Network Risk and Key Players: A Structural Analysis of Interbank Liquidity

Edward Denbee* Christian Julliard*†
Ye Li‡ Kathy Yuan*†

*London School of Economics, †CEPR ◊Bank of England ‡Columbia University

10th Journée of the Fondation Banque de France

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.
Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Traditional regulatory tools focused on bank-specific variables (e.g. capital ratios) and risk (e.g. default probabilities).

Macro-prudential regulation seeks tools to quantify the systemic implication of individual bank’s behavior ⇒ e.g. banks that generate more systemic risk could face more stringent requirements.

Our paper: develops such a tool using a network framework.

Using a linear quadratic model, we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.
Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry. Traditional regulatory tools focused on bank-specific variables (e.g. capital ratios) and risk (e.g. default probabilities). Macro-prudential regulation seeks tools to quantify the systemic implication of individual bank’s behavior ⇒ e.g. banks that generate more systemic risk could face more stringent requirements.

Our paper: develops such a tool using a network framework. Using a linear quadratic model, we can identify:
1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.
The Case Study: Intraday Liquidity in Payment System

- On average, in 2009, £700bn of transactions were settled every day across the two UK systems, CREST and CHAPS: the UK nominal GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.
  ⇒ Almost all banks in CHAPS regularly have intraday liquidity exposures in excess of £1bn to individual counterparties. For larger banks these exposures are regularly greater than £3bn.
  ⇒ We study banks’ intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.
The Case Study: Intraday Liquidity in Payment System

- On average, in 2009, £700bn of transactions were settled every day across the two UK systems, CREST and CHAPS: the UK nominal GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.
  - Almost all banks in CHAPS regularly have intraday liquidity exposures in excess of £1bn to individual counterparties. For larger banks these exposures are regularly greater than £3bn.

  \[\Rightarrow\] We study banks’ intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.
Outline

1. Theoretical Framework
   - Network Specification
   - Bank Objective Function and Nash Equilibrium
   - Risk, and Level, Key Players

2. Empirical Analysis
   - Empirical Specification
   - Network and Data Description
   - Estimation Results

3. Conclusions

Appendix
Outline

1. Theoretical Framework
   - Network Specification
   - Bank Objective Function and Nash Equilibrium
   - Risk, and Level, Key Players

2. Empirical Analysis
   - Empirical Specification
   - Network and Data Description
   - Estimation Results

3. Conclusions

Appendix
Network Specification

- A directed and weighted network of $n$ banks.

Network $g$ : characterized by $n$-square adjacency matrix $G$ with elements $g_{i,j}$, and $g_{i,i} = 0$.

$g_{i,j \neq i}$ : the fraction of borrowing by Bank $i$ from Bank $j$.

$\Rightarrow$ $G$ is a (right) stochastic matrix and is not symmetric.
Bank Objective Function

- **Bank $i$ decision variables:**

$$l_i := q_i + z_i$$ is the observable liquidity holding of bank $i$, where:

- $z_i$ is the **network component** of liquidity buffer stock.

- $q_i$ is the liquidity level of bank $i$ **absent bilateral effects**, given by

$$q_i = q_i(x) := \alpha_i + \sum_{m=1}^{M} \beta_m x_i^m + \sum_{p=1}^{P} \beta_p x^p$$

  - Fixed effect
  - Characteristics
  - Common factors
Bank Objective Function cont’d

- A quadratic payoff function for buffer stock liquidity $z_i$

$$u_i(z_i | g) = \hat{\mu}_i \left( z_i + \psi \sum_j g_{ij} z_j \right) - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + z_i \delta \sum_j g_{ij} z_j$$

\[ \hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d \left( 0, \sigma_i^2 \right) \]

Note: $g$ predetermined at decision time (but can change over time).
Bank Objective Function cont’d

