = \gamma'_b, \gamma''_b = \gamma''_b \) and \( \lambda_t = \lambda_t \). This also implies that time series techniques are not able to identify pricing rules (3a) and (3b) from each other. However, parameter estimates of \( \theta \) and \( \delta \) are unaffected by this indeterminacy. On the positive side, we may conclude that this indeterminacy increases the generality of the specified equations (7) and (8). Indeterminacy concerns only the identification of \( \omega \); different estimates are obtained conditional on the assumption concerning the information lag.

The only situation where this indeterminacy disappears is in the limiting case of completely flexible price setting, i.e. \( \theta = 0 \). In the context of the one period information lag backward looking rule (3a), equation (7) reduces to the backward-looking error correction form: \( \pi_t = \omega \cdot \pi_{t-1} + (1 - \omega) [mcn_t^f + \mu - p_{t-1}] \), while in the context of no information lag backward looking rule, we end up with \( p_t = mcn_t^f + \mu \), i.e. prices are set optimally although part of the firms do not base their price-setting behaviour on optimisation.

When the share of the backward looking firms is zero, \( \omega = 0 \), then both (7) and (8) reduce to,

\[ \pi_t = \delta E_t(\pi_{t+1}) + (1 - \theta) (1 - \delta \theta) \theta^{-1} (mcn_t^f + \mu - p_t) \]  

(9)

Notably, specifications (7)-(9) cannot be estimated before operationalising the marginal cost variable \( mcn_t^f \). We define two approaches. The first – what might be called the standard approach – derives (and calibrates) marginal cost from a highly simplified production
framework. In the second – the one followed here – the fundament is determined by a fully-specified supply-side system based on aggregation across sectors. We relax many of the constraints typically found in this literature and by allowing key parameters on both the supply and demand side to differ across sectors and goods. In the following (Section 3), we first review this standard approach and, in particular, we show that real marginal costs as calibrated by Galí et al. (2001) does not pass the standard requirements of cointegration and in addition we show that their calibration appears to contain a scaling error with strong effects on estimation results. When corrected, in terms of theoretical priors, estimates deteriorate markedly.

3 Estimation of NKPCs: Existing Studies.

Here we evaluate some aspects of existing NKPC studies. The nature of our critique is threefold. First, we examine the implications for the estimated equation of treating capital as exogenous. In particular, we show that the modification to the estimated equation typically made in that regard, requires an unrealistic assumption concerning capital allocation. Second, we show that the driving variable used by Gali et al. (2001) contains a level error, which can be associated with either a mistaken form of underlying identity or an ad-hoc addition of a constant in the estimated regression. Third, we show that the assumption underlying such a standard calibration for the driving variable will not be compatible with euro-area data.

3.1 Treatment of the Capital Stock

The standard approach derives (and calibrates) the marginal cost from a highly simplified production framework where either labour is the only factor (Galí et al., 2001) or where capital is included but determined outside the optimisation framework (Sbordone, 2002). Given Cobb-Douglas technology \( Y_t = A_t N_t^{-\alpha} K_t^\alpha \) and assuming that each firm is a monopolistic competitor producing a differentiated good and faces the iso-elastic demand curve for its products, given by, \( Y_{it} = Y_i(P_{it}/P_t)^{\epsilon} \), the real marginal cost \( mc_{it} = mcn_{it} - p_i \), specified in terms of aggregate marginal cost, is:

\[
mc_{it} - mc_i = -(y_{it} - y_i) + (n_{it} - n_i)
\]

\[
= -(y_{it} - y_i) + \frac{1}{1-\alpha} \left[ (y_{it} - y_i) - \alpha (k_{it} - k_i) \right]
\]

\[
= -\frac{\alpha}{1-\alpha} \left[ (y_{it} - y_i) - (k_{it} - k_i) \right]
\]

(10)
Thus, (10) shows that, if capital is exogenous, an assumption as to how the capital stock is allocated across firms is required. To end up with a formula without capital, the term \((k_i^f - k_f)\) must vanish. Two simple but unrealistic cases are:

(a) \(k^f = k = 0\): This implies that capital is not included in the theoretical framework (e.g., Gali et al. (2001)).

(b) \(k^f = k\): This implies that all capital is held by those forward-looking firms able to reset their prices at period \(t\).

In both cases, capital disappears and after inserting the demand function into (10) we derive:

\[
mc_i^f + \mu = \frac{1 - \alpha}{1 + \alpha(\epsilon - 1)}[mc_i + \mu] = \xi[mc_i + \mu] \quad ; \quad \xi = \frac{1 - \alpha}{1 + \alpha(\epsilon - 1)}
\]  

(11)

Where typically \(\alpha\) and \(\xi\) are calibrated to ensure a pre-set mark up. In the case \(\alpha = 0\) we have \(\xi = 1\) and thus \(mc_i^f = mc_i\).

Two other (more realistic) assumptions about the allocation of the exogenously-determined capital stock, is to assume that the capital-output (c) or capital-labour ratio (d) is equal across all firms:

(c) \(k_i^f - y_i^f = k_i - y_i\)

It is then straightforward to show that (10) collapses to \(mc_i^f = mc_i\). Hence, independent of \(\alpha\), no scaling of \(\xi\) different from unity is needed. Similarly for,

(d) \(n_i^f - n_i = k_i^f - k_i\)

after substituting into the production function:

\[\]

---

5 Sbordonne (2000) suggests that for the disappearance of the capital stock, it is sufficient to say that it is excogeneous. However, equation (10) shows that this is neither a necessary or sufficient condition.
\[ y_{t}^{f} - y_{t} = (1 - \alpha)(n_{t}^{f} - n_{t}) + \alpha(k_{t}^{f} - k_{t}) = k_{t}^{f} - k_{t} \]

again, we see this implies \( mc_{t}^{f} = mc_{t} \) and that \( \xi = 1 \) with \( \alpha \in [0,1] \). \(^6\) In fact, we derive the same result when capital is endogenous and part the optimisation framework.\(^7\)

The difference is that in the capital-endogenous case, this holds for all future values, whereas when the capital stock is exogenously given, this identity between aggregate and dis-aggregated marginal cost does not hold for expected future values. However, from the point of view of estimated specification (7) or (8) does not matter, whilst this is not the case if one tries to estimate in present-value form.

To conclude, the motivation for introducing this scaling parameter, \( \xi \), to be different from unity, is not well founded. On one hand, if capital is treated exogenously and constant returns are assumed, this requires unrealistic assumptions about the allocation of capital (a, b). On the other hand, for non-constant returns, pinning down a value for \( \xi \) is difficult because there is no straightforward way to calibrate the size of the returns to scale and, due to the difficulties to disentangle the speed of technical progress from non-unitary returns to scale, its estimation is very problematic.

### 3.2 Estimation Issues: Labour share Identities and regression constant

Here we show that the driving variable used by Gali \textit{et al.} (2001) contains a level error, which can be associated with either a mistaken form of underlying identity or an ad-hoc addition of a constant in the estimated regression. Thus, although there are two ways to operationalise their driving variable – and thus reproduce their results – both approaches are potentially flawed. Although these deviations from correct form may appear minor, they have dramatic effects on the parameter estimates of the estimated equations.

First, let us derive the driving variable based on the Gali \textit{et al.} (2001) calibration. Given our production function, static profit maximisation in the absence of price-adjustment, implies:

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\(^6\) Thus, the deviation of \( \xi \) from unity requires that the production function exhibits non-constant returns to scale.

