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Résumé

Cet article développe un modèle de formation de réseau d'expositions dérivées, où prix et quantités sont négociés de gré-à-gré (OTC). La modélisation génère comme variables endogènes le réseau d'expositions, les montants bruts et nets échangés, ainsi que le collatéral délivré via les marges initiales et les marges de variation, en fonction du niveau de risque de contrepartie idiosyncratique et du cadre réglementaire en matière de collatéral et de compensation. Le modèle est utilisé pour analyser numériquement la taille du marché des dérivés, la demande totale de collatéral et les prix. Trois types de configuration sont analysés : *(i)* différents niveaux de collatéralisation des transactions non compensées, *(ii)* la réhypothécation du collatéral reçu et *(iii)* la compensation via une contrepartie centrale. Les effets dynamiques dus à l'endogénéité du réseau d'expositions dérivées ont des conséquences significatives tant sur les volumes que sur les prix des contrats échangés. Les activités d'intermédiation et la liquidité du marché sont réduites par l'application de contraintes plus sévères en matière de collatéral, et améliorées par la réhypothécation. Parallèlement, le potentiel de contagion est réduit. Ne pas prendre en compte ces effets dynamiques peut conduire (dans les conditions de marché actuelles) à surestimer la demande de collatéral induite par la compensation centrale jusqu'à 22%.

Mots-clés: collatéral, dérivés de crédit, contrepartie centrale (CCP), réseau endogène
Code JEL: G11, G17, G28

Abstract

This paper proposes a network formation model of an OTC derivatives market where both prices and quantities are bilaterally negociated. The key feature of the framework is to endogenize the network of exposures, the gross and net notional amounts traded and the collateral delivered through initial and variation margins, as a function of idiosyncratic counterparty risk and regulatory collateral and clearing requirements. Using the framework, we investigate numerically the size of the derivatives network, the aggregate collateral demand and the pricing of the contracts under the following schemes: *(i)* various levels of collateralization for uncleared transactions, *(ii)* rehypothecation of received collateral and *(iii)* clearing through a central clearing party (CCP). Overall results suggest that dynamic effects due to the endogeneity of the derivative network to the collateralization and clearing requirements have sizeable consequences on both contract volumes and prices. Intermediary trading and market liquidity are reduced by higher collateralization requirements and enhanced by rehypothecation, while the potential for contagion is reduced. Not accounting for dynamic effects in current market conditions may lead to over-estimate collateral demand induced by mandatory central clearing by up to 22%.

Keywords: Collateral, Credit derivatives, Central Clearing Party (CCP), Network formation
JEL: G11, G17, G28

Non technical summary

Over-the-counter (OTC) derivatives markets will undergo significant changes as major jurisdictions implement the regulatory reforms initiated by the G20 in its 2009 Pittsburgh meeting. Relative to the pre-crisis regulatory environment, new regulations mandate clearing by a central counterparty (CCP) for standardized derivatives contracts, and enhanced collateral requirements.

This paper presents a network formation model of an OTC derivatives market to analyze the impact of this type of reforms on market activity. The main feature of the analysis in comparison to the extant literature is to endogenize the network of derivative exposures, rather than work with a given matrix of exposures. For concreteness, the framework is developed in the context of credit default swap (CDS) contracts on a single risky entity, and incorporates realistic features including counterparty risk concerns, the opportunity cost of collateral posted to comply with margin requirements, and the possibility of rehypothecation. The network of exposures emerges as the aggregate outcome of OTC bilateral contracts where dealers set trade-specific prices and quantities, taking as given regulatory requirements and the level of counterparty risk.

We use the framework to simulate networks of credit derivative exposures and investigate numerically the size of the derivatives network, the aggregate collateral demand and the pricing of the contracts under the following regulatory schemes: *(i)* various levels of collateralization for uncleared transactions, *(ii)* rehypothecation of received collateral and *(iii)* clearing through a CCP. Among our main results, we document a sizeable reduction in the gross CDS market size when tougher collateral requirements are imposed, either through higher collateralization levels on uncleared exposures or through novation to a CCP. The reduction in total *net* CDS exposures is, however, much smaller, so that the net-over-gross ratio increases when tougher collateral requirements or increased central clearing are imposed. An increase in the net-over-gross notional ratio signals a shrinkage of intermediary trading, thus of "market liquidity" in a broad sense. We thus provide a benchmark to analyze market liquidity under various market structures. Rehypothecation, in contrast, increases intermediary trading while leaving net exposures almost unchanged (and reducing collateral demand to a large extent).

Even though the CDS market size shrinks as a consequence of increased CCP clearing, the total collateral demand increases as more trades get centrally cleared. When shifting from full bilateral clearing to full central clearing, the gross market size shrinks by 13.7% whereas total initial margins demand increases so as to represent 21.3% of net exposures. The potential for bank-to-bank contagion is thus likely reduced to a sizeable extent. A calibrated version designed to match end-2012 market conditions suggests that ignoring adjustments in the network of exposures could lead to over-estimate the rise in collateral demand induced by mandatory central clearing by up to 22%.

Furthermore, the benefits of collateralization are found to be significantly higher at times of high counterparty risk. When dealer banks' probabilities of failure increase, a shrinkage of market gross and net notional amounts is observed, which is

mitigated for high collateralization levels or when full central clearing is in place. In high-counterparty risk environments, collateral—whether collected bilaterally or centrally—enables preserving a well-functioning derivatives market.

While the current work focuses on the impact of regulatory measures on trading in the derivatives market, the setup provides a first step towards a cost-benefit analysis of regulatory measures in OTC markets. In particular, future work could extend the model to explore the impact on the stability or contagion properties of the network of derivatives exposures.

1 Introduction

The regulatory reforms of OTC derivative markets initiated in 2009 by the G20 and coordinated by the Financial Stability Board are about to be implemented in the United States (as part of the Dodd-Frank act) and in Europe (through the EMIR directive). These reforms raised an intense debate about the costs and benefits of increased collateral requirements due to mandatory central clearing of standardized contracts. Among policymakers, an increased role for the clearing of derivatives through central clearing parties (CCPs) is widely expected to enhance the resilience of the financial system at times counterparty risk increases (IMF 2010) and to alleviate the adverse consequences of the considerable growth of the notional amounts traded on derivative markets (from 6,395 to 25,068 bn. USD between end-2004 to end-2012 for the CDS market) during the past decade (BIS 2013).[1]

So far, however, there does not exist any full-fledged model to assess the consequences that a change in the regulatory collateral and clearing requirements may have on the derivatives market size and structure, on collateral demand and on the pricing of the derivative contracts. It is not clear either how intermediary activities and counterparty screening will likely be affected. Most of the existing theoretical and empirical contributions, as we shall emphasize below, study collateral demand or clearing schemes (bilateral vs. central) while considering the network of derivative exposures as exogenously given and unaffected by such regulatory requirements. *De facto*, the fact that the network of exposures itself is endogenous to—and dynamically dependent from—the regulatory environment has been neglected to a large extent.

The contribution of the paper in this respect is twofold. First, from a theoretical perspective, we propose an OTC network formation model where both idiosyncratic counterparty risk and the regulatory framework regarding trade collateralization and clearing are accounted for in banks' decision to open and adjust derivative exposures. Second, we use the model to simulate networks of credit derivative exposures under three regulatory collateral frameworks and focus on their gross and net notional size, on collateral demand and on the pricing of the contracts. We investigate *(i)* various levels of trade collateralization for uncleared exposures, *(ii)* rehypothecation of received collateral and *(iii)* clearing through one central clearing party. To specify the contracts' payoffs, we consider credit default swaps (CDS), but our framework could incorporate *mutatis mutandis* other contract specifications.

Theoretically, whereas the extant research (Duffie and Zhu 2011; Heller and Vause 2012; Sidanius and Zikes 2012; Duffie et al. 2014) considers an exogenously given network of exposures in order to assess various collateralization and clearing schemes—including a CCP—the size and structure of the network of exposures here depend dynamically on the regulatory collateral scheme. This feature is important both theoretically and practically, as the expected net benefit of opening a new derivative exposure crucially depends on the cost of collateral to be posted as initial margins (in the present) and variation margins (in the future) or to be received (mitigation of

[1] A recent overview of the related policy issues can be found in Banque de France (2013).

counterparty risk). In this sense, an exogenous change in the regulatory framework amounts to a new weighting of costs (foregone returns on collateral delivered) and benefits (higher recovery value in case of counterparty default) which needs to be accounted for by trading institutions. Dynamic effects of this nature have been highlighted by Bliss and Kaufman (2006), who use a counterfactual reasoning—absent any model—to assert that the systemic risk-mitigating role of netting, collateral and close-out is not clear once their effects on the size and structure of the derivatives market are accounted for. In the present paper, the expected utility costs and benefits of the regulatory collateral and clearing rules are a key component of a bank's objective function. During the trading process, idiosyncratic probabilities of failure are also accounted for, so that, it is optimal *ceteris paribus* for a bank to reduce its exposure to a direct counterparty whenever its probability of failure rises. This effect, however, can be mitigated by higher collateralization levels, which secure higher recovery payoffs in case of counterparty failure. Finally, our dynamic OTC network formation model accounts for several features of real-world markets, including preferential attachment and bargaining on both notional amounts and prices.

With regards to the literature on endogenous financial networks, this paper is in contrast with models of interbank loans and deposits, as it features a model of *derivative* exposures, where one essential difference lies in the fact that the upfront payment to be made at the time a transaction is agreed upon is low compared to the notional amount that may have to be settled in later periods. One other key difference with models of the interbank market is that variation margins are delivered only at a later date in case the market value of a position is non-zero. In that respect, the framework we propose accounts for the expected future costs and benefits of collateral posting (including potential rehypothecation benefits) in the link formation decisions. To our knowledge, there are no models in the literature comparable to the present OTC derivative network formation model.

Numerical simulations are used to investigate the comparative dynamics of derivative markets under several regulatory schemes. Among our main results, we document a sizeable reduction in the gross CDS market size when tougher collateral requirements are imposed, either through higher collateralization levels on uncleared exposures or through novation to a CCP. The reduction in total *net* CDS exposures is, however, much smaller, so that the net-over-gross ratio increases when tougher collateral requirements or increased central clearing are imposed. An increase in the net-over-gross notional ratio signals a shrinkage of intermediary trading, thus of "market liquidity" in a broad sense. We thus provide a benchmark to analyze market liquidity under various market structures. Rehypothecation, in contrast, increases intermediary trading while leaving net exposures almost unchanged (and reducing collateral demand to a large extent).

Even though the CDS market size shrinks as a consequence of increased CCP clearing, the total collateral demand increases as more trades get centrally cleared. When shifting from full bilateral clearing to full central clearing, the gross market size shrinks by 13.7% whereas total initial margins demand increases so as to represent 21.3% of net exposures. The potential for bank-to-bank contagion is thus likely reduced to a sizeable extent. Absent any account of the dynamic market adjustment in a model calibrated with parameter values prevailing in end-2012 market conditions, the rise in collateral demand induced by mandatory central clearing could be over-estimated by up to 22%. We also document an increase in CDS prices when collateralization requirements are tightened, as part of the collateral costs are passed onto CDS buyers—or, alternatively, as buyers are provided with higher recovery payoffs in the event of a counterparty failure.

Furthermore, the benefits of collateralization are found to be sizeably more important at times of high counterparty risk. When banks' probabilities of failure increase, a shrinkage of market gross and net notional amounts is observed, which is mitigated for high collateralization levels or when full central clearing is in place. In high-counterparty risk environments, collateral—whether collected bilaterally or centrally—enables preserving a well-functioning derivatives market. Our findings are consistent with those of theoretical papers on clearing (Koeppl et al. 2011; Biais et al. 2012), who model lax counterparty screening when counterparty credit risk is mutualized at a CCP.

Finally, we consider the effects of a reduction in the safe asset supply. Its effects on market size are sizeably larger than those of exogenous changes in collateralization requirements. In contrast with the market shrinkage induced by higher collateral requirements, where net exposures decrease less than gross exposures, net exposures are here found to react to a larger extent. The conversion of riskless into risky assets (interpretable as the downgrade of previously creditworthy pledgeable bonds) indeed implies portfolio rebalancing decisions for banks, which are *ceteris paribus* less willing to take on more credit risk when risky assets are in larger supply in the first place, or need to be compensated for it. Not surprisingly, higher CDS prices are observed when the safe asset supply is reduced.

