I'M OKAY. I LANDED ON A TAXPAYER.
Crowded Risk as a Systemic Concern for Central Clearing Counterparties

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July 3, 2014
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References
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   1.1 BIS-IOSCO (2004)
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   1.2 BIS-IOSCO (2012)
       “Principles for Financial Market Infrastructures.”
ESRB annual report 2012, p. 16:

*Structural reforms being promoted across the globe have paved the way for improved risk management throughout the financial system. In particular, the mandatory move to clearing standardised over-the-counter (OTC) derivatives trades via CCPs will help to reduce counterparty risk between financial institutions, ...*
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ESRB annual report 2012, p. 16:

Structural reforms being promoted across the globe have paved the way for improved risk management throughout the financial system. In particular, the mandatory move to clearing standardised over-the-counter (OTC) derivatives trades via CCPs will help to reduce counterparty risk between financial institutions, . . .

However, the more prominent role of CCPs will also introduce new systemic risks. Mandatory clearing will turn CCPs into systemic nodes in the financial system, with unknown, but possibly far-reaching, consequences.
Exhibit 1: 
CPSS-IOSCO Technical Committee 
Recommendations for Central Counterparties (CCPs)

1. Legal risk
A CCP should have a well founded, transparent and enforceable legal framework for each aspect of its activities in all relevant jurisdictions.

2. Participation requirements
A CCP should require participants to have sufficient financial resources and robust operational capacity to meet obligations arising from participation in the CCP. A CCP should have procedures in place to monitor that participation requirements are met on an ongoing basis. A CCP’s participation requirements should be objective, publicly disclosed, and permit fair and open access.

3. Measurement and management of credit exposures
A CCP should measure its credit exposures to its participants at least once a day. Through margin requirements, other risk control mechanisms or a combination of both, a CCP should limit its exposures to potential losses from defaults by its participants in normal market conditions so that the operations of the CCP would not be disrupted and non-defaulting participants would not be exposed to losses that they cannot anticipate or control.

4. Margin requirements
If a CCP relies on margin requirements to limit its credit exposures to participants, those requirements should be sufficient to cover potential exposures in normal market conditions. The models and parameters used in setting margin requirements should be risk-based and reviewed regularly.
Motivation

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2. They scale with a member’s yet-to-clear trade portfolio times volatility.
3. For example, 54 exchanges and clearing houses use SPAN developed by Chicago Mercantile Exchange (CME).
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2. If so, can one come up with a reasonable measure of crowding?
3. And, is there an alternative way to calculate margins so that (systemic) CCP risk is allocated appropriately across members?
Objective

1. Do crowded trades constitute a hidden risk to a CCP? Yes!
2. If so, can one come up with a reasonable measure of crowding? Yes!
3. And, is there an alternative way to calculate margins so that (systemic) CCP risk is allocated appropriately across members? Yes!
Findings

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   1.1 It uses the “aggregate exposure” measure of Duffie and Zhu (2011).
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   1.3 Sensitivity to any security/risk factor is based on an analytic result.
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3. To account for crowded-trade risk the paper proposes the following:

   3.1 A crowding index, \texttt{CrowdIx}, to measure the size of crowded-trade risk.
   3.2 A new margin methodology, \texttt{Margin(A)}, to appropriately account crowded-trade risk.
1. **CCP vs. OTC**

2. **Counterparty risk monitoring**

3. **Systemic risk in trades**

4. **CCP risk management**

5. **Crowded trades**

6. **Systemic risk allocation**
   Brunnermeier and Cheridito (2014), . . .
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\[ R \sim \text{N}(0, \Omega). \]
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3. $n_j$ is the vector of yet-to-settle trade portfolio of member $j$. 
Measure

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$$R \sim N(0, \Omega).$$

3. $n_j$ is the vector of yet-to-settle trade portfolio of member $j$.
4. Let $X_j = n_j'R$ be the P&L on member $j$’s trade portfolio, then

$$X \sim N(0, \Sigma), \quad \Sigma = N'\Omega N, \quad N = [n_1, \cdots, n_J].$$
Measure

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4. Let \( X_j = n_j' R \) be the P&L on member \( j \)'s trade portfolio, then

\[
X \sim N(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \cdots, n_J].
\]

5. CCP aggregate exposure to trade portfolios of all members is defined as

\[
A = \sum_j E_j \quad \text{with } E_j = - \min (X_j, 0).
\]
Measure

1. Duffie and Zhu (2011, p. 78): “For given collateralization standards, the risk of loss caused by a counterparty default is typically increasing in average expected exposure.”
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2. Can the standard deviation of exposure also be derived analytically?
Absolute Moments in 2-dimensional Normal Distribution

By Seiji Nabeysa

Let \( x \) and \( y \) be distributed according to the following 2-dimensional normal distribution,

\[
\frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left\{ - \frac{1}{2(1 - \rho^2)} \left( \frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2} \right) \right\} \, dx \, dy.
\]

