Insight from a Time-Varying VAR Model with Stochastic Volatility of the French Housing and Credit Markets
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ABSTRACT

Through a time-varying VAR model with drifting parameters and stochastic volatilities (Cogley and Sargent, 2005, Primiceri, 2005), we explore nonlinearities on the French housing and credit markets, which give rich insights on the persistent bubble of the 2000s. While the price increase took place during a period of low shock variance, shock persistence increased during this period, as well as the elasticity relative to demography and income. Low reactivity of the housing stock to housing prices may create construction bottlenecks and explain these nonlinearities. However, even though our framework is very flexible, part of the price increase remains unexplained.

Keywords: R20, G21, C32.
JEL classification: housing bubble; housing credit; housing demand; housing supply; time-varying VAR, stochastic volatility.

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NON-TECHNICAL SUMMARY

Real estate prices have been a major trigger mechanism in systemic crises, which calls for placing their analysis at the heart of macroprudential policy. Understanding their behavior is crucial in the context of the post-crisis regulation.

Housing market empirical frameworks usually assume linear relationships with time invariant parameters between macroeconomic fundamentals and housing supply or demand. However, theoretical models of the housing market reveal some nonlinearities in the interaction between credit conditions and income shocks. These results question the relevance of linear models with time invariant parameters in the analysis of the joint dynamics of housing and credit markets. Indeed, these kinds of models may not be able to capture the instability of the parameters and uncertainty. In addition, the recent empirical literature highlights the evidence that macroeconomic and financial variables are affected by structural breaks. These breaks require a non-linear system of equations to describe the behavior of housing and credit markets.

Such a flexible approach is particularly interesting for the French housing market. Housing prices increased tremendously in the first half of the 2000s in France, as in many other advanced countries. However, unlike most of them, the housing prices were persistent in the mid-2000s: housing prices increased 2.5-fold from 1998 to 2008 and are only 3.5% below their peak level by end-2014. The specific French situation of “persistent bubble” questions the underlying factors behind this increase. The housing prices seem clearly described by a combination of different processes.

Usual fundamentals (households’ income, user cost of housing, demographic changes, housing supply, etc.) cannot fully explain this increase (Antipa and Lecat, 2013). Several hypotheses may then be expressed. A basic and intuitive bubble definition would put forward the dynamics of expectations as an explanation: “if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when “fundamental” factors do not seem to justify such a price – then a bubble exists” (Stiglitz, 1990). The absence of a bust in housing prices does not discard this statement: bubbles may be persistent, especially on the housing market, as emphasized for example by Glaeser (2013), on long US data series.

Another hypothesis relates to the impact on housing prices of a change in credit conditions and in particular to the lengthening of credit duration, which is the most notable change in credit condition in France over the period along with the sharp decline in the housing loan interest rate (Antipa and Lecat, 2013). Finally, the elasticity of housing prices relative to its usual fundamentals may change over time. It is another plausible explanation of the persistence of the bubble. The future evolution of prices and the risks of a sharp correction of the housing market rely on this diagnosis: expectations but also credit conditions may swiftly reverse, whereas demographic factors or other real-economy fundamentals are particularly inert.

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Our model of the French housing and credit markets is based on an empirical VAR combined with drifting parameters and stochastic volatilities. We show that a time-varying VAR representation of the housing market does a better job at predicting the dynamics of the housing price index than a VAR with fixed parameters and constant volatility.

Indeed, as traditional models fail to explain the price increase during the 2000s, this framework allows taking into account of the instability of the coefficients and the nonlinearities of the links between the endogenous variables and their fundamentals. In addition, this makes it possible to describe the variance and persistence of the shocks. It is also particularly relevant to handle endogeneity biases and macroeconomic time-series, with a limited number of observations.

We show that variances of the innovations did not increase during the bubble period, emphasizing that the increased shock variance is not at the root of the bubble. Auto-regressive coefficients increased somewhat for housing prices and credit, testifying for greater shock persistence and maybe for stronger price expectations dynamics. The coefficients of the demographic indicator and income moved upward during the period, which makes up for the nonlinearity we were looking for. The low elasticity of the housing stock to prices may explain this nonlinearity as housing prices may overreact to a demand shock in case of construction bottlenecks. However, even in such a flexible setting, a part of the price increase remains unexplained.
RÉSUMÉ : LES ENSEIGNEMENTS D’UN MODÈLE VAR À COEFFICIENTS VARIABLES ET VOLATILITÉ STOCHASTIQUE SUR LES MARCHÉS DE L’IMMOBILIER ET DU CRÉDIT EN FRANCE


Mots-clés : bulle immobilière, crédit à l’habitat, demande immobilière, offre immobilière, VAR à coefficients variables dans le temps, volatilité stochastique.
1. Introduction

Real estate prices have been a major trigger mechanism in systemic crises, as demonstrated in Hartmann (2015), which calls for placing their analysis at the heart of macroprudential policy. Understanding their behavior is crucial in the context of the post-crisis regulation.

Housing market empirical frameworks usually assume linear relationships with time invariant parameters between macroeconomic fundamentals and housing supply or demand. However, theoretical models of the housing market reveal some nonlinearities in the interaction between credit conditions and income shocks (Ortolo-Magné and Rady, 2006). These results question the relevance of linear models with time invariant parameters in the analysis of the joint dynamics of housing and credit markets. Indeed, these kinds of models may not be able to capture the instability of the parameters and uncertainty. In addition, the recent empirical literature highlights the evidence that macroeconomic and financial variables are affected by structural breaks. These breaks require a non-linear system of equations to describe the behavior of housing and credit markets. Besides, it is now recognized that the performance of this dynamic non-linear system is improved by adding random instabilities through stochastic volatility to this system (Jore et al., 2010, Clark, 2011, Clark and Ravazzolo, 2015).

To illustrate the relationship between credit and housing markets, it is worth noting that easy credit constitutes a positive demand shock on the housing market. A booming housing market impacts the credit market as well, via a collateral channel affecting standard parameters of credit risk hedging (probability of default and loss given default). Studying the recent crisis episode in the US, Mian and Sufi (2009) show that the increase in mortgage defaults in 2007 in the US was significantly amplified in subprime ZIP codes, which experienced an unprecedented relative growth in mortgage credit. This growth was dissociated from income growth and this dissociation is closely correlated with the increase in securitization of subprime mortgages.

According to Duca et alii (2011), the credit constraints for first-time home buyers play a major role in the dynamics of housing prices. Indeed, while the financial innovations helped to drive the boom years in the housing market, the highly restrictive lending policies adopted by the major banks exacerbated the subsequent downturn. The empirical literature uses regulation shocks to identify the causal role of credit on housing prices. For instance, Favara and Imbs (2015) show that since 1994, branching deregulations in the US have significantly affected the supply of mortgage credit and ultimately housing prices: these prices rise with branching deregulation, particularly in areas with inelastic construction. Landier, Sraer and Thesmar (2013) calculate correlations between the housing prices for all the different states of the USA and the geographic integration of the US banking market. They also show that banking integration explains up to one third of the rise in these correlations between 1976 and 2000.

In this paper, we study the joint dynamics of the housing and credit markets. We run a simultaneous estimation of housing demand, housing supply and housing credit, with an adequate treatment of endogeneity biases. Indeed, we jointly examine the demand and supply side of the housing market in order to make the analysis more realistic.

