TANK Models with Amplification and no Puzzles: the Magic of Output Stabilization and Capital

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The era of HANK has arrived

- For decades, the monetary policy was done with RANK.
- Now, HANK incorporate heterogeneity and study the role of monetary policy in redistributinal effects (interactions with fiscal policy);
- *In general, HANK models* are of Krusell-Smith type:
  - distributions affect aggregate dynamics;
  - numerical solutions; e.g., Ahn et al. (2018), Bayer and Luetticke (2019)
- But numerical solutions are non-trivial to compute and they are not always conclusive.
- In this paper, we show novel closed-form solutions for empirically relevant HANK.
- *We obtain sharp predictions about both aggregate and distributive implications of HANK.*
Analytical HANK in the literature

- **TANK** *(Bilbiie, 2008, 2019a)*
  
  _Assumptions:_
  - limited asset market participation;
  - liquid and illiquid assets with zero liquidity limits;
  - redistribution of profits.
  
  _Predictions:_
  - impossible to get both a monetary policy amplification and resolve forward guidance puzzle (Catch-22)

- **PRANK** *(Acharya and Dogra, 2018)*
  
  _Assumptions:_
  - CARA utility ⇒ no need to keep track of wealth distribution;
  - uninsurable idiosyncratic uncertainty ⇒ HANK.
  
  _Predictions:_
  - not quantitatively large differences from RANK;
  - can aggravate the forward guidance puzzle.
Analytical HANK in the literature (cont.)

- **THANK** (Bilbiie, 2019b, Bilbiie and Ragot, 2017)
  
  **Assumptions:**
  - uninsurable idiosyncratic uncertainty;
  - full insurance within type but limited insurance across types;
  - exogenous switches between types ⇒ TANK.
  
  **Predictions:**
  "the Catch-22" is resolved by adding procyclical risk to model with countercyclical income inequality.

Broad conclusion of the analytical HANK literature:

- Need both:
  
  1. Limited asset market participation. $\Rightarrow$ Amplification.
  2. Uninsurable idiosyncratic uncertainty. $\Rightarrow$ Discounted effects of future policy. $\Rightarrow$ No forward guidance puzzle.

- We revisit the findings of the literature.
This paper: magic of output stabilization and capital

• We show that idiosyncratic uncertainty is not the only way to solve the forward-guidance puzzle.

• We consider two TANK models (⇒ no idiosyncratic risk):
  – TANK with no capital.
  – TANK with capital.

• In the no-capital TANK, we show that
  – forward-guidance puzzle occurs under very special (empirically implausible or socially suboptimal) policy rules.
  – empirically relevant Taylor rules do not lead to forward-guidance puzzle.
  – output stabilization in the Taylor rule is the key!

• In the TANK with capital, we show that:
  – forward guidance puzzle is resolved even in the absence of output stabilization
  – capital itself is the key and is sufficient for solving forward guidance puzzle!
This paper: distributive predictions are also magic

- In HANK, we also need to produce meaningful distributive implications.
- The deterministic no-capital TANK model predicts
  - consumption of constrained agents increases with aggregate income.
- The stochastic no-capital TANK model predicts
  - comovement between consumption inequality and technology is nearly zero (inequality follows markups);
  - consumption and income inequality move one-for-one.
- We introduce capital into TANK with shocks to account for business cycles and show
  - low adjustment costs generate countercyclical consumption inequality;
  - movement between consumption and income inequality will not be one-for-one.
How do we get these magic results?

- Maliar and Taylor (2018) derive closed-form solutions to RANK.
- In this paper, we construct parallel closed-form solutions to HANK.
  - solutions to homogeneous equations are well known;
  - our main contribution are solutions to non-homogeneous equations;
- We distinguish not only determinacy versus indeterminacy regions but also real versus complex-root regions.
- We show that the size of the smallest root (or eigenvalue) captures model dynamics better than the underlying parameters.
- We use closed-form solutions to characterize both the aggregate and distributional predictions of HANK.
- We discuss the implications of our analysis to the case of liquidity trap.
Consumer side