- A quadratic payoff function for buffer stock liquidity $z_i$

$$u_i(z_i|g) = \hat{\mu}_i \left( z_i + \psi \sum_j g_{ij} z_j \right) - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + z_i \delta \sum_j g_{ij} z_j$$

$\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d (0, \sigma_i^2)$

Note: $G$ predetermined at decision time (but can change over time).
Bank Objective Function cont’d

- A quadratic payoff function for buffer stock liquidity $z_i$

$$u_i(z_i|g) = \hat{\mu}_i \left( z_i + \psi \sum_j g_{ij} z_j \right) - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + z_i \delta \sum_j g_{ij} z_j$$

Accessible Liquidity

Collateralized Liquidity

$$\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d \left( 0, \sigma_i^2 \right)$$

Note: $\mathbf{G}$ predetermined at decision time (but can change over time).
(Decentralized) Equilibrium Outcome

**Eq.** \( \text{um} \) : (Nash) If \(|\phi| < 1\)

\[
z_i^* = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i
\]

\[
\Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{M(\phi, G)\}_i. \mu
\]

where \(\mu := \gamma^{-1} [\hat{\mu}_1, ..., \hat{\mu}_n]'\), \(\{\}_i\) is the row operator, and

\[
M(\phi, G) := I + \phi G + \phi^2 G^2 + \phi^3 G^3 + ... = \sum_{k=0}^{\infty} \phi^k G^k.
\]

\[
\phi := \frac{\delta}{\gamma} - \psi
\]

If \(\phi > 0\) complementarity (reciprocate/herding/leverage stacks e.g. Moore (2011)).

If \(\phi < 0\) substitutability (free ride à la Bhattacharya and Gale (1987)).
(Decentralized) Equilibrium Outcome

Eq.\textsuperscript{um} : (Nash) If \( |\phi| < 1 \)

\[ z_i^* = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i \]

\[ \Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{M(\phi, G)\}_i \mu \]

where \( \mu := \gamma^{-1} \left[ \hat{\mu}_1, \ldots, \hat{\mu}_n \right]' \), \( \{ \} \) is the row operator, and

\[ M(\phi, G) := I + \phi G + \phi^2 G^2 + \phi^3 G^3 + \ldots = \sum_{k=0}^{\infty} \phi^k G^k. \]

\[ \phi := \frac{\delta}{\gamma} - \psi \]

\[ \text{If } \phi > 0 \text{ complementarity (reciprocate/herding/leverage stacks e.g. Moore (2011)).} \]

\[ \text{If } \phi < 0 \text{ substitutability (free ride à la Bhattacharya and Gale (1987)).} \]
Key Players

The total liquidity originating from the network externalities is

\[ \sum_{i=1}^{n} z_i^* = \begin{pmatrix} 1' M(\phi, G) \bar{\mu} \\ 1' M(\phi, G) \nu \end{pmatrix} \]

where \( \bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]' \), \( \nu \equiv [\nu_1, ..., \nu_n]' \)

\( \Rightarrow \) tradeoff: both terms increasing in \( \phi \) (for \( \bar{\mu} > 0 \)).

Risk Key Player: (the one to worry about...)

\[ \max_i \frac{\partial 1' z^*}{\partial \nu_i} \sigma_i = \max_i 1' \{ M(\phi, G) \}_{i,i} \sigma_i \rightarrow \text{outdegree centrality} \]

Level Key Player: (the one you might want to bailout...)

\[ \max_i E [1' z^* - 1' z^*_i] = \max_i \{ M(\phi, G) \}_{i,i} \bar{\mu}_i + 1' \{ M(\phi, G) \}_{i,i} \bar{\mu}_i - m_{i,i} \bar{\mu}_i \]

\( \rightarrow \) indegree centrality + shock analogous – correct double counting.
Key Players

The total liquidity originating from the network externalities is

\[ \sum_{i=1}^{n} z_i^* = 1'M(\phi, G) \bar{\mu} + 1'M(\phi, G) \nu \]

where \( \bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]' \), \( \nu \equiv [\nu_1, ..., \nu_n]' \)

\( \Rightarrow \) tradeoff: both terms increasing in \( \phi \) (for \( \bar{\mu} > 0 \)).

