Reconsidering the Microeconomic Foundations of Price-Setting Behavior

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Abstract

Although the Dixit-Stiglitz aggregator is the workhorse specification of monopolistic competition, this framework and related variants are fundamentally inconsistent with a key stylized fact from empirical studies of consumer behavior and product marketing, namely, that the price elasticity of demand for a given brand is primarily determined by the extensive margin (i.e., changes in the number of customers purchasing that product) rather than the intensive margin (i.e., changes in the specific quantity purchased by each individual customer). In this paper, we analyze household scanner data to confirm the salient empirical results, and we then proceed to formulate a new dynamic general equilibrium framework that captures both the intensive and extensive margins of demand. Our theoretical framework involves a two-dimensional product space and incomplete household information, giving rise to an equilibrium price distribution with customer search. Assuming uniform distributions for the individual-specific search costs and for the firm-specific productivity shocks, we obtain analytical expressions for the equilibrium price distribution and the optimal price-setting behavior of each producer, and we show that the implications of the model are consistent with the key stylized facts. Finally, we discuss how this new approach holds substantial promise for future research on price-setting behavior, not only in enhancing the linkages between micro data and macro models but in building stronger connections to ongoing research in marketing and consumer behavior.

JEL classification: E30; E31; E32

Keywords: Customer Search; Product Differentiation; Extensive and Intensive Margin; Quasi-Kinked Demand Curve

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1 Introduction

Over the past several decades, monopolistic competition has played a crucial role in pushing out the frontiers of analytical and empirical work across a wide range of economic fields. In nearly all of those studies, monopolistic competition has been represented in terms of the household preference aggregator introduced by Dixit and Stiglitz (1977) or more recent variants.\footnote{Such variants include the generalized preference aggregator introduced by Kimball (1995) and the deep habits formulation of Ravn, Schmitt-Grohe, and Uribe (2007).} In particular, the Dixit-Stiglitz aggregator has been a key building block in the development of New Keynesian economics; prominent examples include Rotemberg (1982), Blanchard and Kiyotaki (1987), Dornbusch (1987), Benhabib and Farmer (1994), Rotemberg and Woodford (1997, 1999), McCallum and Nelson (1999), and Khan, King, and Wolman (2003).\footnote{See also Yun (1996) and Erceg, Henderson, and Levin (2000).} This specification has also been used in a number of seminal studies in international trade, endogenous growth, and economic geography; see Krugman (1980, 1991), Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Bernard et al. (2003), and Melitz (2003).

In recent years, the increased accessibility of highly disaggregated economic data has contributed to a growing interest in developing models of price-setting behavior that can provide closer links between the theory and the empirical evidence.\footnote{See Bils and Klenow (2003), Klenow and Kryvstov (2008), and Nakamura and Steinsson (2008).} In macroeconomics, of course, that interest also reflects the ongoing quest to address the Lucas (1976) critique by building models with sufficiently deep micro foundations that are reasonably invariant to changes in the policy regime; moreover, the specific characteristics of the micro foundations can turn out to be crucial in determining the features of the welfare-maximizing policy and in assessing the welfare costs of alternative regimes.\footnote{See Levin et al. (2005), Schmitt-Grohe and Uribe (2005), Levin, Lopez-Salido and Yun (2007), and Levin et al. (2008).} Even in very recent studies, however, the Dixit-Stiglitz aggregator has continued to serve as the workhorse specification of monopolistic competition; see Broda and Weinstein (2006), Golosov and Lucas (2007), Klenow and Willis (2007), Gertler and Leahy (2008), and Mackowiack and Wiederholt (2008a,b).

Nevertheless, the Dixit-Stiglitz specification and related variants are fundamentally inconsistent with a key stylized fact from empirical studies of consumer behavior and product marketing, namely, that the price elasticity of demand for a given brand is primarily determined by the extensive margin (i.e., changes in the number of customers purchasing...
that product) rather than the intensive margin (i.e., changes in the specific quantity purchased by each individual customer). Indeed, the extensive margin is completely absent from the Dixit-Stiglitz framework, which assumes that every household purchases output from every producer and hence that the elasticity of demand faced by the firm is identical to the own-price demand elasticity of households. Moreover, this framework imposes a very tight link between the steady-state demand elasticity and the markup of price over marginal cost; thus, given the available evidence that average price markups generally fall in the range of 5 to 25 percent, the Dixit-Stiglitz specification is typically calibrated using a demand elasticity of about -5 to -20. In contrast, the empirical literature on consumer demand, including seminal studies by Stone (1954), Theil (1965), and Deaton and Muellbauer (1980), has consistently obtained very low estimates (around unity or below) for households own-price elasticity of demand, even for extremely narrow product categories. At the same time, the empirical marketing literature has documented the extent to which the elasticity of demand for a firms product mainly depends on consumers choice of brand within that product category.

In this paper, we analyze household scanner data to confirm the key set of stylized facts, and we then proceed to formulate a new dynamic general equilibrium framework that captures both the intensive and extensive margins of demand. Our empirical analysis documents that consumers typically only purchase a single brand within each narrow product category. We also document that the own-price elasticity of household demand is quite low compared with the demand elasticity for each individual brand within that product category.

Our theoretical framework involves a two-dimensional product space and incomplete household information, giving rise to an equilibrium price distribution with customer search. The first dimension of the product space represents the set of narrow categories of consumer goods and services—such as bathsoap, breakfast cereal, toothpaste, haircuts, and automotive repairs—and the second dimension represents the set of nearly-identical producers within each category. Moreover, consistent with the typical pattern observed in the micro data, we assume that each household purchases items from only a single producer within each product category. To motivate an equilibrium with customer search over a non-trivial distribution of prices, the firms within each category are assumed to face idiosyncratic productivity shocks; the households are assumed to have full knowledge of this distribution but do not have any ex ante knowledge of the characteristics of individual producers and hence have an intrinsic search motive. There is a continuum of households,
and each household is comprised of many individual members; each member is responsible for searching across the producers within a single product category, and the fixed cost per search is assumed to vary randomly across the members of the household.

By assuming uniform distributions for the individual-specific search costs and for the firm-specific productivity shocks, we obtain analytical expressions for the equilibrium price distribution and the optimal price-setting behavior of each producer. We show that each firm faces a continuous downward-sloping demand curve, where the steady-state elasticity of demand can be represented as the sum of the elasticity at the intensive margin (which is determined by the households own-price elasticity of demand for that product category) and the elasticity at the extensive margin (which is determined by the distribution of search costs). Using an empirical reasonable calibration, this framework can readily match the low elasticity of demand on the part of households along with the relatively high demand elasticity faced by each producer. Furthermore, the extensive margin of demand implies that each producer faces a quasi-kinked demand curve, where the degree of curvature is roughly consistent with the mode of the distribution of recent estimates from firm-level scanner data; cf. Dossche et al. (2006). Finally, we discuss how this new approach holds substantial promise for future research on price-setting behavior, not only in enhancing the linkages between micro data and macro models but in building stronger connections to ongoing research in marketing and consumer behavior.

Although the analysis of customer search has been relatively quiescent in recent years, our investigation builds directly on the earlier work of MacMinn (1980), who analyzed sequential search in a partial-equilibrium setting with firm-specific idiosyncratic shocks, heterogenous search costs, and completely inelastic demand by each consumer (and hence no intensive margin). Moreover, the role of customer search in generating a kinked or quasi-kinked demand curve for each firm was emphasized by Stiglitz (1979, 1987). Finally, a large but now-dormant literature studied the incidence of relative price dispersion in search models with nominal rigidities.

