Increasing Returns in the Inter Bank Liquidity Market*

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Abstract

This paper proposes a framework to analyze the functioning of the inter-bank liquidity market and the occurrence of liquidity crises when.banks can face liquidity shocks on their assets. When such shocks occur, banks are confronted with adverse selection -asset quality is private information- and moral hazard -the quality of assets can be improved through unobservable effort-. Under these conditions, the market for liquidity is shown to work as an increasing return to scale technology, i.e. the equilibrium (risk-adjusted) return on liquidity increases with the aggregate volume of liquidity in the economy. As a consequence, banks always provision too little liquidity compared to the social optimum. Within this framework we derive two main results. First credit rationing on the inter-bank market is more likely to happen when the individual probability for a bank to face a liquidity shock is lower. Second ex ante competition between banks on illiquid long term investment can hamper the functioning of the inter-bank market.

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1 Introduction

The financial market turmoil that has been under way since the summer of 2007 hit the core of the global financial system, the inter-bank market for liquidity. This aspect of the crisis has manifested itself through episodes of widening spreads on inter-bank interest rates (vs. policy rates), together with evidence of plummeting volumes in inter-bank lending transactions. While part of this phenomenon has been ascribed to a reassessment of credit risk involved in dealing with banking counterparties, a large share of the premium that has emerged on inter-bank rates has been attributed to "liquidity risk".[1] To be sure, liquidity needs on behalf of banks were to some extent related to concerns by some institutions over their own balance sheets dynamics in the face of credit losses. More generally, banks certainly needed liquidity as they prepared for re-intermediation of special investment vehicles and other conduits that had previously been funded off-balance sheet.[2]

Competitive aspects notwithstanding, we argue that at least two key features of the recent crisis must be taken into account so as to yield a satisfactory understanding of the liquidity market seizure. First the so called "originate and distribute" banking business model may have provided the wrong incentives regarding the monitoring of underlying asset quality (e.g. subprime mortgages): such a concern strikes us – and many other observers – as a clear-cut case of *moral hazard* mechanism. Second, in a context of uncertainty regarding the location of credit losses, the inter-bank market freeze-up was associated with generalized suspicion regarding banking counterparty quality. Rumor and stigma effects affecting banks that would need to access discount window facilities were ubiquitous. The prior that institutions borrowing liquidity on the inter bank market may be the ones that are in bad shape appears as evidence of *adverse selection* effects. This paper does not endeavour to account for all the features of the recent crisis, be it hard evidence

[1] Indeed, early estimates of credit losses related to subprime mortgages investments, which were recognized as the trigger of the crisis, while large in absolute terms, were not enough to justify a systemic concern over the credit quality of financial intermediaries. As a consequence strains on the inter-bank market had to be related to a surge in liquidity needs as such.

[2] Yet among market participants, there was a general feeling that something else was at stake in the liquidity hoarding behavior of banks. At some point, the buzz even had it that it was "payback time"; in other words, some financial intermediaries may have been unwilling to provide funding to competitors that had cut into their market share. Such strategic behavior would be hard to document. However, it sounds very likely that some banks may have held some extra liquidity in order to be in a position to seize latter opportunities if competitors were forced to fire sales. Historical precedent in mentioned by Kindleberger (1996), in the context of financial crises: « Outsiders particularly suffered. The Bank of the United States was allowed to fail in New York in December 1930 by a syndicate of banks, not the Federal Reserve System, amid accusations that the Bank was being punished for its pushy ways » (p 158).

or casual stories about the motivations of market players. However, it argues that a proper modelling of the collapse in the market for liquidity involves both moral hazard and adverse selection mechanisms in the inter-bank market.

In this context, the inter bank market for liquidity is shown to work as an increasing return to scale technology: the risk adjusted return on liquidity lending increases with aggregate ex ante liquidity provision. The mechanism behind this property is fairly straightforward.

With large ex ante liquidity provision, the moral hazard problem is mitigated as banks facing the liquidity shock deliver a large effort. Since banks finance reinvestment mostly on their own funds, they pay particular attention to improving the probability that reinvestment be successful. Moreover the large volume of liquidity supply in the economy reduces the nominal interest rate on liquidity lending. This improves the quality of the pool of banks looking to borrow liquidity which raises the risk adjusted return to liquidity lending on the inter-bank market. Given these two effects, ex ante liquidity provision is highly profitable and this gives banks incentives to provision ex ante a large volume of liquidity. Note however that due to adverse selection, the risk adjusted return on liquidity lending increases with the *aggregate* volume of liquidity in the economy. Banks do not fully internalize the positive impact of ex ante liquidity provision on its expected return.

By contrast, with low ex ante liquidity provision, the argument is reversed: the moral hazard problem is amplified and banks deliver a low effort. Due to the relative scarcity of liquidity supply, the nominal interest rate on liquidity borrowing is large. This deteriorates the quality of the pool of banks on the borrowing side of the liquidity market. The risk adjusted return to liquidity lending is then lower. This reduces bank incentives to provision liquidity ex ante. If the risk adjusted return is sufficiently low, the inter-bank liquidity market collapses as banks supplying liquidity prefer to store it (at the central bank or in any other risk free asset).

In this model, the volume of liquidity provision in the decentralized equilibrium is always inefficiently low. In the case of multiple equilibria, the welfare loss associated with the "low liquidity provision" equilibrium is larger than the welfare loss associated with the "high liquidity provision" equilibrium. Interestingly, the former equilibrium is more likely when the individual probability for banks to face the liquidity shock is lower. Hence the liquidity crisis is more likely in "rosy environments" because such an environment induces banks to reduce their liquidity holdings which turns out to be harmful to banks that do face a liquidity shock, as the inter-bank market for liquidity is more likely to collapse.

The liquidity crisis is also more likely when competition between banks on long term illiquid investments is higher. With more intense competition ex ante, the negative impact of each individual bank capital supply on the return to illiquid investment is lower. As a result, banks tend to increase their long term illiquid investments and thereby reduce their ex ante liquidity provision.

Our interest in understanding liquidity crises is of course not a purely positive one. Another salient feature of the recent events is the variety of responses that have been implemented by major central banks. Liquidity injections have reached a massive scale. Some central banks, namely the Eurosystem, have stressed the distinction between: i) market intervention for financial stability purposes; ii) an unchanged monetary policy stance with respect to the price stability objective. By contrast, other central banks, chiefly the Federal Reserve System, have resorted to large interest rate cuts early on, although such policy action has been motivated by risks to the macroeconomic outlook within a dual-mandate framework. The effects of these diverse central bank reactions to the liquidity crisis have been uneven across markets and over time: the benefits of liquidity injections have often been modest or short-lived, while it is probably too early to conclude over the consequences over monetary policy easing. A key purpose of this paper is to provide a framework to understand under what circumstances liquidity injections may suffice to thwart a liquidity crisis.

In particular, liquidity injections by the central bank can play a role in restoring second best efficiency when they are expected from market participants. However, the central bank is shown to possibly suffer a time inconsistency problem. Announcements of liquidity injections are useful in order to influence ex ante banks' decision as to how much liquidity they need to provision. However once liquidity provision decisions have been taken, the central bank should optimally refrain from injecting liquidity as this tends to amplify the moral hazard problem for banks facing the liquidity shock. As a result, in the absence of commitment, central bank announcements of future liquidity injections can be relatively ineffective in restoring second best efficiency.