- Constrained ("Keynesian") and unconstrained agents.
- \( \lambda \) = share of constrained agents.
- Unconstrained agent:

\[
\max_{\{C_t^U, N_t^U, B_t^U, F_t^U\}} E_0 \sum_{t=0}^{\infty} \beta^t Z_t \left[ \frac{(C_t^U)^{1-\sigma} - 1}{1 - \sigma} - \frac{(N_t^U)^{1+\varphi} + 1}{1 + \varphi} \right]
\]

\[
C_t^U + \frac{B_t^U}{P_t} + Q_t F_t^U = \frac{B_{t-1}^U (1 + i_{t-1})}{P_t} + \frac{W_t}{P_t} N_t^U + (Q_t + (1 - \delta) D_t) F_{t-1}^U + T_t^U,
\]

\( Z_t \) = preference shock following an AR(1) process;
\( F_t^U \) = shares; \( T_t^U \) = government transfers;
\( D_t \) = dividends from ownership of firms;
\( \delta \in [0, 1] \) = share of illiquid profits allocated across agents.

- Constrained agents consume labor income and get government transfers \( T_t^K \).
Supply side

- Wage is a constant share of the ratio of the marginal utility of consumption to the marginal utility of leisure
- Firms: $Y_t (i) = A_t N_t (i)$, with $A_t = \text{productivity level following an AR}(1)$ process
- Rotemberg pricing
- Phillips curve

$$\Pi_t (\Pi_{t-1}) = E_t \left\{ \Lambda^U_{t,t+1} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} (\Pi_{t+1}-1) + \frac{\varepsilon}{\xi} \left( \frac{1}{M^p_t} - \frac{1}{M^p} \right) \right\}$$

$\Pi_t = \text{gross price inflation}; \quad \Lambda^U_{t,t+1} = \text{stochastic discount factor};$

$M^p_t = \frac{A_t}{W_t} = \text{average gross markup}; \quad M^p = \frac{\varepsilon}{\varepsilon-1}.$
Fiscal policy

- As in Debortoli and Gali (2018).
- Government re-distributes the illiquid profits of the firms $\delta D_t$, so that $(1 - \lambda) T^U_t + \lambda T^K_t = \delta D_t$.
- Redistribution rules:

  $$T^U_t = \left(1 + \frac{\tau \lambda}{1 - \lambda}\right) \delta D_t,$$
  $$T^K_t = (1 - \tau) \delta D_t,$$

  $1 - \tau$ = share of profits distributed to the constrained agents.
- $\tau = 0$ uniform-based & $\tau = 1$ wealth-based redistribution rules.
- $\{\lambda, \tau, \delta\}$ = redistribution (heterogeneity) parameters.
Linearized no capital TANK model

- Phillips curve and IS equations are:

\[
\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t, \\
\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \frac{1}{\sigma(1 - \Phi)} \left( \hat{i}_t - E_t[\pi_{t+1}] - \hat{r}_t^n \right),
\]

where \( \Phi \) represents a function of heterogeneity and other parameters

\[
\Phi = f \left( \lambda, \delta, \tau, \sigma, \varepsilon, \gamma, \varphi \right)
\]

- \( \lambda = \) share of constrained agents; \( \delta = \) share of allocated illiquid profits; \( \tau = \) related to share of profits of unconstrained.
- \( \gamma = \) steady-state consumption gap \((\gamma_t \equiv 1 - \frac{C_t^K}{C_t^U})\).
- \( \tilde{y}_t = \) output gap; \( \pi_t = \) inflation; \( \hat{i}_t = \) nominal interest rate; \( \hat{r}_t^n = \) natural rate of interest; \( \kappa = \) slope of the Phillips curve.
- \( \Phi = 0 \) when \( \lambda = 0 \) \( \Rightarrow \) RANK.
Linearized no capital TANK model (cont.)

- We study stylized Taylor rule with output stabilization:
  \[
  \hat{i}_t = i_t^* + \phi_\pi \pi_t + \phi_{E\pi} E_t \pi_{t+1} + \phi_y \hat{y}_t + \nu_t,
  \]
  output stabilization

  \(i_t^*\) = desired interest rate; \(\phi_\pi, \phi_{E\pi}\) and \(\phi_y\) = constant coefficients; \(\nu_t\) = disturbance.

