Repo Markets, Counterparty Risk, and the 2007/2008 Liquidity Crisis[1]

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Abstract. A standard repurchase agreement between two counterparties is considered to examine the endogenous choice of collateral assets, the feasibility of secured lending, and welfare implications of the central bank's collateral framework. As an important innovation, we allow for two-sided counterparty risk. Our findings relate to empirical characteristics of repo transactions and have an immediate bearing on market developments since August 2007. *JEL classification:* G21, G32, E51.

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Non-technical summary

Why is it the case that illiquid and risky assets are used so little as collateral in the interbank market? How could it happen that, during the summer turbulences of 2007, interbank lending secured by certain types of collateral (such as structured assets) became so much less common in the money market? Why do commercial banks have a preference for using relatively illiquid assets as collateral vis-à-vis the central bank? And what are the welfare implications of the central bank's collateral framework?

To explore these and related questions, the present paper takes a closer look at the role of collateral in the interbank lending relationship. A scenario is analyzed in which two commercial banks, a borrower and a lender, negotiate simultaneously about (a) the collateral assets to be used, (b) the haircut, and (c) the repo rate. In contrast to the existing theoretical literature, we allow for two-sided credit risk, i.e., the possibility that also the lender may default. The following results are obtained.

First, we demonstrate that it will always be efficient to share risks between the two counterparties. That is, typically both counterparties in a repo transaction will be exposed to non-trivial counterparty risk. This central result has the implication that if two commercial banks agree to transact, they always agree to use the most liquid and the least risky assets of the borrower as collateral first. Thus, in a bilateral transaction between two commercial banks that may each default with positive probability, good collateral drives out bad collateral.

Second, if the most liquid and least risky assets of the borrower are still relatively illiquid or risky, then the two banks may, under certain conditions,

not be able to agree on a transaction at all. This outcome occurs in particular if default probabilities are non-negligible and collateral assets have the potential to become illiquid. The break-down of the market under two-sided credit risk is a potentially important result as it can explain why there is hardly any interbank repo market in which risky or illiquid asset types are used as collateral. It also complements existing structural explanations of the microstructure of the money market based on asymmetric information, and last but not least allows us to apply an important theoretical argument that has been put forward recently by Kashyap, Rajan, and Stein (2002).

Finally, we study the welfare implications of the central bank's collateral policy. It is shown that an expansion of the set of collateral eligible for central bank operations may indeed lead to a welfare improvement for market participants. However, the expansion of the set of eligible collateral will typically be accompanied by a replacement of liquid collateral by illiquid collateral, i.e. bad collateral drives out good collateral in lending relationships with the central bank. Moreover, such replacement is not likely to be stopped by an adjustment of haircuts.

Our findings offer a potential rationale for the willingness of major central banks to broaden the range of assets accepted as collateral during the market turmoil. In the specific case of the Eurosystem, with its already very broad range of eligible collateral, the analysis comes to the conclusion that a widening of the set of eligible collateral would not necessarily be or have been supportive for a resolution of the credit crunch in the interbank market. Indeed, there is no evidence that too much high quality collateral is bound in operations conducted by the Eurosystem. We also argue that the situation might have been different in the US and in the UK, where policy measures included the expansion of the set of assets accepted by the Federal Reserve and the Bank of England, respectively.

Introduction

Standard (sale and) repurchase agreements, or repos (RPs) in short, are used by both private and public counterparties to conveniently swap cash against collateral for a pre-defined period of time. In any such contract, the cash lender is usually compensated in the form of a repo rate, which is essentially an interest on the gross value of the transaction. Moreover, a haircut is applied to the collateral to limit the exposure for the cash lender in the case that the borrower is unable to repay the principal amount plus interest, and at the same time the liquidation value of the collateral declines below the creditor's claim.[4]

The theoretical analysis of repurchase agreements started with the seminal contribution by Duffie (1996) who has pointed out that when owners of a specific asset incur frictional costs from using the asset as collateral, the repo rate for the asset may fall significantly below the repo rate charged for general collateral. Moreover, through its impact on funding conditions, such specialness is predicted to add a premium to the asset's market price. In a number of recent papers, this theoretical prediction on competitive repo markets has been empirically confirmed from different perspectives.[5]

[4] Over the last few years, the repo segment has gained considerable importance in international money markets. For instance, the euro repo market has been growing on average by 17% annually since 2002, while the unsecured market segment has been expanding only moderately over the same period (cf. ECB, 2007a). The growth of repo markets is often attributed to an anticipated benefit under Basle II capital regulation and to an increasing reliance on the instrument in central banks' implementation frameworks. There is also an increasing interest in national repo markets. See for instance papers by Baba and Inamura (2004), Fan and Zhang (2007), Jordan and Kugler (2004), and Wetherilt (2003).

[5] Jordan and Jordan (1997) validate specialness in repo rates using daily data for the US Treasury repo market of overnight general collateral rates and special financing rates. Based on a data set for the German money market, Buraschi and Menini (2002) reject the rational expectations hypothesis of the term structure for the repo market, and find empirical evidence for a time-varying liquidity risk premium. Krishnamurthy (2002) identifies

An assumption underlying this existing theory of the repo market is that there is an investor (the "Short") who seeks to get hold of a well-specified asset through the repo market transaction. However, it has been noted at various places that in general the repo market is open both to investors in search of a specific security and to investors in search of cash. That is, there are also repurchase agreements that are driven mainly by the funding motive, with the choice of collateral being of secondary importance.[6] As a practical matter, this difference in the motive for approaching the market is not only reflected by the side that initiates the trade (i.e., who is calling whom), but also in differences in the margining (either in cash or in collateral). Moreover, in the case of cash-driven repos, the repo rate for less liquid collateral may also exceed the rate for general collateral.[7] The present paper aims at exploring the determinants of collateral in such cash-driven repurchase agreements. To this end, we introduce counterparty risk into a partial equilibrium model of bilaterally negotiated repurchase agreements.

Two empirical regularities have motivated this route of inquiry. One observation is that typically, only collateral of the highest quality is accepted in the interbank market. This can be seen by comparing the collateral usage of the private repo market with uses of collateral in the large-scale repo trans-

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repo specialness as a cost-of-carry that renders convergence trading much less profitable than suggested by bond spreads.

[6] As far as we know, there is so far no empirical evidence on the share of interbank repo transactions in the euro area that is cash-driven. For instance, in Comotto's (2007, p. 17) product analysis, there is no split-up for the bulk of the repo desk activity. However, the evidence for the US market surveyed by Buraschi and Menini (2002, p. 253) suggests a role for the funding motive in repo markets even under normal market conditions.

[7] For instance, Griffiths and Winters (1997) document an average spread for repos on collateral issued by the Government National Mortgage Association (GNMA) over government bonds of 8.5 basis points during the period February 1984 through January 1985.

actions conducted by central banks. For instance, as shown in Table I, the collateral used during 2006 in the private euro repo market has been mostly government bonds. Illiquid and risky assets such as asset-backed securities (ABS) are *not* commonly employed as collateral in the private bilateral repo market. This situation stands in stark contrast with the composition of collateral held with the European Central Bank (ECB) that accepts a wide range of asset types including government bonds, bank bonds (both uncovered and covered), corporate bonds, ABS, other marketable securities, and credit claims. During 2006, only about 29 percent of assets deposited for use as collateral in Eurosystem credit operations were issued by governments. More generally, Table I shows that vis-à-vis the central bank, highly liquid and safe assets such as government bonds have played a subordinate role, while uncovered bank bonds and asset-backed securities have been forwarded extensively.

Table I about here

The second regularity in the data is related to more recent developments that have impacted also on interbank credit relationships. Following the summer 2007 financial market turbulences, requirements on collateral assets imposed by cash-lenders in the interbank market became even stricter than they usually are. Indeed, recent data by Clearstream (2007, p. 15) shows that the share of structured securities used as collateral in tri-party repos has fallen from 35 percent to 25 percent between June 1 and September 14, 2007, with ABS Auto, Card, CDOs, and MBS the most affected through the subprime crisis. This is consistent with observations by Comotto (2008, p. 19) who writes that "Concern over the quality of collateral could explain the reduction in the share of tri-party repos, which has been the preferred way of managing non-government collateral. It definitely explains [...] the unusually high share of government bond collateral in tri-party repos." In contrast, the composition of central bank collateral has shown just the opposite development. Indeed, media reports suggest that the share of illiquid and relatively risky assets such asset-backed securities has increased significantly since the beginning of the turbulences in August 2007.[8]

To better understand these observations, the present paper takes a closer look at the role of collateral in interbank lending relationships. A hypothetical scenario is studied in which two counterparties, a borrower and a lender, negotiate simultaneously about (a) the collateral assets to be used, (b) the haircut, and (c) the repo rate. Extending the existing theoretical framework, we allow for two-sided counterparty risk, i.e., the possibility that the borrower and likewise the lender may default. This has potentially important consequences for the economic determinants of collateral. Moreover, the analysis will enable us to study the welfare consequences of the central bank's collateral policy.

It turns out that with two-sided credit risk, the bilateral negotiation between borrower and lender achieves a subtle balance of interests. On the one hand, the lender may be willing to accept a somewhat lower haircut in exchange for a somewhat higher repo rate, as a higher haircut obviously implies better protection for the lender. Conversely, the borrower may be willing to

[8] Similar developments have been documented for the US. For instance, the Federal Reserve Bank of New York (2008, p. 21) writes that "in recent years the distribution by collateral tranche of outstanding RPs has been weighted heavily toward the Treasury tranche...until financial market strains appeared in short-term funding markets. At that point dealers' propositions against agency and MBS collateral tranches that it accepts on its RPs became more attractive on a relative basis."

provide somewhat more collateral for a somewhat lowered repo rate. This is not costless, however, because there is the real risk that collateral deposited by the borrower may get lost in the lender's insolvency mass.[9] Optimal risk sharing is achieved, therefore, by making the marginal rate of substitution between haircut and repo rate congruent between the two counterparties. It turns out that, as a consequence, if collateral is not perfect, i.e., if price fluctuation or illiquidity is possible, then it is typically optimal to expose both parties to non-trivial counterparty risk.

The efficiency of risk sharing is what ultimately drives our first main result. This result says that if two counterparties agree to transact, they always agree to use the most liquid and the least risky assets of the borrower as collateral first. Thus, in a bilateral transaction between two counterparties that may each default with positive probability, good collateral drives bad collateral out of circulation, suggesting an intuitive analogy with Gresham's law for commodity money.

