Capital, Income Inequality, and Consumption: the Missing Link

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A topic with some tradition ...
Motivation

- Investment in Physical Capital ($K$) central to macro: growth, inequality, optimal taxation, business cycles

- Real Business Cycles (RBC): General Equilibrium model
  - $K$ investment $\sim$ 4 times more volatile than GDP, etc.

- New Keynesian (NK) ... No $K$
  - side show, does not matter much (Appendix G, robustness)
  - ingredient in DSGE versions, but back seat ... for $C$
    Caveat: "financial accelerator" notwithstanding

- Missing Link RBC—NK: "to $K$ or not to $K"
Monetary Policy Transmission

- What happens in response to monetary policy?
- Recent extension of Representative-Agent (RA)NK:
  - Heterogeneous-agent (TA&HA)NK: Consumption $C$
    - common theme: general equilibrium $GE$ of the essence
  - "$GE" = core of RBC ... through $K$
Our Result(s)

- Heterogeneity puts $K$ back at the center in NK!
- Novel **complementarity** $K$ – income ($Y$) inequality

<table>
<thead>
<tr>
<th>Amplification of Mon. Pol. Effects on $C$</th>
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</thead>
<tbody>
<tr>
<td>no $Y$ Inequality</td>
<td>$Y$ Inequality</td>
</tr>
<tr>
<td>no $K$</td>
<td>=repres.-agent</td>
</tr>
<tr>
<td>$K$</td>
<td>small</td>
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<td></td>
<td><strong>LARGE</strong></td>
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</table>

- Interaction w/ fiscal redistribution: essential **what $K$ income**
  - **opposite effects** of redistributing physical vs monopoly $K$
  - $T_{(ractable)}$-HANK w/ $K \rightarrow$ **analytics** (novel even for RANK)
- Role of wage rigidity
*ANK: Heterogeneity/Inequality Matters

- **Empirical**: Campbell Mankiw; Hall; ...; Kaplan Violante; Cloyne Ferreira Surico

- **RANK+K**: Dupor, Carlstrom Fuerst, Sveen Weinke, Woodford, Rupert Sustek...

- **2000s TANK**: Galí Lopez-Salido Vallés; Bilbiie...
  - **borrower-saver**: Iacoviello; Nistico; Eggertsson Krugman; Bilbiie Monacelli Perotti

- **2010s HANK focus on C**: Auclert; Kaplan, Moll, Violante; Gornemann Kuester Nakajima; McKay Nakamura Steinsson; Guerrieri Lorenzoni; Bayer Luetticke Pham.Dao Tjaden; Auclert Rognlie; Deboortoli Galí; Ravn Sterk; Den Haan Rendahl Riegler; Luetticke; McKay Reis; Challe Matheron Ragot Rubio; Oh Reis; Hagedorn Manovskii Mitman; Auclert Rognlie Straub; Acharya Dogra; Werning; Cantore Freund; Bilbiie Ragot; Cui Sterk; Bhandari Evans Golosov Sargent; Bilbiie

- **now**: HANK with focus on K
  - Auclert Rognlie Straub; Alves Kaplan Moll Violante; this paper ...

- **Sticky Wages**: TANK Colciago ... HANK Broer, Hansen, Krusell, Oberg ...
A Tale of Two Inequalities

- This paper:

\[ C^j + S^j = (WN)^j + (rK + D)^j + T^j \equiv Y^j \]
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- **Bilbiie 2008 JET ("TANK"), many HA models**

- **\( K \), no \( Y \) inequality (but still MPC heterogeneity): LHS**

\[ C^j + S^j = Y \]

Saver-spender: Gali Lopez-Salido Valles 2007, Mankiw 2000, any HA model with assets in positive net supply
A(n Outrageous) Model

- $\lambda$ hand-to-mouth $H$, $1 - \lambda$ savers $S$ w/ bonds Euler (loglin)

$$c_t^S = E_t c_{t+1}^S - r_t$$

+ Solow (1956) investment $I_t = \alpha\beta Y_t$

$$\alpha\beta < 1$$ is the savings rate

$$i_t = y_t$$

+ (Loglin) individual:

$$\begin{align*}
(1 - \alpha\beta)c_t^S + \frac{\alpha\beta}{1 - \lambda} i_t &= y_t^S \\
(1 - \alpha\beta)c_t^H &= y_t^H = \chi y_t,
\end{align*}$$

the key $\chi$ = a model of the income distribution.

