Optimal Life-Cycle Asset Allocation with Housing as Collateral

Rui Yao and Harold H. Zhang *

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ABSTRACT

We investigate household finance (asset allocation decisions among risk free asset, stock, housing and mortgage) in a life-cycle model with costly mortgage refinancing and default. Our analysis demonstrates that a household’s liquid wealth is the most important determinant of both home and stock ownership. Our model can explain empirically documented negative (positive) relation between stock investment in networth and home value in networth (mortgage balance relative to networth), as well as generating hump-shaped home and stock ownership patterns. Extensive comparative static analysis suggests that it remains a challenge for life cycle models to match empirically observed magnitude of equity proportions.

Key Words: portfolio choices, life–cycle model, housing, mortgage

JEL Classification Codes: D91, E21, R21

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1. Introduction

This paper investigates household finance (asset allocation decisions among money market, stock, housing and mortgage) in a life-cycle model with costly mortgage refinancing and default. Existing studies have demonstrated that introducing housing and mortgage decisions plays an important role in understanding household finance (such as Cocco (2005) and Yao and Zhang (2005), among others). However, these studies assume that homeowners can costlessly refinance their home mortgage and maintain a fixed level of home equity. Costlessly mortgage refinancing allows homeowners to tap into their home equity whenever they need cash. This increases homeowners’ ability to buffer liquidity shocks. The latter assumption makes the initial downpayment requirement as a maintenance margin and effectively rules out mortgage default. This forces homeowners to hold a minimum level of illiquid home equity and reduces their ability to buffer liquidity shocks. Thus, introducing mortgage refinancing cost and default will have an impact on households’ liquid asset holdings and overall asset allocation decisions.

For the vast majority of the U.S. households, a house is the largest and most important single asset and mortgage is the largest consumer debt in their portfolio. For instance, the Survey of Consumer Finances (SCF) reveals that from 1989 to 2001, the average home value ranges from 55 percent to 60 percent of a homeowner’s total asset and home equity accounts for half of household net worth, while mortgage accounts for 62 percent to 70 percent of the total household debt. Households have also become more leveraged in housing assets now than they were more than a decade ago. In year 1989 the average mortgage balance relative to home value (the loan–to–value ratio) is 25.9 percent. However, in year 2001, the loan–to–value ratio climbed to 34.8 percent.

Over the time period 1989–2001, stock ownership has increased from 17.8 percent to 34.1 percent for renters and from 41.6 percent to 62.8 percent for homeowners. Among those who own stocks, the average stock investment increased from $29,900 to $38,500 for renters and from $51,300 to $149,000.
for homeowners.\textsuperscript{1} The fraction of net worth invested in stock has increased from 20.7 percent to 37.0 percent for renters and from 12.2 percent to 26.7 percent for homeowners.

Empirical analysis on the Panel Study of Income Dynamics (PSID) data has uncovered several important characteristics on household stock investment and housing and mortgage choices (see for example Kullmann and Siegel (2003) and Yao and Zhang (2005)). First, both home ownership and stock ownership are hump-shaped in age, with the home ownership peaking at around 90 percent for the 65-75 age group, and the stock ownership peaking around 50 percent at younger age group of 45-55. Second, households with a low home value or a high mortgage in their net worth are more likely to own stocks. Third, households with a large home value in net worth invest less in stock and households with a high mortgage balance relative to net worth invest more in stock relative to net worth. Since a large home value and high mortgage balance in net worth implies that the household holds most of its net worth in liquid financial assets, this finding suggests that households’ stock investment is positively related to the fraction of household net worth in liquid assets.

A few recent theoretical and numerical studies attempt to understand the effect of housing on an investor’s life-cycle stock investments. Cocco (2005) introduces durable housing to a standard life-cycle consumption and portfolio choice model. He assumes that the investor consumes housing services only from owning a house, i.e., no housing rental market is available. His study suggests that housing price risk “crowds out” stockholding. He also finds that house selling cost reduces the frequency of housing adjustments and the investor’s exposure to stock.

Yao and Zhang (2005) explicitly incorporate a rental market for housing services. They find that when indifferent between renting and owning, the homeowner holds a lower equity proportion in his net worth, yet a higher equity proportion in his liquid portfolio. They also demonstrate that the investor holds a suboptimal portfolio and suffers a large utility cost by following the alternative policy of acquiring housing services either only from renting or only from owning a house.

\textsuperscript{1}All stock investments are denominated in 1989 dollars.
Hu (2005) studies portfolio choices for homeowners in the presence of a house rental market in a five-period model. She finds that young and middle-aged households, regardless of whether they are currently homeowners, have much less stock holdings than predicted by traditional models without housing.

Our paper extends existing studies in several important dimensions. First, we explicitly incorporate a refinancing cost. Both Cocco (2005) and Yao and Zhang (2005) assume that the investor can costlessly access home equity by borrowing up to a fraction of home value anytime.\(^2\) While this assumption greatly simplify the numerical solution of the problem by eliminating mortgage as a separate state variable and, they have some undesirable implications. When the lending rate is lower than the borrowing rate, as assumed in Cocco (2005), the investor will not hold any bond until he completely pays off his mortgage. For low net worth investors with a positive mortgage balance, this leads to either zero liquid financial asset holdings for non-stockholders or an all-equity portfolio for stockholders. When the lending and borrowing rates are equal, as assumed in Yao and Zhang (2005), the investor is indifferent between paying down mortgage debt and holding extra investments in a riskless bond. This leads to an indeterminacy between mortgage debt and bond holding.\(^3\) A refinancing cost leads to infrequent access to home equity and induces simultaneous holdings of mortgage and bond. Hence it separates net worth into liquid financial assets and illiquid home equity. Further, in the absence of refinancing charges, the investor can costlessly access home equity accumulated through house price appreciation or payment of mortgage principals. Hence the investor is less inclined to invest liquid assets conservatively.

Second, we apply the collateral borrowing constraint only to the newly initiated loans but not to the existing mortgage contracts and allow homeowners to optimally default on their mortgage if their home value decreases sufficiently. Both Cocco (2005) and Yao and Zhang (2005) assume that the collateral borrowing constraint needs to be satisfied every period. By applying the collateral

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\(^2\)Hu (2005) assumes that home equity is illiquid and the refinancing cost is the same as the housing adjustment cost. This implies that if a homeowner wants to cash out home equity, he has to sell the house and then buy it back and pay house liquidation cost.

\(^3\)To resolve the indeterminacy, Yao and Zhang (2005) assume that the investor always carries the maximum mortgage balance allowed. If the investor’s home equity rises above the level required by the collateral requirement, the excess equity is cashed out and invested in stocks or bond.
borrowing constraint to both new loans and ongoing loan contracts, an investor is forced to inject liquid funds to reduce mortgage balance when his home equity falls below the collateral requirement. Yet in reality, mortgage loans are subject to an “initial margin” but not a “maintenance margin” requirement. As long as an investor makes his scheduled interest and principal payments, the lender cannot take possession of the borrower’s property even when home equity is negative. The “maintenance margin” assumption prevents an investor from holding a home equity position lower than the collateral requirement in a housing market downturn. Therefore it eliminates the option to default on the investor’s mortgage. In reality, default option offers an important protection for homeowners from severe housing depreciation.\(^4\) Given a documented positive correlation between shocks to housing value and labor income (see Campbell and Cocco (2003) and Ortalo-Magnè and Rady (2003)), this assumption significantly reduces available liquid asset when the investor suffers simultaneously negative shocks to both his income and his housing asset, and reduces the investor’s willingness to take equity risk.