The model in this paper builds on the standard literature on adverse selection. It is well known since Akerlof (1970), that adverse selection can cause a market to break down. The investigation of this problem in the context of the credit market is mostly based on Stiglitz and Weiss (1981). Their work however restricted the set of issuable securities to debt contracts, so that a more general framework is required in order to insure optimality of financial contracts. The moral hazard problem is modelled in a basic, standard fashion, similar to that of Holmström and Tirole (1997), whereby the effort choice by the agent (in our case the bank) has an impact on the project's probability of success. This paper is connected to the literature on interbank markets, as a mechanism for managing, and potentially eliminating, risks stemming from idiosyncratic liquidity shocks. Bhattacharya and Gale (1987) in particular studied the case where neither banks investments in the illiquid technology, nor liquidity shocks are observable. Rochet and Tirole (1996) adapted the Holmström-Tirole framework to the interbank market in order to study systemic risk and "too-big to fail" policy. The existence of interbank market imperfections has been established empirically by Kashyap and Stein (2000), which showed the role of liquidity positions, the so-called "liquidity effect". Building on such evidence, Freixas and Jorge (2008) analyze the functioning of the interbank market in order to show the consequences of its imperfections for monetary policy. In particular, they establish the relevance of heterogeneity in banks' liquid asset holdings for policy transmission. Our work is also related to work on liquidity crises. In particular Caballero and Krishnamurthy (2008) provide a model of crises that features liquidity hoarding, and provides a motivation for lender of last resort intervention. However, their approach is primarily based on Knightian uncertainty that leads each agent to hedge against the worst-case scenario. Recent research on industry hedging behaviour shows that hedging decisions are related to the degree of competition, with more heterogeneity in hedging in the more competitive industries (Adam, Dasgupta and Titman, 2007). Liquidity provisions on behalf of banks may be seen as a form of hedging; however we are not aware of any work studying the impact of bank competition on liquidity ratios. Acharya, Gromb and Yorulmazer (2008) have studied the consequences of imperfect competition in the interbank market for liquidity. In a model where there are frictions in the money and asset markets, if banks that provide liquidity have market power, they may strategically under-provide liquidity, and thus precipitate fire sales. Their model does not however feature interbank liquidity crises in the sense of a market breakdown. A common feature of this literature is that the public provision of liquidity, such as liquidity injections, can often improve on the allocation of liquidity resulting from the decentralized outcome.

The paper is organized as follows. The following section lays down the main assumptions of the model. The behavior of intact and distressed banks is analyzed in section 3. Section 4 derives the decentralized equilibrium of the economy. Section 5 looks at the implications of imperfect competition between banks. Conclusions are drawn in section 6.

2 Assumptions and Technologies

We consider an economy with a unit mass continuum of firms and a unit mass continuum of banks. Banks are risk neutral and maximize expected profits. The economy lasts for three dates; 0, 1 and 2. At date 0 banks have a unitary capital endowment each and two investment possibilities. They can invest in a liquid technology: a unit of capital invested in the liquid technology at date t yields r_t units of capital at date $t+1$. Hence the date 2 return to a unitary investment made at date 0 in the liquid technology is $r_0 r_1$. The volume of capital that a bank invests at date 0 in the liquid technology is noted l. Alternatively each bank can lend capital to a unique firm. Firms have access to an illiquid technology but have no capital. The volume of capital a bank lends to a firm at date 0 is noted k_0 and R_L is the interest rate a bank charges to a firm on loans extended at date 0. Each bank hence faces a date 0 resource constraint, $k + l = 1$.

Firm projects are illiquid because they require investment at date 0 but do not pay-off before date 2. In particular, illiquid projects do not yield any output at date 1 contrary to date 0 liquid investments. Illiquid projects may face liquidity shocks at date 1. When a project does not face the liquidity shock at date 1, the bank which has financed it at date 0 is said to be "intact". On the contrary, when the project faces the liquidity shock at date 1, the bank which has financed it at date 0 is said "distressed". Liquidity shocks are assumed to be non observable. A project does not face the liquidity shock with a probability $1-q$. Then the date 2 marginal return to an illiquid project is R. Assuming that banks get a given fraction β of the project's surplus, the interest rate R_L banks charge to firms on loans extended at date 0 is then equal to βR. Without loss of generality we set β at one in what follows.[3]

A project faces the liquidity shock at date 1 with a probability q. Then the project is worthless unless the bank that has financed the project at date 0 makes a reinvestment at date 1. This implies that banks whose illiquid investments do not face any liquidity shock cannot directly invest in projects facing a liquidity shock. These investments need to go through the inter-bank liquidity market.[4] The total volume of capital a

[3] This is equivalent to assuming perfect competition among firms with a zero reservation value for firms. Relaxing these assumptions has not proved to modify the main results of the paper. In section 5, we introduce competition between banks.

[4] Note that the alternative arrangement under which banks would sign ex ante insurance contracts against liquidity shock is not possible here. If banks receive a payment when they declare to be distressed then "intact" banks may have incentives to report untruthfully their situation as "distressed" since (i) liquidity shock are unobservable and (ii) all banks can invest from date 1 to date 2 in the liquid technology the proceeds of the insurance contracts and consume the output at date 2.

distressed bank reinvests at date 1 is noted k_1 and is assumed to be limited by the size of the initial project: date 1 reinvestment cannot be larger than initial date 0 investment: $k_1 \leq k_0$. If a bank reinvests k_1 units of capital at date 1 and makes an effort e, then reinvestment is successful with probability e and the project then yields Rk_1 at date 2. With a probability $1 - e$, reinvestment is unsuccessful and the project then yields no output. A bank faces a non pecuniary cost $c_i(e, k_1)$ to undertake an effort e. This cost c_i is increasing in effort and reinvestment. Effort can take two values; low effort is noted e_l and high effort is noted e_h with $e_l < e_h$. The effort e a bank delivers at date 1 when the project it has financed undergoes a liquidity shock is private information and is hence a source of moral hazard. The non pecuniary cost function $c_i(.,.)$ drawn at the beginning of date 1 determines the type of a bank faced with a liquidity shock. Moreover it is each bank's private information. Distressed banks can be of two types. "Good" type banks do not face any cost to deliver effort, $c_i(e, k_1) = 0$. "Bad" type banks do face a cost to deliver effort, $c_i(e_h, k_1) > 0$. The proportion of distressed banks of the "good" type is α, $1 - \alpha$ being the share of distressed banks of the "bad" type. The type of a distressed bank is private information and is hence a source of adverse selection. Parameters of the economy are however common knowledge to all agents in the economy.

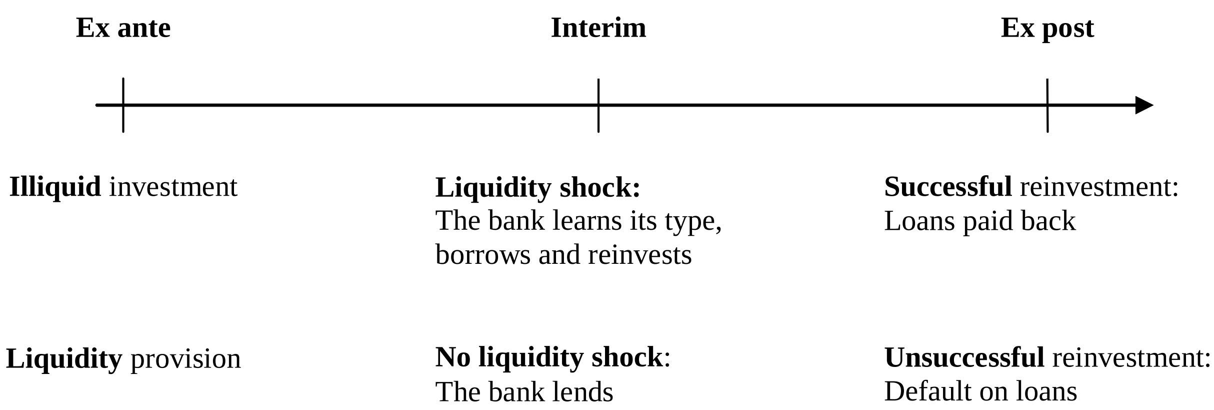

Figure 1: Timing of the model

The timing of the model is as follows. At date 0, each bank decides its capital allocation between liquid and illiquid assets. At date 1, a proportion q of banks face the liquidity shock and distressed banks learn their type. The inter-bank market then opens. Lending banks then propose a menu of financial contracts to borrowing banks. Borrowing banks then choose a contract in the set of financial contracts proposed by lending banks and borrow according to this contract. They reinvest in their illiquid project and finally deliver some effort. Finally at date 2, banks which have reinvested in their illiquid project learn whether reinvestment has been successful or not. They pay their liabilities back when reinvestment is successful.

3 Intact and distressed banks

The model can be solved by standard backward induction. We begin with the problem of intact and distressed banks at date 1 in terms of optimal decisions as to lending/borrowing, effort and reinvestment. Then in the following section, we turn to the problem of the optimal ex ante liquidity provision policy of a bank at date 0.