It is our purpose to express absolute moments in terms of elementary functions. Putting \( E(|x^n y^m|) = (m, n) \) for simplicity, we have

\[
(m, n) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^n y^m| \exp \left\{ - \frac{1}{2(1 - \rho^2)} \right\} \times \left( \frac{x^2}{\sigma_x^2} - 2\rho \frac{xy}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2} \right) \, dx \, dy
\]

\[
= \frac{2^{\frac{m+n}{2}} \sigma_x^m \sigma_y^n}{\pi} (1 - \rho^2)^{\frac{m+n}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^n y^m| \exp \left( - x^2 + 2\rho xy - y^2 \right) \, dx \, dy
\]

\[
= \frac{2^{\frac{m+n}{2}} \sigma_x^m \sigma_y^n}{\pi} (1 - \rho^2)^{\frac{m+n+1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^n y^m| e^{-x^2 - y^2} \sum_{k=0}^{\infty} \frac{(2\rho xy)^k}{k!} \, dx \, dy
\]

\[
= \frac{2^{\frac{m+n}{2}} \sigma_x^m \sigma_y^n}{\pi} (1 - \rho^2)^{\frac{m+n+1}{2}} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{m+1}{2} + k\right) \Gamma\left(\frac{n+1}{2} + k\right)}{(2k)!} (2\rho)^k
\]

\[
= \frac{2^{\frac{m+n}{2}} \sigma_x^m \sigma_y^n \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right)}{\pi} (1 - \rho^2)^{\frac{m+n+1}{2}} \times F\left(\frac{m+1}{2}, \frac{n+1}{2}; \frac{1}{2}; \rho^2\right)
\]

\[
= \frac{2^{\frac{m+n}{2}} \sigma_x^m \sigma_y^n \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right)}{\pi} F\left(-\frac{m}{2}, -\frac{n}{2}; \frac{1}{2}; \rho^2\right).
\]

Here

\[
F(\alpha, \beta; \gamma; z) = 1 + \frac{\alpha \beta}{1! \gamma} z + \frac{\alpha(\alpha + 1) \beta(\beta + 1)}{2! \gamma(\gamma + 1)} z^2 + \ldots
\]

is the hypergeometric function, which reduces to the polynomial of \( z \) if \( \alpha \) or \( \beta \) is a non-positive integer and \( \gamma \) is positive. Thus, when at least one of the integers \( m, n \) is an even number, \( (m, n) \) reduces to the polynomial of \( \rho^2 \) multiplied by \( \sigma_x^m \sigma_y^n \).
The case where both \( m \) and \( n \) are odd may be treated as follows. Put
\[
x = \sqrt{2(1 - \rho^2)} \sigma_1 r \cos \theta, \quad y = \sqrt{2(1 - \rho^2)} \sigma_2 r \sin \theta.
\]
When \( m - n = 2q \), where \( q \) is a non-negative integer, we have then
\[
(m, n) = \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| x^m y^n \exp \left\{ -\frac{1}{2(1 - \rho^2)} \right\} \right. \\
\times \left( \frac{x^2}{\sigma_1^2} - 2\rho \frac{xy}{\sigma_1 \sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right| dx \, dy
\]
\[
= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} (1 - \rho^2)^{\frac{m+n+1}{2}} \int_0^{2\pi} \int_0^{\infty} r^{m+n+1} |\cos^m \theta \sin^n \theta| \\
\times \exp \left\{ -r^2(1 - 2\rho \cos \theta \sin \theta) \right\} \, dr \, d\theta
\]
\[
= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma \left( \frac{m+n+1}{2} \right) (1 - \rho^2)^{\frac{m+n+1}{2}} \\
\times \int_0^{2\pi} \frac{|\cos^m \theta \sin^n \theta|}{(1 - 2\rho \cos \theta \sin \theta)^{\frac{m+n+1}{2}}} \, d\theta
\]
\[
= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma \left( \frac{m-n+1}{2} \right) (1 - \rho^2)^{\frac{m+n+1}{2}} \\
\times \frac{d^n}{d\rho^n} \int_0^{2\pi} \frac{|\cos^m \theta|}{(1 - 2\rho \cos \theta \sin \theta)^{n+1}} \, d\theta.
\]
As the last integral may be calculated in the elementary fashion, \((m, n)\) may be evaluated.