We go on and study the general feasibility of secured contracting under market stress. It is shown that if the most liquid and least risky assets of the borrower are still relatively illiquid or risky then the two counterparties may, even under symmetric information and zero opportunity costs of collateral,

[9] The institutional literature has repeatedly stressed this issue. For instance, Stigum (1989, p. 325) writes that "Sophisticated managers of large bond portfolios exercise extreme care in determining to whom they will reverse out their *valued* bonds" (emphasis in the original). Corrigan and de Terán (2007, p. 76) emphasize the same point: "It is often mistakenly thought that the provider of cash has the greater credit risk but this is not necessarily so." An example illustrating the symmetric nature of counterparty risk in collateralized transactions is the failure of the securities dealer Drysdale in 1982. According to Garbade (2006, p. 32), "it was quickly evident that firms that had lent securities to Drysdale were inadequately margined and were going to be left with far less cash than the replacement cost of their securities."

not be able to agree on a transaction at all. This outcome occurs in particular if default probabilities are perceived as non-negligible by market participants, which relates our analysis to the developments in the money markets following August 2007. The break-down of the market under two-sided credit risk also adds to existing structural explanations of the microstructure of the money market based on asymmetric information, and explains the existence of central counterparties. Last but not least, this second result allows us to relate our analysis to an important theoretical argument that has been put forward recently by Kashyap, Rajan, and Stein (2002).

The final part of the analysis explores the question how the central bank's collateral policy might affect overall welfare. It is shown that the expansion of the set of collateral eligible for central bank operations may lead to a welfare improvement for market participants. However, as we also show, the expansion of the set of eligible collateral is typically accompanied by a replacement of liquid collateral by illiquid collateral in the primary market. I.e., in contrast to the prediction obtained for market transactions, bad collateral drives out good collateral in lending relationships with the central bank. More generally, the framework allows discussing the collateral framework of central banks both in the context of fiscal discipline of euro area member countries and in the context of the subprime crisis.

The analysis relates to further strands of the theoretical literature. One is concerned with credit rationing and collateral under one-sided credit risk. Stiglitz and Weiss (1981) have shown that credit rationing may occur as a consequence of asymmetric information either at a pre- or post-contracting stage. Bester (1985) has argued that in the case of pre-contracting asymmetric information, the adverse selection problem may be resolved when commitment to costly collateral is feasible for entrepreneurs with relatively low risks. Berger and Udell (1990) remark that existing theoretical and empirical approaches to the use of collateral still have to be reconciled, but see Cocco (1999) for a potential resolution. Flannery (1995) has examined the breakdown of the unsecured money market due to adverse selection in a crisis situation.[10]

The rest of the paper is structured as follows. Section I introduces the model and discusses efficient risk sharing in standard repurchase agreements involving two-sided credit risk. Section II studies the possibility of a market break-down. Section III elaborates on central bank policy and welfare consequences. Section IV concludes. All proofs are relegated to the Appendix.

I. The basic model

Consider a money market over three dates, date 0, date 1, and a terminal date 2. There are altogether $1 + m$ assets, cash and $m \geq 1$ collateral assets $j = 1, ..., m$. Cash is riskless and does not carry interest. Collateral assets may be either risky or illiquid or both.[11] There are two counterparties in the market.[12] In the sequel, we will mainly think of these as commercial banks, but the model applies with minor changes in the interpretation likewise to

[10] Still another strand of literature related to the present study is concerned with rediscounting and payments. Freeman (1996) considers a model with overlapping generations in which fiat money is used both for consumption and for repayment of loans. It is shown that an elastic provision of liquidity within the period can resolve temporary tensions in liquidity demand without affecting price levels for the consumption good. Mills (2006) considers liquidity provision from a mechanism design perspective, and shows in particular that distortions may occur when the central bank requires collateral that has alternative benefits for the borrowers.

[11] Kocherlakota (2001) uses risky collateral to rationalize deposit insurance.

[12] An extension to more than two counterparties is not attempted here.

other financial and non-financial institutions.

Bank $i = 1, 2$ has an exogenous initial endowment of cash and collateral assets at date 0. Each bank is required to hold a certain amount of cash (potentially zero) at the end of date 1. Cash held in excess of these minimum reserve requirements will be of no value, i.e., there is no carry-over provision. Moreover, initial endowments in cash are such that reserve requirements would be fulfilled without slack in the absence of further transactions. For $i = 1, 2$, let bank i's utility function be denoted by $u_i(.)$. The function $u_i(.)$ is assumed to be weakly concave and differentiable with $u_i'(.) > 0$. Each bank $i = 1, 2$ maximizes expected utility from terminal payoffs.

The time structure of the model is as follows (cf. also Figure 1). Between dates 0 and 1, there is a publicly observable random customer request to transfer an amount $\lambda > 0$ of cash at date 1. With equal probability, the transfer will be from Bank 1 to Bank 2 or vice versa from Bank 2 to Bank 1. The absolute size λ of the liquidity shock may also be random. However, without loss of generality, λ will initially be normalized to one "unit." To compensate for the liquidity shock, the bank receiving the transfer, bank i_L, will seek to become the lender in the money market, while the bank sending the funds, bank i_B, will seek to become the borrower.

By definition, if not defaulted, a commercial bank in the role of the borrower (lender) is equipped at date 2 with sufficient assets to repay principal and interest (to redeliver the collateral). Without loss of generality, there are then three states of nature: In state $\omega = G$, neither the lender nor the borrower defaults (this is the "good" state); in state $\omega = B$, only the borrower defaults; and in state $\omega = L$, only the lender defaults. Denote by $\pi_\omega = \pi_\omega(i_B, i_L)$ the probability that state ω realizes at date 2, where $\omega \in \{G, B, L\}$. Clearly, $\pi_G + \pi_B + \pi_L = 1$. The utility in case of own default is normalized to zero.

The following assumption is fundamental to all what follows. To our knowledge, it also marks the departure from the existing theoretical literature on collateralized lending.

Assumption 1. (Two-sided credit risk) $\pi_B > 0$, $\pi_L > 0$.

To mitigate two-sided credit risks, banks might in principle want to write complicated contracts that condition on all the information observable and verifiable at date 2. However, to make progress, we shall instead consider an institutional form of the repo contract.[13] Specifically, it is assumed that counterparties may sign a *standard repurchase agreement (SRA)* $C = (y, h, r)$, which is composed of a collateral composition y, a haircut $h \geq -1$, and a repo rate r. Here and later on, a *composition* is a collection $y = (y_1, ..., y_m)$ of weights $y_j \geq 0$ for individual assets j such that $\sum_{j=1}^{m} y_j = 1$. The agreement foresees that the lender promises to transfer one unit of cash at date 1. The borrower in turn promises to deposit collateral of composition y with the lender at date 1.[14] Moreover, the common haircut h is applied to all assets.[15] At date 2, in the good state, the borrower will repay the principal

[13] Lacker (2001) derives conditions for collateralized debt contracts being optimal.

[14] For cash-driven repos considered here, the lender typically leaves the borrower a certain discretion (typically upwards in quality) concerning the collateral. This discretion, however, seems to be motivated by practical issues (such as coordination problems across trading desks), which are absent from our model.

[15] Equivalently, but more demanding in terms of notation, the contract could specify an individual haircut for each collateral asset used in the transaction, where the collateral composition should be adjusted correspondingly.

plus an interest (rate) r. The lender, in turn, redelivers the collateral to the borrower.

So far, the contract would be incomplete, as no provisions are made for the cases of the borrower's or the lender's default. Specifically, as the interbank contract matures, the lender's claim on repayment of principal and interest would stand against the borrower's non-monetary claim on the collateral. Without documented provisions, the lender would have no legal basis for liquidating the collateral asset in case of the borrower's default. In the worst case, the insolvency agent of the defaulting borrower would decide to refuse payment, while claiming delivery of the collateral. Likewise, without provisions, the borrower would have no right to withhold repayment of principal and interest when the lender does not render the collateral.

The institutional reply to this problem is to allow for setting-off (or netting) of mutual claims in case of insolvency of one counterparty.[16] Netting involves transforming the borrower's claim for delivery of the collateral into a monetary claim. Following standard legal practice, we will assume that the size of the monetary claim is determined by market conditions at the time when the default occurs. Let $\widetilde{v}_b$ denote the liquidation value of the collateral portfolio at date 2, conditional on the borrower's default. Similarly, let $\widetilde{v}_a$ denote the replacement cost of the collateral portfolio at date 2, conditional on the lender's default.[17]

[16]The contract form provided by The Bond Market Association (2000) foresees a set-off of mutual claims in case of one-sided insolvency, where collateral claims are evaluated by the non-defaulting party either by actual, quotes, or estimated market prices. This contract has been used prevalently in major repo markets (cf. Garbade, 2006, and Comotto, 2007).

[17]Alternatively, there is no market available at date 2, and prices reflect the respective second-best alternative. For instance, when no buyer can be found for the collateral, then

Assumption 2. **(Netting)** *In state $\omega = B$, the borrower's claim on the collateral is replaced by a claim of payment of $\widetilde{v}_b$. In state $\omega = L$, the borrower's claim is replaced by a claim of payment of $\widetilde{v}_a$. Subsequently, the claim of the non-defaulting party vis-à-vis the defaulting party may be used to set off the claim of the defaulting party vis-à-vis the non-defaulting party.*

For instance, in state $\omega = B$, the lender's claim on repayment of principal plus interest is protected by the collateral only if the realized liquidation value v_b of the collateral portfolio at date 2 covers $1 + r$. Thus, the lender incurs a potential loss of $\min\{v_b - (1 + r); 0\} \leq 0$ compared to state G. Similarly, in state $\omega = L$, the borrower has a potential loss of $\min\{(1+r) - v_a; 0\} \leq 0$, where v_a is the realized replacement cost of the collateral portfolio at date 2. In reality, the extent to which such a potential loss becomes an actual loss depends on several factors including whether the insolvency assets of the defaulting party have some market value, and whether the net claim of the non-defaulting party is senior to claims by third parties. The following assumption is made for simplicity.

Assumption 3. **(Subordination)** *Any net claim of the non-defaulting party vis-à-vis the defaulting party will be completely lost.*

As an additional matter, the agreement must be specific about what happens when the defaulting party has a claim that exceeds the claim of the non-defaulting party. A very primitive form of netting would imply that the non-defaulting party ends up with a windfall profit. For instance, in the case

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$\widetilde{v}_b$ should be replaced by the risk-adjusted present value of the cash flow generated for the lender by holding the collateral until maturity net of costs of funding, all projected conditional on the borrower's default.

of the borrower's default, the lender could sell the collateral and keep any potential interim increase in the market price. Similarly, in the case of the lender's default, the borrower would profit from a decline in the collateral value. This primitive form of netting is not applied in the money market.[18]

Assumption 4. (No windfall profits) *If the defaulting party has a net claim vis-à-vis the non-defaulting party then the non-defaulting party has the obligation to pay the net claim (to the insolvency agent of the defaulting party).*

Assumptions 2 through 4 complement the contract and thereby determine conditional expected utilities for the two counterparties. Write $u_L(.) = u_{i_L}(.)$ and $u_B(.) = u_{i_B}(.)$. Let $\widetilde{u}_L$ and $\widetilde{u}_B$, respectively, denote the lender's and the borrower's uncertain terminal utility at the time of contracting. Then the lender's expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B E[u_L(\min\{\widetilde{v}_b - 1; r\})], \tag{1}$$

where $E[.]$ denotes the unconditional expectation operator. Similarly,

$$E[\widetilde{u}_B] = \pi_G u_B(-r) + \pi_L E[u_B(\min\{1 - \widetilde{v}_a; -r\})] \tag{2}$$

will be the borrower's expected utility at the time of contracting.[19]

[18] In practice, no-fault termination is excluded in standard repurchase agreements, i.e., no single counterparty can just walk away from the contract before maturity.

