Modeling Credit Contagion via the Updating of Fragile Beliefs

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The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve System or the Board of Governors
CDS spreads for Euro zone countries, sample period: 2004/02/12-2011/11/14

Source: Markit Financial Information Services.
Contagion Risk and EU Sovereign Spreads

"Contagion and Euro" returns over 750,000 Google Search Results

- Contagion fears hit Euro, *WSJ, Nov 30, 2010*
- the Euro crisis: a contagious Irish disease, *The Economist Nov 25, 2010*
- Euro Zone: warned of contagion - *Reuters, May 5 2010*
- Contagion fears push the Euro to fresh low, *FT, Nov 29 2010*
“Contagion and Euro” returns over 750,000 Google Search Results

Yet, the notion of contagion remains controversial

– John Cochrane, “Contagion and other Euro Myths,” *WSJ, Dec 2, 2010*

The bailout is being justified on grounds of containing “contagion.” This is nonsense. The notion is that news of an Irish restructuring would scare investors in Spanish bonds, who would start looking at Spain’s ability to repay its debts and then demand higher interest rates. But haven’t investors in Spanish bonds already noticed that there’s a bit of a problem? And wouldn’t news of a giant bailout make these investors question Spanish finances as much as would news of debt restructuring?

Often, contagion implies prices ‘overshoot’ beyond rational response to changing macro-fundamentals
• “Contagion and Euro” returns over 750,000 Google Search Results

• Yet, the notion of contagion remains controversial
  – Often, contagion implies prices ‘overshoot’ beyond rational response to changing macro-fundamentals

• Questions:
  – What is contagion risk, and what are its economic sources?
  – Is there a risk premium associated with contagion risk?
  – To what extent is correlation of sovereign spreads driven by contagion?
Summary

• GE model of contagion across defaultable bonds with two main ingredients:

  1. A ‘hidden state’ whose true value must be learned
     - Beliefs impact expected consumption growth, default probabilities

  2. Representative agent with ‘fragile beliefs’ (Hansen-Sargent)
     - Distinguish between risk and uncertainty
     - No official EU policy regarding the bail-out of member-states
     - Agents formulate beliefs about ‘hidden state’ with scant information to base beliefs on
     - They are uncertain about both the underlying state and the posterior probabilities associated with these states (they have fragile beliefs)
     - Agents display preference for robustness
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• Several nice implications:
  – Levels of sovereign credit spreads well above expected losses
    * Helps explain the so-called ‘credit spread puzzle’
  – Although we use *contagion* label, prices stem from rational representative agent
    * Correlations in credit spreads in excess of correlations in ‘fundamentals’
Summary

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- **Estimate model on a panel of sovereign CDS spreads for 11 Euro zone countries**
  - Incorporate info in economic fundamentals and global financial uncertainty
  - Filter posterior probability of hidden state ⇒ measure of contagion risk
  - Via contagion mechanism, model captures important nonlinearities in CDS spreads
    - Model outperforms (linear) affine benchmark
  - Decompose portion of CDS spreads attributed to preference for robustness
Summary

- **Two additional results:**

1. **Identify tractable defaultable bond price formula in face of ‘multi-directional’ contagion**
   
   - model falls outside of “doubly stochastic” or “Cox-process” framework

2. **Identify conditions for which fragile beliefs produces time-consistent state prices in GE framework generated from marginal utility $U'(\cdot)$ of representative agent:**

\[
U'(t)V(t) \triangleq E_t \left[ U'(t + 1)[V(t + 1) + D(t + 1)] \right] \\
\triangleq E_t \left[ \sum_s U'(t + s)D_{t+s} \right]
\]

**Definition:** A model is *time-consistent* if, using marginal utility as state price density in GE framework, there are no dynamic arbitrage opportunities.
Related Literature

- Robustness and Fragile Beliefs (learning about hidden states)

- Learning
  - Detemple (1989), Gennotte (1986)

- Regime Switching - Hidden States

- Contagion (theoretical)
  - King and Wadhwani (1990), Kodres and Pritsker (2002)

- Contagion (empirical)
  - Lang and Stulz (1992), Bae, Karolyi and Stulz (2000)

