Endogenous Market Making and Network Formation

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Core-Periphery Structure in OTC

Figure: Observed Interbank Network (Blasques et al. 2015)

- Stylized Facts (Li & Schurhoff (2011), Bech & Atalay (2010)...)  
  - “Customers” trade through “Dealers”  
  - Heterogeneity in dealers’ connectedness  
    - A few highly interconnected banks (Implications on financial stability)
Core-Peripherhey Structure in OTC

Figure: Observed Interbank Network (Blasques et al. 2015)

“In the current crisis, ... financial firms ... become too interconnected to fail .... Due to the complexity and interconnectivity of today’s financial markets, the failure of a major counterparty has the potential to severely disrupt many other financial institutions, their customers, and other markets.”
– Charles Plosser, 03/06/09
Core-Periphery Structure in OTC

Figure: Observed Interbank Network (Blasques et al. 2015)

Q: Why is this the equilibrium structure?

Existing approaches:
- Random Search (non-directional)
- Network (*mostly* exogenous links)
Core-Periphery Structure in OTC

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This paper: all trading links are formed optimally
Basic Ingredients

- Both market makers and customers are exposed to uncertainty about asset values.

- Market makers provide insurance to customers against the uncertainty.

- Traders with less exposure to uncertainty have a comparative advantage to be market makers.
Result: One-Round

- Volatile types trade through stable types

Figure: Equilibrium Trading Structure (N=1)
Result: N-Round Trade & Network Formation

- Stable types have most connections & highest gross trading volume

**Figure:** Equilibrium Trading Structure (N=7)
Model

A continuum of traders

- Endowment: \( a \) units of asset, unlimited numeraire goods
- Capacity constraint: asset holding \( \bar{a} \in [0, 2a] \).
- Preference: \( u(a) = \varepsilon_\sigma a + t \).
  - \( \sigma \): volatility of preference, \( \sigma \sim G(\cdot) \).
  - \( \varepsilon_\sigma \): i.i.d. shocks

\[
\varepsilon_\sigma = \begin{cases} 
  y + \sigma, & \text{w.p. 0.5} \\
  y - \sigma, & \text{w.p. 0.5}
\end{cases}
\]

- \( t \): transfer of numeraire goods
Market Structure

- $t = 0$: **bilateral matching**
  - Choose counterparty based on *observables* $z$
    
    $$z = (\text{volatility type } \sigma, \text{ asset holding } a)$$
  
  - Agree on *state contingent* and *feasible* asset allocation & transfer
    
    $$\psi = \{ a(z, s), t(z, s), a(z', s), t(z', s), \forall s = (\varepsilon_z, \varepsilon_{z'}) \}$$
  
  - Preference shocks are realized
- $t = 1$: **bilateral trade** takes place according to agreement $\psi$
Market Structure

- **$t = 0$**: bilateral matching
  - Choose counterparty based on *observables* $z$
    \[ z = (\text{volatility type } \sigma, \text{ asset holding } a) \]
  - Agree on *state contingent* and *feasible* asset allocation & transfer
    \[ \psi = \{ a(z, s), t(z, s), a(z', s), t(z', s), \forall s = (\varepsilon_z, \varepsilon_{z'}) \} \]
  - Preference shocks are realized
- **$t = 1$**: bilateral trade takes place according to agreement $\psi$
  - The expected payoff of a trader of type $z$:
    \[ W(z|z') = \mathbb{E}_{s=(\varepsilon_z, \varepsilon_{z'})} [\varepsilon_z a(z, s) + t(z, s)] \]
  - Pairwise surplus:
    \[ \Omega(z, z') = \max_{\psi} W(z|z') + W(z'|z) \]
    \[ = \mathbb{E}_{s=(\varepsilon_z, \varepsilon_{z'})} \varepsilon_z a(z, s) + \varepsilon_{z'} a(z', s) \]
Autarky Allocation

asset allocation

\[ a_0 = a \]

preference

y
First Best Allocation

Implementation

- Centralized Walrasian market, with an auctioneer without capacity constraint
- Bilateral matching based on realized preferences
Constrained Efficient Allocation