[19] In general, counterparties' actual returns may differ from expressions given in (1) and (2) as a consequence of accounting rules. More specifically, Griffiths and Winters (1997, p. 819) report that in the US, government and agency repos do not affect required reserves for a depository institution, whereas private-issue repos are exempt from Federal Reserve Board Regulation D. This might imply an indirect cost of using private-issue collateral in the US repo market. This effect, however, is absent in the Euro area, because the Eurosystem generally applies a zero reserve ratio to all repo liabilities (cf. ECB 2005, p. 57).

From the explicit expressions for the counterparties' expected utilities, it is immediate that the two-sided credit risk could be effectively eliminated by using a type of collateral that shares the desirable properties of cash in terms of risklessness and liquidity. However, such collateral is very unlikely to exist in reality. To account for imperfections of collateral, we shall assume a linear ordering of collateral assets along a joint liquidity/riskiness dimension. This ordering is inspired by strict second-order stochastic dominance, but is technically somewhat stronger than standard definitions. Let $\widetilde{p}_b^j$ (and $\widetilde{p}_a^j$) denote the liquidation value (replacement cost) of asset j, conditional on the borrower's (lender's) default.

Assumption 5. (Liquidity ranking) *There are constants* $\mu_1 > 0, ..., \mu_m > 0$*, and a collection of random variables* $\widetilde{\varepsilon}_b^1, \widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_b^m, \widetilde{\varepsilon}_a^m$*, all of which possess densities, such that* $\widetilde{\varepsilon}_b^1, ..., \widetilde{\varepsilon}_b^m$ *are mutually independent,* $\widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_a^m$ *are mutually independent, and such that*

$$\begin{aligned} \widetilde{p}_b^j &\equiv \mu_j \widetilde{p}_b^{j-1} - \widetilde{\varepsilon}_b^j \text{ with } E[\widetilde{\varepsilon}_b^j] > 0 \qquad (j = 1, ..., m), \text{ and} \\ \widetilde{p}_a^j &\equiv \mu_j \widetilde{p}_a^{j-1} + \widetilde{\varepsilon}_a^j \text{ with } E[\widetilde{\varepsilon}_a^j] > 0 \qquad (j = 1, ..., m), \end{aligned}$$

where $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$.

Here, the symbol $\equiv$ denotes equality in distribution. The constant μ_j takes account of the possibility that "relative market values" of collateral assets, measured for instance by the midpoint of ask and bid prices, may change between the time of contracting and date 2.[20] The random variables $\widetilde{\varepsilon}_b^j$ and

[20] Our analysis does not presuppose marketability of collateral assets at the time of contracting. However, there is one interpretation of the model in which all collateral assets are perfectly liquid at the time of contracting and possess a market value of 1 at

$\widetilde{\varepsilon}_a^j$ capture both illiquidity and liquidity risk on the sell side (buy side) of the asset market at date 2, conditional on the default of the lender (of the buyer). Thus, a collateral asset with a higher index is assumed to be less liquid and subject to more liquidity risk than any collateral asset with a lower index. Clearly,

$$\widetilde{v}_b = (1+h)\sum_{j=1}^{m} y_j \widetilde{p}_b^j, \tag{3}$$

and

$$\widetilde{v}_a = (1+h)\sum_{j=1}^{m} y_j \widetilde{p}_a^j. \tag{4}$$

It is assumed in the sequel that the multivariate conditional distributions of the vectors $(\widetilde{p}_b^1, ..., \widetilde{p}_b^m)$ and $(\widetilde{p}_a^1, ..., \widetilde{p}_a^m)$ are common knowledge among market participants.

A scenario will be considered now in which lender and borrower bargain to an efficient outcome. Let $q_j^i \geq 0$ denote bank i's initial endowment of collateral asset j, for $i = 1, 2$ and $j = 1, ..., m$. Apparently, the bargaining set for borrower and lender will consist of all standard repurchase agreements (y, h, r) with collateral composition $y = (y_1, ..., y_m)$ that satisfy

$$y_j(1+h) \leq q_j^{i_B} \qquad (j = 1, ..., m). \tag{5}$$

An SRA that satisfies (5) will be called *valid.* A valid SRA is *efficient* when the pair of counterparties' expected utilities resulting from the contract is not dominated, in the Pareto sense, by expected utilities resulting from any other valid SRA.

that stage. Note also that if collateral assets are assumed to be marketable both at the time of contracting and in the good state, outright trading becomes an alternative to the repo, and expected round-trip costs may impose a bound on implicit opportunity rates (cf. Section II).

Given our assumptions, it turns out that any efficient SRA will expose both the lender and the borrower to non-trivial credit risk.

Theorem 1 (Risk sharing). *Under Assumptions 1 through 5, for any efficient SRA,* $\mathrm{pr}(\widetilde{v}_b < 1 + r) > 0$ *and* $\mathrm{pr}(\widetilde{v}_a > 1 + r) > 0$.

Indeed, under the assumptions made, it cannot be efficient to protect one counterparty fully. To see why, assume that the lender, say, is fully protected against any losses. Then a marginal decrease of the haircut may lead to an infinitesimally small loss for the lender, but this loss occurs only with an infinitesimally small probability. As a consequence, the expected utility of a fully protected lender is not lowered by a marginal concession in the haircut. However, for the borrower, who is not fully protected, a marginal decrease in the haircut reduces losses that occur with strictly positive probability. Hence, when the lender is fully protected, the relative willingness to pay (in terms of utility) for a concession in the repo rate compared to a concession in the haircut is infinite for the lender, but finite for the borrower. Thus, full protection of the lender cannot be efficient. A similar argument shows that full protection of the borrower likewise cannot be Pareto optimal. Thus, optimal risk sharing must be true risk sharing.[21]

From this general theoretical insight, the following testable characterization of the market contract can be derived.

Theorem 2 (Gresham's law for collateral, market version). *Under*

[21] Interior risk sharing may break down when the conditional liquidation value (replacement cost) is bounded from below (above) by a mass point. For instance, when there is a partial guarantee for the collateral by a third party, then it may be optimal to fully insure the lender against the residual counterparty risk. A mass point at zero, however, does not invalidate the conclusion of Theorem 1.

Assumptions 1 through 5, any efficient SRA (y^*, h^*, r^*) *entails the collateral composition*

$$y^* = \left(\frac{q_1^{i_B}}{1+h^*}, ..., \frac{q_{j^*-1}^{i_B}}{1+h^*}, 1 - \frac{\sum_{j=1}^{j^*-1} q_j^{i_B}}{1+h^*}, \underbrace{0, ..., 0}_{m-j^*\ times}\right),$$

where j^* *is the smallest index such that* $\sum_{j=1}^{j^*} q_j^{i_B} \geq 1 + h^*$.

Proof. See the Appendix. □

Thus, provided that rational counterparties reach an efficient outcome, good collateral is used up first in the interbank lending relationship. Illiquid collateral is not used in the market because it would not allow counterparties to share their risks resulting from the agreement as efficiently as liquid collateral. Theorem 2 thereby offers an explanation for the empirical finding mentioned in the Introduction that interbank repos are so much concentrated on liquid collateral.

The intuition for Theorem 2 is as follows. Assume that borrower and lender consider a collateral composition that contains some relatively illiquid collateral even though the borrower would be able to offer somewhat more of a relatively liquid collateral. I.e., it would be possible for the counterparties to replace a fraction of the illiquid collateral by a portion of the more liquid collateral. As we show in the Appendix, there is then a joint adjustment to composition and haircut that strictly reduces the exposure of both counterparties to counterparty risk. This Pareto ranking can be achieved essentially because counterparty's indirect utility functions (1) and (2) are weakly concave. Any improvement in the liquidity of the collateral portfolio thereby weakly raises expected utilities for both lender and borrower at the time of contracting. In fact, Theorem 1 implies that the kinks in the indirect utility functions are hit by realizations of the uncertainty with positive probability, so that in fact a strict gain in utility is achieved for both counterparties. This argument shows, therefore, that it is optimal to fully use up the most liquid collateral first in the interbank lending relationship.[22]

If credit risk is one-sided only (and collateral is ample), the economic characteristics of the collateral asset should play a subordinated role. For instance, if the lender cannot default then the borrower could in many cases offer even very illiquid assets as collateral. Indeed, provided that the liquidation value of the collateral asset is bounded away from zero, a sufficiently large haircut would fully protect the lender against any credit risk.[23] Conversely, when the borrower cannot default, no collateral is needed in the first place.

II. Feasibility of the market transaction

In this section, it is shown that interbank lending may not be feasible even if collateral causes no opportunity costs, information is symmetrically distributed, and physical transaction costs are zero. Sufficient conditions for a market break-down are that both banks default with positive probability and that assets that are available as collateral are not perfectly liquid or else not absolutely risk-free.

But indeed, counterparties will approve a contract only when it is indi-

[22] Also security-driven repurchase agreements tend to concentrate on liquid assets. This is because of dynamic shorting strategies that rely on the trader's ability to close the position potentially at very short notice. We are grateful to Darrell Duffie for pointing this out to us.

[23] Otherwise, the borrower of cash would of course offer all available collateral.

vidually rational to do so. We assume outside options guaranteeing utilities of $\underline{u}_L = (\pi_G + \pi_B)u_L(r^D)$ to the lender and $\underline{u}_B = (\pi_G + \pi_L)u_B(-r^L)$ to the borrower, respectively, where $r^D = r^D(i_L)$ is the lender's implicit risk-free opportunity deposit rate, and $r^L = r^L(i_B)$ is the borrower's implicit unsecured opportunity lending rate. In practice, effective outside options might include capital market transactions (a bond issue, say), outright transactions (provided that collateral assets are marketable at the time of contracting, cf. footnote 20), money market transactions with non-banks, recourses to the central bank's standing facilities, renegotiation of contractual obligations, accepting a contractual penalty, etc. In the worst case, banks might become even more reluctant to offer credit to non-banks.