We adopt a time-varying VAR representation of the French housing and credit markets combined with the stochastic volatility process in order to introduce non-linear relationships between macroeconomic fundamentals. This improves the performance of the model developed by Avouyi-Dovi et al. (2014), which highlighted that the interconnections between both markets are important triggers and drivers of the housing prices dynamics. We notably show that a time-varying VAR representation of the housing market does a better job at predicting the dynamics of the housing price index than a VAR with fixed parameters and constant volatility.

The French housing market is particularly interesting to explore. Housing prices increased tremendously in the first half of the 2000s in France, as in many other advanced countries. However,
Unlike most of them, the housing prices were persistent in the mid-2000s: housing prices increased 2.5-fold from 1998 to 2008 and are only 3.5% below their peak level by end-2014. The specific French situation of “persistent bubble” questions the underlying factors behind this increase. The housing prices seem clearly described by a combination of different processes.

Usual fundamentals (households’ income, user cost of housing, demographic changes, housing supply, etc.) cannot fully explain this increase (Antipa and Lecat, 2013). Several hypotheses may then be expressed. A basic and intuitive bubble definition would propose the dynamics of expectations as an explanation: “if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when “fundamental” factors do not seem to justify such a price – then a bubble exists” (Stiglitz, 1990). The absence of a bust in housing prices does not discard this statement: bubbles may be persistent, especially on the housing market, as emphasized for example by Glaeser (2013), on long US data series. Another hypothesis relates to the impact on housing prices of a change in credit conditions and in particular to the lengthening of credit duration, which is the most notable change in credit condition in France over the period along with the sharp decline in the housing loan interest rate (Antipa and Lecat, 2013). Finally, the elasticity of housing prices relative to its usual fundamentals may change over time. It is another plausible explanation of the persistence of the bubble. The future evolution of prices and the risks of a sharp correction of the housing market rely on this diagnosis: expectations but also credit conditions may swiftly reverse, whereas demographic factors or other real-economy fundamentals are particularly inert.

We treat these issues in the time-varying VAR set-up with drifting coefficients and stochastic volatilities developed by Cogley and Sargent (2005) and Primiceri (2005). This approach provides us with a diagnosis on the change in parameters and the persistence of shocks, on the variance of the shocks, and, on the uncertainty.

The time-varying VARs are related to the well documented strand of literature on modeling and estimating time variation in multivariate linear models (Canova, 1993, Sims, 1993, Stock and Watson, 1996, Cogley and Sargent, 2001). In this framework, the time variation governs the entire coefficients of the VAR and the variance covariance matrix of the model innovations. According to Clark and Ravazzolo (2015), the VAR specifications combined with conventional stochastic volatility (i.e. the random walk stochastic volatility) overall dominate other volatility specifications in terms of point forecasting and, especially, density forecasting. In addition, the stochastic volatility models are the most suitable volatility models to theoretical models in finance.

Cogley and Sargent (2005) framework and Primiceri (2005) developed a VAR model with drifting parameters combined with a multivariate stochastic volatility model. However, the non-diagonal elements of the variance covariance matrix specified by Cogley and Sargent are time invariant and the diagonal elements are time-varying in order to introduce the drifting innovation variance hypothesis. Therefore, the constant conditional correlations and the time-varying variances assumptions of the residuals are accepted. On the other hand, Primiceri imposed the time variation in the coefficients of the VAR with the changes in all elements of the conditional variance and covariance matrix of the residuals. Both the entire VAR coefficients and the variance covariance matrix of the shocks are time-varying. Primiceri’s approach is hence less parsimonious and more demanding in terms of observation numbers to get precise estimates. Due to data limitations, we privilege Cogley and Sargent’s approach in our applications. Besides, the Bayesian estimation methods bring some additional information through the prior distributions. Therefore, Bayesian estimates are well-suited in this study.

1 For the impact of credit conditions on housing prices, see Duca et alii (2011) and Dell’Ariccia et alii (2012) for the US case
2 More precisely, Clark and Ravazzolo (2015) (see also Poon and Granger (2003) for a literature review) compared the performance of many specifications of time-varying volatility: random walk stochastic volatility (conventional stochastic volatility), stochastic volatility with stationary AR process, stochastic volatility combined with fat tails, GARCH and the mixture of innovation models). They found that the conventional stochastic volatility displays the best performance in terms of point and density forecasting.
3 Some innovative estimation methods are recently introduced. Koops and Korobolis (2013) developed methods for estimation and forecasting in large time-varying VARs. Their methods are based on the idea drawn from the dynamic model averaging literature. Their approaches allow for achieving reduction in the computational major drawbacks through forgetting factors. More recently, Kapetanios,
Our main contributions are the following: i) we do not observe an increase in the variances of innovations during the price increase period, which seems to be taking place during a “great moderation” episode; ii) variances of innovations and global uncertainty on the contrary, strongly increased during the financial crisis, when housing prices slightly decreased; iii) some increase in the persistence of the shocks, possibly reflecting the dynamics of price expectations, has taken place; iv) the most salient results in relation with price increases are, however, the increase in the elasticity of prices to population and gross disposable income, as well as a limited reactivity of the housing stock to prices; v) this limited reactivity may indeed explain this non-linearity in the reaction to demographics and income, as constrained availability of land creates bottlenecks in construction. Part of the increase remains unexplained, despite our very flexible framework.

In section 2, we set the theoretical framework; in section 3, we present the empirical strategy; in section 4, we describe data collection and treatment; in section 5, we explain the empirical results; and in section 6, we do the sensitivity analysis of the results.

2. Theoretical Framework

a. A brief theoretical snapshot

We aim at giving the intuition of the theoretical background to the equations estimated in this paper. For this purpose, we present a simple one-period model of the housing and credit markets.

Consider an economy with three homogenous types of agents, homeowners, banks and building firms. The period is defined as the timeframe between the origination of the loans used to finance the construction of the houses and their repayments after the sale of the houses. This period can be divided into two sub periods: the housing construction period and the consumption period. During the first sub period, households borrow from the banks to pay the building firms which deliver the houses at the end of the sub period. Households receive an endowment from which they consume and pay the user costs of housing (in particular interest on mortgage loans) during the consumption sub period. Finally, at the end of the period, homeowners die and banks get the housing loan capital back from the sale of the house on a secondary market.

Homeowners maximize their utility via an arbitrage between the amount of goods and housing services consumed, subject to his endowment and the cost of housing. Hence, the homeowner problem is:

\[
\max U = u \left( \frac{h}{n} ; \frac{g}{n} \right) \quad \text{(Eq. 1)}
\]

Subject to the budget constraint: \( c h + g = n e \),

with \( \frac{\partial u}{\partial h} > 0 \); \( \frac{\partial u}{\partial g} > 0 \) and \( \frac{\partial^2 u}{\partial h^2} < 0 \); \( \frac{\partial^2 u}{\partial g^2} < 0 \),

where \( h \) is the amount of housing services consumed; \( c \), the housing user cost; \( g \), the other consumption goods (the numeraire); \( n \) is the population ((uniformly spread in an exogenous number of households) and \( e \) the per capita endowment. The number of persons per household and the endowment are assumed here to be exogenous, although causality may be at play from housing prices to fertility, separation rates or wages.

The housing user cost encompasses housing-related net costs. In our simple frameworks, they involve financing costs minus potential capital gains.