3.1 Intact banks optimal lending

Let us consider bank i which has invested at date 0 in the liquid technology l_i units of capital and has lent $k = 1 - l_i$ to a firm endowed with an illiquid project. If it is intact at date 1, then it reaps $(1 - l_i)R$ at date 2 and can lend at date 1 the product of its liquid investments $l_i r_0$ to banks which face a reinvestment need. Noting r the interest rate on the inter-bank market for liquidity and θ the average probability that banks which borrow liquidity at date 1 effectively pay back their debts contracted on the inter-bank market, an intact bank enjoys date 2 expected profits equal to

$$\pi_g = (1 - l_i)\, R + l_i r_0 \max\{r\theta, r_1\} \tag{1}$$

since an intact bank can always invest at date 1 in the liquid technology the proceeds of its liquid invest investments $l_i r_0$ undertaken at date 0. Hence intact banks do supply their liquid holdings on the inter-bank market if and only if $r\theta \geq r_1$.

3.2 Distressed banks optimal effort and borrowing

Let us now turn to the case where bank i is distressed at date 1 and denote d_i the volume of capital it borrows at date 1, and e_i the effort it undertakes assuming bank i reinvests in its illiquid project. Its date 2 expected pecuniary profit writes as $e_i\left[\left(l_i r_0 + d_i\right)R - rd_i\right]$. The distressed bank uses output from its liquid investments $l_i r_0$ undertaken at date 0 and the capital d_i borrowed at date 1 to reinvest in the firm it has financed at date 0. Hence reinvestment size k_1 is equal to $l_i r_0 + d_i$. The date 2 output, conditional on success, is $\left(l_i r_0 + d_i\right)R$, the face value of its liabilities is rd_i and e_i is the probability of success. Note that the interest rate r is independent of bank i decisions and in particular its effort e_i, because effort is unobservable.[5] To simplify notations, we write the non pecuniary cost to deliver effort $c_i\left(e, k_1\right)$ as $c_i\left(e_i, k_1\right) = e_i\left(1 - \phi_i\right)Rk_1$. The parameter ϕ_i is always equal to 1 for distressed banks of the good type. As to distressed banks of the bad type, the parameter $\phi_i\left(e_i\right)$ is equal to 1 if the low effort is delivered $e_i = e_l$ and equal to ϕ in $]0;1[$ if the high effort is delivered $e_i = e_h$. The expression for date 2 expected total profits (net of non pecuniary costs) for a distressed bank therefore writes as

$$\pi_b = e_i\left[\left(l_i r_0 + d_i\right)\phi_i\left(e_i\right)R - rd_i\right] \tag{2}$$

and the problem at date 1 of a distressed bank which reinvests in its illiquid project consists in

$$\begin{aligned} &\max_{d_i, e_i} \pi_b\left(d_i, e_i\right) \\ &\text{s.t. } l_i r_0 + d_i \leq 1 - l_i \end{aligned} \tag{3}$$

The constraint on the volume of capital d_i that is borrowed on the inter-bank market stipulates that total reinvestment $\left(l_i r_0 + d_i\right)$ cannot be larger than initial investment $\left(1 - l_i\right)$. The upper bound on the volume of capital a distressed bank can borrow $\left(1 - l_i - l_i r_0\right)$ is denoted $\overline{d_i}$ in what follows. Moreover to fix ideas,we assume that the high effort e_h is the efficient effort for all banks. Expected output from a unitary reinvestment

[5] More formally, this expression is based on the assumption that lenders are unable to discriminate between borrowers of different types. Put differently, the contract on the inter-bank market is a "pooling" contract. We provide in appendix a sufficient condition under which this property holds in equilibrium.

is always larger when delivering high effort: $\phi_i(e_h) \leq \phi_i(e_h)$ and $e_h\phi_i(e_h) \geq e_l\phi_i(e_l)$. We can then derive the following proposition.

Proposition 1 *If bank i is distressed and reinvests in its illiquid project then optimal borrowing d^* writes as*

$$d^* = \overline{d_i}\mathbf{1}\left[\phi_i(e^*) R > r\right] \tag{4}$$

and optimal effort e^* writes as

$$e^* = \begin{cases} e_h \text{ if } \frac{e_h\phi_i(e_h) - e_l\phi_i(e_l)}{e_h - e_l} R(l_i r_0 + d^*) \geq r d^* \\ e_l \text{ if } \frac{e_h\phi_i(e_h) - e_l\phi_i(e_l)}{e_h - e_l} R(l_i r_0 + d^*) \leq r d^* \end{cases} \tag{5}$$

Proof. cf. appendix ■

In what follows we assume that parameters of the model are such that the interest rate on the inter-bank market r will be such that $r < \phi R$. Under this assumption borrowing from the inter-bank market is profitable for all distressed banks. As a result optimal decisions for distressed banks will be as follows. Distressed banks of the good type always deliver high effort e_h and borrow from the inter-bank market . Distressed banks of the bad type can be in one of two different cases. First, distressed banks of the bad type borrow and deliver high effort when ex ante liquidity provisioning l_i is such that

$$\frac{lr_0}{1 - l_i} > \frac{e_l(R - r) - e_h(\phi R - r)}{(e_h - e_l) r}$$

Secondly distressed banks of the bad type borrow from the inter-bank market and deliver low effort when ex ante liquidity provisioning l_i is such that

$$\frac{lr_0}{1 - l_i} < \frac{e_l(R - r) - e_h(\phi R - r)}{(e_h - e_l) r}$$

This illustrates the trade-off an agent faces when it can borrow capital while facing moral hazard. Increased borrowing raises profits but reduces incentives to deliver effort. With large ex ante liquidity provision, borrowing is lower and incentives to deliver effort -which decrease with borrowing- are larger. Hence banks prefer to deliver the high effort and possibly refrain from borrowing. On the contrary with low liquidity provision, borrowing is large and incentives to deliver effort -which decrease with borrowing- are lower. Banks then prefer to borrow and deliver low effort. Note also that in the case of large ex ante liquidity provision, banks all deliver the same effort and the quality of all borrowers is identical -in spite of differences in types- which eliminates the adverse selection problem. Large ex ante liquidity provision tends to give incentives to all borrowing agents to deliver high effort and the quality of borrowing banks eventually ends up identical.

Now that we have determined optimal borrowing and optimal effort decisions conditional on reinvestment, we turn to the question of whether distressed banks prefer to reinvest in their illiquid assets or to lend their liquid holdings on the inter-bank market and give up their illiquid project. The following lemma derives the optimal occupational choice of distressed banks.

Proposition 2 *Distressed banks reinvest in their illiquid project if and only if it is optimal to borrow conditional on reinvestment.*

Proof. cf. appendix. ■

Given that borrowing from the inter-bank market is always profitable for all distressed banks, proposition 2 says that reinvestment is therefore always more profitable than lending on the inter-bank market. As a result, the choice for distressed banks simply consists in determining the optimal effort they want to deliver.

4 The decentralized equilibrium

In the previous section we have established the behavior of intact and distressed banks at date 1 in terms of lending, borrowing, and effort. Based on these results, we now examine the problem of the optimal liquidity provision policy at date 0. Given the assumed restrictions on parameters, distressed banks always borrow from the inter-bank market the maximum amount that they can afford. However distressed banks of the "good" type always deliver the high effort e_h while distressed banks of the "bad" type can either deliver the high effort e_h or the low effort e_l. We call the former situation the case of "tranquil times" and the latter the case of "crisis times".

4.1 Optimal ex ante liquidity provision under tranquil times

Let us first consider the case where distressed banks choose to reinvest in their illiquid project, borrow from the inter-bank market and carry out the high effort e_h. The problem of a bank at date 0 then consists in

$$\max_{l_i} (1-q)\,\pi_g + q\left(\alpha\pi_b\left(\phi_i = 1\right) + (1-\alpha)\,\pi_b\left(\phi_i = \phi\right)\right)$$
$$\text{s.t.} \begin{cases} d_i = 1 - (1+r_0)\,l_i \\ e_i = e_h \end{cases} \tag{6}$$

Two points are worth being noticed here. First banks do not know at date 0, their type ϕ_i in case the project they have invested at date 0 faces a liquidity shock at date 1. Hence banks determine their optimal liquidity provision considering the average profit they could reap from reinvestment across the different possible types. This assumption helps simplify the model as banks are symmetric ex ante. Second we assume that the inter-bank market for liquidity works, in the sense that intact banks accept to provide distressed banks with all the capital they ask for borrowing. Third we consider the case where all distressed banks deliver the high level of effort $e_i^* = e_h$ and borrow from the inter-bank market which amounts to assuming. We can then derive the following result.