**Risk Key Player:** (the one to worry about...)

\[ \max_i \frac{\partial 1'z^*}{\partial \nu_i} \sigma_i = \max_i 1' \{M(\phi, G)\}_i \sigma_i \rightarrow \text{outdegree centrality} \]

**Level Key Player:** (the one you might want to bailout...)

\[ \max_i E [1'z^* - 1'z_{i}] = \max_i \{M(\phi, G)\}_i \bar{\mu} + 1' \{M(\phi, G)\}_i \bar{\mu}_i - m_{i,i} \bar{\mu}_i \]

\( \rightarrow \) indegree centrality + shock analogous – correct double counting
Key Players

The total liquidity originating from the network externalities is

$$\sum_{i=1}^{n} z_i^* = 1' \mathbf{M}(\phi, G) \bar{\mu} + 1' \mathbf{M}(\phi, G) \nu$$

where $\bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]'$, $\nu \equiv [\nu_1, ..., \nu_n]'$

⇒ tradeoff: both terms increasing in $\phi$ (for $\bar{\mu} > 0$).

Risk Key Player: (the one to worry about...)

$$\max_i \frac{\partial 1' z^*}{\partial \nu_i} \sigma_i = \max_i 1' \{\mathbf{M}(\phi, G)\}_i \sigma_i \rightarrow \text{outdegree centrality}$$

Level Key Player: (the one you might want to bailout...)

$$\max_i E [1' z^* - 1' z^*_{\backslash i}] = \max_i \{\mathbf{M}(\phi, G)\}_i \bar{\mu} + 1' \{\mathbf{M}(\phi, G)\}_i \bar{\mu}_i - m_{i,i} \bar{\mu}_i$$

→ indegree centrality + shock analogous − correct double counting
Outline

1. Theoretical Framework
   - Network Specification
   - Bank Objective Function and Nash Equilibrium
   - Risk, and Level, Key Players

2. Empirical Analysis
   - Empirical Specification
   - Network and Data Description
   - Estimation Results

3. Conclusions

Appendix
Empirical Model

**SEM:** the theoretical framework is matched by a **Spatial Error Model**

\[
I_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \beta_p^{time} x_t^p + z_{i,t}
\]

\[
z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \quad \nu_{i,t} \sim iid \left(0, \sigma_i^2\right),
\]

where \(g_{i,j,t}, x_{i,t}^m\) and \(x_t^p\) are predetermined at time \(t\).

**Note:**
1. Network as a shock propagation mechanism
2. (average) Network Multiplier: \(1 / (1 - \phi)\)
3. Total liquidity, \(L_t \equiv 1' [L_{1,t}, \ldots, L_{n,t}]\), is heteroskedastic:

\[
\text{Var}_{t-1} (L_t) = 1'M (\phi, G_t) \text{ diag } \left(\{\sigma_i^2\}_{i=1}^{n}\right) M (\phi, G_t)' 1.
\]

3. Can perform Q-MLE (\(\phi\) overidentified if \(\text{rank} (M (\phi, G_t)) > 2\))
Empirical Model

**SEM:** the theoretical framework is matched by a **Spatial Error Model**

\[
I_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_{m}^{bank} x_{i,t}^{m} + \sum_{p=1}^{P} \beta_{p}^{time} x_{t}^{p} + z_{i,t}
\]

\[
z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \ \nu_{i,t} \sim iid \left(0, \sigma_i^2\right),
\]

where \(g_{i,j,t}, x_{i,t}^{m}\) and \(x_{t}^{p}\) are predetermined at time \(t\).