In the following we shall give the obtained formulae for the cases \( m \geq n \). The formula of \((m, n)\) for \( m \leq n \), is obtained by exchanging \( \sigma_1 \) and \( \sigma_2 \) in the formula \((n, m)\).

\[
(1, 0) = \sqrt{\frac{2}{\pi}} \sigma_1,
\]
\[
(2, 0) = \sigma_1^2,
\]
\[
(1, 1) = \frac{2}{\pi} \left( \sqrt{1 - \rho^2 + \rho \sin^{-1} \rho} \right) \sigma_1 \sigma_2,
\]
\[(3, 0) = 2\sqrt{\frac{2}{\pi}} \sigma_i^3,
(2, 1) = \sqrt{\frac{2}{\pi}} (1 + \rho^4) \sigma_i^3 \sigma_5,
(4, 0) = 3\sigma_i^4,
(3, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (2 + \rho^2) + 3\rho \sin^{-1} \rho \right\} \sigma_i^5 \sigma_2,
(2, 2) = (1 + 2\rho^2) \sigma_i^5 \sigma_2^3,
(5, 0) = 8\sqrt{\frac{2}{\pi}} \sigma_i^6,
(4, 1) = \sqrt{\frac{2}{\pi}} (3 + 6\rho^2 - \rho^4) \sigma_i^6 \sigma_2,
(3, 2) = 2\sqrt{\frac{2}{\pi}} (1 + 3\rho^2) \sigma_i^6 \sigma_2^3,
(6, 0) = 15\sigma_i^8,
(5, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (8 + 9\rho^2 - 2\rho^4) + 15\rho \sin^{-1} \rho \right\} \sigma_i^8 \sigma_2,
(4, 2) = 3(1 + 4\rho^2) \sigma_i^8 \sigma_2^2,
(3, 3) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (4 + 11\rho^2) + 3\rho (3 + 2\rho^2) \sin^{-1} \rho \right\} \sigma_i^8 \sigma_2^3,
(7, 0) = 48\sqrt{\frac{2}{\pi}} \sigma_i^7,
(6, 1) = 3\sqrt{\frac{2}{\pi}} (5 + 15\rho^2 - 5\rho^4 + \rho^6) \sigma_i^6 \sigma_2,
(5, 2) = 8\sqrt{\frac{2}{\pi}} (1 + 5\rho^2) \sigma_i^6 \sigma_2^3,
(4, 3) = 6\sqrt{\frac{2}{\pi}} (1 + 6\rho^2 + \rho^4) \sigma_i^6 \sigma_2^3,
(8, 0) = 105\sigma_i^8,
(7, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (48 + 87\rho^2 - 38\rho^4 + 8\rho^6) + 105\rho \sin^{-1} \rho \right\} \sigma_i^8 \sigma_2,
(6, 2) = 15(1 + 6\rho^2) \sigma_i^8 \sigma_2^2,
(5, 3) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (16 + 83\rho^2 + 6\rho^4) + 15\rho (3 + 4\rho^2) \sin^{-1} \rho \right\} \sigma_i^8 \sigma_2^3,
(4, 4) = 3(3 + 24\rho^2 + 8\rho^4) \sigma_i^8 \sigma_2^3,
(9, 0) = 384 \sqrt{\frac{2}{\pi}} \sigma_i^3,

(8, 1) = 3 \sqrt{\frac{2}{\pi}} (35 + 140\rho^2 - 70\rho^4 + 28\rho^6 - 5\rho^8) \sigma_i^3 \sigma_2,

(7, 2) = 48 \sqrt{\frac{2}{\pi}} (1 + 7\rho^2) \sigma_i^2 \sigma_2^3,

(6, 3) = 6 \sqrt{\frac{2}{\pi}} (5 + 45\rho^2 + 15\rho^4 - \rho^6) \sigma_i^3 \sigma_2^3,

(5, 4) = 24 \sqrt{\frac{2}{\pi}} (1 + 10\rho^2 + 5\rho^4) \sigma_i^2 \sigma_2^4,

(10, 0) = 945\sigma_i^0,

(9, 1) = \frac{6}{\pi} \left( \sqrt{1 - \rho^2} (128 + 325\rho^2 - 210\rho^4 + 88\rho^6 - 16\rho^8) + 315\rho \sin^{-1}\rho \right) \sigma_i^3 \sigma_2,

(8, 2) = 105 (1 + 8\rho^2) \sigma_i^4 \sigma_2^3,

(7, 3) = \frac{2}{\pi} \left( \sqrt{1 - \rho^2} (96 + 741\rho^2 + 120\rho^4 - 12\rho^6) + 315\rho \sin^{-1}\rho \right) \sigma_i^3 \sigma_2^3,

(6, 4) = 45 (1 + 12\rho^2 + 8\rho^4) \sigma_i^4 \sigma_2^4,

(5, 5) = \frac{2}{\pi} \left( \sqrt{1 - \rho^2} (64 + 607\rho^2 + 274\rho^4) + 15\rho (15 + 40\rho^2 + 8\rho^4) \sin^{-1}\rho \right) \sigma_i^5 \sigma_2^5,

(11, 0) = 3840 \sqrt{\frac{2}{\pi}} \sigma_i^9,

(10, 1) = 15 \sqrt{\frac{2}{\pi}} (63 + 315\rho^2 - 210\rho^4 + 126\rho^6 + 45\rho^8 + 7\rho^{10}) \sigma_i^9 \sigma_2,

