SMALL AND LARGE FIRMS OVER THE BUSINESS CYCLE

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Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the view of the US Census Bureau and all results have been reviewed to ensure that no confidential information is disclosed.
OVERVIEW

Questions:

1. Are small firms more sensitive to business cycles?
2. Does this excess sensitivity magnify aggregate fluctuations? (Gertler and Gilchrist (1994); Moscarini and Postel-Vinay (2012), Fort et al. (2013), Chari, Christiano and Kehoe (2013))
3. Is it evidence of a “financial accelerator” mechanism? (Bernanke, Gertler and Gilchrist (1999))

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Contribution: new firm-level data

Representative, quarterly information on income statements & balance sheets of all (private and public) US manufacturing firms, 1977-2014
Results

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   - estimates of ES invariant to controlling for proxies for financial strength
   - no ES of debt issuance, as theory would predict
   - in response to identified MP shock, limited and statistically insignificant ES
Plan

The data

Aggregation

Excess sensitivity and external financing
THE QUARTERLY FINANCIAL REPORT (QFR)

Survey structure:

- stratified random sample; manufacturing, retail, wholesale
- all firms filing tax forms 1120 and 1120S
- those over $250m in assets are surveyed each quarter
- those btw $250k and $250m in assets are surveyed for 8 quarters
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Constructed data:

- panel of about 1.5m firm-quarter obs, 0.9m in manufacturing
- balance sheet items add up > 99.9% of observations
- financial statements articulate for > 98% of observations
  - similar to Compustat
- zero change in quarterly sales: 0.7% of observations
### Summary Statistics by Size Groups

<table>
<thead>
<tr>
<th>Size group</th>
<th>[0, 90]</th>
<th>[90, 99]</th>
<th>[99, 99.5]</th>
<th>99.5+</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets (2009 $ mil.)</td>
<td>2.0</td>
<td>48.8</td>
<td>626.0</td>
<td>6766.3</td>
<td>2877.4</td>
</tr>
<tr>
<td>Net leverage</td>
<td>0.20</td>
<td>0.19</td>
<td>0.23</td>
<td>0.22</td>
<td>0.13</td>
</tr>
<tr>
<td>Short-term to total debt</td>
<td>0.33</td>
<td>0.33</td>
<td>0.20</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>Bank to total debt</td>
<td>0.48</td>
<td>0.57</td>
<td>0.43</td>
<td>0.28</td>
<td>n.a.</td>
</tr>
<tr>
<td>Trade credit to tot. liab.</td>
<td>0.32</td>
<td>0.27</td>
<td>0.17</td>
<td>0.13</td>
<td>0.21</td>
</tr>
</tbody>
</table>
The cyclical behavior of sales

Sales growth (equal-weighted)

-8 -6 -4 -2 0 2 4 6
%-8 -6 -4 -2 0 2 4 6

1980q1 1985q1 1990q1 1995q1 2000q1 2005q1 2010q1

small (bottom 99% by assets) large (top 1% by assets)
THE SIZE EFFECT: SALES

Average marginal effect of GDP growth on sales growth

\[ g_{i,t} = \sum_{J} \alpha_J 1_{\{i \in J\}} + \sum_{J} \beta_J (1_{\{i \in J\}} \times \Delta GDP_t) + (\text{ind. controls}) + \epsilon_{i,t} \]
Investment

Inventory growth (de-meaned)

Fixed investment (de-meaned)
### The size effect: investment

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<th>Sales growth</th>
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<tr>
<td>GDP growth</td>
<td>3.700***</td>
<td>2.650***</td>
<td>0.912***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>[90, 99] × GDP growth</td>
<td>−0.160*</td>
<td>−0.107</td>
<td>−0.299*</td>
</tr>
<tr>
<td></td>
<td>(0.260)</td>
<td>(0.538)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>[99, 99.5] × GDP growth</td>
<td>−0.251*</td>
<td>−0.299*</td>
<td>−0.687***</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.097)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>[99.5, 100] × GDP growth</td>
<td>−0.600***</td>
<td>−0.730***</td>
<td>−1.257***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
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</table>

| N        | ≈ 460000     | ≈ 460000         | ≈ 460000         |
| nr. firms| ≈ 60000      | ≈ 60000          | ≈ 60000          |
| adj. $R^2$ | 0.005       | 0.005            | 0.003            |
| industry controls | yes       | yes              | yes              |
| s.e. clustering    | firm-level | firm-level       | firm-level       |
ROBUSTNESS: AGE; EXIT; METHODOLOGY

