Monetary Policy, Endogenous Inattention, and the Volatility Trade-off*

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Abstract

This paper considers the interaction of optimal monetary policy and agents' beliefs. We assume that agents choose their information acquisition rate by minimizing a loss function that depends on expected forecast errors and information costs. *Endogenous inattention* is a Nash equilibrium in the information processing rate. Although a decline of policy activism directly increases output volatility, it indirectly anchors expectations, which decreases output volatility. If the indirect effect dominates then the usual trade-off between output and price volatility breaks down.

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1 Introduction

The "Sticky-Information" model of Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005) has recently been proposed as an alternative to the New Keynesian Phillips curve employed, for example, by McCallum and Nelson (1999) and Clarida, Gali, and Gertler (1999), and developed in detail by Woodford (2003b). The New Keynesian approach, which rests on the "Calvo assumption" that only a proportion of firms each period have an opportunity to adjust their prices, delivers a forward-looking expectational Phillips curve. The sticky-information model replaces this with the assumption that each period a fixed proportion of firms update their information set, and yields a backwards-looking expectational Phillips curve arising from the slow diffusion of information through the economy. Ball, Mankiw, and Reis (2005) argue that the sticky-information approach is more consistent with widely accepted views about inflation persistence and the effects of monetary policy, e.g. about the output costs of disinflation.[1]

Both of these approaches treat the proportion of agents that fully adjust each period as exogenous to the model. This is convenient as a simplification, but endogenizing the proportion is desirable both from a theoretical perspective and from the viewpoint of increased realism. In this paper we examine this point in detail and argue that the consequences for monetary policy can be far-reaching. We develop our analysis as an extension of the Ball, Mankiw, and Reis (2005) model because it fits neatly with our "bounded rationality" viewpoint that the frequency with which agents update and utilize new information should depend on the benefits relative to the costs of doing so. One way to view our contribution is that we study the implications of applying the "Lucas critique" to the rate of information acquisition as well as to expectation formation.

Our approach has a number of natural applications to monetary policy, but to illustrate its potential importance we restrict attention to one: the output-inflation volatility trade-off that is implicit in most monetary policy models. In many models, a renewed focus on inflation stabilization will lead monetary policy to produce higher output volatility.[2] Although Bernanke (2004), Svensson (2003), and others, conjecture that if policymakers can more tightly pin down inflation expectations then they will achieve economic stability, the specific channels for this effect are left open.

By extending the model of Ball, Mankiw, and Reis (2005) to endogenize the rate at which firms update their information, the current paper develops a framework in which to study the joint determination of optimal monetary policy and private sector expectations, and the connection of this joint relationship to the trade-off between price and output volatility. We study the intimate connection between optimal mon-

[1] See also Fuhrer and Moore (1995), Mankiw (2001) and Mankiw and Reis (2002). Versions of the New Keynesian approach that yield inflation persistence are developed in Woodford (2003b).

[2] See Woodford (2003b) for examples.

etary policy and the equilibrium anchoring of price expectations that arises through the endogenous response of private sector information acquisition or 'attentiveness.' Our key insight is that if monetary authorities follow policies that stabilize the aggregate price path, then this allows firms to update information less frequently, reducing the sensitivity of the economy to exogenous shocks.[3]

Our principal argument is that monetary policy has both direct and indirect effects on output and price volatility[4]: the direct effect gives the usual trade-off – by moving away from activist policy the Fed tends to decrease price volatility and increase output volatility; the indirect effect is channeled through expectation formation – policy that stabilizes price will anchor price expectations and thereby induce agents to be less reactive to intrinsic shocks, reducing both output and price variability. Thus there is a tension between the direct and indirect effects of policy; and which effect dominates determines the existence of a volatility trade-off. The novelty of our paper is the development of a model that can address this issue as an equilibrium response.

Our resolution of the policy tension begins with a relatively new approach to bounded rationality that endows agents with a correct model of the economy, but which assumes it is costly to acquire and process information. Recent proponents of this approach in macroeconomics are Sims (2003), Woodford (2003a), Mankiw and Reis (2002), Ball, Mankiw, and Reis (2005), and Reis (2005). These models assume that agents form conditional expectations, as in RE, but that the information set on which they condition may include only past data. Ball, Mankiw, and Reis (2005) (hereafter BMR) assume that agents have a time-invariant probability for updating their information in any given period. The resulting model is a sticky information version of the Calvo pricing model emphasized by Woodford (2003b).[5]

In the current paper we take the BMR model as a laboratory in which to study

[3]This is very close in spirit to the first type of "stability-enhancing" change to the "economic environment ... induced by improved monetary policies" listed by Bernanke (2004), p. 6. One mechanism specifically mentioned by Bernanke, citing Sims' approach, is the possibility that "... the dynamic behavior of the economy would change – probably in the direction of greater stability and persistence – in a more stable pricing environment, in which people reconsider their economic decisions less frequently."

[4]Attentive readers will notice that we refer interchangeably to inflation and price volatility. There is a continuing debate among experts in monetary policy about the precise form of the price stability objective that is appropriate for policymakers to pursue (see Woodford (2003) for references). This question, though of considerable importance, is essentially orthogonal to the issue under study, and we therefore take a pragmatic approach, and follow BMR's framework where optimal monetary policy is formulated in terms of the variability of the price level around an arbitrary trend. We suspect that an alternative formulation of our ideas could be developed in terms of inflation variability.

[5]A model of sticky information in wages is developed in Koenig (2004). Yetman (2003) compares symmetric Nash equilibria in sticky information and sticky price models. Reis (2003) studies optimal endogenous inattention for consumers with an exogenous income stream. Adam (2004) analyzes optimal monetary policy when firms have finite capacity to process information as in Sims (2003). Dotsey, King and Wolman (1999) and Bonomo and Carbalho (2004) endogenize the timing of price responses in a fixed price model. Dupor and Tsuruga (2005) compare different updating assumptions.

the interaction of optimal policy, information acquisition and private sector expectations. We take their motivation of costly updating seriously and assume that agents choose the rate at which they acquire new information by minimizing a quadratic loss function. A key insight of our approach is that this loss function depends on the information updating rate of the other agents. We define *Endogenous Inattention* as a symmetric Nash equilibrium in information updating together with the associated stationary stochastic processes for aggregate price-level and output.[6] In this Nash Equilibrium we treat the monetary authorities as following the optimal monetary policy recommended by BMR, given the equilibrium updating rate.

The BMR model is a simple model of monopolistically competitive firms combined with a quantity equation aggregate demand relation. Optimal monetary policy is a stochastic process for the money supply that minimizes a second-order approximation to the social welfare function. Because optimal policy in this model depends on the equilibrium rate of information updating, in our formulation monetary policy and the updating frequency, or 'attentiveness' of agents, are jointly determined.

The joint determination of Endogenous Inattention and optimal policy has important implications. We model the 'activism' of monetary policy by parameterizing policymaker preference for low price variance relative to output variance. The usual result is that as the policy authority becomes less 'activist' (i.e. places a higher weight on price variance) then the reduction in price volatility is accompanied by higher output volatility. We show that this trade-off is indeed present in the sticky information model of BMR.

Our main result is that the nature and existence of a trade-off between price and output stability depends on the joint determination of the rate of information processing and optimal policy. If policymakers are more activist, the direct effect, including the adjustment of rational expectations, is a reduction of output volatility and increased price-level volatility. However, an indirect effect on expectations arises from the increase in price level volatility which, in turn, induces agents to become more 'attentive'. This greater attentiveness tends to increase the volatility of output. Whether there is a trade-off between inflation and output volatility thus depends on whether the indirect or direct effect of policy dominates. We show that which effect dominates depends on how strongly the equilibrium level of attentiveness responds to the higher price-level volatility.

In contrast to the implications of the BMR model, we show that for relatively low costs of information accrual, the policy frontier can be non-monotonic. As the government switches from activist to less activist policy, there need be no trade-off between price and output variance – both can be lowered simultaneously.[7] However, as policy

[6]Some readers would find the term "endogenous attention" more natural, but the concept of "rational inattention" was introduced by Sims (2003) and used by BMR.

[7]A policy frontier is a set of inflation-output volatility pairs indexed by the activism parameter.

becomes increasingly vigilant against price volatility a trade-off between price and output variance can emerge. Our results, showing the possibility of a decline in both output and price volatility, provide a theoretical basis for many proposed explanations of the 'Great Moderation' – the empirical finding of a decline in inflation and output volatility, e.g. McConnell and Quiros (2001), Blanchard and Simon (2001), Stock and Watson (2003).

The cause of the Great Moderation is an important and open question, not to be settled here. Some authors have attributed the decline in economic volatility to a fundamental shift in the focus of monetary policy.[8] Orphanides and Williams (2003b) maintain that monetary authorities concerned themselves primarily with output stabilization ('activist policy') during the late 1960's and 1970's and then switched their emphasis to price stability in subsequent years. Bernanke (2004) contends that monetary policy during the 1970's exhibited 'output optimism' and 'inflation pessimism'. According to Bernanke's hypothesis, an overplaced emphasis on exploiting a (perceived) Phillips curve trade-off, and a mistaken belief that monetary policy was unable to control inflation, led to higher volatility in both output and inflation – confirming the positive correlation in Blanchard and Simon (2001).[9] Bernanke conjectures that a movement away from activist monetary policy anchored inflation expectations and produced lower volatility in *both* inflation and output. Stock and Watson (2003) and Ahmed, Levin, and Wilson (2002) attribute the Great Moderation to both improved monetary policy and a fortuitous sequence of shocks than the 1970's.

Only a few mechanisms have appeared in the literature to explain a possible connection between changed monetary policy objectives and lower economic volatility. In Orphanides and Williams (2003a) the trade-off can disappear when agents engage in 'perpetual learning' and policymakers have the appropriate preferences on inflation and output volatility. An alternative story, given in Clarida, Gali and Gertler (2000), retains rational expectations, but relies on multiple equilibria.

We propose a complementary but distinct approach that requires neither sunspot equilibria nor persistent deviations from rational expectations and emphasizes a plausible mechanism in the spirit of Bernanke (2004). Our finding that the policy frontier may be non-monotonic is consistent with Bernanke's hypothesis.

The organization of our paper is as follows. In Section 2 we summarize the BMR model and show how to extend it to endogenize the rate of "inattention." In Section 3 we then prove the existence of an equilibrium with endogenous inattention and explore the comparative statics results. Section 4 explores the policy implications of

[8]The evidence for a one-time permanent shift in monetary policy, and for a similar shift in macroeconomic volatility, is open to other interpretations. Cogley and Sargent (2005) and Sims and Zha (2006) present evidence of drifting and regime switching over much of the post-WWII period.

[9]Sargent (1999) develops a model in which the central bank mistakenly exploits a Phillips curve even though the natural rate hypothesis holds. Orphanides (2002) emphasizes poor natural rate estimates on the part of policymakers.

our comparative statics results. Finally, Section 5 concludes.

2 The Model

We begin by briefly reviewing the model developed in Ball, Mankiw, and Reis (2005). In this review, we assume, as did BMR, that the probability of information updating, $\bar{\lambda}$, is exogenous and fixed. This allows us to use their results on optimal monetary policy to obtain equilibrium paths of price and output for a given set of structural parameters. Then, taking as given both monetary policy and the updating frequency $\bar{\lambda}$, we consider the incentive for a single agent to deviate from $\bar{\lambda}$, where this incentive is measured by expected squared forecast error plus a cost reflecting the choice λ. An equilibrium occurs when each agent does not have an incentive to deviate from the aggregate $\bar{\lambda}$.