- Aggregate Euler (use $c_t = i_t = y_t$)

$$c_t = E_t c_{t+1} - \frac{(1 - \alpha\beta)(1 - \lambda)}{1 - (\alpha\beta + \lambda\chi)} r_t.$$
Isolating Income (Y) Inequality

- no K ($\alpha \beta = 0$), all RHS: Bilbiie 2008 → 2019

$$c^H_t = \chi y_t; \quad c^S_t = \frac{1 - \lambda \chi}{1 - \lambda} y_t.$$ 

- Aggregate Euler ~ Aggregate Demand

$$c_t = E_t c_{t+1} \quad - \quad \frac{1 - \lambda}{1 - \lambda \chi} r_t$$

amplification iff $\chi > 1$ (counter-cyclical Y ineq.)

full-HANK general. Auclert JMP; evidence: Patterson 19; Slacalek Tristani Violante 19; Heathcote Perri Violante 10

- New Keynesian Cross:

Aggregate MPC(ish): $\lambda \times 1 \times \chi + ...$

"indirect effect" Kaplan Moll Violante
The New Kenesian Cross \( c_t = \omega \hat{y}_t - (1 - \omega) \Omega r_t + ... \)

ERC: \( c_t = \hat{y}_t \)

PE: \( c_t = c(\hat{y}_t, r_t, g_t) \)
Isolating K Inequality (LHS): This Paper

- Assume that income is perfectly redistributed $\chi = 1$:

\[
(1 - \alpha \beta)c^S_t + \frac{\alpha \beta}{1 - \lambda}i_t = y_t;
\]

\[
(1 - \alpha \beta)c^H_t = y_t.
\]

- Aggregate Euler - Demand

\[
c_t = E_t c_{t+1} - \frac{(1 - \alpha \beta)(1 - \lambda)}{1 - (\alpha \beta + \lambda)} r_t
\]

\[
\text{multiplier}
\]

- Another Keynesian-cross multiplier ($\lambda < 1 - \alpha \beta$):

the savings rate (of $S$) acts as an MPC (of $H$)

S’s saving-investment $\rightarrow$ K income, redistribution $\rightarrow$ H, not saving

- Novel analytical isolation, translates to any HA w/ some K (net saving)
- ex.: Auclert Rognlie Straub: turn off $\chi$ but other channels (inattention, liquidity)
The Multiplier ... of the Multiplier

- both $K$ and $Y$ inequality

\[
\left| \frac{\partial c_t}{\partial r_t} \right| = \frac{(1 - \alpha \beta)(1 - \lambda)}{1 - (\alpha \beta + \lambda \chi)} = \frac{1 - \lambda}{1 - \lambda \chi \frac{1}{1 - \alpha \beta}}
\]

- Complementarity if $Y$ ineq. counter-cyclical $\chi > 1$:

\[
\left| \frac{\partial c_t}{\partial r_t} \right|_{K, \ Y \ ineq} > \left| \frac{\partial c_t}{\partial r_t} \right|_{no \ K, \ Y \ ineq} \times \left| \frac{\partial c_t}{\partial r_t} \right|_{K, \ no \ Y \ ineq}
\]

- Intuition 1: aggreg MPC $\sim \lambda \chi + \alpha \beta$ so multiplier $\frac{1}{1 - (\alpha \beta + \lambda \chi)}$

- Intuition 2: $K$ multiplier $\frac{1}{1 - \alpha \beta}$ "inside" $Y$ ineq. multiplier $\frac{1 - \lambda}{1 - \lambda \chi}$ at each round
A picture worth \(1/(1-x)\) words

Figure: \(C\) multipliers as a function of \(\lambda\) \((\alpha = 0.33, \beta = 0.99, \chi = 1.7)\).
Simple Testable Predictions

1. $Y$ and $C$ inequality:

$$y^S_t - y^H_t = \frac{1 - \chi}{1 - \lambda} y_t$$

$$c^S_t - c^H_t = \frac{1}{1 - \alpha \beta} (y^S_t - y^H_t) - \frac{\alpha \beta}{(1 - \alpha \beta)(1 - \lambda)} i_t$$

$$\text{Solow} = \frac{1 - \chi - \alpha \beta}{(1 - \lambda)(1 - \alpha \beta)} y_t$$

both counter-cyclical iff $\chi > 1 - \alpha \beta$. General condition:

$$\chi + \frac{I}{Y} \frac{\partial i_t}{\partial y_t} > 1$$

2. $C$ ineq. more counter-cyclical than $Y$ ineq. if $i_t$ procyclical (dah)