At the outset, we shall clearly state that financial stability and contagion *per se* are ruled out and left for future work. The contagion potential is captured only through the ratio of collateral posted over net notional exposures. A full-fledged model of contagion through derivative exposures would require specifying properties of the resolution mechanism in banks' objective functions. We focus instead on prices and quantities on the derivatives market, on collateral demand and on the extent of intermediary trading.

The remainder of the paper is structured as follows. In section 2, we briefly review the relevant theoretical and empirical literature. In section 3, the model is presented. The calibration and results are described in section 4. Section 5 investigates the robustness of the results once other parameters relevant for the cost-benefit weighting of collateral are varied.

2 Relevant literature

The present paper relates to two strands of the literature. First, there exists a growing literature on endogenous network formation and on OTC market models. Dynamic network formation models have first been developed to study social networks. They have been reviewed by Dutta and Jackson (2003) and, more recently, by Jackson and Zenou (2013). Few contributions attempt at applying these methodologies to financial networks, especially to the interbank network (Acemoglu et al. 2013; Battiston et al. 2012; Bluhm et al. 2013). Babus (2013) studies risk-sharing mechanisms and the potential for contagion in a network formation game, whereas Georg (2013) proposes a dynamic multi-agent model to investigate the effects of particular interbank network structures on contagion. Closely related to the literature on networks is the one on OTC markets, where bilateral relationships are studied, absent any account of network effects. Seminal theoretical contributions include Duffie et al. (2005, 2007).

Second, our paper complements an emerging research on collateral and central counterparties in OTC derivative markets. From a theoretical perspective, CCPs have been studied with regard to their impact on both collateral demand and on the incentives of trading institutions to alter their exposure to aggregate risk or to engage in moral hazard. Papers studying incentives provided by CCPs have recently been surveyed by Biais et al. (2013). Acharya and Bisin (2013), for instance, argue that central counterparty clearing can increase investment efficiency by eliminating a "counterparty risk externality" inherent in OTC markets. Koeppl and Monnet (2010) show that CCP clearing can increase traders' incentives to load on aggregate risk, and explore the implications for efficient clearing. Biais et al. (2012) show that when protection buyers need to search for robust counterparties, the efficient CCP arrangement features *partial* insurance against counterparty risk.

The impact of central clearing on collateral demand has been investigated theoretically and numerically by Duffie and Zhu (2011), who show that imposing a CCP for only one class of derivatives reduces netting efficiency, whereas efficiency may be improved when the number of derivative classes cleared increases. This work has been extended by Cont and Kokholm (2014), who focus on assets that are heterogeneous with respect to their risk characteristics and by Anderson et al. (2013) who study netting efficiency with linked and unlinked CCP configurations. One limitation of these articles, which we address in this paper, is that they consider the matrix of derivative bilateral exposures as exogenously given, i.e. they do not account for the dynamic effects that a particular regulatory framework regarding trade collateralization may have both on the size and on the structure of the derivatives network.

From an empirical perspective, a handful of papers discuss or assess the consequences of mandatory central clearing. Heller and Vause (2012) and Sidanius and Zikes (2012) estimate the amount of collateral that CCPs should demand—leaving the network of exposures unchanged—to clear safely all interest rate swap and CDS positions after the introduction of mandatory central clearing. Duffie et al. (2014) improve on their estimates by using bilateral exposure data and by investigating a variety of clearing schemes. Hull (2010) discusses the possibility for all derivatives to be cleared, Sidanius (2012) the criterions for CCP eligibility, whereas Bliss and Steigerwald (2006) compare CCPs to alternative structures and Singh (2010a, 2013) describes the changing collateral space.

3 The network formation model

This section presents the OTC derivative network formation model. A simpler version of the model, without collateral, is first described (section 3.3) before the full problem is introduced (section 3.5).

We consider a set $\Omega = \{1, ..., n\}$ of n financial institutions[2] indexed by i and three dates denoted $t = \{0, 1, 2\}$. In a nutshell, date 0 is the date at which financial institutions enter into derivative contracts; date 1 can be thought of as an interim date where public information arrives; and date 2 is the date at which contracts and exposures mature. The timing of the model is detailed further below.

3.1 Portfolios

For any bank $i \in \Omega$, its total assets are composed of risky credit exposures a_i and of riskless securities c_i. All credit exposures a_i are homogenous, i.e. there is one unique underlying reference entity $\mathcal{A}$ on which CDS contracts can be written. At $t = 2$, a unit of credit exposure yield a gross return $R > 1$ with probability $(1 - \delta_1)$, where $\delta_1 \in [0, 1]$ is the probability of default of the reference entity, as of date $t = 1$. With probability δ_1, the gross yield on a at $t = 2$ is zero.[3] To capture a key aspect of derivative contracts, the default probability δ_1 is observed at $t = 1$ but uncertain at date 0. Formally, it is common knowledge that δ_1 is drawn according to a probability density $\pi(\delta_1)$ with c.d.f. Π and support $[0, 1]$. As of date 0, the expected probability of default of the risky credit exposure is thus $\delta_0 \equiv \int_0^1 \delta_1 \pi(\delta_1)\, d\delta_1$, while it changes to δ_1 at $t = 1$. Thus δ_0 is an imperfect signal for δ_1, and the observation of the date 2 probability of default δ_1 at date 1 amounts to the arrival of a new piece of information. Given that both assets and banks may default in this model, we thereafter consistently refer, for clarity, to assets' probabilities of *default* whereas we refer to banks' probabilities of *failure*. With probability one, riskless securities c yield a gross return r_0 at $t = 2$, where $1 < r_0 < R$. At $t = 0$, c_i can be used by any bank i to buy credit derivatives contracts and used as collateral to post initial margins, while at $t = 1$ it can be used to deliver variation margins.

The portfolio $\{a_i, c_i\}$ is exogenously given at $t = 0$, while credit derivatives exposures on $\mathcal{A}$ are endogenously chosen by each bank i. In the following, we consider credit derivatives, but other derivative classes could be considered alternatively within the same framework with minor changes. Imposing a particular contract type is essentially a way to specify present and future cash flows, so to specify a functional form for the banks' objective function. Credit derivatives in this paper resemble the contracts modeled by Atkeson et al. (2013).

A credit derivative (CDS contract for short) on the reference entity $\mathcal{A}$ is an OTC contract between two parties i and j defined by the tuple $\{\omega_{ij}, p_{ij}\}$. The first

[2] Even though OTC markets are typically organized according to a dealer/customer dichotomy, it is here neglected as all institutions are part of the same set. The demand for (or supply of) derivative contracts does not arise from exogenous cusomer/dealer relations, but from each institution's own portfolio rebalancing needs.

[3] To simplify, we specify a zero recovery rate in case of a credit event. This is with no loss in generality.

term $\omega_{ij} \in \mathbb{R}$ is the gross notional amount of the contract, whereas $p_{ij} > 0$ is the premium per unit of notional to be paid by the buyer at $t = 0$. By convention, we define $\omega_{ij} > 0$ in case i sells the contract and $\omega_{ij} < 0$ in case it buys it. Therefore $\omega_{ij} = -\omega_{ji}$. Conversely, $p_{ij} = p_{ji}$. The contract $\{\omega_{ij}, p_{ij}\}$ implies two streams of payments. The fixed leg of the contract is paid at $t = 0$ by the buyer of the contract (here assumed to be i), implying a transfer $-\omega_{ij}p_{ij}$ to the seller. The contingent leg of the contract is paid upon default at $t = 2$ and equals $-\omega_{ij}R$. The seller of the contract invests the payment in the safe asset, which yields $\omega_{ji}p_{ij}r_0$ at $t = 2$.

There exists a vector of idiosyncratic bank probabilities of failure $\Lambda = \{\lambda_1, ..., \lambda_n\}$, with $\lambda_i \in [0, 1]$ for all i. Given our primary focus is neither on contagion nor on the loss distribution conditional on a shock, we make a simplifying assumption about the joint distribution of bank failures.

Assumption 1. *Idiosyncratic bank failures are independent. Therefore, the joint probability of i and j failing at $t = 2$ is $\lambda_i\lambda_j$.*

A bank failure implies its inability to honor its promised CDS repayments at date 2, if any. Potential bank failures provide a rationale for collateral posting at dates 0 and 1.

3.2 Timing

At $t = 0$, each institution i observes its own asset portfolio $\{a_i, c_i\}$ and the trading of credit derivatives takes place. The weight of each asset class within a bank's portfolio is exogenously given. Such exogeneity leaves room for a genuine market for credit derivatives, which enable adjusting a portfolio's risk profile when risky assets are illiquid or in limited supply. It can be interpreted as resulting from an unexpected change in an asset's creditworthiness, so that a previously optimized portfolio structure does no longer maximize a bank's expected utility, i.e. it would like to be more or less exposed to the risky or to the riskless asset. We make the following important assumption, which implies that rather than trading directly a for c, banks trade credit derivatives ω (written on $\mathcal{A}$) for c and may therefore adjust their exposure to each of these two asset classes:

Assumption 2. *The risky asset a is illiquid and in fixed supply.*

The trading of credit derivatives takes places in an OTC market where banks meet bilaterally. The date $t = 0$ is divided into a large number of sub-periods during which two randomly chosen banks are offered the opportunity to trade and may negotiate both prices and quantities. The premium for credit derivatives bought by any bank i is paid out of its cash holdings c_i and is invested by its counterparty j in riskless securities yielding r_0. All trade takes place before the arrival of new public information on the likelihood of a credit event, that is before δ_1 is realized. The change in creditworthiness from δ_0 to δ_1 is essential to the model's structure as it gives ground for collateral posting through variation margins on top of initial margins paid at $t = 0$.

Importantly, when choosing to trade credit derivatives, a bank takes a given regulatory collateral framework into account. The rationale for costly collateral posting is that each bank i may fail with a probability $\lambda_i > 0$ from a purely idiosyncratic risk that does not depend on the common risk exposure. Both initial and variation margins are modeled. At $t = 0$, all banks have to post initial margins whenever they are net sellers of credit derivatives. At $t = 1$, after δ_1 is observed, collateral requirements are adjusted through variations margins, which are either called on the net sellers of CDS (if $\delta_1 > \delta_0$) or paid back to them from their initial margin requirement (if $\delta_1 < \delta_0$). Various regulatory collateral frameworks are later studied, including varying levels of trade collateralization, rehypothecation and central counterparties. The collateral posted by any bank i must be paid out of its pool of riskless securities c_i. Finally, all assets mature at $t = 2$.

3.3 Bilateral trade without collateral

Let us first focus on a unique trade between any two institutions i and j in the absence of collateral. We later introduce the sequential trading problem and several regulatory collateral schemes. All institutions in Ω aim at maximizing a Von Neumann-Morgenstern expected utility function $V = \mathbb{E}[U]$ with U continuously differentiable, $U'(.) > 0$ and $U''(.) < 0$. The objective function can be interpreted as capturing the standard problem of (risk-adverse) bank shareholders maximizing their *ex post* net wealth. Given that all payoffs are obtained at $t = 2$, discount factors are neglected without loss of generality.

One assumption when defining the objective function is that, when an institution i fails (which occurs with probability λ_i), it does not honour any of its promised CDS payments and its utility is normalised to $U(0)$. Failures as a choice are ruled out: failures occur only as exogenously triggered events. In any pair of institutions $\{i, j\}$, the expected utility for i when being offered a trade $\{\omega_{ij}, p_{ij}\}$ with j is:

$$\begin{aligned} V_i(\omega_{ij}, p_{ij}) \quad = \quad & \int_0^1 \pi(\delta_1) \Big[\lambda_i U(0) + (1-\lambda_i) \Big[(1-\delta_1) U(C_i + a_i R) \\ & + \delta_1 \Big[(1-\lambda_j) U(C_i - \omega_{ij} R) \\ & + \lambda_j U(C_i - \max\{\omega_{ij} R, 0\}) \Big] \Big] \mathrm{d}\delta_1, \end{aligned} \tag{1}$$

where

$$C_i \equiv (c_i + \omega_{ij} p_{ij}) r_0.$$

With probability λ_i, bank i fails and gets a payoff of zero. Conditional on i not failing (with probability $1-\lambda_i$), the underlying asset does not default with probability $(1 - \delta_1)$ and i gets payoffs on both the riskless and the risky asset, irrespective of j failing or not. On the contrary, if $\mathcal{A}$ defaults (with probability δ_1) the potential failure of j is accounted for in i's objective function 1. In case j does not default, all contractual CDS repayments are settled. In case it does, j cannot honor its repayment (if any) whereas i must do so.