2.1 The Ball-Mankiw-Reis Model

The economy is populated by a continuum of yeoman farmers. Each farmer uses its own labor to produce a good to sell in a monopolistically competitive market. The instantaneous utility of agent i is given by

$$U(C_{it}, Y_{it}) = \frac{(C_{it})^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma} - \frac{\hat{A}Y_{it}^{1+\zeta}}{1+\zeta}, \tag{1}$$

where C_{it} is the usual consumption index defined in terms of the CES aggregator:

$$C_{it} = \left[\int_0^1 \left(C_{it}^j\right)^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{\gamma}{\gamma-1}}.$$

The last term in (1) captures the disutility of labor. The production function is $Y = AL$ with labor L, technology A to be normalized later for convenience, and $\hat{A} = A^{-(1+\zeta)}$.

Agents choose sequences of consumption and labor in order to maximize the expected discounted utility stream subject to their budget constraint, which includes a government levied proportional sales tax τ_t assumed to follow a stationary process. The consumer problem leads to a demand that, in log form, is given by

$$y_{it} = y_t - \gamma(p_{it} - p_t) \tag{2}$$

where p_t is the log of the usual price index and $\gamma > 1$ is the elasticity of substitution between different goods. To obtain this form of demand, we follow BMR by assuming the presence of complete markets for risk; this allows the agents to insure themselves against idiosyncratic information shocks, and allows us to identify consumption and output.[10]

The producer's pricing problem may now be solved, taking the computed demand as given, resulting in an optimal price (given full information) of the form

$$p_{it}^{*} = p_t + \alpha y_t + u_t \tag{3}$$

where $\alpha = (\zeta + \sigma) / (1 + \gamma\zeta)$. We have chosen the technology constant A to normalize the log natural output level to zero.[11] u_t is a stationary stochastic process deriving its structure, for example, from the sales tax τ_t. We follow BMR by interpreting u_t as capturing mark-up shocks and take it to have an AR(1) structure: $u_t = \rho u_{t-1} + \varepsilon_t$ with $0 < \rho < 1$.[12] In a sticky-price model similar in spirit to BMR, Ireland (2004) interprets the mark-up shocks as arising from variation in the substitutability of differentiated products. Mark-up (or supply) shocks are standard in the literature and are taken to represent shifts in the Phillips curve; for further discussion see Woodford (2003b).[13]

Whereas the above model is fairly standard – see for example Woodford (2003b) – BMR introduce a novel information structure that fundamentally alters equilibrium outcomes. Combining the probabilistic friction of Calvo (1983) with the limited information capacity notion of Sims (2003), these authors assume that agents update their information with exogenous probability $0 < \lambda < 1$ each period, and that each agent sets a price path optimally every period, subject to their information constraint.[14] Thus an individual who last updated information k periods ago will set price equal

[10] The co-existence of complete financial markets and heterogeneous information is often thought to be inconsistent because the prices in these markets should reveal all relevant information. To avoid this potential criticism, we could alternatively assume the presence of a benevolent insurance planner who collects all income and redistributes the average to all agents. Other approaches include Preston (2005) who develops a model of bounded rationality assuming labor market and profit sharing which allows agents to hedge risk. There is also an extensive literature which studies competitive equilibria in incomplete markets. In these models, wealth dynamics matter for equilibrium allocations. We follow BMR and abstract from these considerations by assuming the existence of a risk-sharing mechanism; we leave the interesting issues raised by a more careful modeling of incomplete markets to future research.

[11] BMR allow A to form a stochastic process, thus allowing for drift in the natural rate as well as for the analysis of productivity shocks. We abstract from this here to focus attention on the impact of mark-up shocks, which are the usual source of volatility tradeoffs. It is straightforward to incorporate productivity as well as preference shocks into this framework.

[12] BMR allow u_t to have general $MA(\infty)$ form.

[13] Benigno and Woodford (2006) also introduce pure cost push shocks by assuming wage mark-up shocks in the firm's profit function.

[14] This is the idiosyncratic risk mentioned earlier. An individual's income will vary with respect to average output depending on her most recent information. The insurance market for risk assures the agent a yearly consumption level equal to average output, regardless of her income level.

to $E_{t-k}p_t^*$.[15] Equilibrium price is given by

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j}(p_t + \alpha y_t + u_t). \tag{4}$$

Note that this identification requires approximating the price index as an average, $p_t = \int p_{it} di$.

Equation (4) is a Phillips curve and represents the aggregate supply relationship in the economy. Aggregate demand is derived from a cash-in-advance constraint and takes the form

$$y_t = \hat{m}_t - p_t + e_t,$$

where $\hat{m}_t$ is the policy instrument set in time $t-1$ and e_t is a white noise money demand shock assumed orthogonal to ε_t.[16] BMR conclude with the clever observation that there is a linear relationship between $E_{t-1}p_t$, $\hat{m}_t$, and other information available at $t-1$; thus, we may assume that policymakers set $E_{t-1}p_t$.[17]

The model is closed by specifying monetary policy, which, as we just noted, is equivalent to specifying a (stochastic) time path for $E_{t-1}p_t$. BMR assume that the preferences of policymakers are captured by a quadratic loss in output and cross-sectional relative price variance, as given by

$$\mathcal{L} = Var(y_t) + \omega E\left(Var_i(p_{it} - p_t)\right). \tag{5}$$

This equation can be derived as a second order approximation to average cross-sectional utility. When this approximation is taken seriously, the associated value of ω is $\gamma^2(\zeta + \gamma^{-1})/(\zeta + \sigma)$, though BMR consider varying values of ω for fixed structural parameters, and we will as well. We attach the interpretation of 'activism' to this parameter: as ω increases the policymaker places a higher relative loss on cross-section price variation and less on unconditional output variance. Policymakers with low values of ω are "activist" in the sense that they place a relatively high weight on reducing output volatility.[18]

[15] It would be interesting for future research to extend this model to a Taylor-type contracting environment such as Dupor and Tsuruga (2005).

[16] The quantity equation can also be derived from a money in the utility function specification where the utility function is specified so that money demand is interest inelastic. Following Walsh (2003) then e_t can also be interpreted as a composite shock which includes preference shocks.

[17] Ireland (2004) assesses the relative contributions of mark-up and demand shocks to the aggregate time series in the context of a New Keynesian model. A similar analysis based on the BMR model would be useful for interpreting the results of this paper.

[18] "Activism" is also sometimes used to mean a lower weight on the output gap in an interest rate rule. Because of the quantity theory form of aggregate demand used in the BMR model, there is no IS curve and consequently monetary policy is formulated in terms of $\hat{m}_t$ or $E_{t-1}p_t$ rather than an interest rate rule. In the current context our use of the term "activist policy" seems the most natural.

Having specified the government's objective, BMR analytically solve the optimal policy problem. They show that when optimal policy is followed, the first-order condition

$$E_{t-1}p_t = -\frac{1}{\alpha\omega}E_{t-1}y_t \tag{6}$$

must be satisfied. Solving for the equilibrium paths of price and output then yields

$$p_t = \sum_{j=0}^{\infty}\phi_j\varepsilon_{t-j} + \alpha\phi_0 e_t \tag{7}$$

$$y_t = \sum_{j=0}^{\infty}\varphi_j\varepsilon_{t-j} + (1-\alpha\phi_0)e_t \tag{8}$$

with

$$\phi_j = \frac{\rho^j}{\alpha^2\omega + \frac{(1-\lambda)^{j+1}}{1-(1-\lambda)^{j+1}}}, \quad \varphi_j = -\alpha\omega\phi_j \text{ for } j>0, \text{ with} \tag{9}$$

$$\phi_0 = \frac{\lambda}{1-\lambda(1-\alpha)} \text{ and } \varphi_0 = -\phi_0. \tag{10}$$

Equations (7) and (8) imply the usual trade-off between σ_p^2 and σ_y^2, the unconditional (time-series) variances of price and output. This can be seen as follows. For $0<\lambda<1$, an increase in ω reduces $|\phi_j|$, for all $j>0$, and increases $|\varphi_j|$, for all $j>0$. It follows immediately that an increase in ω reduces $\sigma_p^2 = Var(\varepsilon_t)\sum_{j=0}^{\infty}\phi_j^2$ and increases $\sigma_y^2 = Var(\varepsilon_t)\sum_{j=0}^{\infty}\varphi_j^2$. (In the extreme case $\lambda=1$, σ_y^2 becomes independent of ω and the trade-off is vertical.) Finally, notice that changing ω does not alter the impact of σ_e^2 on output or price variance; this follows from the monetary policy timing assumption.

For $0<\lambda<1$, as ω continues to increase, price-level variance and output variance will converge to positive, finite values. These insights will be important for discussion of the output-price volatility trade-off when λ is determined endogenously. For this reason, we summarize this discussion in the following remark.

Remark: *Consider the BMR model with exogenous λ.*

$$\lim_{\omega\to\infty}\sigma_p^2 = \phi_0^2(\sigma_\varepsilon^2 + \alpha^2\sigma_e^2)$$

$$\lim_{\omega\to\infty}\sigma_y^2 = \left(\phi_0^2 + \frac{1}{\alpha^2}\frac{\rho^2}{1-\rho^2}\right)\sigma_\varepsilon^2 + (1-\alpha\phi_0)^2\sigma_e^2.$$

Intuitively, in the presence of a (positive) markup shock, price will rise and output will fall due to the fact that policy is lagged one period and thus cannot respond contemporaneously to the shock. An option for policymakers is to return price to its mean the following period, but, pursuing such a policy would exacerbate the impact of the shock on output. In order for agents to lower prices in the presence of a markup shock whose influence is still felt due to serial correlation, output must fall further. The form of the government's objective function makes such a policy suboptimal as policymakers prefer to allow prices to capture some of the economy's volatility. This trade-off is consistent with the sticky-price model of Woodford (2003b) and appears in most models with mark-up or supply shocks. Below, we focus on the generality of this result to the case where λ is determined endogenously.

The key to our results will involve the endogenous response of λ. At this stage it is therefore helpful to obtain the effects of an exogenous change in λ on σ_p^2 and σ_y^2. Incorporating also the results just stated, we have:

Proposition 1 *Consider the BMR model with exogenous* λ.

1. $\omega \uparrow \Rightarrow \sigma_p^2 \downarrow,\ \sigma_y^2 \uparrow,\ E(Var_i(p - p_i)) \downarrow$.
2. $\lambda \uparrow \Rightarrow \sigma_p^2 \uparrow$
3. *If* σ_e^2 *is sufficiently small then* $\lambda \uparrow \Rightarrow \sigma_y^2 \uparrow$.

The effect of ω on σ_p^2 and σ_y^2 was shown above, and the impact on $E(Var_i(p-p_i))$ is shown in Appendix A. Note that the impact on the expected cross-sectional price variation of an increase in λ is ambiguous. This is intuitive as the cross-sectional variance will be zero when λ is zero or one. The second set of results, giving the impact of λ, are straightforward. Increases in λ can be seen to increase both $|\phi_j|$ and $|\varphi_j|$, for all j, and hence, provided $\sigma_e^2 = 0$, to increase both σ_p^2 and σ_y^2. Intuitively, in the absence of demand shocks, as λ increases there is a greater price, and hence output, response to new information. On the other hand, whether or not they are observed, demand shocks impact output volatility, and those agents who observe these shocks will shift some of this volatility to price. Provided the size of the demand shocks is large compared to the mark-up shocks, increasing λ may reduce output variance. This discussion highlights the critical role λ plays in the stochastic properties of the economy and motivates the remaining sections of the paper. In the sequel, we will often speak of results holding for small enough demand shocks: by this we will mean small enough so that part 3 of Proposition 1 holds.