Our analysis is also reminiscent of the burgeoning literature on the role of search in labor markets, although that literature has mainly focused on the extensive margin of demand. In the case of customer search, of course, there are no direct parallels to the incidence of unfilled job vacancies or of workers who remain unemployed while searching for a new job; hence, specifying an aggregate matching function for customers and firms might

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5The need for such connections has recently been emphasized by Rotemberg (2008).
provide a useful approximation for analyzing inflation dynamics, as in Hall (2008), but might not be as useful for interpreting micro evidence on price-setting behavior. Finally, while the assumption of an exogenous separation rate may be useful for interpreting U.S. labor market data over the past couple of decades, specifications that incorporate on-the-job search and endogenous separation may well have greater potential for cross-fertilization with the analysis of customer search.\footnote{For recent analysis of labor market separation patterns, see Shimer (2005), Hall (2006), and Davis et al. (2006).}

The remainder of this paper is organized as follows. Section 2 analyzes household scanner data. Section 3 specifies our analytical framework. Section 4 concludes.

2 Stylized Facts from Household Scanner Data

In this section, we confirm the key stylized fact that the price elasticity of demand for a given brand is primarily determined by changes in the number of customers rather than changes in the specific quantity purchased by each individual.

2.1 Brand Choices of Households in Narrow Product Categories

In this section, we use ERIM data sets from A.C. Nielsen. The ERIM data set tracks information for each UPC at several stores in two markets between January 1985 and June 1987 for narrowly defined product categories such as ketchup, canned tuna, peanut butter, stick margarine and toilet tissue.

Our interest is to see the following stylized facts in the Erim data sets.

- Each consumer faces a multiplicity of producers (brands) of each specific good/service, but typically purchases the item from a single producer (e.g., haircuts, toothpaste).

- Household expenditures generally exhibit own-price elasticities of demand near/below unity, even at extremely narrow levels of disaggregation (e.g., haircuts, toothpaste).

- Firms face relatively high demand elasticities (consistent with low markups), reflecting the dominant role of the extensive margin (number of customers) rather than the intensive margin (quantity purchased by each customer).
Figure 1: Consumer Choices in Narrow Product Categories

Note: this figure shows how many different brands households on average each month between January 1985 and June 1987.

In order to see the first stylized fact, we investigate how frequently a typical household changes producers each month for narrow product categories. We can see from the A.C. Nielsen ERIM dataset (Chicago GSB) that a wide diversity of brands tends to exist in narrow product categories. For example, creamy peanut butter includes a variety of brands such as Arrowhead, Peter Pan, Billy Boy, Robb Ross, Elam’s, Skippy, Hallam’s, Smucker’s, Home Brand, Sun Gold, JIF, Superman, and 26 chain-specific ”private label” brands. Nevertheless, as shown in Figure 1, more than 90 percent of households who purchased creamy and chunky peanut butters 18 oz buy the item from a single producer. Figure 2 shows essentially the same brand choice behavior of households for ketchup 32 oz and tuna 6.5 oz as well.

2.2 Demand Equations for Brands

The ERIM data suggest that firms face relatively high demand elasticities, consistent with low markups. In order to show this, we follow the approach of Hausman, Leonard and Zona (1994) that takes account of a three stage demand system in estimating demand for
Figure 2: Consumer Choices in Narrow Product Categories

Note: this figure shows how many different brands households on average each month between January 1985 and June 1987.

differentiated products. The top level correspond to overall demand for the product such as beer. The middle level corresponds different segments for the product. The bottom level of the demand system corresponds to competition among brands in a given segment.

In particular, we are interested in the bottom level of the demand system. The reason for this is that this elasticity measures the price elasticity within a narrow product category and thus reflects changes in the demand due to changes in the number of customers. For each brand within the market segment, the demand specification is

\[ s_{int} = \alpha_i + \omega_i d_{nt} + \beta_i \log(y_{Gnt}/P_{nt}) + \sum_{j=1}^{J} \gamma_{ij} \log p_{jnt} + \epsilon_{int} \]

where \( s_{int} \) is the revenues share of total segment expenditure of the \( i \)th brand in city \( n \) in period \( t \), \( y_{Gnt} \) is overall segment expenditure, \( P_{nt} \) is the price index and \( p_{jnt} \) is the \( j \)th brand in city \( n \). We use this specification of demand for creamy and chunky peanut butter. For the creamy peanut butter, the conditional own elasticities are -3.2 to -7. For example, JIF has a conditional own elasticity of -5.16, Peter Pan is -3.21 and Skippy is -
7.01. Similarly, estimates of the chunky peanut butter covers a range of -3.7 and 6.1. As a result, the ERIM data indicate that firms face relatively high demand elasticities consistent with low markups. This result is also consistent with elasticity estimates for beer shown in Hausman, Leonard and Zona (1994). For example, the conditional own-price elasticities are in the range of -3.5 to -5.0 for the premium beer.
Table 1: Brand Share Equations - Creamy Peanut Butter 18 oz.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JIF</td>
<td>Peter Pan</td>
<td>Skippy</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.02</td>
<td>0.34</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.29)</td>
<td>(0.31)</td>
</tr>
<tr>
<td>log(Y/P)</td>
<td>0.05</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>log(P_{JIF})</td>
<td>-0.93</td>
<td>0.74</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.40)</td>
<td>(0.43)</td>
</tr>
<tr>
<td>log(P_{Peter Pan})</td>
<td>0.22</td>
<td>-0.81</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.19)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>log(P_{Skippy})</td>
<td>0.63</td>
<td>0.26</td>
<td>-1.13</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.27)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>City Dummy</td>
<td>0.01</td>
<td>-0.006</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Own</td>
<td>-5.16</td>
<td>-3.21</td>
<td>-7.01</td>
</tr>
<tr>
<td>Price Elasticity</td>
<td>(1.29)</td>
<td>(0.54)</td>
<td>(1.56)</td>
</tr>
</tbody>
</table>

Note: The regression equation for each brand share implies that the own-price elasticity (=\(\epsilon_i\)) and the own-price coefficient (\(\beta_i\)) in the regression equation for brand \(i\) should satisfy \(\epsilon_i = \beta_i \left( \frac{1}{T} \sum_{t=1}^{T} s_{it}^{-1} \right) - 1\), where \(s_{it}\) is the expenditure share of brand \(i\) at period \(t\). The controlled includes controlled brands and other small brands. The numbers in parenthesis are standard errors.
Table 2: Brand Share Equations - Chunky Peanut Butter 18 oz.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>JIF</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.21</td>
<td>0.40</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.32)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>log(Y/P)</td>
<td>0.004</td>
<td>-0.06</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>log(PJIF)</td>
<td>-0.92</td>
<td>1.21</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.43)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>log(PPeter Pan)</td>
<td>0.54</td>
<td>-0.97</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.30)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>log(PSkippy)</td>
<td>0.50</td>
<td>0.36</td>
<td>-1.20</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.28)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>City Dummy</td>
<td>0.04</td>
<td>0.07</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Own</td>
<td>-5.84</td>
<td>-3.71</td>
<td>-6.11</td>
</tr>
<tr>
<td>Price Elasticity</td>
<td>(1.29)</td>
<td>(0.54)</td>
<td>(1.56)</td>
</tr>
</tbody>
</table>

