Optimal Monetary Policy with Heterogeneous Agents

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How does household heterogeneity affect the optimal conduct of monetary policy?

Emerging positive literature about the redistributive effects of monetary policy in incomplete-markets models with non-trivial heterogeneity


Less progress on the normative front: the entire wealth distribution is a state in the policy-maker’s problem

This paper: we solve the optimal monetary policy with commitment in a model with non-trivial heterogeneity
The economy in a nutshell
An open-economy à la Huggett with nominal, long-term, non-contingent financial assets

- Incomplete markets economy à la Huggett (1993)
  - Nominal, long-term, non-contingent financial assets

- Small open-economy with risk-neutral foreign investors

- Disutility costs of inflation (nominal rigidities)
Transmission channels of monetary policy

- **Redistributive channels:**

  1. **Fisherian:** current inflation redistributes wealth from creditors (positive net nominal positions, $\text{NNP} > 0$) to debtors ($\text{NNP} < 0$)

  2. **Unhedged interest rate exposure (URE):** Households with net financing needs ($\text{URE} < 0$) benefit from low future inflation ($\rightarrow$ higher bond price today) at the expense of bond-purchasing households ($\text{URE} > 0$)

- **Costly inflation** (due to nominal rigidities)
Optimal monetary policy features inflation front-loading
Reflecting the interaction of Fisherian-driven ‘redistributive inflationary bias’ vs URE-driven disinflationary force.

\[
\hat{x}'(\pi_t) = \mathbb{E}_t(a,y) \begin{bmatrix}
\text{net liabilities} & \text{marginal utility of cons.} \\
(-) NNt(a,y) & u'(c_t(a,y))
\end{bmatrix} + \mu_t Q_t,
\]

\[
\mu_t = -\int_0^t e^{-\int_s^t(\bar{r}+\pi_z+\delta-\rho)dz} \frac{1}{Q_s} \mathbb{E}_s(a,y) \begin{bmatrix}
\text{net financing needs} \\
(-) URE_s(a,y)u'(c_s(a,y))
\end{bmatrix} ds,
\]

Under certain conditions, both redistributive motives cancel out asymptotically: zero optimal long-run inflation
Related literature

1. Ramsey policies in incomplete-market models in which the policy-maker does not need to keep track of the wealth distribution or the latter is finite-dimensional

2. Ex-ante parametric form for the optimal policy + numerical optimization
   - Dyrda and Pedroni (2018) and Itskhoki and Moll (2019)

3. Finite dimensional Lagrangian methods
   - Bhandari et al. (2018), Açikgöz et al. (2018)

4. Infinite-dimensional calculus in problems with non-degenerate distributions
   - Davila et al. (2012), Lucas and Moll (2014) and Nuño and Moll (2018)
A heterogeneous-agent model with long-term nominal debt
Model: output and prices

- Household $k \in [0, 1]$ is **endowed** with $y_{kt}$ units of output
  - $y_{kt}$ follows a 2-state Poisson process, $y_1 < y_2$, with intensities $\lambda_1$ and $\lambda_2$

- Domestic price level $P_t$ follows

\[
dP_t = \pi_t P_t dt.\]
Assets

- **Long-term bond** issued at time $t$ pays stream of geometrically-decaying *nominal* coupons $\left\{ \delta e^{-\delta(s-t)} \right\}_{s \geq t}$

- **Nominal face value** of net wealth follows

  $$dA_{kt} = (A_{kt}^{\text{new}} - \delta A_{kt}) \, dt.$$ 

- **Budget constraint**

  $$Q_t A_{kt}^{\text{new}} = P_t (y_{kt} - c_{kt}) + \delta A_{kt}.$$ 

- **Define** $a_{kt} \equiv A_{kt}/P_t$: *real* face value of net wealth
Household problem

