State and Time dependence on price setting

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Two hypothesis to explain infrequent price changes

- **menu costs** models: fixed cost of changing prices
  
  Example: Golosov and Lucas (JPE 2007).

  *State* dependence rules.

- **observation cost** models: cost of gathering and processing information
  
  Example Reis (ReStud 2006)

  *Time* dependence rules.
Effect of aggregate shocks in time vs. state dependent models

- **Small** aggregate shocks: *same* effects in both type of models.

- **Large** aggregate shocks: *higher* effect in time dependent models.
Time vs State Dependence

- Small Aggregate shocks:
  - A sufficient statistic for cumulative IRF of once-and-for-all small monetary shock
    \[ \text{Kurtosis} \text{ of size of price changes to } \text{Frequency} \text{ of price changes} \]
  - irrespective of time or state dependence nature of the decision rules.

- Large aggregate shocks:
  - Kurtosis/ Frequency is NOT a sufficient statistic
  - State and Time dependent models react differently to large shocks.
  - Unrelated additional evidence for models w/state & state dependence.
General Equilibrium Set Up

- GE setup: Woodford, Midrigan, Golosov-Lucas model

\[
\text{Lifetime Utility : } \int_0^\infty e^{-rt} \left( \frac{c(t)^{1-\epsilon} - 1}{1 - \epsilon} - \alpha \ell(t) + \log \frac{M(t)}{P(t)} \right) dt
\]

\[
\text{CES aggregate : } c(t) = \left( \int_0^1 \sum_{i=1}^n (A_{ki}(t) c_{ki}(t))^{\frac{1}{\eta}} \right)^{\frac{\eta}{\eta - 1}}
\]

- Intra-temp. subst. elasticity $\eta$ (= for firms $k$ & products $i$); $n \geq 1$ goods

- Linear technology $c_{ki}(t) = \ell_{ki}(t) / Z_{ki}(t)$ and $Z_{ki}(t) = \exp(\sigma W_{ki}(t))$.

- Fundamental shocks: $W_{ki}$, standard Brownian Motion.
General Equilibrium Set Up

• GE setup: Woodford, Midrigan, Golosov-Lucas model

\[
\text{Lifetime Utility} : \quad \int_{0}^{\infty} e^{-r t} \left( \frac{c(t)^{1-\epsilon} - 1}{1 - \epsilon} - \alpha \ell(t) + \log \frac{M(t)}{P(t)} \right) dt
\]

\[
\text{CES aggregate} : \quad c(t) = \left( \int_{0}^{1} \sum_{i=1}^{n} \left( A_{ki}(t) c_{ki}(t) \right)^{1-\frac{1}{n}} dk \right)^{\frac{n}{n-1}}
\]

- Intra-temp. subst. elasticity \( \eta \) (= for firms \( k \) & products \( i \)); \( n \geq 1 \) goods

- Linear technology \( c_{ki}(t) = \ell_{ki}(t) / Z_{ki}(t) \) and \( Z_{ki}(t) = \exp(\sigma W_{ki}(t)) \).

- Fundamental shocks: \( W_{ki} \), standard Brownian Motion

- Equilibrium: constant nominal interest rate & wages \( W(t) = a M(t) \).
Price setting frictions (technology)

Two frictions considered (in isolation or combined):

- “Menu cost”: $\psi$ units of labor paid to change the nominal price.
- “Observation cost”: $\theta$ units of labor paid observed relevant firm’s state.
- Fixed cost, in the sense that they independent on size of price changes.
- Allow versions with random Markovian adjustment costs: $\psi, \theta$
Based upon:

- Only menu cost $\psi > 0$ (no observation cost, $\theta = 0$)
  - Multiproduct $n$ (w/Lippi, Etc 2014)
  - Multiproduct $n$ and random menu cost (w/Lippi & LeBihan, AER-RR)
    - Menu cost is either zero with prob. $\lambda \, dt$ or $\psi > 0$ with prob. $(1 - \lambda \, dt)$