Note: The regression equation for each brand share implies that the own-price elasticity ($\epsilon_i$) and the own-price coefficient ($\beta_i$) in the regression equation for brand $i$ should satisfy $\epsilon_i = \beta_i((1/T) \sum_{t=1}^{T} s_{it}^{-1}) - 1$, where $s_{it}$ is the expenditure share of brand $i$ at period $t$. The controlled includes controlled brands and other small brands. The numbers in parenthesis are standard errors.
Table 3: Price Elasticities for Individual Brands of Beer

<table>
<thead>
<tr>
<th>Beer Brand</th>
<th>Conditional on Expenditures</th>
<th>Total Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budweiser</td>
<td>-3.5 (0.1)</td>
<td>-4.2 (0.1)</td>
</tr>
<tr>
<td>Molson</td>
<td>-5.1 (0.2)</td>
<td>-5.4 (0.2)</td>
</tr>
<tr>
<td>Labatts</td>
<td>-4.3 (0.3)</td>
<td>-4.6 (0.3)</td>
</tr>
<tr>
<td>Miller</td>
<td>-4.2 (0.3)</td>
<td>-4.5 (0.2)</td>
</tr>
<tr>
<td>Coors</td>
<td>-4.6 (0.2)</td>
<td>-4.9 (0.2)</td>
</tr>
<tr>
<td>Old Milwaukee</td>
<td>-4.8 (0.1)</td>
<td>-5.3 (0.1)</td>
</tr>
<tr>
<td>Genesee</td>
<td>-3.8 (0.1)</td>
<td>-4.2 (0.1)</td>
</tr>
<tr>
<td>Milwaukee’s Best</td>
<td>-5.8 (0.2)</td>
<td>-6.2 (0.2)</td>
</tr>
<tr>
<td>Busch</td>
<td>-5.7 (0.3)</td>
<td>-6.1 (0.3)</td>
</tr>
<tr>
<td>Piels</td>
<td>-4.0 (0.5)</td>
<td>-4.1 (0.5)</td>
</tr>
<tr>
<td>Genesee Light</td>
<td>-3.2 (0.1)</td>
<td>-3.8 (0.1)</td>
</tr>
<tr>
<td>Coors Light</td>
<td>-4.2 (0.1)</td>
<td>-4.6 (0.1)</td>
</tr>
<tr>
<td>Old Milwaukee Light</td>
<td>-5.9 (0.1)</td>
<td>-6.1 (0.1)</td>
</tr>
<tr>
<td>Lite</td>
<td>-4.8 (0.1)</td>
<td>-5.0 (0.1)</td>
</tr>
<tr>
<td>Molson Light</td>
<td>-5.7 (0.2)</td>
<td>-5.8 (0.2)</td>
</tr>
</tbody>
</table>

Note: The estimates of price elasticities for each beer brand are taken from Jerry Hausman, Gregory Leonard, and J. Douglas Zona (1994).
3 Consumer Search and Quasi-Kinked Demand Curve

In this section, we derive a quasi-kinked demand curve from the optimizing behavior of consumers when they have imperfect information about location of different prices. It is shown that the demand curve facing an individual firm depends on not only the purchasing behavior of individual households but also their search behavior.

3.1 Economic environment

The economy is populated by a lot of infinitely-lived households, while different types of goods are produced and sold in different islands. Thus, members of each household visit different islands to purchase different goods. Labor services are traded in a perfectly competitive labor market and wages are fully flexible. Moreover, production of each firm in an island is subject to idiosyncratic shocks whose distribution is identically and independently distributed across islands and over time. The presence of such shocks creates a non-degenerate price distribution in each type of different goods.

Consumers do not know realized values of productivity shocks that hit individual firms. A non-degenerate price distribution then gives households incentive to find a seller with the lowest price for each type of goods, while a continuum of firms, indexed by \([0, 1]\), produce the same type of goods. Although the number of search is not limited, we assume that any visit to a particular seller requires a fixed cost. Specifically, each visit to a seller incurs a fixed nominal amount of \(z_{jt} = z_j L_t\) for type \(j\) goods where \(L_t\) is the total cost of purchasing goods and \(Z_j(z_j)\) be the distribution function for search cost. For simplicity, we assume that search costs are uniformly distributed on \([z_j, \bar{z}_j]\), so that \(Z_j(z_j) = (z_j - z_j)/(\bar{z}_j - z_j)\).

Furthermore, we assume that members of individual household do not communicate each other while they visit different islands. Although this assumption may be rather restrictive, it leads consumers to follow a simple reservation-price strategy for each type of goods even when consumers are supposed to search a lot of different goods in each period. Specifically, search continues until each consumer finds a seller who quote price at or below his or her reservation price for each type of goods.

Finally, a fraction of firms can have their prices grater than the maximum reservation price of consumers if their productivity shocks are very low. In this case, firms that undergo these situations are assumed to shut down their production activities temporarily until their prices go back within the range of reservation prices of consumers.
3.2 Household Optimization

Each period is divided into two sub-periods. The first-half is search stage and the second half is spending stage. In the spending stage, households make actual purchases of goods after they have decided on which sellers they trade with. The level of actual spending is determined as a result of utility optimization. Specifically, households choose the use of time and the level of consumption to maximize their utilities and then their searches for sellers help to minimize the cost of maintaining the optimized level of consumption without deterring the efficient use of time.

3.2.1 Decisions on Consumption Spending and Use of Time

The preference of each household at period 0 is given by

\[ \sum_{t=0}^{\infty} E_0[U(C_t, \bar{H} - H_t)], \]  

where \( C_t \) is the consumption at period \( t \), \( \bar{H} \) is the amount of time endowment available for each household, \( H_t \) is the amount of hours worked at period \( t \). The instantaneous utility function \( U(C_t, \bar{H} - H_t) \) is continuously twice differentiable and concave in consumption and leisure.

Households aggregate differentiated goods to produce composite goods using the Dixit-Stiglitz aggregator. Specifically, composite goods are produced by the following Dixit-Stiglitz type aggregator:

\[ C_t = \left( \int_{j=0}^{1} C_{jt}(i)^{\theta-1} \, dj \right)^{\frac{\theta}{\theta-1}}, \]  

where \( C_t \) is the real amount of the composite goods and \( C_{jt}(i) \) is the amount of type \( j \) goods that each household purchases from a seller \( i \).

We also assume that there is a complete financial market in which all agents trade contingent claims. In addition, wages are fully flexible in a perfectly competitive market. Given these assumptions, the period budget constraint of each household can be written as

\[ \int_{0}^{1} (P_{jt}(i)C_{jt}(i) + z_{jt}X_{jt}) \, dj + E_t[Q_{t,t+1}B_{t+1}] \leq W_t^N H_t + B_t + \Phi_t, \]  

where \( P_{jt}(i) \) is the dollar price at period \( t \) of good \( j \) at seller \( i \), \( X_{jt} \) is the number of search that an individual household has made in order to determine a seller \( i \), \( z_{jt} \) is the nominal cost of each visit to a seller, \( Q_{t,t+1} \) is the stochastic discount factor used for computing
the dollar value at period \( t \) of one dollar at period \( t + 1 \), \( W_t^N \) is the nominal wage rate, and \( \Phi_t \) is the dividend distributed to households.