Note:
1. Network as a shock propagation mechanism
   \(\Rightarrow\) (average) **Network Multiplier:** \(1/(1 - \phi)\)
2. Total liquidity, \(L_t \equiv 1'[l_{1,t}, ..., l_{n,t}]\), is heteroskedastic:
   \[
   Var_{t-1}(L_t) = 1'M(\phi, G_t) \text{diag} \left(\{\sigma_i^2\}_{i=1}^{n}\right) M(\phi, G_t)' 1.
   \]
3. Can perform Q-MLE (\(\phi\) overidentified if rank \((M(\phi, G_t)) > 2)\)
Network Impulse-Response Functions

- The network impulse-response of total liquidity, \( L_t := \sum_{i=1}^{n} l_{i,t} \), to a one standard deviation shock to bank \( i \) is

\[
NIRF_i (\phi, G_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = 1' \{ M (\phi, G_t) \} .i \sigma_i
\]

**NIRFs:**
1. are pinned down by the outdegree centrality and
   
   Risk Key Player \( \equiv \arg \max_i NIRF_i (\phi, G_t, \sigma_i) \)
2. account for all direct and indirect links among banks since
   \[
   1' \{ M (\phi, G_t) \} .i = 1' \{ I + \phi G_t + \phi^2 G_t^2 + \ldots \} .i = 1' \left\{ \sum_{k=0}^{\infty} \phi^k G_t^k \right\} .i
   \]
3. are a natural decomposition of total liquidity variance
   \[
   Var_{t-1} (L_t) \equiv \text{vec} \left( \{ NIRF_i (\phi, G_t, \sigma_i) \}_{i=1}^{n} \right)' \text{vec} \left( \{ NIRF_i (\phi, G_t, \sigma_i) \}_{i=1}^{n} \right).
   \]
Network Impulse-Response Functions

- The network impulse-response of total liquidity, 
  \( L_t := \sum_{i=1}^{n} l_{i,t} \), to a one standard deviation shock to bank \( i \) is

  \[
  NIRF_i (\phi, G_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = 1' \{ M(\phi, G_t) \}.i \sigma_i
  \]

NIRFs:

1. are pinned down by the outdegree centrality and

   \[
   \text{Risk Key Player} \equiv \arg\max_i NIRF_i (\phi, G_t, \sigma_i)
   \]

2. account for all direct and indirect links among banks since

   \[
   1' \{ M(\phi, G_t) \}.i = 1' \{ I + \phi G_t + \phi^2 G_t^2 + \ldots \}.i = 1' \left\{ \sum_{k=0}^{\infty} \phi^k G_t^k \right\}.i
   \]

3. are a natural decomposition of total liquidity variance

   \[
   \text{Var}_{t-1} (L_t) \equiv \text{vec} \left( \{ NIRF_i (\phi, G_t, \sigma_i) \}^{n}_{i=1} \right)' \text{vec} \left( \{ NIRF_i (\phi, G_t, \sigma_i) \}^{n}_{i=1} \right).
   \]
Sample: from Feb 2006 to Sept 2010, daily data.

Network Banks: all CHAPS members (non CHAPS banks must channel their payments through these banks)

Network Proxy: $g_{i,j,t} =$ the fraction of bank $i$’s loans borrowed from bank $j$ (computed as monthly averages in previous month)

Dependent Variable: liquidity available at the beginning of the day (account balance plus posting of collateral)

Macro Controls: (aggregate risk proxies, lagged) LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

Banks Characteristics: (lagged) Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS.
Network and Other Data Description

**Sample:** from Feb 2006 to Sept 2010, daily data.

**Network Banks:** all CHAPS members (non CHAPS banks must channel their payments through these banks)

**Network Proxy:**  \( g_{i,j,t} \) = the fraction of bank \( i \)'s loans borrowed from bank \( j \) (computed as monthly averages in previous month)

**Dependent Variable:** liquidity available at the beginning of the day (account balance plus posting of collateral)

**Macro Controls:** (aggregate risk proxies, lagged) LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

**Banks Characteristics:** (lagged) Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS.
Estimation Results

Two types of estimation:

1. **Subsample estimations:**
   - (good times) Pre Hedge Fund Crisis/ Northern Rock
   - (fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement
   - (Q.E.) Post Asset Purchase Program Announcement

2. Rolling estimations with 6-month window ⇒ allow $\phi$ to change at higher frequency.
Two types of estimation:

1. **Subsample estimations:**
   - (good times) Pre Hedge Fund Crisis/ Northern Rock
   - (fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement
   - (Q.E.) Post Asset Purchase Program Announcement

2. **Rolling estimations with 6-month window** ⇒ allow $\phi$ to change at higher frequency.
## SEM Estimation

### Network Effect: $\phi$

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.640*</td>
<td>0.166*</td>
<td>−0.151*</td>
</tr>
<tr>
<td></td>
<td>(52.44)</td>
<td>(7.06)</td>
<td>(−6.45)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>69.11%</td>
<td>89.71%</td>
<td>85.54%</td>
</tr>
</tbody>
</table>

(average) Network Multiplier

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.77*</td>
<td>1.12*</td>
<td>0.87*</td>
</tr>
</tbody>
</table>
Period 1: $NIRF^e(\phi, \bar{G}, 1)$ – Risk Key Players

Pre Northern Rock/Hedge Fund Crisis

Excess NIRF
+/- 2 s.e. C.I.
Excess network multiplier
+/- 2 s.e. C.I.
Period 1: Net Borrowing
Period 1: Network Borrowing/Lending Flows
\( \hat{\phi} \): SEM Rolling Estimation (6-month window)
Outline

1 Theoretical Framework
   • Network Specification
   • Bank Objective Function and Nash Equilibrium
   • Risk, and Level, Key Players

2 Empirical Analysis
   • Empirical Specification
   • Network and Data Description
   • Estimation Results

3 Conclusions

Appendix
Conclusions

We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

Empirical Findings:

1. First estimation of network risk multiplier ⇒ a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it’s basically zero post Bearn Stearns ⇒ rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).

Denbee, Julliard, Li and Yuan Network Risk and Key Players
Conclusions

We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

Empirical Findings:

1. First estimation of network risk multiplier $\Rightarrow$ a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it’s basically zero post Bearn Stearns $\Rightarrow$ rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).
Appendix

4 Additional Data Info
- Second Largest Eigenvalue of $G_t$
- Average Clustering Coefficient
- Other Variables

5 Additional Estimation Result
- Full SEM Results
- Specification Test

6 Network Evolution
- NIRFs
- Net Borrowing and Flows
Outline

4 Additional Data Info
  • Second Largest Eigenvalue of $G_t$
  • Average Clustering Coefficient
  • Other Variables

5 Additional Estimation Result
  • Full SEM Results
  • Specification Test

6 Network Evolution
  • NIRFs
  • Net Borrowing and Flows

Appendix
The Second Largest Eigenvalue of $G_t$
Cohesiveness

**Q:** How cohesive is this network?

**A:** Average Clustering Coefficient (Watts and Strogatz, 1998)

\[
ACC = \frac{1}{n} \sum_{i=1}^{n} CL_i(G),
\]

\[
CL_i(G) = \frac{\#\{jk \in G \mid k \neq j, j \in n_i(G), k \in n_i(G)\}}{\#\{jk \mid k \neq j, j \in n_i(G), k \in n_i(G)\}}
\]

where \( n \) is the number of members in the network and \( n_i(G) \) is the set of players between whom and player \( i \) there is an edge.