- Default Risk

- Correlated Default risk (theoretical)
• Correlated Default risk (empirical/numerical)

• Sovereign Credit Risk (theoretical)

• Sovereign Credit Risk (empirical)

• General Equilibrium and Credit Risk
  – Chen, Collin-Dufresne and Goldstein (2009), Chen (2010), Bhamra, Kuehn, and Strebulaev (2010a,b), Gomes and Schmid (2010)

• Dynamic Principal Component Analysis
The Economy

- True state of the economy $\tilde{S}$ unknown; agents form prior

$$\pi_s(t) \equiv \text{Prob}(\tilde{S} = s | \mathcal{F}_t)$$

- $N$ defaultable entities (firms, countries) indexed by $i \in (1, N)$ with random default times $\tau_i$

- The default probability over the next interval $dt$, conditional on being in state-$s$, is

$$\text{Pr} \left[ d1_{\{\tau_i < t\}} = 1 \mid \tilde{S} = s, \mathcal{F}_t \right] \equiv \mathbb{E} \left[ d1_{\{\tau_i < t\}} \mid \tilde{S} = s, \mathcal{F}_t \right]$$

$$= \lambda_{is}(t^-) 1_{\{\tau_i > t\}} dt$$

- $\lambda_{is}(t^-)$ is the date-$t$ default intensity for country-$i$, conditional on being in state-$s$.

- Investors don’t know hidden state; conditional on their information, the default intensity is

$$\bar{\lambda}_i^P(t^-) = \sum_{s=1}^{M} \pi_s(t) \lambda_{is}(t^-)$$
Updating Beliefs by Observing Defaults

- Investors update estimates \( \{\pi_s(t)\} \) conditional upon default observations \( dt \):

\[
\frac{d\pi_s(t)}{\pi_s(t^-)} = \sum_{i=1}^{N} \left( \frac{\lambda_{is}(t^-)}{\lambda^P_i(t^-)} - 1 \right) dM_i(t)
\]

where

\[
dM_i(t) \equiv \left( d1_{\{\tau_i \leq t\}} - \bar{\lambda}^P_i(t) 1_{\{\tau_i > t\}} dt \right)
\]

- Process has many intuitive properties:
  
  - If prior \( \pi_s(t) = 1 \) for some state-\( s \), then no updating
  
  - When no default is observed, investors revise downward the ‘high-default’ states of nature (i.e., those \( s \) with \( \lambda_{is}(t^-) > \bar{\lambda}^P_i(t^-) \))

\[\pi_s(t) \equiv E \left[ \hat{S} = s \mid \mathcal{F}_t \right] \text{ is a } \mathbb{P}\text{-martingale in that } E_t \left[ d\pi_s(t) \right] = 0\]
• Investors also observe continuous signals that provide information about the state

\[
d\Omega_k(t) = \mu_{k,s} \, dt + \sigma_k \, dZ_k(t) \\
= \bar{\mu}_k \, dt + \sigma_k \, dZ_k(t) \quad k \in (0, K),
\]

where

\[
\bar{\mu}_k(t) \equiv \frac{1}{dt} E \left[ d\Omega_k(t) | \mathcal{F}_t \right] = \sum_s \pi_s(t) \mu_{k,s}
\]

\[
dZ_k(t) = dZ_k(t) + \left( \frac{\mu_{k,s} - \bar{\mu}_k(t)}{\sigma_k} \right) dt
\]

• Agent observes both continuous signals \(d\Omega_k(t)\) and defaults \(d1_{t_i > t} \Rightarrow\) updating equation follows:

\[
\frac{d\pi_s(t)}{\pi_s(t^-)} = \sum_{i=1}^{N} \left( \frac{\lambda_{is}(t^-)}{X_i(t^-)} - 1 \right) \, dM_i(t) + \sum_{k=0}^{K} \left( \frac{\mu_{k,s} - \bar{\mu}_k(t)}{\sigma_k} \right) \, dZ_k(t)
\]
Updating Beliefs

• Rational investors form prior $\pi_s(t)$ on hidden states.