Household solves

\[ v_t(a, y) = \max_{\{c_s\}_{s \in [t, \infty)}} \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \left[ u(c_{ks}) - x(\pi_s) \right] ds \]

subject to

\[ \dot{a}_{kt} = s_t(a_k, y_k) = \frac{1}{Q_t} \left[ \underbrace{(y_{kt} - c_{kt} + \delta a_{kt})}_{UR\tilde{E}_t(a, y)} - (\delta + \pi_s) \underbrace{Q_t a_{kt}}_{NNP_t(a, y)} \right], \]

and exogenous borrowing limit

\[ a_{kt} \geq \phi, \quad \phi \leq 0. \]
International investors (bond pricing)

- Risk-neutral investors can invest elsewhere at riskless real rate $\bar{r}$
- Unit price of the nominal non-contingent bond

$$Q(t) = \int_{t}^{\infty} \delta e^{-(\bar{r}+\delta)(s-t)} - \int_{t}^{s} \pi_s du \, ds.$$
Density $f_{it}(a) \equiv f_t(a, y_i)$ dynamics given by Kolmogorov Forward equation

$$\frac{\partial f_{it}(a)}{\partial t} = -\frac{\partial}{\partial a} [s_{it}(a) f_{it}(a)] - \lambda_i f_{it}(a) + \lambda_j f_{jt}(a),$$

$i, j = 1, 2, j \neq i$. 

Dynamics of the income-wealth density
Central bank

- Central bank can trade a short-term nominal claim with foreign investors
  - It sets the instantaneous nominal rate $R_t$ of that facility. By no-arbitrage: $R_t = \bar{r} + \pi_t$.
  - Equivalent to assume that the central bank chooses directly the inflation rate $\{\pi_t\}_{t \geq 0}$

- Central bank’s utilitarian welfare criterion

$$U_{0}^{CB} \equiv \sum_{i=1}^{2} \int_{\Phi} \infty v_0 (a, y_i) f_0 (a, y_i) \, da$$

$$= \mathbb{E}_{f_0(a, y)} [v_0 (a, y)]$$

$$= \int_{0}^{\infty} e^{-\rho t} \mathbb{E}_{f_t(a, y)} [u (c_t (a, y), \pi_t)] \, dt.$$
Optimal Monetary Policy
The Ramsey problem is

\[ J^R [f_0 (\cdot)] = \max_{\{\pi_t\}_{t\geq 0}} U^C_{0B} \]

subject to
- the law of motion of the distribution
- the bond pricing equation
- the individual HJB equation
- the first-order condition of households

\[ J^R \text{ and } \pi \text{ are not ordinary functions, but functionals as they map a distribution } f_t (\cdot) \text{ into } \mathbb{R} \]

The problem is time-inconsistent
How can we solve it?

▶ We construct a functional Lagragian $\mathcal{L}_0 [f, \pi, Q, \nu, c]$

▶ This is a problem of constrained optimization in an infinite-dimensional Hilbert space $\rightarrow$ Gateaux derivative:

▶ Example, the Gâteaux derivative with respect to density $f$ is

$$\lim_{\alpha \to 0} \frac{\mathcal{L}_0 [f + \alpha h, \pi, Q, \nu, c] - \mathcal{L}_0 [f, \pi, Q, \nu, c]}{\alpha}$$

where $h$ is an arbitrary function in the same function space as $f$. 
Optimal inflation

\[ \chi'(\pi_t) = \underbrace{\text{Domestic Fisherian motive}}_{\text{COV}_{f_t(a,y)} [-NNP_t(a,y), MUC_t(a,y)]} + \underbrace{\text{Cross-border Fisherian motive}}_{E_{f_t(a,y)} [-NNP_t(a,y)] E_{f_t(a,y)} [MUC_t(a,y)]} + \mu_t Q_t, \]

and

\[ \mu_t = \int_0^t e^{-\int_s^t (\bar{r} + \pi_z + \delta - \rho) dz} \frac{1}{Q_s} \left\{ \underbrace{\text{Domestic interest rate exposure motive}}_{\text{COV}_{f_s(a,y)} [URE_s(a,y), MUC_s(a,y)]} + \underbrace{\text{Cross-border interest rate exposure motive}}_{E_{f_s(a,y)} [URE_s(a,y)] E_{f_s(a,y)} [MUC_s(a,y)]} \right\} ds, \]

where \( MUC_t(a,y) \equiv u'(c_t(a,y)) \) denotes the marginal utility of consumption.
Redistributive inflationary bias at time 0