Proposition 3 *When all distressed banks deliver the high effort e_h, then denoting $\overline{\phi} = \alpha + \phi\,(1-\alpha)$, the equilibrium interest rate on the inter-bank market r writes as*

$$r = r_h \equiv \frac{1 - q + q\overline{\phi}e_h}{r_0 + q}\frac{R}{e_h} \tag{7}$$

Proof. Program (6) is linear in ex ante liquidity provision l_i. Hence the solution writes as an indifference condition yielding the equilibrium interest rate r on the inter-bank market as given by (7). ■

Expression (7) shows that the equilibrium interest rate on the inter-bank market r_h decreases with the individual probability of a liquidity shock q. The expected return to liquidity provisioning is therefore lower when the liquidity shock is more likely. When the probability q to face the liquidity shock is lower,= banks have more incentives to invest in the illiquid technology because the date 0 expected return to liquidity provision is lower. As a result, the interest rate on the inter-bank market needs to increase in order to raise the expected return on liquidity lending and thereby raise incentives to provision liquidity ex ante. The important property to note here is that when all distressed banks deliver the high effort e_h the expected profits from liquidity provisioning increases with the interest rate r on the inter-bank market.

$$\pi_h = \left[\left(1 - q + qe_h\overline{\phi}\right) R - qe_h r\right] (1 - l_i) + e_h r l_i r_0$$

Consequently, an improvement in the profit stemming from illiquid investments relative to those stemming from liquid investments needs to be compensated through an increase in the interest rate on the inter-bank market r. Equilibrium expected profits then write as

$$\pi_h = \frac{r_0}{q + r_0} \left(1 - q + qe_h\overline{\phi}\right) R \tag{8}$$

and the equilibrium of the inter-bank market is simply

$$r_0 \int_{[0;1]} l_i = q \left(1 - \int_{[0;1]} l_i\right) \tag{9}$$

Given that banks are all symmetric ex ante, they all choose ex ante the same volume of investment in the liquid technology l_i and the optimal ex ante volume of liquidity provision l^* is simply

$$l^* = \int_{[0;1]} l_i = \frac{q}{r_0 + q} \tag{10}$$

Individual rationality constraints end up the description of this equilibrium. On the one hand the optimal policy of any distressed bank consists in delivering high effort e_h and borrowing from the inter-bank market.

The first condition -delivering high effort e_h- writes as

$$\frac{lr_0}{1-l_i} > \frac{e_l\left(R-r_h\right)-e_h\left(\phi R-r_h\right)}{\left(e_h-e_l\right)r_h} \tag{11}$$

and the second one -borrowing must be profitable $\phi R > r$- simplifies as

$$\phi e_h > \frac{1-q+q\alpha e_h}{r_0+q\alpha} \tag{12}$$

On the other hand, the individual rationality condition for intact banks states that lending on the inter-bank market should be, on average, more profitable than investing in the liquid technology. This writes as

$$\frac{1-q+qe_h\overline{\phi}}{r_0+q}R \geq r_1$$

We will assume in what follows that the individual rationality constraint for intact lending banks $e_h r_h > r_1$ is verified: when all distressed banks deliver the high effort e_h, the expected return from lending liquidity on the inter-bank market is strictly larger than the return from outside option for intact lending banks.[6]

4.1.1 Equilibrium under tranquil times

The situation where all distressed banks deliver high effort e_h and borrow from the inter-bank market is an equilibrium if and only if no bank has any incentive to deviate. The only possible deviation that a bank can consider consists in delivering the low effort e_l -and provisioning a lower level of liquidity so that lowe effort e_l is the optimal effort. However given that liquidity provisioning is observable by intact lending banks, a lower level of liquidity provisioning entails a larger interest rate on borrowing to compensate for the higher risk associated with lower effort.[7] Denoting $r_{d,h}$ the interest rate charged to a distressed bank delivering low

[6] This is a weak assumption because in the absence of this assumption there would be no equilibrium on the inter-bank market.

[7] Even if liquidity provisioning was not observable, the borrowing bank which considers possible deviations would reveal through the volume of capital it borrows on the inter-bank market, its liquidity provisioning policy since $d = 1 - l_i\left(1+r_0\right)$. Hence the deviating bank would trade-off the drawback of a mimicking strategy -where it would borrow the same volume of capital as non deviating banks- against the benefits of deviations stemming from the low interest rate on the inter-bank market.

effort e_l when other banks all deliver the high effort e_h, the interest rate $r_{d,h}$ verifies $e_l r_{d,h} = e_h r_h$. Hence expected profits of a deviating bank $\pi_{d,h}$ would write as

$$\pi_{d,h} = [(1 - q + q\theta) R - qe_h r_h] (1 - l_i) + l_i e_h r_h r_0 \tag{13}$$

From (13) expression, it is straightforward to note that expected profits under deviation are strictly increasing in liquidity provisioning l_i since

$$\frac{\partial \pi_{d,h}}{\partial l_i} = (1 - \alpha) q (\phi e_h - e_l) R > 0$$

Consequently, given that the condition under which low effort e_l is optimal writes as

$$r_{d,h} r_0 \frac{l_i}{1 - l_i} < (1 - \phi) R \frac{e_h}{e_h - e_l} - (R - r_{d,h}) \tag{14}$$

optimal liquidity provisioning l_i under deviation is defined by the volume of liquidity l such that the constraint (14) binds. In any case however given that $l_i \leq 1$, we have $\pi_{d,h} \leq \pi_{d,h} (l_i = 1) = r_0 r_h e_h$. Given the expression (7) of the equilibrium interest rate on the inter-bank market, the last inequality simplifies as

$$\pi_{d,h} \leq \frac{r_0}{r_0 + q} \left[1 - q + qe_h \overline{\phi}\right] R$$

The inequality $\pi_{d,h} \leq \pi_h$ is therefore always correct and this precludes any profitable deviation. We can then write down the following result.

Proposition 4 *Assuming that*

$$\phi e_h > \frac{1 - q + q\alpha e_h}{r_0 + q\alpha} \quad \textit{and} \quad \frac{1 - q + qe_h \overline{\phi}}{r_0 + q} R \geq r_1$$

then when individual optimal ex ante liquidity provisioning l_i is sufficiently large, i.e.

$$\frac{l_i}{1 - l_i} r_0 > 1 - e_h \frac{e_h \phi - e_l}{e_h - e_l} \frac{r_0 + q}{1 - q + q\overline{\phi} e_h}$$

the situation where all distressed banks reinvest in their illiquid project, deliver the effort e_h and borrow from the inter-bank market is an equilibrium.

The individual rationality constraints for distressed banks (12) and (11) are both more likely to be verified when the individual probability q of the liquidity shock is larger. In other words the tranquil times equilibrium is more likey to hold in deteriorated environments. We have shown above that a larger probability q to face the liquidity shock reduces the interest rate r_h on the inter-bank market. The reduction in the cost of capital for distressed banks has two conseqence: it raises distressed banks incentives to borrow from the inter-bank market and it also raises incentives to deliver high effort e_h. Hence the equilibirum where all distressed banks deliver high effort is more likely when fundamentals are deteriorated, i.e. when the individual probability q to face the liquidity shock is larger. The question we need to answer now is what happens when individual rationality condition (12) and/or (11) are not verified. We look more closely at this question in the next paragraph.

4.2 Optimal ex ante liquidity provision under crisis times

In the previous case all distressed banks -including the bad type ones- deliver high effort. While distressed banks of the "good" type always deliver the high effort e_h, a distressed bank of the "bad" type may reinvest in its illiquid project, borrow from the inter-bank market but deliver the low effort e_l. Assuming the interest rate on the inter-bank market is sufficiently low, $\phi R > r$, this situation happens when ex ante liquidity provision l_i is sufficiently low, i.e.

$$\frac{lr_0}{1-l_i} < \frac{e_l\left(R-r\right) - e_h\left(\phi R - r\right)}{\left(e_h - e_l\right) r}$$

In this case, the program of an individual bank i at date 0 consists in choosing the volume of ex ante liquidity provision l_i which solves the following problem

$$\max_{l_i} (1-q)(1-q)\pi_g + q(\alpha\pi_b(\phi_i = 1) + (1-\alpha)\pi_b(\phi_i = \phi))$$
$$\text{s.t.} \begin{cases} d_i^* = 1-(1+r_0)l_i \\ e_i^* = e_l + (e_h - e_l)\mathbf{1}[\phi_i = 1] \end{cases} \tag{15}$$

Distressed banks all borrow the same volume of capital d_i^*. However only distressed banks of the "good" type deliver the high effort e_h while distressed banks of the "bad" type deliver the low effort e_l. We can then derive the following result.