(9, 2) = 384 \sqrt{\frac{2}{\pi}} (1 + 9\rho^2) \sigma_i^4 \sigma_2^3,

(8, 3) = 6 \sqrt{\frac{2}{\pi}} (35 + 420\rho^3 + 210\rho^4 - 28\rho^6 + 3\rho^8) \sigma_i^4 \sigma_2^3,

(7, 4) = 48 \sqrt{\frac{2}{\pi}} (3 + 42\rho^3 + 35\rho^4) \sigma_i^4 \sigma_2^4,

(6, 5) = 120 \sqrt{\frac{2}{\pi}} (1 + 15\rho^3 + 15\rho^4 + \rho^6) \sigma_i^4 \sigma_2^5,
(12, 0) = 10395 \sigma_1^10 ,

(11, 1) = \frac{6}{\pi} \left[ \sqrt{1 - \rho^2} (1280 + 4215 \rho^4 - 3590 \rho^6 + 2248 \rho^8 - 816 \rho^{10} + 128 \rho^{12}) + 3465 \rho \sin^{-1} \rho \right] \sigma_1^1 \sigma_2^1 ,

(10, 2) = 945 (1 + 10 \rho^2) \sigma_1^1 \sigma_2^1 ,

(9, 3) = \frac{6}{\pi} \left[ \sqrt{1 - \rho^2} (256 + 2639 \rho^4 + 690 \rho^6 + 136 \rho^8 + 16 \rho^{10}) + 315 \rho (3 + 8 \rho^2 \sin^{-1} \rho) \right] \sigma_1^3 \sigma_2^3 ,

(8, 4) = 315 (1 + 16 \rho^2 + 16 \rho^4) \sigma_1^3 \sigma_2^3 ,

(7, 5) = \frac{6}{\pi} \left[ \sqrt{1 - \rho^2} (128 + 1779 \rho^4 + 1518 \rho^6 + 40 \rho^8) + 105 \rho (5 + 20 \rho^2 + 8 \rho^4 \sin^{-1} \rho) \right] \sigma_1^5 \sigma_2^5 ,

(6, 6) = 45 (5 + 90 \rho^3 + 120 \rho^4 + 10 \rho^6) \sigma_1^5 \sigma_2^5 .

In another paper we shall treat the 3-dimensional case by a unified but more complicated method.

Institute of Statistical Mathematics.
Measure

1. Results for the folded and truncated normal distribution are used to calculate the mean and standard deviation of $A$ (Nabeya, 1951; Rosenbaum, 1961):

\[
\text{mean } (A) = \sum_j \sqrt{\frac{1}{2\pi}\sigma_j} (Duffie and Zhu, 2011)
\]

\[
\text{std } (A) = \sqrt{\sum_{k,l} \left(\pi - \frac{1}{2}\pi\right)\sigma_k\sigma_l M(\rho_{kl}) M(\rho)} = \left[\frac{1}{2}\pi + \arcsin (\rho)\right] \rho + \sqrt{1 - \rho^2 - \frac{1}{2}\pi - \frac{1}{2}\pi}
\]
Measure

1. Results for the folded and truncated normal distribution are used to calculate the mean and standard deviation of $A$ (Nabeya, 1951; Rosenbaum, 1961):

2. 

$$
mean(A) = \sum_{j} \sqrt{\frac{1}{2\pi} \sigma_j} 
$$

(Duffie and Zhu, 2011)
Measure

1. Results for the folded and truncated normal distribution are used to calculate the mean and standard deviation of $A$ (Nabeya, 1951; Rosenbaum, 1961):

2. \[
mean(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j \quad \text{(Duffie and Zhu, 2011)}
\]

3. \[
std(A) = \sqrt{\sum_{k,l} \left(\frac{\pi - 1}{2\pi}\right) \sigma_k \sigma_l M(\rho_{kl})}
\]
Measure

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$$\text{mean}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$$  
(Duffie and Zhu, 2011)

3. 

$$\text{std}(A) = \sqrt{\sum_{k,l} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k \sigma_l M(\rho_{kl})}$$

$$M(\rho) = \frac{\left[ \frac{1}{2}\pi + \arcsin(\rho) \right] \rho + \sqrt{1 - \rho^2} - 1}{\pi - 1}$$
Measure

\[ M(.) \]
Noncrowded trades

security/

risk factor 2

security/

risk factor 1
Simple example noncrowded trades

1. 

\[ N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \]
Simple example noncrowded trades

1. \[ N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \]

2. \[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & 0 & 0 \\ -1 & \pi - 1 & 0 & 0 \\ 0 & 0 & \pi - 1 & -1 \\ 0 & 0 & -1 & \pi - 1 \end{pmatrix} \]
Simple example noncrowded trades

1. 

\[ N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \]

2. 