- Provided the aggregate NNP is non-positive,
  \[ \mathbb{E}_{f_0(a,y)} \left[ -NNP_0(a,y) \right] / Q_0 = -\bar{a}_0 \geq 0, \]
  optimal inflation at time-0 is strictly positive, \( \pi_0 > 0 \).

  - Even if economy’s aggregate NNP is zero, as long as \( u''(c) < 0 \) (concave preferences) and there is net wealth dispersion, the central bank has a reason to inflate.

  - Different from classical ‘inflationary bias’ in NK models.
Optimal long-run inflation

In the limit as $\rho \to \bar{r}$, the optimal steady-state inflation rate under commitment tends to zero: $\lim_{\rho \to \bar{r}} \pi_\infty = 0$.

- Reminiscent of same result in NK models, but very different reason (two counteracting redistributive motives).
Numerical analysis
Numerical solution and calibration

- Given an initial distribution, the model is solved using finite difference methods as in Achdou et al. (2017)

- Calibrate to a prototypical European small open economy, time unit = 1 year

- \( u(c) = \log(c) \), \( x(\pi) = \frac{\psi}{2}\pi^2 \) (Rotemberg pricing)

- \( f_0(\cdot) = f_{ss}(\cdot) \) under \( \pi = 0 \)

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<th>Description</th>
<th>Source/Target</th>
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<td>world real interest rate</td>
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<td>( \psi )</td>
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<td>( \phi )</td>
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Time-0 optimal policy

Figure: Transitional dynamics.
Understanding the redistributive motives

Figure: Equilibrium objects $t = 0$. 
Redistributive effects of optimal inflation

Figure: Consumption density at time 0.
Welfare analysis

Aggregate welfare is defined as

$$E_{f_0(a,y)}[v_0(a,y)] = \int_0^\infty e^{-\rho t} E_{f_t(a,y)}[u(c_t(a,y)) - x(\pi_t)] \, dt \equiv W[c].$$

Welfare losses of a zero-inflation policy relative to the optimal commitment

<table>
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<th>Economy-wide</th>
<th>Lending HHs</th>
<th>Indebted HHs</th>
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<td>0.05</td>
<td>-0.17</td>
<td>0.22</td>
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Note: welfare losses are expressed as a % of permanent consumption
The importance of debt duration

Figure: Optimal inflation under different debt durations.
Let individual income now be given by \( \{y_1 Y_t, y_2 Y_t\} \), with

\[
dY_t = \eta_Y (1 - Y_t) \, dt + \sigma \, dZ_t,
\]

where \( Z_t \) a Brownian motion (\( \eta_Y = 0.5, \sigma = 0.01 \)).

Boppart, Krusell and Mittman (2018) show how this is equivalent to analyze an MIT shock with amplitude \( \sigma \).

We consider policy 'from a timeless perspective', in the sense of Woodford (2003).

- the initial wealth distribution is the stationary distribution implied by the optimal commitment \( f_0 (\cdot) = f_\infty (\cdot) \).
- the initial condition \( \mu_0 = 0 \) is replaced by \( \mu_0 = \mu_\infty \).
Optimal response to a negative TFP shock

Figure: Generalized impulse response function of an aggregate income shock.
Conclusions

- Analyze **optimal monetary policy** in economies with nontrivial household heterogeneity
- Uncover **redistributional** motives that drive optimal inflation
  - **Inflationary bias**: the central bank gives more weight to debtors (NNP<0) as they have larger marginal utility of consumption (MUC)
  - **Inflation front-loading**: avoid inflation expectations from hurting issuers of new bonds (URE<0) also with higher MUC
- **Technical contribution**: Novel methodology based on infinite-dimensional calculus