- The previous analytical TANK literature uses "reduced" Taylor rules.

  \[\rightarrow\] Bilbiie (2008), Bilbiie (2019a), McKay et al. (2015):
  \[
  \hat{i}_t = i_t^* + E_t \pi_{t+1} + \nu_t.
  \]

  \[\rightarrow\] Bilbiie (2017), Acharya and Dogra (2018):
  \[
  \hat{i}_t = i_t^* + \phi_\pi \pi_t + \nu_t.
  \]

  We show that switching from "reduced" to more general Taylor rules helps to solve the forward guidance puzzle.
Second-order difference equation

- Combining the Taylor rule, IS and Phillips curves:

\[ E_t [\pi_{t+2}] + b \pi_{t+1} + c \pi_t = -X_t, \]

where

\[ b \equiv -1 - \frac{1}{\beta} + \frac{1}{\beta \sigma (1 - \Phi)} \left( \phi E \pi \kappa - \beta \phi y - \kappa \right); \]

\[ c \equiv \frac{1}{\beta} + \frac{\kappa \phi \pi + \phi y}{\beta \sigma (1 - \Phi)}; \]

\[ X_t \equiv -\frac{\kappa}{\beta \sigma (1 - \Phi)} (\hat{r}_t^n - u_t); \]

\[ u_t \equiv \phi y \frac{1 + \varphi}{\sigma + \varphi} a_t + i_t^* + v_t; \]

\[ \hat{r}_t^n = (1 - \rho_z) z_t - \sigma (1 - \rho_a) \frac{1 + \varphi}{\sigma + \varphi} a_t. \]
Roots to characteristic equation

- The roots to the characteristic equation $m^2 + bm + c = 0$ are given by $m_{1,2} = \frac{-b\pm\sqrt{b^2-4c}}{2}$.
- Three solution cases:
  1. If both roots are real and distinct, $m_1 \neq m_2$;
  2. If both roots are real and the same, $m_1 = m_2 = m$;
  3. If both roots are complex, $m_{1,2} = \mu \pm \eta \nu$. 
Homogenous and non-homogenous equations

- **Non-homogeneous** 2d order difference equation \((z_t \neq 0)\) with constant coefficients:

\[
E_t [\pi_{t+2}] + b\pi_{t+1} + c\pi_t = -X_t. \tag{1}
\]

- Its **homogeneous equation** is

\[
\pi_{t+2} + b\pi_{t+1} + c\pi_t = 0. \tag{2}
\]

- A solution of \((1) = A\) general solution to \((2) + a\) particular solution to \((1)\).

- Homogeneous equations of type \((2)\) are well studied in the field of differential equations.
  - Their solutions will contain integration constants \(C_1\) and \(C_2\).

- No general approach delivering particular solutions to the studied non-homogenous equations.
  - "Guess and verify" method.

- **Our contribution is to show closed form solutions for both deterministic and stochastic settings.**
Theorem 1: closed-form solutions

We focus on forward stable equilibria – conventional equilibria not exploding in the future. Consider solutions for the deterministic case.

1. Two distinct real roots $m_1 \neq m_2$:
   
   i) **Indeterminate** with $C_1 = 0$, $C_2$ any, if $|m_1| > 1$ and $|m_2| < 1$.

   \[
   \pi_t = C_1 m_1^t + C_2 m_2^t + \frac{1}{m_1 - m_2} E_t \left[ \sum_{s=t}^{\infty} m_1^{t-1-s} X_s + \sum_{s=-\infty}^{t-1} m_2^{t-1-s} X_s \right]
   \]

   general solution

   particular solution

   ii) **Unique** with $C_1 = 0$, $C_2 = 0$, if $|m_1| \geq 1$ and $|m_2| \geq 1$.

   \[
   \pi_t = C_1 m_1^t + C_2 m_2^t + \frac{1}{m_1 - m_2} E_t \left[ \sum_{s=t}^{\infty} m_1^{t-1-s} X_s - \sum_{s=t}^{\infty} m_2^{t-1-s} X_s \right]
   \]
Theorem 1: closed-form solutions