For any given r^D, denote by $\rho^D(h)$ the lowest repo rate that a lender would be willing to accept for a given haircut h. Clearly, $\rho^D(h) \geq r^D$. Similarly, for any given opportunity rate r^L, denote by $\rho^L(h) \leq r^L$ the highest repo rate that the borrower would accept for a given haircut h. Figure 2 illustrates $\rho^D(h)$ and $\rho^L(h)$ for a numerical example. Note that the cut-off rate for both lender and borrower is declining in the haircut because a higher haircut implies improved (weakened) protection for the lender (borrower) that must be compensated by a lower (lower) repo rate.

Figure 2 about here

Theorem 3. (Market break-down) *Let Assumptions 1 through 5 be satisfied. Then, for any $r_0 > 0$, there is an implicit unsecured lending rate r^L for the borrower and an implicit risk-free deposit rate r^D for the lender such that $r^L > r_0 > r^D$, and such that with these opportunity rates, no market transaction is individually rational for both lender and borrower.*

Proof. See the Appendix. □

Theorem 3 offers an explanation for the observation that in times of financial distress and mutual distrust, financial institutions may not be willing to exchange liquidity against relatively illiquid collateral. In reality, such a break-down may be driven by several, mutually reinforcing factors. First, banks may perceive a higher probability of an individual default. Second, perceptions of potential illiquidity and riskiness may increase, making it more difficult to achieve conditions that are individually rational for both sides of the contract. Third, counterparties may also become more risk-averse. Fourth, there may be the fear that liquidity needs still increase. Finally, even if a counterparty would be willing to give cash for collateral today, this counterparty may be less confident that the collateral will be accepted tomorrow. The joint effect of such developments may lead to a disruption even of the "secured" segment of the interbank market.

Theorem 3 captures the fact that even in the repo market, a counterparty benefits significantly from contracting with a counterparty that has a good credit rating. In reality, this benefit should be reflected in the topology of the interbank network. Two types of regularities are predicted. First, counterparties with an excellent rating may be able to intermediate in the repo market. In practice, this should lead to a *two-tiered structure* of the repo market, just as predicted for the unsecured market by Freixas and Holthausen (2004). The second regularity should be the emergence of *central counterparty* trading. Indeed, while restricted to dominant players, central counterparty trading has recently gained momentum in the euro area.[24]

[24] Theorem 3 also provides a rationale for the use of maintenance margins in markets

The comparative statics for feasibility is as suggested by intuition. The lower the default probabilities of lender and borrower, respectively, the more liquid and less risky the collateral, and the less attractive the outside options, the more likely is the market transaction. Vice versa, the worse the rating of lender and borrower, the less liquid and more risky the collateral, and the more acceptable the outside options, the more likely is a break-down of the market relationship.[25]

Example 1. As an illustration, assume that both lender and borrower are risk-neutral. Assume also that the liquidation value and the replacement cost of the only available collateral asset is known to be p_b and p_a with certainty at date 1, respectively, where $p_a > p_b > 0$. Consider first the lender. Expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G r + \pi_B \min\{(1+h)p_b - 1; r\}.$$

It is not difficult to see that in any contractible (i.e., efficient) agreement (y, h, r), the lender will not be overprotected, i.e.,

$$(1+h)p_b \leq 1 + r. \tag{6}$$

This is because overprotection would be without value for the lender, but costly for the borrower. Thus,

$$E[\widetilde{u}_L] = \pi_G r + \pi_B((1+h)p_b - 1).$$

for repurchase agreements. Maintenance margins are an instrument that keeps collateral deposits for a cash-driven repo abreast with the development of the market price of the collateral. As our analysis suggests, this is a useful instrument to balance the interests of both counterparties also in transactions that have a longer maturity, i.e., term repos. Thus, intuitively, maintenance makes feasibility easier to achieve.

[25] Maybe interestingly, the analysis suggests that a borrower may find it easier to transact in the interbank market by offering the lender a collateral whose market value is positively correlated with the lender's survival, such as the lender's own uncovered bonds.

Comparing these expressions with the available outside option $\underline{u}_L$ for the lender yields that for a deposit rate r satisfying condition (6) of at least

$$\rho^D(h) = r^D + \frac{\pi_B}{\pi_G}(1 + r^D - (1+h)p_b),$$

the lender would be willing to contract against a haircut of h. On the other hand, when (6) is not satisfied, then the lender would be overprotected, and expect at least r^D. Thus, in general,

$$\rho^D(h) = r^D + \frac{\pi_B}{\pi_G}(1 + r^D - (1+h)p_b)^+, \tag{7}$$

where, as usual, $(x)^+ = x$ for $x > 0$ and $= 0$ otherwise. Using completely analogous arguments, one can see that the borrower would be willing to contract against a haircut of h if and only if the repo rate is at most

$$\rho^L(h) = r^L + \frac{\pi_L}{\pi_G}(1 + r^L - (1+h)p_a)^-, \tag{8}$$

where $(x)^- = x$ for $x < 0$ and $= 0$ otherwise.

Apparently, a repurchase agreement (r, h) is contractible between borrower and lender if and only if $\rho^D(h) \leq \rho^L(h)$ for some h. As the expressions (7) and (8) are piecewise linear, one can check that a contract is not feasible if and only if conditions

$$\rho^D(\frac{1+r^L}{p_a} - 1) > r^L$$

and

$$\rho^L(\frac{1+r^D}{p_b} - 1) < r^D$$

are simultaneously satisfied. Rewriting these conditions yields

$$\frac{\pi_B}{\pi_G + \pi_B} \cdot \frac{p_a - p_b}{p_a} > \frac{r^L - r^D}{1 + r^L} \tag{9}$$

and

$$\frac{\pi_L}{\pi_G + \pi_L} \cdot \frac{p_a - p_b}{p_b} > \frac{r^L - r^D}{1 + r^D} \tag{10}$$

as intuitive conditions for contractibility. That is, in the case of risk-neutrality and risk-free but illiquid collateral, contracting is impossible if and only if both (9) and (10) are satisfied.[26]

Illiquidity of collateral assets might have played a role in recent market developments. On August 9, 2007, problems with subprime loans in the US led, among other things, to a sudden dry-out of the market for asset-backed commercial paper, which has served as a source of funding for so-called structured investment vehicles. Banks with credit commitments vis-à-vis such vehicles had an unexpected increase in liquidity needs. Long-term assets held by the vehicles, such as collateralized debt obligations, could no longer serve as collateral. At the same time, those investors that had refused to roll over commercial paper have received significant cash transfers to their bank accounts.

Kashyap, Rajan, and Stein (2002) have put forward the argument that commercial banks have the unique ability to pool imperfectly correlated liquidity risks resulting from loan commitments and deposit contracts. Gatev and Strahan (2006) find empirical support for a similar mechanism in the context of the commercial paper market. The stylized facts mentioned above might relate our analysis to the pooling argument. Specifically, one could argue that before the turbulences, numerous banks might have decided to

[26] A closer inspection of Example 1 also shows that with a degenerate price distribution and with risk-neutrality, it can be efficient to protect one party fully against any credit risk.

specialize and to exploit the synergies identified by Kashyap et al. across the money market, assuming that liquidity risks can be shared effectively with other banks. Then, during the turbulence, some of those banks (e.g., investment banks) would have to satisfy a loan commitment, while others would receive a liquidity inflow in the form of additional deposits. However, in view of Theorem 3, a market transaction that matches supply and demand may not be guaranteed. Thus, using the terminology introduced by Kashyap et al., with specialized banks, synergies across banks may become a prerequisite to synergies across the two sides of the balance sheet.

III. Welfare implications

In the previous sections, it has been shown that with two-sided credit risk, counterparties seek to use the most liquid and least risky assets as collateral first. A policy issue may arise here when central bank operations have the potential to withhold liquid collateral assets from uses in the interbank market. To address this issue, an extension of the basic model will be considered in which banks forward collateral also to the central bank. Examined will be the consequences on welfare of changing the central bank's collateral policy.[27]

Thus, in contrast to the set-up considered so far, it is assumed now that from date 0 onwards, Bank 1 and Bank 2 have debt of $D_1 \geq 0$ and $D_2 \geq 0$, respectively, outstanding vis-à-vis the central bank (cf. Figure 1). We also assume throughout this section that the size of the liquidity shock λ is the realization of a random variable $\widetilde{\lambda}$ with full support on $\mathbb{R}_{>0}$.

Figure 1 about here

It is assumed that the central bank exerts its influence on the money

[27] The question of why apparently all central banks do require collateral is not addressed in this paper. For a comprehensive discussion of this point, see ECB (2007b).

market by two policy choices. At date 0, the central bank decides about its *collateral policy*. Specifically, it is assumed that the central bank chooses a set $J = \{1, ..., m^{\mathrm{CB}}\}$ of eligible assets, where $1 \leq m^{\mathrm{CB}} \leq m$. Only assets contained in the set J will be accepted as collateral in central bank operations. Let $\eta_j \geq 0$ denote the exogenous haircut applied by the central bank to asset $j \in J$. In contrast to the interbank market, these haircuts are not subject to negotiation.

Second, the central bank exerts influence on the money market by affecting the relative bargaining power of lender and borrower in the market. For specificity, it is assumed that the central bank chooses, immediately following the liquidity shock, a *liquidity policy* $\alpha = (\alpha_L, \alpha_B)$ such that $\alpha_L \geq 0$, $\alpha_B \geq 0$, and $\alpha_L + \alpha_B = 1$; counterparties then determine the terms of the SRA at the contracting stage using the Nash bargaining solution, where α_L becomes the bargaining power of the lender, and α_B becomes the bargaining power of the borrower.

Denote by $\theta^i = \theta^i(J)$ the composition of bank i's collateral deposits, net of haircuts, with the central bank at date 0. Note that by the definition of the collateral policy, $\theta^i_j = 0$ for $j \notin J$. In line with the institutional environment in the euro area, it is assumed that each bank i may at any point in time change the collateral composition with the central bank as long as the total market value of the collateral net of haircuts remains at least D_i. Such replacement may indeed occur, in particular when the bank i_B that turns out to be the borrower wishes to replace liquid by illiquid collateral at the time of contracting to free liquid collateral for an interbank transaction.

Substitution is not necessary, though. In our framework, there are in principle two reasons why relatively liquid collateral may be kept with the central bank. One potential reason is that the maximal size of the liquidity shock expected in the interbank market is small, so that there is no need to optimize collateral usage vis-à-vis the central bank. The reader will note that we have chosen to exclude this possibility by imposing a full-support assumption on $\widetilde{\lambda}$, but it is clear that dropping this assumption would yield partial indeterminacy of central bank collateral. The second potential reason for not optimizing the portfolio of collateral held vis-à-vis the central bank is that the secured market is rationally expected to break down. To exclude this possibility, we impose another assumption. For simplicity, we will assume that a market break-down may occur only when the liquidity shock is excessively large, i.e., only when the shock exceeds the maximum quantity of liquidity that the central bank is willing to lend to the counterparties.