Our model identifies liquid wealth as the most important determinant for an investor’s home and stock ownership decisions. A young investor yet to accumulate enough wealth remains as a renter and a non-stockholder. As the investor ages, he is more likely to become a homeowner first, reflecting the large benefit that home ownership offers compared to stock investment when total wealth is relatively low. A homeowner can become a stockholder after accumulating enough liquid wealth.

Furthermore, a homeowner with a large illiquid home equity position may hold much less stock as a fraction of his net worth than desired, even with an all equity portfolio in his liquid wealth. When facing such a liquidity constraint, the homeowner may optimally choose to pay a fee to refinance his mortgage contract and cash out a fraction of his home equity to increase his overall equity exposure.

Our numerical analysis demonstrates that our model can reconcile the documented empirical relations between a household’s stock market participation and asset allocations and its housing and

\(^4\)While Hu (2005) does not impose the housing downpayment to be satisfied every period, she explicitly rules out default as an option for homeowners.
mortgage decisions. Specifically, given wealth and income, an investor with a lower house value or a higher mortgage balance is more likely to participate in the stock market. This is attributable to the fact that the investor with a larger fraction of wealth in liquid form can more easily overcome the cost associated with investing in stock. Further, the stock ownership rate is hump-shaped as observed in the data, reflecting some investors at advanced ages exiting from stock market to avoid the per period maintenance cost when their investable liquid wealth is low.

Given wealth, income, and house value, the investor with a large mortgage balance (a low home equity proportion) also has a large amount of liquid wealth. Conditional on owning stock, he optimally chooses to allocate a higher fraction of net worth to equity and yet hold a lower equity proportion in liquid portfolio. This reflects the combined effects of the investor’s desire to hold a portfolio close to the optimal choice without borrowing restriction on one hand and the availability of liquid wealth without triggering costly refinance and/or house liquidation on the other.

Using simulation analysis, Cocco (2005) documents a positive correlation between the equity proportion in net worth and the mortgage–net worth ratio. However, there is an important distinction between his result and ours. In Cocco (2005), the mortgage balance is not a state variable. The positive relation between the equity proportion and the mortgage–net worth ratio is contemporaneous. It is due to homeowners not holding any bond or holding an all-equity portfolio until they completely pay off their mortgage, leading to high mortgage balance being associated with high equity proportion for stockholders. The positive relation between equity investment and mortgage balance is caused by the model assumption that homeowners hold an all-equity portfolio until they fully pay off their mortgage debt.

In our model, due to mortgage refinancing charges and the default option, existing mortgage balance becomes a state variable and directly affects an investor’s portfolio choices. Homeowners will only invest in stocks if they have sufficient liquid financial wealth. Since a higher mortgage balance implies that a homeowner has less wealth tied to illiquid home equity and more liquid wealth for investing in financial assets, the homeowner will more likely participate in the stock market.
and also hold more stock investment. This leads to a positive relation between equity proportion and mortgage balance in net worth. Our study thus offers a mechanism for understanding the empirically documented positive relation between equity investment and mortgage balance.

Our analysis suggests that assuming zero refinancing cost biases upward households’ stock ownership and equity exposure in total wealth. On the other hand, imposing maintenance margin on home equity, which eliminates mortgage default, induces more liquid savings before home ownership. It thus increases stock ownership rate among young renters, yet discourages equity market participation and stock holdings among homeowners by reducing their liquid wealth. Introducing a borrowing and lending spread drastically accelerates home equity accumulation by inducing faster mortgage pay-downs. This reduces the percentage of stock ownership and equity exposure.

Our analysis on optimal decision rules also indicates that the relation between a homeowner’s liquid asset equity proportion and the value of his home in net worth is complex and dependent upon the homeowner’s mortgage balance relative to home value. Therefore, the insignificant effect of the home value–net worth ratio on the liquid asset equity proportion reported in existing empirical studies is likely caused by empirical mis-specification. Future empirical model should allow more flexible forms of the relation between households’ stock allocations and the house value–net worth ratio and the mortgage–net worth ratio. Empirical analysis should also explicitly model both the one-time stock market entry cost and the per period maintenance cost to correctly identify the life-cycle pattern of stock ownership.

The rest of the paper is organized as follows. Section 2 discusses empirical findings on the effects of housing and mortgage decisions on the investor’s portfolio choice. Section 3 introduces the model, while section 4 discusses the numerical results. Section 5 provides concluding remarks.
2. Empirical Evidence on Household Housing and Portfolio Choice

To understand households’ housing and portfolio choices, we first describe the sample averages for key housing and portfolio choice related variables for households in the Survey of Consumer Finances between 1989 and 2001. We then review the main empirical findings on the relations between households’ asset allocations and their housing and mortgage positions.

Table 1 reports percentage of home ownership (OWNHOUSE), average house value–asset ratio (PH/ASSET) and house value–networth ratio (PH/NW), percentage of homeowners owing mortgage (OWEMORT), average mortgage–total debt ratio (MORT/DEBT), mortgage loan–to–value ratio (LTV), home equity–networth ratio (HE/NW), percentage of stock ownership (OWNSTOCK), average equity proportions in total asset (S/ASSET), networth (S/NW), and liquid financial assets (S/SB), across age groups and various years.

Home ownership rate exhibits a hump–shape over an investor’s life cycle, peaking at 84.7 percent just before retirement. The home ownership rate is lowest at 19.3 percent for the youngest age group and remains high at 77.7 percent for the oldest age group (75 years and beyond). Homeowners’ house value–networth ratio decreases in age, reflecting households’ wealth accumulation as human capital is gradually realized. The average loan–to–value ratio declines drastically as homeowners pay off their mortgage debt. The home equity in networth is slightly U–shaped in age and lowest for households reaching retirement age. In the sample period between 1989 and 2001, home ownership has increased slightly. Households also increasingly depend on leverage to finance their home ownership. The average mortgage balance relative to home value increases from around 25 percent to about 35 percent from 1989 to 2001. Consistent with the increasing loan–to–value ratios, the home equity–networth ratio declines over the same period.

Overall, stock ownership rate is much higher for homeowners than that of renters and also demonstrate a hump–shape in age, highest among age groups 35-44 for renters and 45-54 for homeowners. Among stockholders, the fraction of stock investment in total asset increases from a low 20 percent
to around 30 percent for renters, and from about 5 percent to almost 20 percent for homeowners as investors age. The fraction of stock investment in networth is rather flat over life cycle, at around 30 percent for renters and 20 percent for homeowners across age groups. The share of stock investment in the liquid financial assets however shows somewhat a hump-shape for both renters and homeowners. It peaks at 42.6 percent for ages 35-44 for renters and at 47.2 percent for 45-54 for home owners. Over the sample period, there is a strong positive trend in stock ownership for both renters and home owners, increasing from 18 percent to 34 percent for renters, and from 42 percent to 62 percent for homeowners. Similarly, stock investments as a fraction of total asset, networth, or liquid assets also exhibit a strong positive time trend for both renters and homeowners.

Up until recently, empirical studies on the relation between housing and household portfolio choices have been scarce. When examining the effect of entrepreneurial income risk on portfolio choice and asset prices with OLS regressions using the 1992 Survey of Consumer Finances (SCF) data, Heaton and Lucas (2000a) find that mortgage balance relative to financial networth has a positive effect on the shares of stock in liquid assets, financial assets, and total assets, while the relative value of residential real estate has a negative effect. They interpret the positive effects of mortgage on equity proportions as “suggesting that some stockholdings are indirectly financed via mortgage debt.” The negative effects of real estate are attributed to a risk “crowding out” effect, where “households bearing more risk from other assets cut back on stockholdings,” or “a pure substitution effect,” where “households with more in other assets hold less stock.” However, their empirical model specification does not account for limited stock market participation among households in the data.