**Numerator:** \# of pairs of banks linked to \( i \) that are also linked to each other

**Denominator:** \# of pairs of banks linked to \( i \)
Average Clustering Coefficient of the Network

- 20070201: Subprime Default
- 20070809: Northern Rock/Hedge Fund Crisis
- 20080311: Bear Stearns
- 20080914: Lehman Brothers
- 20090919: Asset Purchase Programme Announced

Time:
- 060525
- 061016
- 070308
- 070801
- 071220
- 080516
- 081007
- 090227
- 090723
- 091211
- 100510
- 100929

Average Clustering Coefficient (Weekly Average):
- 20070809: Northern Rock/Hedge Fund Crisis
- 20070201: Subprime Default
- 20080311: Bear Stearns
- 20080914: Lehman Brothers
- 20090919: Asset Purchase Programme Announced
Aggregate Liquidity Available at the Beginning of a Day

Graph showing changes in aggregate liquidity over time, with key events noted:
- 20070201: Subprime Default
- 20070809: Northern Rock/Hedge Fund Crisis
- 20080311: Bear Stearns
- 20080914: Lehman Brothers
- 20090919: Asset Purchase Programme Announced

The graph also indicates a period of increase in liquidity, possibly related to the Asset Purchase Programme Announced in 2009.
Interest Rate in Interbank Market

- 20070201: Subprime Default
- 20070809: Northern Rock Hedge Fund Crisis
- 20080311: Bear Stearns
- 20080914: Lehman Brothers
- 20090919: Asset Purchase Programme Announced

Data
Additional Estimation Result
Network Evolution
Second Largest Eigenvalue of $G_t$
Average Clustering Coefficient
Other Variables
Cross-Sectional Dispersion of Interbank Rate

20070201: Subprime Default
20070809: Northern Rock/Hedge Fund Crisis
20080311: Bear Stearns
20080914: Lehman Brothers
20090919: Asset Purchase Programme Announced
Intraday Volatility of Aggregate Liquidity Available

20070201: Subprime Default
20070809: Northern Rock/Hedge Fund Crisis
20080311: Bear Stearns
20080914: Lehman Brothers
20090919: Asset Purchase Programme Announced

GBP

Time
Turnover Rate in the Payment System

20070201: Subprime Default
20070809: Northern Rock/Hedge Fund Crisis
20080311: Bear Stearns
20080914: Lehman Brothers
20090919: Asset Purchase Programme Announced
Right Kurtosis of Aggregate Payment Time

20070201: Subprime Default
20070809: Northern Rock/ Hedge Fund Crisis
20080311: Bear Stearns
20080914: Lehman Brothers
20090919: Asset Purchase Programme Announced
Outline

4 Additional Data Info
- Second Largest Eigenvalue of $G_t$
- Average Clustering Coefficient
- Other Variables

5 Additional Estimation Result
- Full SEM Results
- Specification Test

6 Network Evolution
- NIRFs
- Net Borrowing and Flows

Appendix
# SEM Estimation

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>69.11%</td>
<td>89.71%</td>
<td>85.54%</td>
</tr>
<tr>
<td>Network Effect: $\phi$</td>
<td>0.6400*</td>
<td>0.1660*</td>
<td>$-0.1510^*$</td>
</tr>
<tr>
<td></td>
<td>(52.44)</td>
<td>(7.06)</td>
<td>$(-6.45)$</td>
</tr>
</tbody>
</table>