Proposition 5 *When distressed banks of the good type deliver the high effort e_h, but distressed banks of the bad type deliver low effort e_l, then denoting $\overline{\theta} = \alpha + (1-\alpha)\frac{e_l}{e_h}$, the equilibrium interest rate on the inter-bank market r writes as*

$$r = r_p \equiv \frac{1}{\overline{\theta}} \frac{1-q+q\overline{\theta}e_h}{q+r_0} \frac{R}{e_h} \tag{16}$$

Proof. As in the previous case banks' expected profits are linear in the volume of ex ante liquidity provision l_i. Solving for the optimal liquidity provision policy therefore yields an indifference condition through which the equilibrium interest rate r on the inter-bank market can be derived as (16). ■

As in the previous case, the equilibrium interest rate r_p decreases with the individual probability q to face the liquidity shock, the reason behind this result being the same as previously: a larger probability to face the liquidity shock raises incentives to invest in liquid assets. The expected return on liquid assets then needs to decrease to restore equilibrium. Moreover the equilibrium interest rate r_p decreases with the average quality of the pool of borrowers θ while the risk adjusted return increases with this average quality θ. Hence a deterioration in the average quality of the borrowing banks tends to raise the interest rate on the inter-bank market but decreases the expected return on liquidity lending on the inter-bank market. As a confirmation, it can be verified that the risk adjusted return on liquidity lending $r_h e_h$ is larger when all distressed banks deliver high effort e_h than $r_p\overline{\theta}e_h$ in the case where only distressed banks of the "good" type deliver high effort since $\phi e_h > e_l$.Given (16) the expected profits of a bank at date 0 write as

$$\pi_p = \frac{r_0}{q + r_0} \left(1 - q + q\overline{\theta} e_h\right) R \tag{17}$$

and it can be easily verified that $\pi_p < \pi_h$. The equilibrium of the inter-bank market is similar to the previous case given that supply and demand are unchanged

$$r_0 \int_{[0;1]} l_i = \left(1 - \int_{[0;1]} l_i\right) q \tag{18}$$

Last, the individual rationality constraint for distressed banks states that borrowing on the inter-bank and delivering the low effort for distressed banks of the bad type shoud be optimal. Borrowing is optimal for all distressed banks if and only if the interest rate on the inter-bank market r_p is lower than the return to capital ϕR; this simplifies as

$$r_0 \overline{\theta} e_h > 1 - q \tag{19}$$

Delivering the low effort e_l is optimal for distressed banks of the bad type if and only if

$$\frac{l_i r_0}{1 - l_i} < \frac{e_l \left(R - r_p\right) - e_h \left(\phi R - r_p\right)}{\left(e_h - e_l\right) r_p} \tag{20}$$

As the individual rationality constraint for intact bank, the risk adjusted return on the inter-bank market must be larger than the return to the liquid technology: $\overline{\theta} e_h r_p > r_1$. As can be noted from expressions (11) and (20), the equilibrium where all distressed banks carry out the high effort e_h and the equilibrium where only distressed banks of the "good" type carry out the high effort e_h are not *a pirori* mutually exclusive because the interest rate in the former equilibrium r_h is lower than the interest rate in the latter equilibrium r_p. This is due to two different reasons. First, the risk adjusted return to ex ante liquidity provision increases with the average repayment probability. As a result, the larger the repayment probability the lower the interest rate needed for banks to be indifferent as to the volume of ex ante liquidity provision. Second, when distressed banks of the "bad" type do carry out the high level of effort, they undergo an additional cost when reinvesting in the illiquid project. Hence the expected return to ex ante liquidity provision is lower and this is compensated through a lower interest rate as to raise expected profits from reinvestment and thereby give incentives to banks to provision liquidity ex ante. To simplify the model we assume that intact banks individual rationality condition $\overline{\theta} e_h r_p > r_1$ is not verified and the "pooling" situation does not constitute an equilibrium:

$$\theta e_h r_p < r_1 < e_h r_h$$

Lending liquidity on the inter-bank market is not profitable for intact banks because the proportion of borrowing banks with a low repayment probability is too large.

4.3 Credit rationing on the inter-bank market

Given that effort increases with liquidity provisioning l_i and decreases with liquidity borrowing d_i, intact lending banks can reduce the volume of capital they lend on the inter-bank market as to reduce incentives for distressed banks to delivering low effort and thereby raise the average quality of the pool of banks borrowing liquidity on the inter-bank market. Distressed banks of the good type always deliver the high effort e_h. Hence to raise the average quality of the pool of banks borrowing liquidity, intact lending banks need to focus on distressed banks of the bad type. Given the expected profits function (2) of distressed banks, one of the bad type is indifferent between delivering the high effort e_h and delivering the low effort e_l if and only if these two strategies yield the same level of expected profits

$$e_l \left(\left(l_i r_0 + d \right) R - dr \right) = e_h \left(\left(l_i r_0 + d \right) \phi R - dr \right)$$

This defines a threshold $\widehat{d_i}$ for borrowing below which distressed banks of the bad type deliver the high effort effort e_h and above which they deliver low effort e_l. This threshold writes as

$$\widehat{d_i} = \frac{\left(\phi e_h - e_l \right) R}{\left(e_h - e_l \right) r - \left(\phi e_h - e_l \right) R} l_i r_0$$

Note that this constraint applies to all distressed banks including those of the "good" type for which borrowing does not have any impact on the choice of effort. This is because intact lending banks cannot discriminate between "good" type and "bad" type banks. The program of an individual bank i at date 0 therefore consists in choosing the volume of ex ante liquidity provision l_i such that

$$\max_{l_i} \begin{array}{c} (1-q)\left((1-l_i)R + l_i r r_0 \widehat{\theta}\right) + q\alpha e_h\left[(l_i r_0 + d_i)R - d_i r\right] + \\ q(1-\alpha)\left[\mu e_l\left[(l_i r_0 + d_i)R - d_i r\right] + (1-\mu)e_h\left[(l_i r_0 + d_i)\phi R - d_i r\right]\right] \\ \text{s.t. } d_i \le \widehat{d}_i \end{array} \tag{21}$$

where μ is the proportion of distressed banks of the "bad" type which deliver the low effort e_l. In this case the average repayment probability θ writes as $\widehat{\theta} = \alpha e_h + (1-\alpha)(\mu e_l + (1-\mu)e_h)$ and the interest rate r verifies by definition $\widehat{\theta} r = r_1$. We can then derive the following proposition.

Proposition 6 *Denoting $\overline{e} = \alpha e_h + (1-\alpha)e_l$, the equilibrium of the inter-bank market under credit rationing is such that the equilibrium interest rate on the inter-bank r_c writes as*

$$r_c = \frac{\phi e_h - e_l}{\left(1 - \frac{q}{1-q}\frac{R}{R - r_0 r_1}\overline{e} r_0 (1-\phi)\right)e_h - e_l} R$$

Proof. The problem (21) of an individual bank is linear in the individual volume of ex ante liquidity provisioning. Consequently, the first order condition to this problem writes as an indifference condition determining the interest rate r on the inter-bank market as follows

$$\frac{1-q}{q r_0}(R - r_0 r_1) = e_h(1-\phi)R\frac{(\alpha e_h + (1-\alpha)e_l)r}{(e_h - e_l)r - (\phi e_h - e_l)R}$$

where $\overline{e} = \alpha e_h + (1-\alpha)e_l$. Solving this equation in r yields the equilibrium interest rate r_c when distressed banks are credit constrained. as

$$r = \frac{\frac{1-q}{q r_0}(R - r_0 r_1)(\phi e_h - e_l)R}{\frac{1-q}{q r_0}(R - r_0 r_1)(e_h - e_l) - e_h(1-\phi)R\overline{e}}$$

