Price plans and the real effects of monetary policy

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Many temporary price changes in micro data

- Should we “count” the transitory price changes or ignore them?
Many temporary price changes in micro data

Many temporary price changes in micro data

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REVIEW OF ECONOMIC STUDIES

GOLDBERG & HELLERSTEIN
LOCAL CURRENCY PRICE STABILITY

– Should we “count” the transitory price changes or ignore them?

This paper:

• build a model w/ temporary price changes matching “the data”
• compare the effect of monetary shocks w/ and w/o transitory price changes
We study this question using an analytical model based on Eichenbaum, Jaimovich, Rebelo (2011)

Problem setup:

- Firms track a desired profit maximizing price $p^*(t)$
- they can post a price from the current “Price Plan” $\mathcal{P} = \{p^L, p^H\}$
- Price changes within the plan $p^L \rightleftharpoons p^H$ are costless
- Changing the Plan $\mathcal{P}$ requires paying a fixed cost $\psi$

Objective is to characterize:

- optimal plan, stopping times, aggregation, shock-propagation
Main results

- **Analytics of the firm’s choices (make complex problem tractable)**
- **Decreasing Hazard rate price of changes**
- **Plans increase price flexibility (reduce output effects of monetary shocks)**
  - differs from Guimares-Sheedy, Eichenbaum et al., Kehoe-Midrigan
GE setup: Woodford, Midrigan, Golosov-Lucas model with 2-price Plan

\begin{align*}
\text{Lifetime Utility} : & \quad \int_0^\infty e^{-rt} \left( \frac{c(t)^{1-\epsilon} - 1}{1 - \epsilon} - \alpha \ell(t) + \log \frac{M(t)}{P(t)} \right) dt \\
\text{CES aggregate} : & \quad c(t) = \left( \int_0^1 c_k(t) \frac{n-1}{n} dk \right)^{\frac{n}{n-1}}
\end{align*}

- Linear technology \( c_k(t) = \ell_k(t) / Z_k(t) \) and \( Z_k(t) = \exp(\sigma Z_k(t)) \).

- Equilibrium: constant nominal interest rate \& wages \( W(t) = aM(t) \).
General Equilibrium Set Up

• GE setup: Woodford, Midrigan, Golosov-Lucas model with 2-price Plan

\[
\int_0^\infty e^{-rt} \left( \frac{c(t)^{1-\epsilon} - 1}{1 - \epsilon} \right) dt - \alpha \ell(t) + \log \frac{M(t)}{P(t)}
\]

CES aggregate:
\[
c(t) = \left( \int_0^1 c_k(t)^{\frac{n-1}{n}} dk \right)^{\frac{n}{n-1}}
\]

• Linear technology \(c_k(t) = \ell_k(t) / Z_k(t)\) and \(Z_k(t) = \exp(\sigma Z_k(t))\).

• Equilibrium: constant nominal interest rate & wages \(W(t) = a M(t)\).

• Firm’s profits: \(\Pi(p - p^*, c) \approx -B (p - p^*)^2\) where \(B = \frac{n(n-1)}{2}\).

• \(\psi \ell\) units of labor paid to change the Price Plan
The firm problem

process for desired price: \[ dp^*(t) = \sigma \, dZ(t) \]

where \( \sigma \) shock volatility

Value function:

\[
V(p^*, p^L, p^H) = \min_{\{\tau_i, P_i\}_{i=1}^{\infty}} \mathbb{E} \left[ \sum_{i=1}^{\infty} \int_{\tau_{i-1}}^{\tau_i} e^{-rt} \min_{p(t) \in P_{i-1}} B(p(t) - p^*(t))^2 \, dt \right]
\]

\[
+ \sum_{i=1}^{\infty} e^{-r\tau_i} \psi \left\| p^* = p^*(0), P_0 = \{p^L, p^H\} \right\|
\]
process for desired price: \[ dp^*(t) = \sigma \, d\mathcal{Z}(t) \] where \( \sigma \) shock volatility

Value function:

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V(p^*, p^L, p^H) = \min_{\{\tau_i, P_i\}_{i=1}^\infty} \mathbb{E} \left[ \sum_{i=1}^\infty \int_{\tau_{i-1}}^{\tau_i} e^{-rt} \min_{p(t) \in P_{i-1}} B(p(t) - p^*(t))^2 \, dt \right] \\
+ \sum_{i=1}^\infty e^{-r\tau_i} \psi \left| p^* = p^*(0), P_0 = \{p^L, p^H\} \right|
\]