• They observe
  
  – History of defaults
  
  – Continuous signals that provide information about the state

• They use Bayes rule to update prior

• **Updating equation follows**

$$\frac{d\pi_s(t)}{\pi_s(t^-)} = \sum_{i=1}^{N} \left( \frac{\lambda_{is}(t^-)}{\lambda_i^P(t^-)} - 1 \right) dM_i(t) + \sum_{k=0}^{K} \left( \frac{\mu_{k,s} - \tilde{\mu}_k(t)}{\sigma_k} \right) dZ_k(t)$$
Model for intensity conditional on the state

- Specify observable state vector $X(t)$ which contains both country specific variables and common variables.

- Assume $X(t)$ follows multi-dimensional Gaussian affine process.

  $$dX(t) = [\psi - \kappa X(t)] \, dt + \Sigma \, dW(t)$$

- Specify default intensity of entity $i \in (1, N)$ conditional on being in state $s$ as:

  $$\lambda_{is}(t) = \alpha_{is} + \beta'_{is} \, X(t)$$
General Equilibrium with Fragile Beliefs

• Conditional on state-\(s\), aggregate (log-) endowment process is

\[
d \log y = \mu_{0,s} \, dt + \sigma_0 \, dZ_0
\]

• Preferences:

  – Conditional upon being in state \(s \in (1, M)\), agent has logarithmic-preferences

\[
V(\{C(\cdot)\} | F_0, s) = E \left[ \int_0^\infty \beta \, dt \, e^{-\beta t} \, \log C(t) \bigg| F_0, s \right]
\]

  – To rank consumption streams unconditionally, agent displays fragile beliefs

\[
V(\{C(\cdot)\} | F_0) = \min_{\{\xi_s(0)\} > 0} \left\{ \sum_{s=1}^M \pi_s(0) \left( \xi_s(0) V(\{C(\cdot)\} | F_0, s) + \zeta \xi_s(0) \log \xi_s(0) \right) \right\}
\]

subject to the constraint

\[
1 = \sum_s \pi_s(0) \xi_s(0)
\]
General Equilibrium with Fragile Beliefs

- Specify “conditional pricing kernel”

\[
\frac{d\Lambda^s(t)}{\Lambda^s(t)} = -r_s \, dt - \sigma_0 \, dZ_0(t)
\]

with constant state-contingent spot rates \( \{r_s\} \):

\[
r_s = \beta + \mu_{0,s} - \frac{\sigma_0^2}{2}
\]

- Price \( V_t^{D(\omega_T)} \) of a security with contingent cash flows \( D(\omega_T) \) is linear in (adjusted) probabilities!!:

\[
V_t^{D(\omega_T)} = \sum_s \pi^Q_s(t) \mathbb{E} \left[ \left( \frac{\Lambda^s(T)}{\Lambda^s(t)} \right) D(T) | \mathcal{F}_t, s \right]
= \sum_s \pi^Q_s(t) e^{-r_s(T-t)} \mathbb{E}^{Q_s} [D(T) | \mathcal{G}_t, s]
\]

with risk adjusted probabilities:

\[
\pi^Q_s(t) \equiv \pi_s(t) \xi_s(t)
\]

- Note: \( \xi_s(t) \propto e^{-\mu_s/(\beta\zeta)} \) are larger for worse hidden states
Time consistency of Representative agent

- To prove that the price system is sensible (and that the representative agent is time-consistent), we show that there exists a strictly positive stochastic process $\Lambda_t$, such that the price of a claim to payoff $X_T$ can be written as:

$$E_t[\frac{\Lambda(T)}{\Lambda(t)} X_T] = \sum_s \pi^Q_s(t) E[\Lambda^s(T) X_T | s].$$  \hfill (1)

- The right-hand side is the arbitrage-free price in an economy supported by the valid price-system $\Lambda_t$.

- The left-hand side is the ‘shadow value’ of the claim for the agent with Fragile beliefs.

- Thus there exists a no-trade equilibrium in this arbitrage-free economy populated by agents with fragile beliefs.