- compare to evidence (Coibion Gorodnichenko Kueng Silvia 2017)
A Tractable HANK (THANK) with K: This Paper

- extend Bilbiie (2019) with **illiquid K**
- idiosyncratic uncertainty: two states $S \leftrightarrow H$
  \[
  \lambda = \frac{1 - s}{2 - s - h}
  \]

- assets: *liquid bonds, illiquid K & shares* (claims to profits)
- labor union (sticky wages later)
- the government redistributes K and/or profits
- **Reminder**: THANK matches micro moments: iMPCs, income risk, Rognlie Straub, income risk, leptokurtosis, left-skewness, cyclical skewness, etc.
- Novel **analytical** solution in special case (incl. RANK!)
A Tractable HANK (THANK) with K: This Paper

- Illiquid K ($Q_t \equiv (\Phi'(.))^{-1}$ Tobin's marginal Q)

\[
Q_t = \beta E_t \left\{ \left( \frac{C^{S}_{t+1}}{C^S_t} \right)^{-\frac{1}{\sigma}} \left[ (1 - \tau^K) R^K_{t+1} + Q_{t+1} (1 - \delta + \Phi_{t+1} - \frac{I_{t+1}}{K_{t+1}} \Phi'_{t+1}) \right] \right\}
\]

\[
K_{t+1} = (1 - \delta) K_t + \Phi \left( \frac{I_t}{K_t} \right) K_t
\]

- Liquid bonds

\[
(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1 + r^n_t}{1 + \pi_{t+1}} \left[ s(C^S_{t+1})^{-\frac{1}{\sigma}} + (1 - s)(C^H_{t+1})^{-\frac{1}{\sigma}} \right] \right\}
\]

- Redistribution $C^H_t = w_t N_t + T^H_t$

\[
\lambda T_{H,t} = \tau^D D_t + \tau^K r^K_t K_t
\]

- No Y ineq. $\chi = 1$: perfect redistribution

\[
\tau^D = \tau^K = \lambda
\]
### Parameterization: Standard

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.33</td>
<td>Capital share of output</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.025</td>
<td>Depreciation rate per quarter</td>
</tr>
<tr>
<td>(\omega)</td>
<td>10</td>
<td>Elasticity of investment to (Q)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>(s)</td>
<td>1 / 0.98</td>
<td>Probability of staying unconstrained</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>1</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>(1/\varphi)</td>
<td>1.00</td>
<td>Frisch elasticity</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0 / 0.27</td>
<td>Share of hand-to-mouth</td>
</tr>
<tr>
<td>(\tau^D, \tau^K)</td>
<td>(\begin{cases} 0 &amp; \text{no redistribution} \ \lambda &amp; \text{full redistribution} \end{cases})</td>
<td>Taxes on profits and capital</td>
</tr>
<tr>
<td>(\psi)</td>
<td>0.050</td>
<td>Slope of PC</td>
</tr>
<tr>
<td>(\psi_w)</td>
<td>(\infty / 0.075)</td>
<td>Slope of PC wages</td>
</tr>
<tr>
<td>(\phi_n)</td>
<td>1.50</td>
<td>Taylor rule coefficient</td>
</tr>
<tr>
<td>(\rho_i)</td>
<td>0.60</td>
<td>Persistence MP shock</td>
</tr>
</tbody>
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Quantifying the Complementarity

<table>
<thead>
<tr>
<th></th>
<th>No $Y$ ineq.</th>
<th>$Y$ ineq.</th>
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<tbody>
<tr>
<td><strong>no-$K$</strong></td>
<td>1 (=RANK)</td>
<td>1.51</td>
</tr>
<tr>
<td><strong>$K$</strong></td>
<td>1.15</td>
<td><strong>2.25</strong></td>
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(Bilbiie 2008)
Quantifying the Complementarity

<table>
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<tr>
<th>Amplification of Mon. Pol. Effects on C</th>
<th>RANK</th>
<th>No $Y$ ineq.</th>
<th>$Y$ ineq.</th>
<th>+ Risk</th>
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<tbody>
<tr>
<td>no-$K$</td>
<td>1</td>
<td>1(=RANK)</td>
<td>1.51</td>
<td>1.60</td>
</tr>
<tr>
<td>$K$</td>
<td>0.66</td>
<td>1.15</td>
<td><strong>2.25</strong></td>
<td><strong>2.62</strong></td>
</tr>
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</table>