Marcellino and Venditti (2016) provided us with a non-parametric estimates method for a large TVP-VAR. these methods are the good alternatives ones to that applied in this paper.
where $P$ is the price per housing service unit, relative to the numeraire, $r^h$ represents the housing loan interest rates and $\omega$, the capital gains (or losses) rate realised on the secondary market.

Capital gains being perfectly known by all agents, the households borrow against capital gains to consume.

Homeowners do not have any capital endowment at the beginning of the period and finance their housing acquisition through a housing credit, without down payment.

$$l^h = h(P + \omega) \quad \text{(Eq. 3)},$$

where $l^h$ is the housing loan stock.

Building firms supply housing under perfect competition and are owned by outside investors. Their profit functions can be written as follows:

$$\pi^f = hP - cc(h) \quad \text{(Eq. 4)},$$

where $\pi^f$ is firms’ profits and $cc$, construction cost per housing service unit. Construction costs are increasing in housing service units and convex ($cc'(h) > 0$; $cc''(h) > 0$).

From the equalisation of marginal cost to marginal revenues we have:

$$cc'(h) = P \quad \text{(Eq. 5)}.$$

Banks provide housing loans to the households and choose optimally their balance sheet structure. As for the building firms, Banks are assumed to be owned by outside investors and operate under conditions of perfect competition. Banks’ profits on housing loans can be written as follows:

$$\pi^b = r^h l^h - r^b (l^h - k) - r^k (k) \quad \text{(Eq. 6)},$$

where $\pi^b$ is banks’ profits, $r^b$, banks borrowing costs, $r^k$, banks cost of capital stock ($r^k > r^b$), and, $k$, capital stock.

We assume that the cost of financing, both for debt financing and equity financing, is increasing with refinancing needs ($\frac{\partial r^f}{\partial l^h} > 0$ for $i = b$ and $i = k$). This assumption notably relates to the idea of risk hedging through portfolio diversification (Markowitz (1952)). Due to prudential regulations, we also assume that the capital stock $k$ satisfies $k \geq \rho l^h$ with $\rho$, the solvability ratio. Given that $r^k > r^b$, for any level of loans, banks substitute capital as much as they can for bonds and, in equilibrium, we have $k = \rho l^h$.

The first order condition gives the following marginal condition:

$$r^h = (1 - \rho) \frac{\partial r^b}{\partial l^h} [(1 - \rho) l^h] + \rho \frac{\partial r^k}{\partial l^h} (\rho l^h) \quad \text{(Eq. 7)}.$$

If the households utility function is linear,

$$u \left( \frac{1}{n}, \frac{2}{n} \right) = \alpha \ln \left( \frac{1}{n} \right) + \beta \ln \left( \frac{2}{n} \right),$$

where $\alpha$ and $\beta$ are, respectively, parameters reflecting households’ preference for housing service and goods. From (Eq. 2), we obtain the reverse housing market demand equation:
\[ P = \frac{1}{r^n} \left\{ \frac{\alpha}{\alpha + \beta} \frac{ne}{n} + (1 - r^h)w \right\} \]  
(Eq. 8).

Housing prices depend positively on population, endowment, preference for housing services and capital gains, negatively on the provision of housing services and the interest rate on housing credit.

If \( cc(h) = \theta h^\iota \) where \( \iota > 1 \), \( \theta \) and \( \iota \) govern the housing building costs and in order to satisfy the convexity condition, we impose \( \iota > 1 \), then, the housing supply equation is:

\[ h = \frac{p}{i^\theta} \]  
(Eq. 9).

Housing supply is positively correlated to housing prices and negatively to the building costs.

If \( r^b = r + \nu' \) and \( r^k = r + \nu' \nu'' \), where \( r \) is the risk free rate, \( \nu' > \nu \) and for the sake of simplicity \( \nu' = \gamma \), from (3), (6) and (7), the equation for demand of housing credit can be written:

\[ l^h = \left( \frac{\alpha}{\alpha + \beta} \frac{ne + wh}{\nu(1-\nu') + \nu' \nu''} \right)^{\frac{1}{\nu}} \]  
(Eq. 10)

Demand for housing credit has similar determinants as housing prices, but for the positive impact of capital stock. The level of the prudential regulation and the rates at which the banks finance their operation both negatively affect the amount of housing credit.

b. A bridge between the theoretical intuition and the empirical model

Several specific features of the housing and credit markets must be taken into account in order to estimate the empirical model.

First, according to Di Pasquale and Wheaton (1994), the housing market slowly adjusts, both on the demand and supply sides. Traditional stock-flow models exhibit positive serial correlation, due to a gradual market-clearing process. Disequilibria are slowly corrected by prices and supply adjustments. This stylized fact relies on both legal or physical constraints and the dual nature of housing which is both consumption and investment goods. This finding translates in our empirical strategy into a time-varying VAR specification.

Second, as a large share of purchases and construction are financed through a banking loan, there are strong interactions between the housing and credit markets. For example, in France, about 87% of second-hand transactions involve a loan.

Finally, we impose nonlinear relationships between macroeconomic fundamentals and housing supply and demand. These nonlinearities may be rooted in several features of the housing markets: the scarcity of land, which is observed in tense areas in France; the interaction between credit conditions and income shocks (Ortolo-Magné and Rady, 2006); the changes in behaviors linked to the sharp increase in prices over the period (2.5-fold from 1998 to 2008).

The inverted housing demand equation (Eq. 8) enables us to relate housing prices positively to gross disposable income (GDI), demographic factor (hereafter population) and negatively to housing credit rates and the housing stock. Gross disposable income is the relevant measure of the endowment in our model, a proxy for credit-constrained households and a proxy for permanent income for other households. As a consumption service, housing demand should increase with income. We expect a positive relationship between housing price and GDI. A growing population should lead to a growing share of housing service consumption for a given GDI level. An alternative proxy of the demographic factor could also be the number of households or a weighted measure of population according to age propensity to buy. However, age propensity to buy changes over time and the time-varying
coefficients framework should be able to capture this feature; the number of households is endogenous, as households formation depends on housing prices. We may note that the number of households has increased faster than population over the estimation period due to socio-demographic factors such as ageing or divorce. We expect a positive elasticity of prices to population which may grow over time due to these socio-demographic factors.

The housing credit rate stands for the user cost of housing capital. This is a more assessable measure as tax costs and benefits or price expectations would be included in a full-fledged user cost. These elements could hence translate into changes in our coefficients. As price expectations are not easy to catch, this is a flexible way to take into account this determinant of housing demand. Endogenous determinants are housing supply and housing loans. We expect a negative elasticity of housing prices to supply, as supply shocks would translate into a price decrease, and a positive elasticity of prices to credit as easier credit conditions would support housing demand.

The housing stock, drawn from the housing supply equation (Eq. 9), depends on building costs, tax credit conditions and new housing prices. Increase in building costs reduces construction profits of firms and constitutes a negative supply shock. The French government has been very active in implementing tax incentives for building-to-let constructions, leading to cycles with large amplitudes in this component of housing supply. Finally, new housing prices are introduced alongside second-hand prices. Housing supply should be positively impacted by prices as higher prices would lead to convert more land into residential use and are usually associated with higher profits for construction firms. New housing prices, although they are imperfectly measured over part of the period, should be better related to housing supply than second-hand prices, which stand for all demand determinants from the inverted demand equation. Housing loans are included as the other endogenous determinants and stand for construction financing and overall credit conditions.