The possibility that a reduction in activism (increased ω) could lead to greater stability in both output and prices can be seen to arise if it is accompanied by a reduction in λ. We now turn to the endogenous determination of λ in an equilibrium setting.

2.2 Endogenizing Inattention

BMR take λ as exogenous to the model. We propose extending their model by making $0 \leq \lambda \leq 1$ a choice variable. In our framework, agents choose an intensity with which to gather and analyze information and this chosen intensity yields a probability of obtaining and processing current information. To model this choice, we assume agents choose λ to minimize mean squared forecast error, as discussed below. Not surprisingly, the mean squared forecast error is decreasing in λ and so if gathering information is costless, the choice for agents is quite simple: choose $\lambda = 1$. However, we argue that information gathering and processing is not costless, and instead assume a cost function that is quadratic in λ. A purely quadratic cost function allows for increasing marginal costs, with marginal cost tending to zero as $\lambda \to 0$. This implies that it is always optimal to choose a non-zero probability of updating information.

The choice of λ for a given agent depends on the equilibrium stochastic processes of price and output, which in turn depend on structural parameters, the monetary policy parameter ω, and the intensity with which other agents gather information. Given the monetary policy dictated by ω, the optimal choices of λ by private agents are interdependent. Thus the correct equilibrium concept for our model is Nash, and we focus on Nash equilibria that are symmetric with respect to the private agents. Note also that the stochastic processes for price and output depend, in turn, on the Nash equilibrium value of λ.

We need to be explicit also about the policy assumptions. As just indicated, we take ω to be exogenous, and we make the assumption that policymakers follow the optimal monetary policy recommended by BMR, so that price and output processes are given by (7)-(8) with coefficients (9) and (10). In effect, policymakers treat the equilibrium rate of information gathering by private agents as given, and thus our equilibrium value of λ is a Nash equilibrium in choices of private agents *and* the policymaker. This has important implications for comparative statics and is discussed further below.

Let $\bar{\lambda}$ be the economy-wide probability of updating information and define $p_t^*(\bar{\lambda})$ as the optimal price given the economy wide $\bar{\lambda}$, that is

$$p_t^*(\bar{\lambda}) = p_t(\bar{\lambda}) + \alpha y_t(\bar{\lambda}) + u_t,$$

where $p_t(\bar{\lambda})$ and $y_t(\bar{\lambda})$ are the equilibrium price level and output given that all agents use $\bar{\lambda}$.

Now let $\hat{p}_t(\lambda)$ be the price set by a firm at time t given that the firm updates its information with probability λ. Note, $\hat{p}_t(\lambda)$ is a random variable that depends not only on the process of shocks hitting the economy, but also on a process determining whether updating occurs. It may help to think of $\hat{p}_t(\lambda)$ as depending on the process

s_t, which takes on the value 1 with probability λ and zero otherwise. Then

$$\hat{p}_t(\lambda) = \begin{cases} p_t^*(\bar{\lambda}) & \text{if} \quad s_t = 1 \\ E_{t-k}p_t^*(\bar{\lambda}) & \text{if} \quad s_{t-k+1}, \cdots, s_t = 0 \text{ and } s_{t-k} = 1 \end{cases} \tag{11}$$

Note also that $\hat{p}_t(\lambda)$ is firm specific.

The firm's loss function is taken to be the expected squared forecast error:

$$L(\lambda, \bar{\lambda}) = E\left(\hat{p}_t(\lambda) - p_t^*(\bar{\lambda})\right)^2. \tag{12}$$

This loss function is standard in statistical settings, but requires comment here. Private agents maximize utility by setting prices at the conditionally expected optimal level, given their information set. In principle, we could ask that agents choose the rate of information gathering λ also on the basis of expected utility maximization.[19] Having agents instead minimize expected squared forecast error for prices has the advantage for us of technical simplicity, but it also has a natural interpretation in terms of bounded rationality. Agents are in effect splitting their decision problem into separate optimization and forecasting problems, a procedure that is often followed, for example, in the least-squares learning literature.[20]

There is a further sense in which agents minimizing (12) are boundedly rational. In principle, agents might choose a time-varying rate of information gathering that depends on their information set. In endogenizing the rate of information acquisition, we are less demanding of our agents, but in a way that we find particularly plausible. Private agents are required to choose a rate λ that minimizes the unconditional mean squared forecast error, including costs of information acquisition, given the actual stationary price process. Such a choice could plausibly arise as the outcome of a stable adaptive learning process by comparing average mean squared errors for different rates.[21]

Noting that the mean of both $p_t^*(\bar{\lambda})$ and $\hat{p}_t(\lambda)$ is zero, we see that to compute the loss value, it is sufficient to compute the variance of $p_t^*(\bar{\lambda})$, $\hat{p}_t(\lambda)$ and their covariance. Using the equilibrium price paths for p and y together with (3) we obtain

$$p_t^*(\bar{\lambda}) = \sum_{j=0}^{\infty} \bar{\theta}_j \varepsilon_{t-j} + A(\alpha, \bar{\lambda})e_t, \tag{13}$$

where

$$A(\alpha, \bar{\lambda}) = \alpha((1-\alpha)\phi_0 + 1),$$

[19] An approach based on expected utility maximization merits future research. This raises additional issues, sidestepped in the present study, and is the subject of work in progress.

[20] Moreover, loss proportional to squared errors makes sense in settings with firms maximizing profits conditional on prices set incorrectly.

[21] Reis (2006) develops the microfoundations of endogenous inattention and appears to provide a foundation for our simpler, tractable approach.

and $\bar{\theta}_j = \bar{\phi}_j(1-\alpha^2\omega) + \rho^j$ if $j > 0$ and $\bar{\theta}_0 = (1-\alpha)\bar{\phi}_0 + 1$. We use the notation $\bar{\theta}$ and $\bar{\phi}$ to emphasize that these parameters depend on the economy-wide $\bar{\lambda}$. Now set

$$\Omega(k) = \begin{cases} \sum_{j=k}^{\infty} \bar{\theta}_j \varepsilon_{t-j} & \text{if} \quad k \geq 1 \\ \\ \sum_{j=k}^{\infty} \bar{\theta}_j \varepsilon_{t-j} + A e_t & \text{if} \quad k = 0. \end{cases}$$

Note $\Omega(k) = E_{t-k} p_t^*(\bar{\lambda})$. Then $Var(p_t^*(\bar{\lambda})) = Var(\Omega(0))$ and

$$Var(\hat{p}_t(\lambda)) = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j Var(\Omega(j)).$$

Also, noting

$$\begin{aligned} Cov(p_t^*(\bar{\lambda}), \hat{p}_t(\lambda)) &= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j Cov(\Omega(0), \Omega(j)) \\ &= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j Var(\Omega(j)) = Var(\hat{p}_t(\lambda)) \end{aligned}$$

we get that

$$L(\lambda, \bar{\lambda}) = Var(p_t^*(\bar{\lambda})) - Var(\hat{p}_t(\lambda)). \tag{14}$$

Let

$$\bar{\psi}_k = \begin{cases} \sigma_\varepsilon^2 \sum_{j=k}^{\infty} \bar{\theta}_j^2 & \text{if} \quad k \geq 1 \\ \\ \sigma_\varepsilon^2 \sum_{j=k}^{\infty} \bar{\theta}_j^2 + A^2 \sigma_e^2 & \text{if} \quad k = 0. \end{cases}$$

We the obtain with the following result:

$$L(\lambda, \bar{\lambda}) = (1-\lambda)\bar{\psi}_0 - \lambda \sum_{j=1}^{\infty} (1-\lambda)^j \bar{\psi}_j. \tag{15}$$

Also, it is not difficult to show all infinite sums considered are absolutely convergent, so there are no existence issues. We have the following:

Lemma 1 *The function $L(\lambda, \bar{\lambda})$ is monotonically decreasing in λ.*

The proof of this lemma is contained in the Appendix.

If information gathering and processing were costless then the optimal choice would be $\lambda = 1$ so that the loss would be zero. BMR motivate sticky-information by a cost to information gathering. Along these lines assume that the cost to information gathering and processing is $C\lambda^2$ where $C \geq 0$. Define the function

$$T(\bar{\lambda}) = \arg\min_{0 \leq \lambda \leq 1} \left(L(\lambda, \bar{\lambda}) + C\lambda^2 \right).$$

$T(\bar{\lambda})$ is a best-response function: for fixed $\bar{\lambda}$ and resulting equilibrium processes, $T(\bar{\lambda})$ delivers an agent's optimal choice of λ. Existence of a solution to this optimization problem is guaranteed by the compactness of the choice set, and uniqueness can be demonstrated by directly computing that $\frac{\partial^2 \hat{L}}{\partial \lambda^2} > 0$, where $\hat{L} = L + C\lambda^2$: the proof of this is contained in the Appendix. A fixed point of this map is a symmetric Nash equilibrium and is our desired notion of Endogenous Inattention.

3 Existence and Comparative Static Analysis

The previous section showed that there exists a mapping from aggregate information flows, through a loss function defined by the associated equilibrium stochastic process, into an individual 'inattentiveness' rate.

Definition. *Endogenous Inattention is the symmetric Nash equilibrium defined by the fixed point* $\lambda^* = T(\lambda^*)$.

3.1 Existence Result

Note that $T : [0, 1] \to [0, 1]$. Moreover, from above, it is apparent that T is a well-defined and continuous function. From Brouwer's theorem we know that a fixed point exists. The value λ^* is a symmetric Nash equilibrium in λ, taking into account the policy reaction to aggregate λ. We summarize existence as a proposition.

Proposition 2 *Endogenous Inattention exists in the BMR model.*

Some comments are in order.

1. We will say that λ^* is a stable equilibrium if $T'(\lambda^*) < 1$ since in that case if $\bar{\lambda} \neq \lambda^*$ then (locally) an individual will have an incentive to adjust λ toward λ^*. Our focus is on equilibria that are stable, but below we will highlight existence of unstable equilibria as well.

2. An increase in λ^* may result in an increase in price and output variance, which may yield increased incentive for a given agent to choose a higher λ. This potentially self-fulfilling behavior suggests that multiple Nash equilibria may be present, and indeed we will see that this can arise.

3. Raising ω, and thereby decreasing the equilibrium price variance, gives an individual agent the incentive to lower her choice of λ and thus potentially reduces output variance and further reduces price variance. The usual trade-off between the price and output volatility may therefore break down.

3.2 Comparative Static Analysis

Endogenous Inattention is a fixed point of the map T, and the fixed points of this mapping depend on the deeper parameters of the model $\alpha, \rho, C, \omega, \sigma_\varepsilon, \sigma_e^2$. This subsection examines how the fixed points depend on these underlying parameters. In particular, we characterize the direction in which λ^* moves for infinitesimal changes in each parameter.