- Only observation cost $\theta > 0$ (no menu cost, $\psi = 0$)
  - Random observation cost $\theta$ (w/Lippi & Paciello, RES 2015)
    - Observation cost are Markov with transition: $Q(\theta' | \theta)$

- Both observation and menu cost ($\theta > 0$, $\psi > 0$)
  - $\alpha = \psi / \theta$ (w/Lippi & Paciello, QJE 2011 and JEEA-RR)
Based upon:

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Steady State and Impulse Response

- Solve firms’ problem in steady state
  - Characterize nature of decision rules.
  - Show how decision rules depends on parameter of models.
  - As function of parameters of decisions rules, solve for
    - distribution of size of price changes and
    - distribution of time to adjustments (hazard rates)
- Solve for impulse response of a once-and-for-all shock
- Relate IRF to simple steady state statistics
Log optimal static profit maximizing price = random walk w/innovation $\sigma^2$.

Price “gap” $p$:

difference between actual price and optimal static profit maximizing one.

In all models at a price change: close the gap, i.e. set optimal static price.

All models feature tradeoff between frequency and size of adjustments

\[
\frac{N(\Delta p_i)}{\text{# price changes}} \times \frac{\text{Var}(\Delta p_i)}{\text{variance of price changes}} = \sigma^2
\]
Optimal decision rules

- **Menu cost models:**
  - Change prices when price gap reaches optimally determined barrier $\bar{p}$
  - By symmetry we can write the barrier on $p^2 \leq \bar{p}^2$.
  - Multiproduct: $n$-dimensional optimally determined barrier on $\sum_{i=1}^{n} \frac{p_i^2}{n} \leq \bar{y}$.
  - Every time there is a zero menu cost: adjust price regardless of gap.

- **Observation cost models:**
  - Upon each observation *always* change price (close gap)
  - Upon observation set time until new observation of the state, as function of current cost $\theta$.

- **Model w/observation and menu cost**
  - Upon observation change price *only if* price gap is large enough.
  - Upon observation set time until new observation of the state
Distribution of price changes

- **Menu cost models:**
  - shape depends on number of products $n$ and $\lambda \frac{\bar{y}/n}{\sigma^2} \sim \text{"Calvo"}$ and $\text{"Golosov-Lucas"}$.
  - Fixing $n$, the value of $\lambda \frac{\bar{y}/n}{\sigma^2}$ controls peakness (Kurtosis).
    - As $\lambda \frac{\bar{y}/n}{\sigma^2} \to \infty \in (0, \infty)$ distribution goes from Bernoulli to Laplace.
  - Fixing $\lambda = 0$, as $n$ increases distribution goes from Bernoulli to Normal.
  - Different combination of $n$, the value of $\lambda \frac{\bar{y}/n}{\sigma^2}$ imply same kurtosis.

- **Observation cost models:**
  - Distribution of price changes is a mixture of normals.
  - Kurtosis = 3 if expected value of $\theta$ is constant,
  - Kurtosis > 3 if expected value of $\theta$ is random.

- **Model w/observation and menu cost**
  - From Bernoulli (pure state dependent) to normal (pure time dependent)
  - Kurtosis increases with $\alpha \equiv \frac{\theta}{\psi} = \frac{\text{Rational inattention cost}}{\text{Menu Cost}} \sim \text{"Reis"}$ and $\text{"Golosov-Lucas"}$. 
Distribution of price changes: multiproduct Calvo

\[ \lambda \frac{N(\Delta p_i)}{\bar{y}/n} = 0.2 \]

\[ \lambda \frac{N(\Delta p_i)}{\bar{y}/n} = 0.8 \]

\[ \lambda \frac{N(\Delta p_i)}{\bar{y}/n} \text{ maps one-to-one with } \lambda \frac{\bar{y}/n}{\sigma^2} \]

\[ \text{each line for different number of products } n \]
Distribution of price changes for different $\alpha = \frac{\theta}{\psi}$