- Interestingly, we show in a two-period binomial world, that this is not necessarily true for all fragile beliefs preferences. For example, using CRRA utility with EIS different from 1, will in general not lead to time consistent (i.e., arbitrage-free) prices if using the same approach.
Pricing Risky and Risk-free Bonds

- Risk-free Bond: 
  \[ P(\pi(t), T-t) = \sum_s \pi_s^Q(t) e^{-r_s(T-t)} E^Q_s [1|G_t, s] \]
  \[ = \sum_s \pi_s^Q(t) e^{-r_s(T-t)} \]

- Risky Bond (no recovery)
  \[ B^i(\pi(t), X(t), T-t) = \sum_s \pi_s^Q(t) e^{-r_s(T-t)} E^Q_s \left[ 1_{\{\tau_i > t\}} | G_t, s \right] \]
  \[ = 1_{\{\tau_i > t\}} \sum_{s=1}^M \pi_s^Q(t) e^{-r_s(T-t)} B^i_s(X(t), T-t), \]

where

\[ B^i_s(X(t), T-t) \equiv E^Q_s \left[ e^{-\int_t^T du \lambda_{i,s}(X(u))} | F_t, s \right] \]
\[ = e^{M_{i,s}(T-t) - N_{i,s}(T-t) X(t)} \]

with deterministic coefficients \( M_{i,s}(T-t) \) and \( N_{i,s}(T-t) \) available in closed form

⇒ Bond prices are a weighted sum of terms, each of which can be expressed in a “reduced-form structure” Conditional on being in state-\( s \), we are in a doubly stochastic framework!
Contagion in the Euro-Zone Sovereign CDS Market

- Fit the model to sovereign CDS data from 2004-2010 with a two-step approach

- First, identify country-specific and common variables to include in state $X$, which drives jump intensity:

$$
\lambda_{is}(t) = \alpha_{is} + \beta_{is}' X(t)
$$

and has dynamics:

$$
dX(t) = [\psi - \kappa X(t)] \, dt + \Sigma \, dW(t)
$$

- Natural candidates:

  1. Country-specific macroeconomic variables
      \( \Rightarrow \) Construct macroeconomic conditions indices for Euro-zone countries

  2. Common financial markets factor(s)
      \( \Rightarrow \) VIX as a Proxy for global market conditions (Longstaff-Pan-Pedersen-Singleton, Pan-Singleton)

- Pre-estimate coefficients for $X$ dynamics

- Second, fit hidden state probability $\pi$ and other coefficients using first-step inputs and CDS data
Macroeconomic Conditions Indices (MCI)

- Dynamic factor model to estimate a latent economic activity variable (Aruoba-Diebold-Scotti, Harvey)
  - The state of an economy is about the dynamics and interactions of many economic variables
  - Incorporate business conditions indicators measured at different frequencies
    * monthly (e.g., inflation, political risk, ...); quarterly (e.g., GDP);
      and yearly (e.g., Government debt, surplus/deficit)
  - Solve optimal filtering problem: Mixed data frequency; stock vs. flow variables

- State space model for country $i$, $i = 1, \ldots, 11$:
  $y_{i,t}^j = \beta_i^j MCI_{i,t} + \Gamma_i^j W_{i,t} + \varepsilon_{i,t}^j$, $\varepsilon_{i,t}^j \sim N(0, Q_i^j)$, $j = 1, \ldots, J$
  $MCI_{i,t+1} = \rho_i MCI_{i,t} + \eta_{i,t+1}$, $\eta_i \sim N(0, R_i)$

- Maximum likelihood estimation via the Kalman filter
MCIs: The Exciting Details

• Mixed data frequency

  – Country $i$’s variable $j, y_{i,t}^j$, evolves daily but is not observed at that frequency

  – Let $\tilde{y}_{i,t}^j$ be the same variable observed at a lower frequency

  1. For stock variables observed at $D^j$-day intervals:

     $\tilde{y}_{i,t}^j = \begin{cases} y_{i,t}^j = \beta_i^j MCI_{i,t} + \Gamma_i^j y_{i,t-D^j}^j + \varepsilon_{i,t}^j & \text{if } y_{i,t}^j \text{ is observed} \\ N.A. & \text{otherwise} \end{cases}$