It is useful to rewrite the T-map to emphasize its dependence on model parameters. Denote $\xi = (\alpha, \rho, C, \omega, \sigma_\varepsilon^2, \sigma_e^2)'$. We now define the T-map to be

$$T(\bar{\lambda}; \xi) = \arg\min_{\lambda} \left(L(\lambda, \bar{\lambda}; \xi) + C\lambda^2 \right).$$

Fixed points are $\lambda^* = T(\lambda^*; \xi)$. Comparative statics require computing, for each element of ξ,

$$(T' - 1)d\lambda^* + T_{\xi_i} d\xi_i = 0$$

where $T' \equiv \partial T / \partial \bar{\lambda}$, $T_{\xi_i} \equiv \partial T / \partial \xi_i$. As mentioned above we focus on stable equilibria so that $T' < 1$. In a neighborhood of a stable fixed point, the effect of a change in one of the parameters on the fixed point is determined by $sign(T_{\xi_i})$. In particular, $sign\left(\frac{d\lambda^*}{d\xi_i}\right) = sign(T_{\xi_i})$.[22] We have the following result:

Proposition 3 *Let $\lambda^* < 1$ denote a stable symmetric Nash equilibrium. Assume $\alpha \leq 1$. For $\xi = (\alpha, \rho, C, \omega, \sigma_\varepsilon^2, \sigma_e^2)'$ the effect of a change in a component of ξ on λ^* is as follows:*

1. $\frac{d\lambda^*}{dC} < 0, \frac{d\lambda^*}{d\rho} > 0, \frac{d\lambda^*}{d\omega} < 0, \frac{d\lambda^*}{d\sigma_\varepsilon^2} > 0, \frac{d\lambda^*}{d\sigma_e^2} > 0.$

2. *If σ_e^2 is small enough, then* $\frac{d\lambda^*}{d\alpha} < 0.$

The proof is contained in the Appendix. We focus on $\alpha \leq 1$, in which pricing decisions are strategic complements, because this is the case examined in the literature,[23] but extending the analysis to $\alpha > 1$ would clearly be of theoretical interest. Proposition 3 provides comparative static results for interior endogenous inattention equilibria. If $\lambda^* = 1$ then the impact on the equilibrium inattention level will either be as given in the proposition or zero, depending on the sign of the change in the parameter and on whether the associated first order condition holds with equality. The intuition behind the proposition is given below, together with graphical representations of equilibria.

[22] Using stability in this way is closely related to the observation made in Evans and Honkapohja (2003b) in a different context.

[23] For example, in their numerical illustrations BMR set $\alpha = 0.1$.

To illustrate the results of this proposition, and to elaborate on the existence of equilibria, we turn to a numerical analysis. We give a graphical representation of the results, in particular, to demonstrate the possibility of multiple equilibria. Although Proposition 3 gives analytical details on comparative statics, in the policy discussion below it will be useful to have greater intuition on the comparative statics of ω and C.

We plot the T-function for various parameter values. For a vector of parameter values $(C, \omega, \rho, \alpha, \sigma_\varepsilon^2, \sigma_e^2)$, we plot an agent's optimal choice of λ given that all other agents choose $\bar{\lambda}$. A few brief comments are warranted. First, as mentioned above, we treat ω as an exogenous policy parameter and use changes in its value to study the impact of the changes in policy 'activism' recently detailed in Orphanides and Williams (2003b). An alternative interpretation, if instead ω is regarded as a function of deeper preference parameters of the agents, is that one of those preference parameters has changed.[24] However, our preferred interpretation is to view changes in ω as reflecting changing priorities of policymakers. Second, our interest is not in calibration but in the implications of the model with Endogenous inattention.

In order to conduct the numerical analysis we need a baseline parameter valuation. Our baseline parameterization sets $\alpha = .1$, $\rho = .8$, $C = 5$, $\sigma_\varepsilon^2 = .1, \sigma_e^2 = .1$.[25] We choose these values as the baseline because they deliver results suitable for comparative static analysis, i.e. intermediate and not extreme results. They are not baseline in the sense of being calibrated to actual data, though they are consistent with the values in BMR. We also choose σ_e^2 in accordance with the comparative static results of Propositions 1 and 3.

Figure 1 below graphs the T-map and resulting equilibria for the baseline calibration and $\omega = 20$. Recall that the T-map takes the aggregate attentiveness parameter and maps it into an individual choice of λ. Any point on this curve that crosses the 45-degree line is a Nash equilibrium. The various comparative static results of Proposition 3 are summarized in Figure 1, which shows the way in which the T-map is altered by changing one of the parameters of the model.

INSERT FIGURE 1 HERE

Figure 1 shows that multiple equilibria can exist, though here only one equilibrium is stable.[26] In all of our numerical calculations, only one stable interior equilibrium is observed. In the baseline case there are equilibria at about .11 and at 1. The

[24]The parameter α is also a function of deeper parameters, but there are enough degrees of freedom so that α and ω can be chosen independently. In particular, $\alpha\omega = \gamma$.

[25]BMR use $\alpha = .1$, $\rho = .8, \omega = 1$, and implicitly $\sigma_\varepsilon^2 = 1$.

[26]The possibility of multiple equilibria with low C is related to the presence of multiple equilibria in the degree of rigidity, found in the earlier literature on nominal rigidity and coordination failures. See (Ball and Romer 1991).

equilibrium at .11 is stable since $T' < 1$. Note that in this case full rationality – in the sense of full information, i.e. $\lambda^* = 1$ – does constitute an equilibrium. As we will see below, it is not always the case that full rationality is an equilibrium. The existence of a full-information equilibrium even though it produces higher volatility may initially seem surprising, but the result is intuitive. If all agents respond fully to contemporaneous shocks then price and output volatility will be higher. The higher volatility here reinforces agents' decisions to coordinate on full-information, making the point an equilibrium. However, $\lambda^* = 1$ is not a stable equilibrium: for values .11 $< \bar{\lambda} < 1$ agents have an incentive to reduce λ. There are parameterizations in which $\lambda^* = 1$ is the only stable equilibrium; an example is given below.

Having established a baseline result, we turn to comparative statics. First we alter C while holding $\rho, \alpha, \omega, \sigma_\varepsilon^2, \sigma_e^2$ fixed. Figure 2 plots T-maps for various values of C. The arrow indicates the direction of change in the graph of the T-map, given that C is increasing. The comparative static direction is intuitive, since the optimal choice of λ, for fixed $\bar{\lambda}$, will decrease as its cost increases.

INSERT FIGURE 2 HERE

The thick horizontal line at the top of the figure is a plot of the T-map when $C = 0$. In this case, $\lambda^* = 1$ is the unique equilibrium, and it is stable. This result is as expected since whenever the cost to acquiring and processing information is sufficiently low we should expect to see full-information rational expectations arise. Figure 2 also demonstrates that as the cost increases the possibility for multiple equilibria arises. Moreover, for medium-sized costs there exists a stable interior fixed point. Clearly, for a particular value of C it is possible to generate BMR's choice of $\lambda = .25$. For very low $C > 0$ there are two stable equilibria as well as an unstable equilibrium.[27] As C continues to rise, the full-information equilibrium disappears and the only equilibrium is the stable sticky information equilibrium.[28]

Figure 3 plots the comparative statics as the policy parameter ω is varied. According to Figure 3, for fixed $\bar{\lambda}$, as ω rises firms have less incentive to update their information since the higher ω is associated with a monetary policy that decreases price volatility and, as a result, reduces the value of new information.

INSERT FIGURE 3 HERE

Low values of ω imply a unique full information equilibrium. As ω increases, the full information equilibrium becomes unstable and a stable interior equilibrium emerges. As already shown, further increases in ω lead to lower rates of information processing.

[27]Note that the unstable equilibrium has counterintuitive comparative statics. For example, higher costs will increase the value of λ^* in the unstable equilibrium.

[28]The connection between C and information technology is not obvious. For instance, one might expect that the development of the internet manifests as a lower C. However, since we interpret C as processing costs, the greater availability of information may just increase noise.

3.3 A Note on Welfare

In the absence of demand shocks, it is optimal, from an aggregate welfare perspective, for agents to coordinate on an information acquisition rate of zero. In this case, prices and output would stay constant at their equilibrium values of zero, so that the government's loss function, which approximates aggregate welfare, would be zero. The apparently paradoxical result that less information is welfare enhancing stems from the fact that the mark-up shocks are distortionary in nature and so, on aggregate, are best ignored. In the presence of aggregate demand shocks, it is no longer socially optimal for agents to set $\lambda = 0$, since if they do, while price levels will be set to zero, output will follow a white noise process. It is welfare improving for agents to observe this process with positive frequency and therefore have price movements capture some of the variance in the economy.

4 Policy Implications

The previous section on comparative statics revealed that the number and nature of the equilibria in our model is strongly impacted by parameter values. We turn now to our central application, which is how the relationship between output and price volatility depends, through endogenous changes in λ, on the activism of optimal policy. The framework in this paper is the first to allow for an equilibrium study of this issue. The novel implication of our approach is that policy 'activism' has both direct and indirect effects on unconditional price and output variance. Above we noted that the Bernanke Hypothesis is a conjecture on the tension between these effects. The current Section examines this relationship.

4.1 Policy Implication Results

Result one of Proposition 1 obtained the usual trade-off between σ_p^2 and σ_y^2 in the BMR model with exogenous λ. Increasing ω leads policy to reduce price variation. Because λ has not changed, the real mark-up shocks are observed with the same regularity, and if prices do not move to accommodate them then output must. Combining all results of Proposition 1 with the result for $\frac{d\lambda^*}{d\omega}$ in Proposition 3, indicates the potential shape of the trade-off in case of endogenous inattention and small demand shocks. For an interior equilibrium we know that $\frac{d\lambda^*}{d\omega} < 0$. It is thus unambiguous that an increase in ω will reduce price volatility. However, while for fixed λ, increasing ω directly increases output volatility, raising ω may indirectly decrease output volatility as a result of the equilibrium reduction in λ. Thus the effect of an increase in ω on output volatility in the case of endogenous inattention is ambiguous. The results of Proposition 1 and Proposition 3 therefore suggest that the usual trade-off between output and price volatility may not always obtain. In this section we investigate this issue numerically and show that it is indeed possible, over at least part of the range of ω, for the usual trade-off to disappear, and that decreased policy activism may lead to a decline in both price and output volatility. However, our numerical results also indicate that an output-price volatility trade-off will emerge for sufficiently high ω and/or for sufficiently high σ_e^2.

Policy in this model is pinned down by the Central Bank's objective function. We alter policy by varying the relative weight ω in the central bank's preferences. For each chosen value of ω, we compute the unconditional equilibrium output and price variance and plot the relationship between σ_p^2 and σ_y^2. This relationship is a "policy frontier" in the sense that it describes the equilibrium outcome for each level of policy activism.

By way of comparison, we present the policy frontier first for the BMR model with exogenous λ and then for our model which endogenizes λ. We choose the parameters as $\alpha = .1$, $\rho = .85$, $C = 5$, $\sigma_\varepsilon^2 = .1, \sigma_e^2 = .1$, which are close to our benchmark values.[29] Figure 4 sets $\lambda = .25$ and thus provides an illustration of the BMR model with exogenous λ. The figure contains four panels describing, for $3 \leq \omega \leq 30$, (clockwise, starting from the NW corner) the frontier, the exogenous value of λ, and the values of σ_y^2 and σ_p^2 as ω varies. The arrow indicates the direction of motion along the frontier as ω is increased. The downward sloping nature of the frontier represents the usual trade-off between output and price variance. As ω is increased, policy is chosen to reduce price variance, and the equilibrium response is to increase output variance.

INSERT FIGURE 4 HERE

In Figure 5 we consider the impact of increasing ω when λ is chosen endogenously as in our model. For each value of ω we compute the associated stable fixed point of the T-map and the resulting equilibrium variances. The frontier is described in the northwest panel of Figure 5. The arrow indicates the direction of motion along the frontier as $13 \leq \omega \leq 30$ is increased. For $\omega < 13$ the shape of the frontier becomes quite steep and so, except for the λ panel, we omit this range for clarity of presentation.[30]

INSERT FIGURE 5 HERE

Unlike when λ is fixed exogenously, the frontier in the case of endogenous inattention is non-monotonic and takes the shape of a 'nose'. The usual trade-off between price

[29]The value for α is the one used by BMR. The values of ρ, σ_e^2 and σ_ε^2 are chosen to roughly match observed values of σ_p^2 and σ_y^2 for our choice of C.