\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad var(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & 0 & 0 \\ -1 & \pi - 1 & 0 & 0 \\ 0 & 0 & \pi - 1 & -1 \\ 0 & 0 & -1 & \pi - 1 \end{pmatrix} \]

3. 

\[ E(A) = 4\sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad std(A) = 2\sqrt{\frac{\pi - 2}{2\pi}} \approx 0.85 \]
Crowded trades

security/

risk factor 2

security/

risk factor 1

n_1

n_2

n_3

n_4
Simple example crowded trades

1.

\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N' \Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]
Simple example crowded trades

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\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N' \Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]

2.

\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \\ \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \end{pmatrix} \]
Simple example crowded trades

1. 

\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N' \Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]

2. 

\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \\ \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \end{pmatrix} \]

3. 

\[ E(A) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2 \sqrt{\frac{\pi - 2}{\pi}} \approx 1.21 \]
Histogram aggregate exposure for four members (N=4)
A crowded-trade risk thermometer?

Is there a natural “thermometer” for crowded-trade risk?

---

1 A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
A crowded-trade risk thermometer?

Is there a natural “thermometer” for crowded-trade risk?

Definition

CrowdIx for $\Sigma$ is defined as

$$CrowdIx = \frac{\text{std}(A)}{\text{std}(\tilde{A})}$$

where $\tilde{A}$ is CCP aggregate exposure when all members’ trades are re-allocated to a single risk factor to the maximum extent possible.\(^1\)

\(^1\)A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
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where $\tilde{A}$ is CCP aggregate exposure when all members’ trades are re-allocated to a single risk factor to the maximum extent possible. ¹

Lemma

$$\text{Crowdlx} \geq \sqrt{\frac{1}{\tilde{J}/2}} \quad \text{where} \quad \tilde{J} = 2 \lfloor J/2 \rfloor J$$

¹A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
A crowded-trade risk thermometer?

1. CrowdIx in the simple example is
   \[ \begin{cases} \sqrt{1/2} = 0.71 \quad \text{in the noncrowded case.} \\ 1 \quad \text{in the crowded case.} \end{cases} \]
An alternative margin methodology?

Prelude: Standard (member by member) margin methodologies base margins on the tail risk in a trade portfolio.

1. A standard tail risk measure is value-at-risk (VaR).
2. VaR is often calculated by the “delta-normal method” (Jorion, 2007, p. 260).
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Let $\text{Margin}(A)$ be the total margin a CCP should collect to protect against tail risk:

$$\text{Margin}(A) := E(A) + \alpha \text{std}(A).$$
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Definition

Let Margin(A) be the total margin a CCP should collect to protect against tail risk:

\[
Margin(A) := E(A) + \alpha \text{std}(A).
\]

Claim: Margin(A) is the “aggregate” approach extrapolated from existing member by member approaches.
An alternative margin methodology?

1. Homogeneity of degree one of mean($A$) and std($A$) implies that Margin($A$) naturally decomposes across members (Euler’s homogeneous function theorem).
   
   1.1
   
   $\text{mean}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$
   
   1.2
   
   $\text{std}(A) = \sum_k \sigma_k \frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl})$
An alternative margin methodology?

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$$\text{mean}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$$

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$$\text{std}(A) = \sum_k \sigma_k \frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl})$$

2. Therefore,

$$\text{Margin}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j \quad \text{Member-specific part (old)}$$

$$\quad + \quad \alpha \sigma_j \sum_l \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl}) \quad \text{Crowded-trade part (new)}$$
An alternative margin methodology?

1. To identify risk factor(s) on which members’ trades crowd, the following results are useful:

1.1

$$
\frac{\partial}{\partial \sigma_f} \mathbb{E}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \frac{\sigma_f}{\sigma_j} B_{jj}
$$

1.2

$$
\frac{\partial}{\partial \sigma_f} \text{std}(A) = \left( \frac{\pi - 1}{4\pi} \right) \frac{\sigma_f}{\sigma^A} \sum_{k,l} \left[ M'(\rho_{kl}) B_{kl} + \rho_{kl}^2 \left( 1 - 2\sqrt{1 - \rho_{kl}^2} \right) \left( \frac{\sigma_l}{\sigma_k} B_{kk} + \frac{\sigma_k}{\sigma_l} B_{ll} \right) \right]
$$

with

$$
B_{kl} := n_k' \beta \beta' n_l \quad \text{and} \quad \beta = \text{cov}(R, r^f)/\text{var}(r^f)
$$
An alternative margin methodology?

1. The sensitivity of $\text{Margin}(A)$ to a particular risk factor is naturally described by the following elasticity:

$$e^\text{Margin(A)}_{\sigma_f} = \frac{\sigma_f}{\text{Margin}(A)} \left( \frac{\partial}{\partial \sigma_f} E(A) + \alpha \frac{\partial}{\partial \sigma_f} \text{std}(A) \right).$$
Outline

Motivation

Objective

Measure+Allocation

Illustration

Conclusion

Appendix

References
Data

1. A European Multilateral Clearing Facility (EMCF) sample of “trade reports” filed by its (anonymized) members.
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2. It contains all trades in stocks listed in Denmark, Finland, and Sweden.
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2. It contains all trades in stocks listed in Denmark, Finland, and Sweden.