  2. Flow variables cumulate over the $D^j$-day interval:

     $\tilde{y}_{i,t}^j = \begin{cases} \sum_{s=1}^{D^j-1} y_{i,t-s}^j = \beta_i^j \sum_{s=1}^{D^j-1} MCI_{i,t-s} + \Gamma_i^j \sum_{s=1}^{D^j-1} y_{i,t-D^j-s}^j + \sum_{s=1}^{D^j-1} \varepsilon_{i,t-s}^j & \text{if } y_{i,t}^j \text{ is observed} \\ N.A. & \text{otherwise} \end{cases}$

  – $D^j$ time varying

• Kalman filter remains valid with missing data

  – If all elements of $y_{i,t}$ are missing, skip updating step

  – If some elements of $y_{i,t}$ are missing, exclude them from the measurement equation
## Economic/political risk indicators and the MCIs

<table>
<thead>
<tr>
<th></th>
<th>EU</th>
<th>Austria</th>
<th>Belgium</th>
<th>Finland</th>
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<tr>
<td>Pol. stability</td>
<td>−</td>
<td>+ ✓</td>
<td>+</td>
<td>−</td>
<td>−</td>
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<td>M3/GDP</td>
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<td>−</td>
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<td>−</td>
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<td>Reserves/GDP</td>
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<td>−</td>
<td>+ ✓</td>
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<td>Real GDP/Pop.</td>
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<td>−</td>
<td>+ ✓</td>
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<td>+</td>
</tr>
</tbody>
</table>

**Legend:**

(+) $\Rightarrow \beta_i^j > 0$

(−) $\Rightarrow \beta_i^j < 0$

✓ $\Rightarrow \beta_i^j$’s $p$-val < .1

Higher MCI level $\Rightarrow$ worse economic conditions
1) \[ CDS_i = b_{0,i} + b_{1,i} MCI_i + b_{2,i} MCI_{EU_{i}} + b_{3,i} \log VIX + b_{4,i} \text{BB-BBB spread} + b_{5,i} PC_{CDS_{-i}}^{1} + \varepsilon_i \]

2) \[ \Delta CDS_i = b_{1,i} \Delta MCI_i + b_{2,i} \Delta MCI_{EU_{i}} + b_{3,i} \Delta \log VIX + b_{4,i} \Delta \text{BB-BBB spread} + b_{5,i} \Delta PC_{CDS_{-i}}^{1} + \varepsilon_i \]

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Panel C: Germany</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MCI_i )</td>
<td>0.81</td>
<td>0.15</td>
<td>-0.27</td>
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<td></td>
<td>(3.72)</td>
<td>(0.94)</td>
<td>(-1.07)</td>
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<tr>
<td>( MCI_{EU_{i}} )</td>
<td>1.81</td>
<td>0.02</td>
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<td></td>
<td>(7.85)</td>
<td>(0.14)</td>
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<td>( \log VIX )</td>
<td>29.40</td>
<td>7.75</td>
<td>10.43</td>
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<td></td>
<td>(9.21)</td>
<td>(1.66)</td>
<td>(4.15)</td>
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<td>( \text{BB-BBB spread} )</td>
<td>9.99</td>
<td>(15.02)</td>
<td>5.87</td>
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<td>(4.10)</td>
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<td>(2.77)</td>
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<td>(11.95)</td>
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<td>( \text{Adj. } R^2 )</td>
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<td>( \text{mean}(\varepsilon) )</td>
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<tr>
<td>( \text{max}(\varepsilon) )</td>
<td>64.27</td>
<td>51.53</td>
<td>41.38</td>
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</table>

- Linear model with MCIs, \( \log VIX \), and one latent factor captures fluctuations in CDS spreads well
- It provides benchmark for the fit of our contagion model
Linear Regression Benchmark: Greece

1) \( \text{CDS}_i = b_{0,i} + b_{1,i} \text{MCI}_i + b_{2,i} \text{MCI}_{EU_i} + b_{3,i} \log \text{VIX} + b_{4,i} \text{BB-BBB spread} + b_{5,i} \text{PC}_{CDS-i}^1 + \varepsilon_i \)