Firm size vs. firm age
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Firm size vs. firm age

- Sample of firms observed over more than 5 years
- Excess sensitivity is $\approx 20\%$ smaller than unconditionally
- Young firms only account for a small share of employment in manuf.
Robustness: Age; Exit; Methodology

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- Treat all un-anticipated non-response as exit ($\approx 3.5\%$ quart.)
- Higher excess sensitivity, but $\Delta$ to baseline not statistically significant
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- Pre vs. post-1990; exclude outlier episodes (1980’s recovery, Great Recession)
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Excess sensitivity and external financing
CONTRIBUTION TO AGGREGATE GROWTH

\[ G_t = \hat{g}_t^{(\text{large})} + s_{t-4} \left( \hat{g}_t^{(\text{small})} - \hat{g}_t^{(\text{large})} \right) + \hat{\text{cov}}_t \]

\[ \text{growth of large firms} \]

\[ \text{excess sensitivity of small firms} \]

\[ \text{covariance term} \]

where:

\[ s_{t-4} = \text{lagged share of small firms in total sales} \]

\[ \hat{\text{cov}}_t = \text{within-group covariance between initial sales and growth} \]

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The covariance term \( \hat{\text{cov}}_t \) is positive but almost acyclical.
Counterfactuals: sales

- Counterfactual 1: \( \hat{g}_t^{(\text{small})} = \hat{g}_t^{(\text{large})} \)

- Counterfactual 2: \( \hat{g}_t^{(\text{small})} = \hat{g}_t^{(\text{large})} \) and \( \hat{\text{cov}}_t^{(\text{small})} = \hat{\text{cov}}_t^{(\text{large})} \)
The share of the bottom 99%
**Counterfactuals**

\[ G_t = \alpha + \beta \Delta (GDP_t) + \epsilon_t \]

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External validity: employment; other sectors - firms with 2500+ emp. account for 43% of employment, down from 55% in 1980 - sales skewness may not be as extreme in other sectors

Comparison with BDS data on employment
## Counterfactuals

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### Excess sensitivity after controlling for financial strength

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<tr>
<th>Additional control</th>
<th>Baseline</th>
<th>Leverage</th>
<th>Liquidity</th>
<th>Div. issuance</th>
<th>Bank dep.</th>
<th>Mkt. access</th>
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<tr>
<td>[90, 99]</td>
<td>-0.160</td>
<td>-0.195</td>
<td>-0.162</td>
<td>-0.176</td>
<td>-0.189</td>
<td>-0.193</td>
</tr>
<tr>
<td>[99, 99.5[</td>
<td>-0.251*</td>
<td>-0.321**</td>
<td>-0.282*</td>
<td>-0.247</td>
<td>-0.257*</td>
<td>-0.490***</td>
</tr>
<tr>
<td>99.5+</td>
<td>-0.600***</td>
<td>-0.675***</td>
<td>-0.640***</td>
<td>-0.594***</td>
<td>-0.563***</td>
<td>-1.097***</td>
</tr>
</tbody>
</table>

| N                  | ≈ 460000  | ≈ 460000  | ≈ 460000  | ≈ 460000      | ≈ 460000  | ≈ 460000    |
| nr. firms          | ≈ 60000   | ≈ 60000   | ≈ 60000   | ≈ 60000       | ≈ 60000   | ≈ 60000     |
| adj. R²             | 0.025     | 0.025     | 0.025     | 0.025         | 0.025     | 0.025       |
| ind. contr.         | yes       | yes       | yes       | yes           | yes       | yes          |
| s.e. clust.         | firm-level| firm-level| firm-level| firm-level    | firm-level| firm-level   |

Additionally, the size effect has the same magnitude within low-leverage/low bank share/high-liquidity/dividend-paying groups.
**Excess sensitivity after controlling for financial strength**