Equivalent value function representation

\[
\nu(g; \tilde{g}) \equiv V(p^*, p_0^* - \tilde{g}, p_0^* + \tilde{g}) \quad \text{where} \quad g(t) \equiv p^*(t) - p^*(\tau_i)
\]
benefits of a 2-price plan

flow cost: \[ B \min \{ (g - \tilde{g})^2 , (g + \tilde{g})^2 \} \]

HJB equation: \[ r \nu(g; \tilde{g}) = B \left( g^2 + \tilde{g}^2 - 2 |g| \tilde{g} \right) + \frac{1}{2} \sigma^2 \nu''(g; \tilde{g}) \]
Example of a policy: $p(t) \in \{p^L, p^H\}, p^*(t)$

Notice changes “of Plans” vs changes “within Plans”
Characterization of the Optimal policy: $\tilde{g}$ , $\tilde{g}$

HJB equations

$$r \nu(g; \tilde{g}) = B \left( g^2 + \tilde{g}^2 - 2 |g| \tilde{g} \right) + \frac{1}{2} \sigma^2 \nu''(g; \tilde{g})$$

(1)

$$\nu(g; \tilde{g}) \leq \psi + \min_{\tilde{g}'} \nu(0; \tilde{g}')$$

optimality conditions

$$\nu(\bar{g}; \tilde{g}) = \psi + \nu(0, \tilde{g})$$, value matching

(2)

$$\nu'(\bar{g}; \tilde{g}) = 0$$, smooth pasting

(3)

$$\frac{\partial}{\partial \tilde{g}} \nu(0; \tilde{g}) = 0$$, optimality of $\tilde{g}$

(4)
Optimal decision rules: $\tilde{g}, \bar{g}$ (Prop 1 and Prop 2)

Optimal prices within a plan (recall $p(t) = p_i^* \pm \tilde{g}$)

$$
\tilde{g} = \bar{g} \rho \left( \frac{r\bar{g}^2}{\sigma^2} \right) \approx \bar{g} \frac{1}{3} \quad \text{as} \quad r \to 0
$$

Optimal width of the plan (adjust when $|g| \equiv |p^*(t) - p_i^*| > \bar{g}$)

$$
\bar{g} = f \left( \frac{r \psi}{B}, \frac{r}{\sigma^2} \right) \approx \left( 18 \frac{\psi}{B \sigma^2} \right)^{\frac{1}{4}} \quad \text{as} \quad r \to 0
$$
Frequency of price changes: model vs data

- Total (expected) number of price changes $N = N_T + N_p$ where $N_p = \frac{\sigma^2}{\bar{g}^2}$

  $N_T$ depends on "model period" $\Delta$: \[
  \begin{cases} 
  \text{if } \Delta \to 0 & \text{then } N \to \infty \\
  \text{if } \Delta > 0 & \text{then } N \approx \sqrt{N_p/\Delta}
  \end{cases}
  \]

- Reference Price (EJR) = mode of prices within a period (say a quarter)
Frequency of price changes: model vs data

- Total (expected) number of price changes $N = N_T + N_p$ where $N_p = \frac{\sigma^2}{\bar{g}^2}$

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- Reference Price (EJR) = mode of prices within a period (say a quarter)

- Data: Prices spend large fraction of time at reference price ; $N$ large

- Model with $N_p > 0$ (but small) and $\Delta > 0$:
  - prices spend almost all the time at the “reference” price and $N \to \infty$. 