As our focus is on endogenous network formation in the OTC market, ω_{ij} and p_{ij} are negotiated when i is matched with j. This has several implications. First, credit derivatives ω cannot be bought or sold in unlimited quantities, so the portfolio choice by each bank i cannot result from it solving a maximization program with respect to ω_i. Therefore, i can only buy or sell ω_{ij} to the extent j is willing and able to supply or demand it. Second, none of the institutions is buyer or seller of CDS as such; conversely any institution may act as buyer or seller depending on the price that is offered. Third, in the most general case, there is no unique contract $\{\omega_{ij}, p_{ij}\}$ that increases both V_i and V_j compared to an autarkic (i.e. $\omega_{ij} = 0$) benchmark situation. We assume that, in any pair $\{i, j\}$ of traders, the objective function is the Nash product of both utility gains with equal bargaining power. In addition, both traders are subject to budget and participation constraints. The problem solved by both traders is :

$$\{\omega_{ij}, p_{ij}\} \in \arg\max \quad \left(V_i\left(\omega_{ij}, p\right) - V_i\left(0, .\right)\right)\left(V_j\left(\omega_{ji}, p\right) - V_j\left(0, .\right)\right) \tag{2}$$

subject to the constraints:

$$\begin{cases} -p_{ij}\omega_{ij} \leq c_i \\ \left(c_j + p_{ij}\omega_{ji}\right) r_0 \geq \omega_{ji} R \\ V_i\left(\omega_{ij}, p_{ij}\right) - V_i\left(0, .\right) \geq 0 \\ V_j\left(\omega_{ji}, p_{ij}\right) - V_j\left(0, .\right) \geq 0 \end{cases}$$

The first constraint is the budget constraint for i, here assumed to be the buyer of the contract. The second constraint is the budget constraint for the seller j, who cannot sell credit derivatives above some threshold determined by its holdings of riskless assets (i.e. by its ability to honor its payments in case the reference entity defaults). Therefore a CDS seller is always able to honor its CDS payments unless it fails exogenously. The third and fourth constraints are the participation constraints for i and j respectively. Finally, we do not require CDS contracts to insure truly held underlying exposures, i.e. naked credit derivative holdings are allowed to the extent permitted by the budget constraints.

3.4 Matching

The first period $t = 0$ is assumed to be divided into a large number S of sub-periods indexed by s. At each instant s, a randomly chosen pair of institutions is matched, i.e. offered the opportunity to trade credit derivatives. Our model features preferential matching between institutions, which mirrors real-world features such as relationship banking or prime brokerage services offered by dealers. In this section, we propose a general framework for preferential attachment at each instant s, where the probability of any bank i to be matched with any bank $j \neq i$ depends on the number of times i and j have been trading before s, and on a parameter θ measuring the strength of the preferential attachment.

Let us define a $n \times n$ matrix $\mathbf{A}^s$ whose each element $\mathbf{A}^s(i,j)$ denotes the number of times i and j have been trading before date s. From $\mathbf{A}^s$, one can create a probability map $\mathcal{P}^s$ whose each element $\mathcal{P}^s(i,j,\theta)$ is the probability of i to be matched with j at instant s, conditional on i being drawn first (which occurs with probability $1/n$). For any $i \neq j$, we have:

$$\mathcal{P}^s(i,j,\theta) = \frac{\mathbf{A}^s(i,j) + \theta}{\sum_{j \neq i} [\mathbf{A}^s(i,j) + \theta]} \tag{3}$$

This definition of $\mathcal{P}^s$ ensures that $\sum_j \mathcal{P}^s(i,j,\theta) = 1$ and that $\mathcal{P}^s(i,i,\theta) = 0$ for all i. In addition, $\theta > 0$ is a parameter inversely related to the strength of the preferential attachment. For θ low or close to zero, there exists a strong preferential attachment, so that the probability of any bank i to be matched with a bank j with which it has never been trading is low. Conversely, as θ increases, preferential attachment becomes weaker and $\lim_{\theta \to \infty} \mathcal{P}^s(i,j,\theta) = 1/(n-1)$, i.e. the probability that would prevail if any pair was matched with equal probability. When an institution i has never been trading before s, it is matched with any bank j with equal probability whatever the value of θ.

Moreover, we introduce a fixed cost of creating a link. Two institutions that have never been trading in the past have both to incur a fixed cost κ, which can be interpreted as the cost of setting up a master agreement, a credit support annex or as the cost incurred to learn the quality of a given counterparty (i.e. λ_j). Denote Ξ^s a $n \times n$ matrix whose each element (i,j) equals 1 if κ has been paid before instant s (i.e. if there is at least one ongoing contract between i and j at date s). Ξ^s is symmetric with $\Xi^s(i,i) = 0$ for all i, so that it is the adjacency matrix of an undirected network at date s. It is updated with the trading process.

In modelling the sequential matching problem, one must set assumptions about the rationality of each trading institution. Assuming perfect rationality (one example being Afonso and Lagos (2012)), each trading institution perfectly foresees the consequences of its own behaviour and, moreover, may be willing to reject a profitable trading opportunity in the present if the probability of an even more profitable trading opportunity in the future is high enough. We adopt instead a simplifying assumption of imperfect rationality on banks' behalf. We assume that, whenever faced with a trading opportunity yielding an expected net trading surplus, a bank decides to trade. This amounts to assuming that, whenever matched with a potential counterparty, a bank considers the match as the last trading opportunity before S.

3.5 The sequential trading problem with collateral

This section first defines collateral requirements in the absence of central clearing or rehypothecation, then introduces the traders' objective function and constraints once exogenous regulatory collateral rules are accounted for.

3.5.1 Definition of collateral requirements

Let us introduce collateralization and clearing rules in the sequential trading problem. The rationale for collateralization is the existence of a vector of idiosyncratic bank probabilities of failure Λ. In the baseline model and in most of the later specifications, collateralization and clearing rules are considered exogenously given by the regulator. Such exogeneity of the rule reflects tightened regulatory requirements in recent years and is grounded theoretically on the fact that market participants typically do not internalize the system-wide (or systemic) cost that their failure may have on other institutions.

In this setup, collateral posting is a requirement for bilateral net CDS sellers only. There is no collateral posting by bilateral net CDS buyers, as all their payments occur at date 0, so there is no future uncertainty to be mitigated. Both initial margins and variation margins are considered. Initial margins are posted at date 0 (eventually to a CCP, see below), whereas variation margins are posted at date 1 whenever the creditworthiness of the underlying reference entity deteriorates.

Initial margins are meant to cover the losses of a CDS buyer i in case one of its counterparties j fails at date 2, and are computed on the basis of the observed δ_0. On financial markets, initial margins are computed based on an asset's volatility or on the estimation of the tail of a returns distribution (e.g. worst loss over a time period). We assume a particular functional form for the computation of initial margins, which is given by definition 1.

Definition 1. *A bilateral exposure is fully collateralized through initial margins whenever initial margins paid by a bilateral net seller j to its counterparty i at $t = 0$ equal $f^{IM} \cdot \sigma(\delta_1)\,\omega_{ji}$ and assume that banks only collateralize a fraction $\tau^{IM} \in [0; 1]$ of it. Denoting χ_{ji}^{IM} the initial margin posted by a seller j to a buyer i, we have:*

$$\chi_{ji}^{IM} = \tau^{IM} f^{IM} \cdot \sigma(\delta_1) \cdot \max\{\omega_{ji}, 0\}, \tag{4}$$

where $\sigma(\delta_1)$ is the standard deviation of δ_1 in period 1 and $f^{IM} > 0$ a constant.

Defining full collateralization, even though it requires positing a particular function form in this setup, is a way of defining a benchmark model with respect to which partial collateralization can later be studied. The full collateral amount ($\tau^{IM} = 1$) is later assumed to be the collateral requirements imposed by central clearing parties. Definition 1 ensures $\chi_{ij}^{IM} = 0$ whenever $\chi_{ji}^{IM} > 0$, i.e. only one party may be posting a strictly positive amount of collateral in any pair of banks. Margins are posted by each bank (if strictly positive) out of its pool of available riskless asset.

In addition to initial margins, banks post variation margins when the creditworthiness of a changes from δ_0 to δ_1 between dates 0 and 1 and are computed according to 2. Even though they are paid for uncleared trades, variation margins play a non-trivial role once alternative clearing schemes are introduced. The utility cost of maintaining a buffer stock of riskless asset for variation margins payments is indeed not the same when initial margins have been paid or not, as the marginal utility of a unit of riskless asset is then not the same.

Definition 2. *A bilateral exposure between any two parties i and j, where j denotes the net seller, is fully collateralized through variation margins whenever it equals $f^{VM} \cdot (\delta_1 - \delta_0) \cdot \omega_{ji}$. Variation margin calls, assumed to be fully collateralized and denoted χ_{ji}^{VM}, equal:*

$$\chi_{ji}^{VM}(\delta_1) = f^{VM} \cdot (\delta_1 - \delta_0) \cdot \max\{\omega_{ji}, 0\}, \tag{5}$$

where $f^{VM} > 0$ a constant.

Note that variation margins are contingent on the date 1 public information captured by δ_1. Definition 2 ensures that, whenever $\chi_{ji}^{VM} > 0$, the seller j has to post additional collateral to i as the creditworthiness of $\mathcal{A}$ dropped compared to δ_0. Whenever it is negative, i returns part of the collateral to j, which can then obtain a gross return r_0 on it at $t = 2$.

Total initial margins and variation margins to be posted by any bank i to all its counterparties are denoted respectively χ_i^{IM} and χ_i^{VM} and equal:

$$\chi_i^{IM} = \sum_j \chi_{ij}^{IM}, \tag{6}$$

$$\chi_i^{VM}(\delta_1) = \sum_j \chi_{ij}^{VM}(\delta_1). \tag{7}$$

One feature which makes collateral posting costly is that the gross return r_0 is not earned on any amount of riskless asset pledged as collateral. Therefore, one institution i posting one euro of riskless asset as collateral incurs an opportunity cost $r_0 - 1\,(> 0)$.

3.5.2 The sequential trading problem

Before setting up the sequential trading problem, we shall refine our notations so as to distinguish between a single *trade* between any i and j and the *exposure* resulting from all past trades between i and j. Whereas ω_{ij} denotes a CDS contract that is currently being negotiated between i and j, $\tilde{\omega}_{ij}^s$ denotes the sum of all (adding up or offsetting) agreed-upon trades before instant s. Furthermore, p_{ij}^s denotes the volume-weighted average price of these trades.