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Leverage</th>
<th>Liquidity</th>
<th>Div. issuance</th>
<th>Bank dep.</th>
<th>Mkt. access</th>
</tr>
</thead>
<tbody>
<tr>
<td>[90, 99]</td>
<td>−0.160</td>
<td>−0.195</td>
<td>−0.162</td>
<td>−0.176</td>
<td>−0.189</td>
<td>−0.193</td>
</tr>
<tr>
<td>[99, 99.5]</td>
<td>−0.251*</td>
<td>−0.321**</td>
<td>−0.282*</td>
<td>−0.247</td>
<td>−0.257*</td>
<td>−0.490***</td>
</tr>
<tr>
<td>99.5+</td>
<td>−0.600***</td>
<td>−0.675***</td>
<td>−0.640***</td>
<td>−0.594***</td>
<td>−0.563***</td>
<td>−1.097***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>nr. firms</th>
<th>adj. $R^2$</th>
<th>ind. contr.</th>
<th>s.e. clust.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>≈ 460000</td>
<td>≈ 60000</td>
<td>0.025</td>
<td>yes</td>
<td>firm-level</td>
</tr>
<tr>
<td></td>
<td>≈ 460000</td>
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<td>≈ 460000</td>
<td>≈ 60000</td>
<td>0.025</td>
<td>yes</td>
<td>firm-level</td>
</tr>
</tbody>
</table>

Full results

Additionally, the size effect has the same magnitude **within** low-leverage/low bank share/high-liquidity/dividend-paying groups.
A SIMPLE MODEL

- Dynamic investment problem with simple financial constraint:

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \]

- Same investment opportunities \( z_t \); idiosyncratic exit shocks
A SIMPLE MODEL

- Dynamic investment problem with simple financial constraint:
  \[ b_{i,t+1} \leq b(n_{i,t}; z_t) \]

- Same investment opportunities \( z_t \); idiosyncratic exit shocks

Model details

Sales growth (cumul.)

Investment rate (cumul.)

Div. issuance rate

Change in debt to assets (cumul.)
A SIMPLE MODEL

- Dynamic investment problem with simple financial constraint:

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \]

- Same investment opportunities \( z_t \); idiosyncratic exit shocks

- Constrained firms experience a bigger contraction in investment...
A SIMPLE MODEL

- Dynamic investment problem with simple financial constraint:

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \]

- Same investment opportunities \( z_t \); idiosyncratic exit shocks

- Constrained firms experience a bigger contraction in \textit{investment}...
- ... if and only if they experience a bigger contraction in \textit{debt flows}
Recessions at small and large firms

\[ g_{i,t} = \alpha + \beta 1_{\{i \in S_{t-4}\}} + \sum_{k=-4}^{10} \left( \alpha_k + \beta_k 1_{\{i \in S_{t-4}\}} \right) \times 1_{\{t+k \in H\}} + \epsilon_{i,t} \]
Recessions at small and large firms

\[ g_{i,t} = \alpha + \beta 1_{\{i \in S_{t-4}\}} + \sum_{k=-4}^{10} \left( \alpha_k + \beta_k 1_{\{i \in S_{t-4}\}} \right) \times 1_{\{t+k \in \mathcal{H}\}} + \epsilon_{i,t} \]
The response to identified monetary policy shocks

- Financial accelerator may be more potent after shocks that directly affect cost of capital

Bernanke, Gertler and Gilchrist (1999)
THE RESPONSE TO IDENTIFIED MONETARY POLICY SHOCKS

- Financial accelerator may be more potent after shocks that directly affect cost of capital
  
  Bernanke, Gertler and Gilchrist (1999)

- Monetary policy shocks, identified using analysis of FOMC meetings
  
THE RESPONSE TO IDENTIFIED MONETARY POLICY SHOCKS

- Financial accelerator may be more potent after shocks that directly affect cost of capital

Bernanke, Gertler and Gilchrist (1999)

- Monetary policy shocks, identified using analysis of FOMC meetings


- Estimate response by size group using Jorda (2005) projection:

\[
\Delta y_{i,t,t+h} = \sum_{j \in \mathcal{J}} \left( \alpha_{j}^{(h)} + \beta_{j}^{(h)} r_{r-1,t} + \phi_{j}^{(h)} (L) X_t \right) 1_{i \in \mathcal{I}_{t}(j)} 
+ \text{(industry controls)} + \text{(seasonal controls)} + \epsilon_{i,t,h}
\]

\[ j \in \mathcal{J} = \{[0, 99], [99, 100]\} \]

\[ h = 0, 1, ..., 7 \text{ quarters} \]
THE RESPONSE OF REAL VARIABLES TO IDENTIFIED MP SHOCKS