4. It spans almost all exchanges: NASDAQ-OMX, Chi-X, Bats, Burgundy, and Quote MTF (Turquoise not included).

5. Sample consists of 1.4 million trades by 57 clearing members in 242 securities across 228 days.
### Clearing members

<table>
<thead>
<tr>
<th>Clearing members</th>
<th>Source: Zhu (2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABN AMRO Clearing Bank N.V.</td>
<td>Numis Securities Ltd</td>
</tr>
<tr>
<td>BNP Paribas Securities Services S.A.</td>
<td>UBS Ltd</td>
</tr>
<tr>
<td>Bank of America Merrill Lynch</td>
<td>Barclays Capital Securities Ltd.</td>
</tr>
<tr>
<td>Citibank Global Markets and Citibank International</td>
<td>Alandsbanken Abp</td>
</tr>
<tr>
<td>JPMorgan Securities Ltd.</td>
<td>Alandsbanken Sverige AB</td>
</tr>
<tr>
<td>Goldman Sachs International</td>
<td>Amagarbanken A/S</td>
</tr>
<tr>
<td>Skandinaviska Enskilda Banken</td>
<td>Arbejdernes Landsbank A/S</td>
</tr>
<tr>
<td>KAS BANK N.V.</td>
<td>Avanza Bank AB</td>
</tr>
<tr>
<td>Parel S.A.</td>
<td>Carnegie Bank A/S</td>
</tr>
<tr>
<td>Deutsche Bank AG</td>
<td>Dexia Securities France</td>
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<tr>
<td>Citigroup</td>
<td>E-Trade Bank</td>
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<td>Eik Bank A/S</td>
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<td>ABG Sundal Coller Norge</td>
<td>FIM Bank Ltd.</td>
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<tr>
<td>DnB NOR Bank</td>
<td>GETCO Ltd.</td>
</tr>
<tr>
<td>Deutsche Bank (London Branch)</td>
<td>Handelsbanken</td>
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<tr>
<td>HSBC Trinkaus &amp; Burkhardt</td>
<td>Jefferies International Ltd.</td>
</tr>
<tr>
<td>Istituto Centrale delle Banche Popolari Italiane SpA</td>
<td>Knight Capital Markets</td>
</tr>
<tr>
<td>Interactive Brokers</td>
<td>Lan &amp; Spar Bank A/S</td>
</tr>
<tr>
<td>KBC Bank N.V.</td>
<td>Nordnet Bank AB</td>
</tr>
<tr>
<td>Nordea</td>
<td>Nomura International Plc</td>
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<td>Swedbank</td>
<td>Nykredit A/S</td>
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<tr>
<td>Credit Agricole Cheuvreux</td>
<td>Pohjola Bank</td>
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<tr>
<td>Credit Suisse Securities (europe) Ltd</td>
<td>RBC Capital Markets</td>
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<tr>
<td>Morgan Stanley International Plc</td>
<td>Saxo Bank A/S</td>
</tr>
<tr>
<td>RBS Bank N.V.</td>
<td>Spar Nord Bank A/S</td>
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<tr>
<td>Instinet europe Ltd.</td>
<td>Sparekassen Kronjylland A/S</td>
</tr>
<tr>
<td>Morgan Stanley Securities Ltd.</td>
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References:
## Summary statistics

<table>
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<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
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<tr>
<td><strong>Panel A: Overall summary statistics</strong></td>
<td></td>
<td></td>
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<td>Daily number of reports</td>
<td>6,293.6</td>
<td>699.0</td>
<td>1,135.0</td>
<td>6,426.5</td>
<td>7,663.0</td>
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<tr>
<td>Daily volume (in mln shares)</td>
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<td>42.1</td>
<td>8.1</td>
<td>155.5</td>
<td>342.4</td>
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<td>Daily volume (in mln euro)</td>
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<td>475.1</td>
<td>272.4</td>
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<td>Volume per report (in 1000 shares)</td>
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<td>114.1</td>
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<td>18,631.8</td>
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<tr>
<td>Volume per report (in 1000 euro)</td>
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<td>142,271.3</td>
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<td><strong>Panel B: Cross-sectional summary statistics, based on clearing-member averages</strong></td>
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<td>Daily number of reports</td>
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<td>Daily volume (in mln shares)</td>
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<td>4.2</td>
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<td>20.8</td>
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<td>Daily volume (in mln euro)</td>
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<td>46.9</td>
<td>0.0</td>
<td>7.8</td>
<td>222.4</td>
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<td><strong>Panel C: Cross-sectional summary statistics, based on stock averages</strong></td>
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<td>Daily number of reports</td>
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<td>Daily volume (in mln shares)</td>
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<td>Daily volume (in mln euro)</td>
<td>7.5</td>
<td>14.6</td>
<td>0.0</td>
<td>0.9</td>
<td>124.0</td>
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</table>
Aggregate daily margin: actual margin and Margin\(_\text{(A)}\)

![Graph showing aggregate daily margin with actual margin and Margin\(_\text{(A)}\).]