When solving for the sequential problem, each time a financial institution trades a credit derivative contract, its riskless asset holdings as well as its exposure at default change. In a sequential perspective, where each bank is matched with several counterparties at different dates s, both the expected utility from trade and the budget constraints are functions of s. The expected utility from a new trade $\{\omega_{ij}, p_{ij}\}$ of an institution i once collateral posting is accounted for can be re-written:

$$
\begin{aligned}
V_i(\omega_{ij}, p_{ij}) \quad = \quad & \int_0^1 \pi(\delta_1)\Bigg[\lambda_i U(0) + (1-\lambda_i)\Bigg[(1-\delta_1)\,U\Bigg(C_i + a_i R + \chi_i\Bigg) \\
& +\delta_1\Bigg[\sum_{\Phi\subseteq\Omega\backslash\{i\}} \prod_{k\in\Phi}\lambda_k \prod_{k\notin\Phi}(1-\lambda_k)\cdot U\Bigg(C_i \\
& + \sum_{k\in\Phi}\left[-\max\left\{\tilde{\omega}^s_{ik}, 0\right\} + \chi_{ki}\right] - \sum_{k\notin\Phi}\left[\tilde{\omega}^s_{ik}R + \chi_{ik}\right]\Bigg)\Bigg]\Bigg]\Bigg]\mathrm{d}\delta_1,
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
C_i &\equiv \left(c_i - K_i - \chi_i + \omega_{ij}p_{ij} + \sum_k \tilde{\omega}^s_{ik}p^s_{ik}\right) r_0, \\
K_i &\equiv \kappa \sum_k \mathbb{1}_{\{\Xi^s(i,k)=1\}}, \\
\chi_i &\equiv \chi_i^{IM}\left(\omega_{ij} + \sum_k \tilde{\omega}^s_{ik}\right) + \chi_i^{VM}\left(\omega_{ij} + \sum_k \tilde{\omega}^s_{ik}, \delta_1\right), \\
\chi_{ij} &\equiv \chi_{ij}^{IM} + \chi_{ij}^{VM}.
\end{aligned}
$$

In equation 8, C_i is the total payoff derived from the riskless asset, K_i the number of times the fixed trading cost κ has been paid at date s, whereas notations related to collateral are redefined. Moreover, the term $\sum_k \tilde{\omega}^s_{ik}$ represents the sum of all exposures previously contracted by i (before date s), excluding the exposure ω_{ij} negotiated in the current trading meeting with j. With probability λ_i, institution i fails and gets a payoff of zero. Conditional on it not failing (with probability $1-\lambda_i$), the underlying asset a does not default with probability $(1-\delta_1)$, so that no CDS repayments are to be made and idiosyncratic bank probabilities of failure Λ do not enter the payoff. A bank's total payoff is composed of its payoff on both the riskless and the risky asset, plus the collateral posted at dates 0 and 1, which it gets back. The opportunity cost of posting collateral is implicit in the fact that χ_i, posted at dates 0 and 1 as collateral, does not get paid r_0.

The last part of equation 8 describes the various possible outcomes when a defaults (which occurs with probability δ_1), conditional on i not failing. Each counterparty $j \in \Omega$ fails with probability λ_j, so that there are 2^{n-1} payoffs to be specified. Each of these payoffs is composed of a riskless part C_i and of a part contingent on counterparty failures. Conditional on both the default of a and on i not failing, the contingent payoff of any bank i on any exposure ω_{ij} is:

$$
\begin{cases}
-\max\left\{\omega_{ij}R, 0\right\} + \chi_{ji} & \text{if} \quad j \text{ fails}, \\
-\omega_{ij}R & \text{if} \quad j \text{ does not fail}.
\end{cases}
$$

Whenever a bank which is net CDS seller fails, it leaves to its counterparty all collateral posted at dates 0 and 1. In case it is net buyer, it enjoys the benefit from the protection bought, even though it is failed.

The expected utility described in equation 8 is updated with the trading process through Ξ^s and through the summation term over past exposures. It depends importantly on the vector of bank probabilities of failure Λ and on the collateral to be posted through initial and variation margins, i.e. on τ^{IM} and on the probability distribution $\pi(\delta_1)$. This has two important implications. First, the incentive to take on more or less exposure ω_{ij} vis-à-vis any counterparty j crucially depends, *ceteris paribus*, on the regulatory collateral framework, which provides incentives through the level of collateralization. Alternative collateral schemes are later considered. Second, a bank takes the probability of failure of each of its counterparty into account, and is willing to reduce—for given collateralization levels—its exposure towards a counterparty j whose probability of failure λ_j is increasing.

In addition to the objective functions, the budget constraint for any bank i is updated with the trading process as the sequential problem is solved for. The time-s budget constraint for i is given by equation 9 (resp 10) if it aims at buying (resp. selling) credit derivatives to its counterparty j.

$$-p_{ij}\omega_{ij} \leq c_i - \kappa \sum_k \mathbb{1}_{\{\Xi^s(i,k)=1\}} + \sum_k \tilde{\omega}^s_{ik} p_{ik} - \chi^{IM}_{ik} \tag{9}$$

$$C_i + \omega_{ij} p_{ij} \geq \left(\omega_{ij} + \sum_k \tilde{\omega}^s_{ik}\right) R \tag{10}$$

Finally the two participation constraints for institutions i and j are simply updated with the new value of the objective function 8 at each s.

At date 1, a bank is able to meet its variation margin calls (if any, i.e. if $\delta_1 > \delta_0$) if its available riskless assets are greater than its collateral requirement, i.e. if $\chi^{VM}_i(\delta_1) < c^1_i$, where c^1_i denotes the cash available at date 1 (but determined at date 0) for any bank i. We rule out such failures from collateral shortage and thus impose a constraint (dubbed risk-management rule) to be followed by all banks.

Assumption 3. ***(Risk management rule)*** *An institution $i \in \Omega$ cannot enter into credit derivative positions which drive its probability to fail from collateral shortage above $\bar{\nu}$. Thus credit derivative exposures must satisfy:*

$$\Pr\left[\chi^{VM}_i(\delta_1) > c^1_i\right] \leq \bar{\nu}, \tag{11}$$

where $c^1_i = c_i - K_i - \chi^{IM}_i + \sum_k \tilde{\omega}^S_{ik} p_{ik}$. We further specify $\bar{\nu} = 0$, ruling out failure from collateral shortage at $t = 1$.

Imposing $\bar{\nu} = 0$ implies that all banks are able to meet their variation margin calls even if the highest possible value for δ_1 is realized at date 1. Conversely, $\bar{\nu} > 0$ would imply that a subset of banks may fail from collateral shortage at date 1 with a strictly positive probability. Resolution rules would thus need to be specified, which do not lie within the scope of this paper.

4 Clearing schemes and results

This section investigates the comparative dynamics of the model with bilateral collateral posting and alternative margining and clearing schemes. It finally calibrates the model to reproduce the ongoing migration from bilateral to mandatory central clearing.

4.1 The banking sector, assets and portfolios

Our baseline banking sector is composed of $n = 5$ institutions. For computational reasons, we restrict attention to a relatively low number of institutions which, however, are fit to represent the set of CDS dealers active in one geographical area such as the United States.[4] Each of them has a utility function U calibrated as a CRRA utility function. The elasticity of substitution is calibrated as $\gamma = 0.3$, a parameter value which falls within the interval estimated in dynamic macroeconomic models (see Rabanal and Rubio-Ramirez 2005; Casares 2007). To replicate a stylized fact from the empirical banking literature, we assume the total assets of each institution i to be drawn from a power law distribution with minimum value 100 and shape parameter $\alpha = 1.8$ (consistent with empirical distributions, see Janicki and Prescott (2006)). The breakdown of total assets between a_i and c_i for each bank i is obtained through the random assignment of shares β and $(1 - \beta)$ to each of these two asset classes respectively, with β drawn from a uniform distribution with support $[0; 1]$. Therefore banks are heterogeneous both with regards to their total assets and to the share of each asset class within their balance sheets.

Idiosyncratic bank probabilities of default are a key component of the model as they affect the cost-benefit weighting of increased margin requirements. In the baseline case, we consider a low level of bank probabilities of default, which are assumed to be drawn out of a beta distribution with shape parameters 25 and 60000. Thus 95% of bank probabilities of default fall between $3.1 \cdot 10^{-4}$ and $5.3 \cdot 10^{-4}$. consistent with the empirical rating-implied average probability of default of US corporates.[5]. These probabilities are meant to mimic un-stressed states of the financial system. Increased idiosyncratic bank probabilities of default are investigated below (section 5.1).

Collateralization parameters are chosen to be conservative. Initial margins are designed to cover the potential future exposure of any counterparty. It is typically computed as the standard deviation of a portfolio value over a several days-horizon (needed by the non-defaulted party to liquidate and replace defaulted derivative exposures). We here assume $f^{IM} = 10$. Variation margins cover current exposures

[4] The CDS market is centered around 14 main dealers (Brunnermeier et al. 2013), 6 of which are in the United States: Bank of America, Citigroup, Goldman Sachs, J. P. Morgan, Morgan Stanley and Wells Fargo.

[5] Ratings for the senior debt provided by Standard & Poors and of September 2013 range between A- and A+ for the six dealers listed in footnote 4 The corresponding empirical average probability of default over the 1981-2011 period are provided by Standard and Poors (2012)

and are called at a more regular frequency, typically daily. We assume $f^{VM} = 2$ for conservativeness.

The gross yield on the riskless asset is calibrated as the 10-Year U.S. Treasury Constant Maturity Rate as of end-2012, i.e. $r_0 = 1.029$. The gross yield on the risky asset is calibrated as the long-term arithmetic average of the S&P 500 return estimated on the 1928-2012 period. Thus, $R = 1.158$. The data is retrieved from the Federal Reserve Bank of Saint-Louis. All calibrated parameters are summarized in table 1.

4.2 Outcome metrics

We restrict attention to a subsample of metrics and distinguish between market outcomes and contract characteristics.

- The aggregate size of the market is captured through the market gross notional amount computed as $\sum_i \sum_s \sum_j \left|\omega_{ij}^s\right| /2$, i.e. the sum of all trades between all banks at all instants s, where $|.|$ denotes the absolute value.
- The exposure at default assuming no idiosyncratic bank failure is captured through the market-wide multilateral net notional exposure, which is computed as $\sum_i \left|\sum_j \sum_s \omega_{ij}^s\right| /2$.
- In addition, the ratio of the multilateral net over the gross notional exposure provides information about the extent to which the trading activity creates genuine bilateral exposure at default. It can be interpreted as the extent of intermediary trading or as "market liquidity", as a trader who holds large gross but low net exposures typically buys from one counterparty and sells to another, earning the price differential.
- The extent to which traders are hedged against counterparty credit risk is computed as the total collateral delivered system-wide through initial margins, i.e. $\sum_i \chi_i^{IM}$. Variation margins are not included as they do not only depend on the network of exposures but also on particular realizations of δ_1.
- We also compute the ratio of margins to the net multilateral exposure, which is interpreted as the extent to which contagion is likely mitigated in the event of a counterparty failure.

All outcome metrics are averaged across simulations.

4.3 Bilateral clearing with various levels of collateralization

This section and the following two examine the comparative dynamics of the model when key collateralization and clearing parameters are varied. First, pure bilateral clearing in the decentralized OTC market described by equations 8 to 11 is analysed for various levels of collateralization τ^{IM}. Rehypothecation is not possible ($\rho = 0$).

The outcome metrics for the level of collateralization $\tau^{IM} \in [0,1]$ are presented on figure 1. Increasing the required level of collateralization from 0 to 1 induces a decrease of the CDS market size, as the system-wide gross exposure is found to be about 21.0% smaller when there is full collateralization, as compared to the no-collateralization case. Thus, a sizeable share of trades for which the benefits of CDS contracts no longer outweigh the cost of collateral disappears.

The decrease in traders' exposure at default, as captured by the market multilateral net notional exposure is, however, of lower magnitude (about 13.0%). Thus the multilateral net over gross notional ratio is an increasing function of the level of trade collateralization. Whereas a sizeable share of the market activity is eliminated when collateralization requirements increase, ultimate net credit risk exposures diminish only to a lesser extent. One factor explaining this result relates to the trading patterns by institutions acting as intermediary (i.e. buying CDS protection from some traders and selling protection to others) : intermediary trading creates potentially large gross exposures but low multilateral net exposures, and yields profit only as long as the price differential between CDS bought and CDS sold is not offset by the cost of collateral to be posted on the positions where the intermediary is a net seller. Increased collateral requirements may therefore reduce gross notional amounts faster than net notional amounts.

System-wide initial margins demand is found to be an increasing function of the collateralization level, even though gross and net exposures are smaller in magnitude. This stems from the fact that the elasticity of the (gross or net) network size to the collateralization level is much smaller than that of collateral demand. When τ^{IM} increases from 0 to 1, the ratio of initial margins to net notional exposures at a system level increases from 0 to 21%.