Three recent studies provide more in-depth empirical investigation on how housing influences portfolio choice. Cocco (2005) applies cross-sectional Tobit regressions to account for limited stock market participation in the 1989 wave of the Panel Study of Income Dynamics (PSID) data to

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5The effects of mortgage and real estate are statistically insignificant in Heaton and Lucas (2000a) when stockholdings are measured relative to a narrow definition of wealth, i.e. liquid asset, which is defined as the sum of direct and indirect holdings of cash, bonds, bills, stocks. Financial asset is liquid asset plus housing and other real estate, proprietary businesses, pensions, and trusts. Total asset is financial asset plus capitalized labor, social security, and pension income. Financial networth is defined as financial assets minus various types of debt including mortgages and consumer loans.
examine the relation between stockholdings and housing related variables. Similar to Heaton and Lucas (2000a), he finds that mortgage debt relative to financial networth has a positive effect on share of stock in liquid assets, financial assets, and total assets, while relative house value has a negative effect.\textsuperscript{6} However, equity ownership and conditional equity proportions may have different determinants. The same explanatory variables could also have opposite effects on stock market participation versus conditional equity proportions. Tobit regression does not allow two effects to be different.

Using the PSID data from 1984 to 1999, Kullmann and Siegel (2003) examine portfolio choice as a function of household exposure to real estate risks. Their study uses Heckman’s sample selection technique to account for limited stock market participation. They also use first differencing in both dependent and explanatory variables to deal with unobservable household-specific heterogeneity in stockholdings. Kullmann and Siegel (2003) find that house value–networth ratio has a negative effect on both stock market participation and stock (or risky financial asset) holdings relative to liquid financial asset, while mortgage–networth ratio has a positive effect on stock market participation and stockholdings (or risky financial asset holdings). These results are robust to dynamic specifications allowing for lagged dependent variable and endogeneity in income and wealth. Kullmann and Siegel (2003) interpret the findings as being consistent with home ownership presenting a background risk.

In Kullmann and Siegel (2003), the conditional error in the stockholding equation is assumed to be a linear function of the error term in the participation equation so that the unobserved time-invariant heterogeneity effect can be removed after taking the first difference in all variables. Violation of this assumption may lead to inconsistent estimates. To provide consistent estimates on the relation between equity proportion and variables on housing asset in the presence of limited market participation and unobserved heterogeneity, Yao and Zhang (2005) apply the two-step estimation procedure proposed by Kyriazidou (1997) to the PSID data from 1984 to 2001. The coefficients of the stock market participation (selection) equation are first consistently estimated using a conditional Logit

\textsuperscript{6}The effects of mortgage and housing on equity proportions are statistically significant for all three portfolio definitions in Cocco (2005).
regression. The estimates are then used to construct a kernel weight to consistently estimate the equity proportion equation.

Yao and Zhang (2005) find that all housing–related variables have a significant impact on homeowners’ stock market participation decisions. Home value–networth ratio affects stock market participation negatively and mortgage–networth ratio has a positive effect on homeowners’ stock market participation, both at a decreasing rate. Further, home value–networth ratio has a negative effect on share of stock in networth while mortgage–networth ratio has a positive effect at a decreasing rate. However, the effects of home value–networth ratio and mortgage–networth ratio on stockholdings relative to liquid financial assets are ambiguous due to lack of statistical significance.

Both Kullmann and Siegel (2003) and Yao and Zhang (2005) find that home value–networth ratio and mortgage–networth ratio affect households’ stock market participation decision in opposite directions but with similar magnitude. Given that home equity is illiquid, this finding suggests that households’ stock market participation decision is largely determined by the size of households’ available liquid assets relative to their networth. Indeed when liquid financial wealth is included in the Probit regression for stock market participation using pooled SCF data from 1989 to 2001, home value–networth ratio and mortgage–networth ratio are no longer significant.\(^7\) The effects of home value–networth ratio and mortgage–networth ratio on stockholdings in either liquid financial assets or networth, however, could not be combined into that of home equity. While the signs of the two variables on equity proportions remain opposite of each other, as in the case of stock ownership, the magnitudes in absolute values and statistical significance are very different. This indicates that not only the size of illiquid home equity but also the composition of home equity, i.e., the leverage ratio, affects a household’s asset allocations.

Overall, empirical evidence suggests that house value and mortgage leverage ratio affects stock market participation through changing the amount of liquid financing assets. Further, home value–networth ratio has a significant negative effect while mortgage–networth ratio has a significant pos-

\(^7\) Regression results are not reported. We thank Annette Vissing-Jøgenson for the comment.
itive effect on stockholdings relative to broader definitions of wealth. The effects of home value–
networth ratio and mortgage–networth ratio on a narrow definition of equity proportion in liquid
financial assets are much weaker and statistically inconclusive.

3. The Economic Model

Our economic model extends the setup in Cocco (2005) and Yao and Zhang (2005) by introducing
costly refinancing and default options to residential mortgage contracts, and both a one–time entry
cost and a per period participation cost to stock market investing. Our description on the model
below is focused on the extensions to the existing literature. The detailed derivation of the model
and the solution algorithm are provided in the appendix.

The investors in our economy face positive per period mortality rate and live for at most \( T > 0 \)
periods. In each time period, the investor receives nonfinancial income \( Y_t \). Before retirement at age
\( J \), \( Y_t \) represents labor income given by

\[
Y_t = P_t^\varepsilon \varepsilon_t, \quad \text{and} \quad P_t^\varepsilon = \exp\{f(t)\} P_{t-1}^\nu \nu_t \quad \text{for } t = 0, \ldots, J - 1, \tag{1}
\]

where \( P_t^\varepsilon \) is the permanent component of the investor’s labor income, \( \nu_t \) and \( \varepsilon_t \) are persistent and
temporary shocks to labor income, respectively. We assume that \( \ln \varepsilon_t \) and \( \ln \nu_t \) are i.i.d. with mean
\(-0.5\sigma_{\varepsilon}^2, -0.5\sigma_{\nu}^2\) and variance \( \sigma_{\varepsilon}^2 \) and \( \sigma_{\nu}^2 \), respectively.\(^8\)
\( f(t) \) is a function of household age. \( \ln P_t^\nu \) thus follows a random walk with a deterministic drift. The growth rate of an individual labor income
evolves according to

\[
\Delta \ln Y_t = f(t) + \ln \varepsilon_t + \ln \nu_t - \ln \nu_{t-1}, \tag{2}
\]

\(^8\)The corresponding mean values for \( \varepsilon_t \) and \( \nu_t \) are one.
with unconditional variance $\sigma^2_\varepsilon + 2\tau^2_\nu$. After retirement at age $J$, $Y_t$ represents payments from investors’ pension or retirement accounts, including social security, that replaces a constant fraction ($\theta$) of his pre-retirement permanent labor income $P^Y_{J-1}$.\footnote{In our numerical analysis, we implement this specification by setting the investor’s permanent income in retirement at his pre-retirement level, while setting the transitory shock to his income replacement ratio, i.e., $\exp\{f(t)\}\nu_t = 1$ and $\varepsilon_t = \theta$ for $t \geq J$.}