The credit demand equation (Eq. 10) enables us to relate housing credit volume to the spread of housing loan rates over government bonds and loan duration. The spread represents the premium on housing loans and banks’ financing conditions, which reflect, among others, specific prudential regulation on housing loans and banks’ assessment of housing loans risk. Longer loan duration tend to relax credit conditions, as a higher amount can be borrowed for the same monthly loan service. Loan duration has increased over the period, significantly supporting credit demand. Regarding endogenous determinants, housing prices should be positively related to loans due to a nominal effect, as housing loans are deflated by the private consumption deflator. The housing stock should also be positively related to housing loans, due to a volume effect.

3. Empirical strategy

a. Motivation

Time series are often impacted by structural breaks. This could stem from supply or financial shocks, technological or policy change. For example, following some recent findings, monetary policy has become more transparent and aggressive against inflation in the last few decades. This evolution can yield some changes in the behavior of firms and households.

Furthermore, it has now been accepted that the long-run trends in some macroeconomic or financial variables can dramatically change over time and shifts in relationships between economic variables can arise.

These new phenomena necessitate the introduction of certain assumptions in the modeling of the dynamics of economic variables. The starting point of the procedure consists in testing linearity against a permanent structural change. In order words, whether an economic time series should be described by some linear or nonlinear models needs to be tested. For example, nonlinear dynamic models (such as threshold and Markov-switching models) could help capture such changes. However,
one of the most recent empirical nonlinear dynamic models consists in modeling time variation in the parameters of the VAR models. These VAR models are applied mostly to the analysis of the monetary policy. However, as far as we know, they have not been applied to the joint dynamics of housing and credit markets.

Besides, a growing number of papers have validated the hypothesis of time-varying volatility especially in advanced economies. For example, according to Clark and Ravazzolo (2015), Clark (2011), Jore et al. (2010), the combination of time-varying parameters for the conditional mean and stochastic volatilities improves the accuracy of forecasts (point and density). D’Agostino et al. (2013) provided us with some systematic comparisons between the prediction performances of many models including the time-varying VAR with Stochastic Volatility, the fixed coefficients VARs, the time-varying AR with Stochastic Volatility and the naïve random walk model. They performed their predictions for three US variables: the inflation rate, unemployment rate and interest rate. They concluded that the TVP-VAR combined with a stochastic volatility model systematically delivers accurate forecasts and perform better than the other models in terms of inflation prediction. Besides, they also found that both time-varying coefficients and stochastic volatilities play a key role in the predictability of the variables. These findings enhance and complete the findings of Clark (2011) who highlighted the role of volatilities. The results by Pettenuzzo and Timmermann (2015) are in the same vein.

In addition, the motivation of the use of this combination can be found in the fact that (d’Agostino et al., 2013): a) the variations of the coefficients (predictable components) can stem from the changes in the structural dynamic links among macroeconomic factors; b) the stochastic volatility (unpredictable component) can be associated with the variations in the size of the exogenous shocks or their effects on macroeconomic variables. Furthermore, the real effects of volatility are more and more significant in the macroeconomic literature (Christiano, Motto and Rostagno, 2014).

On top of that, to model time variation in the volatility of macroeconomic variables is crucial to the accuracy of some types of inferences (for example, the improvement in the efficiency of the General Least Squares Estimates (GLSE) or the accuracy of density forecasts).

The above discussion leads us to test the ability of a combination of a VAR model with drifting parameters and a stochastic volatility model to describe the joint dynamics of housing and credit markets. Both changes in the parameters and instabilities in volatility allow us to question or challenge structural changes in these markets since the housing and credit markets have been impacted by dramatic changes in the last few decades. Therefore, these markets should be good candidates to experiment the performance of these kinds of models. Lastly, to our knowledge, there are no empirical works regarding these models applied to the French housing and credit markets.

c. The empirical specification of the model

The baseline equation of the model is as follows:

\[ y_t = c_t + B_{1t} y_{t-1} + B_{2t} y_{t-2} + B_{3t} z_t + u_t \]  

(Eq.11),

where:

- \( y_t \) is an \( n \times 1 \) vector of endogenous variables;
- \( c_t \) is an \( n \times 1 \) vector of time-varying coefficients that scale up the intercept term;
- \( z_t \) is a \( k \times 1 \) vector of exogenous variables;
- \( B_{1t} \), \( i = 1, 2 \) are \( n \times n \) matrices of time varying coefficients of the lagged endogenous variables;
- \( B_{2t} \) is an \( n \times k \) matrix of time varying coefficients of the lagged endogenous variables.

The usual tests validated a VAR (2) for describing the joint dynamics of housing, housing stock and housing loans.
$u_t$ are the heteroskedastic unobservable shocks with variance covariance matrix $\Omega_t$. The common triangular reduction of $\Omega_t$ allows us for guaranteeing the existence of a positive definite matrix. So, $\Omega_t$ is factored as:

$$ A \Omega_t A = \sum_t \Sigma_t^t $$

(Eq.12),

where $A$ is a time invariant lower triangular matrix whose elements ($\alpha_{ij}$, $i,j = 1, ..., n$) represent the invariant conditional covariance of the innovations of the model. $A$ is defined as follows:

$$ A = \begin{pmatrix}
1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
\alpha_n & \cdots & 1
\end{pmatrix}.$$ 

$\Sigma_t$ is a time-varying diagonal matrix whose elements are the variances for the innovations of the models ($\sigma_{it}$, $i=1, ..., n$). $\Sigma_t$ is defined as follows:

$$ \Sigma_t = \begin{pmatrix}
\sigma_{1t} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sigma_{nt}
\end{pmatrix}. $$

We denote $\Sigma_t^t$ the transpose of the matrix $\Sigma_t$. These variances are driven by stochastic volatility processes. The baseline equation can be laid out as follows:

$$ y_t = c_t + B_{1t} y_{t-1} + B_{2t} y_{t-2} + B_{3t} z_t + A^{-1} \Sigma_t \varepsilon_t $$

(Eq.13),

where $\varepsilon_t$ is a standard normal random vector and $Var(\varepsilon_t) = I_n$.

The previous equation can be written in matrix form (Primiceri, 2005):

$$ y_t = X_t^t B_t + A^{-1} \Sigma_t \varepsilon_t $$

(Eq.14),

where $X_t^t$ represents the transpose of a matrix $X_t$ of endogenous and exogenous variables built as follows: $X_t = I_n \otimes \{y_{t-1}; y_{t-2}; z_t; 1\}$ and $B_t$ is a vector where all the RHS time-varying coefficient are stacked.

The processes driving the time varying parameters are specified as follows:

$$ B_t = B_{t-1} + \theta_t $$

(Eq.15)

$$ \log \sigma_t = \log \sigma_{t-1} + \mu_t $$

(Eq.16),

where the innovation $\theta_t$ is normal with mean zero and variance covariance matrix $Q$; $\sigma_t$ is the vector of diagonal elements of $\Sigma_t$; the innovation of the volatility, $\mu_t$, is normal mean zero and variance $W$.

The elements of $B_t$ are specified as random walk with no drift. Nevertheless, we follow Cogley and Sargent (2005) and we introduce a reflecting barrier that restrict and truncate the random walk prior. Therefore, the roots of the associated VAR polynomial are inside the unit circle, and as a result, the stability of the VAR is validated. Parameters $\sigma_t$ follow geometric random walks.