The demand of each different type of goods is then determined by solving a cost-minimization of the form:

\[
\min \{ \int_{j=0}^{1} P_j(i) C_{jt}(i) dj + \Lambda_t\{C_t - (\int_{j=0}^{1} C_{jt}(i)^{\theta-1} dj)^{\frac{\theta}{\theta-1}}) \} \},
\]

where \( P_j(i) \) is the dollar price at period \( t \) of good \( j \) at seller \( i \) and \( \Lambda_t \) is the Lagrange multiplier of this cost minimization. As a result of this cost-minimization, the demand curve facing a seller \( i \) that sells type \( j \) goods can be written as

\[
C_{jt}(i) = (P_j(i)/\Lambda_t)^{-\theta} C_t.
\]

The cost-minimization also implies that the Lagrange multiplier can be written as

\[
\Lambda_t = (\int_{j=0}^{1} P_j(i)^{1-\theta} dj)^{\frac{1}{1-\theta}}.
\]

In addition, letting \( L_t(\{P_j(i)\}) = \int_{j=0}^{1} P_j(i) C_{jt}(i) dj \) denote the nominal consumption expenditures of each household, we can see that the following equation holds:

\[
L_t(\{P_j(i)\}) = \Lambda_t C_t.
\]

It then follows from this equation that the specification of the period budget constraint described above is consistent with the Dixit-Stiglitz type aggregator. The demand function for individual goods as well as optimization condition of households have been widely used in much of the recent macro-economic literature that allows for monopolistic competition in goods markets.

Furthermore we can rewrite the nominal flow budget constraint of the household as follows:

\[
\Lambda_t C_t + \int_{0}^{1} z_j t X_j t dj + E_t[Q_{t,t+1} B_{t+1}] \leq W_t^N H_t + B_t + \Phi_t.
\]

As a result, the utility maximization of each household leads to the following optimization conditions:

\[
U_2(C_t, \bar{H} - H_t) = (W_t^N / \Lambda_t)U_1(C_t, \bar{H} - H_t),
\]

\[
Q_{t,t+1} = \beta \frac{U_1(C_{t+1}, \bar{H} - H_{t+1}) \Lambda_t}{U_1(C_t, \bar{H} - H_t) \Lambda_{t+1}},
\]

where \( U_1(C_t, \bar{H} - H_t) \) is the marginal utility of consumption and \( U_2(C_t, \bar{H} - H_t) \) is the marginal utility of leisure.
Finally, it should be noted that the demand function specified above is valid only under the condition that households do not change sellers. More precisely, as noted earlier, each household does not know exact locations of individual prices, though their true distribution is publicly known. Therefore, this demand function is valid after households finish their searches for the lowest price for each type of goods. In the next, we analyze the search behavior of each household under the assumption that each type of goods has the intensive margin demand curve specified above once household determines a seller for each type of differentiated goods.

3.2.2 Search Decision

In this section, we consider search behaviors of households. It is important to note that their search behaviors should be fully consistent with their spending decisions described above, though households complete their searches for the lowest price in the first sub-period. The reason for this is that households should not deviate from their decisions made at the the first sub-period when actual spending is carried out at the next stage.

In order to see this, it is necessary to demonstrate that households have incentive to find a seller that gives the lowest price for each type of goods, given that they purchase each type of goods according to the demand function specified in (5). In particular, notice that the cost-minimization of households in the spending period leads to the following consumption expenditure function: $L_t(\{P_{jt}(i)\}) = \Lambda_t C_t$. In addition to this, we point out that the partial derivative of the consumption expenditure function with respect to the relative price of an individual price is positive: $\partial L_t(\{P_{jt}(i)\})/\partial P_{jt}(i) = (P_{jt}(i)/\Lambda_t)^{-\theta} C_t$.

Hence, to the extent that $\theta$ is not very big and search does not require an arbitrary large amount of costs, households have incentive to search for sellers with the lowest price for each type of goods. As a result, the standard Dixit-Stiglitz model without search can be viewed as implicitly assuming that search costs are arbitrarily large so that no consumer wants to search.

**Household’s Information on Prices:** Having shown that individual households have incentive to search for the lowest price of each type of goods, we briefly discuss the information of each household on prices. Individual households are supposed to know the true distribution of nominal prices denoted by $F_{jt}(P_{jt})$, where $F_{jt}(P_{jt})$ represents the measure of firms that set their nominal prices equal to or below $P_{jt}$.

We assume that each individual household has a lot of shopping members. Since each
shopping member is supposed to search the lowest price for each type of goods, each individual household send a continuum of shopping members to each island. For example, a shopping member visits an island \( j \) in order to search for the lowest price for type \( j \) goods. In this case, observations on prices are independent random variables drawn from the true price distribution. After each observation, each shopping member decides to continue search or determine a particular seller, while a positive search cost limits the number of observations. But these shopping members do not communicate each other after they depart from their households until all of them determine a seller for each type of goods. Because of this assumption, households’ search decisions turn out to resemble the search process of one single product, though they purchase a continuum of goods at the same time.

**Derivation of the Objective Function of Search:** Notice that the nominal consumption expenditure function for each household at the spending stage is given \( L_t(\{P_{jt}(i)\}) = \Lambda_t C_t \). Since we assume that shopping members do not communicate each other, a shopping member’s behavior affects only a slice of the consumption expenditure function while the consumption expenditure function is defined in terms of integral. In order to formulate this feature, notice that the consumption expenditure function can be viewed as a function of \( P_{jt} \) if only nominal price of type \( j \) goods changes but all other prices are fixed at \( P_{st}(i) = P_{st} \) for all \( i \). We thus define a new function \( L(P_{jt}) \) reflecting this situation. Then, \( L(P_{jt}) \) is affected by a shopping member’s decision on the determination of a particular seller. In addition, \( z_{jt} = z_j \Lambda_t C_t \) is a realized level of nominal search cost for type \( j \) goods that a particular household should pay each time the household visits a seller at period \( t \).

**Determination of Reservation Prices:** We now explain how households determine reservation prices for their sequential searches, denoted by \( R_{jt}(z_j) \). We assume that individual households adopt a reservation price strategy for their searches. Specifically, shopping members stop searching for any price observation \( P_{jt} \leq R_{jt}(z_j) \), while for any \( P_{jt} \geq R_{jt}(z_j) \), they continue to search.