- No menu cost (time dependent)
- No observation cost (state dependent)
- Mixed case with $\psi > 0$ and $\theta > 0$. 
Effect of aggregate shocks
Once and for all monetary shock at $t = 0$. 
Impulse response of Price Level

\[ P(t) \]

percent deviation from steady state \( \delta \) vs time \( t \)
Impulse response of output $d \log c_t = \frac{1}{\epsilon} d \log(M_t/P_t)$
Aggregate Shocks

Area under impulse response of output: $\mathcal{M}$

$$\mathcal{M}(\delta) \equiv \frac{1}{\epsilon} \int_0^\infty (\delta - \mathcal{P}_t) \, dt$$
Aggregate Shocks

Area under impulse response of output: $\mathcal{M}$

$$\mathcal{M}(\delta) = \frac{1}{\epsilon} \int_0^\infty (\delta - P_t) \, dt$$

$$\approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p_i)}{6 N(\Delta p_i)}$$
Sufficient Statistic

- $\mathcal{M}(\delta) \equiv$ cumulative IRF of output to a (small) monetary shock $\delta$

$$\mathcal{M}(\delta) \approx \frac{\partial \mathcal{M}(0)}{\partial \delta} \delta = \frac{\delta}{\epsilon} \frac{Kurt(\Delta p_i)}{6 \cdot N(\Delta p_i)}$$

- Frequency of price changes $N(\Delta p_i)$ has a first order effect,

- Kurtosis price changes $Kurt(\Delta p_i)$ has a first order effect.

- $Kurt(\Delta p_i)$ measure frequency of very small & very large price changes.

- $Kurt(\Delta p_i)$ controls for selection of time and size of price changes.

- $Kurt(\Delta p_i)$ can be the same in models with different implied distributions of size of price changes and of time to adjustment.

- Result only for small monetary shock $\delta$ (first order approximation).
Large vs Small Aggregate Shocks

- Cumulative IRF of Output for small depends only on Kurtosis/Frequency.
- Kurtosis can be the same in state dependent or time dependent models.
- Three models with Kurtosis equal 3 (as in a normal distribution):
  - $\psi = 0, \theta = 0, (n, \frac{\lambda}{N}) = (\infty, 0)$, multiproduct w/ many goods & Calvo shocks.
  - $\psi = 0, \theta = 0, (n, \frac{\lambda}{N}) = (1, 0.9)$, one product but lot’s of Calvo type shocks.
  - $\psi = 0, \theta > 0, (n, \frac{\lambda}{N}) = (1, 0)$ one product, observation cost.
- These three models have the same cumulative IRF for small shocks $\delta$.
- These three models have different IRF for large shocks $\delta$.

- $\frac{\text{Kurt}(\Delta p_i)}{6 N(\Delta p_i)}$ is NOT a sufficient statistic for large shocks $\delta$. 

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Large vs Small Aggregate Shocks

Define: impact effect on the aggregate price IRF of a shock \( \delta \) at time \( t \):

\[
\Theta(\delta) \equiv \lim_{\epsilon \downarrow 0} P(t + \epsilon; \delta) - P(t)
\]

\( \Theta(\delta) \) = immediate jump on the price level at the time of the shock

By definition \( \Theta(0) = 0 \). All models described so far have:

\[
\Theta'(0) \equiv \frac{\partial \Theta(0)}{\partial \delta} = 0 \implies \Theta(\delta) \approx \Theta(0) + \Theta'(0) \delta + \frac{\Theta''(0)}{2} \delta^2
\]

Caballero and Engel (CE) refer to \( \Theta'(0) \) as the *Price Flexibility Index*.

All models so far have \( \Theta'(0) = 0 \), equally rigid for small shocks.
Large vs Small Aggregate Shocks

Define: impact effect on the aggregate price IRF of a shock $\delta$ at time $t$:

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Caballero and Engel (CE) refer to $\Theta'(0)$ as the *Price Flexibility Index*.