The last three equations (Eq. 14, Eq. 15 and Eq. 16) are the core relations of the model which explain the evolution of the endogenous variable $y_t$; the innovations are associated with the following vectors: $\varepsilon_t, \theta_t, \mu_t$. According to Primiceri (2005) and Cogley and Sargent (2005), the innovations of the model are jointly and normally distributed but not correlated with each other. The variance covariance matrix of the innovations is as follows:
\[
\text{Var}(\varepsilon_t, \theta_t, \mu_t) = \begin{pmatrix}
I_n & 0 & 0 \\
0 & Q & 0 \\
0 & 0 & W
\end{pmatrix} = V
\]

As mentioned before, \(I_n\) is an \(n \times n\) identity matrix. The matrices \(Q\) and \(W\) are positive definite. Due to data limitations (the sample spans on 1993Q2 to 2013Q3), we impose the zero-blocks restrictions in this paper.\(^4\)

\[d. \quad \text{Bayesian inference}\]

Bayesian methods are implemented to assess the posterior distributions of the entire unknown parameters of interest (\(B_t\), \(A\), \(\Sigma_t\), the hyperparameters of the variance covariance matrices of the models, the blocks elements of \(V\)- Primiceri, 2005, Cogley and Sargent, 2005). Bayesian methods avoid the pile-up problem or ease high dimensionality and nonlinearity problems and allow circumventing the difficulties related to both the formalization and maximization of the likelihood function of the different models of this kind.

The posterior estimates of the parameters of the models are obtained using a variant Gibbs sampling (Kim and Nelson, 1999). A way to implement the Gibbs sampling is presented in Primiceri (2005) and Cogley and Sargent (2005) which we did not report here.

Prior distributions are chosen by privileging economic intuition and empirical tractability. For the sake of simplicity and parsimony, in empirical works, it is common to impose the independency hypothesis between the initial states of all the coefficients. Besides, regarding the initial states of the time-varying parameters, we assume that they are normally distributed (Sims and Zha, 1998, Smith and Kohn, 2002).

The initial means and variances are drawn from a time-invariant VAR using data covering five years\(^5\). To complete the identification of these distributions, we assess their means, variances, degrees of freedom and scales matrices, following the proposals of Primiceri (2005) and Cogley and Sargent (2005).

The priors for the hyperparameters, the elements of the matrices \(Q\) and \(W\), are distributed as inverse-Wishart under the independence hypothesis. For example, the prior for the matrix \(Q\) is inverse-Wishart defined as follows:

\[
f(Q) = IW(Q^{-1}, T_0),
\]

where \(T_0\) is the degree of freedom of inverse-Wishart and \(T_0\) \(\tilde{Q}\) a scale matrix.

In their baseline applications, Cogley and Sargent suggested to set out \(T_0\) (which has to be greater than \(\text{dim}(B_t)\)) and \(\tilde{Q}\) as follows: \(T_0 = \text{dim}(B_t) + 1\) and \(\tilde{Q} = y^2 \tilde{P} ; \tilde{P}\) is the asymptotic variance covariance matrix of \(B_t ; y^2 = 3.5e^{-04}\).

Therefore, an analysis of the sensitivity of the empirical results could be done by changing the value of \(y\) or \(T_0\). As mentioned by Cogley and Sargent (2005), the size of the drift in posterior estimates of the elements of \(B_t\) depends on the degree of freedom and the scale matrix by Primiceri (2005) and Cogley and Sargent (2005).

\[^{4}\] All the zero-blocks could be substituted by the non-zero-blocks with only a few modifications of the estimation procedure (Primiceri, 2005).

\[^{5}\] The effect on the results of the length of the period used to calibrate the previous parameters could be evaluated through an analysis of sensitivity; we report the results of this analysis in the robustness checks.
In this model, the VAR ordering may affect the results because of how VAR innovation variances depend on the stochastic volatilities. For this reason, it is crucial to test the effect of all possible orderings on our results. We present all the results in the robustness section but, following Cogley and Sargent (2015), we focus on the ordering that minimizes the rate of drift in $B$, especially in the core part of the paper. In addition, these exercises could be completed by a comparison of different estimates across parallel chains set on different initial conditions.

4. Data

We use quarterly and seasonally-adjusted over the period 1993:Q1-2013:Q4, drawn from the French National Institute of Statistics and Economic Studies (INSEE) database.

We use three endogenous variables in our VAR representation, namely housing prices, housing capital stocks and housing credit. We also use various exogenous variables that are presented in this section.

The main housing prices series are existing housing seasonally-adjusted hedonic price indices from INSEE. Existent housing prices indices are privileged as an acquisition of second-hand accommodation makes up for three quarters of the value of all owner-occupier acquisitions as no hedonic price series for new accommodation existed before 2006. Differences between the two series can result from composition effects (new home prices are not hedonic) but they may also be due to evolutions of building costs. Therefore, to capture the impact of new home prices in the building sector dynamics, we put up an index for newly-built property prices based on the INSEE index available from 2006 onwards; they have been retropolated since 1993 using the quarterly variations of the average square meter sales price of newly-built flats from a survey conducted by the Housing Ministry. Real housing prices are obtained by deflating nominal housing prices series by the private consumption deflator.

The housing capital stock is drawn from the INSEE yearly household wealth account, excluding land; it is deflated by residential investment deflator. The INSEE housing stock series is built mixing a specific survey and a permanent inventory methodology (INSEE, 2008). The quarterly series of the housing stock is built using the residential investment quarterly profile. The housing stock does not only reflect the number of square meters available but also takes into account the quality adjustment stemming from residential investment, which is consistent with the hedonic housing price indices. In the housing supply equation, we use the seasonally-adjusted building cost index from INSEE. As mentioned above, private investments in housing is widely supported by public authorities (tax reductions granted for rental investments, reduced VAT rate, subsidised loans) aiming at fostering the private supply of rental housing and the share of owner-occupant as well as the quality of private housing. The overall support to private housing suppliers accounted for 19.2 billion euros in 2012 – close to 1% of the French GDP. Such public interventions are likely to affect the housing market dynamics all the more since the public spending dedicated to those policies has not been constant over the studied period.

Nevertheless, using the annual series of the public subsidies to the private housing sector is arguably inadequate to understand their causal impact on the residential housing market for at least two reasons. First, the date when the subsidy affects the agents’ behaviour and the date when it is effectively recognised as a cost in the public accounts may differ - notably because of tax credits that can be claimed over a long period. Second, the public expenditures can result from deadweight effects which are not informative to understand their causal impacts. Instead, we use a measure of the attractiveness of the rental investment incentive schemes by computing the actual value tax savings that can be expected by a representative household given the applicable incentive scheme at each date. The series is deflated by the private consumption deflator.

6 From 1993 to 1995, INSEE series are not available; we retropolated them on a real estate agents network (FNAIM) data. This is one of the series used by INSEE to assess housing prices in the wealth accounts. For retropolation purposes, an alternative way consists of using INSEE housing prices regarding only Paris. However, there are some empirical evidences showing that there is a significant divergence between Paris and the rest of France housing prices over the period spanning from 1993 to 1995. Therefore, we privilege FNAIM data.
Housing credit here is the stock of credit to households contracted to buy an accommodation, new or old. Hence, they are not fully consistent with the existing housing price indices used. They include securitised amounts; and as a consequence, they are not disrupted by the securitisation policies of the banks. In order to be consistent with the treatment of housing prices, we deflate housing credit by the private consumption deflator. Our housing credit rates are annual interest rates charged on new housing credit to resident households net of the contemporaneous annualised variations of the private consumption deflator.