The investor acquires housing services by either renting or owning a house. If the investor rents housing services in period $t-1$, he can either keep renting, or buy a house and become a homeowner at period $t$. To rent, the investor pays a fraction ($\alpha$) of the market value of the rental house to the landlord. To become a homeowner, the investor needs to pay at least a fraction ($\delta$) of the house value as a down payment at loan initiation and finance the rest through a fixed-rate mortgage at real rate $r_m$. A home buyer also incurs search and closing costs associated with identifying houses to purchase and securing mortgage and title, assumed to be proportional to the house value ($\phi$). A homeowner also spends a fraction ($\psi$) of the house value on repair and maintenance in order to keep house quality constant. Let $H_t$ be the unit of housing services and $P^H_t$ be the price per unit of housing services at time $t$. A newly initiated mortgage, $M_t$, therefore needs to satisfy:

$$0 \leq M_t \leq (1 - \delta)P^H_t H_t.$$ \hspace{1cm} (3)

At the beginning of each period, the investor incurs an exogenous moving shock, represented by $D^m_t$, which takes the value of 1 if the investor moves for exogenous reasons such as job related relocation and 0 otherwise. While a renter can move without incurring any transaction costs, a homeowner who experiences the moving shock ($D^m_t = 1$) is forced to liquidate his house through either voluntary selling or involuntary foreclosure by defaulting on mortgage payment. Selling a house entails a transaction cost—assumed to be a fraction ($\phi$) of the market value of the house—which is borne by the seller. The full balance of mortgage is due upon house sale. In case of a default, we assume that the debtor incurs a one-time cost equal to $\kappa$ fraction of the house value, which captures the damage to the debtor’s reputation and future access to credit market. Upon
defaulting, the investor is relieved of his mortgage liability and the lender forecloses the investor’s house. Let $\tilde{L}_t$ be the investor’s beginning-of-period loan-to-value ratio

$$\tilde{L}_t = \frac{M_{t-1}(1 + r_m)}{P_t H_{t-1}}.$$  \hfill (4)

Conditional on house liquidation, the investor will default if his loan–to–value ratio is sufficiently high:

$$-\kappa < 1 - \tilde{L}_t - \phi.$$ \hfill (5)

The right–hand–side of the inequality is the proceeds upon house sale after paying off mortgage debt and selling costs. The left-hand-side, on the other hand, is the cost of default. Upon liquidating his house, a homeowner faces the same decision as a renter.

If a homeowner does not have to move for exogenous reasons, he needs to decide whether to stay in the same house or liquidate the house and move endogenously. If he chooses to stay in the same house, he has the option to convert illiquid home equity to liquid financial assets through a “cash-out” refinancing. However, refinancing entails a closing cost ($\xi$) as a percentage of the house value. The mortgage balance after refinancing also needs to satisfy the collateral constraint given by equation (3). If a homeowner chooses not to refinance, he is required to pay down his mortgage balance according to the mortgage amortization schedule. Denote $\iota$ the deterministic inflation rate, and $r_m^N = (1 + r_m)(1 + \iota) - 1$ the nominal mortgage rate, then the real mortgage balance needs to satisfy

$$M_t = \frac{1}{1 + \pi} \left[ M_{t-1}(1 + r_m^N) - \frac{M_{t-1}}{\sum_{j=t}^{T}(1 + r_m^N)^{t-j-1}} \right]$$

\hfill (6)

$$= M_{t-1}(1 + r_m) \left[ 1 - \frac{r_m^N/(1 + r_m^N)}{1 - (1 + r_m^N)^{T-t-1}} \right].$$

If a homeowner cannot make the scheduled principal and interest payments, then he has to either refinance his mortgage or liquidate his house.
We assume that the real gross return on housing assets ($\tilde{R}_t^H$) follows a stochastic (binomial) process, which can be correlated with stock returns and pre-retirement labor-income growth rate. The investor can invest in two financial assets: a riskless bond ($B_t$) and a risky stock ($S_t$). However, if the investor has not entered the stock market previously, he needs to pay a one-time entry cost in order to participate in the equity market. The cost is assumed to be a fraction ($\eta$) of the investor’s current permanent income, $P_Y^t$, reflecting the opportunity cost in acquiring information about stock investments. Following Vissing-Jøgenson (2005), we assume that the investor also faces a per period maintenance cost to participate in the stock market for the current period, which is assumed to be a fraction ($\varrho$) of the investor’s permanent income. No other costs are incurred for trading financial assets. The real gross return on the riskless bond is denoted $R_f = 1 + r_f$ and is assumed to be constant over time. The real gross return on the risky stock is denoted $\tilde{R}_t^S$ and is assumed to follow a stochastic (binomial) process that can be contemporaneously correlated with labor income shocks and the house appreciation return. Borrowing is allowed only through a mortgage by collateralizing one’s house. Short sale of stock is prohibited.

The investor derives utility from consuming a numeraire good ($C_t$) and housing services ($H_t$). Intratempral utility is assumed to be of Cobb-Douglas form, modified to allow for demographic effects such as family size. The investor’s intertemporal preference is characterized by the recursive Epstein-Zin utility function (Epstein and Zin (1989)) with $\gamma$ and $\zeta$ denoting the curvature parameter and the intertemporal elasticity of substitution, respectively. The investor also has a bequest motive represented by a function of bequeathed wealth net of house liquidation cost and the unit price of housing services.

The investor maximizes his recursive utility of lifetime numeraire good consumption, housing services, and bequest, subject to the intertemporal budget constraint, given the initial asset holdings, non-financial income endowment, and previous stock market participation situation. To reduce the dimension of the state space, we normalize all continuous variables by the investor’s spendable

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10 The proportionality of participation cost with respect to permanent income greatly simplifies the numerical solution. A similar assumption is made in Haliassos and Michaelides (2003) and Gomes and Michaelides (2005).
resources including labor income \((Q_t)\) or house value \((P^H_t H_t)\).\(^{11}\) As a result of this normalization, the investor’s optimization problem involves the following choice variables: the stock market entry choice, \(D^e_t\); the stock market participation choice, \(D^p_t\); the house tenure choice, \(D^o_t\); the mortgage refinancing choice, \(D^r_t\); and the house liquidation decision through selling, \(D^s_t\) or default, \(D^d_t\); the numeraire good consumption–wealth ratio, \(c_t = C_t/Q_t\); the house value–wealth ratio, \(h_t = P^H_t H_t/Q_t\); the mortgage loan-to-value ratio after housing and mortgage choices, \(l_t = M_t/Q^H_t H_t\); the fraction of wealth allocated to bonds, \(b_t = B_t/Q_t\); the fraction of wealth allocated to stocks, \(s_t = S_t/Q_t\).

The relevant state variables for the normalized optimization problem are stock market entry status dummy, \(D^e_{t-1}\); home ownership status dummy, \(D^o_{t-1}\); moving shock dummy, \(D^m_t\); beginning-of-period total wealth–labor-income ratio, \(q_t = Q_t/P^Y_t\); beginning-of-period house value–total wealth ratio, \(h_t = P^H_t H_{t-1}/Q_t\); and beginning-of-period mortgage loan–to–value ratio, \(l_t = M_{t-1}(1+r_m)/P^H_t H_{t-1}\).