Let us focus on the channels driving the reduction in gross market size as well as on the pricing of the contracts. Figure 2 plots the average price of a CDS contract traded (weighted by the contract notional amount), the average number of trades and the average trade notional amount for a set of collateralization levels ranging between 0 and 1. First, we do find that increased collateral requirements τ^{IM} imply a reduction of the average number of trades (from 15.0 to 7.6 when the collateralization level rise from 0 to 1) and an increase in the average notional amount traded (from 1.7 to 3.0). The shrinkage in market size is driven by the disappearance of trades with a low notional amount, which are no longer profitable on expectation for at least one party. This pattern provides further evidence for the reduction in market liquidity induced by higher collateralization requirements.

As the marginal costs and benefits of collateral change with exogenous requirements, they are to be priced by traders. We do find increased collateral costs *ceteris paribus* to be priced on the market. CDS buyers pay higher prices both because they enjoy a higher recovery payoff in case of counterparty failure, and to partly compensate sellers for the foregone returns on the riskless asset delivered as margins. An increase of the collateralization ratio τ^{IM} from 0 to 1 translates into an increase of the market CDS spread by 2.2%. While policy debates typically focus on the collateral costs for CDS sellers, the overall cost of collateralization is also partly supported by CDS buyers through higher prices.

4.4 Rehypothecation

Through rehypothecation, traders re-use part of the collateral received to meet their own collateral calls. It is a widespread practice on derivative markets (Singh 2010b). Usually not all collateral received can be repledged. We assume that only a fraction $\rho \in [0;1]$ can be rehypothecated. Both initial and variation margins are assumed to be eligible for rehypothecation, so that rehypothecation may occur at dates 0 and 1.

At $t = 0$, initial margins received by i equal $\sum_j \chi_{ji}^{IM}$, whereas initial margins delivered equal $\sum_j \chi_{ij}^{IM}$. Collateral received cannot be used by a bank to buy additional CDS, but only to meet its own collateral calls. Whenever $\rho \sum_j \chi_{ji}^{IM}$ can be rehypothecated to pay for $\sum_j \chi_{ij}^{IM}$, the budget constraint 9 is relaxed.

At $t = 1$, both initial and variation margins can be rehypothecated. For any bank i, the pool of cash that can be rehypothecated at date 1 is $\rho \sum_j \left[\chi_{ji}^{IM} + \chi_{ji}^{VM}\right]$. This increases the pool of riskless assets c_i^1 and thus relaxes the risk-management rule (assumption 3) accordingly.

Rehypothecation is introduced in the setup of the preceding section (bilateral clearing with no CCP), where ρ is varied parametrically for a given collateralization level. Figure 3 plots the outcome metrics for $\rho \in [0;1]$ and a level of collateralization $\tau^{IM} = 1$. As can be seen, increasing the level of rehypothecation increases market-wide gross notional exposures. When ρ goes from 0 to 1, gross exposures rise by about 13.7%. Net notional exposures, however, remain close to unchanged or tend to decrease slightly. An important consequence follows. Rehypothecation is thus found to favour intermediary activities, i.e. trading patterns at an institution level whereby large gross notional exposures are being opened but virtually no net notional exposures. Market liquidity in a broad sense is enhanced. Given these results, the net-over-gross ratio decreases from 0.56 to below 0.5 when ρ increases from 0 to 1. Most noticeable is the drop in the system-wide collateral demand, which decreases by 44.1%, therefore leading the collateral-over-net notional ratio to fall from 0.21 to 0.12. Policy debates regarding rehypothecation should therefore focus on the respective costs and benefits of collateralization and of well-developed intermediary activities.

In terms of magnitude, we do find that allowing for greater rehypothecation does not fully compensate for the shrinkage in market size when higher collateralization levels are imposed. A market with full rehypothecation and $\tau^{IM} = 1$ is 10.8% smaller in gross notional size than a market with no collateralization requirements when idiosyncratic counterparty risk is low. Finally, CDS prices are higher to a limited extent when rehypothecation is not allowed, as the opportunity cost of a unit of riskless asset pledged is then higher.

4.5 Central clearing party

We add one central clearing party (CCP) to which any trade ω_{ij} between any i and j satisfying an eligibility criterion has to be novated. Ongoing reforms require derivative contracts to be centrally cleared based on a standardization criterion. In this model we do have one homogenous CDS contract on one unique reference entity. Instead, we assume that trades with a notional amount above a threshold $\bar{T}$ have to be novated to a CCP. In turn, if $\omega_{ij} < 0$, the novation of a trade amounts to the CCP buying $\omega_{CCP,j} = \omega_{ij}$ from j and selling $-\omega_{CCP,j}$ to i (conversely whenever $\omega_{ij} > 0$). In terms of collateral amounts to be posted, the case where one central party clears all positions above a threshold $\bar{T}$ differs in several respects from the case where collateral is posted bilaterally.

First, collateral is posted on the basis of the multilateral net exposure to or from the CCP, i.e. trades with different initial counterparties may enjoy netting benefits when centrally cleared. The total notional exposure of any bank i sold to, or bought from, the CCP, is $\tilde{\omega}^s_{i,CCP} = \sum_j \omega_{ij} \mathbb{1}_{\{|\omega_{ij}|>\bar{T}\}}$. On the basis of these exposures for all i, the collateral to be posted by any institution i to the CCP is computed using equation 4 and 5 with $\tau^{IM} = 1$. Thus the CCP is assumed to hold a consistently flat portfolio with full collateralization of current and potential future exposures. Moreover, the CCP does not post initial margins to its members.

Second, the CCP imposes one additional requirement onto its members in the form of a contribution to a default fund. Default fund contributions are typically computed from CCP members' tail risks. We adopt a simplified approach (as in Heller and Vause 2012) and set default fund contributions as a fixed percentage of a bank's initial margins. It generally ranges—depending on the portfolio structure of the clearing member—between 4% and 25% of its initial margin (Rafi 2012). We consider that a multiple $\psi = 0.15$ of the initial margins must contribute to the default fund.

Figure 4 presents the outcomes metrics, as well as the share of cleared CDS notional for $\bar{T}$ ranging between 0 (full central clearing) to 60 (no central clearing) and a collateralization level $\tau^{IM} = 0$ on non-cleared contracts. The system-wide effect of a CCP balances the benefits of cross-counterparty netting with the cost of additional collateral to be delivered. Which effect dominates the other depends both on the collateralization levels for uncleared trades and of the share of CCP-cleared trades, i.e. on the threshold $\bar{T}$.

From the base case, novating a larger share of trades to a CCP has a contractionary effect on the market. When all trades are novated to the CCP, the market gross notional amount is 34.1% lower than in the absence of a CCP. This shrinkage is more pronounced than when raising τ^{IM} on all bilaterally cleared trades, in which case the market size is still 21.6% larger.

Other effects documented earlier for an increase in the collateralization levels on uncleared trades (section 4.3) are magnified. The decrease in the market net notional exposure is of lower magnitude than the drop in gross exposures, yielding a sizeable increase in the net-over-gross notional ratio (from 0.52 to 0.66). Intermediary trading is reduced, and these results give ground for concerns—expressed in policy debates—about the reduction in market liquidity due to mandatory clearing.

Collateral represents 18.2% of the net exposure when all exposures are cleared.[6] Finally, a shift from no clearing to full clearing implies an increase in prices by 4.5%, compared to 2.2% when collateralization levels on bilateral trades were set to 1.

4.6 Calibration and the role of market size adjustment

Finally, we define a market-based calibration, aimed at reproducing the ongoing regulatory migration from full bilateral clearing (or "pre-reform") to full central clearing (or "post-reform"). Our main purpose is not to provide a proper empirical estimation of increased collateral needs induced by mandatory central clearing—which lies outside the scope of the paper—, but rather to convey information on the potential over-estimation of such needs if dynamic ajustments of the market structure are not accounted for (Heller and Vause 2012; Sidanius and Zikes 2012; Duffie et al. 2014). All outcome metrics are here compared with a benchmark in which changes in network size and structure are not accounted for, thus akin to the existing literature. The comparison of collateral demand in both cases provides an estimate of the potential over-estimation of the rise in margins demand in the existing empirical literature (where the network is considered exogenously given).

The parameters related to trade collateralization in the pre-reform case are calibrated using available market data as of end-2012. Our main sources are ISDA (2012, 2013). Descriptive statistics in these surveys are provided for two groups of institutions, "Large" and "Medium/Small".[7] As our set of institutions is meant to represent the most active CDS dealers, we use figures for large institutions. As regards the collateralization of initial margins in the pre-reform case, we consider alternatively $\tau^{IM} = 0$ and $\tau^{IM} = 0.88$. In the first specification, dealers are assumed not to post initial margins between themselves, as anecdotal evidence suggests. In the second case, banks do not enjoy any dealer privilege and initial margins are posted. In the latter case, we consider both $\rho = 0$ (no rehypothecation) and $\rho = 0.75$ (partial rehypothecation). All calibrations are summarized in table 1.

The main outcome metrics for the calibrated model and for several base cases are presented in table 2. From a base case where dealers do not post initial margins, the increase in collateral demand represents a substantial share (17.7%) of the market net notional. However, if the adjustment in the market size and structure were not to be accounted for, estimated collateral needs would be 47% larger, implying a sizeable over-estimation. Qualitatively similar results hold for alternative base cases, however to a lesser extent. In the case where dealers do post initial margins ($\tau^{IM} = 0.88$) and rehypothecation ($\rho = 0.75$), initial margins demand increases by 12.0% when

[6] Note, however, that the multilateral net exposure is here computed, for the sake of comparability, on the bilateral market before novation to the CCP. Because of the cross-counterparty netting benefits of central clearing, net exposures are reduced in the process of novation, so that the ratio of collateral to net exposures increases to 24.6%.

[7] Institutions qualify as "Large" or "Medium/Small" depending on the number of collateral agreements which they signed.

dynamic effects on the network size are accounted for, whereas it would rise by 22% otherwise.

5 Robustness

This section presents further evidence on the dynamics of the model as other parameters affecting the cost-benefit weighting of collateral and clearing schemes are varied. In particular, we focus on increased bank idiosyncratic probabilities of default and on the market-wide supply of riskless assets.

5.1 Probabilities of default

Ceteris paribus, the benefits provided by initial and variation margins to CDS buyers depend on each counterparty's probability of default. Therefore the relative cost-benefit weighting of collateral depends on the level and distribution of bank probabilities of default. This section explores the dynamics of the model when the mean of the distribution from which bank idiosyncratic probabilities of default are drawn rise. Such a scenario is akin to a system-wide downward shift in banks' creditworthiness.

Figures 5 and 6 plot the outcome metrics for two levels of bank probability of default, with changing collateralization levels on the bilateral market and with increased central clearing respectively. Several results are obtained. First, gross and net notional exposures system-wide are smaller when counterparty risk is higher. However, the shrinkage of market size depends importantly on the collateralization level or on the scope of central clearing.

Focusing first on changes in the collateralization level on uncleared bilateral trades (figure 5), the different dynamics of gross and net notional exposures is to be noted. Even with full collateralization of bilateral exposures, the gross market notional remains significantly lower when probabilities of default are higher (14.3% lower in this case); on the contrary, increased collateralization levels reduce the impact of probabilities of default on net notional exposures, i.e. on ultimate exposure at default. The gap between the multilateral net notional for two levels of bank probabilities of default is found to narrow when collateralization levels increase. CDS buyers are then compensated to a larger extent in case of counterparty default, and are thus willing to take on larger net exposures. Collateral mitigates the shrinkage in market size.

Furthermore, prices are overall lower when counterparty risk is higher, reflecting the fact that states of the world in which CDS buyers will not receive CDS payoffs are more likely to occur. This is particularly true for low collateralization levels, i.e. when collateral cannot compensate for the expected lost payoff in these states. For higher collateralization levels ($\tau^{IM} > 0.6$), the price differential narrows.

Turning to changes in the scope of central clearing (figure 6), the introduction of a CCP is found to limit to a large extent the negative impact of higher probabilities of default on the network size. When no trades are centrally cleared (right side of the charts), differences in gross and net notional sizes are large depending on the level of bank probabilities of default. When, however, a larger share of trades are cleared (left side of the chart) these differences narrow, so that both gross and net notional due to counterparty risk disappear to a large extent. This is consistent with existing theoretical results according to which mandatory CCP clearing may reduce the incentive to screen counterparties (Koeppl and Monnet 2010; Biais et al. 2012): differences in counterparty risk no longer play a sizeable role for the market structure once trades are novated to a CCP; only higher collateralization requirements imposed by the CCP have a first-order effect on market outcomes.