2. Two repeated real roots $m_1 = m_2 = m$:
   iii) **Unique** with $C_1 = 0, C_2 = 0$, if $|m| > 1$.

\[
\pi_t = (C_1 + C_2 t) m^t + \frac{1}{m} E_t \left[ (t - 1) \sum_{s=t}^{\infty} m^{t-1-s} X_s - \sum_{s=t}^{\infty} s m^{t-1-s} X_s \right]
\]

3. Complex roots $m_{1,2} = \mu \pm \eta \iota$ with $r \equiv \sqrt{\mu^2 + \eta^2} > 1$:
   iv) **Unique** with $C_1 = 0, C_2 = 0$.

\[
\pi_t = C_1 r^t \cos (\theta t) \\
+ C_2 r^t \sin (\theta t) + \frac{1}{\eta} E_t \left[ \sum_{s=t}^{\infty} r^{t-1-s} \sin (\theta (t - 1 - s)) X_s \right],
\]

where $\theta = \arctan \left( \frac{\eta}{\mu} \right)$.

Complex roots are not exotic but the most relevant case, e.g., the standard Taylor rule with $\phi_\pi = 1.5, \phi_y = 0.5$!
## Theorem 2: boundaries on the parameters

Taylor rule with output gap, expected and actual inflation

\[ \hat{i}_t = i^*_t + \phi_\pi \pi_t + \phi_{E\pi} E_t \pi_{t+1} + \phi_y \hat{y}_t + \nu, \]

Restrictions on the model’s parameters:

<table>
<thead>
<tr>
<th>i) 2 distinct real / indeterm.</th>
<th>[ \phi_{E\pi} &lt; \phi_{E\pi}^1 \quad &amp; \quad \phi_{E\pi} &gt; \phi_{E\pi}^4 ]</th>
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\[ \phi_{E\pi}^1 \equiv \frac{\phi_y}{\kappa} (\beta - 1) + 1 - \phi_\pi; \]
\[ \phi_{E\pi}^2 \equiv \sigma (1 - \Phi) \left[ \frac{\beta + 1}{\kappa} - 2 \frac{\beta}{\kappa} \sqrt{\frac{1}{\beta} + \frac{\kappa}{\beta} \frac{1}{\sigma (1 - \Phi)} \left( \phi_\pi + \frac{\phi_y}{\kappa} \right)} \right] + \frac{\beta}{\kappa} \phi_y + 1; \]
\[ \phi_{E\pi}^3 \equiv \sigma (1 - \Phi) \left[ \frac{\beta + 1}{\kappa} + 2 \frac{\beta}{\kappa} \sqrt{\frac{1}{\beta} + \frac{\kappa}{\beta} \frac{1}{\sigma (1 - \Phi)} \left( \phi_\pi + \frac{\phi_y}{\kappa} \right)} \right] + \frac{\beta}{\kappa} \phi_y + 1; \]
\[ \phi_{E\pi}^4 \equiv 2 \frac{\beta}{\kappa} \sigma (1 - \Phi) + 2 \frac{1}{\kappa} \sigma (1 - \Phi) + \phi_\pi + \left( \frac{1 + \beta}{\kappa} \right) \phi_y + 1. \]

**Corrolary:** Upper indeterminacy bound depends on heterogeneity!
Theorem 3: boundaries on the parameters

Taylor rule with output gap and actual inflation

\[ \hat{i}_t = i^*_t + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t, \]

Restrictions on the model’s parameters:

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<td>( \phi_\pi &gt; \phi_\pi^2 )</td>
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\[ \phi_\pi^1 \equiv \frac{\phi_y}{\kappa} (\beta - 1)+1, \]
\[ \phi_\pi^2 \equiv \frac{1}{4} \frac{\beta}{\kappa} \sigma (1-\Phi) \left(-1 - \frac{1}{\beta} - \frac{\phi_y}{\sigma(1-\Phi)} - \frac{\kappa}{\beta \sigma(1-\Phi)}\right)^2 - \frac{\sigma(1-\Phi)}{\kappa} - \frac{1}{\kappa} \phi_y. \]

Corollary: \textit{Indeterminacy regions do not depend on heterogeneity parameters!}
Impulse responses for one-time anticipated shock

The literature often study one-time anticipated shock.