All models so far have $\Theta'(0) = 0$, equally rigid for small shocks.

But state dependent models have $\Theta'(\delta) > 0$ for $\delta > 0$.

Indeed, $\Theta'(\delta) = 1$ as $\delta \to \infty$ (full flexibility for large shocks).
Large vs Small Aggregate Shocks

- Time dependent models have **zero** price impact for any $\delta$
  (under the assumption that agents only learn when they review prices)

- State dependent models have **positive** impact effect for a large $\delta$.

- What type of evidence can be brought to bear? *Exchange rate shocks*
  - Argue that they are similar to once-and-for-all changes $\delta$.
  - Short term pass through coefficient of exchange rate changes
    as function of *size of exchange rate change*

- Recapitulating Theory:
  - Zero short term impact of small changes (zero flexibility CE index)
  - Impact effect of large changes is evidence of state dependence.
Exchange rate changes as measure of $\delta$

- Consider countries with low inflation (model have zero inflation)
- For flexible exchange rates, $\text{FX} \approx \text{random walk}$
- For fixed exchange rates, $\text{FX}$ change in step side devaluations.
- Both cases are approximately as once-and-for-all changes in cost.
- Many pitfalls:
  - Anticipation effects
  - Exogeneity of exchange rate changes
  - Exchange rates only a fraction of the cost (attenuation bias)
Non-linear effect of short term pass-through

- IFS monthly data on nominal ER, output, ER regime (fixed, flexible)

- 33 low inflation countries (below 10%), post-1990

- $\pi_{i,t,t+h}$: country $i$ CPI inflation between $t$ and $t+h$

- $\Delta e_{i,t-1}$: country $i$ nominal exchange rate change during the month $t$

$$\pi_{i,t,t+h} = \alpha_i + \delta_t + \beta_0 \Delta e_{i,t-1} + \beta_1 \mathcal{I}(|\Delta e_{i,t-1}| > K) \Delta e_{i,t-1} + \beta_6 X_{it} + \epsilon_{it}$$

$h = 1, 3, .., 12$ months and $K = 5\%, 10\%, 20\%$

regression includes controls, time and fixed effects.

- Theory “implies" $\beta_0 = 0, \beta_1 > 0$
Preliminary evidence for $K = 10\%$

### Impact of a 10% change in ER

<table>
<thead>
<tr>
<th>VARIABLES</th>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
<td>1-month devaluation</td>
<td>0.002</td>
<td>0.025***</td>
<td>0.031***</td>
<td>0.068***</td>
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<td></td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.012)</td>
<td>(0.018)</td>
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<tr>
<td>1.big_dev10#c.dev</td>
<td>0.031***</td>
<td>0.035</td>
<td>0.009</td>
<td>-0.030</td>
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<td></td>
<td>(0.009)</td>
<td>(0.022)</td>
<td>(0.021)</td>
<td>(0.026)</td>
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<tr>
<td>Fixed ER</td>
<td>-0.032</td>
<td>-0.067</td>
<td>-0.067</td>
<td>-0.174</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.059)</td>
<td>(0.080)</td>
<td>(0.113)</td>
</tr>
<tr>
<td>GDP per capita, PPP</td>
<td>0.022***</td>
<td>0.070***</td>
<td>0.141***</td>
<td>0.286***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.014)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.027</td>
<td>0.076</td>
<td>-0.371</td>
<td>-0.205</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.304)</td>
<td>(0.468)</td>
<td>(0.664)</td>
</tr>
</tbody>
</table>

- Observations: 3,228
- R-squared: 0.249, 0.378, 0.502, 0.578
- Country Fixed Effects: Yes, Yes, Yes, Yes
- Time Fixed Effects: Yes, Yes, Yes, Yes

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$
Evidence of both time and state dependence $\alpha > 0$