4. Numerical Results

We use the following parameter values for our baseline analysis. The investor is assumed to make decisions annually starting at age 20 \((t = 0)\) and lives for at most another 60 years \((T = 80)\). The annual mortality rate is calibrated to the 1998 life table for the total U.S. population from the National Center for Health Statistics (Anderson (2001)). We use similar labor income and security returns processes as in the portfolio choice literature. Specifically, we assume that the investor retires at age 65. After retirement, the investor receives 68% of his labor income at age 64. This replacement ratio, as well as the age-dependent deterministic labor income growth rate before retirement, \(f(t)\), is based on the empirical estimation of Cocco, Gomes and Maenhout (2005) by fitting a third-order polynomial to the labor income of high school graduates using the PSID data. Standard deviations of permanent and temporary shocks to labor income are set at 10% and 25%, respectively. The risk-free rate is set at 2%, while the stock return is assumed to have a mean of 6%, and a standard deviation of 18%, as in Gomes and Michaelides (2005). We also assume that the mortgage borrowing

\(^{11}\) We call \(Q_t\) “wealth” for the ease of discussion. The exact expression of the investor’s spendable resources depends on his housing and mortgage positions is provided in the appendix.
rate \( r_m \) is the same as the lending rate that the investor can earn on holding bond \( r_f \) under the baseline case.\(^{12}\)

We set rental costs and housing maintenance costs at 6% and 1.5% of house value, respectively, as in Yao and Zhang (2005). We assume a mean of 0% for real house price appreciations, and a standard deviation of 11.5%, the latter an estimate based on individual residential houses in PSID by Cocco (2005). We set the one-time market entry cost at 10%, as in Haliassos and Michaelides (2003), and set the per period stock investment maintenance cost at 2% of an investor’s permanent income. Refinancing charge is set at 0.8% of house value, or 1% of mortgage balance for a loan with 20% down payment, within the range of the empirical estimate (Benett, Peach, and Peristiani (2001)). Housing purchase cost is set at 3% to take into account the additional search costs. The default penalty is set at 15% of house value so that the model generates overall default rate consistent with the observed default rate for residential mortgages (see Deng, Quigley, and Van Order (2000)).

We set the correlation between the permanent component of labor income and housing returns at 0.2, and the correlation between equity and housing returns at 0.0, as in Yao and Zhang (2005). The exogenous moving shock is calibrated to the average annual migration rate between March 2000 to March 2001 for non-housing related reasons, as reported in the Current Population Survey by the U.S. Census Bureau (2003). For preference parameters, we set the investor’s risk aversion curvature parameter at 5 as in Yao and Zhang (2005) and Cocco (2005). Time discount rate, elasticity of intertemporal substitution, and number of beneficiaries are set at \( \beta = 0.93 \), \( \zeta = 0.5 \), and \( L = 8 \), respectively, to generate life-cycle wealth and home ownership profiles consistent with households’ behavior in the PSID. A complete list of baseline parameter values are summarized in Table 2.

Under the assumption of costless stock market participation and zero mortgage refinancing cost, Yao and Zhang (2005) provide a detailed analysis of the optimal housing and portfolio choices of

\(^{12}\)In doing so, we eliminate the financial incentives of refinancing to take advantage of lower market mortgage rate, and focuses instead exclusively on the consumption smoothing and portfolio allocation-driven reasons of refinancing. Also under the assumption of equal lending and borrowing rates and costly refinancing, it is optimal for the investor to borrow the maximum allowed loan amount upon mortgage initiation and pay the minimum amount required by the amortization schedule to reduce the chance of triggering refinancing cost and increase the option value of defaulting. Hence, the mortgage decision is reduced to a binary choice of whether to refinance. The effect of a positive spread between mortgage rate and riskfree rate is investigated in the comparative static analysis section.
an investor who enters the current period as a renter. Conditional on stock market participation, a renter’s optimal housing tenure choice, housing service consumption, and asset allocations under the current setup are qualitatively similar to the results in Yao and Zhang (2005). In general, the renter chooses to become a homeowner when his wealth–income ratio is sufficiently high. The investor’s optimal consumption–wealth ratio and house value–wealth ratio are both decreasing in wealth–income ratio, consistent with permanent income hypothesis. The renter’s optimal stock holdings in both liquid wealth and total wealth are decreasing functions of his wealth–labor income ratio, reflecting the role of future earnings as a close substitute for explicit bond holdings. Moreover, when indifferent between renting and owning, the homeowner holds less stock as a proportion of total wealth, reflecting the substitution effect, yet a higher equity proportion in his liquid wealth, reflecting the diversification benefit afforded by low correlation between housing and equity returns.

However, the presence of stock market participation cost further divides renters into stockholders and non-stockholders. To demonstrate how home ownership and stock market entry and participation decisions interact, we plot the joint home ownership and stock ownership decisions as a function of the wealth–income ratio \( Q_t/P_Y \) and age in the top panel of figure 1 for an investor who does not own a house in the previous time period and has no previous stock market entry. To own both a house and stock requires higher wealth relative to permanent income than to own either one alone. Except at very young ages, it takes less wealth to trigger home ownership than stock ownership. At very young ages, the investor chooses to become a stock owner before becoming a home owner. The shape of the home and stock ownership trigger bounds are primarily driven by the investor’s life–cycle earnings profile and bequest motive.

A young investor faces a steep earnings profile and therefore desires a large house relative to his current permanent income \( P_Y \). Because of the collateral borrowing constraint in the housing market, a larger house requires more wealth on hand to finance the housing down payment. A young investor also faces high exogenous mobility risk, leading to higher likelihood of triggering house liquidation costs, which in turn reduces the incentive to own a house and increases the amount wealth on hand necessary to induce home ownership. Yet a young investor can benefit more from
paying the one-time stock market entry cost since he has a longer investment horizon to recover the initial entry cost. A young investor also invests a higher proportion of liquid wealth in equity, due to the diversification effect of life-time human capital. By delaying home purchase, a young investor with moderate amount of wealth can potentially earn higher average return and afford a house of desired size in the future.

From middle age and on, the stock ownership boundary increases monotonically, reflecting the reduction in time to recoup one-time entry cost. The home ownership trigger bound, however, first becomes lower for a middle-aged investor as his income increases and exogenous mobility risk decreases. So starting at this stage of the life cycle, home ownership surpasses stock ownership as a primary concern to a household. Immediately after retirement, the uncertainty about future non-financial income is resolved, which lowers the home ownership trigger bound. Yet as an investor approaches his terminal date, he is more concerned with leaving a bequest with minimum liquidation cost. Therefore, at this stage of his life he is less likely to initiate home ownership and the wealth-permanent income trigger bound increases sharply.\textsuperscript{13}

In our discussion below, we first investigate a homeowner’s optimal housing, mortgage refinancing and default, and stock market participation decisions (figure 1b). We then focus on a homeowner’s optimal asset allocation decisions when he chooses to stay in the existing house, conditional on stock market entry (figure 2 to figure 4). For each continuous choice variable, we present both a three-dimensional figure (fixing the wealth–labor income ratio), and a two-dimensional figure (fixing both the wealth–labor income ratio and the house value–wealth ratio). Also, since refinancing cost is assumed to be proportional to house value and is independent of the mortgage balance before and after mortgage refinancing, the optimal decision rules conditional on refinancing are invariant to the investor’s beginning-of-period loan-to-value ratio. We present the simulation results under the baseline parameter, a borrowing and lending rate spread, and comparative static analysis at the end of this section.

