Endogenous Derivatives Networks.

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Question: How is the network for OTC derivatives likely to change as a consequence of the OTC derivatives market reform?

- Central clearing set to become mandatory (Dodd-Frank, EMIR).
- **First channel:** Increased demand for safe collateral assets to meet increased margin requirements.
- **Neglected channel:** Changes in network size and structure.

The model

- OTC derivatives network formation model.
- Both prices and quantities are negotiated.
- Both **idiosyncratic counterparty risk** and (exogenous) collateralization and clearing requirements are accounted for in banks' decision to trade.
Motivation

- Same network of net CDS exposures.
  - No central clearing (left chart), full central clearing (right chart).
CCP clearing and collateral requirements

- **Wide range of estimates** for required additional margin requirements.
  - Heller and Vause (2012): 720 bn USD for IR swaps and CDS.

- **Main limitation of previous studies**: Neglect of the fact that the network of exposure is endogenous to the collateralization and clearing requirements.
Focus instead on changes in the market structure.

The net benefit of opening a derivative exposure depends on the relative weighting of costs and benefits of collateral and clearing.

*Costs:* Exposures that have the lower net expected marginal benefit may disappear if increased collateral requirements are imposed *ceteris paribus.*

*Benefits:* Or, alternatively, increased collateral requirements may induce some counterparties to trade more, as counterparty risk is then mitigated.

Results underpinned by bank trading behaviour

What if counterparty risk increases?

What if the safe asset becomes scarcer?

Rough assessment of collateral demand mis-estimation when network exogeneity is assumed.
New model of OTC derivatives network formation.
- Accounts for features of derivative networks (upfront payment, off-balance sheet exposure).
- Collateralization and clearing requirements enter banks’ objective functions.
- Counterparty credit risk as a key ingredient.
- Both prices and quantities negotiated.

Investigate the effect of three regulatory schemes.
- Increased collateralization of bilateral exposures.
- Rehypothecation or not of collateral received.
- Increased scope for central clearing.
Main results.

- Market liquidity / intermediary activity decreases with higher collateral requirements.
- Some end-users are no longer able to obtain credit risk protection.
- Rehypothecation does not change the ability of end-users to obtain credit risk protection, but increases market liquidity.
- Overall: Redistributive effects.

Additional results.

- Increased counterparty risk reduces market size to a large extent. Collateralization can mitigate the market shrinkage.
- A drop in the safe asset supply has a first-order effect on market outcomes. Market changes due to collateralization requirements are second-order in this context.
Endogenous network formation

- **Network models:** Acemoglu et al. (2013), Battiston et al. (2012), Bluhm et al. (2013), Babus (2013).
  - Reviews by Dutta and Jackson (2003), Jackson and Zenou (2013).
- **OTC markets:** Duffie et al. (2005, 2007).

Central clearing

- **Empirical:** Heller and Vause (2012), Sidanius and Zikes (2012), Singh (2013), Duffie et al. (2014).
• Set $\Omega = \{1, ..., n\}$ of financial institutions indexed by $i$.
• Three dates $t = \{0, 1, 2\}$.
• Stylized balance sheet of $i \in \Omega$.
  • **Riskless securities** $c_i$.
    • Gross yield $r_0 > 1$ at $t = 2$ with probability one.
  • **Risky credit exposures** $a_i$.
    • Gross yield $R > r_0 > 1$ at $t = 2$ with probability $(1 - \delta_1)$.
    • Gross yield 0 with probability $\delta_1$.
    • $\delta_1$ observed only at $t = 1$.
    • Imperfect signal $\delta_0 = \mathbb{E} [\delta_1]$ at $t = 0$.
• The portfolio $\{a_i, c_i\}$ is exogenously given at $t = 0$.
• Vector of idiosyncratic bank probabilities of default $\Lambda = \{\lambda_1, ..., \lambda_n\}$.
  • Provides a rationale for collateralization
  • **Assumption**: Independence of any $\lambda_i$ and $\lambda_j$.
  • The joint probability of $i$ and $j$ failing at $t = 2$ is $\lambda_i \lambda_j$. 

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Assumption: The risky asset \(a\) is illiquid and in fixed supply. Thus, instead of trading \(a\) for \(c\), \(c\) is traded against credit derivatives written on \(a\).

Consider CDS contracts on reference entity \(a\).

OTC contract between \(i\) and \(j\) defined by the tuple \(\{\omega_{ij}, p_{ij}\}\).