1. Sales

2. Inventory

3. Fixed capital

Bottom 99% minus Top 1%

Quarters
The response of debt to identified MP shocks

Total debt to assets

Bank debt to assets

Short-term debt to assets

- The graphs show the response of total debt to assets, bank debt to assets, and short-term debt to assets, respectively, to identified MP shocks.
- The data is represented in quarters, with the bottom 99% and top 1% categories.
- The bottom 99% minus top 1% is also plotted for each category.
Recessions at financially strong and weak firms

Dividend issuance

![Graphs showing sales, inventory, and fixed investment for firms with and without dividends.](image)
Conclusion

1. Small firms are more sensitive to business cycles
2. But their excess sensitivity contributes little to aggregate volatility
3. And it is likely not evidence of financial amplification
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2. But their excess sensitivity contributes little to aggregate volatility
3. And it is likely not evidence of financial amplification

▸ If not financing, then what is ES due to?
  - correlated with export share & downstream diversification
**Conclusion**

1. Small firms are more sensitive to business cycles
2. But their excess sensitivity contributes little to aggregate volatility
3. And it is likely not evidence of financial amplification

- If not financing, then what is ES due to?
  - correlated with export share & downstream diversification

- Broader testable cross-sectional implications of the financial accelerator
  - structural approach, matching cross-sectional + business-cycle moments
Plan

More
RECESSIONS AT FINANCIALLY STRONG AND WEAK FIRMS
ZERO VS. POSITIVE LEVERAGE

- **Sales**
- **Inventory**
- **Fixed investment**

Zero leverage (+/- 2 se) vs. Positive leverage (+/- 2 se)
Recessions at financially strong and weak firms
High vs. low liquidity

Cash to assets > 0.2 (+/- 2 se)  Cash to assets < 0.2 (+/- 2 se)
RECESSIONS AT FINANCIALLY STRONG AND WEAK FIRMS

BOND MARKET ACCESS

Bond market access (+/- 2 se)  No bond market access (+/- 2 se)
Recessions at financially strong and weak firms
Bank dependence

Sales
Inventory
Fixed investment

- Bank share < 0.9 (+/- 2 se)
- Bank share > 0.9 (+/- 2 se)
**RECESSIONS AT FINANCIALLY STRONG AND WEAK FIRMS**

**DIVIDEND ISSUANCE**

![Graphs showing sales, inventory, and fixed investment during recessions at financially strong and weak firms.](image)

- **Dividends (+/- 2 se)**
- **No dividends (+/- 2 se)**
EMPLOYMENT IN MANUFACTURING

-0.120
-0.100
-0.080
-0.060
-0.040
-0.020
0.000
0.020
0.040
0.060
0.080

Top 1% Emp. Growth (>= 2500 emps)
Total Emp. Growth (>=10 emps)
COMPARISON WITH GERTLER-GILCHRIST (1994)

Romer-Romer dates
equal-weighted growth rates

Bottom 99%
Top 1%

Quarters
Comparison with Gertler-Gilchrist (1994)

Romer-Romer dates
value-weighted growth rates

Bottom 99%
Top 1%
Aggregate

Quarters

-15 -10 -5 0 5 10

Bottom 99%
Top 1%
Aggregate

-15 -10 -5 0 5 10
COMPARISON WITH GERTLER-GILCHRIST (1994)

Romer-Romer dates
value-weighted growth rates

Quarters
Bottom 30% of sales
Top 70% of sales
Aggregate
Romer-Romer dates
value-weighted growth rates
COMPARISON WITH GERTLER-GILCHRIST (1994)

Romer-Romer dates
equal-weighted growth rates (no detrending)

-15 -10 -5 0 5 10 15
Quarters

Bottom 99%
Top 1%

-15 -10 -5 0 5 10 15
-10 -5 0 5 10

Quarters
Exact growth decomposition (1/2)