■

The expected profits of a bank in this situation then writes as $\pi_c = (1-q)R$. This situation is an equilibrium if and only if four conditions are satisfied. First the agregate supply of liquidity $(1-q)\int_{[0;1]} r_0 l_i$ must be equal to or larger than the aggregate demand for capital $q\int_{[0;1]} d_i$. Assuming that borrowing is profitable for all distressed banks, i.e. $\phi R > r_c$ then the borrowing constraint $d_i \leq \widehat{d}_i$ is binding and the inequality $(1-q)\int_{[0;1]} r_0 l_i \geq q\int_{[0;1]} \widehat{d}_i$ simplifies as

$$R \leq \frac{(1-q)(e_h - e_l) r_0 r_1}{(1-q)(e_h - e_l) - (1-\phi)\bar{e} r_0 e_h}$$

Secondly borrowing is profitable both for distressed banks which deliver low effort e_l and for banks which deliver high effort e_h if and only if $\phi R > r$. This condition simplifies as

$$\frac{(1-q) r_0 r_1}{(1-q) e_l - q\phi\bar{e} r_0 e_h} \leq R$$

Thirdly the borrowing constraint $d_i \leq \widehat{d}_i$ should be binding in the sense that (i) reinvestment is not fully carried out: $l_i r_0 + \widehat{d}_i < 1 - l_i$ and (ii) in case full reinvestment is carried out, distressed banks of the bad type should deliver low effort e_l. These two conditions simplify as

$$l_i < \frac{e_h}{(1+r_0) e_h + \lambda (\phi e_h - e_l) r_0}$$

with

$$\lambda = \frac{1}{(1-\phi)\bar{e}} \frac{1-q}{q} \frac{R - r_0 r_1}{r_0 r_1}$$

Finally there should be no profitable deviation for distressed banks. The only possible deviation a bank can consider consists in provisioning a larger volume of liquidity and deliver high effort. The expected profit of a deviating bank writes as

$$\pi_{d,p} = (1-q)((1-l_i)R + l_i r_0 r_1) + q e_h \left[(1-l_i)\left(\bar{\phi} R - r_{d,p}\right) + l_i r_{d,p} r_0\right]$$

where the interest rate $r_{d,p}$ charged to the deviating bank verifies the no arbitrage condition $e_h r_{d,p} = r_1$. Hence the expression for expected profits of the deviating bank simplifies as

$$\begin{aligned}\pi_{d,p} &= (1-l_i)\left[\left(1-q+qe_h\overline{\phi}\right)R - qr_1\right] + r_1 l_i r_0 \\ &= \left[(1-q)R + q\left(e_h\overline{\phi}R - r_1\right)\right](1-l_i) + r_1 l_i r_0\end{aligned}$$

with $\overline{\phi} = \alpha + (1-\alpha)\phi$. If expected profits from deviation increase with liquidity provision, i.e. $\left(1-q+qe_h\overline{\phi}\right)R < r_1(1+r_0)$ then expected profits from deviation write as $\pi_{d,p} = r_1 r_0$ and deviating is unprofitable if and only if $(1-q)R > r_1 r_0$.

On the contrary if expected profits from deviation decrease with liquidity provision, i.e. $\left(1-q+qe_h\overline{\phi}\right)R > r_1(q+r_0)$, then deviation is profitable if and only if

$$(1-l_i)\left[\left(1-q+qe_h\overline{\phi}\right)R - qr_1\right] + r_1 l_i r_0 > (1-q)R$$

This inequality is true if and only if the volume of ex ante liquidity provision is sufficiently low:

$$\frac{l_i r_0}{1-l_i} < q\frac{e_h\overline{\phi}Rr_0 - r_1 r_0}{(1-q)R - r_1 r_0}$$

However according to proposition 1, delivering high effort e_h is optimal if and only if ex ante liquidity provision is sufficiently large

$$\frac{l_i r_0}{1-l_i} > \frac{e_l(R-r_c) - e_h(\phi R - r_c)}{(e_h - e_l)r_c}$$

Consequently, if $e_h\overline{\phi}R < r_1$ then deviation is never profitable while if $e_h\overline{\phi}R > r_1$, a profitable deviation is incompatible with the large effort e_h being optimal if and only if

$$\frac{e_h}{e_h - e_l}\frac{R}{R - r_0 r_1}\overline{e} > \frac{1-q}{1-\phi}\frac{e_h\overline{\phi}R - r_1}{(1-q)R - r_1 r_0}$$

We can therefore derive the following proposition.

Proposition 7 *When the return to the illiquid asset R verifies the following conditions*

$$\frac{1}{(1-q)\,e_l - q\phi\bar{e}r_0e_h} \leq \frac{R}{(1-q)\,r_0r_1} \leq \frac{e_h - e_l}{(1-q)\,(e_h - e_l) - (1-\phi)\,\bar{e}r_0e_h}$$

and

$$\frac{e_h}{e_h - e_l}\frac{R}{R - r_0r_1}\bar{e} > \frac{1-q}{1-\phi}\frac{e_h\overline{\phi}R - r_1}{(1-q)\,R - r_1r_0}$$

then if individual liquidity provision l_i is sufficiently low

$$l_i < \frac{e_h}{(1+r_0)\,e_h + \lambda\,(\phi e_h - e_l)\,r_0}$$

the credit rationing situation is an equilibrium of the inter-bank maket which features the following charactersitics:

(i) distressed banks are credit constrainted and cannot acheive total reinvestment

(ii) the equilibrium risk adjusted return on liquidity lending is equal to r_1

(iii) some of the distressed banks of the bad type deliver the low effort e_l and some others deliver the high effort e_h

Proof. cf. appendix ■

The equilibrium interest rate on the inter bank market r_c when distressed banks face credit rationing increases with the individual probability q for banks to face the liquidity shock. When banks face the liquidity shock with a large probability, the expected return to investing in the illiquid asset is low, first because the probability to carry out till maturity the illiquid project is low and second because under credit rationing, distressed banks cannot borrow as much as they would like to even if they could pay for a larger interest rate. As a result, banks have incentives to invest more capital in the liquid asset when the probability to face the liquidity shock is larger. Consequently, to restore equilibrium the interest rate on the inter-bank market needs to increase as to reduce the borrowing capacity of distressed banks and thereby reduce expected profits from ex ante liquidity provisioning. This property contrasts with the case of tranquil times where the equilibrium interest rate r_h decreases with the probability q to face the liquidity shock. While a deterioration in fundamentals, i.e. an increase in the probability q, reduces the cost of capital on the inter-bank liquidity market in tranquil times, it tends on the contrary to raise the cost of capital on this market in times of credit rationing. This difference can be examined from a somewhat different angle: in the tranquil times equilibrium, banks expected profits stemming from liquidity provisioning increase with the interest rate on the inter-bank market

$$\frac{\partial\left(\partial \pi_h / \partial l_i\right)}{\partial r} > 0$$

since expected profits in the tranquil times equilibrium π_h write as

$$\pi_h = \left[\left(1 - q + q e_h \overline{\phi}\right) R - q e_h r\right] (1 - l_i) + e_h r l_i r_0$$

However in the credit rationing equilibrium, because the borrowing capacity of distressed banks decreases with the interest rate on the inter-bank market, a larger interest rate decreases banks expected profits stemming from liquidity provisioning

$$\frac{\partial\left(\partial \pi_c / \partial l_i\right)}{\partial r} < 0$$

since expected profits in the credit rationing equilibrium π_c write as

$$\pi_c = (1 - q)(1 - l_i) R + \left[(1 - q) r_1 + q \frac{\overline{e} e_h (1 - \phi) R r}{(e_h - e_l) r - (\phi e_h - e_l) R}\right] l_i r_0$$

This is the reason why the cost of liquidity r is counter-cyclical in tranquil times - it is low when the probability of the liquidity shock is large- but pro-cyclical in crisis times -it is large when the probability of the liquidity shock is large-.

Secondly the equilibrium interest rate on the inter bank market r_c when distressed banks face credit rationing increases with the share α of distressed banks of the good type. This result is counter-intuitive because a higher share α corresponds to an improvement in the fundamentals of the economy in the sense that the default probability of distressed banks is lower every thing else equal. However conditional on being in the credit rationing equilibrium, a larger share α of distressed banks of the good type raises banks incentives ex ante to provision liquidity because the expected return to liquidity provisioning is larger. Hence to restore equilibrium, the interes rate on the inter-bank market r_c needs to increase in order to reduce the borrowing capacity $\widehat{d}$ and the profits stemming from liquidity borrowing $\phi_i(e)R - r$ and thereby reduce banks incentive to provision liquidity.