\(\omega_{ij} \in \mathbb{R}\): Gross notional amount.

- \(\omega_{ij} > 0\) in case \(i\) sells the contract.
- \(\omega_{ij} = -\omega_{ji}\)

\(p_{ij} > 0\): Premium per unit of notional

\(p_{ij}\) (fixed leg) paid at \(t = 0\) by the buyer.

Proceeds invested in the safe asset, yielding \(\omega_{ji}p_{ij}r_0\) at \(t = 2\).

In case of default at \(t = 2\) (w. p. \(\delta_1\)), the seller \(j\) pays \(-\omega_{ij}R\).
**Model - Timing**

- **Initial period: \( t = 0 \)**
  - Each bank \( i \) observes its portfolio \( \{a_i, c_i\} \).
  - \( \delta_0 \in [0; 1] \) is observed and the p.d.f. of \( \delta_1 \), denoted \( \pi (\delta_1) \) (with c.d.f. \( \Pi \)), is common knowledge.
  - Banks are matched and credit derivatives are traded over a large number of subperiods \( s = \{1, ..., S\} \).
  - Initial margins are posted (with the riskless asset \( c \)).

- **Interim period: \( t = 1 \)**
  - True asset probability of default \( \delta_1 \) is observed.
  - Change from \( \delta_0 \) to \( \delta_1 \) gives ground for variation margins posting.

- **Final period: \( t = 2 \)**
  - All assets mature.
  - In case a defaults, CDS payments take place.
Banks maximize VNM expected utility $V = \mathbb{E} [U]$. Utility of a bank when it fails normalized to $U(0)$. Expected utility for $i$ when being offered a contract $\{\omega_{ij}, p_{ij}\}$ with $j$:

$$V_i(\omega_{ij}, p_{ij}) = \int_0^1 \pi(\delta_1) \left[ \lambda_i U(0) + (1 - \lambda_i) \left[ (1 - \delta_1) U(C_i + a_i R) ight. ight.$$

$$+ \left. \delta_1 \left[ (1 - \lambda_j) U(C_i - \omega_{ij} R) ight. ight.$$

$$+ \left. \lambda_j U(C_i - \max\{0; \omega_{ij} R\}) \right] d\delta_1$$

With $C_i \equiv (c_i + \omega_{ij} p_{ij}) r_0$. Both asset default probability and counterparty default probability are accounted for.
Both $\omega_{ij}$ and $p_{ij}$ are negotiated.

Any bank $i$ can thus be both buyer and seller.

Objective: **Nash product with equal bargaining power:**

$$\{\omega_{ij}, p_{ij}\} \in \arg\max \quad (V_i(\omega_{ij}, p) - V_i(0, .))(V_j(\omega_{ji}, p) - V_j(0, .))$$

Subject to budget and incentive-compatibility constraints:

$$\begin{cases} 
-p_{ij}\omega_{ij} \leq c_i \\
(c_j + p_{ij}\omega_{ji})r_0 \geq \omega_{ji}R \\
V_i(\omega_{ij}, p_{ij}) - V_i(0, .) \geq 0 \\
V_j(\omega_{ji}, p_{ij}) - V_j(0, .) \geq 0
\end{cases}$$

**Note:** Naked CDS buying is allowed.
Period $t = 0$ is divided into a large number $S$ of sub-periods indexed by $s$.

At each $s$, a randomly chosen pair of banks is matched.

Preferential attachment is modeled and parameterized.

Fixed cost $\kappa$ of creating a first link with a counterparty.

**Limited rationality:** A bank considers each match as the last trading opportunity before $S$. 

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Rationale for collateralization: idiosyncratic probabilities of default $\Lambda$.

Collateralization requirements are exogenously given.

**Initial margins**
- Meant to cover *potential future exposure*.

$$\chi_{ji}^{IM} = \tau^{IM} f^{IM} \cdot \sigma(\delta_1) \max \{0; \omega_{ji}\}$$

- $\tau^{IM} \in [0; 1]$: Collateralization level.
- $\sigma(\delta_1)$: Standard deviation of $\delta_1$.
- $f^{IM}$: Constant.

**Variation margins**
- Meant to cover *current exposure*.

$$\chi_{ji}^{VM} = \tau^{VM} f^{VM} \cdot (\delta_1 - \delta_0) \max \{0; \omega_{ji}\}$$

- $f^{VM}$: Constant.