Let $x_{i,t}$ be the variable of interest, e.g., sales. For any group of firms, defined by an initial set of indices $\mathcal{I}_{t-4}$:

$$G_t(\mathcal{I}_{t-4}) = \hat{g}_t(\mathcal{I}_{t-4}) + \hat{c}\text{ov}_t(\mathcal{I}_{t-4})$$

where:

$$G_t(\mathcal{I}_{t-4}) \equiv \frac{\sum_{i \in \mathcal{I}_{t-4}} x_{i,t}}{\sum_{i \in \mathcal{I}_{t-4}} x_{i,t-4}},$$

$$\hat{g}_t(\mathcal{I}_{t-4}) \equiv \frac{1}{\#\mathcal{I}_{t-4}} \sum_{i \in \mathcal{I}_{t-4}} \frac{x_{i,t}}{x_{i,t-4}},$$

$$\hat{c}\text{ov}_t(\mathcal{I}_{t-4}) \equiv \sum_{i \in \mathcal{I}_{t-4}} \left( s_{i,t-4} - \frac{1}{\#\mathcal{I}_{t-4}} \right) (g_{i,t} - \hat{g}_t),$$

and:

$$g_{i,t} = \frac{x_{i,t}}{x_{i,t-4}},$$

$$s_{i,t-4} = \frac{x_{i,t-4}}{\sum_{i \in \mathcal{I}_{t-4}} x_{i,t-4}}.$$
Aggregate growth of all firms can be then be decomposed as:

$$\hat{G}_t = s_{t-4} \hat{G}_t(I_{t-4}^{(small)}) + (1 - s_{t-4}) \hat{G}_t(I_{t-4}^{(large)}),$$

where:

$$s_{t-4} = \frac{\sum_{i \in I_{t-4}^{(small)}} x_{i,t-4}}{\sum_{i \in I_{t-4}^{(small)} \cup I_{t-4}^{(large)}} x_{i,t-4}}$$

is the initial share of small firms in total sales.
How much does the covariance term contribute to the cyclicality of aggregate growth $G_t$?

<table>
<thead>
<tr>
<th></th>
<th>Small firms</th>
<th>Large firms</th>
<th>All firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(G_t, Y_t)$</td>
<td>0.68</td>
<td>0.62</td>
<td>0.65</td>
</tr>
<tr>
<td>$\frac{\sigma_{cov_t}}{\sigma_{G_t}}$</td>
<td>0.54</td>
<td>0.45</td>
<td>0.41</td>
</tr>
<tr>
<td>$corr(cov_t, Y_t)$</td>
<td>$-0.32$</td>
<td>$-0.05$</td>
<td>$-0.15$</td>
</tr>
<tr>
<td>contribution</td>
<td>$-0.17$</td>
<td>$-0.02$</td>
<td>$-0.06$</td>
</tr>
</tbody>
</table>

$Y_t =$ year-on-year GDP growth.
The covariance between size and growth (2/2)

Sales - all firms

Sales - bottom 99% by assets

Sales - top 1% by assets

Growth decomposition
### Complete Regression Results for Financial Proxies

<table>
<thead>
<tr>
<th>Additional control</th>
<th>Baseline</th>
<th>Bank dep.</th>
<th>Leverage</th>
<th>Liquidity</th>
<th>Mkt. access</th>
<th>Div. issuance</th>
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<tbody>
<tr>
<td>([90, 99] \times \Delta GDP_t)</td>
<td>-0.160</td>
<td>-0.189</td>
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<td>-0.193</td>
<td>-0.176</td>
</tr>
<tr>
<td>([99, 99.5] \times \Delta GDP_t)</td>
<td>-0.251*</td>
<td>-0.257*</td>
<td>-0.321**</td>
<td>-0.282*</td>
<td>-0.490***</td>
<td>-0.247</td>
</tr>
<tr>
<td>(99.5^+ \times \Delta GDP_t)</td>
<td>-0.600***</td>
<td>-0.563***</td>
<td>-0.675***</td>
<td>-0.640***</td>
<td>-1.097***</td>
<td>-0.594***</td>
</tr>
<tr>
<td>Bk ([0.10,0.90] \times \Delta GDP_t)</td>
<td>0.300</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bk &lt; 0.10 \times \Delta GDP_t</td>
<td>-0.315</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev ([0.15,0.50] \times \Delta GDP_t)</td>
<td>-0.126</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev ((0,0.15] \times \Delta GDP_t)</td>
<td>-0.474*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev = 0 \times \Delta GDP_t</td>
<td>-0.630***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liq ([0.01,0.20] \times \Delta GDP_t)</td>
<td>0.228</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liq &gt; 0.20 \times \Delta GDP_t</td>
<td>-0.101</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mkt \times \Delta GDP_t</td>
<td>0.826**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Div. \times \Delta GDP_t</td>
<td>0.087</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- \(N \approx 460000\) for all outcomes.
- \(\text{nr. firms} \approx 60000\) for all outcomes.
- \(\text{adj. } R^2 = 0.025\) for all outcomes.
- Industry controls are included for all outcomes.
- Standard errors are clustered at the firm level for all outcomes.