- **Apr 22:** Nokia publishes Q1 results
- **May 2:** Eurozone and IMF agree to bailout Greece
Aggregate daily margin: actual margin and Margin(A)

- Apr 22: Nokia publishes Q1 results
- May 2: Eurozone and IMF agree to bailout Greece

Graph showing the trend of Margin(A) and CrowdIx (right) from April 21 to May 12, 2010.
Aggregate exposure distribution ‘Greek Bailout’

- **Greek bailout**
- **Median CrowdIx day benchmark**
- **Min CrowdIx day benchmark**
# Aggregate exposure distribution ‘Greek Bailout’

<table>
<thead>
<tr>
<th>Date</th>
<th>CrowdIx</th>
<th>Q(0.90) (million euro)</th>
<th>Q(0.99) (million euro)</th>
<th>Q(0.999) (million euro)</th>
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<tbody>
<tr>
<td>Greek bailout</td>
<td>May 10, 2010</td>
<td>0.62</td>
<td>267</td>
<td>405</td>
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<tr>
<td>Median CrowdIx day benchmark</td>
<td>Jul 29, 2010</td>
<td>0.46</td>
<td>196</td>
<td>289</td>
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<tr>
<td>Min CrowdIx day benchmark</td>
<td>Nov 12, 2009</td>
<td>0.31</td>
<td>175</td>
<td>233</td>
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Actual margin versus Margin(A)

CrowdIx = 0.46

Member margin actually posted (million euro)
Margin(A), model-implied member margin (million euro)
## Actual margin versus Margin(A)

This figure contains three scatterplots of the margin that members actually posted versus the model-implied margin, Margin(A). The plots correspond to three days in the sample: the median-CrowdIx day and the two days for which the CCP charged highest aggregate margin. The exhibit below the scatterplots contains the ten largest positions in the trade portfolio of a member in the top-left corner and a member in the bottom-right corner.

### Clearing member 41

<table>
<thead>
<tr>
<th>Stock</th>
<th>NetPos (mln €)</th>
<th>AbsNetPos (mln €)</th>
<th>AbsNetPos (%)</th>
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</thead>
<tbody>
<tr>
<td>ER</td>
<td>23.1</td>
<td>23.1</td>
<td>13.7</td>
</tr>
<tr>
<td>SHBA</td>
<td>14.5</td>
<td>14.5</td>
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<tr>
<td>NOVNB</td>
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<td>ASSAB</td>
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### Clearing member 6

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Actual margin versus Margin(A)

CrowdIx = 0.62
### Actual margin versus Margin(A)

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</thead>
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<td>TLS1V</td>
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<td>3.9</td>
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<tr>
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<td>3.8</td>
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<th>Stock</th>
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<th>AbsNetPos (%)</th>
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<td>SAND</td>
<td>13.8</td>
<td>13.8</td>
<td>3.3</td>
</tr>
</tbody>
</table>
Actual margin versus Margin(A)

Crowdlx = 0.72

Member margin actually posted (million euro)

Margin(A), model-implied member margin (million euro)
## Actual margin versus Margin(A)

### Clearing member 41

<table>
<thead>
<tr>
<th>Stock</th>
<th>NetPos (mln €)</th>
<th>AbsNetPos (mln €)</th>
<th>AbsNetPos (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOKI</td>
<td>-84.7</td>
<td>84.7</td>
<td>20.7</td>
</tr>
<tr>
<td>ER</td>
<td>64.8</td>
<td>64.8</td>
<td>15.8</td>
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<tr>
<td>FUM1V</td>
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<td>9.6</td>
</tr>
<tr>
<td>NDA1V</td>
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<td>VOLB</td>
<td>16.2</td>
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<tr>
<td>HMB</td>
<td>15.5</td>
<td>15.5</td>
<td>3.8</td>
</tr>
<tr>
<td>STERV</td>
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<tr>
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<td>9.8</td>
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<td>2.4</td>
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<tr>
<td>OUT1V</td>
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<td>2.2</td>
</tr>
<tr>
<td>SEN</td>
<td>-8.3</td>
<td>8.3</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### Clearing member 12

<table>
<thead>
<tr>
<th>Stock</th>
<th>NetPos (mln €)</th>
<th>AbsNetPos (mln €)</th>
<th>AbsNetPos (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOLB</td>
<td>35.7</td>
<td>35.7</td>
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<td>TLS1V</td>
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<tr>
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<tr>
<td>ABBN</td>
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<td>VWS</td>
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<td>9.2</td>
<td>3.2</td>
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<td>TEL2B</td>
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<td>BOLI</td>
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<td>2.2</td>
</tr>
</tbody>
</table>
# Margin(A) sensitivity