Finally, the interest rate on the inter-bank market r_c in the credit rationing equilibrium increases with the returns to the liquid technology r_0 and r_1. As previously, banks incentives to proviosion liquidty ex ante increase because the return to liquidity provisioning is larger. Hence to restore equilibrium the interest rate on liquidity lending needs to increase as to reduce the profits stemming reinvestment and thereby reduce incentives to provision liquidity. This result implies that any reduction in the return on liquid assets will reduce the interest rate r_c on the inter-bank market and thereby raise the average quality of the pool of banks applying for liquidity borrowing.

Comparing the equilibrium of tranquil times and the equilibrium of credit rationing shows that the risk adjusted return to liquidity provisioing is higher when the volume of liquidity provisioining is larger. In the former equilibrium the volume of liquidity provisioining is relatively large since it verifies

$$\frac{l_i r_0}{1-l_i} > \frac{e_l(R-r_h) - e_h(\phi R - r_h)}{(e_h - e_l) r_h}$$

and the date 0 risk adjusted return on liquidity provisioning is $\rho_h = r_0 e_h r_h$. However in the latter equilibrium, the volume of liquidity provisionning is relatively small given that it must verify

$$\frac{l r_0}{1-l_i} < \frac{e_l(R-r_c) - e_h(\phi R - r_c)}{(e_h - e_l) r_c}$$

Moreover in the credit rationing equilibrium, the date 0 risk adjusted return on liquidity provisioning is $\rho_c = r_0 r_1$ which is lower than ρ_h by definition of the individual rationality constraint for intact lending banks. Hence the expected return on liquidity lending ρ increases in the volume of liquidity that banks provision ex ante. This can be interpreted as the fact that the inter-bank market for liquidity lending works at the aggregate level as an increasing return to scale technology in the sense that the expected return to liquidity provision ρ for a bank which does not face the liquidity shock is a positive function of the total volume of liquidity that banks have provisioned. This property has an important consequence. The decentralized equilibrium can be inefficient because banks do not fully internalize the positive effect of liquidity provision on the return to liquidity. The economy may be short of liquidity in the decentralized equilibrium compared to the social optimum. The next sections derive formally this case.

5 The impact of competition on inter-bank market functionning

Up to now, we have solved the equilibrium of the economy where technologies are constant returns to scale. This has the advantage of simplifying the resolution of the model. However there is a drawback related to the indeterminacy of the volume of liquidity provision provided the equilibrium conditions are satisfied. For instance, in the tranquil times equilibrium, individual optimal liquidity provision l_i verifies two conditions: it must be sufficiently large

$$\frac{l_i r_0}{1-l_i} > \frac{e_l\left(R-r_h\right)-e_h\left(\phi R-r_h\right)}{\left(e_h-e_l\right)r_h}$$

and the inter-bank market must be in equilibrium

$$\int_{[0;1]} l_i = \frac{q}{r_0+q}$$

Hence provided these two conditions as satisfied, each bank can provision an indeterminate volume of liquidity. To bypass this limitation we introduce some concavity in the problem of banks through the introduction of competition on investment in illiquid assets and we focus on the possible impact of ex ante competition between banks on the likelihood of a credit rationing equilibrium on the inter-bank market. To do so, we slightly modify the framework considered up to now as follows:

We consider a continuum of local markets: each firm with an illiquid project can be financed by n different banks. Conversely, each bank can finance n different firms. When $n=1$, each bank is local monopoly and a larger number n of banks implies that each bank has a lower monopoly power on the interest rate R_L it can charge to a given firm. To simplify the problem of a representative bank, we assume that the interest rate R_L that can be charged to a firm is linear and decreasing in the volume of capital that it borrows. Noting I_n the set of n banks that can lend to a given firm we have

$$R_L = R_0 - R_1 \left(1 - \frac{1}{n}\left(l_i + \sum_{j \in I_n \backslash i} l_j\right)\right)$$

$I_n \backslash i$ being the set of all banks that can lend to the considered firm apart from bank i. Given that a bank can lend to n different firms and that firms are ex ante identical, bank i divides its capital supply $1 - l_i$ equally among the n firms. As a result total capital lent to a given firm is

$$k = 1 - \frac{1}{n} \sum_{j \in I_n} l_j \equiv 1 - E_n l_j$$

In this context, if all distressed banks deliver the high effort e_h, the optimal ex ante liquidity provision policy l_i of bank i solves the problem

$$\max_{l_i} (1-q)\left[(1-l_i)\left[R_0 - R_1(1 - E_n l_j)\right] + l_i r_0 r e_h\right] + q e_h \left[(1-l_i)\left(\overline{\phi} R - r\right) + r l_i r_0\right] \qquad (22)$$

The expected profit expression makes one important assumption: although banks play a Cournot game ex ante to determine the interest rate R_L on illiquid loans, each bank lends its capital dedicated to illiquid loans to a unique firm. Hence once the interest rate R_L is fixed, each firm is randomly allocated to a unique bank which lends it capital. This implicit assumption has two consequences: first banks cannot diversify away liquidity risk by lending to a large number of firms. Second, this allows us to abstract from modeling the game banks would play when a firm which has been financed by more than one bank faces a liquidity shock.

Under these assumptions, we can derive the following proposition

Proposition 8 *The interest rate on the inter-bank market r_{ic} that emerges at the imperfect competition* *equilibrium of the economy is such that*

$$r_{ic} = \frac{1}{e_h}\frac{1-q}{r_0+q}\left(R_0 - \frac{1+n}{n}R_1\frac{r_0}{r_0+q}\right) + \frac{q}{r_0+q}\overline{\phi}R \tag{23}$$

Proof. The first order condition to problem (22) writes as

$$(1-q)\left[-\left[R_0 - R_1\left(1-E_n l_j\right)\right] + \left(1-l_i\right)\frac{R_1}{n} + r_0 r e_h\right] + q e_h\left[-\left(\overline{\phi}R - r\right) + r r_0\right] = 0$$

Summing this equality across all banks of the economy, we end up with

$$1 - \int_{[0;1]} l_i = \frac{n}{1+n}\frac{(1-q)R_0 + q e_h\left(\overline{\phi}R - r\right) - r_0 r e_h}{(1-q)R_1}$$

Hence the equilibrium of the inter-bank market determines the equilibrium interest rate r as

$$r_{ic} = \frac{1}{(r_0+q)e_h}\left[(1-q)\left(R_0 - \frac{1+n}{n}R_1\frac{r_0}{r_0+q}\right) + q e_h\overline{\phi}R\right]$$

and assuming a symmetric equilibrium where banks make ex ante similar decisions, the optimal volume of liquidity provision l^* at the equilibrium on the inter-bank market is simply

$$l^* = \frac{q}{r_0+q}$$

■

As is clear from expression (23), the equilibrium interest rate r_{ic} on the inter-bank market an increasing function of the number of competitors n each bank faces on its illiquid investments. With more intense competition, banks are willing to invest more capital in illiquid assets because competition tends to dampen the negative effect of illiquid investment on the return to illiquid investment. Hence with more capital invested in illiquid assets, the inter-bank market may be in excess demand for liquidity and the equilibrium is restored with a larger interest rate r_{ic} on liquidity lending as to raise incentives to provision liquidity ex ante. However The tranquil times situation where all distressed banks deliver the high effort e_h is an equilibrium if and only if individual liquidity provision is sufficiently large, or put differently if the interest rate on the inter-bank market is sufficiently low:

$$\frac{lr_0}{1-l_i} > \frac{e_l\left(R-r\right)-e_h\left(\phi R-r\right)}{\left(e_h-e_l\right)r}$$

Given above expressions for individual liquidity provision and equilibrium interest rate on the inter-bank market, this condition writes in the case of an imperfect competition equilibrium as

$$\frac{1+n}{n}R_1\frac{r_0}{r_0+q} > R_0+\left(\overline{\phi}q-\frac{e_h\phi-e_l}{e_h-e_l}\frac{r_0+q}{1-q}\right)\frac{e_hR}{1-q}$$

which defines an upper bound on the number of competitors n each bank faces on its illiquid investments above which the tranquil times equilibrium becomes impossible. The reason for this result is fairly intuitive: with large ex ante competition, each bank knows that the negative impact on the return to illiquid investments due to an increase in the volume of capital dedicated to illiquid investments is lower. This is due to the fact that each bank represents a smaller share of total capital lent to a given firm. Given that this negative impact is lower, banks have incentives to increase their illiquid investments. This contributes to raise the return they can expect on liquid assets as to ensure the equilibrium of the inter-bank market. However the increase in the interest rate on the inter-bank market reduces incentives of distressed banks to deliver effort in as much a larger share of increased expected output coming from effort delivering goes to creditors. At some point the interest rate on the inter-bank market is so high that distressed banks stop delivering the high effort and the tranquil times equilibrium disappears.