- Frictions face
  - Bilateral matching based on observables - volatility $\sigma$
    Uncertainty of counterparty's valuation before making the call
- Planner's choice:
  - Matching based on observables - volatility $\sigma$
  - Within-pair asset allocation based on preferences
Constrained Efficiency: Within-Pair Allocation

- Within-pair asset allocation maximizes pairwise surplus

\[ \Omega(\sigma, \sigma') = E_{s=(\varepsilon_z, \varepsilon_{z'})} \varepsilon_z a(s) + \varepsilon_{z'} a'(s) \]

- When the trader of type \( \sigma' > \sigma \) has high preference

![Diagram showing asset allocation](image)

- \( a' = 2a \)
- \( a_0 = a \)

- w.p. 1/2
- w.p. 1/2

- preference

- y - \( \sigma \)
- y
- y + \( \sigma \)
- y + \( \sigma' \)
Constrained Efficiency: Within-Pair Allocation

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Within a pair, the trader of more stable type “makes market” and may not receive efficient allocation.

\[
\Omega(\sigma_1, \sigma_2) + \Omega(\sigma_3, \sigma_4) < \Omega(\sigma_1, \sigma_3) + \Omega(\sigma_2, \sigma_4)
\]

Trading through stable types minimizes the overall misallocation.

Stable types have comparative advantages at making the market.
Constrained Efficient Allocation

\[ \sigma^* \] is such that \( G(\sigma^*) = 1/2 \).
Equilibrium

Definition

An equilibrium is an allocation function \( f : \mathbb{Z} \times \mathbb{Z} \to R_+ \) and equilibrium payoff \( W^*(\cdot) : \mathbb{Z} \to R_+ \) satisfying the following conditions:

1) Optimality for Traders:

\[
W^*(z) = \max_{\tilde{z} \in \mathbb{Z}} \Omega(z, \tilde{z}) - W^*(\tilde{z})
\]

and for any \( f(z, z') > 0, z' \in \arg \max_{z \in \mathbb{Z}} \{ \Omega(z, z') - W^*(z) \} \).

2) Feasibility constraint:

\[
\int f(z, \tilde{z})d\tilde{z} = h(z) \text{ for } \forall z,
\]

where \( h(z) \) is the density function of \( z \).

The solution concept is related to pair-wise stability.
Decentralization of Constrained Efficient Allocation

- Customers’ payoff: \( W(\sigma) = \frac{1}{2}(y + \sigma)2a + \frac{1}{2} \cdot 0 - T \)
- Competition across market makers: they charge the same expected transfer \( T \)
- Traders with volatility type below \( \sigma^* \):
  
  Welfare loss from misallocation < \( T \)

- Traders with volatility type above \( \sigma^* \):
  
  Welfare loss from misallocation > \( T \)
Implementation of Transfer: Bid Price and Ask Price

- “Market makers” buy at $q^b$ and sell at $q^a$
- The payoff of a market maker:

$$W(\sigma_m) = \frac{1}{2} [2aE(\tilde{\varepsilon}|\varepsilon_{\sigma_m} > \varepsilon_{\sigma_c}) - q^b a] + \frac{1}{2} q^a a$$

$$= ya + a \frac{q^a - q^b}{2}$$

- Revenue from the bid-ask spread

$$T = a \left( \frac{q^a - q^b}{2} \right)$$
Takeaway

- Trading through stable types minimizes the cost of misallocation

- Stable types
  - act as market makers
  - are compensated by a bid-ask spread
Setup: $N$ Rounds of Trade

- Traders receive the payoff from holding assets at the moment they leave the market.
- Discount Factor $\beta \leq 1$
Setup: $N$ Rounds of Trade