Failures from collateral shortage (inability to post variation margins) are ruled out through an additional constraint.
Model - Sequential trading problem

- Refine notations:
  - $\omega_{ij}$: CDS contract currently being negotiated between $i$ and $j$.
  - $\tilde{\omega}_{ij}^s$: sum of all (adding up or offsetting) agreed-upon trades between $i$ and $j$ before instant $s$.

- **Expected utility** for $i$ at date $s$ when matched with $j$:

$$V_i(\omega_{ij}, p_{ij}) = \int_0^1 \pi(\delta_1) \left[ \lambda_i U(0) + (1 - \lambda_i) \left[ (1 - \delta_1) U \left( C_i + a_i R + \chi_i \right) \right. \right. $$

$$\left. \left. + \delta_1 \left( \sum_{\Phi \subseteq \Omega \setminus \{i\}} \prod_{k \in \Phi} \lambda_k \prod_{k \notin \Phi} (1 - \lambda_k) \cdot U \left( C_i \right. \right. \right. $$

$$\left. \left. \left. + \sum_{k \in \Phi} \left[ -\max \{0; \tilde{\omega}_{ik}^s\} + \chi_{ki} \right] - \sum_{k \notin \Phi} \left[ \tilde{\omega}_{ik}^s R + \chi_{ik} \right] \right) \right] \right] d\delta_1$$

- Some simplifying notations have been introduced.

- More on notations.

- Budget constraints are also updated with the trading process $s$.

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Endogenous Derivatives Networks
Calibration

- **Banks**
  - $n = 5$ banks (main dealers in one area).
  - $U$: CRRA utility with elasticity of substitution $\gamma = 0.3$.
  - Total bank assets drawn from a Power law distribution.
  - Random assignment of shares $\beta$ and $(1 - \beta)$ for $a_i$ and $c_i$.
  - Bank probabilities of default drawn from a beta distribution (support $[0;1]$) calibrated from credit ratings.

- **Assets**
  - $r_0 = 1.029$ (10-Year U.S. Treasury Constant Maturity Rate)
  - $R = 1.158$ (Long-term arithmetic average of S&P 500 return).

- **Collateral**
  - $f^{IM} = 10$ (Potential future exposure)
  - $f^{VM} = 2$ (Current exposure)
Restrict attention to key metrics.

**Gross notional amount:** Size of the market.

**Multilateral net notional:** Exposure at default.

**Net over gross ratio:** Extent to which trading activity generates exposure at default. Proxies for “market liquidity”.

**Total collateral and collateral over net notional ratio:** Counterparty risk mitigation.

**Average volume-weighted price:** Pricing of the contracts.
Results - Bilateral clearing, varying $\tau$

- Outcome metrics for changing collateralization levels on bilateral trades.
Results - Bilateral clearing, varying $\tau$

- Gross market size shrinks by 21%, but net exposure decreases to a lower extent (13%).
  - The net-over-gross ratio increases: Market liquidity / intermediary activity are lower.
  - Some end-users no longer get credit risk protection.
  - Thus, sizeable effects for both “intermediaries” and “end-users”.

- Increased collateral requirements are priced: higher spreads paid by CDS buyers.

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Rehypothecation: re-use of collateral.

Common practice on OTC derivative markets (Singh 2010)

Assume a fraction $\rho \in [0; 1]$ of collateral received can be rehypothecated.

At $t = 0$, rehypothecation of initial margins.
Results - Rehypothecation

- Outcome metrics with bilateral trades and changing rehypothecation, $\tau^{IM} = \tau^{VM} = 1$.

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Results - Rehypothecation

- Rehypothecation increases gross market size by about 13.7%, but net notional remains unchanged.
  - The ability to rehypothecate does not increase the ability of end-users to get credit risk protection.
  - But intermediary activities increase to a large extent.
- Collateral over net ratio drops (from 0.21 to 0.12).
  - Policy debates on rehypothecation should weigh the costs and benefits of extensive collateralization and of well-developed intermediary activities.
  - Trading volumes by end-users are not a first-order concern.
- Prices decrease slightly as rehypothecation increases.

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Bilateral exposures satisfying a size criterion are novated to a CCP.

- All exposures above $\bar{T}$ have to be centrally cleared.
- Size criterion $\bar{T}$ can be interpreted as a proxy for standardization.