Alvarez and Lippi (UofC, EIEF)
Summary statistics on price setting behavior (weekly model $\Delta = \frac{1}{52}$)

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong># Price changes per year</strong> ($N = N_p + N_w$)</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5.1</td>
</tr>
<tr>
<td><strong># Plan changes per year</strong> ($N_p$)</td>
<td>5.8</td>
<td>3.3</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td><strong># Reference Price changes per year</strong>*</td>
<td>4.1</td>
<td>3.3</td>
<td>2.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

*Reference price \(\downarrow\) modal price within a quarter

\(\downarrow\) fraction time spent ref. price: EJR: 0.62, Dominick's: 0.77

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Alvarez and Lippi (UofC, EIEF)

Sticky plans
## Cross section predictions

Summary statistics on price setting behavior (weekly model $\Delta = \frac{1}{52}$)

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<tr>
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<thead>
<tr>
<th></th>
<th>0.35</th>
<th>0.27</th>
<th>0.18</th>
<th>0.10</th>
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<tbody>
<tr>
<td>Freq. of Price changes (per week)</td>
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<td></td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.29</td>
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<tr>
<td>Freq. of Reference price change (per quarter)</td>
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</table>

### Fraction of time spent at reference price**

<table>
<thead>
<tr>
<th></th>
<th>0.50</th>
<th>0.63</th>
<th>0.79</th>
<th>0.95</th>
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<tbody>
<tr>
<td>Fraction of time spent at reference price</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.17</td>
<td>0.10</td>
<td>0.02</td>
</tr>
<tr>
<td>Fraction of time spent below the reference price</td>
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</table>

* reference price $\equiv$ modal price within a quarter

** fraction time spent ref. price: EJR : 0.62, Dominick’s: 0.77
Hazard Rate of Price changes

Hazard: probability (per unit of time) of price change as function of duration

- Hazard for all price changes $h(t)$ with $\lim_{t \to 0} h(t) t = \frac{1}{2}$ or $h(t) \approx \frac{1}{2t}$
- Hazard for plan changes $h_p(t)$

Model

Data (Campbell and Eden)
Impulse response analysis

- Start with invariant cross-section distribution of desired prices: $h(g)$
- Shock: permanent increase of money supply $\delta \%$ (nominal wages)
- Simplification: decision rules in transition same as in St. St.

Agg. price level IRF

$$P(t, \delta) = \Theta(\delta) + \int_0^t \theta(s, \delta) ds$$

Cumulated output response

$$M(\delta) \equiv \int_0^\infty Y(t, \delta) dt = \frac{1}{\epsilon} \int_0^\infty [\delta - P(t, \delta)] dt$$
Impulse responses for models with same $\bar{g}$, $\sigma^2$

Models have identical number of Plan changes $N_p = \sigma^2/\bar{g}^2$

Cumulated output response

$$\mathcal{M}(\delta) \equiv \int_0^\infty Y(t, \delta) \, dt = \frac{1}{\epsilon} \int_0^\infty [\delta - \mathcal{P}(t, \delta)] \, dt$$
Mechanics of the impulse response: shift $h(g)$

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Why this difference? Non-negligible impact effect!

A mass of firms increase prices from $-\tilde{g}$ to $\tilde{g}$ on impact! on impact the price level jumps by $2/3$ of $\delta$
Impulse response
Output effect following a small shock $\delta$ (analytics)

For small shocks we use

$$\mathcal{M}(\delta) = \delta \mathcal{M}'(0) + o(\delta)$$

Cumulated output effect in menu cost model without plans

$$\mathcal{M}'_{MC}(0) = \frac{\bar{g}^2}{6 \sigma^2} = \frac{1}{6 N}$$

Cumulated output effect in menu cost model with plans

$$\mathcal{M}'(0) = \frac{\bar{g}^2}{18 \sigma^2} = \frac{1}{18 N_p}$$
Application of main result

$N$: number of price changes , $N_p$: number of PLAN changes

$M$ is approximate cumulated output after small shock $\delta$

<table>
<thead>
<tr>
<th>Theory</th>
<th>W/ Price Plans</th>
<th>W/out Price Plans</th>
<th>Ratio $\frac{M(\delta)}{M(\delta)}$</th>
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<tr>
<td>$M(\delta) = \frac{\delta}{6} \frac{1}{3N_p}$</td>
<td>$M(\delta) = \frac{\delta}{6} \frac{1}{N}$</td>
<td>$\frac{M(\delta)}{M(\delta)} = \frac{N}{3N_p}$</td>
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Alvarez and Lippi (UofC, EIEF)
**Application of main result**

\(N\): number of price changes \(, \ N_p\): number of PLAN changes

\(\mathcal{M}\) is approximate cumulated output after small shock \(\delta\)

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<th>Ratio W/ With Plans</th>
<th>Without Plans</th>
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<td>(\mathcal{M}(\delta) = \frac{\delta}{6} \frac{1}{3N_p})</td>
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<td></td>
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**Calibration**