[30]Under this parameterization there exists a unique stable equilibrium.

and output variance exists for sufficiently large ω but, most interestingly, the 'nose' implies that for some range of ω the output-price variance trade-off is eliminated. In particular, we find that in this range a decrease in activism reduces both output variance and price variance. When the policymaker's preferences shift toward lower activism, the unconditional variance of price will decline accordingly. For fixed λ, this would increase output volatility. However, the decrease in price level volatility lowers the firms' incentive to pay for information and decreases λ^*, as is seen in the northeast panel of Figure 5.

The decrease in equilibrium λ^* associated with this range of ω acts to decrease output volatility. The northeast and southwest panels illustrate that for $13 \leq \omega \leq 39$ the indirect effect – whose strength is measured by the responsiveness of λ^* to changes in ω – is greater than the direct effect and so output variance falls sharply. As ω increases beyond 39, the associated point on the frontier moves onto the downward sloping portion corresponding to the usual trade-off. As the northeast panel clearly demonstrates this occurs when λ^* adjusts slowly to its lower bound. At this point, the direct effect of ω on output variance outweighs the indirect expectation formation effect; hence, the southwest panel indicates an increase in output variance. We conclude that by decreasing policy activism, the central bank may be able to *jointly* lower the volatility of the price level and output. This unequivocal gain to reduced activism is not without bounds, however, as eventually a volatility trade-off emerges. Below we present further discussion of the implications of Figure 5 for government policy and social welfare.

The intuition behind the results above suggest that, depending on the responsiveness of λ^* to changes in ω, the slope of the frontier could be positive or negative. While the frontier is upward sloping for $C/\sigma_\varepsilon^2 = 50$ and sufficiently small ω, for a sufficiently high C (or σ_e^2) the impact on the information accrual rate will be small and the frontier will be everywhere downward sloping. This conjecture is verified in Figure 6 which takes the same parameter values as Figure 5 except that it sets $C/\sigma_\varepsilon^2 = 200$.[31] By increasing the relative costs of updating by a factor of four, the usual trade-off exists over the entire range.[32] Figure 6 illustrates that if the marginal cost of information acquisition increases sufficiently rapidly in λ then the results are close to the BMR case of exogenous λ. Similar results obtain for large σ_e^2.

INSERT FIGURE 6 HERE

Non-monotonic policy frontiers exist also in Orphanides and Williams (2003a). In their model, private agents forecast inflation using a constant gain version of recursive

[31] In the figure we have scaled up σ_ε^2 in order to roughly match observed price and output variances.

[32] In this case there are two stable equilibria for low values of ω: $\lambda^* = 1$ and $0 < \lambda^* < 1$. Figure 6 plots the results for the choice of the stable interior equilibrium. Choosing $\lambda^* < 1$ is in the spirit of BMR and, thereby, appropriate for examining the policy implications of endogenous inattention.

least squares. The constant gain learning produces greater persistence in response to exogenous shocks. Orphanides and Williams (2003a) study the implications of this greater persistence for the conduct of optimal monetary policy. They find that optimal policy should be more vigilant against inflation when agents engage in least-squares learning. In our model, the response to mark-up shocks depends on the equilibrium value of λ, which depends on the activism of policy. Less activist policy lowers the optimal attentiveness of agents and consequently can lower economic volatility as observed in the 'Great Moderation'. However, our results in Figure 5 caution policymakers that there may be a limit to the reduced output volatility, resulting from heightened vigilance against price volatility, since eventually a trade-off may emerge.

The key intuition to this cautionary insight is the effect ω has on the equilibrium value of λ^*. Successively higher values of ω will decrease λ^*, as detailed in Proposition 3. It can be shown that, as $\omega \to \infty$, $\lambda^*(\omega)$ converges to a positive value. One might therefore expect that the direct effect of ω on σ_y^2 will dominate for sufficiently large ω, leading to an eventual trade-off.[33]

4.2 Discussion of Policy Implications

The results illustrated in Figures 5-6 are new and important. Previous work on optimal monetary policy has either not taken into account the costs of processing and collecting information or has ignored the endogenous feedback between policy and the degree of inattention. Our results show that if the policy authority decreases its output activism it induces agents to reduce the rate at which they gather new information. This has the effect of lowering the unconditional variance of the economy. This result is at odds with what is generally found in the literature, but is consistent with the empirical evidence of the 'Great Moderation.'

Whether the policy frontier is upward or downward sloping depends crucially on the costs of updating and processing information. We have shown that for relatively low costs the usual trade-off between price level and output volatility gives way to an upward sloping frontier over a wide range of the policy parameter ω. However, for sufficiently high ω it appears the policy frontier is always eventually downward sloping.

It may seem odd that the mechanism for the reduction in economic volatility is a reduction in the rate of information acquisition by private agents. Intuitively, one might expect a higher rate of information gathering to be socially optimal. This is not necessarily the case in our set-up since the mark-up shocks are distortionary. It

[33]Our numerical investigations suggest that there is always a trade-off for sufficiently high ω (at least for $\alpha < 1$). Investigating this issue theoretically is not straightforward since $\partial\sigma_y^2/\partial\omega$ vanishes as $\omega \to \infty$.

can therefore be welfare improving to reduce the effect of these shocks on the pricing and output decisions of firms. By reducing the price volatility associated with these shocks, private agents are induced to reduce their intensity of information acquisition and diminish their response to the distortionary shocks.

Our finding that a stronger response to markup shocks not only lowers price variance, but also provides an incentive for agents to update their information less often, is related to Svensson's (2003) hypothesis about inflation targeting. Svensson argues that by targeting an inflation rate agents' expectations will be anchored and economic volatility reduced. In our model, the policymaker becomes less 'active' and as a result the *equilibrium* outcome is that agents' expectations are anchored. This is an intuitively appealing result that supports the inflation/price targeting hypothesis through the equilibrium response in a model with information updating costs.

We note that it is not the case that the policy frontier is analogous to a production possibilities frontier or a budget constraint. The points on the frontier are equilibrium outcomes resulting from the joint determination of optimal monetary policy and Endogenous Inattention. The possibility of a positively sloped policy frontier does, however, raise the possibility that there may be gains to commitment analogous to the gains in other set-ups from appointing a conservative central banker.

To pursue this line of thought, imagine that the government evaluates outcomes according to the loss function in (5) with weighting parameter ω^* which is not necessarily equal to the parameter ω used to set policy. In other words, the government hires a central banker with activism parameter ω so that the resulting equilibrium outcomes $\sigma_y^2, Var(p_i - p)$ minimize their loss with preferences ω^*. Is appointing a central banker with $\omega > \omega^*$ socially preferable? For the economy illustrated in Figure 5, for all realistic ω^* there is an unambiguous welfare gain to choosing a central banker that moves along the policy frontier, past the 'nose', and onto the usual trade-off portion of the curve. In this case the loss-minimizing policy parameter ω is greater than ω^*. This example suggests that appointing a more conservative central banker and placing the economy along the usual trade-off is socially optimal.[34]

A conservative bias is not a fully general result, however. In Figure 6 there is always a trade-off between output and price volatility. In this case, the socially optimal point on the frontier depends critically on ω^*. For $\omega^* < \hat{\omega} \backsimeq 20$, our numerical results indicate that the government should choose a more conservative (i.e. $\omega > \omega^*$) central banker, while for $\omega^* > \hat{\omega}$ the government benefits by choosing a less conservative central banker. Since this issue is not central to the current paper we reserve further investigation for future work.

As noted in the introduction, a possible interpretation of the "Great Moderation"

[34]We remark that the government's loss function could be adjusted to include costs of information gathering by private agents. This would strengthen the argument for a conservative central banker (and would weaken the counter-example given in the following paragraph).

is the result of a permanent reduction in activism beginning in the early 1980s, moving the economy down along a positively sloped frontier. In proposing this explanation we are going beyond the formal model, and a number of specific interpretations of the shift in policy and the resulting decline in economic volatility are possible, depending on the degree of sophistication that we want to attribute to policymakers. As presented in Sections 2 and 3, the equilibrium described is the usual Nash equilibrium in a simultaneous move game. Within this setting policymakers are fully cognizant of the structure of the economy but, as in Kydland and Prescott (1977), are condemned by the timing protocol to an inefficient equilibrium. An increase in ω leading to a simultaneous decline in output and price volatility might either be the fortuitous result of an exogenous change in policymakers preferences or a more conscious attempt to improve welfare by appointing a conservative banker, following the logic of Rogoff (1985).

The interpretation of the great moderation just described assumed sophisticated policymakers who understood the endogeneity of the information acquisition rate λ, but were hemmed into an inefficient equilibrium by the timing protocol of the economy. If instead the timing protocol is that policymakers first choose the policy rule and that private agents then respond optimally, given this policy, then an alternative interpretation is possible. Suppose that policymakers were initially naive, believing that λ was exogenous, but that over time policymakers began to appreciate the importance of the various channels through which a more stable price level affects the economy. A growing understanding, in particular, that λ is endogenous, could eventually lead policymakers to adopt less activist policies in order to gain the additional benefits of reduced output volatility.

While both of these interpretations are viable, we prefer a third interpretation in which policymakers, as well as private agents, are neither naive nor fully informed rational, but instead are boundedly rational in the spirit of Marcet and Sargent (1989), Sargent (1999) and Evans and Honkapohja (2001). In this interpretation, policymakers follow a policy rule of the form recommended by BMR, but instead of using fully rational forecasts to implement the policy, which would require knowledge of the full structural model, they forecast using a time-series model, updating the parameters over time using recursive least-squares. An analogous bounded rationality assumption is made for private firms, who use consultants to act as information gatherers and provide firms with an estimate of their optimal frequency for information processing as well as with forecasts of the optimal prices to set. Least-squares learning allows both policymakers and firms to track changes in structural parameters that may occur for a variety of reasons.

In a companion paper, Branch et. al. (2006), we develop an adaptive learning formulation of this model. We consider there a system initially in equilibrium and look at the impact of an exogenous increase in ω, i.e. a permanent decrease in policy activism, with the cost of information accrual parameter set at a moderately low level. The numerical results track our theoretical results that a simultaneous decline in price and output volatility is possible, but with one difference. Initially, when the new policy rule is implemented, output volatility rises in line with the "standard" view of a trade-off, reflecting the transitional period in which λ adapts over time to its new lower equilibrium level. However, in the long-run output as well as price volatility decline permanently. These real-time learning results strike us as very natural, reinforcing the model developed in the current paper and indicating that the policy implications are not fragile to the assumed timing protocol of the game between policymakers and the private sector.

5 Conclusion

This paper has studied the implications for monetary policy of an economy in which agents endogenously choose the rate at which they update their information. Following Ball, Mankiw, and Reis (2005) we assume that it is costly for agents to update their information sets each period. We extend their model, however, by explicitly modeling the choice of the rate at which private agents acquire information. We assume that agents choose the frequency with which they update their information sets by minimizing a quadratic loss function that depends on the costs of updating and forecast errors. The aggregate rate at which agents update their information is determined in a Nash equilibrium, among the private agents as well as the policymaker, in which policy is set optimally given the equilibrium rate.

We characterize the set of equilibria and study their comparative statics. Bernanke (2004, p.5) conjectures that a fundamental shift in Federal Reserve objectives could lead to an anchoring of expectations and a reduction in economic volatility. This paper provides a systematic account of this hypothesis. A primary insight of this paper is to elucidate the important interactions between monetary policy and the degree of private agent attentiveness, which in turn determines the relationship between price and output volatility.