## Macro Controls

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Liquidity (log)</td>
<td>$-0.0020$</td>
<td>0.3324*</td>
<td>0.5974*</td>
</tr>
<tr>
<td></td>
<td>$(-0.04)$</td>
<td>(4.59)</td>
<td>(14.73)</td>
</tr>
<tr>
<td>Right Kurtosis of Payments</td>
<td>$-0.1654^*$</td>
<td>0.0265</td>
<td>0.0031</td>
</tr>
<tr>
<td></td>
<td>$(-2.39)$</td>
<td>(1.12)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>Volatility of Liquidity (log)</td>
<td>0.1750</td>
<td>0.1935*</td>
<td>0.0075</td>
</tr>
<tr>
<td></td>
<td>(1.37)</td>
<td>(7.15)</td>
<td>(0.52)</td>
</tr>
<tr>
<td>Turnover Rate</td>
<td>0.0097</td>
<td>0.0055*</td>
<td>0.0049*</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(2.87)</td>
<td>(2.07)</td>
</tr>
<tr>
<td>LIBOR</td>
<td>0.6456*</td>
<td>0.3216*</td>
<td>$-0.1633$</td>
</tr>
<tr>
<td></td>
<td>(2.16)</td>
<td>(6.48)</td>
<td>$(-1.12)$</td>
</tr>
<tr>
<td>Interbank Rate Premium</td>
<td>1.9305*</td>
<td>$-0.0505$</td>
<td>0.9514*</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
<td>$(-0.61)$</td>
<td>(2.86)</td>
</tr>
<tr>
<td>Constant</td>
<td>16.0761*</td>
<td>10.7165*</td>
<td>11.7844*</td>
</tr>
<tr>
<td></td>
<td>(5.14)</td>
<td>(5.66)</td>
<td>(9.70)</td>
</tr>
</tbody>
</table>
### Bank Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Estimate 1</th>
<th>Estimate 2</th>
<th>Estimate 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interbank Rate</strong></td>
<td>-0.5096</td>
<td>-0.2977*</td>
<td>0.1414</td>
</tr>
<tr>
<td></td>
<td>(-1.72)</td>
<td>(-6.02)</td>
<td>(1.0428)</td>
</tr>
<tr>
<td><strong>Intraday Payment Level (log)</strong></td>
<td>-0.1558*</td>
<td>-0.1595*</td>
<td>0.0478*</td>
</tr>
<tr>
<td></td>
<td>(-5.73)</td>
<td>(-8.87)</td>
<td>(2.51)</td>
</tr>
<tr>
<td><strong>Right Kurtosis of Payment In</strong></td>
<td>0.0359</td>
<td>-0.0045</td>
<td>-0.0395*</td>
</tr>
<tr>
<td></td>
<td>(1.90)</td>
<td>(-0.26)</td>
<td>(-3.39)</td>
</tr>
<tr>
<td><strong>Right Kurtosis of Payment Out</strong></td>
<td>0.1416*</td>
<td>0.1742*</td>
<td>0.0426*</td>
</tr>
<tr>
<td></td>
<td>(8.17)</td>
<td>(15.89)</td>
<td>(4.16)</td>
</tr>
<tr>
<td><strong>Vol of Liquidity Available (log)</strong></td>
<td>0.0558*</td>
<td>0.0524*</td>
<td>0.0417*</td>
</tr>
<tr>
<td></td>
<td>(39.72)</td>
<td>(50.23)</td>
<td>(36.73)</td>
</tr>
<tr>
<td><strong>Liquidity Used (log)</strong></td>
<td>0.0303*</td>
<td>-0.0023</td>
<td>0.0052</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(-0.34)</td>
<td>(0.68)</td>
</tr>
<tr>
<td><strong>Top 4 Bank in Payment Activity</strong></td>
<td>1.3374*</td>
<td>1.6815*</td>
<td>2.3738*</td>
</tr>
<tr>
<td></td>
<td>(26.97)</td>
<td>(46.31)</td>
<td>(57.18)</td>
</tr>
<tr>
<td><strong>Repo Liability / Assets</strong></td>
<td>-0.7721</td>
<td>0.7401*</td>
<td>0.0575</td>
</tr>
<tr>
<td></td>
<td>(-0.92)</td>
<td>(14.46)</td>
<td>(0.64)</td>
</tr>
<tr>
<td><strong>Change in Deposit / Assets</strong></td>
<td>0.5050</td>
<td>-1.3275*</td>
<td>-1.2503*</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(-6.65)</td>
<td>(-3.70)</td>
</tr>
<tr>
<td><strong>Total Lending and Borrowing (log)</strong></td>
<td>0.1209*</td>
<td>0.0249</td>
<td>-0.3231*</td>
</tr>
<tr>
<td></td>
<td>(3.56)</td>
<td>(0.99)</td>
<td>(-23.70)</td>
</tr>
<tr>
<td><strong>CDS (log)</strong></td>
<td>-0.0652</td>
<td>-0.0274*</td>
<td>0.0514*</td>
</tr>
<tr>
<td></td>
<td>(-1.49)</td>
<td>(-3.17)</td>
<td>(4.55)</td>
</tr>
<tr>
<td><strong>CDS Missing Dummy</strong></td>
<td>-2.1893*</td>
<td>-2.2618*</td>
<td>-0.8502*</td>
</tr>
<tr>
<td></td>
<td>(-11.38)</td>
<td>(-32.04)</td>
<td>(-8.37)</td>
</tr>
</tbody>
</table>
**Specification Test**