Assumption 6. (No crowding-out) *For $i = 1, 2$, there is no market break-down for any $\lambda < \sum_{j \in J} q_j^i / (1 + \eta_j)$.*

The following definition turns out to be useful. For a given central bank policy (J, α), a pair of collateral compositions (θ^1, θ^2) for Banks 1 and 2, respectively, will be called *stable* if there is, for any realization of $i_B = 1, 2$, and for any realized liquidity shock $\lambda > 0$, either a break-down or a Pareto efficient SRA between Banks 1 and 2 that does not imply the replacement of collateral deposited with the central bank. We are ready to formally capture the residual characteristic of central bank collateral.

Theorem 4. (Gresham's law for collateral, central bank version) *Let Assumptions 1 through 6 be satisfied. Assume that $(\theta^1(J), \theta^2(J))$ is stable.*

Then for $i = 1, 2$, the collateral composition is given by

$$\begin{aligned}\theta^i(J) &= (\underbrace{0,...,0}_{j_*(i) \text{ times}}, 1 - \sum_{j=j_*(i)+1}^{m^{\text{CB}}} \frac{q_j^i}{(1+\eta_j)D_i}, \frac{q_{j_*(i)+1}^i}{(1+\eta_{j_*(i)+1})D_i}, ... \\ &\quad ..., \frac{q_{m^{\text{CB}}}^i}{(1+\eta_{m^{\text{CB}}})D_i}, \underbrace{0,...,0}_{m-m^{\text{CB}} \text{ times}}),\end{aligned}$$

where $j_*(i)$ denotes the largest index such that $\sum_{j=j_*(i)}^{m^{\text{CB}}} q_j^i/(1+\eta_j) \geq D_i$.

Proof. See the Appendix. □

Theorem 4 captures the observations discussed in the Introduction by suggesting that commercial banks have an incentive to use less liquid and more risky assets with preference in central bank operations. Indeed, as more liquid and less risky assets allow a better risk sharing in interbank repo transactions, there is an endogenous opportunity cost of using the more liquid and less risky assets vis-à-vis the central bank. Moreover, the residual nature of central bank collateral should become more evident in times of increasing liquidity risks.

Our analysis should also help to clarify the role of haircuts applied by the central bank. Haircuts have always been an instrument of risk management, both for commercial banks and for central banks. However, as Theorem 4 shows, there is only a very limited role for haircuts as an instrument to steer the composition of central bank collateral. Indeed, the opportunity costs of using the least liquid and most risky assets accepted by the central bank will remain negligible as long as the borrower's holdings of such assets are ample enough. Changing haircuts should therefore not be sufficient to induce commercial banks to use more liquid and less risky collateral vis-à-vis the central bank. In particular, haircuts are not an instrument for fine-tuning the composition of collateral along, say, issuing fiscal authorities. This point addresses a question of a significant practical interest (cf. Fels, 2005, for instance).[28]

To evaluate the welfare consequences of the collateral framework, it is useful to note that the central bank is always in the position to effectively limit its exposure from repo operations vis-à-vis counterparties that have ample collateral. Indeed, given its standing as a monetary authority, our earlier remark at the end of Section I should apply also here, i.e., there is no market disruption even when haircuts required to limit the central bank's exposure are relatively large. Motivated by this consideration, we will analyze welfare without explicit reference to the central bank and exclusively in terms of expected utilities for lender and borrower.

Two hypothetical scenarios are compared now where the central bank may either pursue a tight or a generous stance concerning the acceptance of collateral. Moreover, adding realism, we will allow that the borrower's opportunity rate $r^L = r^L(i_B, J)$ may depend also on the central bank's collateral framework.

Theorem 5. (Welfare consequences) *Let Assumptions 1 through 5 be satisfied. Fix some policy (J, α), a collateral set $J' \supseteq J$, and some i_B. Assume that $r^L(i_B, J') \leq r^L(i_B, J)$. Then for $E[\widetilde{u}_B|(J, \alpha)] \geq \underline{u}_B(J')$, there is a liquidity policy α' such that both $E[\widetilde{u}_B]$ and $E[\widetilde{u}_L]$ increase weakly.*

[28] Alternatively, one might want to apply different pricing to different collateral, e.g., by using variable-rate tenders for given quantities in each liquidity basket. However, this strategy may not be practicable under all circumstances (cf. Federal Reserve Bank of New York, 2008).

Proof. See the Appendix. □

Theorem 5 contains a prediction concerning the welfare implications of an expanded collateral set. It says that if an extension of the set of collateral assets accepted by the central bank is accompanied by an appropriate liquidity policy α', this may increase expected utilities for both lender and borrower. The reason is that a less restrictive collateral policy allows counterparties to use more liquid and less risky collateral in the interbank repo market. Maybe it should be stressed at this point that the weak increase of expected utility for both lender and borrower implies that the certainty-equivalent interest rates for the two counterparties move closer together. In fact, by definition, any liquidity policy α' that, compared to the tight collateral regime J combined with liquidity pocliy α, increases the implicit risk-free deposit rate for the lender and decreases the implicit unsecured lending rate for the borrower will produce the welfare gain. The policy change suggested by Theorem 4 is therefore consistent with the view that the central bank is mainly in the market to steer interbank conditions, and that welfare maximization through the collateral framework is subject to this important constraint.

Note that the welfare gain is not certain. Specifically, there might be a loss of expected utility for the lender if the expansion of the collateral set improves the outside option for the borrower. This scenario is more likely when the borrower is close to the outside option with the more restrictive policy. On the other hand, the loss of interim utility for the lender may sometimes be more than compensated from an ex ante perspective when the roles of lender and borrower are not yet assigned.[29]

[29] Collateral policy might affect market activity in other ways than suggested by Theo-

To illustrate Theorem 5, we briefly consider the cases of the US Fed, the Bank of England, the Bank of Canada, and the Bank of Australia. Before the start of the turmoil in August 2007, these central banks generally accepted only a very narrow range of assets, mainly government bonds, as collateral. During the turbulences, however, all of these institutions significantly broadened the range of eligible collateral. Theorem 5 provides a rationale for such policy adjustment.[30]

On the other hand, in view of Theorems 2 and 3, it may well be that the collateral potentially unleashed by an enlargement of the set of eligible collateral will not be used in the market. It could be argued that this is the present situation in the euro area given that the Eurosystem already accepts a very broad list of assets as collateral. Then, it would not be the case that too much precious collateral is bound in transactions with the central bank. Widening the set of eligible collateral would, therefore, be unlikely to re-establish the proper working of the money market. Indeed, the current problems in the repo market seem to be linked rather to a general concern about the quality of collateral assets and a mutual mistrust between

rems 3 and 4. Firstly, the usual *moral hazard* caveat applies. After all, accepting illiquid collateral, especially during times of market stress, works like an insurance of commercial banks against temporary funding problems. Secondly, to the extent that repricing risk of illiquid assets may trigger margin calls, *liquidity risks* of commercial banks might actually increase. Finally, there may be an impact on *relative asset prices*.

[30] For the case $E[\widetilde{u}_B|(J,\alpha)] < \underline{u}_B(J')$ that is not considered in Theorem 5, we informally note that the lender may in principle be worse off following an extension of the range of accepted collateral. There are two potential reasons for why this might be case. One reason is that the lender might have had a very strong bargaining position in the tight environment, which is lost when the central bank changes its policy. Another potential reason is that there may be a crowding-out of the market transaction. Also this may mean a loss for the lender, but again only when his bargaining position under the tight policy had been strong.

commercial banks.[31]

IV. Conclusion

Modern liquidity management increasingly relies on repurchase agreements through which cash is exchanged short-term against collateral assets of longer maturities. Interestingly, almost all such refinancing is based on securities that are very stable in value and actively traded. Market requirements on asset liquidity became even stricter when interbank market conditions tightened, as during the credit crunch following August 2007. On the other hand, there has been a tendency to deposit more and more illiquid assets for use in central banks' liquidity-providing operations.

The present study has derived a number of theoretical predictions that clarify and explain these and related observations. First, it has been shown that if there is a choice of collateral in a market transaction, then the most liquid and least risky asset will allow borrower and lender to achieve the most efficient risk-sharing. However, if the best collateral available is still relatively illiquid or risky, and if there is non-negligible bilateral counterparty risk, then no market transaction may come about at all. This point has allowed us to apply a theoretical argument put forward recently by Kashyap, Rajan, and Stein (2002). As regards to policy implications, it has been shown that a less restrictive collateral policy applied by a central bank may lead to a welfare improvement for market participants. Yet, the analysis also suggests that

[31] To the extent that the *precautionary* demand for collateral that can be used with the Eurosystem is high, as suggested by media reports (cf. Financial Times, 2008), a relaxation of the criteria for collateral would of course help to improve commercial banks' outside option in case of market breakdown. However, the comparative statics of feasibility (cf. Section II) suggests that this would make a market breakdown even more likely.

essentially unaffected by the haircut requirement, the least liquid and most risky assets will be deposited with the central bank.

The analysis provides a rationale for the decisions of several central banks to broaden the range of assets accepted as collateral during the turmoil that started in August 2007. For the euro area, the analysis comes to the conclusion that a widening of the set of eligible collateral would not necessarily be or have been supportive for a resolution of market disruptions. As there is no evidence that too much high quality collateral is bound in central bank operations, the benefit of unleashing collateral of intermediate liquidity into the market might turn out to be very limited. Instead, problems with secured lending seem to be related to a general concern about the quality of collateral assets and to a mutual mistrust in particular between banks.

The situation might have been different in the US. Since the start of the market turbulences, the Federal Reserve System has repeatedly taken measures that aimed at making a broader collateral base available. Moreover, in a quite unconventional move, the Federal Reserve decided, effective on Tuesday, March 11, 2008, to offer primary dealers an amount of $200 bn in Treasury bonds and bills in exchange for mortgage-backed securities after spreads for the latter instruments widened dramatically.[32] As our analysis shows, such measures will be directly beneficial for the banking sector to the extent that illiquidity of collateral assets impairs the functioning of the money market.

[32] Cf. Wall Street Journal (2008). More recently, the Bank of England has implemented similar measures, yet on a smaller scale.