We describe the determination of reservation strategy in the context of dynamic programming. In order to do this, we let \( V_j(P_{jt}) \) represent the value function of search cost for each type of goods when the relative price of his or her first visit is \( P_{jt} \). Then, the optimization of a shopping member whose objective is to find a seller with the lowest price
can be written as
\[ V_j(P_{jt}) = \min \{ L_t(P_{jt}), z_j \Lambda_t C_t + \int_0^{R_{jt}(z_j)} V_j(K_{jt})dF(K_{jt}) \}. \] (11)

The reservation price then satisfies the following condition:
\[ z_j \Lambda_t^{1-\theta} = \int_0^{R_{jt}(z_j)} \{ P_{jt}^{-\theta} F_{jt}(P_{jt}) \} dP_{jt}. \] (12)

The left-hand side of (12) corresponds to the cost of an additional search, while the right-hand side is its expected benefit.\(^8\)

It would be worthwhile to discuss a couple of issues associated with the determination of reservation strategy discussed above. First, it is the case in the search literature that prices do not affect the real amount of goods that each customer purchases, which corresponds to setting \( \theta = 0 \). In this case, the reservation price for each level of \( z_j \) turns out to be
\[ z_j \Lambda_t = \int_0^{R_{jt}(z_j)} F_{jt}(P_{jt})dP_{jt}. \] (13)

Second, when the maximum reservation price is higher than the maximum of actual transaction prices, the reservation price equation specified above can be rewritten as follows:
\[ \bar{z}_j \Lambda_t^{1-\theta} = \frac{R_{jt}^{1-\theta}}{1-\theta} - \frac{P_{jt}^{1-\theta}}{1-\theta} + \int_{P_{jt}^{\min,j,t}}^{P_{jt}^{\max,j,t}} P_{jt}^{-\theta} F_{jt}(P_{jt})dP_{jt}. \] (14)

**Average Cost of Search:** Having specified the determination of reservation strategy, we now discuss the level of search cost that each individual household spends. In doing so, we begin with the assumption that a particular level of search cost is randomly assigned to each household for each type of differentiated goods at the beginning of each period. Meanwhile, a continuum of differentiated goods exists in the economy. We thus rely on the law of large numbers in order to make the expected level of total search cost identical across households.

We now discuss the level of total search cost that each individual household is expected to pay. In particular, we allow for the possibility that households can make infinite number of sequential search. It means that the expected number of search is \( 1/F(R_{jt}(z_j)) \) when the fraction of search cost for type \( j \) goods is \( z_j \). The expected nominal cost of search is

\(^8\)The expected benefit of an additional search is \( \int_0^{R_{jt}} (L(R_{jt}) - L(P_{jt}))dF_{jt}(P_{jt}) \). The reservation price for consumers whose search cost is \( z_j \) should satisfy \( z_j \Lambda_t^{1-\theta} C_t = \int_0^{R_{jt}} (L(R_{jt}) - L(P_{jt}))dF_{jt}(P_{jt}) \). The integration by parts then leads to the formula specified in (12). If the maximum reservation price is higher than the maximum transaction price, it should satisfy \( \bar{z}_j \Lambda_t^{1-\theta} C_t = L(R_{jt}) - L(P_{jt}^{max,j,t}) + \int_{P_{jt}^{min,j,t}}^{P_{jt}^{max,j,t}} L(P_{jt}^{max,j,t}) - L(P_{jt})dF_{jt}(P_{jt}) \).
therefore given by \( \{z_j/F(R_{jt}(z_j))\} \Lambda_t C_t \) when the real search cost for type \( j \) goods is \( z_{jt} = z_j \Lambda_t C_t \).

We also define the ex-ante expected search cost as the level of search cost, denoted by \( \Lambda e_{jt} \), which each household is expected to pay at the time point before realization of search cost. The aggregate expected search cost is then given by \( S_t = \int_{0}^{1} X_{e_{jt}} dz_j \). In order to compute \( X_{e_{jt}} \), notice that there exists a level of \( z_j \) denoted by \( z^*_{jt} \) such that when \( \bar{R}_{jt} > P_{\text{max},jt} \), \( z^*_{jt} \) satisfies

\[
z^*_{jt} \Lambda_t^{1-\theta} = \int_{P_{\text{min},jt}}^{P_{\text{max},jt}} \{P^{-\theta}F(P_{jt})\} dP_{jt},
\]

while \( z^*_{jt} = \bar{z}_j \), when \( \bar{R}_{jt} = P_{\text{max},jt} \). Given the definition of \( z^*_{jt} \), the ex-ante expected search cost can be written as

\[
X_{e_{jt}} = (z_j - \bar{z}_j)^{-1} \left\{ \int_{z^*_{jt}}^{z_j} z_j dz_j + \int_{z_j}^{z^*_{jt}} \frac{z_j}{F(R_t(z_j))} dz_j \right\}.
\]

### 3.3 Demand Curves of Individual Sellers

Having described the reservation strategy of households, we now move onto the discussion on the demand curve facing a seller whose nominal price is \( P_{jt} \). Since consumers use reservation price strategies, any consumers whose reservation prices are greater than \( P_{jt} \) are potential customers for the seller.

In order to derive a demand curve facing an individual seller whose price is \( P_{jt} \), it is necessary to compute the expected number of the seller’s potential customers. In order to do so, we choose a set of consumers whose relative price, denoted by \( R_{jt}(z_j) \), is equal to or greater than \( P_{jt} \). It is then important to note that consumers whose reservation price is \( R_{jt}(z_j) \) are randomly distributed to a set of sellers whose relative prices are less than \( R_{jt}(z_j) \). Moreover, given the uniform distribution of search cost, the measure of consumers whose reservation price is \( R_{jt}(z_j) \) is \((\bar{z}_j - z_j)^{-1}\). As a result, \((\bar{z}_j - z_j)^{-1}\{f_{jt}(P_{jt})/F(R_{jt}(z_j))\}\) is the measure of consumers with their reservation price \( R_{jt}(z_j) \), who are assigned to a group of sellers whose relative price is \( P_{jt} \) \((\leq R_{jt}(z_j))\), where \( f_{jt}(P_{jt}) \) is the measure at period \( t \) of sellers whose relative price is \( P_{jt} \). Since this matching process holds for any consumers whose reservation price is greater than \( P_{jt} \), the total expected number of consumers who purchase their products at a shop with price \( P_{jt} \) is

\[
\frac{1}{\bar{z}_j - z_j} \int_{P_{jt}}^{R_{jt}} \frac{f_{jt}(P_{jt})}{F(R_{jt}(z_j))} dz_j,
\]

where \( R_{jt} \) is the maximum reservation price at period \( t \).
Having specified the total expected number of consumers in terms of search-cost distribution, we express it in terms of price distribution. Specifically, the total differentiation of the optimal reservation price (12) and then the aggregation of the resulting equations over individual households yields the following relation between search cost and reservation price:

$$dz_j = \Lambda_1^{\theta-1} \{F_{jt}(R_{jt}(z_j))R_{jt}(z_j)^{-\theta}\}dR_{jt}(z_j).$$

Substituting this equation into (17) and solving the resulting integral, one can show that the expected total number of consumers for $P_{jt}$ can be written as

$$f_{jt}(P_{jt})\Lambda_1^{\theta-1}(\bar{z}_j - z_j)^{-1}(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})/(1-\theta).$$

Since $f_{jt}(P_{jt})$ is the measure of sellers who charge $P_{jt}$, it implies that the expected number of consumers for each individual seller with its relative price $P_{jt}$, denoted by $N_{jt}(P_{jt})$, is given by

$$N_{jt}(P_{jt}) = (\bar{z}_j - z_j)^{-1}(\bar{R}_{jt}^{1-\theta} - P_{jt}^{1-\theta})\Lambda_1^{\theta-1}. \quad (18)$$

Here, it should be noted that if the maximum reservation price is greater than the maximum of actual prices, an individual seller who sets the maximum of actual prices has a significantly positive measure of customers. But it is possible that a fraction of firms can have their prices grater than the maximum reservation price of consumers if their productivity shocks are very low. Then, firms that undergo these situations are assumed to shut down their production activities temporarily until their prices go back within the range of reservation prices of consumers.