**SDM:** For robustness, we also consider a direct network effect of banks observable characteristic, liquidity decisions, and possible match specific control variables, $x_{i,j,t}$ (Spatial Durbin Model)

$$ l_{i,t} = \bar{\alpha}_i + \sum_{m=1}^{M} \beta^{\text{bank}}_m x^m_{i,t} + \sum_{p=1}^{P} \gamma^\text{time}_p x^p_t $$

$$ + \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t} $$

**Note:** if $x_{i,j,t} := \text{vec}(x^m_{j\neq i,t})'$, $\rho = \phi$, $\theta = -\phi \text{vec}(\beta^{\text{bank}}_m)$,

$$ \gamma^\text{time}_p = (1 - \phi) \beta^{\text{time}}_p \forall p \Rightarrow \text{back to SEM} $$

$\Rightarrow$ this more general spatial structure provides a specification test for our model.
\( \hat{\phi} \) and \( \hat{\rho} \): SEM and SDM Rolling Estimation (6-month window)
Outline

4 Additional Data Info
- Second Largest Eigenvalue of $G_t$
- Average Clustering Coefficient
- Other Variables

5 Additional Estimation Result
- Full SEM Results
- Specification Test

6 Network Evolution
- NIRFs
- Net Borrowing and Flows

▶ Appendix
Period 2: $NIRF^e (\phi, \bar{G}, 1) - Risk Key Players$

Post Hedge Fund Crisis - Pre Asset Purchase Programme

Note: network risk reduction despite increased borrowing & lending
Period 3: $NIRF^e(\phi, \bar{G}, 1) – Risk Key Players$

Post Asset Purchase Programme Announcement

Excess NIRF

-1.2 -1.0 -0.8 -0.6 -0.4 -0.2 0.0

Bank 1 Bank 2 Bank 3
Bank 4
Bank 5
Bank 6
Bank 7
Bank 8
Bank 9
Bank 10 Bank 11

Bank Index

Excess NIRF
+/- 2 s.e. C.I.
Excess network multiplier
+/- 2 s.e. C.I.
**Period 1: Net Borrowing**

![Net Borrowing Graph](image)

- **Bank 1**: Net Borrowing = -1e+11
- **Bank 2**: Net Borrowing = 0e+00
- **Bank 3**: Net Borrowing = 5e+10
- **Bank 4**: Net Borrowing = 1e+11
- **Bank 5**: Net Borrowing = 5e+10
- **Bank 6**: Net Borrowing = 0e+00
- **Bank 7**: Net Borrowing = 5e+10
- **Bank 8**: Net Borrowing = 0e+00
- **Bank 9**: Net Borrowing = 5e+10
- **Bank 10**: Net Borrowing = 0e+00
- **Bank 11**: Net Borrowing = 5e+10
Period 2: Net Borrowing
Period 3: Net Borrowing
Period 1: Network Borrowing/Lending Flows
Period 2: Network Borrowing/Lending Flows
Period 3: Network Borrowing/Lending Flows