\(N_p = 3.33 \ , \ N = 15\)

\(\mathcal{M}(\delta) = \frac{\delta}{6} \frac{1}{10}\)

\(\begin{cases} \mathcal{M}(\delta) = \frac{\delta}{6} \frac{1}{3.33} \\ \mathcal{M}(\delta) = \frac{\delta}{6} \frac{1}{15} \end{cases} \)

\(\begin{align*} \frac{\mathcal{M}(\delta)}{\mathcal{M}(\delta)} &= \frac{1}{3} \\
\frac{\mathcal{M}(\delta)}{\mathcal{M}(\delta)} &= \frac{15}{10} \end{align*}\)
Assume exogenous arrivals of price-change opportunities, of 2 types:

- **Permanent price changes**, with arrival rate $\alpha_P$
- **Temporary price changes**, with duration $T$, with arrival rate $\alpha_T$
Assume exogenous arrivals of price-change opportunities, of 2 types:

- **Permanent price changes**, with arrival rate $\alpha_P$
- **Temporary price changes**, with duration $T$, with arrival rate $\alpha_T$

Implies (recall: standard Calvo has $\alpha_T = 0$):

- **Total Number of adjustments per period**: $N = \alpha_P + \alpha_T \left( 1 + e^{-\alpha_P T} \right)$
- **Cumulated output effect**

$$M(\delta) \approx \delta \frac{1}{\alpha_P + \alpha_T \left( 1 - e^{-\alpha_P T} \right)}$$
Relation to Kehoe-Midrigan (2015); 3 parameters

Assume exogenous arrivals of price-change opportunities, of 2 types:

- **Permanen**t price changes, with arrival rate $\alpha_P$
- **Temporary** price changes, with duration $T$, with arrival rate $\alpha_T$

Implies (recall: standard Calvo has $\alpha_T = 0$):

- **Total Number of adjustments** per period: $N = \alpha_P + \alpha_T \left( 1 + e^{-\alpha_P T} \right)$
- **Cumulated output effect**

$$M(\delta) \approx \delta \frac{1}{\alpha_P + \alpha_T \left( 1 - e^{-\alpha_P T} \right)}$$

- As $T \to 0$: real effects unchanged, but $N$ can diverge
- In K-M: $T = \frac{1}{12}$ and $\alpha_P \approx \alpha_T \approx 0.9 \implies e^{-\alpha_P T} = 1.075$
  Bottomline: $N$ triples while $M$ basically unchanged
Flexibility does not come from changes in plans

number of firms that change plans following a shock same as GL

EJR has more flexibility (state dependence) within a Plan
Flexibility does not come from changes in plans

number of firms that change plans following a shock same as GL

EJR has more flexibility (state dependence) within a Plan

In particular: how do firms react to higher cost?

- More firms will be at the high value of their plan.

- Food for empirical work! do temporary Δp respond to shocks?

Figure 5: Frequency of sales in the U.S. CPI data and unemployment rate
Figure 6
Frequency of Sales and Unemployment Rate

Sales Frequency (left axis)  Unemployment Rate (right axis)
Concluding remarks

price-setting problem in a tractable GE set-up

- temporary price changes dampen real effects of monetary shocks
  state dependence (within-plan selection) boosts price flexibility

- Analogue results occur with exogenous adjustment times for Plans

- Analogue results occur in model with transitory (iid) shocks

- Bonus: decreasing hazard functions and hump-shaped IRFs
Firm maximizes profits given exogenous adjustment times: $\tau_i$

Namely: firm allowed to reset PLAN: $\{p^L, p^H\}$ with rate $\lambda$
Firm maximizes profits given exogenous adjustment times: $\tau_i$

Namely: firm allowed to reset PLAN: $\{p^L, p^H\}$ with rate $\lambda$

Model yields the optimal Plan prices upon reset: $p^*_i \pm \tilde{g}$

where $\tilde{g} = \frac{\sigma}{\sqrt{2(\lambda + r)}}$
Extension 1: Calvo plans

Exponentially distributed plan adjustments ("Calvo")