Furthermore, it is necessary to show that the total expected number of consumers should be equal to one because the measure of households is set to one. It means that

$$\int_{\bar{R}_{jt}}^{R_{jt}} N_{jt}(P_{jt})f(P_{jt})dP_{jt} = 1,$$

where $R_{jt}$ is the minimum reservation price for type $j$ goods. In addition, the minimum reservation price satisfies the following equation:

$$\bar{z}_j\Lambda_1^{1-\theta} = \int_0^{\bar{R}_{jt}} \{P_{jt}^{-\theta}F_{jt}(P_{jt})\}dP_{jt}. \quad (19)$$

Subtracting this minimum reservation price equation from the maximum reservation price equation, we have

$$(\bar{z}_j - z_j)\Lambda_1^{1-\theta} = \int_{\bar{R}_{jt}}^{R_{jt}} \{P_{jt}^{-\theta}F_{jt}(P_{jt})\}dP_{jt}. \quad (20)$$

Consequently, we can see that $\int_{\bar{R}_{jt}}^{R_{jt}} N_{jt}(P_{jt})f(P_{jt})dP_{jt} = 1$.

Having derived the expected number of consumers for each seller, we now move onto the demand curve facing an individual seller. Before proceeding, we assume throughout the paper that after determining sellers in the search process, equation (5) determines the amount of goods that each consumer purchases, namely the intensive margin of total
demand. As shown before, the intensive margin depends on relative prices. It would be more convenient to express total demand in terms of relative price. In doing so, we deflate individual nominal prices by households’ marginal valuation on composite consumption goods, $\Lambda_t$, so that we denote the real price of $P_{jt}$ by $\tilde{P}_{jt}$. We now combine equations (5) and (18) are combined to yield

$$D_{jt}(\tilde{P}_{jt}) = \tilde{P}_{jt}^{1-\theta} (\tilde{R}_{jt}^{1-\theta} - \tilde{P}_{jt}^{1-\theta}) C_t/(z_j - \bar{z}_j),$$

where $D_{jt}(\tilde{P}_{jt})$ is the demand function at period $t$ when a seller sets its relative price at $\tilde{P}_{jt}$ and $\tilde{R}_{jt}$ is the relative price of the maximum reservation price.

An immediate implication of the demand curve (21) is that the elasticity of demand for each good, denoted by $\epsilon(P_{jt})$, depends on its relative price. The main reason for this is associated with the presence of the maximum relative price. In particular, the expected number of consumers turns out to be nil when relative price of each firm exceeds the maximum reservation price. Thus, the logarithm of the expected number of consumers is not linear in the logarithm of the relative price. Specifically, the elasticity of demand can be written as follows:

$$\epsilon(P_{jt}) = \theta + \frac{P_{jt}^{1-\theta}}{\tilde{R}_{jt}^{1-\theta}/(1 - \theta) - P_{jt}^{1-\theta}/(1 - \theta)}.$$

3.4 Discussion on Sources of Monopoly Powers of Firms

In order to see where the monopoly power of firms originates, we compute the value at which demand elasticities converge as idiosyncratic shocks have an arbitrarily small support.  

Before going further, the demand function specified above can be used to show that

---

9When there are both of intensive and extensive margins, the reservation price that is determined as a result of minimizing unit-cost plus search-cost may be the same as the reservation price that minimizes actual transaction cost plus search cost. In our paper, consumers choose a seller for each type of goods that maximizes indirect utility function after they solve their utility maximization problem. Each individual consumer then uses equation (5) to determine the amount that each consumer purchases. But one may wonder if the same consumption demand can be derived when consumers are allowed to solve the utility maximization problem after they choose their sellers. It does not change the functional form of demand function for each seller as specified in (5) because consumers take as given the list of prices posted by sellers.

10Specifically, when prices are fully flexible, the equilibrium distribution of prices degenerates in a symmetric equilibrium especially when there are no idiosyncratic elements among firms. It is therefore subject to the Diamond’s paradox. In order to avoid the Diamond’s paradox, one can include idiosyncratic cost shocks.
the following relation holds:
\[
\frac{(\bar{R}_{jt}/\Lambda_t)^{1-\theta}}{1 - \theta} - \frac{(P_{jt}/\Lambda_t)^{1-\theta}}{1 - \theta} = (\frac{(P_{jt}D_{jt})/(\Lambda_tC_t)}{(P_{jt}/\Lambda_t)^{-1}})(\frac{1}{\bar{z}_j - z_j}).
\] (23)

Substituting this equation into the elasticity of demand specified above, we have the following equation:
\[
\epsilon(P_{jt}) = \theta + \frac{(P_{jt}/\Lambda_t)^{2(1-\theta)}}{\bar{z}_j - \bar{z}_j}((\frac{P_{jt}D_{jt}}{\Lambda_tC_t}))^{-1}.
\] (24)

As a result, we can see that when the support of idiosyncratic shocks is arbitrarily small, the demand elasticity turns out to be
\[
\epsilon_j = \theta + 1/(\bar{z}_j - \bar{z}_j).
\] (25)

It is now worthwhile to mention that the demand of an individual firm comes from not only the demand of an individual consumer but also the number of consumers who decides to purchase. We call the former the demand at the intensive margin and the latter at the demand at the extensive margin. The demand elasticity therefore reflects both of the elasticity of demand at the intensive margin and the elasticity of demand at the extensive margin. For example, the first-term in the right-hand side of (25) is the elasticity of demand at the intensive margin, while the second-term corresponds to the elasticity of demand at the extensive margin.

It is also clear from (25) that the elasticity of demand approaches infinity as \(\bar{z}_j\) gets close to zero. It means that all firms are subject to perfect competition in the absence of consumer search frictions. As a result, we can find that the important source of the monopoly power of firms is the presence of search costs together with imperfect information of consumers about the location of prices.

3.5 Equilibrium Distribution of Prices

In order to generate an equilibrium price dispersion for each type of differentiated goods, we introduce idiosyncratic productivity shocks into the model. Specifically, firm \(i\) in island \(j\) produce its output using a production function of the form:
\[
Y_{jt}(i) = H_{jt}(i)/A_t(i),
\] (26)

where \(A_t(i)\) is the firm-specific shock at period \(t\), \(H_{jt}(i)\) is the amount of labor hired by firm \(i\), and \(Y_{jt}(i)\) is the output level at period \(t\) of firm \(i\). In addition, we assume that
Table 4: Example on the Determination of the Equilibrium Distribution of Prices  
(Uniform Distribution of Idiosyncratic Productivity Shocks)