\textsuperscript{13}This result however does not imply that all investors will revert to renting before death. The finding here says that a renter or a homeowner upon house liquidation will not initiate new home ownership at advanced ages. However if an investor owns his house, he can stay in the existing home and does not have to liquidate.
4.1. Optimal Housing, Mortgage, and Stock Market Participation Decisions

In the bottom panel of figure 1, we plot the discrete housing, mortgage, and stock market participation decisions as a function of beginning–of–period loan–to–value ratio (LTV) and house value–wealth ratio for a homeowner who has entered the stock market previously. For housing and mortgage decisions, the presence of collateral constraints, house liquidation cost (cost of selling or defaulting), and mortgage refinancing charge divides the state space into five different regions of actions: (1) the non–admissible region (N.A.)—the investor can never be in this region because combinations of the loan–to–value ratio and the house value–wealth ratio lead to negative liquid wealth and violation of the positivity constraint on stock and bond holdings, (2) the staying region (Stay)—the investor stays in the existing house without refinancing, (3) the refinancing region (Refi)—the investor stays in the existing house and cashes out a fraction of his home equity, (4) the selling region (Sell)—the investor sells his existing house, and (5) the default region (Default)—the investor defaults on his mortgage payments and the house is foreclosed by the lender.

The non–admissible region is a convex set on the upper left corner, with boundary values of incoming loan–to–value ratio and house value–wealth ratio defined by $(1 - \bar{I}_t - \phi)P^H_t H_{t-1}/Q_t = 1$. The investor is restricted from taking unsecured debt. Hence the size of his home equity cannot exceed total wealth. Intuitively, in order to finance a large house relative to one’s wealth, the investor has to carry a high mortgage balance.

A homeowner experiencing an exogenous moving shock ($D^m_t = 1$) has to liquidate his house by either selling or defaulting. When a homeowner’s mortgage loan–to–value ratio is sufficiently high, i.e., when $\bar{I} > 1 + \kappa - \phi$, he prefers defaulting on his mortgage to selling the house. This criterion is shown in the figure as the dotted line.

When not moving for exogenous reasons, the homeowner stays in the existing house if his house value–wealth ratio does not deviate far from the optimal house value–wealth ratio that a renter with identical wealth–income ratio would have chosen. Before the investor’s loan–to–value ratio reaches

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\(^{14}\) The investor’s wealth–labor–income ratio is set at $Q_t/Y_t = 1.75$ and age set at 50 in figure 1.
the conditional default trigger $l_t = 1 + \kappa - \phi$, the lower bound for selling decreases, while the upper bound increases in the loan-to-value ratio, leading to a wider range of tolerable house sizes at a higher mortgage balance. This result reflects the tradeoff between consuming the optimal amount of housing services and maintaining liquidity. Selling a house with a high loan-to-value ratio reduces liquidity because mortgage balance is due upon home sale. This is particularly important for an investor holding a negative home equity position, who has to inject liquid financial wealth to pay off mortgage balance and brokerage fees not covered by the home sale proceeds. Unlike a margin loan in a brokerage account, a mortgage contract has no maintenance margin requirement. Hence the liquidity provided by a large mortgage balance has a long-lasting effect in providing liquidity for consumption and stock investment.

In the absence of exogenous moving shock, the investor may choose to stay in the existing house even when his mortgage loan-to-value ratio exceeds the conditional default threshold by a significant amount. Intuitively, default can be thought of as an American put option. The investor does not have to default as soon as the put option is in-the-money, i.e. $l_t > 1 + \kappa - \phi$, since there might still be enough time value left in the option. The exact triggering level of the loan-to-value ratio for mortgage default is lower when the investor’s house size deviates too much from its optimal level. This reflects the investor’s desire to adjust his housing services, which strengthens his incentive to strategically default.\textsuperscript{15}

If a homeowner decides to stay in the existing house, he can convert a fraction of illiquid home equity to liquid financial assets through mortgage refinancing when his mortgage loan-to-value ratio is too low. Since there is no interest rate uncertainty or spread between borrowing and lending rates in our baseline specification, a homeowner refinesances entirely for consumption smoothing and/or portfolio rebalancing reasons. Further, because refinancing cost is independent of the size of home equity cashed out, the investor optimally chooses not to refinance immediately after his home equity

\textsuperscript{15}In an unreported exercise, we find that when default results in an exclusion from future access to credit market, the effect of deviations from optimal house size on default become asymmetric. While the trigger level of the loan-to-value ratio for default is still lowered for high house value-wealth ratios, it becomes higher for a low house value-wealth ratio. In the latter case, the benefit of defaulting (write-off of negative home equity), measured as a fraction of the investor’s wealth, is small while the cost in losing access to the credit market in the future is large.
exceeds 20 percent or the loan-to-value ratio falls below 80 percent. In fact, he often waits until his home equity position becomes very sizable before he refinances his mortgage.

Panel (b) in Figure 1 also illustrates how a homeowner’s stock market participation decision interacts with his housing and mortgage positions. When staying in the existing house without refinancing, a homeowner is likely to initiate stock ownership (“Stay w/ Stocks”) when his house value-wealth ratio is low or when his mortgage loan-to-value ratio is high. Intuitively, at a given level of wealth and labor income, the investor with a high mortgage balance or low house value has more wealth in liquid form and less in illiquid home equity. He is thus able to invest a large amount in stock to offset the stock market participation cost. Our findings are consistent with the empirical results discussed in section 2, which indicate that the mortgage-networth ratio has a significant positive effect while the home value-networth ratio has a significant negative effect on homeowners’ stock participation decisions. For the same reason, within the “Refi” region, stock ownership is initiated when the house value-wealth ratio is low.\footnote{16}

### 4.2. Optimal Asset Allocation Decisions

We now turn to the homeowner’s asset allocation decisions. In a complete market economy with unrestricted borrowing, Grossman and Laroque (1990) and Damgaard, Fuglsbjerg, and Munk (2003) find that the investor’s stock holdings as a fraction of wealth is U-shaped in his house value-wealth ratio. Intuitively, around the optimal house value-wealth ratio, the investor takes a more conservative equity position to reduce the need to trigger house liquidation. When close to the adjustment boundaries, however, he takes a riskier position due to option-like payoff around liquidation triggers.\footnote{16}
The difference between our portfolio rules and those of earlier research, therefore, reflects the effects of incomplete market (inability to capitalize future labor income) and borrowing constraints.\textsuperscript{17}

Figure 2 shows the investor’s liquid asset proportion \((S_t + B_t / S_t + B_t + P_H^t H_t - M_t)\) as a function of the beginning–of–period house value–wealth ratio and mortgage loan–to–value ratio. The top panel shows that within the no-adjustment region the proportion of the investor’s liquid assets in his wealth varies substantially with his house value–wealth ratio and loan–to–value ratio. Specifically, when the loan–to–value ratio is less than 100% (the investor has positive home equity), as the house value–wealth ratio increases a larger fraction of the investor’s wealth is tied to illiquid home equity and less is available for investment in liquid financial assets. However, when the loan–to–value ratio is greater than 100%, a larger house is associated with more liquid financial assets because the investor has negative home equity. The bottom panel shows that within the “Stay” region, for a given house value–wealth ratio, the investment in liquid financial assets increases in the investor’s loan–to–value ratio. However, upon refinancing, a fraction of illiquid home equity is converted to liquid financial assets, leading to a higher liquid financial assets–total wealth ratio. It is worth noticing that an investor can have more than 100 percent of his net worth in liquid form when he holds a negative home equity position.