6 Conclusions

to be drawn

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7 Appendix

7.1 Proof of proposition 1

The problem for a distressed bank of the good type is very easy to solve as they do not face any cost to deliver effort. Hence optimal effort is always e_h and it borrows on the inter-bank market if and only if $R > r$. The problem for a distressed bank of the good type is slight more complicated: it delivers the high effort if and only if this strategy yields larger expected profits. Given its program (3), this condition writes as

$$e_h\left[\left(l_i r_0 + d_i\right)\phi R - r d_i\right] \geq e_l\left[\left(l_i r_0 + d_i\right)R - r d_i\right]$$

and it borrows from the inter-bank market if and only if it is profitable, i.e. $\phi_i\left(e_i\right)R > r$.Then if borrowing is profitable, it borrows as much as possible given the constraint given by initial investment:

$$d_i = \left(1 - l_i - l_i r_0\right)\mathbf{1}\left[\phi_i\left(e_i\right)R > r\right]$$

7.2 Optimal contracts

The previous analysis has relied on the assumption that banks provide "pooling" contracts on the inter-bank market for liquidity. This section aims at deriving the conditions under which the "pooling" equilibrium is indeed the optimal equilibrium contract. More precisely, the section derives a necessary condition to obtain a separating equilibrium and discusses what happens when this necessary "separating" condition does not hold.

To do so, let us consider a bank without a liquidity shock which considers lending to banks with a liquidity shock. Moreover let us assume that banks which face a reinvestment need draw the probability p_i from a Bernoulli distribution: with a probability ϕ the probability p_i is equal to p_h and with a probability $1-\phi$ the probability p_i is equal to p_l with $p_h > p_l$. Hence if the problem for a bank with no liquidity shock consists in providing a contract $\{(r_h, L_h), (r_l, L_l)\}$ specifying a volume of capital L_i and a cost of capital r_i such that banks which have drawn the probability p_i choose the contract (r_i, L_i) and make the optimal effort $e_i = \overline{e}$. Let us denote

$$\pi_b = p_i \left[e \left[\left(lr_0 + L_i \right) A_i - rL \right] - \frac{1}{2} k_i e^2 \left(lr_0 + L \right) \right]$$

where $k_i p_i = c_i$ and $A_i p_i = R_i$. As a result the problem of the bank with no liquidity shock writes as

$$\max_{\{(r_1, L_1), (r_2, L_2)\}} \phi e_h p_h r_h + (1 - \phi) e_l p_l r_l$$

$$\text{s.t.} \quad \begin{cases} e_i(r, L) = \arg\max_e \pi_b(p_i, e, r, L) \\ e_i(r_i, L_i) = \overline{e} \\ \pi_b(e_i(r_i, L_i), (r_i, L_i)) \geq \pi_b(e_i(r_j, L_j), (r_j, L_j)) \\ e_i(r_i, L_i) p_i r_i \geq r_1 \\ R \geq p_i r_i \end{cases}$$

The first condition (IR 1) states that banks facing a liquidity shock choose their level of effort as to maximize their profit net of non pecuniary costs. The second condition (IC 1) states that banks facing a liquidity shock should choose the largest level of effort $\overline{e}$. The third condition (IC2) states that contracts should be separating, each banks choosing the contract that has been designed for its type. Finally the two last condition (IR2) and (IR3) are participation constraints on the lending and the borrowing side of the interbank liquidity market. The first condition (IR1) can then be solved as

$$e_i(r, L) = \frac{1}{k_i} \left[A_i - r \frac{L}{lr_0 + L} \right]$$

The expected profit of a borrowing bank net of non pecuniary costs hence writes as

$$\pi_b(e_i(r, L), (r, L)) = \frac{1}{2c_i} \left[R_i - p_i r \frac{L}{lr_0 + L} \right]^2 (lr_0 + L)$$

and the volume of capital L_i -lent to banks of type i- which ensures that type i banks carry out the optimal effort $\overline{e}$ writes as

$$L_i = \frac{A_i - k_i\overline{e}}{r_i - (A_i - k_i\overline{e})} r_0 l$$

Hence the problem of a bank that wants to lend capital on the inter-bank liquidity market with a menu of optimal contracts writes as

$$\max_{\{(r_1,L_1),(r_2,L_2)\}} \phi e_h p_h r_h + (1-\phi) e_l p_l r_l$$

$$\text{s.t.} \begin{cases} e_i(r,L) = \frac{1}{k_i}\left[A_i - r\frac{L}{lr_0+L}\right] \\ L_i = \frac{A_i - k_i\overline{e}}{r_i-(A_i-k_i\overline{e})} r_0 l \\ \left[R_i - p_i\frac{r_iL_i}{r_0l+L_i}\right]^2 (lr_0+L_i) \geq \left[R_i - p_i\frac{r_jL_j}{r_0l+L_j}\right]^2 (lr_0+L_j) \\ \frac{1}{\overline{e}} r_1 \leq p_i r_i \leq R \end{cases}$$

We can then derive the following result.

Proposition 9 *The Equilibrium of the inter-bank liquidity market is a "pooling" equilibrium -where all banks demanding liquidity choose the same contract- if and only if*

$$A_i < A_j \ \textit{and} \ A_i - k_i\overline{e} > A_j - k_j\overline{e}$$

or alternatively

$$A_i > A_j \ \textit{and} \ A_i - k_i\overline{e} < A_j - k_j\overline{e}$$

where $p_iA_i = R_i$ *and* $p_ik_i = c_i$

Proof. To fix ideas, let us assume that $A_i > A_j$. The incentive compatibility condition which ensures that type i banks truthfully reveal their types on the inter-bank liquidity market writes as follows

$$\left[R_i - p_i\frac{r_iL_i}{r_0l+L_i}\right]^2 (lr_0+L_i) \geq \left[R_i - p_i\frac{r_jL_j}{r_0l+L_j}\right]^2 (lr_0+L_j)$$

with

$$L_i = \frac{R_i - c_i\overline{e}}{p_i r_i - (R_i - c_i\overline{e})} r_0 l$$

The incentive compatibility constraint then simplifies as

$$\frac{p_i r_i}{p_i r_i - (R_i - c_i\overline{e})} \geq \left[\frac{R_i - \frac{p_i}{p_j}(R_j - c_j\overline{e})}{c_i\overline{e}}\right]^2 \frac{p_j r_j}{p_j r_j - (R_j - c_j\overline{e})}$$

Hence noting

$$\lambda_i(p_i r_i) = \frac{p_i r_i}{p_i r_i - (R_i - c_i\overline{e})}$$

a separating contract must verify

$$\begin{aligned} \lambda_i(p_i r_i) &\geq \left[\frac{R_i - \frac{p_i}{p_j}(R_j - c_j\overline{e})}{c_i\overline{e}}\right]^2 \lambda_j(p_j r_j) \\ \lambda_j(p_j r_j) &\geq \left[\frac{R_j - \frac{p_j}{p_i}(R_i - c_i\overline{e})}{c_j\overline{e}}\right]^2 \lambda_i(p_i r_i) \end{aligned}$$

Given these two inequalities, a necessary condition to get a separating contract writes as

$$\left[\frac{R_i - \frac{p_i}{p_j}(R_j - c_j\overline{e})}{c_i\overline{e}}\right]^2 \leq \left[\frac{c_j\overline{e}}{R_j - \frac{p_j}{p_i}(R_i - c_i\overline{e})}\right]^2$$

Then noting $p_i A_i = R_i$ and $p_i k_i = c_i$, this condition can then simplify as

$$(A_i - A_j)(A_i - k_i\overline{e} - (A_j - k_j\overline{e})) > 0$$

Given the assumption that $A_i > A_j$ this condition simplifies as $A_i - k_i\overline{e} > A_j - k_j\overline{e}$. Hence in the case where $A_i > A_j$, if $A_i - k_i\overline{e} < A_j - k_j\overline{e}$, there cannot exist any separating contracts and the equilibrium of the interbank liquditt market is a pooling equilibrium. ■