- Assume $v_t = 0$ for all $t$ except $T$ in which the economy faces a shock $v_T = v > 0$.
- Solution for case i) – two distinct real roots $|m_1| \geq 1$ and $|m_2| < 1$:

$$
\pi_t = C_1 m_1^t + C_2 m_2^t + \frac{1}{m_1 - m_2} E_t \left[ \sum_{s=t}^{\infty} m X_s + \sum_{s=-\infty}^{t-1} m_2^{t-1-s} X_s \right].
$$

- Set $C_1 = 0$ (forward-stable equilibrium).
- Recall $X_t \equiv -\frac{\kappa}{\beta \sigma (1-\Phi)} \left( \hat{r}_t^n - \phi_y \frac{1+\varphi}{\sigma + \varphi} a_t - i_t^* - v_t \right)$.
- Let $a_t = 0$, $z_t = 0$, and $i_t^* = \hat{r}_t^n$ $\Rightarrow$ $X_T = \frac{\kappa v}{\beta \sigma (1-\Phi)}$.
- For $t \leq T$, $E_t \left[ \sum_{s=t}^{\infty} m_1^{t-1-s} X_s \right] = m_1^{t-1-T} \frac{\kappa v}{\beta \sigma (1-\Phi)}$ and

$$
\sum_{s=-\infty}^{t-1} m_2^{t-1-s} X_s = 0.
$$

- For $t > T$, $E_t \left[ \sum_{s=t}^{\infty} m_1^{t-1-s} X_s \right] = 0$ and

$$
\sum_{s=-\infty}^{t-1} m_2^{t-1-s} X_s = m_2^{t-1-T} \frac{\kappa v}{\beta \sigma (1-\Phi)}.
$$
Impulse responses for one-time anticipated shock (cont.)

- Two distinct real roots $|m_1| \geq 1$ and $|m_2| < 1$:

  $$
  t \leq T, \pi_t = C_2 m_2^t + \frac{\kappa \nu}{\beta \sigma (1 - \Phi) (m_1 - m_2)} m_1^{t-1-T};
  $$
  $$
  t > T, \pi_t = C_2 m_2^t + \frac{\kappa \nu}{\beta \sigma (1 - \Phi) (m_1 - m_2)} m_2^{t-1-T}.
  $$

- Two identical roots $|m_1| = |m_2| = |m| > 1$:

  $$
  t \leq T, \pi_t = \frac{\kappa \nu}{\beta \sigma (1 - \Phi)m} \left[ (t - 1)m^{t-1-T} - Tm^{t-1-T} \right],
  $$
  $$
  t > T, \pi_t = 0.
  $$

- Complex roots $|r| > 1$:

  $$
  t \leq T, \pi_t = \frac{\kappa \nu}{\beta \sigma (1 - \Phi) \eta} r^{t-1-T} \sin \left( \theta (t - 1 - T) \right),
  $$
  $$
  t > T, \pi_t = 0.
  $$
Determinacy and share of constrained agents: stylized Taylor rule

- Full asset market participation ($\lambda = 0$):
  - Large regions of complex roots.
- More real roots as access is limited ($\lambda > 0$).
Determinacy and redistribution parameters: stylized Taylor rule

- **Too much concentration of profits** $\Rightarrow$ "Inverted Aggregate Demand":
  - $\Phi > 1$ $\Rightarrow$ IS curve slopes upward;
  - $\lambda \approx .40$ under baseline calibration.
- **Forward-looking Taylor rule**:
  - too hawkish central bank $\Rightarrow$ indeterminacy
  - $\lambda = .38$ (Kaplan et al., 2018) $\Rightarrow \phi_{E,\pi} = 3$ leads to indeterminacy.
Real versus complex roots: Does it matter?

- Real roots: output gap (roughly) follows natural rate.
- Complex roots: output gap decreases even as natural rate returns to steady state.