We also build a series for the spread between the rates of new housing credit to resident households and the French ten-year government bond. This spread acts as an opportunity cost of capital; it should be an important driver of the housing credit market as in 2012, 53.2% of the real estate activity (investment and acquisitions) was self-financed. As a result, households are often in a position to arbitrage between housing credits and self-financing. In this way, the spread should be one of the prominent determinants of the result of such an arbitrage. The average duration of the housing loans at origination is also another significant determinant of the housing credit market because it affects the borrowing capacity of the households for a given gross debt service ratio (Tsatsaronis and Zhu, 2004, Antipa and Lecat, 2013). We use the quarterly series of the average duration of new housing loans expressed in months.

The demographic factor introduced in the model is the seasonally-adjusted quarterly population series from Eurostat database. Finally, permanent income is approximated by the current gross disposable income per capita deflated by the private consumption deflator.

The Phillips-Perron test does not allow rejecting the presence of unit roots for all the above-mentioned series except for the spread between the rates of housing credits and the French ten-year government bond. The housing credit rate series is taken in first-difference; all the other series except the population are taken in log-first-difference. As the log-second-difference of population is stationary, it is the variable that we use in our empirical studies.

We report the graph of the series of housing prices, housing loans and housing stocks in Figure 1. French housing prices first declined in the middle of the nineties and then sharply increased, often above 5% per year, over a 10-year period (between the beginning of 1997 and the end of 2007). The growth of housing prices decelerated during 2007 and prices decreased between the first quarter of 2008 and the third quarter of 2009 with a trough at -2.9% during the first quarter of 2009, the overall decrease over the period roughly equalled 10%. Housing prices rebounded between 2010 and 2011; we have witnessed a new drop, albeit at a moderate pace, since 2012. Housing loans tend to co-move with housing prices; the contemporaneous correlation coefficient between the two series equals 0.60 (Appendix A.2). Nevertheless, we did not observe a rapid fall in housing loans similar to the one observed in the housing prices during the financial crisis. This feature reveals a downward stickiness of the housing loans. The fluctuations of the housing stock are less volatile than those of housing prices and housing loans stock. The housing stock is contemporaneously uncorrelated with the two other variables.
The real cost of housing credit in France has sharply decreased since the beginning of the nineties, strongly supporting housing credit. This evolution closely follows the pattern of government long-term financing rates. Indeed, the spread between the housing credit rate and the French ten-year government bond yields is contained within a bracket of 0 to 200 basis points. Over the same period, the average loan duration at origination has significantly increased during the price boom period and has stayed stable around 230 months since 2006. This increase has strongly supported the households’ indebtedness capacity: for a given debt service to income ratio, longer maturity increases reachable credit amount, despite the increase in interest rates along the yield curve. We report the contemporaneous correlations between the different variables as well as the complementary diagnosis statistics in Appendix A.2.

5. Empirical Results

The first issue is whether we are correct in trying to emphasize time variation in the VAR coefficients. We can illustrate this point by examining the posterior histogram for the trace of $Q$ (hereafter tr $(Q)$) along with the trace of the prior value $Q$ (see Figure 2). The sample points toward some time variation in the VAR coefficients, as the posterior distribution of tr $(Q)$ is shifted to the right.
This estimation method allows us to discriminate between the effect of changes in the variances of innovations and changes in coefficients. The standard errors of innovations for the three equations are represented in Figure 3. For housing prices, the building-up period of the bubble (1999-2006) displayed a moderation in the variance of innovations. Hence, larger shocks did not seem to be at the root of the very fast appreciation of housing prices. Innovation variance then peaked in 2010 with the financial crisis. For the housing stock, a similar peak is observed in 2009, which was building up since 2003. For the housing loans, innovation variance has been decreasing ever since 2000, which does not reflect the building-up period of the bubble either. Therefore, innovation shocks do not explain the very fast increase of housing prices during the 2000s, but reflect the impact of the subsequent financial crisis on housing prices and stock.
The pairwise contemporaneous correlations between the VAR innovations are presented in Figure 4. Housing stock innovations are positively correlated with housing prices and negatively with housing loans, but both correlations are very low. This reflects the weak elasticity of construction to housing prices, as housing stock innovations are weakly related to the other two housing variable indicators. Correlations between innovations in housing price and housing loan are positive and much stronger, as could be expected. During the building-up of the housing bubble, correlations tended to decrease between the three variables: exogenous factors seem to be at play here.

**Price-Stock**

**Price-Loans**
Finally, the total predicted variance (see Figure 5), corresponding to the determinant of the matrix $A^{-1} \Sigma_t$, can be interpreted as a “measure of the total uncertainty entering the system at each date” (Cogley, Sargent, 2005). It peaks in 2009, with the financial crisis, increasing from 2003 onwards. Uncertainty has not been particularly high during the building-up of the bubble.

As the explanation of the housing prices increase by stronger shocks can be questionable, we now turn to the change in the VAR coefficients (See Figures 6, 7 and 8 for coefficients prior and posterior means). To evaluate the significance of our estimates, we define credible intervals based on the posterior distributions by choosing the narrowest interval containing 90% of the observations. These intervals are called the highest posterior density (HPD) intervals. Hence, an estimate is assumed to be significant if the HPD does not contain the value 0. Turning first to the auto-regressive coefficients, there is an upward drift in the coefficients of the price, when summing t-1 and t-2 lags (+0.21 from 1998Q2 to 2008Q2). As the coefficients are positive, shocks have become more persistent on this market. This tends to support the idea that the dynamics of expectations may have played a role in the price increase, as the evolution in prices tended to be in line with its past dynamics until 2008.

When looking at the price equation, the sign and magnitude of the coefficients are as expected. Overall, for the housing stock, the coefficient is negative and non-significant. It is quite positive but also not significant enough for loans; regarding gross disposable income, the coefficient is less than 1, positive and significantly different from 0; it displays a negative and significant sign for credit rates.

---

8 Indeed, according to Jurado, Ludvigson and Ng (2015), if the economy has become more or less predictable, it is more or less uncertain. However, it is a common knowledge that no objective measure of uncertainty exists even though common proxies of uncertainty (implied volatility, etc.) are often used in the empirical literature. Nevertheless, Jurado et al. provided us with new measures of uncertainty relatively independent from the structure of some specific theoretical models. We did not implement these new measures in this paper for the sake of simplicity.
Finally, for the population factor, it is positive, but not always significant. Its coefficient has been increasing significantly from 1998 onwards and reached a peak in 2008. Our population indicator is the change in the seasonally-adjusted population growth rate; therefore, higher growth in population leads to a higher price level; average population growth rate is 0.5% over the estimation period and exhibits limited fluctuations. Thus, a non-linearity in the impact of a change in the population growth rate, which could be justified by regulatory and space constraints when demand grows above a threshold, may be at play in the price dynamics. The growth rate of population has peaked twice over the period, from 1998Q2 to 2000Q1 (average change of +12bp each quarter in the quarterly population growth rate) and from 2003Q2 to 2005Q1 (average change of +2bp each quarter). These two episodes have coincided with high increases in the housing prices. The change in the pace of population growth has accounted for 1.5% of the housing prices rise during the first episode (1998Q2 to 2000Q1) but 3.3% during the second episode (2003Q2 to 2005Q1).