**Legend:**
- Bk: Bank
- Lev: Leverage
- Liq: Liquidity
- Mkt: Market
- Div: Dividend
SIZE, FINANCIAL CONSTRAINTS AND EXCESS SENSITIVITY

Individual firm problem:
Individual firm problem:

\[ V_t(k_{i,t}) = \max_{k_{i,t+1}, b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right) \]

\[ d_{i,t} = n_{i,t} - k_{i,t+1} \]

s.t.

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \]
Individual firm problem:

\[ V_t(k_{i,t}) = \max_{k_{i,t+1}, b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right) \]

\[ d_{i,t} = n_{i,t} - k_{i,t+1} \]

s.t.

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \]

\[ d_{i,t} \geq 0 \]
Individual firm problem:

\[
V_t(k_{i,t}) = \max_{k_{i,t+1}, b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right)
\]

\[
d_{i,t} = n_{i,t} - k_{i,t+1}
\]

s.t.

\[
n_{i,t} = z_t k_{i,t} + (1 - \delta) k_{i,t}
\]

\[
d_{i,t} \geq 0
\]
SIZE, FINANCIAL CONSTRAINTS AND EXCESS SENSITIVITY

Individual firm problem:

\[ V_t(k_{i,t}) = \max_{k_{i,t+1},b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right) \]

\[ d_{i,t} = n_{i,t} - k_{i,t+1} \]

s.t.

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \]
\[ d_{i,t} \geq 0 \]

Exiting firms replaced by entrants at \( k_e \), chosen so top 1% are unconstrained in steady-state \((z_t = z)\).
Individual firm problem:

\[
\begin{align*}
V_t(k_{i,t}) &= \max_{k_{i,t+1}, b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right) \\
\quad & \quad s.t. \\
& \quad \quad d_{i,t} = n_{i,t} - k_{i,t+1} \\
& \quad \quad n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \\
& \quad \quad d_{i,t} \geq 0
\end{align*}
\]

Exiting firms replaced by entrants at \( k_e \), chosen so top 1% are unconstrained in steady-state \( (z_t = z) \).
Individual firm problem:

\[
V_t(k_{i,t}) = \max_{k_{i,t+1}, b_{i,t+1}} \eta n_{i,t} + (1 - \eta) \left( d_{i,t} + \frac{1}{1 + r} V_{t+1}(k_{i,t+1}) \right)
\]

\[
d_{i,t} = n_{i,t} - k_{i,t+1}
\]

s.t.

\[
n_{i,t} = z_t k^\zeta_{i,t} + (1 - \delta)k_{i,t}
\]

\[
d_{i,t} \geq 0
\]

Exiting firms replaced by entrants at \(k_e\), chosen so top 1% are unconstrained in steady-state \((z_t = z)\).

From steady-state, construct cross-sectional responses to a decline in \(z_t\):

\[
z_t = \exp(-\rho^t \epsilon)z, \quad t \geq 0.
\]
Model solution

Solution depends on:
Model solution

Solution depends on:

\[ k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{r + \delta} \right)^{\frac{1}{1-\zeta}} \]  

(optimal investment; convex in z_t)
Model Solution

Solution depends on:

\[ k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{r + \delta} \right)^{\frac{1}{1-\zeta}} \]  
(optimal investment; convex in \( z_t \))

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta)k_{i,t} \]  
(net worth; linear in \( z_t \))
Model solution

Solution depends on:

\[ k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{r + \delta} \right)^{\frac{1}{1-\zeta}} \quad \text{(optimal investment; convex in } z_t) \]

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \quad \text{(net worth; linear in } z_t) \]

Two types of firms:
Solution depends on:

\[
k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{r + \delta} \right)^{\frac{1}{1-\zeta}} \quad \text{(optimal investment; convex in } z_t)\]

\[
n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta) k_{i,t} \quad \text{(net worth; linear in } z_t)\]