Unreasonable oscillations – another example of anomalous behavior of NK models.
Individual consumption responses

One time MP shock at $T$

$$\hat{c}_t^U = \frac{1}{\sigma} \left( 1 + \frac{(1-\sigma)\Phi}{\sigma(1-\Phi)} \right) \cdot \nu_T \cdot (-B_T),$$

$$\hat{c}_t^K = \chi \cdot \left[ \frac{1}{\sigma} \left( 1 + \frac{\Phi}{\sigma(1-\Phi)} \right) \right] \cdot \nu_T \cdot (-B_T),$$

$$\chi \equiv 1 + \frac{(1-\lambda)\Phi}{\lambda(1-\gamma)},$$

$B_T$ depends on the horizon of the shock,

$$\left( \frac{\phi_y + \phi_\pi}{\kappa} \right) \left( \frac{1}{m_1 - m_2} \right) \left( -\frac{\kappa}{\beta\sigma(1-\Phi)} \right) \sum_{k=0}^{T} \left( m_1^{-1-k} - m_2^{-1-k} \right) +$$

$$\left( \phi_{E\pi} - 1 - \frac{\beta\phi_y}{\kappa} \right) \left( \frac{1}{m_1 - m_2} \right) \left( -\frac{\kappa}{\beta\sigma(1-\Phi)} \right) \sum_{k=0}^{T} \left( m_1^{-k} - m_2^{-k} \right) + 1$$
"Reduced" Taylor rule, $\phi_{E_\pi} = 1$:

- $-B_T = 1,$

\[
\hat{c}_t^U = \frac{1}{\sigma} \left(1 + \frac{(1 - \sigma)\Phi}{\sigma(1 - \Phi)}\right) \cdot v_T,
\]

\[
\hat{c}_t^K = \chi \cdot \left[\frac{1}{\sigma} \left(1 + \frac{\Phi}{\sigma(1 - \Phi)}\right)\right] \cdot v_T = \chi \cdot \hat{c}_t,
\]

$\chi =$ elasticity of constrained income to aggregate income.

- Response does not depend on how far in the future the shock occurs.

- $\sigma = 1 \implies \hat{c}_t^U$ does not depend on $\Phi \Rightarrow$ Amplification due to presence of constrained, not changes by unconstrained.

**Note:** $\gamma < 1 \implies \chi > 1.$

- This case leads to amplification and the forward guidance puzzle (Bilbiie, 2019a).
Discounting with output stabilization

- In RANK, the forward guidance puzzle because of no discounting in the Euler equation (McKay et al., 2017):

$$\hat{c}_t = - \sum_{k=0}^{\infty} \frac{1}{\sigma} (i_{t+k}^* + \nu_{t+k}) - \left( \frac{1}{\sigma} \right) E_t [z_t].$$

- We show

$$\hat{\beta} = - \sum_{k=0}^{\infty} \beta^{k-1} \left( \frac{1}{\phi_y + \sigma (1 - \Phi)} \right) (i_{t+k}^* + \nu_{t+k})$$

$$- \left( \frac{1}{\phi_y + \sigma (1 - \Phi)} \right) E_t [z_t].$$

- $\tilde{\beta} = \frac{\sigma(1-\Phi)}{\phi_y + \sigma(1-\Phi)} \leq 1$ is a discounting term.
- When $\phi_y = 0 \implies$ no discounting.
- $\tilde{\beta}$ decreasing in $\Phi \implies \tilde{\beta} \downarrow$ when either $\lambda \uparrow$ or $\tau \uparrow$ or $\delta \downarrow$.
- One can achieve "amplification" + "no puzzle" under $\chi > 1$. 
Discounting with output stabilization (cont.)

\[ \hat{r}_t = E[\pi_{t+1}] + \phi_y \tilde{y}_t + \nu_t. \]

- No output stabilization \(\implies\) the forward guidance puzzle remains.
- Given \(\phi_y \neq 0\), as heterogeneity \(\lambda \uparrow\) \(\implies\) smaller the immediate effect of a future shock relative to the peak.
- \(\lambda \uparrow\) \(\implies\) smaller \(\phi_y\) needed for a sizable jump in initial output.
Stochastic solutions

- Technology shock

Real:  \[ \pi_t = \frac{\gamma_a}{m_1 - m_2} \left( \frac{a_t}{\rho a} \right) \left[ \sum_{s=t}^{\infty} \left( \frac{m_1}{\rho a} \right)^{t-1-s} - \sum_{s=t}^{\infty} \left( \frac{m_2}{\rho a} \right)^{t-1-s} \right] , \]

Complex:  \[ \pi_t = \gamma_a \left( \frac{a_t}{\eta \rho a} \right) \left[ \sum_{s=t}^{\infty} \left( \frac{r}{\rho a} \right)^{t-1-s} \sin \left( \theta (t - 1 - s) \right) \right] , \]

\[ \gamma_a = \frac{\kappa}{\beta \sigma (1 - \Phi)} \left( \frac{1 + \varphi}{\sigma + \varphi} \right) \left( \sigma (1 - \rho_a) + \phi_y \right) . \]

- Volatility of \( \pi_t \) depends on \( m_1 \) and \( m_2 \) in the real-root case (or \( r \) in the complex-root case).