The growing contribution of demographic factors to the variations in housing prices, reflected by the increase in the coefficients, can also be appreciated by looking at the contribution of population to the trend reversal in housing prices between 2008Q1 and 2009Q2. This period is included in a timeframe where the population grew more slowly (average change of -3bp each quarter in the quarterly population growth rate). The results suggest that this lack of dynamism has, on average, contributed to a quarter of the price decrease in 2008 and in the first half of 2009 (i.e. 2 percentage points over 8). We also find that the rebound of the housing prices in 2010 was fueled by bullish demographic trends with an average contribution of 10% of the price rise (i.e. 0.6 pp to the 6% increase). This coefficient increase may be rooted in the growing construction bottlenecks. Physical and regulatory constraints on construction (which are particularly stringent in France, Caldera and Johansson, 2013) led to a price overreaction facing these demographic shocks.
This is confirmed by the housing stock equation. As expected, the housing stock is not significantly affected by second-hand housing prices but is positively impacted by new housing prices. However, the elasticity of the housing stock to new home prices is low and has not increased over the period. The housing stock is positively impacted by housing loans and tax credit programs and negatively by building costs.
Tax credit for rental investment

Housing price – new

Figure 7: VAR coefficient in stock equation - Prior and posterior mean. Note: Significant estimates are within the 90% HPD interval’s bands

Turning to housing loans, they are positively impacted by housing prices and by loan duration (although not significantly) and negatively by the spread between housing loans and government bond rates. No significant relationship with the housing stock is detected. The positive impact of the housing price is mostly a nominal effect. Increase in the loan initial duration was one of the main credit conditions loosening over the period and supported the increase in purchasing capacity of households; however, its coefficient tends to weaken over the period. Spreads reflect the credit risk premium over housing credits and may be correlated with overall credit conditions, as banks may grant credits more easily when the risk is low. Spreads were at their lowest at the peak of the bubble and supported the price increase through housing loans. These elasticities have not changed significantly over the period, except for housing prices. The cumulated impact of housing prices on loans is negative, as the nominal effect could have been overcome by the negative impact on credit demand from high housing prices.
From these equations, we can dynamically predict the values for the endogenous variables $y_t$ between 1998Q3 and 2013Q3, conditional on the initial values of the endogenous variables in 1998Q1 and 1998Q2 and the exogenous $z_t$. We rescale the level of housing prices, housing stock and housing credit at 100 in 1998Q3 and graph their predicted dynamics from 1998Q3 onwards, as well as their observed levels, in Figure 9. Housing prices are above the mean predicted housing price level during the whole period, with a peak in 2008. Even with drifting parameters, the gap between observed and mean-predicted levels of housing prices is high (+13.2%) at the peak of the bubble and remains high by the end of the period (+5.9%). However, it is noticeable that allowing for drifting parameters improves the fit of the model with respect to the observed housing price index (see the dashed curve corresponding to predicted levels derived from the estimation of a fixed coefficients homoscedastic VAR). For the housing stock, observed levels appear to be close to mean-predicted levels.

For housing loans, observed levels were close to mean-predicted ones until 2006 and then overcame them. The fixed coefficients homoscedastic VAR does a good job at replicating the housing loan stock at the end of the period but generates a quasi-linear trend that is at odds with the observed series. It is worth noting that even though we estimate fixed coefficients VARs using a rolling or a recursive estimation scheme these latter do not perform as well as the TVP-VAR with stochastic volatility specification in terms of forecasting (d’Agostino et al, 2013). Similar conclusions can be drawn when one compares the performances of a stochastic volatility VAR (with fixed coefficients) and the stochastic volatility TVP-VAR.
Drifting parameters imply that shocks will have different impacts through time. A 100 basis points permanent increase in housing loan rates had a slightly different impact in 2000 and 2008. Housing prices decreased after 2 years by 0.6% in 2000 and 0.4% in 2008 for a shock; these findings are in line with the results of Avouyi-Dovi et alii (2014).
6. Sensitivity Analysis

We run some additional regressions in order to validate the empirical results regarding our baseline model. In their seminal paper, Cogley and Sargent (2005) examined the impact of some key factors or parameters on the estimates. Their tests were made on the analysis of the link between monetary policies and outcomes in the USA. They relied on many studies regarding this topic to check the empirical validity of their results. The context of our study is quite different from that of Cogley and Sargent. To our knowledge, the systematic analysis of sensitivity of a joint model of housing and credit markets to changes in some crucial conditions seems poorly explored as a research topic. Therefore, the sensitivity tests suggested by Cogley and Sargent to check the relevance and the stability of the empirical results are welcome. In addition to the analyses presented below, we have conducted a sensitivity test consisting of estimating the model with a higher number of iterations and we found the results are not sensitive to the number of draws for the Gibbs sampling.

a. The impact of changes in $\gamma$ on the estimates

Based on the findings of Primiceri (2005) and as suggested by Cogley and Sargent (2005), we test the effects of different values of $\gamma$, on the estimates. For the sake of simplicity, we are only reporting here the results directly regarding the innovations in the VAR (prior and posterior distributions of the matrix $Q$, time-varying standard deviation of the innovations, and time-varying correlations between the innovations$^{10}$). We compare the results corresponding with three values of $\gamma$ (0.01; 0.05; 0.10) to those drawn from the benchmark model obtained with $\gamma = (3.5e-04)^{0.5}$.

Regarding the prior and posterior distributions (see Figure 10), we do not notice any significant differences between the patterns of the distributions drawn from the four regressions. However, the largest values of $\gamma$ displayed the most volatile distribution. In fact, the distributions corresponding to the smallest values of the scale parameter are the closest. These distributions, especially the posterior distributions, are not sensitive to the smallest values of $\gamma$.

\[
\text{Benchmark} - \gamma = (3.5e-04)^{0.5} \quad \gamma = 0.01
\]

$^9$ We have notably estimated our model with 15,000 draws for the Gibbs sampling instead of 5,000 draws in the baseline estimation. The results are available upon request.

$^{10}$ Regarding the robustness check exercises, detailed results on the time-varying coefficients of different blocks of equations (the time-varying coefficients of different blocks of equations, etc.) are available upon request.
$\gamma = 0.05$

$$L_0 = 0.05$$

$\gamma = 0.10$

$$L_0 = 0.10$$

Figure 10: Trace of the prior for $Q$ and the posterior distribution of the trace of $Q$ – the four values of $\gamma$

Compared to the time-varying standard errors of the innovations related to the baseline value of $\gamma$, those drawn from the alternative estimations are broadly identical (see Figure 11). Overall, the patterns of the standard errors do not depend on the values of $\gamma$ whatever the innovations. The results are not sensitive to the smallest values of gamma. In addition, the intervals of the variation of the standard deviation only depend on the innovations. In other words, we do not observe any dependency of gamma regarding the time-varying standard deviations. The only differences observed here are related to some specific values of the standard deviations at certain scarce periods. However, these differences do not allow us to reject the general view of the similarities of the distributions of the standard deviation whatever the value of $\gamma$.
Regarding the correlations between the innovations of the VAR associated with the four different values of $\gamma$, we notice some common characteristics (see Figure 12). Overall, whatever the values of $\gamma$, the correlation between the housing prices and housing stock, and that between the housing stock and housing loans are very small (less than 0.10). That is to say, the shocks affecting the housing price and housing stock and the shocks impacting the housing loans and housing stock are uncorrelated, whatever the choice of the value of $\gamma$. Regarding these variables, the correlations do not depend on $\gamma$. In contrast, we cannot reject the hypothesis of the dependence of the shocks affecting the housing price and housing loans. Indeed, the correlation associated with these shocks ranges from 0.22 to 0.33, whatever the value of $\gamma$ and is relatively high for the smallest values of $\gamma$. For instance, for $\gamma = 0.10$, the correlation ranges from 0.22 to 0.26. In this case, the pattern of the time-varying correlation is quite different from those corresponding to the smallest values of $\gamma$. 