2) \( \Delta \text{CDS}_i = b_{1,i} \Delta \text{MCI}_i + b_{2,i} \Delta \text{MCI}_{EU_i} + b_{3,i} \Delta \log \text{VIX} + b_{4,i} \Delta \text{BB-BBB spread} + b_{5,i} \Delta \text{PC}_{CDS-i}^1 + \varepsilon_i \)

<table>
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<tr>
<th>Panel D: Greece</th>
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<tbody>
<tr>
<td>( \text{MCI}_i )</td>
<td>10.62</td>
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<td>( \Delta \text{MCI}_{EU_i} )</td>
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<tr>
<td>( \Delta \text{log VIX} )</td>
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<tr>
<td>( \Delta \text{log VIX} )</td>
<td>(10.94)</td>
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<td>( \Delta \text{BB-BBB spread} )</td>
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<td>( \Delta \text{PC}_{CDS-i}^1 )</td>
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<td>( \Delta \text{PC}_{CDS-i}^1 )</td>
<td>199.49</td>
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</tbody>
</table>

- Linear model with MCIs, log VIX, and one latent factor captures fluctuations in CDS spreads well
- It provides benchmark for the fit of our contagion model
Model Estimation – Second Step

• CDS data on 11 Euro zone countries

• Assume $X$ contains:
  
  – Country-specific macroeconomic information
  
  – A measure of global fin. uncertainty

\[
X = [\ MCI_i, \ i = 1, \ldots, 11, \ \log-VIX \ ]'
\]

• Quasi maximum likelihood estimation via the square-root unscented Kalman filter
  
  – Use first-step estimates for $X$ as inputs
  
  – Estimate hidden state probability $\pi$ and other model coefficients
Model Estimation – State Space Representation

• State vector $\overline{X}_t \equiv [X_t, \pi_t]'$

\[
\overline{X}_{t+1} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi & 0 \\ 0 & 1 \end{bmatrix} \overline{X}_t + \begin{bmatrix} \sum \pi_t \mu_{0,G} - \bar{\mu}_{0,t} \\ \pi_t \mu_{1,G} - \bar{\mu}_{1,t} \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ \eta_{0,t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_{1,t+1} \end{bmatrix} + \pi_t \sum_{i=1}^{N} \left( \frac{\lambda_{G,t}}{\lambda_{i,t}'} - 1 \right) \Delta M_{i,t+1}
\]

- $\bar{\mu}_{1,t} = \pi_t \mu_{1,G} + (1 - \pi_t) \mu_{1,B}$
- $\lambda_{is,t} = \alpha_{is} + \beta_{is}' X_t$, $s =$ Good, Bad states
- $\overline{\lambda}_{i,t}^P = \pi_t \lambda_{iG,t} + (1 - \pi_t) \lambda_{iB,t}$
- $\Delta M_{i,t} = \left( \Delta 1_{\{\tau_i \leq t\}} - \overline{\lambda}_{i}^P(t) 1_{\{\tau_i > t\}} \Delta t \right)$
- $\varepsilon_t, \eta_t \sim i.i.d. N(0, \Delta t)$

• No sovereign defaults in the sample $\Rightarrow \Delta 1_{\{\tau_i \leq t\}} = 0, \forall t$ and $i$

• Measurement equations
  - Non-linear pricing equations with Gaussian $i.i.d.$ measurement errors
    
    \[
    \overline{CDS}_{i,t} = CDS_i(\overline{X}_t) + v_{i,t}, \ v_{i,t} \sim N(0, R_i), \ i = 1, \ldots, N
    \]
    
    - $\overline{MCI}_{i,t}, \ i = 1, \ldots, N$, measured without error
Model Estimation – Dealing with Nonlinearities

- Two sources of nonlinearity: measurement equation and state dynamics

- Unscented Kalman filter estimation, Julier-Uhlmann (1997)
  - Use weighted points to parameterize the mean $\overline{X}_{t|t}$ and covariances $P_{t|t}$ of state probability distribution
  - Propagate points through the non-linear system
  - Approximate means and covariances accurately to the $2^{nd}$ or higher order