Two types of firms:

\[
k_{i,t+1} = \begin{cases} 
  n_{i,t} & \text{if } n_{i,t} < k_{t+1}^* \quad \text{(constrained)} \\
  k_{t+1}^* & \text{if } n_{i,t} \geq k_{t+1}^* \quad \text{(unconstrained)}
\end{cases}
\]
Model solution

Solution depends on:

\[ k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{r + \delta} \right)^{\frac{1}{1-\zeta}} \]  
(optimal investment; convex in \( z_t \))

\[ n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta)k_{i,t} \]  
(net worth; linear in \( z_t \))

Two types of firms:

\[ k_{i,t+1} = \begin{cases} 
  n_{i,t} & \text{if } n_{i,t} < k_{i,t+1}^* \quad \text{(constrained)} \\
  k_{i,t+1}^* & \text{if } n_{i,t} \geq k_{i,t+1}^* \quad \text{(unconstrained)}
\end{cases} \]

Size ↔ financial constraints.
Large/unconstrained firms respond more than small/constrained firms.

\[ n_i, t = z_t \kappa_i, t + (1 - \delta) \kappa_i, t \downarrow \Rightarrow \text{small firms invest less, "one-for-one" with} \]

\[ z_t - k^* t + 1 = (\zeta z_t + 1 + r) 1 - \zeta \downarrow = \Rightarrow \text{large firms divest, more than "one-for-one" with} \]

Back
Large/unconstrained firms respond more than small/constrained firms.
Large/unconstrained firms respond \textit{more} than small/constrained firms.

\[ n_{i,t} = z_t k_{i,t}^\xi + (1 - \delta) k_{i,t} \downarrow \]
Large/unconstrained firms respond *more* than small/constrained firms.

\[
n_{i,t} = z_t k_{i,t}^{\zeta} + (1 - \delta)k_{i,t} \downarrow \quad \implies \quad \text{small firms invest less, "one-for-one" with } z_t
\]
Large/unconstrained firms respond more than small/constrained firms.

- \( n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta)k_{i,t} \downarrow \quad \implies \text{small firms invest less, “one-for-one” with } z_t \)

- \( k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{\delta + r} \right)^{\frac{1}{1-\zeta}} \downarrow \)
Large/unconstrained firms respond more than small/constrained firms.

- \( n_{i,t} = z_t k_{i,t}^\zeta + (1 - \delta)k_{i,t} \downarrow \implies \) small firms invest less, “one-for-one” with \( z_t \)

- \( k_{t+1}^* = \left( \frac{\zeta z_{t+1}}{\delta + r} \right)^{\frac{1}{1 - \zeta}} \downarrow \implies \) large firms divest, more than “one-for-one” with \( z_t \)
Adding procyclical borrowing

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \quad \text{(Borrowing limit)} \]
**Adding procyclical borrowing**

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \] (Borrowing limit)

\[ c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \] (Total financing capacity)
Adding procyclical borrowing

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \] (Borrowing limit)

\[ c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \] (Total financing capacity)

\[ k_{i,t+1} = \begin{cases} 
  c_{i,t} & \text{if } c_{i,t} < k_{t+1}^* \\
  k_{t+1}^* & \text{if } c_{i,t} \geq k_{t+1}^* 
\end{cases} \] (constrained/unconstrained)

Small firms' response now depends on total financing capacity, \( c_{i,t} \), which must move sufficiently with \( z_t \): \( \epsilon b_{i,n} + \epsilon b_{i,z} > 1 - \zeta \) \( \Rightarrow \) more procyclical financing flows at small/constrained firms.
Adding procyclical borrowing

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \] (Borrowing limit)

\[ c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \] (Total financing capacity)

\[ k_{i,t+1} = \begin{cases}  & \text{if } c_{i,t} \geq k_{t+1}^* \ (\text{unconstrained}) \\ c_{i,t} & \text{if } c_{i,t} < k_{t+1}^* \ (\text{constrained}) \\ k_{t+1}^* & \text{if } c_{i,t} \geq k_{t+1}^* \ (\text{unconstrained}) \end{cases} \]