Various sources consistent with model with 2 frictions

Robust finding: no tiny price changes (distribution with “hole” in the middle)

- Experimental evidence: Magnani, Gorry, Opera (2015)
- Micro data w/o measurement errors (Cavallo, 2015)
- Direct evidence on relevance of infrequent information reviews
Experimental Evidence

- **Experiment description**
  - Subjects are paid to solve a quadratic tracking problem
  - Subject must track a random walk with no drift
  - Subjects paid proportional to square deviation of current tracking point
  - Subjects pay a fixed cost to adjust to current position
  - Experiments imitates problem of a firm adjusting prices subject to fixed cost

- **Results**
  - Subject act as if they also have an observation cost
  - Implied distribution of adjustment $\approx$ model w/ observation & menu costs.
Description of Experiment

3.2 Implementation
We implemented the experiment using a custom piece of software programmed in a new Javascript environment called Redwood. The subject display, shown in Figure 3, consists of three panels, each visualizing a different part of the decision problem.

First, on the top panel, we show subjects their current price, $p(t)$ and the optimal price $p^*(t)$ (labeled the "Ideal price" on the screen). Subjects see $p(t)$ as a stable red line and $p^*(t)$ as a point fluctuating over time with previous values drawn as a trailing blue line.

The bottom two panels display detailed information on the real time earnings consequences of subjects' decisions. The middle panel shows subjects their flow profits. Positive flows are shown as regions shaded in green, negative flows as regions shaded in red. The bottom line charts the

19 Though we frame the problem above as an expected discounted loss minimization problem, we present subjects as an isomorphic expected discounted profit-maximization problem. In particular, we provide subjects with a flow constant $C$ from which the instantaneous loss function is subtracted (similarly the menu cost is subtracted from current profits whenever an adjustment occurs). The optimal policy in either case is the same.

In our discretized...
Other evidence

Distributions of adjustment in the lab

Figure 4: Weighted histograms of adjustment states for each treatment.
Distributions of adjustment in the lab

Figure 2: Histogram of simulated adjustments for (a) state dependent agents, (b) time dependent agents and (c) costly attention agents.
This, however, does not prove that a comparable scanner data will necessarily have the same bias, or in a similar magnitude. To make the comparison even more explicit, I purchased scanner data for the exact same retailer, location, and time period. The main challenge was to match the retailer in both samples. The scanner dataset, collected by AC Nielsen, does not explicitly identify the retailers. It only provides a supermarket chain id and the zip code of each store. However, all retailers tend to have a distinctive pattern of stores in different zip codes. By simply counting how many stores each supermarket chain in the scanner had in a given set of zip codes, I was able to find a perfect match to the retailer where my online scraped data was collected.

The last column in Table 4 shows that the scanner data also has a duration of 0.8, identical to the effect of a simple time-averaging and the average of results reported by Eichenbaum, Jaimovich, and Rebelo (2011). Time-averages is therefore all that is needed to replicate the duration results in scanner data.

The effect on the size of price changes is even more striking. Figure 1 shows the distribution of the size of price changes in the original data, in the scanner data for the same retailer, and in the simulated weekly averaged data.

Figure 1: The Distribution of the Size of Price Changes in the US
Notes: The online and scanner data in the US was collected at the same retailer during the same time period. Scanner data was collected by Nielsen and provided by the Kilts Center at Chicago Booth.
Frequency of reviews vs adjustment

- Survey: firms reviewing adequacy of the price of main product.

- Interpreting reviews as observing states

- Pure rational inattention model:
  every review (observation) leads to an adjustment

- Pure menu cost model:
  constantly reviewing (observing), and infrequent adjustment

- Mixed model:
  finite frequency of review and adjustment, but more frequent review.
  ratio of observation/menu cost 1-to-1 to frequency adjustment/review
Frequency of price reviews and price changes

Industry year averages symbols, country colors (Source ECB)