\(^7\) Thus, the Capital-labour ratio is determined by the relative price of inputs, which is common across firms with homogenous labour and capital.
\[1 = (1 + \mu) MC_r = (1 + \mu) \frac{W_r N_r}{(1-\alpha) P_r Y_r} = (1 + \mu) \frac{S_r}{(1-\alpha)} \]  \hspace{1cm} (12)

Where \(1 + \mu = \frac{\varepsilon}{\varepsilon - 1}\). Gali et al. (2001) assumed that euro area labour income share \((S_t)\) is 0.75,\(^8\) and further assumed a mark-up of \(\mu = 0.1\). Now (12) implies:

\[\alpha = 1 - (1 + \mu) \cdot S_r = 1 - 1.1 \cdot 0.75 = 0.175 \text{ and } \xi = 0.3 \]  \hspace{1cm} (13)

However, Galí et al. (2001) mistakenly solved \(\alpha\) from:

\[\alpha = 1 - S_r / (1 + \mu) = 1 - 0.75 / 1.1 = 0.32 \text{ and } \xi = 0.163 \]  \hspace{1cm} (14)

Thus we can see that when the solved \(\alpha\) is substituted back into (12), this implies, with the predetermined values of \(S\) and \(\mu\), that the identity (12) does not hold and the average level of marginal costs in Galí et al. (2001) is 10.3% above unity instead of being 10% below unity (corresponding to the mark-up of 1.1). This is in contrast to the “corrected” case (i.e., equation 13) where the average, real mark up over (corrected) marginal cost is zero.

Thus, we can consider 3 cases. First, real marginal cost \((mc)\) from equation (14). Second, \(\alpha\) from equation (13) – the “corrected” case – and finally the mark-up over marginal cost which is what should be used as opposed to just the marginal cost. These cases can be seen in Figure 1.

Furthermore, it is clear that irrespective of its definition, no measure of real marginal costs appears stationary. Tests confirm this observation (DF-t-test= -0.16 and ADF(5)-t-test= -0.42).

Give this, we can proceed to estimation of the NKPC (such as the un-normalised case):

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\(^8\) In the euro area data used in Galí et al (2001), the GDP share of compensation per employee, was around 0.53. Accordingly, their implicit assumption is that GDP share of self-employed is around 0.22.
This is done with the $mc$ based both on (14) and the corrected case (13). In the latter, we re-estimate including the mark–up so that the average level of the term, $(mc + \mu)$, equals zero:

$$\phi \pi_i - \theta \delta \pi_{t+1} - \omega \pi_{t-1} - \lambda \xi (mc_i) = 0.$$  

(15)

It can be further shown that this mark-up over marginal costs can be reduced to deviations of marginal costs from the steady state and, in the Cobb-Douglas case, of deviation of labour income share from steady state (i.e., sample average). Using the fact that in the steady-state (denoted by a bar above the variable): $\overline{mc}_i = \log(1 + \mu)$. This implies that $mc_i + \log(1 + \mu) = mc_i - \overline{mc}_i = s - \overline{s}$ where $s = \log \left( \frac{wN}{PY} \right)$ denotes the labour share, which further equals in the Cobb-Douglas case,

$$\phi \pi_i - \theta \delta \pi_{t+1} - \omega \pi_{t-1} - (1 - \omega)(1 - \theta)(1 - \delta \theta) \xi (s, -\overline{s}) = 0.$$  

(16)

(17)

Though observationally equivalent to (16), this specification has the advantage that as long as $\xi = 1$, no assumption concerning the production function parameters or mark-up is needed. Clearly there is no role for a constant in this equation. However unless such a constant is included into the regression it is not possible to replicate the Gali et al. (2001) results.

$$\phi \pi_i - \theta \delta \pi_{t+1} - \omega \pi_{t-1} - (1 - \omega)(1 - \theta)(1 - \delta \theta) \xi (s, -\overline{s} + \text{const}) = 0.$$  

(18)

3.3 Empirical Results

In Tables 1 and 2, using the same data, instruments and normalisations we re-estimated the Gali et al. (2001) equations; that is to say, equation (15) with the marginal cost based on the mistaken identity, equation (14), mark-up over corrected marginal cost, equation (16) and deviation of labour income share from average with (18) and without constant (17). Table 1 presents

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9 We have also estimated the case of corrected mark-up. These results qualitatively resemble those of the mark-up over marginal costs but were even more volatile and un-robust. Details available.
estimates of the hybrid-version of the NKPC\textsuperscript{10} when $\xi=1$. In Table 2, we repeat the exercise with $\xi=0.163$ corresponding with $\alpha$ to the wrong identity (14) and with $\xi=0.30$ corresponding with $\alpha$ to correct identity (13). Finally in both tables we present estimate in un-normalised and normalised (with respect to inflation) forms.

[Table 1 and 2 here]

We might note that, in both tables the cases of uncorrected marginal cost and labour income share with constant, results are practically identical as those presented in Gali \textit{et al.} (2001). We see that the estimated values of the constant are around 0.1 which corresponds very closely with the average deviation of the uncorrected marginal cost from zero as shown in Figure 1.\textsuperscript{11} Nevertheless, the introduction of the constant is not implied by theory. However, if we re-scale the driving variable around zero, i.e. by using the deviation of labour income share from average without a constant or the mark-up over corrected real marginal costs as the driving variable, estimation results worsen and are quite different from those in Gali \textit{et al.} (2001). When normalised with respect to inflation, estimation results break down completely. In this case, $\theta$ estimates yield implausibly high values (practically unit) and thus very long durations for price fixing. This aspect is also reflected in the fact that the roots tend to be close to unity. Hence, adjustment times and durations (also reflected in $\lambda$ value) are unreasonably long.

Estimations in un-normalised form work better although the results are strongly affected by the value of predetermined parameter $\xi$. The smaller is $\xi$ the more reasonable the results look. With $\xi=0.163$ (based on wrong identity), expected duration is 4.7 quarters and with $\xi=0.3$ it lengthens to 7.4 quarters. However as argued, the motivation for introducing this scaling parameter, $\xi$, to be different from unity, is not well founded. In the case $\xi=1$ the expected duration of fix-price is lengthened to 18.4 quarters, i.e. over four years, which is un-reasonably long.\textsuperscript{12}

\textsuperscript{10} Estimates for the purely forward-looking variant are suppressed for brevity. Details available.

\textsuperscript{11} Actually, we do not know, which procedure Gali \textit{et al} (2000) used. Their theoretical notation refers to the practice of operationalising the driving variable in terms of the deviation of the labour income share from the sample average. However, their reported estimation results contain no additional constant, which in this case is needed to be able to replicate their results.

\textsuperscript{12} According to US survey evidence, average price duration is around one year (Taylor, 1999).
An interesting feature, however, in estimating equation (17) – i.e., without constant – or equation (16) – mark-up over corrected real marginal costs – is that they give reasonable discount factors (close to unity) and more reasonable than those cases corresponding to the Gali et al. (2001) results where we have discount factors well below unity and high quarterly subjective discount rates – in the range of 7.5%-19.6% implying an annual range of 33.5%-105%.

Let us summarise. The fundament in Gali et al. (2001) appears to have resulted in a level error. Although this might appear minor it has – as shown – substantial implications for the estimates. Accordingly, we found results to be quite sensitive to normalisation, with the un-normalised case appearing to yield economically more sensible results. Similarly the value of $\xi$ (conditional on the calibrated value of the production function parameter, $\alpha$ and the price elasticity of demand, $\varepsilon$, coupled with an omitted or exogenous capital stock) seems to add an extra layer of sensitivity in the estimates. Two conclusions follow: first, it is clear that correction of the level error in the driving variable cannot in itself circumvent the basic problem that there is no apparent cointegration between the actual price level and its constructed long-run equilibrium (i.e., its fundamental value, namely, the mark up over nominal marginal cost or the labour income share). Second, this failing of stationarity essentially points to and highlights the mis-specification of the underlying supply side.

13 In their erratum (GGL, 2003) have corrected the mistaken identity case. However, we are able to reproduce their revised results only with an additional (unreported) constant. Details available. Therefore, the estimation results in GGL (2003) still contain this level error.
3.4 New Keynesian Phillips Curve: Inflation or Price Equation?