- Small firms’ response now depends on total financing capacity, \( c_{i,t} \)
Adding procyclical borrowing

\[
b_{i,t+1} \leq b(n_{i,t}; z_t) \quad \text{(Borrowing limit)}
\]

\[
c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \quad \text{(Total financing capacity)}
\]

\[
k_{i,t+1} = \begin{cases} 
  c_{i,t} & \text{if } c_{i,t} < k^*_t+1 \quad \text{(constrained)} \\
  k^*_t+1 & \text{if } c_{i,t} \geq k^*_t+1 \quad \text{(unconstrained)}
\end{cases}
\]

- Small firms’ response now depends on total financing capacity, \(c_{i,t}\)
Adding procyclical borrowing

\[ b_{i,t+1} \leq b(n_{i,t}; z_t) \]  (Borrowing limit)

\[ c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \]  (Total financing capacity)

\[ k_{i,t+1} = \begin{cases} 
  c_{i,t} & \text{if } c_{i,t} < k^*_{t+1} \quad \text{(constrained)} \\
  k^*_{t+1} & \text{if } c_{i,t} \geq k^*_{t+1} \quad \text{(unconstrained)}
\end{cases} \]

- Small firms' response now depends on total financing capacity, \( c_{i,t} \)

- \( c_{i,t} \) must move sufficiently with \( z_t \):

\[ \epsilon_{b,n} + \epsilon_{b,z} > \frac{1}{1 - \zeta}. \]
**Adding procyclical borrowing**

\[
\begin{align*}
    b_{i,t+1} & \leq b(n_{i,t}; z_t) \quad \text{(Borrowing limit)} \\
    c_{i,t} & = n_{i,t} + b(n_{i,t}; z_t) \quad \text{(Total financing capacity)} \\
    k_{i,t+1} & = \begin{cases} 
        c_{i,t} & \text{if } c_{i,t} < k^*_t+1 \\
        k^*_t+1 & \text{if } c_{i,t} \geq k^*_t+1
    \end{cases} \quad \text{(constrained/unconstrained)}
\end{align*}
\]

- Small firms’ response now depends on **total financing capacity**, \( c_{i,t} \)
- \( c_{i,t} \) must move sufficiently with \( z_t \):

\[
\epsilon_{b,n} + \epsilon_{b,z} > \frac{1}{1 - \zeta}.
\]

- \( \implies \text{more procyclical} \) financing flows at small/constrained firms
Small/constrained firms respond more than large/unconstrained firms.

\[ k^* t + 1 = (\zeta z t + 1 + \delta + r) \]

\[ c_i, t = n_i, t + b (n_i, t; z_t) \]

⇒ same response as before among large firms

So excess sensitivity of small firms requires larger fall in debt flows.

Back
Small/constrained firms respond more than large/unconstrained firms.
Small/constrained firms respond more than large/unconstrained firms.

\[ -k_{i+1}^* = \left( \frac{\zeta z_{i+1}}{\delta + r} \right)^{\frac{1}{1-\zeta}} \]
Small/constrained firms respond more than large/unconstrained firms.

\[-k^*_t = \left( \frac{\zeta z_{t+1}}{\delta + r} \right)^{\frac{1}{1-\zeta}} \ \Downarrow \ \implies \text{same response as before among large firms}\]
Small/constrained firms respond more than large/unconstrained firms.

\[- k_{i+1}^* = \left( \frac{\zeta z_{t+1}}{\delta + r} \right)^{\frac{1}{1-\zeta}} \downarrow \quad \implies \text{same response as before among large firms} \]

\[- c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \downarrow \]
Small/constrained firms respond more than large/unconstrained firms.

- \( k_{i+1}^* = \left( \frac{\zeta z_{i+1}}{\delta + r} \right)^{\frac{1}{1-\zeta}} \downarrow \implies \text{same response as before among large firms} \)

- \( c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \downarrow \implies \text{investment declines, proportionally to financing capacity} \)
Small/constrained firms respond more than large/unconstrained firms.

- \( k_{i+1}^* = \left( \frac{\zeta z_{i+1}}{\delta + r} \right)^{1-\zeta} \) \( \downarrow \) \( \implies \) same response as before among large firms

- \( c_{i,t} = n_{i,t} + b(n_{i,t}; z_t) \) \( \downarrow \) \( \implies \) investment declines, proportionally to financing capacity

- So excess sensitivity of small firms requires larger fall in debt flows