Figures 3 and 4 show the investor’s stock investment as a fraction of his wealth \((S_t / S_t + B_t + P_H^t H_t - M_t)\) and as a fraction of his liquid assets \((S_t / S_t + B_t)\), respectively, as a function of the beginning–of–period house value–wealth ratio and mortgage loan–to–value ratio. The investor’s asset allocation choice reflects the investor’s desire to hold an optimal overall portfolio conditional on his existing housing and mortgage positions. Specifically, at very low levels of the loan–to–value ratio or very high levels of house value–wealth ratios, most of the investor’s wealth is in illiquid home equity, and his liquid wealth is small. He thus holds an all-equity liquid portfolio, yet is still underweighted in stock. Since the investor can only borrow against housing asset, the homeowner’s total equity exposure is limited by the size of his liquid portfolio. His net worth equity proportion declines as his mortgage loan–

\textsuperscript{17}For figure 2 to figure 4, the investor’s wealth worth–labor-income ratio is fixed at \(\frac{Q_t}{P_Y^t} = 7.0\), and his age at 50. The investor’s house value–wealth ratio is fixed at \(\frac{P_H^t H_t - 1}{Q_t} = 1.0\) for panel b of these figures.
to–value ratio decreases and/or his house value–net worth ratio increases. When the exposure to equity risk in his wealth becomes too low, the investor optimally refinances and converts a portion of his home equity into liquid wealth to increase his exposure to equity risk.

At higher levels of loan–to–value ratio or lower levels of house value–wealth ratio, the investor holds an interior mix for his liquid asset portfolio. His net worth equity proportion is relatively flat or slightly U-shaped in the house value–wealth ratio, similar to the findings in Grossman and Laroque (1990) and Damgaard, Fuglsbjerg, and Munk (2003). Given the house value–wealth ratio, while still increasing in his loan–to–value ratio, the investor’s net worth equity proportion increases at a much slower rate. These reflect the fact that when the loan–to–value ratio is high, the investor’s desired equity exposure can be financed by his liquid wealth on hand. Therefore the homeowner can achieve close to the optimal stock exposure at a given housing position relative to the case with unrestricted borrowing.

The investor’s liquid asset equity proportion can be increasing, U-shaped, or decreasing in his house value–wealth ratio, depending on the loan–to–value ratio. At low levels of the loan–to–value ratio, because the liquid wealth decreases in the house value–wealth ratio, the investor allocates more liquid wealth to stock to maintain the optimal exposure to equity in the overall portfolio. At high levels of the loan–to–value ratio—higher than 100 percent—the liquid asset equity proportion decreases in the house value–wealth ratio, because a negative home equity position increases the size of liquid portfolio. In between—when the entire housing asset is largely financed by a mortgage—the liquid asset equity proportion is similar to that of net worth equity proportion and demonstrates a slight U-shape in the house value–wealth ratio. Furthermore, at a given house value–wealth ratio, the investor’s liquid asset equity proportion decreases in his loan–to–value ratio, reflecting the larger fraction of liquid wealth and a smaller fraction of home equity.

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18 The effect of a large home equity position, which causes an all-equity liquid portfolio, is similar to that of lower liquid wealth on hand. At a low level of wealth–income ratio, the entire liquid assets are invested in stock, irrespective of one’s housing or mortgage positions, and the investor’s net worth equity proportion is identical to the fraction of liquid assets in total wealth (figure not shown).
Interestingly, at very high levels of the loan–to–value ratio—when the ratio is close to the default boundary—the investor’s liquid asset equity proportion is relatively flat in his loan–to–value ratio. Since the size of his liquid portfolio becomes larger as the loan–to–value ratio increases, this leads to once again a quickly increasing net worth equity proportion. This captures the effects of option–like payoff around default boundary.

Existing empirical studies document that the house value has a negative effect on households’ net worth equity proportion, while the mortgage balance has a positive effect on the net worth equity proportion. Given that the observed loan–to–value ratio is below one for the vast majority of households, our findings are broadly consistent with those empirical findings. While Heaton and Lucas (2000a) suggest that the positive effect of mortgage balance on equity holdings is caused by investors financing stock investments with mortgages, our analysis offers a detailed mechanism through which housing wealth and mortgage balance interplay with equity investments.

Also consistent with the empirical finding, our model predicts that households’ equity proportion in liquid assets decreases in the loan–to–value ratio. As for the lack of robust empirical conclusion on the effect of households’ housing position on their liquid asset equity proportion, a simple reduced form specification may have difficulty capturing the complicated interactions between asset allocations and housing and mortgage positions, given the complexity of the predicted theoretical relation. The presence of sample selection and unobserved heterogeneity in data further complicates its empirical identification.

4.3. Simulation Analysis

We now examine the investor’s life–cycle housing and asset allocation choices using a simulation analysis. We first simulate permanent and transitory shocks to labor income, the realization of exogenous moving shocks, and stock and housing returns based on a serially uncorrelated Markov process with two outcomes for each variable. We then use the optimal decision rules from our state–space solution to calculate the investor’s optimal consumption, housing, mortgage, and portfolio
choices. Home ownership and stock market entry status, wealth–labor-income ratio, house value–wealth ratio, and loan–to–value ratio are updated each period to determine the investor’s optimal decisions for the next period. The time-series profiles of the optimal decisions are generated by repeating the calculation from \( t = 0 \) (age 20) to \( t = 60 \) (age 80) for 50,000 simulation paths.

Figure 5 shows the home ownership rate (panel a), refinancing rate (panel b), default rate (panel c), and the average loan–to–value ratio for refinanced and defaulted mortgages (panel d). Consistent with empirical evidence, the percentage of home ownership is hump–shaped in age. It rapidly increases from zero to over 60 percent at early 30s. By early 40s, almost all investors own a house. After age 76, the percentage of home ownership decreases reflecting the fact that some investors sell their houses in anticipation of bequeathing their wealth to their beneficiaries. While the simulated life–cycle home ownership rate resembles the overall pattern of the observed data, it is higher than the observed counterpart. The growth of home ownership rate among young households is much faster than its empirical counterpart.\(^{19}\)

The refinancing rate reaches the peak of about 2.6 percent for investors in early 30s. With costly refinancing, an investor just purchased a house is less likely to refinance because he has not accumulated enough home equity (through mortgage payment and/or house price appreciation). Investors in their mid–to–advanced ages are also less likely to refinance because of their large liquid asset accumulation. This allows the investors to smooth consumption and hold desired amount of stock investment without incurring a refinancing charge. Young homeowners face steep income profiles and are more likely to be temporarily liquidity-constrained upon a bad income shock due to large mortgage payment obligations and low level of liquid wealth. They also have high desired equity exposure (because of their high present value of labor income) yet are limited by the amount of liquid asset on hand. Hence they are more likely to refinance to smooth intertemporal consumption and increase stock holdings when their houses appreciate in value.

\(^{19}\)Heterogeneity in home ownership benefits among investors in reality can lead to some investors being lifetime renters. For example, rent controls in New York City provide an incentive for some residents to remain as renters throughout their life time. Tax rates also vary across different wealth and income brackets. Young and low-income investors with lower marginal tax rates have less incentives to become a homeowner since the benefit of mortgage interest deductability is small.
The default rate peaks at around 1.6 percent in the investor’s early 40s and goes down to zero in their early 70s. New homeowners will not default because of the mortgage down payment provides protection for the lender’s equity interest in the property. The desire to maintain liquidity also reduces the incentive to exercise the default option. The default rate is lower for a senior homeowner since the investor on average reduces his mortgage balance over time with principal amortization associated with the mortgage contract. Overall, the predicted (low) level of default rate is consistent with the observed rate as reported in Deng, Quigley, and Van Order (2000) for residential mortgages.