NKPC theory is based on price optimisation by firms. This can be seen in that fact that the rhs of the equation contains besides inflation (the change of price level) also the price level itself, i.e. \( \pi_t = \delta E_t(\pi_{t+1}) + \lambda(p^*_t - p_t) \), where \( p^*_t \) is the profit-maximising frictionless price (i.e., the mark-up over nominal marginal costs, \( mc_n: p^*_t = \mu + mc_n \)). This implies that the equation is one for price level not one for inflation. However, standard interpretation seems to be the latter. A necessary requirement for that interpretation is that real marginal costs (\( mc \)) instead of nominal marginal costs (\( mcm \)) is the true fundament. This is possible if real marginal costs can be directly linked to the output gap. This is possible if certain proportionality conditions hold – for instance, that labour markets are frictionless, that there is no capital stock (or it is at least fixed), that consumption and hours are proportional to one another (Sbordone, 2002). 14 Thus, replacing real marginal costs by the output gap yields \( \pi_t = \delta E_t(\pi_{t+1}) + \kappa \tilde{y}_t \), which, being an inflation equation, is sufficient to alone determine the inflation block of the economy (where \( \kappa \) is the multiplication of \( \lambda \) and the weight of the output gap in the measure of real marginal costs). Furthermore, this implies that the stream of discounted present and future output gaps is the fundamental determinant of inflation which, rather counter-factually, suggests that inflation leads output. Moreover, empirical estimation of this form has proved difficult; typically, estimates of \( \kappa \) have been, if positive, insignificant (Esterella and Fuhrer, 1997) or even significantly negative (GGL, 2001). This implies either that these proportionality conditions are not fulfilled and the reduction of real marginal costs to output gap is not legitimate, or, that these conventional measures of output gap are flawed (see Neiss and Nelson, 2002 Gali, 2001 for discussion). Hence, the latter interpretation implies that some multiple of real marginal costs is the true measure of the output gap. Although possible, from the empirical point of view, at least if real marginal costs are expressed in terms of labour income share, and in the context of euro-area data, this interpretation appears controversial as can be inferred from Figures 2 (2 panels) where we present the unemployment gap (based on Fabiani and Mestre, 2002) and the output gap as used by GGL as instrument (i.e., from a quadratic trend) (upper panel) and the unemployment gap and deviation of labour income share from its average (lower panel).

[Figure 2 here]

Visual observation indicate stronger negative correlation of the deviation of output from quadratic trend with the cyclical component of unemployment than that of labour income share. This observation is corroborated by the Table 3.

---

14 Typical auxiliary assumptions also include a closed economy and no government.
As can be seen, the correlation of labour share with unemployment is quite high (and higher than that of the output gap) but this seems mainly due to the fact that correlation of labour share is more strongly associated with structural (i.e., natural rate) than cyclical unemployment. In turn, the output gap is mainly correlated with the cyclical component; correlation with the structural component is relatively weak. Hence, from a monetary policy point of view, the interpretation of labour share as the true output-gap measures is problematic because conventional interpretation is that structural unemployment is outside the scope of monetary policy.

This evidence, coupled with the assumption of fixed capital stock and perfectly flexible labour markets, shows that the correlation between the “true” output gap and Labour Income share (i.e., real marginal costs) cannot be perfect. Consequently, real marginal costs cannot be the driving variable in this equation; instead it is mark-up over nominal marginal costs and, thus, instead of being an inflation equation it is more properly interpreted as a dynamic price or mark-up equation.

In our approach, therefore, the estimation of aggregate supply-side (including time-varying mark-ups) is essential. As already mentioned, this takes the fundament as being determined from a fully optimising, estimated supply-side framework (including explicitly modelling capital). We relax many of the constraints typically found in this literature and by allowing key parameters on both the supply and demand side to differ across sectors and goods, we reformulate the theoretical framework in a potentially more realistic manner to have greater data-consistency and to provide a more encompassing framework in which to analyse price dynamics. Our reformulation implies that, after aggregating across sectors, we derive at the aggregate, both time-varying mark-ups and factor income shares (even with Cobb-Douglas technology).

4 Aggregate prices and factor demand in a frictionless and staggered-price economy.

In this section, we first explain a frictionless model of the economy which is equivalent to a model of long-run supply based on static optimisation but consistent with observed movements in euro-area aggregate mark-ups. This allows us to revise the theoretical framework underlying the NKPC in a more realistic direction. This model defines output price and factor demands as a three-equation system with cross-equation parameter constraints. After having derived our frictionless optimal price determined by the aggregated supply-side system, we then incorporate it into the NKPC framework.
4.1 Supply Side Considerations: Accounting for observed non-stationary mark-ups.

Here, following Willman (2002) and McAdam and Willman (2002), we illustrate the method of estimating a fully consistent system of long run supply. Notably, our long-run system of supply allows price, income elasticities, mark-ups and output shares between sectors to differ. This implies, for instance, secular developments in the aggregate mark up which very much is an observed feature of the euro area aggregated data. Indeed this phenomenon cannot be satisfactorily explained by deviation of substitutions of capital and labour from unity; see the discussion in Blanchard (1997) and Willman (2002).

Further Galí et al. (2001), amongst others, use Cobb Douglas production technology which implies that marginal labour cost is proportional to nominal unit labour costs. Coupled with a constant price elasticity of demand this means that the output price should depend on nominal unit labour cost with a unit elasticity. This, in turn, implies, as a tautology, that real unit labour costs (or labour income share) should be stationary (or at least trendless). Hence, the non-stationarity of the real marginal cost variable (including also the constant mark-up) implied by the humped-shaped pattern of the labour-income share that has been widely observed at the euro-area level (e.g., see the discussions in Blanchard, 1997, Caballero and Hammour, 1998, Willman, 2002) is in contradiction to the standard theoretical framework.

Our approach applies Cobb-Douglas technology but accounts directly for this non-stationarity in aggregate mark-up, which implies co-integration between the actual and optimal price in the frictionless economy.\textsuperscript{15} Note further that in our framework – unlike Galí et al. (2001) and Sbordone (2002) – the optimal capital stock is both included and determined endogenously. This fully (as opposed to partial) optimising framework has important implications for the correct parameterisation of the model and derivation of plausible New Keynesian Phillips Curve estimates.

5 The Model of Long-Run Supply.

5.1 The aggregation of the supply side of the firm

\textsuperscript{15} Although our theoretical framework is written in terms of a general neo-classical framework, in empirical work we apply the Cobb-Douglas production function. We have also estimated using the CES function and found that a unitary elasticity of substitution between capital and labour could not be rejected (see Willman, 2002).
Consider an economy with \( m \) production sectors. Firms in each sector produce differentiated goods, which are close substitutes within each sector, but not for one another across sectors. Except for the differences in the technological level and the growth rate of technological progress in each sector, all firms use the same production technology. As the analysis in such a friction-free economy is static, the time index \( t \) in the context of variables is suppressed for clarity, unless necessary. Hence the production and demand of firm \( i \) in sector \( j \) are determined by the relations:

\[
Y_i^j = A^j e^{\gamma^j} F(K_i^j, N_i^j) = A^j e^{\gamma^j} f(k_i^j)N_i^j
\]  

where \( Y_i^j \) is output, \( N_i^j \) is the output of sector \( j \) and \( D_j \) is the demand function faced by firms in sector \( j \). Parameter \( \gamma^j \) is technological change and \( \epsilon^j \) is the price elasticity of demand in sector \( j \). Since no cross-sector substitutability between goods is assumed, the aggregate demand for goods produced in sector \( j \) is determined by the demand system,

\[
\frac{Y^j}{Y} = s_i^j = s_0^j + \xi^j \log \left( \frac{Y_i^j}{Y_0^j} \right) \text{ with } \sum_{j=1}^m s_i^j = 1, \sum_{j=1}^m \xi^j = 0
\]  

where \( s_i^j \) represents the output share of sector \( j \) in total output, \( \frac{Y^j}{Y} \), and \( 0 \) refers to the starting (or reference) period values of variables. Equation (17) expresses the demand system in per capita terms. Values of parameter \( \xi^j > 0 (\xi^j < 0) \) imply greater (smaller) than unitary income elasticity of demand. The economy-wide aggregates are determined by the identities:

\[
X = \sum_{j=1}^m X_i^j, \quad Y = \sum_{j=1}^m Y_i^j, \quad P = \sum_{j=1}^m P_i^j, \quad Y = \sum_{j=1}^m Y_i^j, \quad N = \sum_{j=1}^m N_i^j
\]

where \( X_i^j \) is output, \( Y_i^j \) is the output of sector \( j \) and \( P_i^j \) is the price of sector \( j \).
We assume that, in each sector, a fixed share of firms are profit maximisers, while the rest minimise their costs. As factor markets are assumed competitive, each firm \( i \) in each sector \( j \) faces the same nominal wage rate \( w \) and nominal user cost of capital \( q \). Hence, the optimisation problems of the profit maximising, on one hand, and the cost minimising firm, on the other hand, can be defined as: \(^{16}\)

\[
\begin{aligned}
\max_{\prod_i} & \quad P_i^j \cdot Y_i^j - W \cdot N_i^j - q \cdot K_i^j \\
\text{s.t. equations (15) and (16)}
\end{aligned}
\] (24a)

\[
\begin{aligned}
\min_{\prod_i} & \quad C_i^j = W \cdot N_i^j + q \cdot K_i^j \\
\text{s.t. equation (15)}
\end{aligned}
\] (24b)