- Square-root UKF version: Wan-van der Merwe (2001)
  - Propagate $S_{t|t}$, where $S_{t|t}' S_{t|t}' = P_{t|t}$, rather than $P_{t|t}$
  - Better numerical properties

- Useful tool in financial applications
  Bakshi-Carr-Wu (JFE 2008); Binsbergen-Koijen (JF 2010)
  Campbell-Sunderam-Viceira (WP 2011); Christoffersen et al. (WP 2009)
Hidden State Probability

\[
d\frac{\pi_s(t)}{\pi_s(t^-)} = \sum_{i=1}^{N} \left( \frac{\lambda_{is}(t^-)}{\lambda_i(t^-)} - 1 \right) dM_i(t) + \sum_{k=1}^{K} \left( \frac{\mu_{k,s} - \bar{\mu}_k(t)}{\sigma_k} \right) dZ_k(t)
\]
Hidden State Probability: Linkage with News Reports

\[
d\frac{\pi_s(t)}{\pi_s(t^-)} = \sum_{i=1}^{N} \left( \frac{\lambda_{is}(t^-)}{\lambda_i(t^-)} - 1 \right) dM_i(t) + \sum_{k=1}^{K} \left( \frac{\mu_{k,s} - \bar{\mu}_k(t)}{\sigma_k} \right) dZ_k(t)
\]

Sample period: 2004/02/12-2010/09/30
Default Intensities

Sample period: 2004/02/12-2010/09/30
Sovereign CDS spreads: Model vs. Data

Sample period: 2004/02/12-2010/09/30
### CDS Pricing Errors: ‘Contagion’ Model vs. Affine Benchmark

<table>
<thead>
<tr>
<th></th>
<th>Contagion</th>
<th>Affine</th>
<th>Contagion</th>
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<td>Austria</td>
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<td>39.83</td>
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<td>1.27</td>
<td>6.40</td>
<td>9.36</td>
<td>33.17</td>
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<tr>
<td>Germany</td>
<td>1.70</td>
<td>5.76</td>
<td>12.55</td>
<td>30.99</td>
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<tr>
<td>Greece</td>
<td>16.04</td>
<td>44.12</td>
<td>201.23</td>
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<tr>
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<td>2.54</td>
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<td>Portugal</td>
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<td>38.23</td>
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<tr>
<td>Spain</td>
<td>4.66</td>
<td>24.25</td>
<td>38.17</td>
<td>102.51</td>
</tr>
</tbody>
</table>

- States in the **(nonlinear)** contagion model: MCIs and VIX variables, and posterior prob. $\pi_{Good}$
- OLS fit of **(linear)** affine model with same MCIs and VIX variables, and a single latent factor
- Sample period: 2004/02/12-2010/09/30
• Preference for robustness parameter: $\zeta$

• $\text{CDS}(\text{Contagion Model, } \zeta = \hat{\zeta}) - \text{CDS}(\text{Contagion Model, } \zeta \Rightarrow \infty)$

• Pre-crisis, 19% of CDS spreads attributed to uncertainty aversion

• Since January 2008, 36% of CDS spreads attributed to uncertainty aversion
Conclusion

- Provide a tractable model of sovereign bond contagion risk

- Two main ingredients:
  - Hidden state that is learned via defaults, other signals
  - Representative agent with fragile beliefs

- Two technical contributions:
  - Can capture multi-directional contagion tractably
  - Identify conditions for when fragile beliefs are time-consistent

- Empirical Results:
  - Implied non-linearities seem to capture something important
  - One “latent” and several “macroeconomic” variables combined explain sovereign spreads well
On-Going Work

- Extend estimation to include current data

- Euro zone crisis keeps evolving
  - Since summer 2011, correlations of Greek CDS with CDSs on other Euro zone sovereigns ↓
    * Some decoupling of Greece’s faith from the rest of the Euro zone
  - Fall 2011: ‘Voluntary’ restructuring of Greek bonds
    * Not a credit event
  - March 2012: Nonvoluntary restructuring of Greek bonds
    * Value of holdings more than halved
      * International Swaps and Derivatives Association (ISDA) rules a credit event

- Greek default: Idiosyncratic vs. Systemic

- Additional states of nature become identified as news develop