![Figure 11: Posterior mean of the log of VAR innovations variance – the four values of $\gamma$](image)
To sum up, it seems clear that the changes in values of \( \gamma \) do not impact the distributions of the innovations and their dispersion or linkage indicators. We reach a similar conclusion when we look at the time-varying coefficients of the models.

b. The effects of the initialization period on the estimates

The length of the period of the initialization impacts the empirical results. In our baseline model we use the 20\(^{th}\) first quarters (i.e. 1993Q2 to 1998Q1) to obtain our initial states. For homogenization orders, we comment on the figures similar to those of the previous paragraph (see Figures 13 and 14). If we compare the time-varying standard deviations and correlations of the baseline regression to that obtained by increasing the initialization periods to 29 quarters (i.e. 1993Q2 to 2001Q2), some noticeable differences appear. First, the patterns of the standard deviations and those of the correlations are quite different in both cases. Second, the intervals of variation of both the standard deviations and correlations are also different even though the same variables are uncorrelated in the two cases. Third, the results corresponding to the extended period of the initialization are significantly different from those obtained in the previous regressions.

In addition, when we look at the posterior distribution of the \( \text{tr}(Q) \), the differences between the results linked to the two alternative conditions are substantial (see Figure 15). When we examine the time-varying coefficients of the equations of the price, the stock or the credit, we also notice significant discrepancies among the results\(^{11}\). The length of the initialization period has an important impact on the empirical results.

\(^{11}\) Detailed results on the time-varying coefficients of different blocks of equations are available upon request.
Figure 13: Posterior mean of the log of VAR innovations variance – baseline and initialization on 29 quarters
c. The impact of the VAR ordering on the estimates

We test the effects of the ordering of the VAR variables by running the regressions at every possible ordering of the three endogenous variables (price/stock/loans; price/loans/stock; stock/loans/price; stock/price/loans; loans/stock/price and loans/price/stock). As already mentioned, our baseline model is built with the ordering price/stock/loans following the “rule of thumb” proposed by Cogley and Sargent (2005) that consist in minimizing the drift in $B$. The innovations, standard deviations, correlations and posterior distributions are strongly linked to the ordering of the variables of the VAR (see Figures 16, 17 and 18). Overall, there are a few similarities among the standard deviations or the correlations. Sometimes, the convexity of the curves of these indicators is substantially different. For instance, the time varying correlation between the housing prices and the housing loans display is convex for two trinomials and concave for two others (see Figure 18).

We reach more or less the same conclusion regarding the posterior distribution (see Figure 16). The remarks regarding the time-varying coefficients of the VAR variables are completely coherent with the previous analyses. The ordering of the VAR impacts the dynamics of the coefficients. Therefore, it is crucial to look for the optimal ordering before implementing the time-varying VAR.
Figure 16: Posterior mean of the log of VAR innovations variance – All VAR orderings
Figure 17: Posterior mean of the correlation between the VAR innovations – All VAR orderings

Baseline - Price/Stock/Loans

Price/Loans/Stock

Stock/Loans/Price

Stock/Price/Loans

Loans/Stock/Price

Loans/Price/Stock
7. Conclusion

Our model of the French housing and credit markets is based on an empirical VAR combined with drifting parameters and stochastic volatilities. As traditional models fail to explain the price increase during the 2000s, this framework allows taking into account of the instability of the coefficients and the nonlinearities of the links between the endogenous variables and their fundamentals. In addition, this makes it possible to describe the variance and persistence of the shocks. It is also particularly relevant to handle endogeneity biases and macroeconomic time-series, with a limited number of observations.

The French housing market is somewhat of a puzzle, as the sharp price increase during the 2000s was not matched by a mirroring reversal, as in many other advanced countries. This persistent bubble yields many hypotheses, and, among them, drifting parameters for some fundamentals, reflecting nonlinearities, is a particularly prominent one. Others may rely on the dynamics of price expectations or a shock in credit conditions. Identifying the source of the bubble and its persistence is obviously relevant for policy analysis, as it allows us to determine the risk of brutal reversal. Price expectations or credit conditions may indeed deteriorate sharply, while other fundamentals, such as demographic factor or income, are more inert. The presence of nonlinearities is also often neglected by such studies and this model may be relevant for the analysis of other housing markets.

We show that variances of the innovations did not increase during the bubble period, emphasizing that the increased shock variance is not at the root of the bubble. Auto-regressive coefficients increased somewhat for housing prices and credit, testifying for greater shock persistence and maybe for stronger price expectations dynamics. The coefficients of the demographic indicator and income moved upward during the period, which makes up for the nonlinearity we were looking for. The low elasticity of the housing stock to prices may explain this nonlinearity as housing prices may overreact to a demand shock in case of construction bottlenecks. However, even in such a flexible setting, a part of the price increase remains unexplained.

These kinds of models yield a valuable insight on the housing market evolutions, taking into account of the variance and persistence of the shocks and nonlinearities in the relationships between the explained variables and their fundamentals. However, many challenges still remain. In particular, price expectations may play a central role in price dynamics but cannot often be explicitly included or highlighted, remaining among the residuals of the estimates or being reflected in shock persistence. Nevertheless, absent reliable price expectations surveys, flexible settings such as the ones we use appear all the more relevant to analyse the housing market.
References


APPENDIX: DESCRIPTIVE STATISTICS

Table A.1: Complementary statistics

<table>
<thead>
<tr>
<th>Housing prices</th>
<th>Housing prices - new</th>
<th>Housing stocks</th>
<th>Housing loans</th>
<th>Housing rates</th>
<th>Spread</th>
<th>Loan duration</th>
<th>Disposable income</th>
<th>Population</th>
<th>Building costs</th>
<th>Tax credit</th>
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<tbody>
<tr>
<td>Mean</td>
<td>0.0077</td>
<td>0.0043</td>
<td>0.0058</td>
<td>0.0121</td>
<td>0.0066</td>
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<td>0.0054</td>
<td>0.0109</td>
<td>0.0058</td>
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<td>Standard Deviation</td>
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<td>0.0043</td>
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Table A.2: Correlations

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<th>Housing loans</th>
<th>Housing rates</th>
<th>Spread</th>
<th>Loan duration</th>
<th>Disposable income</th>
<th>Population</th>
<th>Building costs</th>
<th>Tax credit</th>
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Relative standard deviation

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<th>0.003</th>
<th>0.839</th>
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