5.2 Riskless asset supply

In this section, we document the sensitivity of our results when the aggregate supply of the riskless asset decreases, leaving the size of each bank unchanged. Assume that a share of the riskless asset holding c_i of each bank i is converted into the risky asset so that, from a portfolio $\{a_i, c_i\}$, each bank i then holds $\{a_i + c_i/\bar{c}, c_i(1 - 1/\bar{c})\}$, where $\bar{c} > 1$ is a constant. Thus the size of each institution remains unchanged, but the ratio of riskless over risky assets decreases to an extent determined by the initial portfolio $\{a_i, c_i\}$. Such a setup resembles the downgrade of an AAA-rated bond, whereby a previously riskless security comes to be regarded as risky.

Figure 7 plots respectively the market outcomes and the contract characteristics for three different supplies of the safe asset, including the base case. Several results are obtained. First, with regards to magnitudes, changes in the supply of the riskless asset—leaving the banking sector size unchanged—has a much larger effect on market size (both gross and net) than a change in the collateralization level. A decrease by 50% in the safe asset supply (from the base case) decreases the gross market size by 46.2%. This result has implications for the procyclicality of the financial system, where gross and net exposures are likely to rise when the supply of assets perceived as riskless is abundant, before contracting when becoming scarcer. Such fluctuations are here shown to be potentially larger than those implied by the change in collateralization framework.

The above result on magnitudes is particularly true for net exposures. If the supply of riskless assets is divided by 4 ($\bar{c} = 1.33$), the gross market size is divided by 2.1, whereas the net market size shrinks by 4.4. Thus the drop in exposures at default is much larger than the contraction is gross market size suggests. This results contrasts with previous results on CCPs and collateralization, where net exposures were found to react to a lesser extent than gross exposures to changes in the collateralization requirements. As a result, the ratio of collateral over net exposures is significantly higher when the supply of riskless assets is lower.

The main explanation behind the response of net notional exposures is the following. In the baseline case, traders willing to increase their exposure to the risky asset take positions as net CDS sellers (by contrast, gross exposures do not generate exposure to the risky asset, only to counterparty default). When part of their riskless asset holdings turn risky, the portfolio allocation that there are targeting changes. The short position they are willing to take is lower, unless CDS spreads increase sufficiently (see below) or they may eventually want to reduce their exposure to the risky asset by buying CDS. Thus the effect of the riskless asset supply on market outcomes goes primarily through net notional exposures which, contrary to gross exposures, enable adjusting one's own exposure to the risky or riskless asset at $t = 2$.

The change in prices induced by the increased scarcity of riskless assets is large, whereas price changes due to increased collateralization levels are by contrast of second order effect. The price of CDS contracts increases, as being guaranteed a safe return at $t = 2$ (i.e. buying a CDS) is more valuable whenever riskless assets are scarcer. To accept a larger exposure to the risky asset, CDS sellers require higher compensation through prices.

5.3 Number of trading institutions

An important parameter for central clearing efficiency, investigated theoretically by Duffie and Zhu (2011) and Cont and Kokholm (2014) and empirically by Duffie et al. (2014), is the size of the netting set—as central clearing trades off increase collateral requirements at an exposure level against multilateral netting benefits at a portfolio level. A driver of multilateral netting benefits is thus the number of active institutions in the netting set. This section presents results when n is varied between 4 and 10.

First, an increase in the number of banks implies higher cross-bank heterogeneity, thus larger risk-sharing opportunities *ceteris paribus*. We run two simulation exercises. First, we increase the collateralization level of initial margins τ^{IM} from 0 to 1. Second, we simulate a shift from no central clearing to full central clearing. In both cases, we focus on the change in gross notional size marketwide. Results are presented in table 3. In both cases, an increase in the number of trading banks reduces the drop in market size induced by higher collateral requirements, both for bilaterally and for centrally cleared trades. In the case where all trades are bilateral, the lower drop in notional market size can be explained by the higher trading opportunities that exist when cross-bank heterogeneity is larger, even when collateralization levels are higher. In the case where a CCP exists, a second effect is at play: a larger number of counterparties increases the multilateral netting opportunities provided by the CCP, which dampens partially the effects of higher collateral standards.

6 Conclusion

Using an OTC network formation model, this paper investigates the comparative dynamics of the credit derivative market when regulatory collateral schemes are exogenously imposed onto trading institutions. Both costs and benefits of collateral posting through initial and variation margins enter banks' objective functions. In the trading process, their relative weight contributes to the determination of market outcomes including exposure sizes and contract prices. Not accounting for such dynamic effects is shown to have sizeable consequences for estimated collateral demand increases provided in the literature (between 22% and 47% over-estimation). Furthermore, the analysis of these effects is refined so as to account for changes in both bank idiosyncratic probabilities of default and safe asset supply, both of which have a sizeable marginal effect on the costs and benefits of collateral.

While our analysis embeds some simplifying assumptions, our model framework offers a first step towards a cost-benefit of regulatory measures in OTC markets. In particular, the following extensions could be worth pursuing. First, the current framework features only one risky asset, and therefore does not account for the return correlation structure that exists in portfolios with multiple assets. Asset correlation is nonetheless important, as a more refined model of initial and variation margins would typically feature benefits from portfolio diversification (Duffie et al. 2014). Second, the robustness of the simulated networks to exogenous shocks, as well as contagion arising from adverse scenarios could be analysed using our framework. Finally, our comparative investigation of collateralization schemes could be complemented by a normative analysis. One pending question, for example, relates to whether the reduction in market size induced by increased collateral requirements is welfare-improving. Related is the question about whether trades for which the expected benefit no longer exceeds the expected cost—i.e. do not take place any longer—were associated with genuine hedging needs or with other purposes.

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Appendices

Parameter	Definition	Baseline	Market	Comment
		Banking sector		
n	Number of banks	5	5	Dealers
Λ	Bank probabilities of failure	Beta (25,60000)	Beta (25,60000)	Implied from credit ratings.
γ	CRRA coefficient	0.3	0.3	Rabanal and Rubio-Ramirez (2005) Casares (2007)
ν	Risk-management rule	0	0	No bank failure from collateral shortage
κ	Cost of creating a link	0.1	0.1	
S	Number of matching opportunities	200	200	
θ	Preferential attachment	1000	1000	
		Assets		
r_0	Risk-free rate	1.029	1.029	10-year Treasury
R	Risky yield	1.158	1.158	S&P 500 average yield
δ_0	Default probability at $t = 0$	0.1	0.1	
$\pi(.)$	Distribution of δ_1	$\text{Unif}\,(\delta_0 - \epsilon; \delta_0 + \epsilon)$	$\text{Unif}\,(\delta_0 - \epsilon; \delta_0 + \epsilon)$	
ϵ	Support of $\pi(.)$	0.1	0.1	
		Collateral		
f^{IM}	Initial margin rule	10	10	Potential future exposure
f^{VM}	Variation margin rule	2	2	Current exposure
τ^{IM}	Initial margin level	$[0; 1]$	0	ISDA (2013)
τ^{VM}	Variation margin level	1	1	ISDA (2013)
ρ	Rehypothecation ratio	0	0.75	ISDA (2013)
		With CCP		
$\bar{T}$	CCP clearing threshold	$+\infty$	$+\infty$	No CCP clearing
ψ	Contribution to default fund	0.15	0.15	Rafi (2012)

Table 1: Calibration of the parameters in the baseline model. This table presents the calibration of the model, both for the baseline case and for the market-calibrated case of section 4.6. The baseline models aims at investigating the comparative dynamics of the model, whereas the market-calibrated cases reproduces the transition from bilateral to central clearing. "log $\mathcal{N}$" denotes a log-normal distribution and "Unif" a uniform distribution. Calibrations are justified in section 4.1.

	Gross notional	Net notional	Initial margins demand	Initial margins w/o size adjustment	Overestimation
Base case 1: $\tau^{IM} = 0,\ \rho = 0$	170.9	90.8	0	23.7	**0.47**
Full CCP from base case 1	-0.19	-0.08	+Inf	-	-
Base case 2: $\tau^{IM} = 0.88,\ \rho = 0$	140.9	83.5	14.4	19.6	**0.22**
Full CCP from base case 2	-0.01	-0.00	0.12	-	-
Base case 3: $\tau^{IM} = 0.88,\ \rho = 0.75$	157.9	83.8	4.0	21.9	**0.36**
Full CCP from base case 3	-0.12	-0.01	3.025	-	-
"True" full CCP	**138.8**	**83.1**	**16.1**	-	-

Table 2: Mandatory central clearing and market outcomes This table presents market outcomes when the model is calibrated to three base cases (or pre-reform cases) and one post-reform case with full central clearing. The "true" outcomes when changes in the market size are accounted for are presented on the last line (in bold). For each base case, all market outcomes are also computed as if CCP collateral requirements were imposed only banks without accounting for changes in the market size. The last column gives the difference between the estimates of collateral demand obtained through this and the "true" collateral demand with full CCP clearing.

Number of banks	4	5	6	7	8	9	10
Bilateral - τ^{IM} from 0 to 1 (%)	24.5	21.0	18.9	17.4	16.4	15.8	15.6
With CCP - From no to full clearing (%)	37.7	34.1	32.4	29.5	27.5	26.1	24.7

Table 3: Results with a changing number of banks. This table presents the main results when the number of trading banks n is varied from 4 to 10. The base case is that with 5 banks. The first line presents the change in gross market size when the level of collateralization τ^{IM} is varied from 0 to 1 (no collateralization to full collateralization). The second line presents the change in gross market size when shifting from no central clearing to full central clearing.

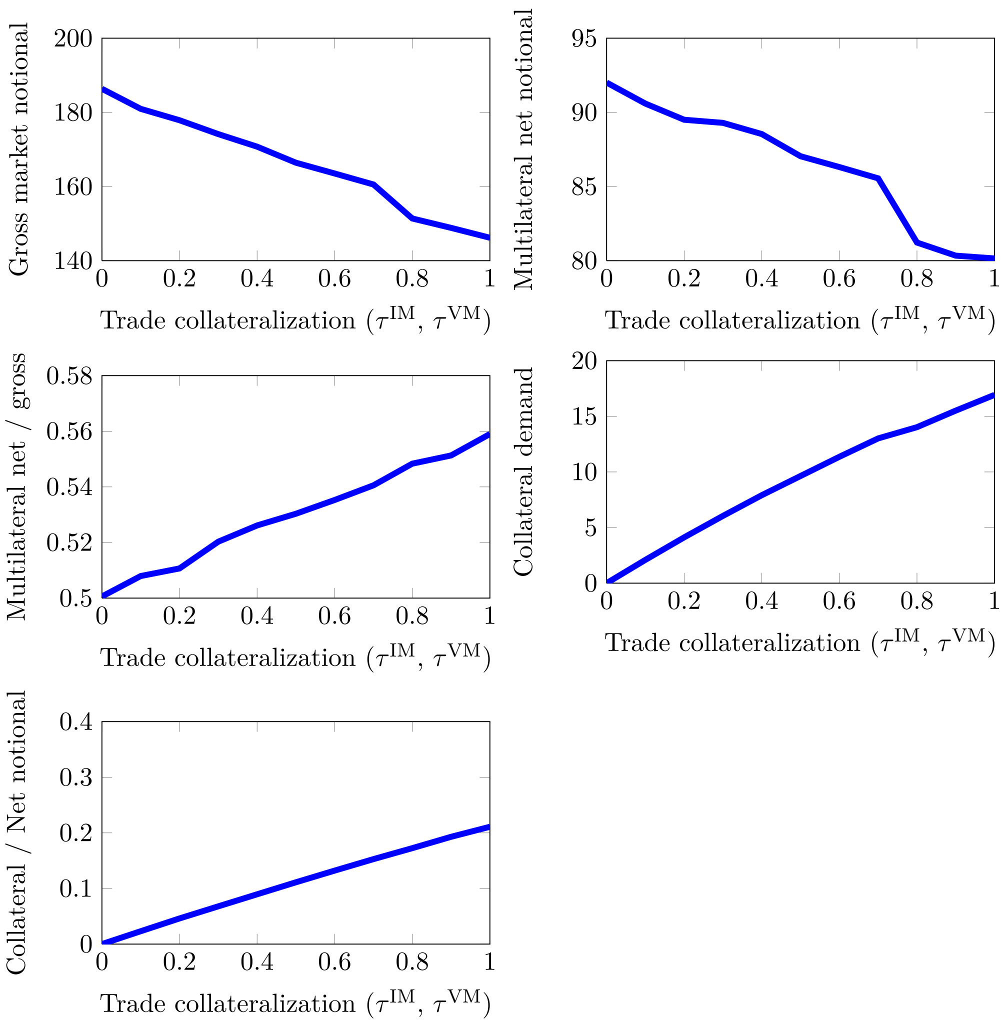

Figure 1: Trade collateralization and market outcomes. This figure presents the market outcomes when the collateralization level τ^{IM} ranges between 0 and 1 while $\tau^{VM} = 1$. The calibrations are those of the baseline case. There is no CCP and rehypothecation is not allowed ($\rho = 0$). Say that the two collateralization levels are identical. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