Preference $\varepsilon$ is realized

Matching decision at $t = 0$:
- Volatility type
- Contingent on asset holding $a_t \in \{0, 2a\}$

Figure: Timeline: $t = 0, 1, \ldots, N$
Constrained Efficient Allocation

\[ \sigma^* \text{ is such that } G(\sigma^*) = \frac{1}{2}. \]
Constrained Efficient Allocation

- $\sigma_1^*$ is such that $G(\sigma_1^*) = \frac{1}{2}$, $\sigma_2^*$ is such that $G(\sigma_2^*) = \left(\frac{1}{2}\right)^2$.
- The constrained efficient solution follows a recursive structure.
Volatile types \((\sigma > \sigma_1^*)\) match with stable types \((\sigma \leq \sigma_1^*)\) 
- Volatile types have reached their efficient allocation
Market Making and Network Formation ($N = 3$)

- “Customers” last period ($\sigma > \sigma_1^*$) do not trade
- Volatile types ($\sigma > \sigma_2^*$) match with remaining stable types ($\sigma \leq \sigma_2^*$)
“Customers” last period \((\sigma > \sigma_2^*)\) do not trade

Volatile types \((\sigma > \sigma_3^*)\) match with remaining stable types \((\sigma \leq \sigma_3^*)\)
Network Structure with $N$ rounds of Trade

- $\sigma > \sigma_1^*$: “customers”
  - receive efficient allocation by trading once

- $\sigma \leq \sigma_N^*$: “central dealers”
  - build most links
  - have highest gross trading volume

- $\sigma_t^* < \sigma \leq \sigma_{t-1}^*$: “peripheral dealers”
  - make the market until $t - 1$
  - trade with more central dealers at $t$
Endogenous Network

- Consistent with Li & Schurhoff (2011):

**Figure:** Hierarchical Core-Periphery Structure
Equilibrium

**Definition**

An equilibrium is an allocation function, \( f_t : \mathbb{Z}^2 \rightarrow \mathbb{R}_+ \), and equilibrium payoff \( W^*_t(\cdot) : \mathbb{Z} \rightarrow \mathbb{R}_+ \), and a participation function \( \chi_t(\cdot) : \mathbb{Z}^2 \rightarrow \{0, 1\}^2 \) for \( t = 1, \ldots, N \) such that the following conditions are satisfied:

1) Optimality of traders’ matching decisions

\[
W^*_t(z) = \max_{\tilde{z} \in \mathbb{Z}} \Omega_t(z, \tilde{z}) - W^*_t(\tilde{z}),
\]

for any \( f_t(z, z') > 0, z' \in \text{arg max}_{z \in \mathbb{Z}} \{ \Omega_t(z, z') - W^*_t(z) \} \).

2) Optimality of traders’ participation decisions

3) Feasibility of the allocation function
Equilibrium Construction: Payoff

- Cutoff type at period $t$: $G(\sigma_t^*) = 2^{-t}$
- Indifference condition for the cutoff type:

$$\underbrace{(y + \sigma_t^*) a - T_t}_{\text{payoff as a period-t customer}} = \underbrace{\beta (y + \sigma_t^*) a - \beta T_{t+1} + T_t}_{\text{payoff as a period-t dealer}}$$

$$2T_t = (1 - \beta) (y + \sigma_t^*) a + \beta T_{t+1}$$
Equilibrium Construction: Payoff

- Cutoff type at period $t$: $G(\sigma^*_t) = 2^{-t}$
- Indifference condition for the cutoff type:

$$
\frac{(y + \sigma^*_t) a - T_t}{\beta (y + \sigma^*_t) a - \beta T_{t+1} + T_t}
= \begin{cases} 
(y + \sigma^*_t) a - T_t & \text{payoff as a period-t customer} \\
\beta (y + \sigma^*_t) a - \beta T_{t+1} + T_t & \text{payoff as a period-t dealer}
\end{cases}
$$