Firm maximizes profits given exogenous adjustment times: $\tau_i$

Namely: firm allowed to reset PLAN: $\{p^L, p^H\}$ with rate $\lambda$

Model yields the optimal Plan prices upon reset: $p_i^* \pm \tilde{g}$

where $\tilde{g} = \frac{\sigma}{\sqrt{2(\lambda + r)}}$

Invariant distribution of desired prices: $h(g) = \frac{\sqrt{2\lambda}}{2\sigma} e^{-\frac{\sqrt{2\lambda}}{\sigma} |g|}$ for all $g$
Impulse response function w/ exponential adjustments

Output deviation from steady state

- Blue line: with Plans
- Red line: without Plans

Time (years)
Cumulated output after a small money shock $\delta$

For small shocks we use

$$M(\delta) = \delta M'(0) + o(\delta)$$

Cumulated output effect in Calvo model without plans

$$M'(0) = \frac{1}{N}$$

Cumulated output effect in Calvo model with plans

$$M'(0) = \frac{1}{2N_p}$$
A model with iid shocks to desired prices

Assume $p^*$ (desired price) is iid with CDF $F(\cdot)$

$$
\nu(p_L, p_H) = \int_x \min \left\{ \min_{p_j \in \mathcal{P}} \left[ B(p_j - x)^2 + \beta \nu(p_L, p_H) \right], \psi + \min_{\{p'_L, p'_H\}} \left( B(\hat{p} - x)^2 + \beta \nu(p'_L, p'_H) \right) \right\} dF(x)
$$
A model with iid shocks to desired prices

Assume $p^*$ (desired price) is iid with CDF $F(\cdot)$

$$
\nu(p_L, p_H) = \int_x \min_p \left\{ \min_{p_j \in P} \left[ B(p_j - x)^2 + \beta \nu(p_L, p_H) \right], \right.

\left. \psi + \min_{\{p'_L, p'_H\}} \left( B(\hat{p} - x)^2 + \beta \nu(p'_L, p'_H) \right) \right\} dF(x)
$$

Simulation result calibrating 2 economies to the same freq. PLAN changes
### Effects of Price-Plans with iid shocks

<table>
<thead>
<tr>
<th>Calibration for both W/ Price Plans</th>
<th>W/ out Price Plans</th>
<th>Ratio $\frac{\mathcal{M}(\delta)}{\mathcal{M}(\delta)} \approx \frac{1}{7}$</th>
</tr>
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<tbody>
<tr>
<td>( N_p = 11 )</td>
<td>( N = 31, N_{ref} = 3.4 )</td>
<td>( N = 11 ), ( N_{ref} = 2.8 )</td>
</tr>
<tr>
<td>( N_{ref} = 1 )</td>
<td>( N = 18, N_p = 0.33 )</td>
<td>( N = 3, N_p = 3 )</td>
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<td>( N = 3, N_p = 3 )</td>
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**Legend:**

- freq. all prices \( N \),
- freq. plans \( N_p \),
- freq. reference prices \( N_{ref} \)

Frequencies per year ; weekly model \( \Delta = 1/52 \)
Hazards rates and monetary shocks

Notice the difference:

- EJR has decreasing hazard rates (consequence of a selection effect)
- KM has constant hazards (no selection effect)
- EJR-plans reduce real effects of monetary shocks more than KM-plans
  the reason is selection (state dependent) nature of adjustments
- Key margin of adjustment is the frequency, not the size of $\Delta p$
  consistent with evidence in Krystov and Vincent (2014)
Approximate profit function

- Define the (log) "desired price" \( p_t^* \equiv \log \left( W_t Z_t \frac{n}{\eta - 1} \right) \)

- Define the "price gap" \( x \equiv p(t) - p^*(t) \) (log prices)