<table>
<thead>
<tr>
<th>Date</th>
<th>CrowdIx</th>
<th>Risk factor</th>
<th>Margin(A) (million euro)</th>
<th>$\Delta$Margin(A) on $\Delta \sigma_f=0.01$ (million euro)</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median CrowdIx day</td>
<td>Jul 29, 2010</td>
<td>0.46</td>
<td>Market 128</td>
<td>81</td>
<td>0.91</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Nokia 128</td>
<td>11</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Telecom 128</td>
<td>46</td>
<td>0.46</td>
</tr>
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<td>Greek bailout</td>
<td>May 10, 2010</td>
<td>0.62</td>
<td>Market 747</td>
<td>307</td>
<td>0.98</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Nokia 747</td>
<td>27</td>
<td>0.14</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Telecom 747</td>
<td>298</td>
<td>0.83</td>
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<tr>
<td>Nokia reports Q1</td>
<td>Apr 26, 2010</td>
<td>0.72</td>
<td>Market 644</td>
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<td>0.19</td>
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<td></td>
<td>Telecom 644</td>
<td>-2</td>
<td>-0.00</td>
</tr>
</tbody>
</table>
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4. The implementation on real data shows that it matters, in particular when the market gets turbulent.
Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU University Amsterdam, Tinbergen Institute, Duisenberg school of finance

July 3, 2014
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Appendix A: Max crowding benchmark, \( \tilde{A} \)

1. If all members would trade the same risk factor, then \( \exists n \in \mathbb{R}^I \) s.t. \( \forall j: \)
   \[
   X_j = \nu_j \times (n'X), \quad \nu_j \in \mathbb{R}.
   \]

2. Then,
   \[
   \Sigma = n'\Omega n \times (\nu_j \nu_j').
   \]

3. Without loss of generality, let \( n'\Omega n = 1 \).

4. For member by member portfolio risks to remain unchanged, one needs \( \forall j: \)
   \[
   \nu_j^2 = \sigma_j^2 \quad \Rightarrow \quad \nu_j = \pm \sqrt{\sigma_j^2}. \tag{1}
   \]

5. In addition, the aggregate (signed) trade is zero:
   \[
   \sum_j \nu_j = 0. \tag{2}
   \]
Appendix A: Max crowding benchmark, $\tilde{A}$

1. The member trade reallocation that yields the maximum crowding benchmark is

$$\arg\max_{\{\nu_1, \nu_2, \ldots, \nu_J\}} \min \left( \sum_j \nu_j^+, \sum_j \nu_j^- \right) \text{ subject to (1),} \quad (3)$$

where

$$\nu_j^+ := \max(\nu_j, 0) \text{ and } \nu_j^- := \max(-\nu_j, 0).$$

2. If $\sum_j \nu_j^+ = \sum_j \nu_j^+$ then trade reallocation is perfect. No portfolio risk is left unallocated.
Appendix A: Max crowding benchmark, \( \tilde{A} \)

1. The trade reallocation is a combinatorial problem that is NP hard.

2. It maps into a one-dimensional bin packing problem (Coffman, Garey, and Johnson, 1996). Can all items be packed into two bins of size \((1/2) \sum_j \sigma_j\)? If not, how much can be packed into two such bins? The minimum of the two bins can be matched, i.e., buyers buy this amount from sellers.

3. First fit descending (FFD) algorithm solves the offline bin packing problem in \(O(J \log J)\) time (brute force requires \(2^J\)).

4. Why FFD instead of alternative approaches?
   4.1 Average-case analysis: If item size is drawn from \(U[0, 1/2]\) for one-unit bins then Coffman, Garey, and Johnson (1996, p. 39) claim “FFD is typically optimal.”
   
   4.2 Worst-case analysis: If all items are smaller than 1/2 then FFD does as well its closest contender MFFD (modified first fit descending) (Coffman, Garey, and Johnson, 1996, p. 16-19).
Appendix B: Q&A

1. **Is it reasonable to assume equity returns are normal?** In the implementation the return distribution is assumed to be *conditionally* normal. Time-varying volatility is accounted for by calculating the covariance matrix as an exponentially weighted average of the outer product of historical daily returns.\(^2\)

2. **Why not divide total risk by the number of members and use that as base for a crowding index?** Such alternative approach does not recognize that individual member portfolio risk is accounted for in existing member by member approaches. Consider the case that only member one and member two engaged in a trade. Dividing total portfolio risk by \( J > 2 \) suggests that there were crowded trades not accounted for. This is not the case here.

---

\(^2\)EWMA(0.94) which is the RiskMetrics standard for daily equity returns.


Hedegaard, Esben. 2012. “How Margins are Set and Affect Prices.” Manuscript, NYU.