- From difference equation  \[ E_t [\pi_{t+2}] + b \pi_{t+1} + c \pi_t = -X_t , \]

  \[ \frac{\partial b}{\partial \tau} = \frac{\left( \phi_E \pi \kappa - \beta \phi_y - \kappa \right)}{(1 - \Phi)^2} \frac{\partial \Phi}{\partial \tau} , \]

  \[ \frac{\partial c}{\partial \tau} = \frac{\left( \phi \pi \kappa + \phi_y \right)}{(1 - \Phi)^2} \frac{\partial \Phi}{\partial \tau} . \]

- As \( \tau \downarrow \rightarrow \) more redistribution \( \rightarrow \ldots \rightarrow \) Volatility of \( \pi_t \downarrow \).
Stochastic solutions (cont.)

- Consumption inequality and markups always move together:
  \[ \widehat{\gamma}_t = \Psi \widehat{\mu}_t^p, \]
  \[ \Psi > 0 \text{ and } \widehat{\mu}_t = \text{markup}. \]

- \textbf{Fact 1} (Kruger et al. (2009)):
  – consumption inequality tends to increases during recessions;
- Current model: \( a_t \uparrow \iff \widehat{\mu}_t \downarrow \) weakly
- \( \Rightarrow \) \textbf{Puzzle in the no-capital TANK model} \( \Rightarrow \) very little decrease in consumption (and income) inequality during expansions.
- \textbf{Fact 2} (Kruger et al. (2009)):
  – increase in consumption inequality is smaller than that of income inequality
- How to break the consumption-income linkage?
- \textbf{Solution}: introduce capital and adjustment costs.
TANK with capital

- Investment adjustment costs.
- Capital income tax $\tau^k$.
- Transfers of illiquid profits and capital taxes.
- Wage determined by unconstrained agent and symmetric hours.
- Consumption gap:

$$\hat{\gamma}_t = \Xi_1 \left( \frac{1}{\alpha} M^P \right) \cdot \hat{\mu}_t - \Xi_2 \frac{1}{\Gamma} \cdot \hat{q}_t + \Xi_2 \delta^k \cdot \hat{R}_t^K,$$

$\Xi_1$, $\Xi_2$ are constants depending on the model’s parameters; $\hat{q}_t =$ value of installed capital in terms of consumption; $\hat{R}_t^K =$ rate of return on capital.
- $\hat{q}_t$ and $\hat{R}_t^K$ work in opposite directions.
TANK with capital: forward guidance

Reduced Taylor rule, capital and adjustment costs $\implies$ no forward-guidance puzzle.

- Multiple assets $\implies$ Equalized returns across assets $\implies$ Contemporaneous feedback between asset returns and output gap.
- Single asset doesn’t allow for portfolio adjustment $\implies$ Output gap jumps.
TANK with capital: consumption inequality

- Cyclicality of consumption inequality depends on $\hat{\mu}_t^p$, $\hat{q}_t$, and $\hat{R}_t^K$.
- No capital $\implies \hat{\gamma}_t$ depends only on $\hat{\mu}_t$.
- Adjustment costs ($\Gamma$) matter for cyclicallity of consumption inequality.

Higher adjustment costs gives procyclical consumption inequality.
Conclusion

- TANK model with no capital can deliver amplification and discounting without idiosyncratic uncertainty \( \Rightarrow \) we just need to add output stabilization in the Taylor rule.

- For a given amount of output stabilization, discounting increases as the economy becomes less redistributive (share of constrained agents increases).

- TANK model with capital generates discounting, even without central-bank output stabilization.
Magic

Thank you!