$$
2 T_t = (1 - \beta) (y + \sigma^*_t) a + \beta T_{t+1}
$$

- Traders’ expected payoff:

$$
W_0^*(\sigma) = \begin{cases} 
\beta (y + \sigma) a - \beta T_1, & \forall \sigma \in [\sigma_1^*, \sigma_H] \\
\beta^t (y + \sigma) a + \sum_{\tau=1}^{t-1} \beta^\tau T_\tau - \beta^t T_t, & \forall \sigma \in [\sigma_t^*, \sigma_{t-1}^*], t \geq 2 \\
\beta^N [y + (2p - 1)\sigma] a + \sum_{\tau=1}^{N} \beta^\tau T_\tau, & \forall \sigma \in [\sigma_L, \sigma_N^*]
\end{cases}
$$
Market structure: Distribution of Links
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Tiered Trading Structure

- Traders within a tier, $\sigma \in (\sigma^*_t, \sigma^*_{t-1}]$ does not trade with each other
- In contrast to random search models: Afonso and Lagos (2014), Hugonnier Lester Weill (2014)
Trade Volume

![Graph showing trade volume distribution]
\[ T_t = (1 - \beta) (c + \sigma_t^*) a + \beta T_{t+1} - T_t \]

- benefit from immediacy
- benefit from saving future payment

\[ G(\sigma) \ (\text{red}) \Rightarrow G(\sigma - \Delta) \ (\text{blue}) \text{ with } \Delta > 0 \]
- upward ⇒ downward bid-ask spread
Application: Interbank Lending

- Interpretation of the model as interbank lending
  - $a =$ capital
  - $\varepsilon =$ return from investment
  - buy $=$ borrow; sell $=$ lend
  - price $=$ as debt repayment in at the end of period $N$
Application: Interbank Lending

- Interpretation of the model as interbank lending
  - $a = \text{capital}$
  - $\varepsilon = \text{return from investment}$
  - buy = borrow; sell = lend
  - price = as debt repayment in at the end of period $N$

- Contagion (Acemoglu et al. (2015)):
  - Negative shocks to investments
  - A bank defaults if it cannot meet its obligations
Implication on Financial Contagion

**Figure:** An Endogenous Network with $N = 3$.

Note: the direction of the links refers to the flow of funds during the day.
Implication on Financial Contagion

**Figure:** An Endogenous Network with $N = 3$.

Note: the direction of the links refers to the flow of funds during the day.

- Vulnerability to Investment Risks & Counterparty Risk depends on
  - Return from its own investment
  - Total loan and liability (interest rates & # of counterparties)
Policy Implication

Table: Contagion through the 8 Interconnected Banks

<table>
<thead>
<tr>
<th>Parameter Value of $y$</th>
<th>Default Chain(s)</th>
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<td>$D - G - E$</td>
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<tr>
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<td>$D - G - E, H - G - E$</td>
</tr>
<tr>
<td>$(\frac{7}{8} \sigma_3^*, \infty)$</td>
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- A Higher Connection ($N$)
  - Benefit: efficient allocation
  - Additional Cost: $G$ are subject to credit risks of $H$ and $F$
    - $F$ is most vulnerable to investment shocks
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- **A Higher Connection ($N$)**
  - Benefit: efficient allocation
  - Additional Cost: $G$ are subject to credit risks of $H$ and $F$
    - $F$ is most vulnerable to investment shocks
- The “Interdependence” matters (not just the number of links)
Related Literature

- OTC Trading based on Random Search:

- OTC Trading with Networks:

- Network Formation:
  - Hojman and Szeidl (2008), Babus (2012), Farboodi (2014)...

Methodology: A dynamic matching model of network formation

Predictions: Hierarchical Core-periphery Structure (Li & Schurhoff (2011))
  - The core: the ones with lower needs for trade
Conclusion

- Contribution: A dynamic matching model of network formation
  - Why does Core-Periphery Structure exist?
  - Implications on price, volume, allocations
  - Implications on financial architecture