Previous studies have emphasized that price and output variance move in opposite directions when a policymaker becomes less activist. These results appear inconsistent with the empirical evidence that, as the Federal Reserve became more aggressive in fighting inflation, both output and price volatility declined. Our model provides a potential explanation of these features of the data by showing that the reduction in price volatility can make it unnecessary for agents to update information as quickly, leading in turn to a reduction in output volatility. This finding is closely related to the idea of inflation targeting advocated by Svensson (2003) and others. At the same time, we show that there is a tension between the direct effect of a policy rule and its indirect effect on the equilibrium attentiveness of agents. At a sufficiently low level of activism, the direct effect can dominate so that a volatility trade-off reappears.

Appendix

Proof of Proposition 1. Notice that ϕ_0 is independent of ω, so that to prove part 1, we may assume $\sigma_e^2 = 0$. These results then follow immediately from the equilibrium descriptions of the price and output processes (7), (8), and from the definitions of ϕ and φ, with the exception of the result concerning cross-sectional variance. Here, we require a result from BMR; they determine that

$$Var_i(p - p_i) = \sum_{j \geq 1} \eta_j \left(p_t - E_{t-j}(p_t)\right)^2$$

where

$$\eta_j = \frac{\lambda(1-\lambda)^j}{(1-(1-\lambda)^j)(1-(1-\lambda)^{j+1})}.$$

Substituting into this expression the equilibrium price path in (7), it follows that

$$Var_i(p - p_i) = \sum_{j \geq 1} \eta_j \left(\sum_{k=0}^{j-1} \phi_k \varepsilon_{t-k} \right)^2$$

Taking unconditional expectations leads to,

$$EVar_i(p - p_i) = \sigma_\varepsilon^2 \sum_{j \geq 1} \eta_j \hat{\phi}_j,$$

where

$$\hat{\phi}_j = \sum_{k=0}^{j-1} \phi_k^2.$$

The result then follows from the fact that $\frac{\partial \phi_k}{\partial \omega} < 0$.

Part 2 follows from the fact that $\frac{\partial \phi_k}{\partial \lambda} > 0$ for $k \geq 0$. To prove part 3, first notice that it holds when $\sigma_e^2 = 0$ using the same argument as for part 2, and then continuity guarantees it holds for small positive values of σ_e^2. ■

To prove Lemma 1 and Proposition 3 we require some notation. It is easiest to decompose the loss function into the parts due to mark-up shocks and demand shocks respectively:

$$L(\lambda, \bar{\lambda}) = L^\mu(\lambda, \bar{\lambda}) + (1-\lambda) A(\alpha, \bar{\lambda})^2 \sigma_e^2,$$

where $\bar{\psi}_k = \sigma_\varepsilon^2 \sum_{j=k}^{\infty} \bar{\theta}_j^2$ and

$$L^\mu(\lambda, \bar{\lambda}) = (1-\lambda)\bar{\psi}_0 - \lambda \sum_{j=1}^{\infty} (1-\lambda)^j \bar{\psi}_j.$$

Notice that L^μ is precisely the loss function with demand shocks set to zero. For the remainder of this Appendix, we assume $\bar{\psi}_k$ are defined as above, that is, with $\sigma_e^2 = 0$.

Recall $T(\bar{\lambda},\xi) = \arg\min_\lambda \hat{L}(\lambda,\bar{\lambda},\xi)$, where $\hat{L} = L^\mu + (1-\lambda)A(\alpha,\bar{\lambda})^2\sigma_e^2 + C\lambda^2$, ξ is the vector of model parameters and $\bar{\lambda}$ is the economy wide value of λ, which is taken as given by individual agents. The equilibrium λ^* is defined by $T(\lambda^*,\xi) = \lambda^*$, so that by the implicit function theorem,

$$\frac{\partial \lambda^*}{\partial \xi_i} = \frac{T_{\xi_i}}{1 - T_{\bar{\lambda}}}.$$

Stability then implies that sign $\left(\frac{\partial \lambda^*}{\partial \xi_i}\right)$ = sign (T_{ξ_i}). To compute T_{ξ_i}, we note that T is defined by the first order condition $\hat{L}_\lambda(T(\bar{\lambda},\xi),\bar{\lambda},\xi) = 0$. Again we may apply the implicit function theorem to obtain

$$T_{\xi_i} = -\frac{\hat{L}_{\lambda\xi_i}}{\hat{L}_{\lambda\lambda}}. \tag{16}$$

We will show that $\hat{L}_{\lambda\lambda} > 0$, so that sign $(T_{\xi_i}) = -$sign $\left(\hat{L}_{\lambda\xi_i}\right)$. Thus it remains to compute the relevant second partials of $\hat{L}$.

We require the following result:

Lemma 2 *Suppose β_i is a decreasing positive sequence and for each real number ν, $\gamma_i(\nu)$ is a sequence with $\sum \gamma_i(\nu) = M$, and $V(\nu) = \sum \gamma_i(\nu)\beta_i < \infty$. If there exists $N(\nu)$ so that $\frac{\partial \gamma_i}{\partial \nu} > 0 \Leftrightarrow i < N(\nu)$ then $V_\nu > 0$.*

Proof. The idea is simple: increase the values of γ_i corresponding to larger weights, and decrease the values corresponding to lower weights. Formally, we have

$$\begin{aligned} V_\nu &= \sum_{i\in\mathbb{N}} \frac{\partial \gamma_i}{\partial \nu}\beta_i = \sum_{i<N(\nu)} \frac{\partial \gamma_i}{\partial \nu}\beta_i + \sum_{i\geq N(\nu)} \frac{\partial \gamma_i}{\partial \nu}\beta_i \\ &> \sum_{i<N(\nu)} \frac{\partial \gamma_i}{\partial \nu}(\beta_i - \beta_{N(\nu)}) + \beta_{N(\nu)} \sum_{i\in\mathbb{N}} \frac{\partial \gamma_i}{\partial \nu} \\ &= \sum_{i<N(\nu)} \frac{\partial \gamma_i}{\partial \nu}(\beta_i - \beta_{N(\nu)}) > 0, \end{aligned}$$

where the last *equality* follows from the fact that $\sum \gamma_i(\nu) = M$ implies the sum of partials equals zero. ■

Now define the following notation:

$$f(\lambda, j) = \frac{(1-\lambda)^{j+1}}{1-(1-\lambda)^{j+1}} \quad \text{and} \quad g(\lambda, j) = \lambda(1-\lambda)^j.$$

Then

$$\bar{\theta}_j = \begin{cases} \frac{1}{1-(1-\alpha)\bar{\lambda}} & j = 0 \\ \frac{(1+f(\bar{\lambda},j))\rho^j}{\alpha^2\omega + f(\bar{\lambda},j)} & j > 0 \end{cases}.$$

and

$$\hat{L} = \bar{\psi}_0 - \sum_{j=0}^{\infty} g(j,\lambda)\bar{\psi}_j + (1-\lambda)A(\alpha,\bar{\lambda})^2\sigma_e^2 + C\lambda^2.$$

The partials we are to compute are then given by

$$\begin{aligned} \hat{L}_\lambda &= -\sum_{j=0}^{\infty} g_\lambda \bar{\psi}_j - A(\alpha,\bar{\lambda})^2\sigma_e^2 + 2C\lambda \qquad (17) \\ \hat{L}_{\lambda\lambda} &= -\sum_{j=0}^{\infty} g_{\lambda\lambda}\bar{\psi}_j + 2C, \\ \hat{L}_{\lambda\xi_i} &= -\sum_{j=0}^{\infty} g_\lambda \frac{\partial \bar{\psi}_j}{\partial \xi_i} - 2\sigma_e^2 \frac{\partial A}{\partial \xi_i}. \end{aligned}$$

Proof of Lemma 1. Setting $C = 0$, this follows from Lemma 2 and the definition of the loss function. ■

Proof of Proposition 3. We now proceed to prove the proposition in a series of steps.

Step 1. $\hat{L}_{\lambda\lambda} > 0$.

First notice that for all λ, $\sum g(\lambda, i) = 1$ so that $\sum g_\lambda(\lambda, i) = 0$. We may compute

$$\begin{aligned} g_\lambda &= (1-\lambda)^{j-1}(1-(j+1)\lambda) \qquad (18) \\ g_{\lambda\lambda} &= -(1+j)(1-\lambda)^{j-1} - (j-1)(1-\lambda)^{j-2}(1-(j+1)\lambda). \end{aligned}$$

We find that

$$g_{\lambda\lambda} < 0 \Leftrightarrow \frac{j+1}{j-1} > \frac{(1+j)\lambda - 1}{1-\lambda},$$

thus implying the existence of $N(\lambda)$ so that $j < N(\lambda) \Leftrightarrow g_{\lambda\lambda} < 0$. Applying the lemma with $V = \hat{L}_\lambda - 2C\lambda$ and $\gamma_i(\nu) = -g_\lambda(\lambda, i)$ yields the result.[35]

Before moving on to the remaining steps, we show the following:

$$\operatorname{sign}\left(\hat{L}_{\lambda\xi_i}\right) = -\operatorname{sign}\left(\frac{\partial \bar{\theta}_j}{\partial \xi_i}\right), \qquad (19)$$

provided $\xi_i = \rho$ or ω, and if $\sigma_e^2 = 0$ then equation (19) holds for $\xi_i = \alpha$.[36] Indeed,

[35]Note that $g_{\lambda\lambda}(\lambda, 0) = 0$, so that the premise of the Lemma is not precisely met. However, it is trivial to modify the proof of the Lemma to account for this minor generalization: just have the premise read $i < N(\nu) \Rightarrow \frac{\partial \gamma_i}{\partial \nu} \geq 0$ with at least one strict inequality, and $i \geq N(\nu) \Rightarrow \frac{\partial \gamma_i}{\partial \nu} \leq 0$, and notice the proof goes through unchanged.

[36]It may be the case that $\frac{\partial \bar{\theta}_0}{\partial \xi_i} = 0$, but this does not impact the result.

notice

$$\frac{\partial \bar{\psi}_k}{\partial \xi_j} = \sum_{j \geq k} 2\bar{\theta}_j \frac{\partial \bar{\theta}_j}{\partial \xi_i}.$$

Assume for the moment that $\frac{\partial \bar{\theta}_j}{\partial \xi_i} < 0$. Then $\beta_j \equiv -\frac{\partial \bar{\psi}_k}{\partial \xi_i}$ form a decreasing positive sequence. Also notice that, from (18), there is a $M(\lambda)$ so that $g_\lambda(\lambda, i) > 0 \Leftrightarrow i < M(\lambda)$. Thus we may apply the Lemma above to $\sum g(\lambda, j)\beta_j$ to get $\hat{L}_{\xi_j \lambda} > 0$. A similar argument applies in case $\frac{\partial \bar{\theta}_j}{\partial \xi_i} > 0$.

To determine the comparative statics for α, ρ and ω, we simply compute the sign of $\frac{\partial \bar{\theta}_j}{\partial \xi_i}$, and then appeal to (19).

Step 2. If $\sigma_e^2 = 0$ then $\hat{L}_{\lambda\alpha} > 0$.