<table>
<thead>
<tr>
<th>Perceived Cumulative Distribution of Real Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ F(P_{jt}) = \frac{(M^e(P_{j,t}) - M^e(P_{min,j,t}))}{(M^e(P_{max,j,t}) - M^e(P_{min,j,t}))} ]</td>
</tr>
<tr>
<td>[ M^e(P_{jt}) = P_{j,t}(2 - (\bar{R}<em>{jt}/P</em>{jt})^{1-\theta})/{\theta((\bar{R}<em>{jt}/P</em>{jt})^{1-\theta} - 1)/(1 - \theta) + 1} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Reservation (Real) Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \bar{z}<em>jA_t^{1-\theta} = (\bar{R}</em>{jt}^{1-\theta} - P_{1-\theta}^{\max,j,t})/(1 - \theta) + \int_{P_{min,j,t}}^{P_{max,j,t}} P_{jt}^{1-\theta} dP_{jt} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Number of Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ N_{jt}(P_{jt}) = {(\bar{R}<em>{jt}^{1-\theta} - P</em>{1-\theta}^{\max,j,t})/(1 - \theta)}{\Lambda_t^{\theta-1}C_t/\bar{z}_j - \bar{z}_j } ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ D_{jt}(\tilde{P}<em>{jt}) = {P</em>{jt}^{1-\theta}(\bar{R}<em>{jt}^{1-\theta} - \tilde{P}</em>{jt}^{1-\theta})/(1 - \theta)}{C_t/\bar{z}_j - \bar{z}_j } ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Profit Maximization Conditions with respect to Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ AW_t(\theta\bar{R}<em>{jt}^{1-\theta} - \tilde{P}</em>{jt}^{1-\theta} - (2\theta - 1)\tilde{P}<em>{jt}^{-2\theta}) = (1 - \theta)(2\bar{P}</em>{jt}^{1-2\theta} - \tilde{P}<em>{jt}^{-\theta}\bar{R}</em>{jt}^{1-\theta}) ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Realized Cumulative Distribution of Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A_{max}W_t(\theta\bar{R}<em>{jt}^{1-\theta} - \tilde{P}</em>{max,j,t}^{-(1+\theta)} - (2\theta - 1)\tilde{P}<em>{max,j,t}^{-\theta}) = (1 - \theta)(2\bar{P}</em>{max,j,t}^{1-\theta} - \tilde{P}<em>{max,j,t}^{-\theta}\bar{R}</em>{jt}^{1-\theta}) ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimum Real Price: Solution to the Following Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A_{min}W_t(\theta\bar{R}<em>{jt}^{1-\theta} - \tilde{P}</em>{min,j,t}^{-(1+\theta)} - (2\theta - 1)\tilde{P}<em>{min,j,t}^{-\theta}) = (1 - \theta)(2\bar{P}</em>{min,j,t}^{1-\theta} - \tilde{P}<em>{min,j,t}^{-\theta}\bar{R}</em>{jt}^{1-\theta}) ]</td>
</tr>
</tbody>
</table>

Notation: \( \tilde{P}_{jt} \) is a profit-maximizing real price at period \( t \) of type \( j \) goods; \( \bar{R}_{max,j,t} \) is the maximum level of profit-maximizing real prices of type \( j \) goods; \( \bar{P}_{min,j,t} \) is the minimum level of profit-maximizing real prices of type \( j \) goods; \( W_t \) is the real wage (\( = W_t^R/\Lambda_t \)); \( C_t \) is the aggregate consumption level; \( \bar{R}_{jt} \) is the relative price of the maximum nominal reservation price; \( \bar{R}_{jt} \) is the maximum of nominal reservation prices.

\( A_t(i) \) is an i.i.d. random variable over time and across individual firms and its distribution is a uniform distribution whose support is \( [\bar{A}, \bar{A}] \).

Given the demand curve and the production function specified above, the instantaneous profit at period \( t \) of firm \( i \) can be written as

\[ \Phi_{jt}(\tilde{P}_{jt}) = \tilde{P}_{jt}^{\theta}(\bar{R}_{jt}^{1-\theta} - \tilde{P}_{jt}^{1-\theta})(\tilde{P}_{jt} - AW_t)C_t/(z_j - \bar{z}_j), \]

when the realized value at period \( t \) of the idiosyncratic shock is \( A_t(i) = \bar{A} \). The maximiza-
tion of this one-period profit with respect to price can be written as
\[
AW_t^N (\theta \bar{R}_{jt}^{1-\theta} - (2 - 1)P_{jt}^{1-\theta}) = (1 - \theta)P_{jt}(2 P_{jt}^{1-\theta} - \bar{R}_{jt}^{1-\theta}).
\] (28)

Furthermore, the optimization condition for prices specified above can be rewritten as
\[
AW_t^N = M(P_{jt}, \bar{R}_{jt}),
\]
where
\[
M(P_{jt}, \bar{R}_{jt}) = \frac{2 - (\bar{R}_{jt}/P_{jt})^{1-\theta}}{1 + \theta((\bar{R}_{jt}/P_{jt})^{1-\theta} - 1)/(1 - \theta)}.
\] (29)

Thus, we can use this representation of profit maximization condition to characterize the distribution of nominal prices denoted by \(F(P_{jt})\). For example, suppose that firm-specific shocks are uniformly distributed over a compact interval, as we did above. Then, the resulting distribution of prices can be written as follows:
\[
F(P_{jt}) = \frac{M(P_{jt}, \bar{R}_{jt}) - M(P_{\text{min},jt}, \bar{R}_{jt})}{M(P_{\text{max},jt}, \bar{R}_{jt}) - M(P_{\text{min},jt}, \bar{R}_{jt})},
\] (30)

where \(P_{\text{max},jt}\) is the price that satisfies the profit maximization condition when \(A = A_{\text{max}}\) and \(P_{\text{min},jt}\) is the price that satisfies the profit maximization condition when \(A = A_{\text{min}}\).

### 3.6 Relative Price Distortion

Having described the distribution of prices, we discuss the distortion that arises because of the price dispersion induced by firm-specific shocks, namely relative price distortion. Specifically, the relative price distortion is defined as the part of output that is foregone because of price dispersion.

Before proceeding further, we define the real aggregate output of type \(j\) goods in terms of the shadow value of composite consumption goods denoted by \(\Lambda_t\). In order to do this, we deflate the nominal output of individual firms by \(\Lambda_t\) and then aggregate these deflated outputs across firms to yield
\[
Y_{jt} = \frac{\Lambda_t^{2\theta-2} C_t}{\bar{z}_j - \bar{z}_j} \int_{P_{\text{min},jt}}^{P_{\text{max},jt}} P_{jt}^{1-\theta} \left( \frac{\bar{R}_{jt}^{1-\theta}}{1 - \theta} - \frac{P_{jt}^{1-\theta}}{1 - \theta} \right) dF(P_{jt}),
\] (31)

where \(Y_{jt}\) is the real aggregate output for type \(j\) goods. The real aggregate output is thus defined as \(Y_t = \int_0^1 Y_{jt} dj\). In addition, when the aggregate market clearing condition, \(C_t(1 + S_t) = Y_t\), holds at an equilibrium, the definition of the aggregate output specified above implies that the following condition holds:
\[
1 = \frac{\Lambda_t^{2\theta-2}}{1 + S_t} \int_0^1 \int_{P_{\text{min},jt}}^{P_{\text{max},jt}} \frac{P_{jt}^{1-\theta}}{\bar{z}_j - \bar{z}_j} \left( \frac{\bar{R}_{jt}^{1-\theta}}{1 - \theta} - \frac{P_{jt}^{1-\theta}}{1 - \theta} \right) dF(P_{jt}) dj.
\] (32)
Furthermore, the relationship between the aggregate output in island $j$ and its total labor input can be written as $Y_{jt} \Delta_{jt} = H_{jt}$, where $\Delta_{jt}$ denotes the measure of relative price distortion and $H_{jt}$ denotes the aggregate labor input for type $j$ goods. Given that individual market clearing conditions hold, the following equation should hold

$$\Delta_{jt} Y_{jt} = H_{jt}(\frac{\bar{R}_{jt}}{1 - \theta} - \frac{P_{jt}^{1-\theta}}{1 - \theta})dF(P_{jt}).$$  \hspace{1cm} (33)