Discounted nominal profit product \( i \) as function of \( x \)

\[
\Pi(x, c(t)) \propto W(t) e^{-r t} c(t)^{1-\eta} e^{-\eta x} \left[ e^x \frac{\eta}{\eta - 1} - 1 \right]
\]

\[
0 = \frac{\partial \Pi(0, c(t))}{\partial x} = \frac{\partial^2 \Pi(0, c(t))}{\partial x \partial c(t)}
\]

where \( c(t) \) aggregate consumption

- Firm’s profits: \( \Pi(x, c(t)) \approx -B (p(t) - p^*(t))^2 \) where \( B = \frac{n(n-1)}{2} \)
Bellman equation (3 parameters: $\psi/B, \sigma, r$)

$$V(p^*, p^L, p^H) = \min_{p(t) \in \{p^L, p^H\}} \mathbb{E} \left[ \int_0^\tau e^{-rt} \min_{p(t) \in \{p^L, p^H\}} B \left( p(t) - p^*(t) \right)^2 \, dt \right]$$

$$+ e^{-r\tau} \left[ \psi + V^*(p^*(\tau)) \right] \bigg|_{p^* = p^*(0)}$$

where

$$V^*(p^*) \equiv \min_{(\bar{p}^L, \bar{p}^H) \in \mathbb{R}^2} V(p^*, \bar{p}^L, \bar{p}^H) = V^* \quad \forall p^*$$

and

$$d\rho^*(t) = \sigma \, d\mathcal{Z}(t)$$

Invariance to translations e.g. $|a| > 0$

$$V(p^*, p^L, p^H) = V(p^* + a, p^L + a, p^H + a) \quad \text{for all} \quad a, p^*, p^L, p^H$$
The optimal price plans are centered and symmetric (Lemma 1):

\[ P_i = \{ p_i^* - \tilde{g}, \ p_i^* + \tilde{g} \} \quad \text{where} \quad p_i^* \equiv p^*(\tau_i) \]

Redefine the state-space (shrink it by one dimension)

\[ v(g; \tilde{g}) = V( p^*, p_i^* - \tilde{g}, p_i^* + \tilde{g}) \quad \text{where} \quad g(t) \equiv p^*(t) - p_i^* \]

The new state \( g \) is the “normalized” desired price
Characterization of cumulated output effect

Expected cumulated output by a firm with \( g(t) = p^*(t) - p^*(\tau_i) \)

\[
m(g) = \mathbb{E} \left[ \int_0^\tau \left( g(t) \right) dt \right] \quad g(0) = g
\]

where \( dg = \sigma \, d\mathcal{Z} \), \( m(0) = 0 \), \( m(-g) = -m(g) \)
Characterization of cumulated output effect

Expected cumulated output by a firm with \( g(t) = p^*(t) - p^*(\tau_i) \)

\[
m(g) = \mathbb{E} \left[ \int_0^\tau \left( \frac{g(t)}{y_t} \right) dt \middle| g(0) = g \right]
\]

where \( dg = \sigma d\bar{Z} \), \( m(0) = 0 \), \( m(-g) = -m(g) \)

Cumulated output \( \rightarrow \quad \mathcal{M}(\delta) = \int_{-\bar{g}}^{\bar{g}} m(g + \delta) h(g) \, dg \)
Characterization of cumulated output effect

Expected cumulated output by a firm with \( g(t) = p^*(t) - p^*(\tau_i) \)

\[
m(g) = \mathbb{E} \left[ \int_0^\tau \left( \begin{array}{c} g(t) \\ y_t \end{array} \right) \ dt \right| g(0) = g
\]

where \( dg = \sigma \, d\bar{Z} \) , \( m(0) = 0 \) , \( m(-g) = -m(g) \)

Cumulated output \( \rightarrow \) \( \mathcal{M}(\delta) = \int_{-\bar{g}}^{\bar{g}} m(g + \delta) \, h(g) \, dg \)

Useful because we have an ODE for \( m(g) \) :

Example w menu cost: \( 0 = g + m''(g) \frac{\sigma^2}{2} \)

with boundary conditions \( m(0) = m(\bar{g}) = 0 \) and known \( h(g) \)
Cumulated output effect with plans

Expected cumulated output by a firm with $g(t) = p^*(t) - p^*(\tau_i)$

$$m(g) = \mathbb{E} \left[ \int_0^T \left( g(t) - \tilde{g} \cdot \text{sign}(g(t)) \right) dt \mid g(0) = g \right]$$

where $dg = \sigma \, d\tilde{Z}$, $m(0) = 0$, $m(-g) = -m(g)$
Cumulated output effect with plans

Expected cumulated output by a firm with \( g(t) = p^*(t) - p^*(\tau_i) \)

\[
m(g) = \mathbb{E} \left[ \int_0^T \left( g(t) - \tilde{g} \cdot \text{sign}(g(t)) \right) dt \ \bigg| \ g(0) = g \right]
\]

where \( dg = \sigma \, d\bar{Z} \), \( m(0) = 0 \), \( m(-g) = -m(g) \)