For $j = 0$ computing the sign of $\frac{\partial \bar{\theta}_j}{\partial \alpha}$ to be negative is straightforward. Let $B_j = \alpha^2\omega + f(\bar{\lambda}, j)$. For $j > 0$ we compute

$$\begin{aligned} \frac{\partial \bar{\phi}_j}{\partial \alpha} &= \frac{-2\rho^j \alpha\omega}{B_j^2} < 0 \\ \frac{\partial \theta_j^*}{\partial \alpha} &= -2\alpha\omega\bar{\phi}_j + (1 - \alpha^2\omega)\frac{\partial \bar{\phi}_j}{\partial \alpha}. \end{aligned}$$

Combining these two equations with the definition of $\bar{\phi}_j$ in terms of B_j yields

$$\frac{\partial \bar{\theta}_j}{\partial \alpha} = 1/B_j^2 \left(-2\alpha\omega\rho^j B_j - 2\alpha\omega\rho^j(1 - \alpha^2\omega)\right).$$

Finally, recognizing $1 - \alpha^2\omega = 1 + f(\bar{\lambda}, j) - B_j$ yields

$$\frac{\partial \bar{\theta}_j}{\partial \alpha} = -\frac{2\alpha\omega\rho^j(1 + f(\bar{\lambda}, j))}{B_j^2} < 0.$$

Step 3. $\hat{L}_{\lambda C} > 0$

This follows easily from the fact that $\hat{L}_C = \lambda^2$.

Step 4. $\hat{L}_{\lambda\rho} < 0$.

Note that

$$\frac{\partial \bar{\theta}_j}{\partial \rho} = \frac{j(1 + f(\bar{\lambda}, j))\rho^{j-1}}{\alpha^2\omega + f(\bar{\lambda}, j)} > 0,$$

for $j > 0$.

Step 5. $\hat{L}_{\lambda\omega} > 0$.

Simply notice

$$\frac{\partial\bar{\theta}_j}{\partial\omega} = -\frac{\alpha^2(1 + f(\bar{\lambda}, j))\rho^j}{(\alpha^2\omega + f(\bar{\lambda}, j))^2} < 0$$

for $j > 0$.

Step 6. $\hat{L}_{\lambda\sigma_e^2} < 0$.

We have $L^{\mu}_{\lambda\sigma_e^2} = 0$ so that

$$\hat{L}_{\lambda\sigma_e^2} = \frac{\partial^2\left((1-\lambda)\,A\left(\alpha, \bar{\lambda}\right)^2\sigma_e^2\right)}{\partial\lambda\partial\sigma_e^2} = -A\left(\alpha, \bar{\lambda}\right)^2 < 0.$$

Step 7. $\hat{L}_{\lambda\sigma_\varepsilon^2} < 0$.

Notice

$$\hat{L}_{\lambda\sigma_\varepsilon^2} = L^{\mu}_{\lambda\sigma_\varepsilon^2} = \frac{\bar{\psi}_0}{\sigma_\varepsilon^2} - \lambda\sum_{j\geq 0}(1-\lambda)^j\frac{\bar{\psi}_j}{\sigma_\varepsilon^2}.$$

Also, $\frac{\bar{\psi}_j}{\sigma_\varepsilon^2}$ is a positive decreasing sequence, so that the proof is completed by applying Lemma 2. ■

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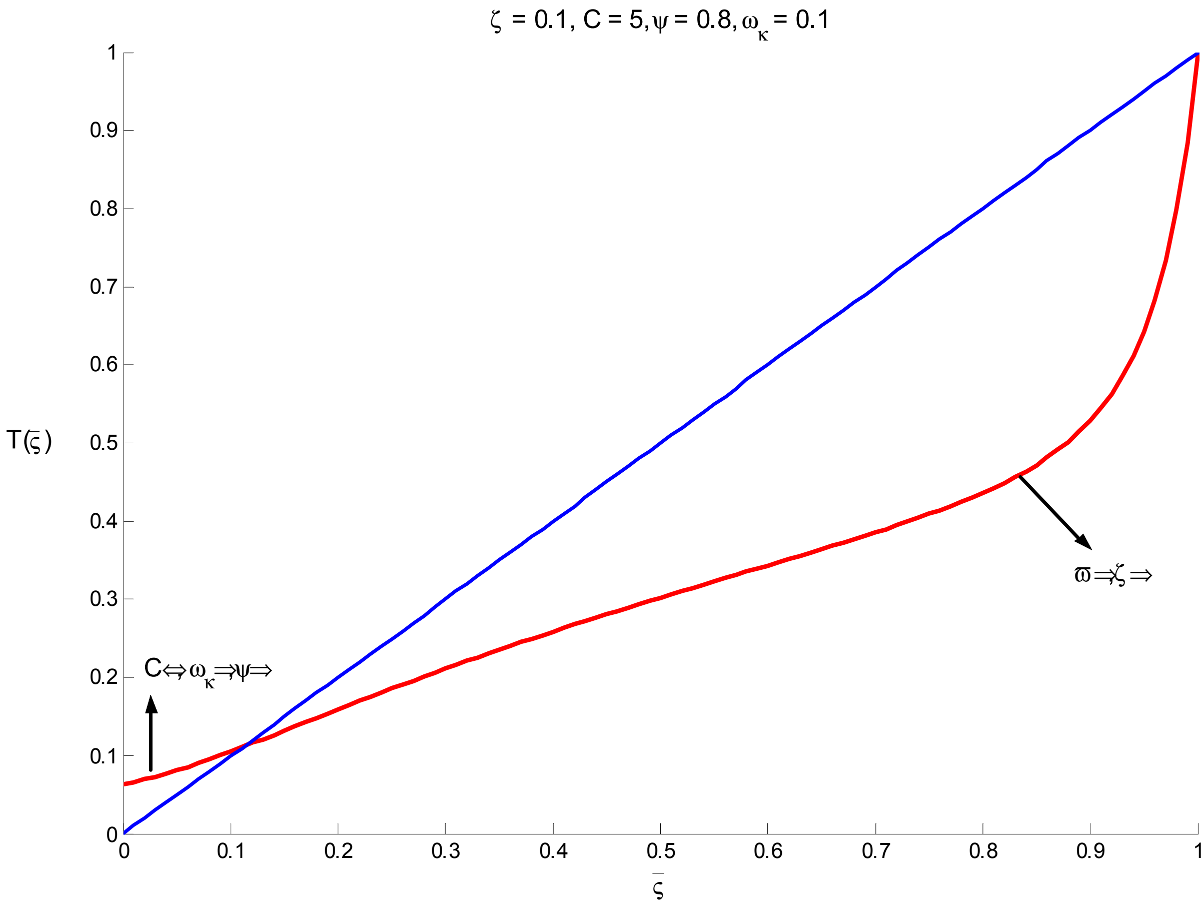

Figure 1. T-map under baseline parameterization. Arrows indicate the way in which T-map is altered by changing a given parameter value.

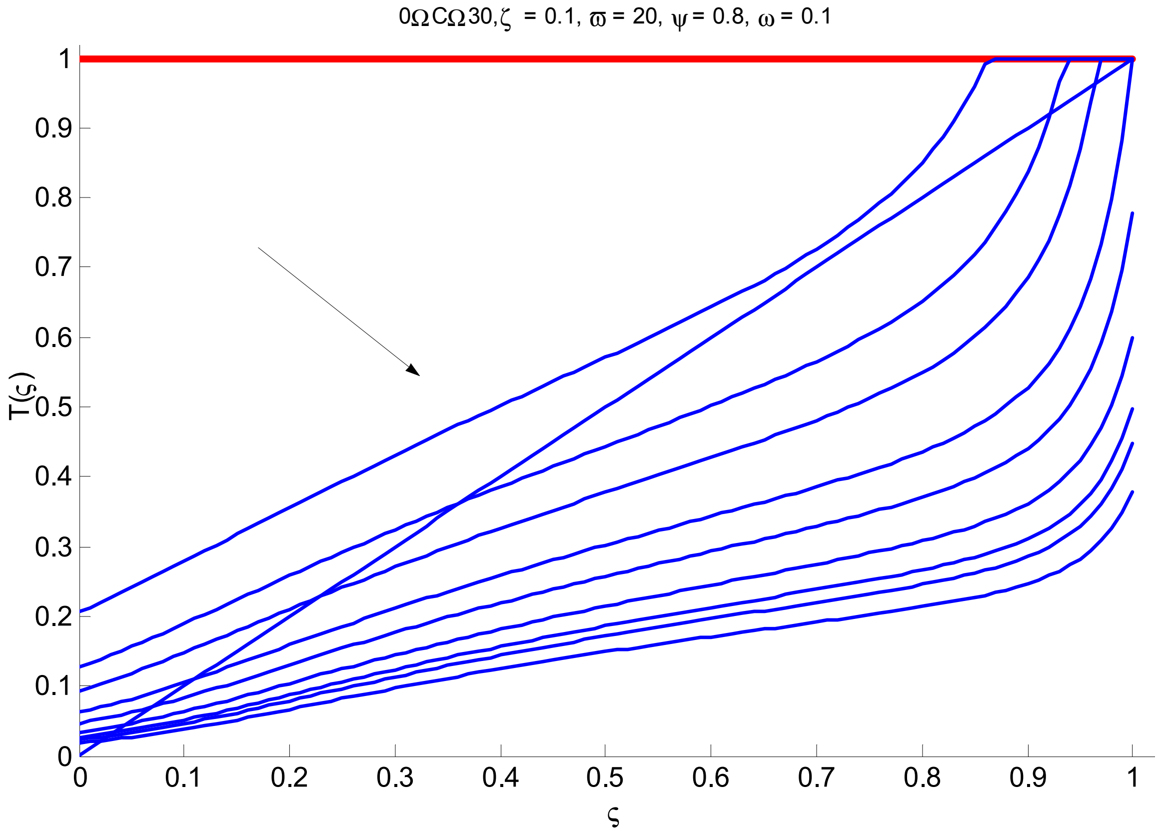

Figure 2. Comparative Statics for $0 \Omega C \Omega 30$.

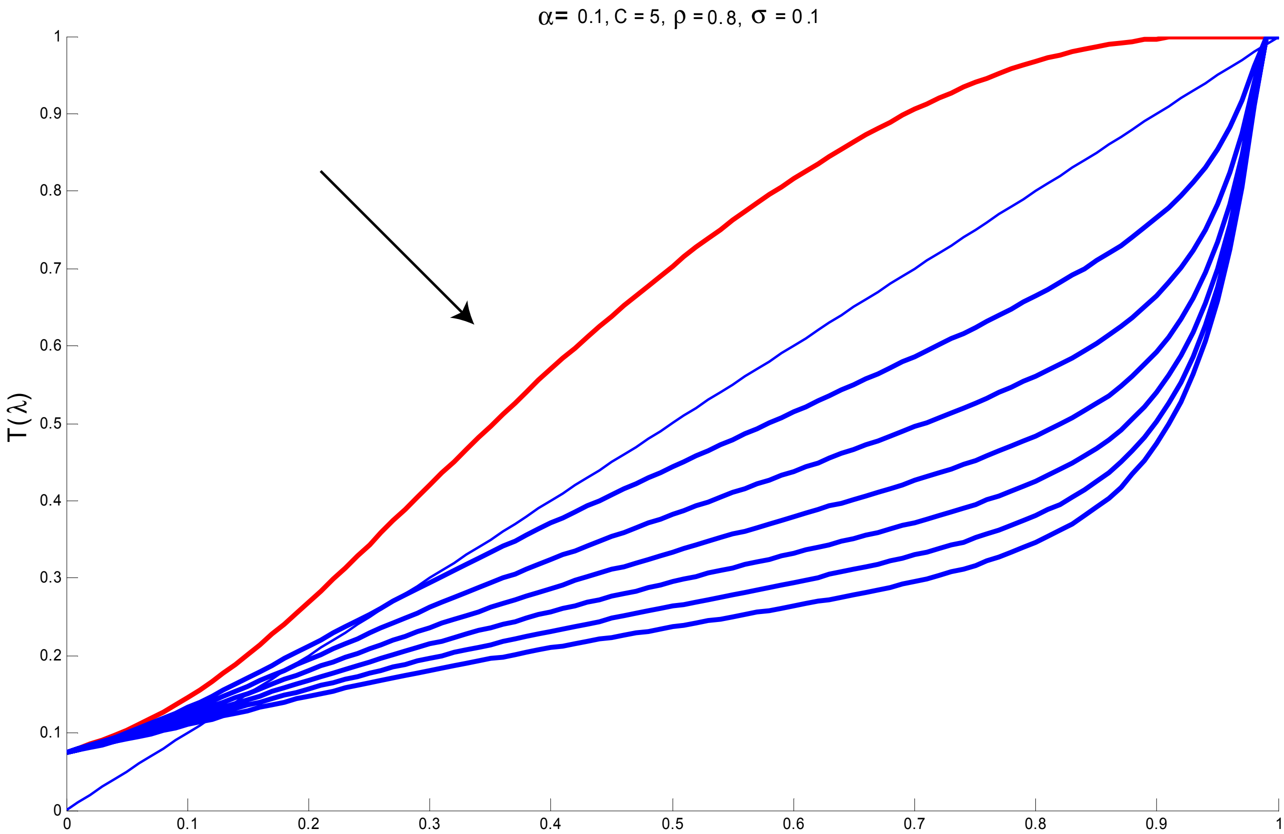

Figure 3. Comparative Statics for 0<ω< 40.