Appendix: Proofs

Proof of Theorem 1. Let $\widetilde{p}_b = \sum_{j=1}^{m} y_j \widetilde{p}_b^j$ denote the conditional liquidation value of the collateral portfolio net of haircuts, and let $F_b(.)$ be the corresponding distribution function. Then, re-writing (1) using integral notation, the lender's expected utility at the time of contracting reads

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \int u_L(\min\{(1+h)p_b - 1; r\}) dF_b(p_b), \qquad (11)$$

where p_b denotes the realized value of $\widetilde{p}_b$. The integrand in (11) will be $u_L(r)$ for all $p_b > p^* = (1+r)/(1+h)$, and $u_L((1+h)p_b - 1)$ otherwise, where $p^* = \infty$ for $h = -1$. Consequently, (11) can be re-written as

$$\begin{aligned} E[\widetilde{u}_L] &= (\pi_G + \pi_B(1 - F_b(p^*)))u_L(r) \\ &\quad + \pi_B \int_{p_b \leq p^*} u_L((1+h)p_b - 1) dF_b(p_b). \end{aligned} \qquad (12)$$

Clearly, by Assumption 5, $F_b(.)$ is differentiable. Therefore, using Leibnitz' rule,

$$\begin{aligned} \frac{\partial E[\widetilde{u}_L]}{\partial r} &= (\pi_G + \pi_B(1 - F_b(p^*)))u_L'(r) - \pi_B \frac{\partial p^*}{\partial r} F_b'(p^*) u_L(r) \\ &\quad + \pi_B \frac{\partial p^*}{\partial r} F_b'(p^*) u_L((1+h)p^* - 1) \\ &= (\pi_G + \pi_B(1 - F_b(p^*)))u_L'(r), \end{aligned}$$

where $F_b'(.)$ denotes the derivative of $F_b(.)$. Likewise,

$$\begin{aligned} \frac{\partial E[\widetilde{u}_L]}{\partial h} &= -\pi_B \frac{\partial p^*}{\partial h} F_b'(p^*) u_L(r) + \pi_B \int_{p_b \leq p^*} p_b u_L'((1+h)p_b - 1) dF_b(p_b) \\ &\quad + \pi_B \frac{\partial p^*}{\partial h} F_b'(p^*) u_L((1+h)p^* - 1) \\ &= \pi_B \int_{p_b \leq p^*} p_b u_L'((1+h)p_b - 1) dF_b(p_b) \end{aligned}$$

Therefore, the marginal rate of substitution between haircut and repo rate for the lender is given by

$$\begin{aligned}\text{MRS}^L_{h,r} &= \frac{\partial E[\widetilde{u}_L]/\partial r}{\partial E[\widetilde{u}_L]/\partial h} \\ &= \frac{(\pi_G + \pi_B(1 - F_b(p^*)))u'_L(r)}{\pi_B \int_{p_b \le p^*} p_b u'_L((1+h)p_b - 1)dF_b(p_b)}.\end{aligned}$$

A completely analogous derivation for the borrower yields the marginal rate of substitution between haircut and repo rate

$$\begin{aligned}\text{MRS}^B_{h,r} &= \frac{\partial E[\widetilde{u}_B]/\partial r}{\partial E[\widetilde{u}_B]/\partial h} \\ &= \frac{(\pi_G + \pi_L F_a(p^*))u'_B(-r)}{\pi_L \int_{p_a \ge p^*} p_a u'_B(1 - (1+h)p_a)dF_a(p_a)} \qquad (13)\end{aligned}$$

for the borrower, where $F_a(.)$ denotes the distribution function of $\widetilde{p}_a = \sum_{j=1}^m y_j \widetilde{p}^j_a$, and p_a the realized value of $\widetilde{p}_a$ at date 2. To provoke a contradiction, assume that $F_a(p^*) = 1$. Then clearly, from Assumption 1, $\pi_G + \pi_L F_a(p^*) > 0$. Moreover, as the distribution of $\widetilde{p}_a$ does not possess any mass points by Assumption 5, the denominator in (13) vanishes. Thus, $\text{MRS}^B_{h,r} = \infty$. On the other hand, by Lemma A.1, $F_b(p^*) > 0$ and therefore $\text{MRS}^L_{h,r} < \infty$. Hence, in any efficient agreement, $F_a(p^*) < 1$, or equivalently,

$$\text{pr}\{\widetilde{v}_a > 1 + r\} = \text{pr}\{(1+h)\widetilde{p}_a > 1 + r\} > 0.$$

Analogously, if $F_b(p^*) = 0$, then $\text{MRS}^B_{h,r} < \infty$, while $\text{MRS}^L_{h,r} = \infty$. Hence, $F_a(p^*) < 1$, which is tantamount to $\text{pr}\{\widetilde{v}_b < 1 + r\} > 0$. Thus, a boundary solution in which one party is fully protected can never be efficient. □

Lemma A.1. *Under Assumption 5, there is no collateral composition* $y = (y_1, ..., y_m)$ *such that* $\text{pr}\{\sum_{j=1}^m y_j \widetilde{p}^j_b \ge p^*\} = \text{pr}\{\sum_{j=1}^m y_j \widetilde{p}^j_a \le p^*\} = 1$ *for some cut-off value* p^*.

Proof. Immediate. □

Proof of Theorem 2. Consider a valid SRA $C = (y, h, r)$ with collateral composition $y = (y_1, ..., y_m)$. Then

$$0 \leq (1 + h)y_j \leq q_j^{i_B} \qquad (j = 1, ..., m), \tag{14}$$

and

$$\sum_{j=1}^{m} y_j = 1. \tag{15}$$

It suffices to show that it is Pareto dominated for lender and borrower to simultaneously use one collateral asset and not fully use up another collateral with a lower index. To provoke a contradiction, assume that

$$y_{k+1} > 0 \text{ and } (1 + h)y_k < q_k^{i_B} \tag{16}$$

for some $k \in \{0, ..., m - 1\}$. By Assumption 5, there are constants $\mu_1 > 0, ..., \mu_m > 0$, and independent random variables $\widetilde{\varepsilon}_b^1, \widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_b^m, \widetilde{\varepsilon}_a^m$, such that for $j = 1, ..., m$,

$$\widetilde{p}_b^j \equiv \mu_j \widetilde{p}_b^{j-1} - \widetilde{\varepsilon}_b^j \tag{17}$$

$$\widetilde{p}_a^j \equiv \mu_j \widetilde{p}_a^{j-1} + \widetilde{\varepsilon}_a^j \tag{18}$$

and such that $E[\widetilde{\varepsilon}_b^j] > 0$, $E[\widetilde{\varepsilon}_a^j] > 0$ and $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$. We will construct a new SRA (y', h', r') with collateral composition $y' = (y_1', ..., y_m')$ that satisfies

$$\begin{aligned} y_k'(1 + h') &> y_k(1 + h) \\ y_{k+1}'(1 + h') &< y_{k+1}(1 + h) \\ y_j'(1 + h') &= y_j(1 + h) \qquad (j \neq k, k + 1). \end{aligned}$$

This can be achieved as follows. Let $\delta > 0$ be small. Define the new SRA $C'(\delta) = (y', h', r')$ by

$$h' = \frac{1+h}{1-(\mu_{k+1}-1)\delta} - 1, \tag{19}$$

$$y'_k = (1-(\mu_{k+1}-1)\delta)y_k + \mu_{k+1}\delta, \tag{20}$$

$$y'_{k+1} = (1-(\mu_{k+1}-1)\delta)y_{k+1} - \delta, \tag{21}$$

$$y'_j = (1-(\mu_{k+1}-1)\delta)y_j \qquad (j \neq k, k+1), \tag{22}$$

and $r' = r$. Clearly, for δ small enough, the haircut h' is well-defined. Moreover, using (14), (16), and

$$1 + h = (1-(\mu_{k+1}-1)\delta)(1+h'),$$

it is straightforward to check that for δ small enough, we have

$$0 \leq (1+h')y'_j \leq q_j^{i_B} \qquad (j = 1, ..., m).$$

Another straightforward calculation exploiting (20) through (22) as well as (15) shows that $\sum_{j=1}^{m} y'_j = 1$. Hence, for δ small enough, the contract $C'(\delta)$ is well-defined and valid. It is claimed now that $C'(\delta)$ achieves a strict Pareto improvement over C. It turns out that in fact, for $\delta > 0$ small enough, the utility level expected at the contracting stage increases strictly for both lender and borrower. To see why, consider first the conditional liquidation value

$$\widetilde{v}'_b = (1+h')\sum_{j=1}^{m} \widetilde{p}_b^j y'_j$$

of the collateral portfolio deposited under the new agreement. Using (20) through (22), one obtains

$$\widetilde{v}'_b \equiv (1+h)\sum_{j=1}^{m} \widetilde{p}_b^j y_j + (1+h')\mu_{k+1}\delta\widetilde{p}_b^k - (1+h')\delta\widetilde{p}_b^{k+1}. \tag{23}$$

Using (3), and subsequently (17) for $j = k + 1$ delivers

$$\widetilde{v}_b' \equiv \widetilde{v}_b + (1 + h')\delta\widetilde{\varepsilon}_b^{k+1}.$$

Note that $\widetilde{v}_b$ and $\widetilde{\varepsilon}_b^{k+1}$ are not independent in general, and we have too little information about the conditional expectation $E[\widetilde{\varepsilon}_b^{k+1}|\widetilde{v}_b]$ to use second-order stochastic dominance at this stage. We shall therefore re-write $\widetilde{v}_b'$ as the sum of two independent random variables. For this, note that a straightforward induction argument involving Assumption 5 shows that

$$\widetilde{v}_b \equiv (1 + h)\sum_{j=1}^{m} y_j \widetilde{p}_b^j \equiv \gamma_0 - \sum_{j=1}^{m} \gamma_j \widetilde{\varepsilon}_b^j,$$

for constants

$$\gamma_j = (1 + h)\sum_{\widehat{j}=j}^{m} \mu_{\widehat{j},j} y_j \qquad (j = 0, ..., m)$$

defined recursively from $y_0 = 0$ and

$$\begin{aligned} \mu_{j,j} &= 1 & (0 \leq j \leq m), \\ \mu_{\widehat{j},j} &= \mu_{\widehat{j}} \cdot \mu_{\widehat{j}-1} \cdot ... \cdot \mu_{j+1} & (0 \leq j \leq \widehat{j} \leq m). \end{aligned}$$

Hence,

$$\widetilde{v}_b' = \widetilde{z} - \delta'\widetilde{\varepsilon}_b^{k+1}, \tag{24}$$

where $\widetilde{z}$ is independent from $\widetilde{\varepsilon}_b^{k+1}$, and

$$\begin{aligned} \delta' &= \gamma_{k+1} - (1 + h')\delta \\ &= \gamma_{k+1} - (1 + h)\frac{\delta}{1 - (\mu_{k+1} - 1)\delta}. \end{aligned}$$

Hence, to prove that the lender is strictly better off with the new agreement $C'(\delta)$ for a sufficiently small $\delta > 0$, it suffices to show that $\partial E[\widetilde{u}_L]/\partial\delta' < 0$, where the derivative is evaluated at $\delta' = \gamma_{k+1}$. Let $G(.)$ and $H(.)$ denote the distribution functions of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. Then, using (24), expected utility (1) for the lender at the time of contracting reads