Cumulated output \( \rightarrow \quad \mathcal{M}(\delta) = \int_{-\tilde{g}}^{\tilde{g}} m(g + \delta) \, h(g) \, dg \)

Useful because we have an ODE for \( m(g) \):

Example (w/ plan): menu cost: \( 0 = g - \tilde{g} \cdot \text{sign}(g) + m''(g) \frac{\sigma^2}{2} \)

with boundary conditions \( m(0) = m(\tilde{g}) = 0 \) and known \( h(g) \)
Lack of sensitivity to Inflation $\mu$

- Inflation has only second order effect around $\mu = 0$ on
  - entire hazard rate $h(t; \mu)$; or: $\frac{\partial h(t;\mu)}{\partial \mu} \bigg|_{\mu=0} = 0$
  - Cumulated output: $M'(0; \mu)$; or: $\frac{\partial M'(\delta;\mu)}{\partial \mu} \bigg|_{\mu=0} = 0$

Main ideas:
- by symmetry $h(t; \mu) = h(t; -\mu)$
- differentiating w.r.t. $\mu$ and evaluating at $\mu = 0$ for any $t$

- by symmetry: $M(\delta; \mu) = -M(-\delta; -\mu)$ where $M'(\delta; \mu) \equiv \frac{\partial M'(\delta;\mu)}{\partial \delta}$
- differentiating w.r.t. $\mu$ and $\delta$ both sides and evaluating at $(\delta, \mu) = (0, 0)$
## Compare Effects of a Monetary Shock:

*given parameters* $\psi / B, \sigma^2, r$

<table>
<thead>
<tr>
<th>Effect M-shock</th>
<th>W/ Price Plans</th>
<th>W/out Price Plans</th>
<th>Ratio $\frac{\mathcal{M}(\delta)}{\mathcal{M}<em>{GL}(\delta)} = \frac{N</em>{GL}}{3 N_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}(\delta) = \frac{\delta}{6} \frac{1}{3 N_p}$</td>
<td>$\mathcal{M}<em>{GL}(\delta) = \frac{\delta}{6} \frac{1}{N</em>{GL}}$</td>
<td>$\frac{\mathcal{M}(\delta)}{\mathcal{M}<em>{GL}(\delta)} = \frac{N</em>{GL}}{3 N_p}$</td>
<td></td>
</tr>
<tr>
<td># plans/prices</td>
<td>$N_p = \frac{\sigma^2}{(\bar{g})^2}$</td>
<td>$N_{GL} = \frac{\sigma^2}{(\bar{g}_{GL})^2}$</td>
<td></td>
</tr>
<tr>
<td>optimal threshold</td>
<td>$\bar{g} = \left(18 \sigma^2 \psi / B\right)^{\frac{1}{4}}$</td>
<td>$\bar{g}_{GL} = \left(6 \sigma^2 \psi / B\right)^{\frac{1}{4}}$</td>
<td></td>
</tr>
<tr>
<td>optimal # plans</td>
<td>$N_p = \frac{\sigma}{\sqrt{18} \psi / B}$</td>
<td>$N_{GL} = \frac{\sigma}{\sqrt{6} \psi / B}$</td>
<td></td>
</tr>
<tr>
<td>Total Effect</td>
<td>$\mathcal{M}(\delta) = \frac{1}{\sqrt{3}} \approx 0.6$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\mathcal{M}$ and $\mathcal{M}_{GL}$ approximations for small value of $\delta$.

$\bar{g}$ and $\bar{g}_{GL}$ approximations for small value of product $(\psi / B) r^2 \sigma^2$. 
Cyclicality of Sales (Kryvstov Vincent, 2015)

Figure 4: The evolution of the size of sales

Figure 5: Frequency of sales in the U.S. CPI data and unemployment rate
Figure 6
Frequency of Sales and Unemployment Rate

This figure illustrates how the optimal regular and sales prices vary according to the marginal cost in the Hendel and Nevo (2013) model. The sales price is calculated as the regular price less discount. The non-discriminatory price is the price that the firm would charge if it was unable to price discriminate between the two periods (charging the same regular price without discounts in both periods).