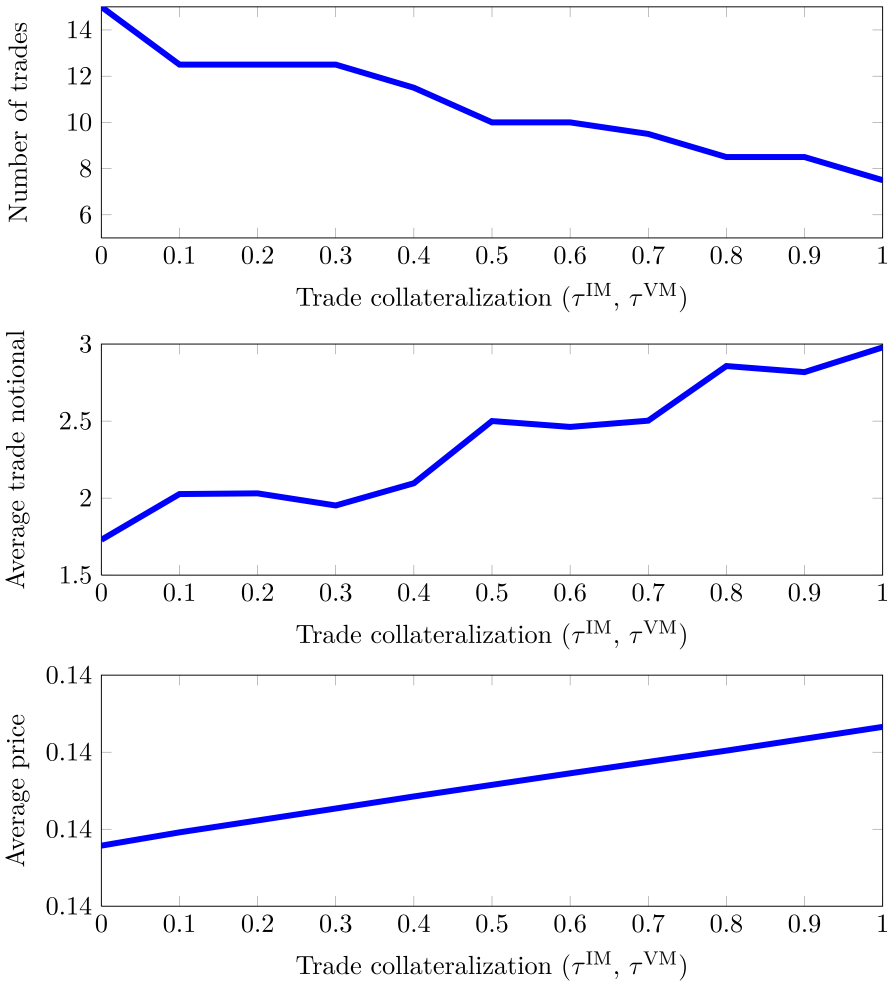

Figure 2: Trade collateralization and contract characteristics. This figure present the average number of trades, the average trade notional and the average CDS price for a collateralization level τ^{IM} ranging between 0 and 1, while $\tau^{VM} = 1$. The average price is weighted by the notional amount of each trade. It is the price of a one-unit notional contract. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

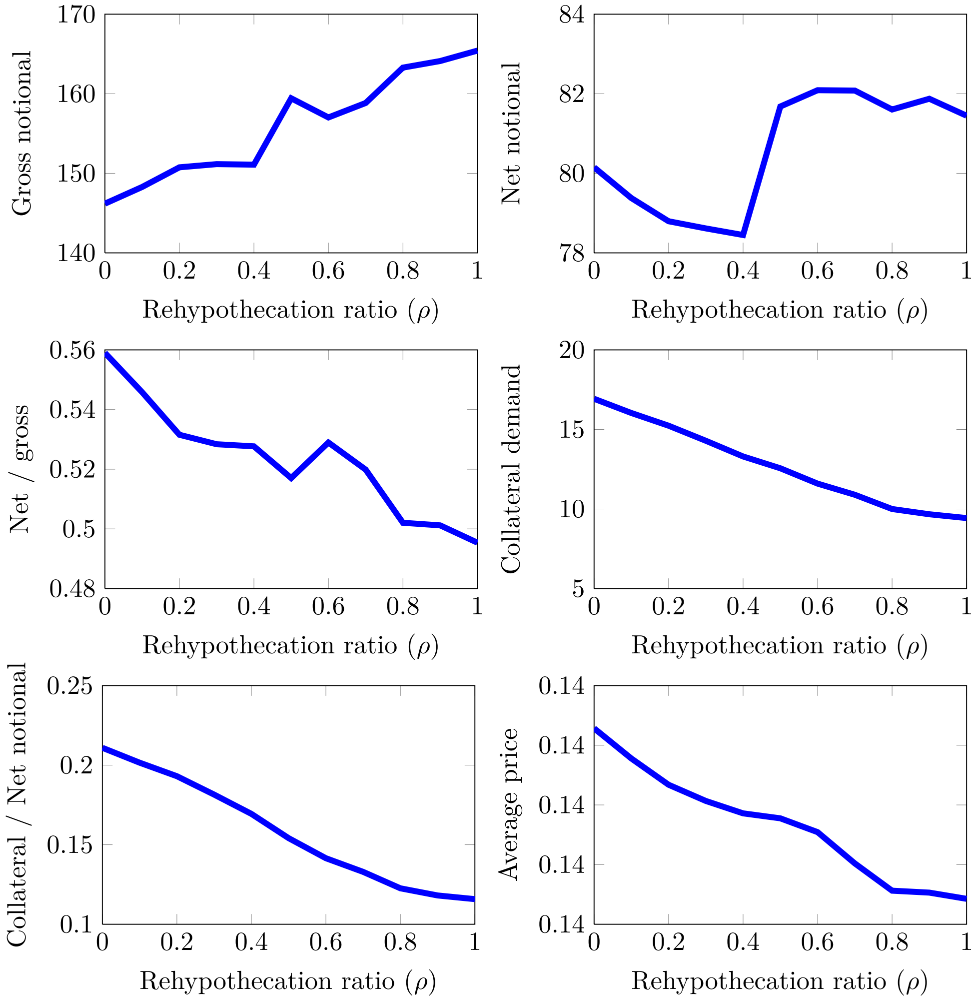

Figure 3: Rehypothecation and market outcomes. This figure presents the outcome metrics for a rehypothecation ratio ρ ranging between 0 and 1. The collateralization level on bilateral trades is assumed to be $\tau^{IM} = 1$, while $\tau^{VM} = 1$. There is no CCP. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

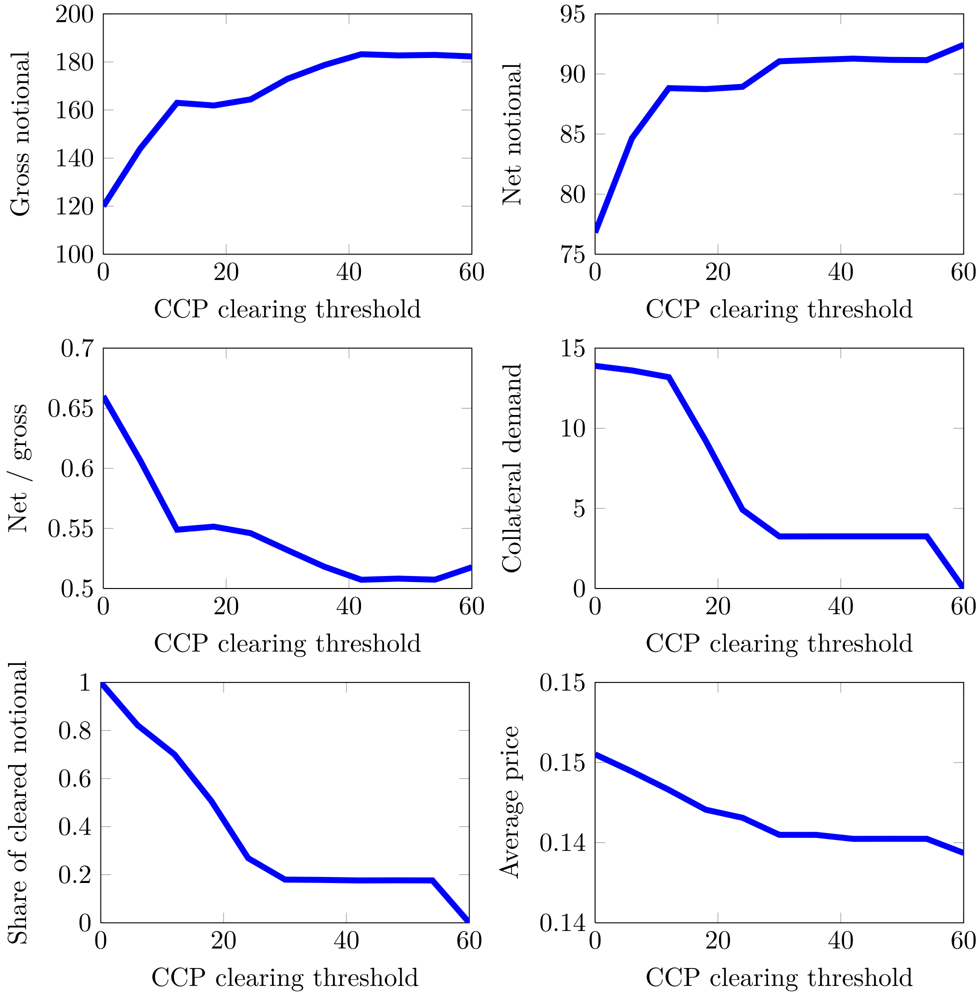

Figure 4: Central clearing and market outcomes. This figure presents the market outcomes when the CCP clearing threshold $\bar{T}$ ranges between 0 and 60. All trades with notional amount above $\bar{T}$ and centrally cleared. Thus results for full central clearing are read at $\bar{T} = 0$. Uncleared trades are assumed not to face collateral requirements, so that $\tau^{IM} = 0$ while $\tau^{VM} = 1$. Collateral includes default fund contributions. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