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \iint u_L(\min\{z - \delta' \varepsilon_b^{k+1} - 1; r\}) dH(\varepsilon_b^{k+1}) dG(z), \quad (25)$$

where z and ε_b^{k+1} denote the realizations of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. We wish to show that $\partial E[\widetilde{u}_L]/\partial\delta' < 0$. The weak inequality would follow from more standard arguments (cf., for instance, Tesfatsion, 1976), but the strict inequality apparently has to be shown directly. The interior integral in (25) reads

$$\begin{aligned} & E[\widetilde{u}_L|\omega = B, \widetilde{z} = z] \\ = & \int u_L(\min\{z - \delta' \varepsilon_b^{k+1} - 1; r\}) dH(\varepsilon_b^{k+1}) \\ = & u_L(r) H(\frac{r - z + 1}{\delta'}) + \int_{\frac{r-z+1}{\delta'}}^{\infty} u_L(z - \delta' \varepsilon_b^{k+1} - 1) dH(\varepsilon_b^{k+1}), \end{aligned}$$

and can be differentiated with respect to δ' at $\delta' = \gamma_{k+1}$. We obtain

$$\begin{aligned} \frac{\partial}{\partial\delta'} E[\widetilde{u}_L|\omega = B, \widetilde{z} = z] &= -\int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} u'_L(z - \delta' \varepsilon_b^{k+1} - 1) dH(\varepsilon_b^{k+1}) \\ &\leq -u'_L(r) \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}), \end{aligned} \quad (26)$$

where the first inequality follows from the fact that $u'_L(.)$ is weakly declining. Now, by Assumption 5,

$$E[\widetilde{\varepsilon}_b^{k+1}] = \int_{-\infty}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) > 0, \quad (27)$$

so that

$$\frac{\partial}{\partial\delta'} E[\widetilde{u}_L|\omega = B, \widetilde{z} = z] \leq 0. \quad (28)$$

It suffices to show that (28) is strict for "sufficiently many" z. Recall that by Theorem 1, efficiency implies

$$\text{pr}\{\widetilde{v}_b' < 1 + r\} < 1.$$

Thus, by (24) and independence,

$$\text{pr}\{\widetilde{v}_b' < 1 + r\} = \int H(\frac{r - z + 1}{\delta'})dG(z) < 1.$$

Therefore, there must be a compact interval Z satisfying $\int_Z dG(z) > 0$ such that for any $z \in Z$, we have $H(\frac{r-z+1}{\delta'}) < 1$. Fix $z \in Z$. From (27) and $H(\frac{r-z+1}{\delta'}) < 1$, clearly

$$\begin{aligned} \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) &= (1 - H(\frac{r - z + 1}{\delta'}))E[\widetilde{\varepsilon}_b^{k+1} | \widetilde{\varepsilon}_b^{k+1} \geq \frac{r - z + 1}{\delta'}] \\ &\geq (1 - H(\frac{r - z + 1}{\delta'}))E[\widetilde{\varepsilon}_b^{k+1}] > 0, \end{aligned}$$

so that by (26), we find indeed that

$$\frac{\partial}{\partial \delta'} E[\widetilde{u}_L | \omega = B, \, \widetilde{z} = z] < 0.$$

Hence, $\partial E[\widetilde{u}_L]/\partial\delta' < 0$. Thus, for small enough $\delta > 0$, the lender's expected utility at the time of contracting is strictly increasing in δ. A completely analogous argument can be used to show that also the borrower's expected utility at the time of contracting is strictly increasing with a change from C to $C'(\delta)$. Hence, the assertion of the theorem follows. □

Proof of Theorem 3. In view of Theorem 2, we may assume without loss of generality that the borrower is equipped amply and exclusively with the most liquid and least risky collateral 1. Define r^L, r^D, h_0 as in Lemma A.3

below. Then $r^L > r_0 > r^D$. Moreover, for any haircut $h \geq -1$, either $h < h_0$ or $h \geq h_0$. If $h < h_0$, then $\rho^D(h) \geq \rho^D(h_0) > r^L \geq \rho^L(h)$, so there is no repo rate for which the market transaction is individually rational for lender and borrower at the same time. If $h \geq h_0$, then $\rho^L(h) \leq \rho^L(h_0) < r^D \leq \rho^D(h)$, and again no market transaction is feasible. This proves the assertion. $\square$

Lemma A.2. *Let Assumption 5 be satisfied. Then for any collateral composition $y = (y_1, ..., y_m)$, there is a cut-off price p^* such that* $\text{pr}\{\sum_{j=1}^m y_j \widetilde{p}_b^j < p^*\} > 0$ and $\text{pr}\{\sum_{j=1}^m y_j \widetilde{p}_a^j > p^*\} > 0$.

Proof. Immediate. $\square$

Lemma A.3. *There is a haircut $h_0 \geq -1$ and interest rates r^L, r^D satisfying $r^L > r_0 > r^D$ such that $\rho^D(h_0) > r^L$ and $\rho^L(h_0) < r^D$.*

Proof. By Lemma A.2, there is a cut-off price p^* for collateral 1 such that $F_b(p^*) > 0$ and $F_a(p^*) < 1$. Define the haircut h_0 by $p^* = (1 + r_0)/(1 + h_0)$. Let $r^D = r_0 - \varepsilon$ and $r^L = r_0 + \varepsilon$ for $\varepsilon > 0$ small. It will be shown that for ε small enough, $\rho^D(h_0) > r^L$ and $\rho^L(h_0) < r^D$. By the definition of $\rho^D(h_0)$,

$$(\pi_G + \pi_B)u_L(r^D) = (\pi_G + (1 - F_b(p_b^*))\pi_B)u_L(\rho^D(h_0)) + \pi_B \int_{p_b \leq p_b^*} u_L((1 + h_0)p_b - 1)dF_b(p_b), \tag{29}$$

where $p_b^* = (1 + \rho^D(h_0))/(1 + h_0)$. Re-arranging (29) yields

$$u_L(\rho^D(h_0)) = u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(p_b^*)\pi_B} \int_{p_b \leq p_b^*} (u_L(r^D) - u_L((1 + h_0)p_b - 1))dF_b(p_b),$$

where the integral is either positive or zero. To provoke a contradiction, assume that $\rho^D(h_0) \leq r^L$ for all small $\varepsilon > 0$. Then $p_b^* \leq \widehat{p}_b = (1+r^L)/(1+h_0)$, and consequently,

$$
\begin{aligned}
& u_L(\rho^D(h_0)) && (30)\\
\geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b \leq p_b^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b).
\end{aligned}
$$

For $p_b < \widehat{p}_a = (1+r^D)/(1+h_0)$, the expression integrated in (30) is positive, while for $p_b \geq \widehat{p}_a$, the expression is negative or zero. Hence, splitting the integral yields

$$
\begin{aligned}
& u_L(\rho^D(h_0)) \\
\geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\
& - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq p_b^*} (u_L((1+h_0)p_b - 1) - u_L(r^D))dF_b(p_b) \\
\geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\
& - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq \widehat{p}_b} (u_L((1+h_0)p_b - 1) - u_L(r^D))dF_b(p_b) \\
\geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\
& - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq \widehat{p}_b} (u_L(r^L) - u_L(r^D))dF_b(p_b).
\end{aligned}
$$

For $\varepsilon \to 0$, we would have $\rho^D(h_0) \to r_0$, and therefore in the limit

$$
\begin{aligned}
& u_L(\rho^D(h_0)) \geq && (31)\\
& u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(p^*)\pi_B} \int_{p_b < p^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b).
\end{aligned}
$$

Conditional prices for collateral have densities, so $F_b(.)$ and $F_a(.)$ are continuous. Hence, for any values $\widehat{p}_b, \widehat{p}_a$ close to p^* it is still true that $F_a(\widehat{p}_b) < 1$ and $F_b(\widehat{p}_a) > 0$. In particular, the integral in (31) is strictly positive. Using Assumption 1, we find a contradiction to the assumption that $\rho^D(h_0) \leq r^L$

for all small $\varepsilon > 0$. Thus, $\rho^D(h_0) > r^L$ for some sufficiently small ε. But for decreasing ε, the interest rate r^L is decreasing, while r^D is increasing so that $\rho^D(h_0)$ is non-decreasing. Hence, $\rho^D(h_0) > r^L$ for *any* sufficiently small ε. An analogous argument can be used to show that also $\rho^L(h_0) < r^D$ for all sufficiently small ε. Hence the assertion. □

Proof of Theorem 4. Assume that θ is stable. To provoke a contradiction, assume that collateral k is used vis-à-vis the central bank, but collateral $k+1 \leq m^{CB}$ is not at all or not exclusively used for the central bank. Formally, $\theta_k^{i_B} > 0$ and

$$D_{i_B}\theta_{k+1}^{i_B} < \frac{q_{k+1}^{i_B}}{1+\eta_{k+1}}. \tag{32}$$

Let λ be such that

$$\sum_{j=1}^{k}(q_j^{i_B} - (1+\eta_j)D_{i_B}\theta_j^{i_B}) < (1+h)\lambda < \sum_{j=1}^{k+1}(q_j^{i_B} - (1+\eta_j)D_{i_B}\theta_j^{i_B}).$$

Such a λ exists because of (32). Moreover, $\lambda < \sum_{j=1}^{m^{\mathrm{CB}}} q_j^{i_B}$. Hence, by Assumption 6, there is no market break-down. Following now the lines of the proof of Theorem 2, it can be seen that both counterparties can strictly gain for this given λ if the borrower replaces a small quantity of collateral k deposited with the central bank by a corresponding quantity of collateral $k+1$. Hence θ cannot be stable. The contradiction proves the assertion. □

Proof of Theorem 5. Fix $J' \supseteq J$. Assume first that there is a market breakdown under policy (J, α). Then lender and borrower obtain their outside option utilities $\underline{u}_L = (\pi_G + \pi_B)u_L(r^D)$ and $\underline{u}_B(J) = (\pi_G + \pi_B)u_L(r^L(J))$, respectively. Choose $\alpha' = \alpha$. If there is also a market-breakdown under policy (J', α'), then $\underline{u}_B(J') \geq \underline{u}_B(J)$ increases weakly, while $\underline{u}_L$ remains unchanged. Hence there is a weak Pareto improvement in this case. If a market transaction comes about under policy (J', α'), then by individual rationality $E[\widetilde{u}_L] \geq \underline{u}_L$, and $E[\widetilde{u}_B] \geq \underline{u}_B(J') \geq \underline{u}_B(J)$. Again, therefore, there is a weak Pareto improvement. Assume now that a market transaction comes about under policy (J, α). The weak enlargement of the set of eligible collateral implies a weak enlargement of the bargaining set, and a weak increase in the borrower's outside option utility. Consider first the case $E[\widetilde{u}_B] \geq \underline{u}_B(J')$. Noting that the bargaining set is convex (possibly as a result of Pareto optimal randomization over SRAs), there is a liquidity policy α' such that a weak Pareto improvement is obtained by changing from policy (J, α) to (J', α'). If, however, $E[\widetilde{u}_B] < \underline{u}_B(J')$, the lender will always be worse off because there is no break-down under (J, α). If the change to collateral framework J' implies an empty intersection of individual rationality constraints and the bargaining set, a crowding-out will result. This proves the assertion. $\square$