The FOC of profit maximisation imply the following 3-equation system, which determines the price of output and the demand for capital and labour conditional on the demand-determined output.

\[
P_i^j = (1 + \mu^j) \left[ \frac{w}{A^j e^{\gamma_j} \left( f(k_i^j) - k_i^j f'(k_i^j) \right)} \right] ; \quad 1 + \mu^j = \frac{\varepsilon^j}{\varepsilon^j + 1} \geq 1
\] (25)

\[
\frac{\partial Y_i^j}{\partial N_i^j} = \frac{f(k_i^j) - k_i^j f'(k_i^j)}{f'(k_i^j)} = \frac{w}{q}
\] (26)

\[
\frac{Y_i^j}{N_i^j} = A^j e^{\gamma_j} f(k_i^j)
\] (27)

The first order conditions of cost minimisation in turn, implies the 2-equation system of (22)-(23). As the relative factor price \( w/q \) in the rhs of equation (22) is the same for all firms that implies that also the capital-labour ratio is also the same across firms equalling the aggregate

\(^{16}\) This partition of decision-makers into profit maximisers and cost minimisers reflects our earlier division of price setters; since a share of agents use rule-of-thumb (i.e., backward-looking) pricing rules they cannot be profit maximisers. We therefore make the alternative operational assumption that they are costs minimisers.
capital-labour rate: \( k' = k, \forall i, \forall j \). Hence aggregated production (or labour demand) and the profit maximising price in sector \( j \) are determined by:

\[
\frac{Y^j}{N^j} = f(k)A^j e^{\gamma^j}
\]  

(28)

\[
P^j = (1 + \mu^j) \left[ \frac{w}{A^j e^{\gamma^j}} \left( f(k) - kf'(k) \right) \right]
\]  

(29)

Aggregation across sectors, as defined by identity (18), implies that the aggregate level supply-system, corresponding to the firm level supply system (21)-(23), can be written as,

\[
\frac{Y}{N} = f(k) \left[ \sum_{j=1}^{m} s^j A^j e^{-\gamma^j} \right]^{-1}
\]  

(30)

\[
g\left[ f(k) - kf'(k) \right] = \frac{1}{w f''(k)}
\]  

(31)

\[
P = \sum_{j=1}^{m} s^j P^j = \frac{w}{f(k) - kf'(k)} \sum_{j=1}^{m} s^j \left( 1 + \mu^j \right) A^{j-1} e^{-\gamma^j}
\]  

(32)

**Proofs of (26) and (28), Appendix One.**

Equations (26) and (27) become more transparent after transforming them into logarithmic form and then linearising the logarithms of the summation terms around the values \( s^j = s_0^j \) and \( t=0 \):

\[
\log \frac{Y}{N} = \log f(k) + \log A + \gamma_A \cdot t - \sum_{j=1}^{m} AA^{j-1} \left( s^j - s_0^j \right)
\]  

(33)

\[
\log P = \log w - \left\{ \log \left[ f(k) - k \cdot f'(k) \right] + \log A + \gamma_A \cdot t - \sum_{j=0}^{m} AA^{j-1} \left( s^j - s_0^j \right) \right\}
\]  

Log of the marginal product of labour

20
\[
+ \log(1 + \mu_A) + \frac{\sum_{j=1}^{m} AA^{-1}(\mu_A - \mu_j)(s_j - s_0)}{1 + \mu_A} - \sum_{j=1}^{m} b^j(\gamma_j - \gamma_A)\cdot t
\] (34)

where \( A = \left( \sum_{j=0}^{m} s_0 A^{-1} \right)^{-1} \), \( \mu_A = \sum_{j=1}^{m} AA^{-1}s_0^j\mu_j \), \( b^j = \frac{AA^{-1}s_0^j(1 + \mu_j)}{\sum_{j=1}^{m} AA^{-1}s_0^j(1 + \mu_j)} \), \( \sum b^j = 1 \), and

\[ \gamma_A = \sum_{j=1}^{m} AA^{-1}s_0^j\gamma_j. \]

**Proofs of (29) and (30), Appendix Two.**

### 5.2 The determination of sectoral output shares

We show in the next step that in a growing economy, with help of the demand system (17), the long-run development of sectoral production shares are reduced to trend. Assume that, in the equilibrium growth path, output and capital grow at a constant rate \( g \). Equation (17) implies that, in this path, the production shares \( \bar{\xi}_j \) are determined as,

\[
\bar{\xi}_j = \tilde{\xi}_0 + \xi_j g \cdot t
\] (35)

Using (35), equation (17) can be written as

\[
s_j - s_0 = \xi_j g \cdot t + \tilde{\xi}_j \left[ \log \left( \frac{Y/Y_0}{N/N_0} \right) - g \cdot t \right] + \xi_j u_t
\] (36)

where \( u_t \) can be assumed to be stationary around zero. After substituting (36) into (29) and (30), we end up with,

\[
\log \frac{Y}{N} = \log f(k) + \log A + \left[ \gamma_A - g \sum_{j=1}^{m} AA^{-1} \xi_j \right] \cdot t + v_{lj}
\] (37)
\[
\log P = \log W - \log \left[ f(k) - k \cdot f'(k) \right] + \log A + \Gamma \cdot t
\]

Log of the marginal product of labour

\[
+ \log(1 + \mu_A) + \left[ g \sum_{j=1}^{m} A A^{j-1} \left( \mu^j - \mu_A \right) \xi^j - \sum_{j=1}^{m} b_j (y^j - \gamma_A) \right] \cdot t + v_{2,t}
\]

Log of the mark-up

where \( v_{1,t} = \left( \sum AA^{-1} \xi^j \right) u_t \) and \( v_{2,t} = \left[ \sum AA^{j-1} \left( \mu^j - \mu_A \right) \xi^j - \sum AA^{-1} \xi^j \right] u_t \).

Since \( v_{1,t} \) and \( v_{2,t} \) are stationary, they are absorbed by the residuals of the estimated long-run aggregated supply system.

5.3. The AIDS demand function, foreign competition and the mark-up

Although the demand function (16) faced by firms is written in general form, the implicit assumption has been that the price elasticities \( \varepsilon^j \ \forall j \) are constant. Let us now relax that and, instead, assume the AIDS demand function. We show that, in the open sectors of the economy, this assumption implies that the mark-up also depends on foreign competitiveness, i.e., the ratio of competing foreign to open sector prices. The following analysis is presented in terms of the export sector, which, for simplicity, is treated as a single aggregate, although similar argument applies to import competing sectors.

Let us approximate the AIDS export demand function as:\(^{17}\)

\[
v = \frac{P^x \cdot X}{P_f \cdot D_f} = a + \theta \cdot \log \left( \frac{P_f}{P^x} \right) \quad ; \theta > 0
\]

---

\(^{17}\) In terms of the AIDS expenditure system, the share of country \( i \) exports in world imports (at current prices) is:

\[
v_i = \frac{P^x_i \cdot X_i}{P_f \cdot D_f} = a_i - \theta_i \cdot \log P^x_i + \sum j (\theta_{ij} \log P^x_j) \quad \text{where} \quad \sum i a_i = 1, \quad \theta_i = \sum j \theta_{ij} \text{ and } P_f \text{ is a weighted index of export prices}
\]

\( P^x_i \) (see Deaton and Muellbauer, 1980).
where \( v \) is the market share of export value, \( X \) is export volume, \( P^* \) is export price, \( P_f \) is the deflator of world exports and \( D_f \) is the volume of world exports.