- RANK w/ $K$ dampening: real (long) rate increases $I$ response, less $C$
- uniform across models: iid shocks: $0.89 \rightarrow 1.55 \rightarrow 3.95$ (w/o $K : 2.02$)
- idiosyncratic risk: adds prec. saving in liquid assets
- $K \rightarrow$ long-run amplification (across models)
Fiscal Redistribution (of Financial Income)

<table>
<thead>
<tr>
<th>Income Redistribution</th>
<th>$D$</th>
</tr>
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<tr>
<td></td>
<td>Yes</td>
</tr>
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<td>$K$</td>
<td>1.15</td>
</tr>
<tr>
<td>No</td>
<td>0.50</td>
</tr>
</tbody>
</table>

- $D$ countercyclical $\rightarrow Y_H$ less cyclical $\rightarrow$ dampening
- $r^K K$ highly procyclical $\rightarrow Y_H$ more cyclical $\rightarrow$ ++amplification
- **But** $D$ procyclical (even wrt MP). One avenue:
Sticky Wages: Some Nuisance

- extend our novel framework of isolating $K$ and $Y$ inequality channels
- derive testable predictions
- role of redistributing capital income and profits
**Sticky Wages**

Amplification of Mon. Pol. Effects on C

<table>
<thead>
<tr>
<th>RANK</th>
<th>No Y ineq.</th>
<th>Y ineq.</th>
<th>+ Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>no-K</td>
<td>1.37 (37%)</td>
<td>1.37 (37%)</td>
<td>1.40 (-13%)</td>
</tr>
<tr>
<td>K</td>
<td>1.29 (95%)</td>
<td>2.10 (89%)</td>
<td><strong>2.43 (8%)</strong></td>
</tr>
</tbody>
</table>

- RA: Further break of neutrality → higher MP effect
- No Y ineq.: also large amplification; conditional-on-\( K \) amplification similar to flex-W (2.1/1.29)
- Conditional on Y ineq. less amplification: dampening w/o \( K \)!
- →Less difference in fiscal redistributions (different \( K \) incomes more similar over cycle)
Empirical Relevance

1. Peak responses to MP

\[ dc < dy < di \]

match both absolute and relative.

2. Both C and Y ineq. are countercyclical, with C ineq. more so
Testable Prediction 1: Peak Responses to MP

Figure: IRFs to expansionary interest rate shock of 25 basis points.
Testable Prediction(s) 2: C & Y Inequality

Figure: IRFs to expansionary interest rate shock of 25 basis points.

- Only HA with both K and Y inequality fits evidence by i.a. Coibion et al (2017)
Conclusions

- Further step toward Macro convergence
  - $K$ back in policy-relevant, monetary models

- Through a multiplier of the multiplier:
  - complementarity $K - Y$ inequality

- Key for MP what $K$ income is fiscally redistributed, machines vs monopoly
  - optimal policy?

- Missing link: 1. RBC–NK & 2. quant.–tractable HANK
Analytics: Closed-form multipliers

- Novel analytical solution:
  \[ \delta = 1; \varphi = 0; \sigma = 1; \kappa \to \infty; \phi_\pi = 1 \]

- RANK Multiplier
  \[ \frac{\partial c_t}{\partial (-r_t)} = 1 - \frac{(1 - \alpha^2 \beta) + (1 - \alpha) \psi^{-1} \frac{\alpha^2 \beta}{1 + \psi^{-1}}}{(1 - \alpha) \psi^{-1} + 1 - \alpha^2 \beta} \leq 1 \]

- TANK fixed-price Multiplier
  \[ \frac{\partial c_t}{\partial (-r_t)} = \frac{1 - \lambda}{1 - \lambda \chi} + \frac{\alpha \beta}{1 - \alpha \beta} \frac{(1 - \alpha) \lambda}{1 - (1 - \alpha) \lambda \chi} \]

- complementarity proof in this special case

THANK
Long-run Amplification

PDV Consumption: Flexible-Wage

- RA, no K
- RA, K
- HA, K, prop. inc.
- HA, K, inc. ineq.

PDV Consumption: Sticky-Wage

Table