Example: Novation of $\omega_{ij} < 0$ to a CCP.

- The CCP buys $\omega_{CCP,j} = \omega_{ij}$ from $j$.
- The CCP sells $-\omega_{CCP,j}$ to $i$.

Multilateral netting of exposures novated to the CCP.

Collateral required by the CCP:

- Full collateralization: $\tau^{IM} = \tau^{VM} = 1$
- The CCP does not post initial margins.
- Default fund contribution: fraction $\psi$ of initial margins.
Results - CCP clearing

- Outcome metrics when increasing CCP clearing (reducing $\bar{T}$)
- Uncleared bilateral trades uncollateralized, $\tau^{IM} = \tau^{VM} = 0.$
Results - CCP clearing

- Effects very similar to those of increased bilateral collateralization.
  - Gross market size decreases by 34.1%.
  - Some end-users trade less in net terms.
  - Net over gross ratio increases from 0.52 to 0.66 (lower market liquidity).
  - Collateral demand increases to represent 18.2% of net exposures.
  - Increased collateral requirements are priced.
Run an experiment with a calibration close to pre-reform parameter values.

**Pre-reform case:**
- No central clearing.
- No bilateral initial margins.
- Variation margins $\tau^V M = 0.88$ (ISDA 2013).
- Rehypothecation ratio $\rho = 0.75$ (ISDA 2013).

**Post-reform case:**
- Full CCP clearing: $\bar{T} = 0$.
- Default fund contribution $\psi = 0.15$ (Rafi 2012).
Not accounting for dynamic effects may lead to an over-estimation of system-wide collateral demand between 22% and 47%.
Role of bank probabilities of default: What if all bank probabilities of default are driven up by an idiosyncratic factor?

- Benefits of collateral posting are larger *ceteris paribus*.
- Low impact of bank PD if full collateralization.
Benefits of central clearing are also larger *ceteris paribus*. Differences due to counterparty risk almost vanish when central clearing is widespread. Lower incentives to screen counterparties.

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Extensions - Safe asset supply

- Keep each bank size constant, reduce the proportion of safe assets.
  - Akin to the **downgrade of safe assets** (≈ safe asset shortage).
  - Blue: base case, red: ÷2, green: ÷4.

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Large consequences on market size, in particular net exposures.

- Portfolio rebalancing: Traders less willing to take on more credit risk when risky assets are in larger supply.

- Much higher net over gross notional ratio when the collateralization level is high enough.

- However, the collateral over net ratio decreases.

- First-order effect on CDS prices, which increase significantly.

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Conclusion

- **Dynamic model of OTC derivatives network formation.**
  - Network is endogenous to collateralization and clearing requirements.
  - Idiosyncratic counterparty risk accounted for.
  - Both prices and quantities are negotiated.

- **Dynamic effects of collateral and clearing on OTC derivatives market size and structure are large.**
  - Reduction in both market liquidity and access to the market for end-users.
  - Between 22% and 47% over-estimation of collateral demand in case dynamic effects are not accounted for.
  - Part of the costs of collateral and clearing are passed onto buyers through higher prices.
  - Sizeable consequences of a change in safe asset supply.
Further research

- **Model with multiple assets**
  - Asset default correlation likely to play a role if initial and variation margins are properly modeled.
  - Benefits from diversification

- **Robustness of simulated networks to contagion**
  - Contagion when accounting for collateral and clearing.
  - Quantify the risk-mitigating role of collateral.

- **Normative (welfare) analysis**
  - Increased collateralization needs not be welfare-improving.
  - Nature of CDS contracts that are no longer traded: true hedging needs or other trades?
Appendix - Notations in the sequential problem

- Notations introduced in setting up the sequential problem.

\[ C_i \equiv \left( c_i - K_i - \chi_i + \omega_{ij} p_{ij} + \sum_k \tilde{\omega}_{ik}^s p_{ik} \right) r_0 \]

\[ K_i \equiv \kappa \sum_k 1 \{ \Xi^s(i,k)=1 \} \]

\[ \chi_i \equiv \chi_i^{IM} \left( \omega_{ij} + \sum_k \tilde{\omega}_{ik}^s \right) + \chi_i^{VM} \left( \omega_{ij} + \sum_k \tilde{\omega}_{ik}^s, \delta_1 \right) \]

\[ \chi_{ij} \equiv \chi_{ij}^{IM} + \chi_{ij}^{VM} \]

- Return.