Equation (39) implies the following price elasticity and the mark-up in the export sector:

\[
\epsilon^* = -1 - \frac{\theta}{v} = -1 - \frac{\theta}{a + \theta \log\left(P_f/P^*ight)} \tag{40}
\]

\[
1 + \mu^* = \frac{\epsilon^*}{\epsilon^* + 1} = 1 + \frac{a}{\theta} + \log\left(P_f/P^*ight) \tag{41}
\]

Equation (40) states that, with \( \theta > 0 \), the price elasticity of exports is \( \epsilon^* < -1 \). We also see that the price elasticity is not constant, but depends on the relative price \( P_f/P^* \). This implies, as shown by (41), that the mark-up of the export sector depends positively on the competing world market prices. For estimation purposes, it is useful to log-linearise (41). Linearising around the point, \( \log\left(P^*/P_f\right) = 0 \),

\[
\log\left(1 + \mu^*\right) \approx \log\left(1 + \bar{\mu}^*\right) + \frac{1}{1 + \bar{\mu}^*} \log\left(P_f/P^*\right); \quad \bar{\mu}^* = \frac{a}{\theta} \tag{42}
\]

The fact that the mark-up of the export sector depends on the competitive pressure of foreign prices allows us to write the economy mark-up as:

\[
\log(1 + \mu_A) = \log(1 + \bar{\mu}_A) + \frac{s^0_i}{1 + \bar{\mu}^*} \log\left(P_f/P^*\right) \tag{43}
\]

where \( \bar{\mu}_A \) is the aggregate mark-up, \( \bar{\mu}^* \) is the export sector (or more generally the open sector) mark-up and \( s^0_i \) is production share of the export (or open) sector in the base (reference) period.

5.4 The specification of the aggregated supply side system with the Cobb-Douglas technology
Before being able to estimate the aggregated supply side system, the functional form of the underlying technology must be specified. From the point of view of estimation the Cobb-Douglas and the CES production function would be natural candidates. However, we constrain our analysis only to the Cobb-Douglas case since, using euro area data, the estimated CES function effectively reduces to the Cobb-Douglas. 18

In the Cobb-Douglas case, we obtain, \( f(k) = \left( \frac{K}{N} \right)^{\beta} \) and \( f(k) - k \cdot f'(k) = (1 - \beta) \left( \frac{K}{N} \right)^{\beta} \). Thus, the aggregated supply system can be written as:

\[
\log \frac{Y}{N} = \beta \log \left( \frac{K}{N} \right) + \log A + \Gamma \cdot t \tag{44}
\]

\[
\frac{qK}{wN} = \left( \frac{\beta}{1 - \beta} \right) \tag{45}
\]

\[
\log P = \log w - \left\{ \log(1 - \beta) + \beta \log \left( \frac{K}{N} \right) + \log A + \Gamma \cdot t \right\} + \log(1 + \bar{\mu}_A) + \chi \log \left( \frac{P}{P^s} \right) + \eta \cdot t \tag{46}
\]

where \( \chi = \frac{\dot{\xi}}{1 + \bar{\mu}^e} \) and \( \eta = \left[ \sum_{j=1}^{m} A A_j^{-1} \left( \mu^i - \mu_A^i \right) \xi_j - \sum_{j=1}^{m} b_j (y^j - y_A^j) \right] \).

It should be noted that relation (43) is also utilised in defining the mark-up behaviour and for notational simplicity stationary terms \( v_{1,t} \) and \( v_{2,t} \), are abstracted from equations (44) and (46).

Aggregate Cobb-Douglas technology (44), implies that the marginal product of labour in equation (46) can be expressed alternatively in the form \( MPL = (1 - \beta) \cdot (Y/N) \). That implies that equation (46) can also be written in that form, where the (inverted) income share is the left-hand variable:

18 See Willman (2002).
\[
\log \left( \frac{pY}{wN} \right) = -\log(1 - \beta) + \log(\text{mark-up})
\]  

Equation (47) shows that the aggregated labour income share \( \frac{wN}{pY} \) is inversely related to the changes in the mark-up and, hence, a non-stationarity of the total economy mark-up also implies the non-stationarity of the aggregated labour income share – even on the assumption of Cobb-Douglas technology. However, simultaneously, equation (45) states that the relative factor income, \( \frac{qK}{wN} \), has to be constant (stationary) for the assumption of the aggregate Cobb-Douglas production function to be true. That implies that the GDP share of capital income, excluding profits, is also inversely related to the aggregate mark up, which may be time-variant, although sectoral mark-ups would be constant. Note that in our estimations, we prefer specification (46) to (47) since the latter essentially assumes that the marginal product of labour is proportional to its average product (i.e., \( \frac{Y}{N} \)). Typically, the average product of labour is found to be pro- instead of counter-cyclical which many commentators (see Roberts, 2001) interpret as meaning that average labour productivity deviates from marginal labour productivity because of, for instance, labour hoarding. Accordingly, we are not open to this well-known critique since our definition of the marginal product of labour – in equation (46) – implies counter-cyclical behaviour if the adjustment of capital to equilibrium is slower than that of labour (commonly considered a stylised fact).

6. Structural estimates

6.1 Data Source

We use data from 1970q1 to 1997q4 taken from the European Central Bank’s Area Wide Model database, Fagan et al. (2001). This data set has been widely used in, inter alia, Fagan and Henry (1998), Stock et al. (2000), Coenen and Wieland (2000), Coenen and Vega (2002), Detken et al. (2002), Vega and Trecroci (2002), Smets and Wouters (2003). Appendix 3 explains important additions to and transformation of the euro-area database that were necessary to estimate the supply side: a correction to include self-employed in labour income, consistent treatment of capital income, a correction to real euro-area interest rates (controlling for the

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19 Our data set used can be downloaded from the ECB’s website, (http://www.ecb.int) and is associated to WP No. 42. Longer data are now available but, for comparability purposes, we retain the dataset used in both Amato and Gerlach (2000) and Galí et al. (2001).
period of financial repression in the 1970s) and, finally, the derivation of price competitive index.

6.2 Estimates of the Supply Side

Estimates of our 3-equation supply-side (44-46) are given in Table 4. The results appear reasonable, significant at the 1% level and in line with our priors. Specifically, we see that the elasticity of output with respect to Capital ($\beta$) is around 0.3, the estimate of technical progress ($\gamma$) implies an annual growth of technical factor progress of 2% and $\eta$ (the parameter capturing the trend in the mark up) deviates significantly from zero. The foreign competing price to export price ($\chi$) also plays a significant role especially in the behaviour of the mark up in the 1970s (e.g., the two oil shocks). Note also the significance of the interest rate dummy capturing real financing costs.

Furthermore, Figure 3 shows our supply-side system estimate of the mark-up over real marginal cost and that from that implied by the Galí et al. (2001) calibration. Notably, the mark-up calculated by our system estimation are stationary around zero (ADF t-test = -3.8) whilst the calibrated variant is non-stationary (ADF t-test = -0.42) and of a relatively higher standard error (0.4 versus 0.2).

20 In our GMM estimations, we use as instruments two and four period lags of inflation, two, four and five periods lags of the mark-up over real marginal costs and the fit of the one period lead of inflation and the mark-up over real marginal costs on the lags of: capacity utilisation rate (as defined by our supply side), unemployment rate, nominal short term interest rate, terms of trade, change of terms of trade, real marginal cost, residual of price equation excluding terms of trade component. The rational behind the formulation of the composite instruments was an attempt in a parsimonious way to maximise the information contained by history. The dataset and (Rats) programmes used to derive our estimates are available.