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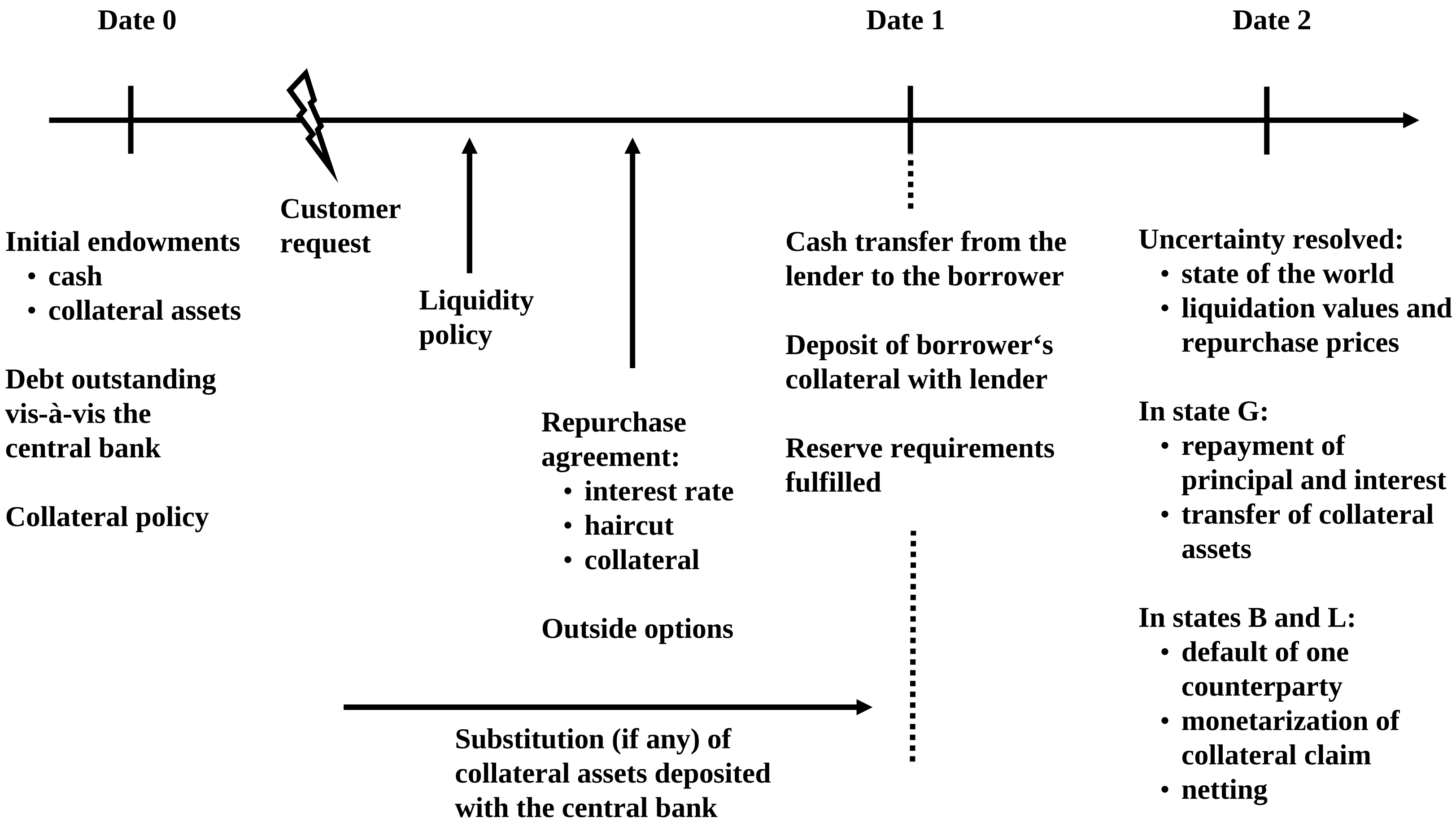
Table I

Eurosystem (Source: ECB data)		Repo market (Source: ICMA)	
Central Gov	22.5%	EU Central Gov	84.2%
Regional Gov	6.6%	Other EU	15.8%
Uncov Bank Bonds	31.5%	Total	100.0%
Cov Bank Bonds	13.2%		
Corporates	6.6%		
ABS	12.1%		
Other marketable	3.8%		
Credit claims	3.7%		
Total	100.0%		

Average Collateral Usage during 2006 in Primary and Secondary Funding

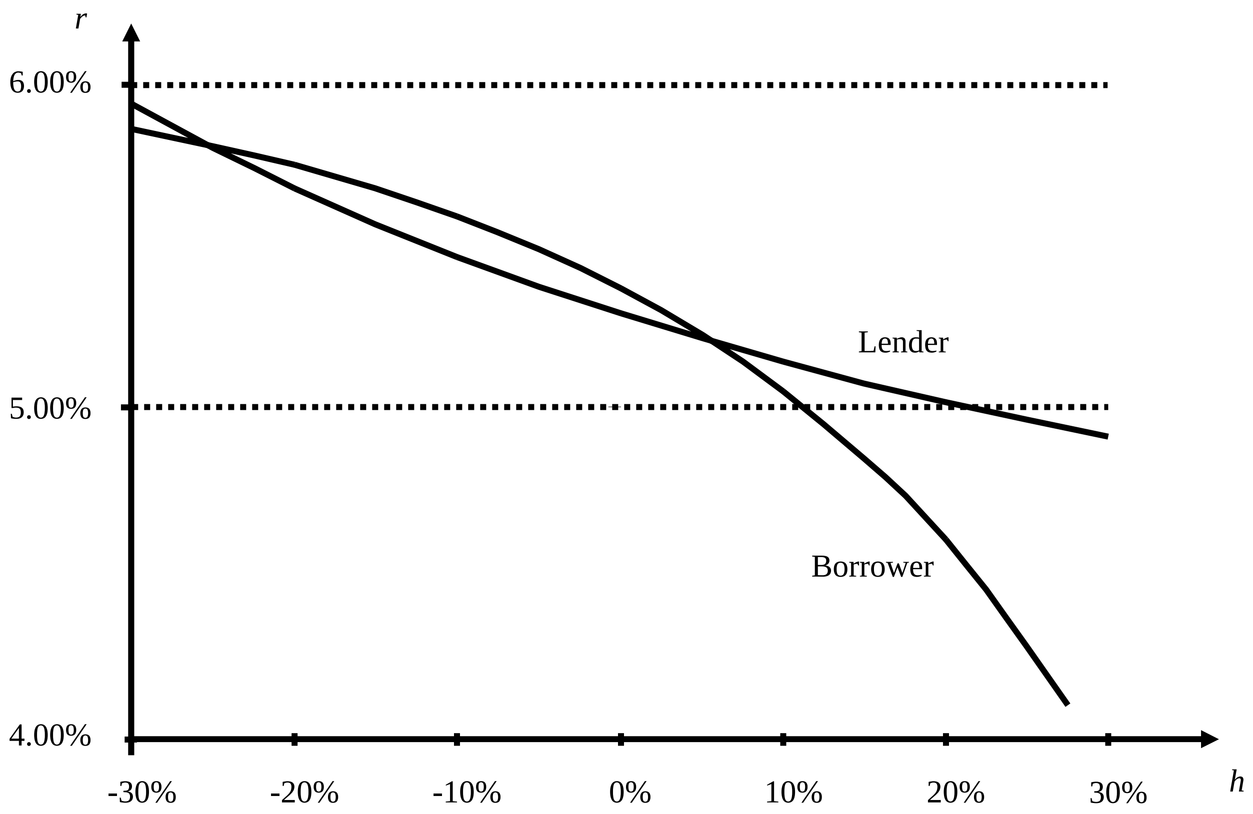
The table on the left-hand side refers to market values of assets, net of haircuts, held as collateral by counterparties with the Eurosystem as an average of monthly data (end-of business, last Friday of the month) for 2006. Shown are the percentage shares of different types of assets eligible as collateral vis-à-vis the Eurosystem. The abbrevations “Central Gov” and “Regional Gov” refer to central government bonds and regional government bonds, respectively. Similarly, “Uncov Bank Bonds” and “Cov Bank Bonds” should be read as uncovered and covered bank bonds, respectively. Vis-à-vis the Eurosystem, collateral can be held either through a pooling or through an earmarking system. Earmarking has been used predominantly in France, Italy, and Ireland. There are some countries where both collateral systems are in use. Most national central banks rely exclusively on the pooling system. In a pooling system, collateral assets may exceed the outstanding credit (i.e., there may be over-collateralization). The table on the right-hand side shows the percentage shares of different types of EU collateral used in the euro repo market. Reported are averages over values reported by 79 (74) financial units as outstanding at close of business for June 14, 2006 and December 13, 2006.

Figure 1



Date 0
Date 1
Date 2
Customer request
Initial endowments
• cash
• collateral assets
Debt outstanding vis-à-vis the central bank
Collateral policy
Liquidity policy
Repurchase agreement:
• interest rate
• haircut
• collateral
Outside options
Substitution (if any) of collateral assets deposited with the central bank
Cash transfer from the lender to the borrower
Deposit of borrower's collateral with lender
Reserve requirements fulfilled
Uncertainty resolved:
• state of the world
• liquidation values and repurchase prices
In state G:
• repayment of principal and interest
• transfer of collateral assets
In states B and L:
• default of one counterparty
• monetarization of collateral claim
• netting

Figure 2



Indifference Curves of Lender and Borrower

The two graphs in the figure show, respectively, the minumum acceptable repo rate for the lender and the maximum acceptable repo rate for the borrower, both as a function of the haircut. In this numerical example, utility functions of lender and borrower have a constant coefficient 1 of absolute risk aversion. The respective conditional distributions for liquidation and repurchase values are identical and of the Erlang-2 type, with mean 1. Probabilities of default are 1% for the lender and 3% for the borrower. The opportunity lending rate is 6%, the opportunity deposit rate is 4%. An agreement can be seen to be individually rational simultaneously for both counterparties for haircuts in the range between about -25% through about +5%. The example illustrates the possibility of negative haircuts even when the lender has the higher default probability than the borrower. The effect is caused by the right-skewness of the Erlang distribution, which exposes especially the borrower to counterparty risk.