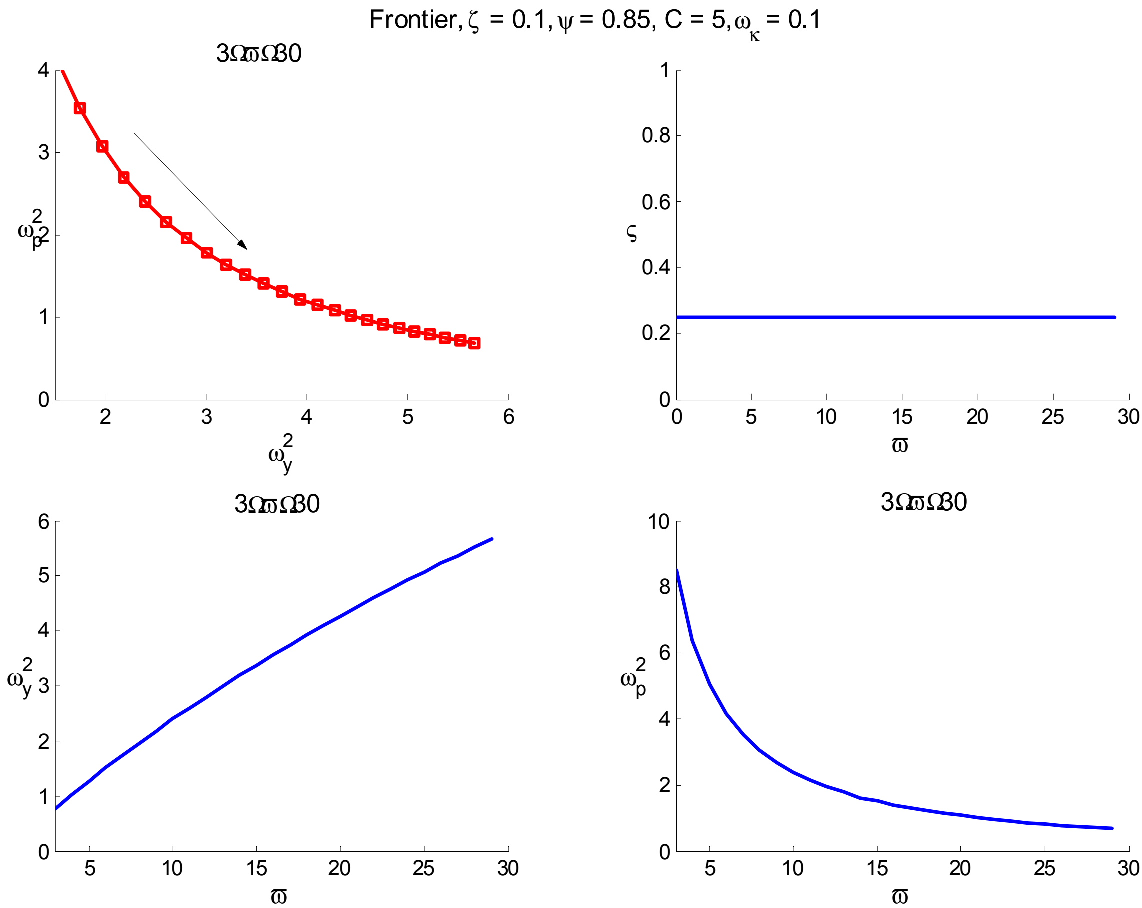

Figure 4. Frontier with fixed ς | .25 .

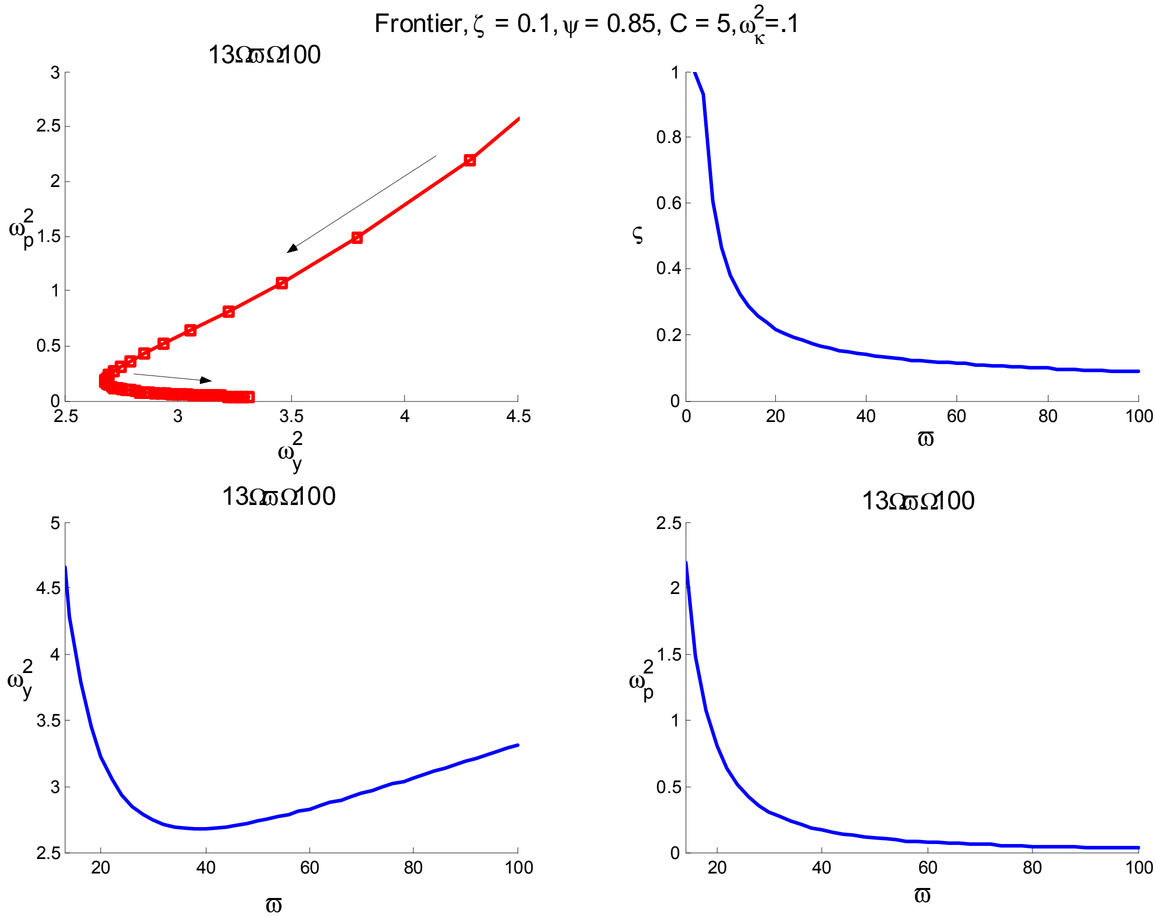

Figure 5. Policy Frontier with Endogenous Inattention and low costs. Northwest, Southwest, and Southeast panels plot $13\ \Omega \varpi\ \Omega 100$. Northeast panel plots $0\ \Omega \varpi\ \Omega 100$.

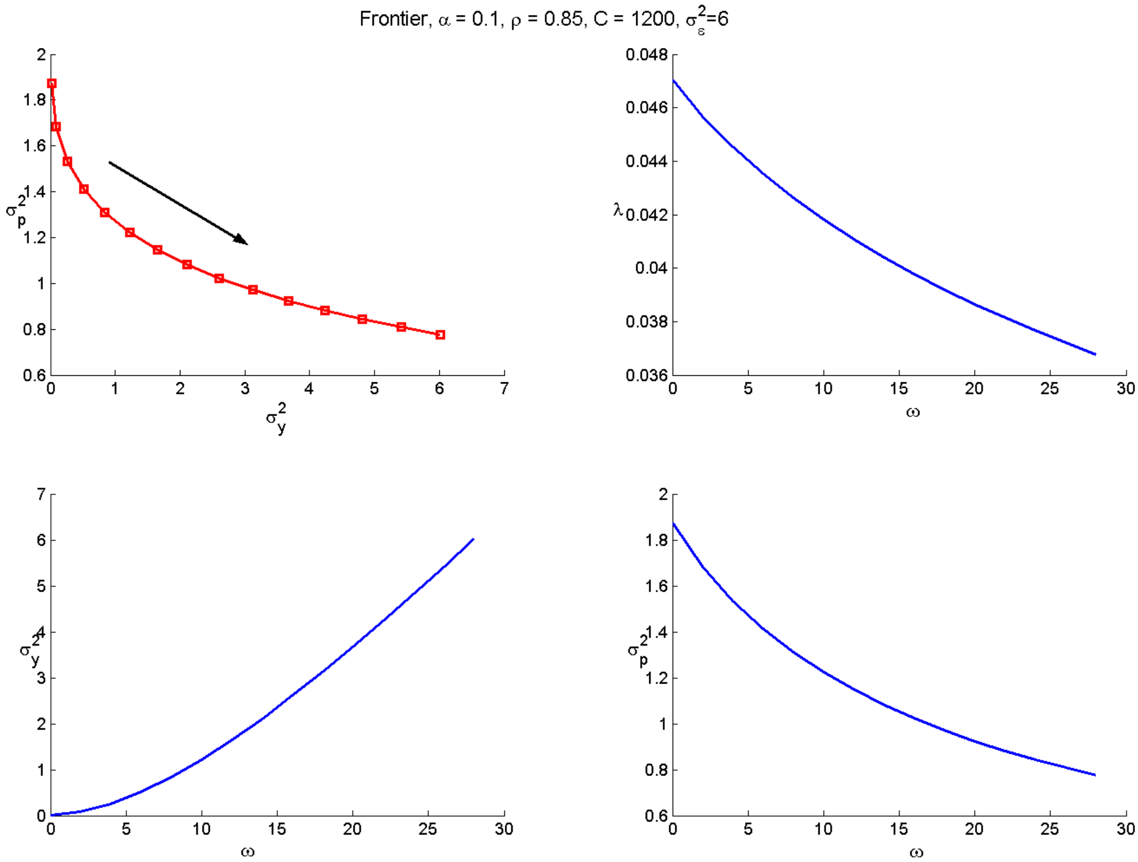

Figure 6. Policy Frontier with Endogenous Inattention and high costs.