21 Our interpretation of this parameter is that it reflects the rise of the output share of the (essentially less competitive) services sector in the euro area and its constituent members. See more closely the discussion in McAdam and Willman (2002).

22 This term plays an essential role in our estimation since otherwise the production function parameters (whether Cobb-Douglas or CES) would not capture the correct relative factor income ratio. This dummy thus provides a link from the period of financial regulation of the 1970s, when persistently negative ex-post real interest rates probably did not properly measure true financing costs, and post 1980s more liberalised period associated with positive real interest rates.

23 Phillips-Perron tests show the same and are available.
6.3. Estimation of inflation equations

We consider two cases: Information Lag (7) and No-Information Lag (8) across five orthogonality cases. Furthermore, the equations are estimated over different samples. As is well known, non-linear estimation using GMM can be sensitive to the way orthogonality conditions are imposed and the choice of instruments. (e.g. Fuhrer et al., 1995). In line with this, we consider five alternative specifications of orthogonality conditions for both equations – un-normalised (A), normalised with respect to inflation (B), one-period ahead inflation (C), the fundament (D), and, finally, with respect to a weighted average of the fundament and inflation (E):

Information Lag (equation 7):

(A) \( E_i \left\{ \left[ \phi \pi_t - \theta \delta \pi_{t+1} - \omega \pi_{t-1} - (1-\omega)(1-\theta)(1-\delta\theta)(mcn_t + \mu_t - p_t) \right] z_i \right\} = 0 \)

(B) \( E_i \left\{ \left[ \frac{\delta \theta}{\phi} \pi_t - \frac{\omega}{\phi} \pi_{t+1} - \frac{(1-\omega)(1-\theta)(1-\delta\theta)}{\phi} (mcn_t + \mu_t - p_t) \right] z_i \right\} = 0 \)

(C) \( E_i \left\{ \left[ \frac{\phi}{\delta \theta} \pi_t - \pi_{t+1} - \frac{\omega}{\delta \theta} \pi_{t-1} - \frac{(1-\omega)(1-\theta)(1-\delta\theta)}{\delta \theta} (mcn_t + \mu_t - p_t) \right] z_i \right\} = 0 \)

(D) \( E_i \left\{ \left[ (\phi \pi_t - \delta \theta \pi_{t+1} - \omega \pi_{t-1}) / (1-\omega)(1-\theta)(1-\delta\theta) - (mcn_t + \mu_t - p_t) \right] z_i \right\} = 0 \)

24 For estimation, two composite instruments were constructed, namely the best fits of inflation and the mark-up over real marginal costs on the following variables (lagged at least by two periods): the residual of long-run system of estimated potential output, the unemployment rate, short-term nominal interest rates, relative foreign-to-competing export price, terms-of-trade, mark-up including residual component implied by the supply-side system and real marginal costs. In addition to these composite instruments, in estimating NKPC equations, two and four period lags of inflation and two, four and five period lags of the mark-up over real marginal costs were used.

25 For robustness, within each case we also considered the auxiliary cases of imposing \( \omega = \gamma_b = 0 \) and \( \delta = 0.9925 \) (i.e., 3% annual discount factor). Details available.

26 Thus, as would be clear from earlier sections, our framework embodies a time varying mark-up, \( \mu_t \).
Where parameters are defined as before, where $Z_t$ is a vector of instruments and 

$$
\sigma_{\pi_{mcn+\mu-p_t}} = \frac{\sigma_{\pi}}{\sigma_{mcn+\mu-p_t}} = 0.0085 \frac{0.0250}{0.34}.
$$

And, equivalently, for No-Information Lag (equation (8)):

(A) $E_t \left[ \left( \pi_t - \theta \delta (1 - \omega (1 - \theta)) \right) \pi_{t+1} - \theta \omega \pi_{t-1} \right] = 0$

(B) $E_t \left[ \left( \frac{\pi_t - \theta \delta (1 - \omega (1 - \theta))}{\zeta} \right) \pi_{t+1} - \frac{\theta \omega \pi_{t-1}}{\zeta} \right] = 0$

(C) $E_t \left[ \left( \frac{\pi_t - \pi_{t+1}}{\delta(1 - \omega(1 - \theta))} \pi_{t+1} - \frac{\omega (1 - \omega (1 - \theta))}{\delta(1 - \omega(1 - \theta))} \right) \pi_{t-1} = 0 

(D) $E_t \left[ \left( \pi_t - \theta \delta (1 - \omega (1 - \theta)) \right) \pi_{t+1} - \theta \omega \pi_{t-1} \right] = 0$

(E) $E_t \left[ \left( \pi_t - \frac{\theta}{1 + \delta \theta \omega} \omega \pi_{t-1} - \delta (1 - \omega (1 - \theta)) \pi_{t+1} - (1 - \omega) \left( 1 - \delta \theta \right) (mcn_t + \mu_t - p_t \right) \right] = 0$

6.4 Results

Case (E) — the weighted case — is motivated by some previous discussion of GMM estimators (e.g., Fuhrer, 1997). Our own justification for this weighted experiment is that in minimising the objective function GMM tends to choose parameter estimates so that weight of the variable which has greatest variance and smallest covariance with other variables tends towards zero. Potentially this is problem if no objective criterion exist about how estimated equation should be normalised. That is why, in models containing leads and lags of endogenous variables (and which is auto-correlated), we may bias the coefficient of the error correction variable towards zero if parameter constraints of the estimated equation allow that. However, if the equation is normalised with respect to the error-correction term that is not possible. Hence, there are two natural ways to normalise the estimated equation, i.e. with respect to the current-period dependent variable or in terms of error correction term. The essential difference between these two is that, in the first case, the objective function is minimised conditional on the variance of inflation and, in the latter, conditional on the variance of error correction term (which is also an endogenous variable). A third possibility is to leave the equation un-normalised. However, in that case there is perhaps a risk that parameters become estimated such that the variable with smallest (largest) variance gets the highest (smallest) weight within the range that parameter constraints allow.

The No-Information Lag case does not (at least in this application) depart from the standard case and accordingly we relegate its full-sample results to Appendix 4. Estimates as before based on sub-samples are available.
In Tables 5 to 6, we document our alternative estimates for the euro-area NKPC. For robustness, we present our five different normalisations across the conventional information lag case and our alternative of No-Information Lag (Appendix 4). We estimate in terms of the price level equation (rather than the inflation equation and derive recursively the composite parameters (e.g., $\gamma_0$). We also solve for the roots of the characteristic equation of each Phillips curve – this gives us information on the dynamic properties of each equation and thus provides a further check on its plausibility. We estimate the hybrid ($\omega \neq 0$) and purely forward looking cases ($\omega = 0$).

[Tables 5 and 6 here]

Our conclusions are:

The discount factor ($\delta$) does not deviate significantly from unity or from 0.99 implying an annual discount rate of 4%. This is a robust result and certainly in line with theory. The parameter of price adjustment, $\theta$, (excluding the fundament and weighted cases) is to be found in a narrow range of 0.80-0.85 and significantly different from zero and unity. This parameter of course determines the duration of price fixing. Over the full sample, using the Hybrid model we find average fixed-price duration of 4.7 quarters and a slightly higher average for the purely forward-looking case of 5.4. However, the deviation from the fundament and weighted normalisations tend to give relatively smaller durations. Excluding these, we find a slightly higher average duration at 5.3 and 6.4 periods respectively. In the light of standard errors, the data supports the Hybrid specification only in the case, when estimated equation is normalised with respect to current inflation. Under other normalisations we find that $\omega$ (though positive and of plausible values) is insignificantly different from zero.