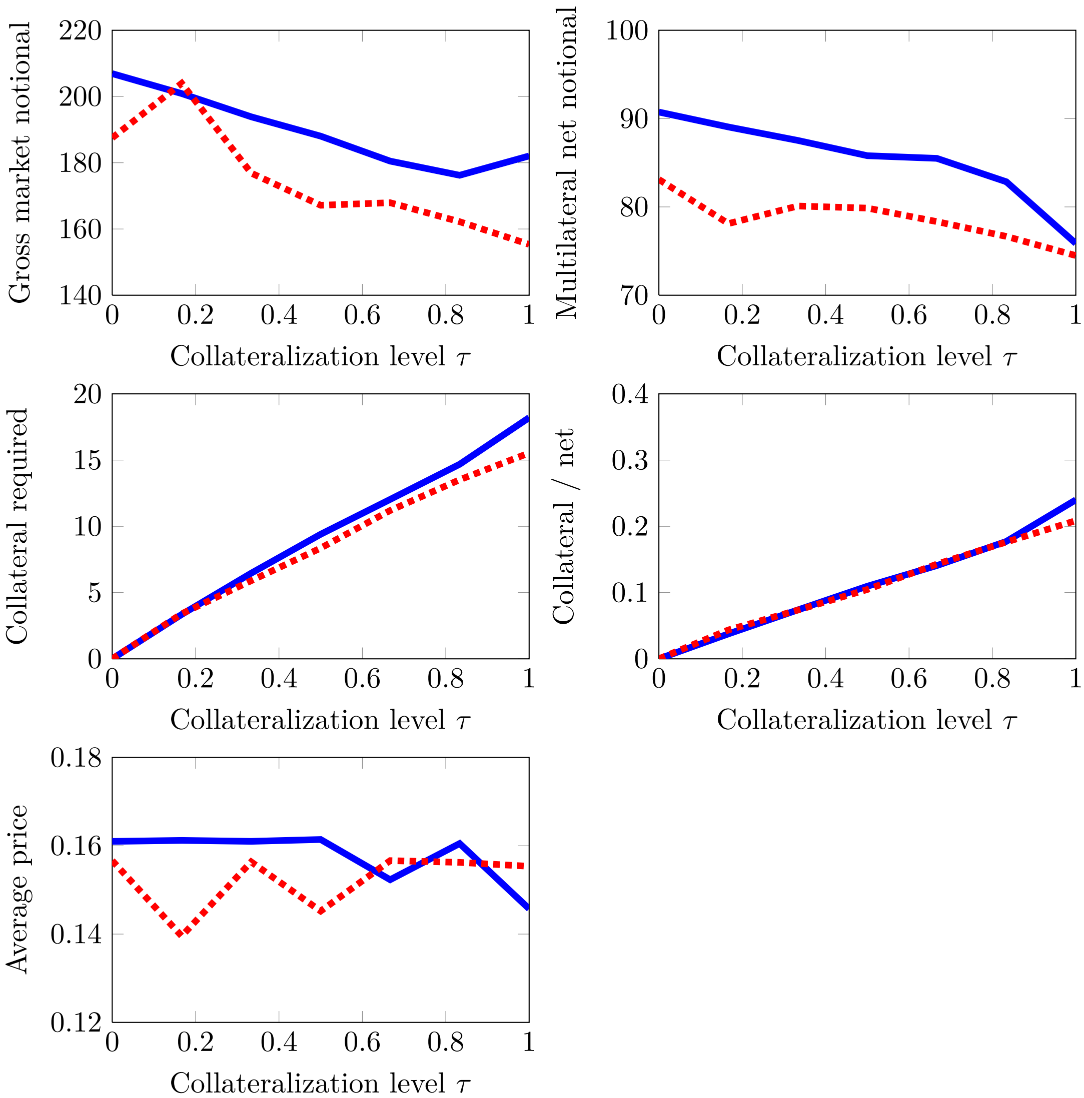

Figure 5: Collateralization level and market outcomes with changing bank probability of default distributions. This figure presents the market outcomes when the collateralization level τ^{IM} ranges between 0 and 1 for two distributions of the bank idiosyncratic probabilities of default, while $\tau^{VM} = 1$. Results for probabilities of default drawn from a beta distribution with parameters $\{25, 50000\}$ (low level) and $\{25, 1000\}$ (higher level) are depicted respectively in blue (solid line) and in red (dotted line). The calibrations are those of the baseline case. There is no CCP and rehypothecation is not allowed ($\rho = 0$). Say that the two collateralization levels are identical. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

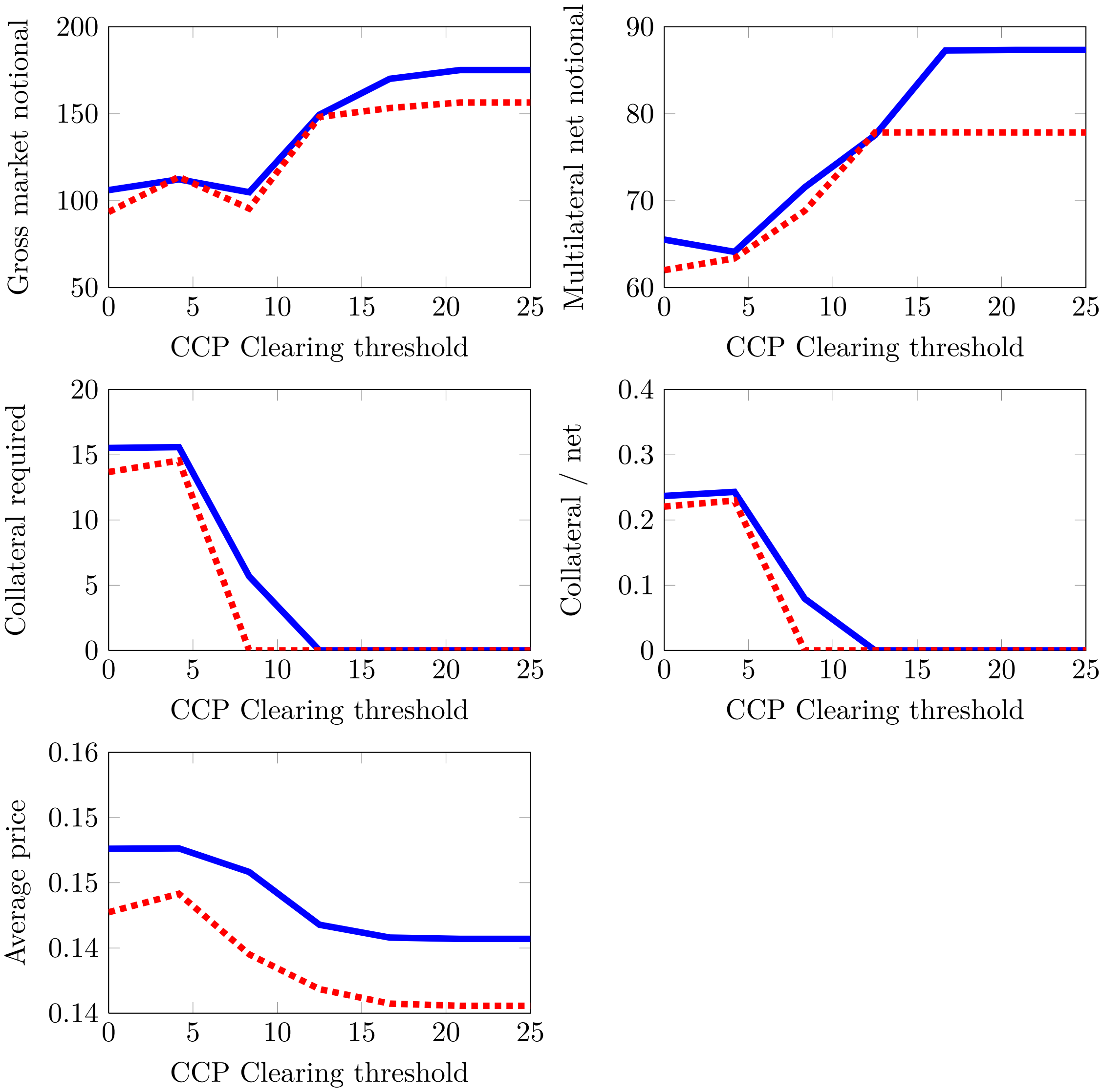

Figure 6: Central clearing and market outcomes with changing bank probability of default distributions. This figure presents the market outcomes when the CCP clearing threshold $\bar{T}$ ranges between 0 and 50 for two distributions of the bank idiosyncratic probabilities of default. Results for probabilities of default drawn from a beta distribution with parameters $\{25, 50000\}$ (low level) and $\{25, 1000\}$ (higher level) are depicted respectively in blue (solid line) and in red (dotted line). All trades with notional amount above $\bar{T}$ and centrally cleared. Thus results for full central clearing are read at $\bar{T} = 0$. Uncleared trades are assumed not to face collateral requirements, so that $\tau^{IM} = 0$, while $\tau^{VM} = 1$. Collateral includes default fund contributions. Each chart is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

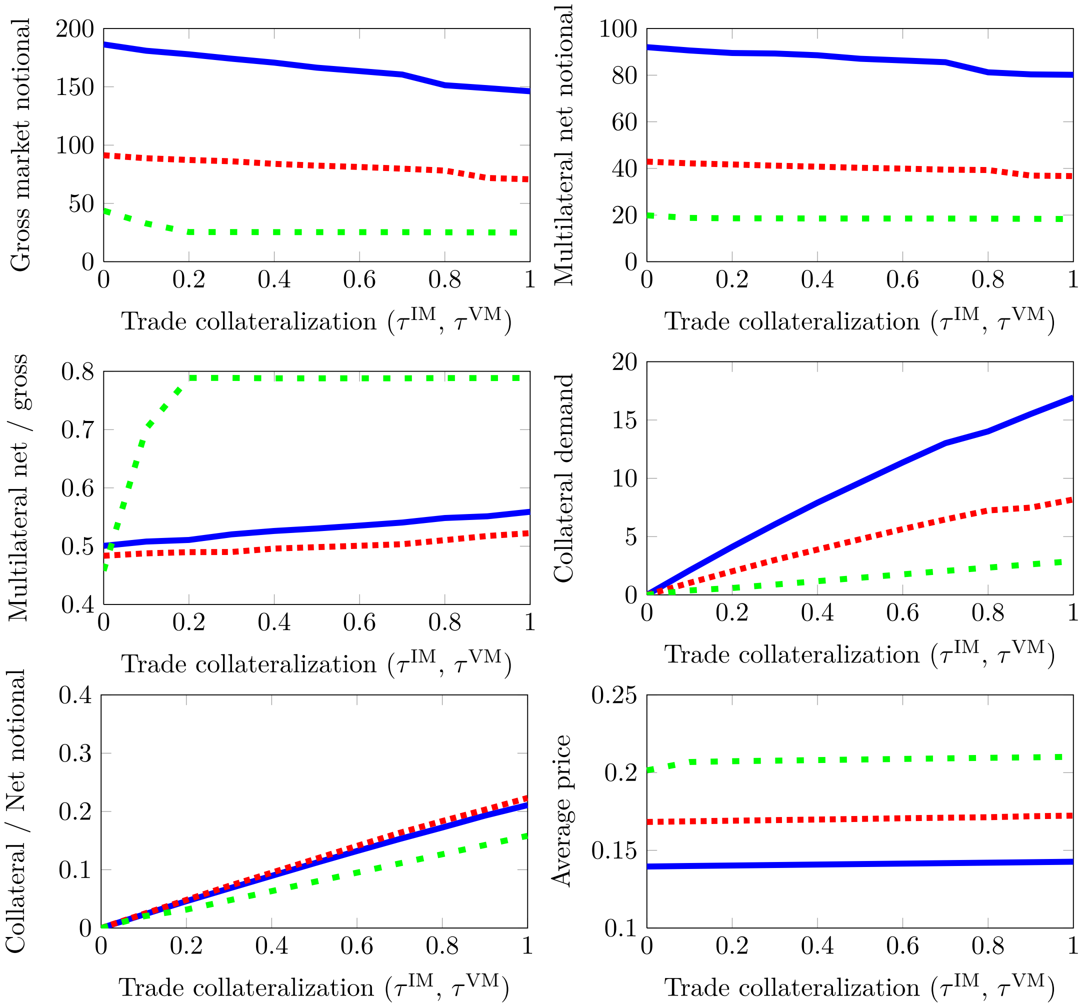

Figure 7: Safe asset supply and market outcomes. This figure presents the market outcomes when the collateralization level τ^{IM} ranges between 0 and 1 for three different supplies of riskless asset while $\tau^{VM} = 1$. The blue/solid curves corresponds to the base case. In the red/dotted (resp. green/dashed) set of curves, the the cash of each institution is divided by 2 (resp. 4), corresponding to $\bar{c} = 2$ (resp. 1.33). The size of each institution is unchanged in all simulations. Other calibrations are those of the baseline case. Each curve is the average over 10 Monte-Carlo simulations where initial balance sheets are identical and the matching process random.

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