Combining these two equations, we have the following equation for the relative price distortion:

$$\Delta_{jt} = \Lambda_t \int_{P_{min,jt}}^{P_{max,jt}} \frac{P_{jt}^{1-\theta}(\bar{R}_{jt} - P_{jt}^{1-\theta})dF(P_{jt})}{\int_{P_{min,jt}}^{P_{max,jt}} P_{jt}^{1-\theta}(\bar{R}_{jt} - P_{jt}^{1-\theta})dF(P_{jt})}. \hspace{1cm} (34)$$

In addition, the aggregate production function can be written as

$$Y_t = H_t/\Delta_t,$$  \hspace{1cm} (35)

where $H_t (= \int_0^1 H_{jt} dj)$ is the aggregate amount of hours worked and $\Delta_t$ is the relative price distortion:

$$\Delta_t = \int_0^1 (Y_{jt}/Y_t)\Delta_{jt} dj. \hspace{1cm} (36)$$

### 3.7 Numerical Example on Quasi-Kinked Demand Curve

In this section, we present a numerical example of the demand curve that is implied by the model. In doing so, we assume that the preference of each household is represented by an additively separable utility between consumption and leisure of the form:

$$U(C_t, \bar{H} - H_t) = \log C_t + b(\bar{H} - H_t).$$  \hspace{1cm} (37)

The utility maximization of each household then leads to the following equation:

$$C_t = bW_t.$$  \hspace{1cm} (38)

It is also possible to have an exact closed-form solution to the model in the case of $\theta = 0$. The resulting equilibrium conditions are described in Table 4.

As shown in Figure 1, we compare search-based and utility-based demand curves of individual firms. In order to do this, we compute equilibrium price distributions for cases in which $\theta = 0$ and $\theta = 1/2$, respectively. The left column corresponds to $\theta = 0$ and the right column corresponds to $\theta = 1/2$. In addition, as a benchmark calibration, we set $\bar{A} = 1.80, A = 0.20$ for the support of idiosyncratic cost shocks and $z_{max} = 0.035$ and $z_{min} =$
Relative Price Distortion
\[ \Delta_t = \frac{9\tilde{R}_t(\tilde{P}_{\text{max},t} + \tilde{P}_{\text{min},t}) - 6\tilde{R}_t^2 - 4(\tilde{P}_{\text{max},t}^2 + \tilde{P}_{\text{min},t}^2 + \tilde{P}_{\text{max},t}\tilde{P}_{\text{min},t})}{3\tilde{R}_t(\tilde{P}_{\text{max},t} + \tilde{P}_{\text{min},t}) - 2(\tilde{P}_{\text{max},t}^2 + \tilde{P}_{\text{min},t}^2 + \tilde{P}_{\text{max},t}\tilde{P}_{\text{min},t})} \]

Marginal Value of Composite Consumption Goods
\[ 1 = (6(\bar{z} - \bar{z})(1 + X_t))^{-1}(3\tilde{R}_t(\tilde{P}_{\text{max},t} + \tilde{P}_{\text{min},t}) - 2(\tilde{P}_{\text{max},t}^2 + \tilde{P}_{\text{min},t}^2 + \tilde{P}_{\text{max},t}\tilde{P}_{\text{min},t})) \]

Share of Search Cost in Aggregate Real Consumption
\[ S_t = \frac{(1/2)(\bar{z} - z_t^2 + (2^{1/2}/3)z_t^{3/2} - \bar{z}^{3/2})\sqrt{\tilde{P}_{\text{max},t} - \tilde{P}_{\text{min},t}}/(\bar{z} - \bar{z})}{(1/2)(\tilde{P}_{\text{max},t} - \tilde{P}_{\text{min},t})} \]

Aggregate Production Function
\[ Y_t = H_t/\Delta_t \]

Aggregate Market Clearing
\[ Y_t = C_t(1 + S_t) \]

Aggregate Labor Supply
\[ C_t = bW_t \]

Maximum Real Price
\[ A_{\text{max}}W_t = 2\tilde{P}_{\text{max},t} - \tilde{R}_t \]

Minimum Real Price
\[ A_{\text{min}}W_t = 2\tilde{P}_{\text{min},t} - \tilde{R}_t \]

Maximum Reservation Price
\[ \bar{z} = \tilde{R}_t - (\tilde{P}_{\text{max},t} + \tilde{P}_{\text{min},t})/2 \]

Note: this table includes 9 equations for 9 variables such as \( \tilde{R}_t, \tilde{P}_{\text{max},t}, \tilde{P}_{\text{min},t}, W_t, Y_t, C_t, \Delta_t, S_t, \) and \( H_t, \) when \( \theta = 0. \)

0.025 for the support of search cost parameter. Under these parameter values, the share of the aggregate search cost in real output turns out to be around 10% for the case of \( \theta = 1/2. \)

Figure 1 indicates that the price-elasticity of the demand curve facing individual firms is larger than that of household’s expenditures on each type of differentiated goods. For example, \( \theta = 0 \) generates \( \epsilon = 3.91, \) while \( \theta = 1/2 \) generates \( \epsilon = 3.52 \) where \( \epsilon \) is the...
price-elasticity measured at the mean of equilibrium relative prices. We thus find that adding search behavior of customers to the model enables us to reconcile the general price inelasticity of household expenditures with measures of markup greater than one in industry data. Figure 1 also shows that the elasticity of each seller’s demand is higher in the case of \( \theta = 0 \) than in the case of \( \theta = 1/2 \). The reason for this is that the number of customers can be more responsive to changes in prices in models with lower own-price elasticities of household expenditures, especially when \( \theta \leq 1 \).

Furthermore, we can see that as price rises, the elasticity of seller’s demand curve becomes more elastic. In particular, the model’s implied demand curves tend to be similar with those derived from the Dotsey-King’s aggregator by setting its curvature parameter \( \eta = -1.1 \). In relation to this, a negative value of \( \eta \) amounts to the presence of a satiation level for each type of differentiated goods under the Dotsey-King’s aggregator and this satiation level helps to reduce increases of consumption expenditures on goods in response to the reduction in their relative prices. This summarizes the mechanism behind quasi-kinked demand curves derived from the Dotsey-King’s aggregator. Meanwhile, potential customers do not know exact locations of price decreases of sellers who they do not trade in models with customer search. As a result, the number of customers who gather because of price decreases is smaller than the number of existing customers who flee from sellers when they raise their prices, thereby leading to quasi-kinked demand curves.

4 Directions for Future Research

We have incorporated product differentiation and consumer search in a general equilibrium model. In the model of this paper, own-price elasticities of household expenditures are completely determined by an elasticity of substitution over differentiated goods. It is thus interesting to allow for the possibility of non-zero cross-price elasticities in household expenditures. An advantage of doing this would be that one can use disaggregated data on household expenditures and transaction prices in the estimation of parameters of the model in order to develop an equilibrium model that better describes responses of households with respect to changes in prices as well as the behavior of price changes.

In this paper, we have presented a very simple example of a state-dependent pricing model in order to focus on the exact solution of the model. But since characteristics of consumer search can affect the timing of price changes for individual firms, it would be interesting to extend the analysis to a more complicated model that include stochastic
exogenous shocks.

Furthermore, this type of modeling strategy can be used in time-dependent pricing models such as Calvo-type, Fisher-type, and Taylor-type staggered price-setting models. In this case, as noted earlier, the incorporation of consumer search into such models can affect endogenous responses of prices with respect to marginal cost.
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