The tables also show the characteristic roots in each specified case – these give us information about stability and dynamic adjustment. For instance, in the case of the hybrid price equation, (7) solved in terms of the deep parameters, we have, the characteristic equation

\[ p_{t+1} = \left[ \frac{1}{\delta \theta} + 2\omega + (1-\omega)\theta \right] p_t + \left[ \frac{1}{\delta} + \frac{2\omega}{\delta \theta} - \frac{\omega}{\delta} + \omega \right] p_{t-1} - \frac{\omega}{\delta \theta} p_{t-2} = 0 \]

with roots

\[ \frac{1}{2} \left( \theta (1-\omega) + 2\omega \pm \sqrt{(\theta (1-\omega) + 2\omega)^2 - 4\omega} \right), \quad \frac{1}{\delta \theta} \]. Saddle-path stability requires one un-stable root.

\[29\text{ By contrast, the Gal\'\c{s} et al. (2001) estimates of the (euro area) discount rate (\(\delta\)) are rather high – in the range of 7.5%-19.6% implying an annual range of 33.5%-105%. Overall, their average figure for \(\delta\) is around 0.88 for both specifications (implying a 57% annualised discount factor).}\]

\[30\text{ This is to be compared to duration estimates from Gal\'\c{s} et al. (2001) of 7.4-8.9 (Hybrid case) and 8.2-9.0 (Purely forward-looking case).}\]
and two stable roots. The third root is clearly unstable for $\delta \cdot \theta < 1$, and it can be shown that the first two roots fulfil the stability condition if inside the open interval, $\lambda_i \in (\omega, 1)$, $i = 1, 2$. In addition, the discount factor ($\delta$) affects only the forward (unstable) root and the backward-looking parameter ($\omega$) only affects the (backward looking) stable root. Note there are two separate backward-looking roots when $\theta(\omega)$ is sufficiently high (low). In the opposite case, the root tends to become complex. This pattern can be seen in our estimates, where sufficiently high $\omega$ produces cyclical adjustment. In cases A, B and C, results suggest a range for the unstable roots between 1.18 to 1.26. (Normalisations D and E, however, give somewhat higher roots, i.e. between 1.32 and 1.43).

Finally, results across the Information Lag and No-Information Lag cases, except estimates of lagged inflation parameter $\omega$, are very similar (in many cases identical as anticipated in section 2). Hence the estimated price dynamics are not sensitive with respect to these specifications alternatives. To conclude, our results appear reasonable and in line with our priors. All specifications pass the orthogonality test (Hansen’s $J$) and the regressions display durations of price stickiness roughly in the region of 5 quarters.

31 For the first two roots, we have the product $\lambda_1 \cdot \lambda_2 = \omega$ and the sum $\lambda_1 + \lambda_2 = \theta(1 - \omega) + 2\omega < 1 + \omega$. Since both roots must be positive, $\lambda - (1 + \omega)\lambda + \omega < 0$ this implies that $\omega < \lambda < 1$, where $\lambda$ denotes both $\lambda_1$ and $\lambda_2$. 
6.5 The Present Value Approach

Recently Rudd and Whelan (2001) criticised the estimation approach typically taken in the literature. They claim that instrumental variable estimates (like GMM) tend to be strongly biased towards forward-looking inflation formation even if the true model contains no such behaviour. This situation occurs when the set of instruments contains variables that belong in the true model for inflation and are erroneously omitted from the estimated specification. In addition, in the case where the lagged value of the dependent variable is (correctly) included in the specification, the inclusion of the lagged inflation also in the set of instruments causes similar bias. Our estimations presented in the previous section are not, however, open to this criticism since all the instruments we used were lagged at least by two periods. Accordingly, although in general our estimates for the lagged inflation were not always very precise, in absolute terms, the point estimates for the lagged inflation parameter were quite high.

As a remedy to discriminate between the competing hypothesis of backward-looking and forward-looking behaviour in expectation formation, Rudd and Whelan (2001) recommend tests based on correlations summarised by the reduced-form Phillips curve, where the lead of inflation is reduced to the sum of the driving variable. In general form,

$$\pi_t = \sum_{j=1}^{J} \omega_j \pi_{t-j} + \left(1 - \sum_{j=1}^{J} \omega_j \right) \sum_{i=0}^{\infty} \beta_i E_t \pi_{t+i} \approx \sum_{j=1}^{J} \omega_j \pi_{t-j} + \left(1 - \sum_{j=1}^{J} \omega_j \right) \sum_{i=0}^{n} \beta_i E_t x_{t+i} \quad (48)$$

where $x$ refers to the fundament (e.g., either output gap or real marginal costs). In their empirical application – as in ours – the infinite sum was approximated by a finite sum of 12 periods. As in case of structural forms, also the reduced form (48) can estimated by instrumental variables. The difference between (48) and the original form is that instead of the lead of the dependent variable (inflation), the instrumented variable is the weighted sum of the leads of the fundament.

---

32 In our “Present Value” estimates, we used as instruments, the fit and its one period lag of the difference of the weighted present value of the fundament (the mark-up over real marginal costs) and the lagged price level on lagged values of the fundament and lagged price level, on, two, five and seven periods lags of the short term interest rate, one period lag of the long-term interest rate, one and three periods lags of relative foreign-to-competing export price, and one and two periods lags of the residual of long-run system of estimated potential output.

33 In this respect, we deviate from Galí and Gertler (1999) and Galí et al. (2002), who included the lagged inflation into the set of instruments.

34 Also in our estimations (not reported here but available), when the lagged inflation was included by the set of instruments the sizes of the point estimates for the lagged inflation dropped dramatically. Also Roberts (2001) in US data, found that estimates of NKPC are sensitive to the inclusion of lagged inflation in the instrument set.
If the NKPC is correctly specified, the present value estimates (48) should give similar results as those based on the structural specification. However, Rudd and Whelan (2001) claim that (48) is less likely to spuriously indicate the presence of forward-looking behaviour. This is because in this case, the term being instrumented for will not have high correlation with variables that have been omitted from the inflation equation. Hence, if the NKPC or its hybrid form were true, then in (48) at most one lag of inflation would be relevant and the parameter estimate the lagged inflation should equal to the estimate of the structural NKPC.

From our earlier discussion two interpretations of the fundament $x$ are possible— as the mark-up over real or nominal marginal costs. The interpretation that real marginal cost is the correct measure of the output gap requires strong proportionality conditions and the non-existence of the capital stock to be true. If these conditions are not fulfilled, then the true driving variable is mark-up over nominal marginal cost and then this inflation equation turns into one for the price level. However, we will show that also under this latter interpretation, the present value form (48) can be presented with $x_{t+1} = (p_{t+1} - p_{t-1})$. Further, under this interpretation also the structural parameters can be identified in the context of present value estimations. Consequently, in this section, we estimate applying both interpretations. This testing procedure helps identify which interpretation of the driving variable is the most appropriate. In contrast to earlier results, using the Present Value method, we find backward-looking component strongly dominating – with the forward-looking component of only marginal importance. When the present-value is expressed in terms of real marginal costs, we get a much more balanced view on the relative importance of backward- and forward-looking components. All in all, these results are well in line with our earlier results, in the context of estimating structural-form which, we think can be interpreted as indirect evidence in favour of the mark-up over nominal marginal costs as the correct fundament.

6.5.1 Real Marginal Cost Interpretation

First, we considered the standard real marginal cost interpretation. Results based on this interpretation are presented in Table 7.

[Table 7 here]

Table (8) summarizes the results obtained from fitting equation (48) conditional on two alternative values of $\beta$ (which equals to the discount factor in the basic (non-hybrid) specification). If the New-Keynesian interpretation of the reduced-form, Phillips curve is correct,
then the inclusion of the present value should result in a substantial reduction in the coefficients on lagged inflation relative to those obtained from purely backward-looking specifications (e.g. the simple regression of inflation on its own lags only as in the bottom row). The results are quite similar as with those of Rudd and Whelan (2001) using US data. The introduction of the present-value term into regression, although statistically significant, lowers only marginally the importance of the lagged inflation, (close to 0.9), suggesting a very limited role for expected future values of the real marginal cost variable. In addition, these results are not compatible with our earlier results based on estimation the structural specification – where the weight on lagged inflation was at most only slightly above 0.5. However, as these results are conditional on the assumption that real marginal costs are the true driving variable, the conclusion that the forward-looking behaviour plays only marginal role price setting is not necessarily correct.

6.5.2 Nominal Marginal Cost Interpretation

In their application to US data, Rudd and Whelan (2001), as an alternative to the output gap, used the labour's share of income (real marginal costs) as a fundament driving variable. This is not, however, correct if a unitary correlation between real marginal costs and the “true” output gap does not hold. If so, the NKPC, when presented in terms of the mark-up over nominal marginal costs, is, as earlier suggested, not an inflation equation but dynamic price one. The correct driving variable, independent from the price setting behaviour of firms, is therefore the mark-up over nominal marginal costs. By denoting \( p_t^* = mcn_i + \mu_i \) and expressing the parameters of our structural NKPCs in terms of their roots, the present value form of our NKPCs can be written as,

\[
1 - (\lambda_1 + \lambda_2) L + \lambda_1 \lambda_2 \sum_{i=0}^{\infty} \left( \frac{1}{\lambda_1} \right)^i (1-\omega)(1-\theta)(1-\delta \theta) E_i p_{t+i}^* \tag{49}
\]

As shown in section 6.4, where the backward looking price setting rule contains information lag, we derive the roots as \( \lambda_1 = 1/\delta \theta, \lambda_1 + \lambda_2 = \theta(1-\omega) + 2\omega \) and \( \lambda_1 \lambda_2 = \omega \). Equation (49) can therefore be transformed into,

\[
\pi_t = \omega \cdot \pi_{t-1} + (1-\omega)(1-\theta)(1-\delta \theta) \sum_{i=0}^{\infty} (\delta \theta)^i E_i \left( p_{t+i}^* - p_{t+i-1} \right) \tag{50}
\]
Let us approximate the infinite sum by the finite sum,
\[ \sum_{i=0}^{\infty} (\delta \theta)^i E_i (p_{t+i}^* - p_{t-1}) = \frac{1}{1-(\delta \theta)^{n+1}} \sum_{i=0}^{n} (\delta \theta)^i E_i (p_{t+i}^* - p_{t-1}). \] Now the estimated form corresponding to (50) can be written as,

\[ \pi_t = \omega \cdot \pi_{t-1} + (1 - \omega) \left\{ \frac{(1-\theta)(1-\delta \theta)}{1-(\delta \theta)^{n+1}} \sum_{i=0}^{n} (\delta \theta)^i E_i (p_{t+i}^* - p_{t-1}) \right\} + \text{cons} \] (51)

Consistently with our earlier results, setting \( \delta = 0.99 \) equation (51) contains two estimated structural parameters \( \theta \) and \( \omega \). We have allowed the specification to include also an additional constant (cons) to take into account the fact that the weighted finite sum of the fundament variable is only an approximation of the infinite sum. We think that the inclusion of freely estimated constant into specification allows less biased estimate for the relative weight of the lagged inflation and the expected present value of the driving fundament variable in the determination of the current inflation rate.

As can be seen (Table 8) when estimating (51) without a constant, \( \omega \) is somewhat higher (0.63) than in the corresponding structural NKPC equation (when normalised on current inflation) (0.53) and is highly significant.

[Table 8 here]

The estimate for \( \theta \), in turn is slightly smaller (0.77 as opposed to 0.80), implying that average fixed-price duration is 4.3 quarters. Moreover, when a constant is included, \( \omega \) and \( \theta \) estimates are very close to the earlier corresponding structural NKPC equation. (0.48 and 0.83 respectively). Again both parameter estimates were highly significant. Overall, these reduced form estimation results are very supportive of our structural form estimates.

The above estimations do not, however, account for the possibility that more than one lag of inflation plays an important role in explaining current inflation. Therefore, for further testing purposes, corresponding to the generalised specification (48), we also estimate the following specification conditional on pre-set values of parameters \( \delta \) and \( \theta \):
Specification (47) can be used for testing whether, in addition to the expected present value of the driving fundamental variable, more than one lag of inflation is needed to explain current inflation. As with Rudd and Whelan (2001), equation (47) is specified conditional on predetermined values of $\hat{\theta}$, $\hat{\delta} \in [0.75,0.9]$, and $\hat{\delta} = 0.99$. Parameters $\omega_j$ and $e$ are freely estimated. Hence, for the NKPC to be true, it is required that parameter estimates for higher than one period lag should not be significant, the sum $\sum_{j=1}^{m} \omega_j$ should be significantly below unity and the parameter estimate of $e$ should be close to unity.

Results (Table 9) suggests that, in addition to significant one-period lag of inflation, two-and three-periods lag of inflation affect current inflation.

The inclusion of three periods lag turned the two periods lagged inflation significant but the three periods lag (negative sign) was borderlinesignificant, i.e. when coupled with high values of $\hat{\theta}$. However, the sum of lagged coefficients were largely the same as in the one-period lag case, i.e. $\sum \omega$ in the range of 0.45-0.54. Further, estimates were partially sensitive to alternative predetermined values of $\hat{\theta}$; higher values of $\hat{\theta}$ were associated with smaller values of $\sum \omega$. Similarly, estimates of $e$ varied around unity and when associated with high (low) predetermined values of $\hat{\theta}$ were above (below). These estimates were not sensitive with respect to the number of lagged inflation terms included. Overall, these results are well in line with our structural NKPC estimates. (With the value of unit estimate for $e$ most strongly associated with our earlier point estimate of $\theta = 0.83$). Overall, two conclusions emerge: first, it would thus appear that the point estimate of $\gamma_j$ is numerically insensitive to the lag specification because introducing more lags largely speaking does not affect the sum of point estimates, $\sum \omega$, second, the present-value approach gives the same point estimates of $\omega$ (in the neighbourhood of 0.5).
but, by diminishing their standard errors, provides statistical support in favour of the Hybrid model.

7. Conclusions

In this paper, we sought to take a holistic approach to evaluating and estimating New Keynesian Phillips curves. Thus, our intention was to take the literature seriously but with three provisos: (1) to estimate a fully-fledged supply side to provide estimates (rather than calibrates) of key supply parameters and real marginal cost indicators and judge their importance, (2) to estimate using different normalisations, sub-samples and present-value estimates to check robustness and (3) with a view to assessing its potential system attributes, examined the adjustment paths of the resulting estimates.

The inclusion of a realistic, data-consistent supply-side to determine technology parameters and real marginal cost proved essential. Indeed, estimation on euro-area data – as well as providing a link to the important contribution of Gali et al. (2001) – emphasises many key but latent issues such as the stability of factor shares, behaviour and correct identification of supply parameters, mark ups and marginal costs. We showed that a standard supply-side approach could not adequately capture the trends and characteristics of the euro-area data.

Furthermore, our different normalisations allowed us to build up robustness. Estimates were neither especially sensitive to normalisation or pricing-rule assumptions (e.g., Information Lag and No-Information Lag). The only outlier in that respect was the normalisation with respect to the fundamental (and to a far lesser extent the weighted normalisation): this produced relatively low estimates of \( \theta \) (and thus low durations), relatively low discount factors (but still insignificantly different from unity), relatively high error correction parameters and (as judged by the highest value stable root) rapid adjustment to equilibrium. Overall, our estimates were reasonable, in line with our priors, and compare favourably with others found in the literature: discount factors insignificantly different from 0.99 (with annualised discount rates of around 4%), reasonable periods of price fixedness, plausible dynamics and support for the Hybrid Phillips curve case, although it was statistically significant only in the context of normalisation with respect to inflation. Also using the Present Value approach, we find quite similar and statistically significant point estimates for the hybrid model with balanced roles of both forward and backward-looking behaviour, when, instead of real marginal costs, the present value the driving variable was expressed in terms of the mark-up over nominal marginal costs. This interpretation, however, implies that the NKPC is not inflation equation but that of price level.
Most macro-models (of whatever size) embody a (Old or New) Phillips-curve. It is therefore reasonable to expect the success of this NKPC literature to depend on how they are incorporated into large-scale models. This would require – amongst other things – that they would be part of a fully (as opposed to partial) optimising framework and thus consistent with whatever long-run was embodied in the full model. And that would require that dynamic (root) analysis becomes one important tool for analysing their successful integration into larger simultaneous equation systems. In that respect, by stressing these system features our paper potentially heralds possible future developments in this literature.
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