The average mortgage balance relative to house value for refinanced mortgages decreases from about 65 percent at late 20s to about 35 percent at late 50s. Investors cash out the excess illiquid home equity only when their loan–to–value ratio falls significantly below the collateral requirement (their home equity substantially exceeds minimum home equity requirement of 20%), due to refinancing cost. The downward drifting in the refinance triggering value of loan–to–value ratio reflects the availability of more accumulated liquid wealth over time for consumption smoothing and equity investment so that the investor does not need to tap into his home equity as often. The average mortgage balance relative to the house value for defaulted mortgage averages around 120%, significantly higher than the conditional default boundary ($l_t = 1 + \kappa - \phi = 109\%$), reflecting the time value of the option to default, and the cost of lost liquidity associated with exercising the default option.

Figure 6 shows the average numeraire good consumption and income (panel a), house value–income ratio and wealth–income ratio (panel b), loan–to–value ratio (panel c), and home equity–income ratio and liquid wealth-income ratio (panel d) across different ages in the life-cycle. The numeraire good consumption tracks the income profile. It exhibits a hump shape and peaks in the investor’s 50 to 60s. The decline accelerates after retirement when the investor stops receiving labor income. Both the average house value–income ratio and the wealth–income ratio for homeowners are also hump-shaped and peaking just before the retirement. Young investors accumulate wealth slowly while renting until early 30s, at which point they start to purchase their first home. For homeowners, the accumulated wealth increases steadily from early 30s until retirement. Investors experiencing bad
income shocks have little wealth on hand and keep renting. After retirement, homeowners’ wealth
decrease as they draw down their wealth accumulated during working years to supplement retirement
income to pay for housing and numeraire good consumption expenses. The house value–income ratio
for homeowners starts at around 3 and reaches the peak value of 4 around retirement, and declines
until the investor reaches the terminal date thereafter.

As the investor ages, he gradually accumulates home equity by making scheduled mortgage
principal and interest payments. The average mortgage balance relative to house value starts at 80
percent at late 20s and declines to zero at age 80. Overall the simulated mortgage loan–to–value
ratio decreases more slowly than what is observed empirically. One possible explanation is that the
benefit of paying down mortgage is much larger in reality than in our baseline model without a credit
spread for mortgage rate.

Consistent with the behavior of the loan–to–value ratio, the average home equity–income ratio
gradually increases. The homeowners’ liquid financial wealth–income ratio tracks the wealth–income
ratio closely. It increases faster than the home equity before retirement. In retirement stage, however,
homeowners draw down liquid savings first to defer mortgage refinancing and home selling costs.
Home equity is accessed only as a last resort.

Figure 7 shows the fraction of investors owning stocks (panel a), the average networth equity
proportion (panel b), the average liquid asset equity proportion (panel c), and the average amount of
stockholding (panel d). Young investors in the early 20s are deterred by the stock market entry and
participation costs because of insufficient liquid wealth. Stock ownership increases in the investor’s
late 20s as renters accumulate wealth by saving a fraction of their income for both precautionary
reasons and future house down payments. The transition to stock ownership then slows down tem-
porarily as some investors become homeowners and convert a significant fraction of liquid wealth to
illiquid home equity.

As investors age and accumulate more liquid financial wealth, the stock market participation rate
quickly increases, reflecting the large lifetime benefit of stock market participation. It reaches 99
percent at age 53, and peaks at close to 100% around retirement. After age 70s, the stock ownership rate quickly declines. By the terminal date, 34% of investors have exited from the stock ownership market, an indication of the investor’s incentive to avoid per period maintenance cost associated with stock investment and the desire to hold safer financial securities. While the predicted peak level of stock ownership rate (close to full ownership) is higher than the observed counterpart, our model still offers a significant improvement over existing studies in generating a lower stock market participation rate as well as exits from stock market. Our results also suggest that other modeling strategies may be required to replicate the empirically documented stock market participation rate.\(^{20}\)

The homeowner’s equity proportion in networth is hump-shaped. It starts around 65 percent at investors’ late 20s and peaks just below 80 percent between ages 40 and 50. It then gradually declines to about 26 percent at age 80. A homeowner’s liquid wealth is entirely concentrated in stocks until his early 40s and declines gradually afterwards. Overall, the behavior of the homeowner’s equity proportion reflects the strong incentive to take advantage of the diversification benefit afforded by non-financial income and home equity. Young homeowners have a large fraction of wealth in home equity, which reduces their networth equity proportions. Yet they also have a high present value of lifetime nonfinancial income that serves as a substitute for bond. Hence, young homeowners concentrate their liquid asset investment in stocks. As they grow older and accumulate more liquid wealth, both the present value of remaining labor income and the proportion of total wealth held in illiquid home equity are reduced, leading to declining equity proportions in both the networth and liquid wealth.

Both the average liquid wealth equity proportion and average networth equity proportion are higher than their observed counterparts in the SCF data. Empirically the homeowners’ liquid wealth equity proportion is rather flat in age and only declines slightly from the peak around 47 percent between age 45 to 50 to about 40 percent for the age group 75 and beyond. For the networth equity proportion, except for the youngest age group, the observed average stock investment in networth

\(^{20}\)In a recent study, Cao, Wang and Zhang (2004) introduce model uncertainty to explain the limited stock market participation with some success.
stays within a narrow range between 19 percent and 23 percent with the peak reached between age 45 and 55. The predicted decline after retirement is much more dramatic than observed in the data.

Panel (d) shows that the average dollar amount of stock investment increases steadily until the investor reaches retirement. It then declines gradually as liquid assets are drawn down to supplement retirement income and the investor becomes more conservative in his liquid wealth portfolio. This pattern is consistent with the observed investors’ behavior in the SCF data. Empirically the stockholding peaks between ages 65 and 75 in the life cycle.

4.4. Borrowing and Lending Rate Spread

The above analysis is based on the assumption that the borrowing rate paid by investors on their mortgage balance is the same as the lending rate received from their bond holding. Given the potential for default and institutional costs in loan underwriting and maintenance, mortgage lenders may require a positive spread between the borrowing rate and the lending rate to compensate them for the extra cost. We now explicitly set the borrowing rate to be 1.5% above the lending rate in the baseline model.\textsuperscript{21}

Figure 8 shows the simulated home ownership rate and average loan-to-value ratio (panel a), stock ownership rate (panel b), and the average equity proportion in liquid financial assets (panel c) and networth (panel d) among stockholders, in the presence of a positive credit spread. Overall, the life-cycle home ownership pattern resembles that in the baseline case. However, investors have a much stronger incentive to pay down mortgage balance with a positive spread than in the baseline case. While the mean loan-to-value ratio stays at around 80 percent from age 30 to age 40 in the baseline model, it decreases drastically from 80 percent to 40 percent. While the mean loan-to-value ratio goes down to about 50 percent by age 70 in the baseline model, it is drastically reduced to 1 percent with a positive credit spread. This reflects the fact that investors pay down mortgage to reduce the extra cost associated with a higher borrowing rate.

\textsuperscript{21}To maintain similar home ownership incentives, we also raise the rental cost by the same amount by setting $\alpha = 7.5\%$. Other parameter values are held the same.
Investors’ stock ownership is significantly reduced when facing a higher borrowing rate. The stock market entry begins in the late 20s, and then grows steadily at about 2.5% per year until peaking at slightly above 85 percent around retirement. It then drops to 62% at retirement. Consistent with the rapidly declining loan–to–value ratio over the life-cycle, a large fraction of the investor’s networth is saved in home equity, reducing the amount of liquid wealth for costly stock investment. The average stock ownership across all age groups is about 50 percent, similar to the rate reported in 2001 SCF data. The diversification benefit associated with a large home equity position allows the investor to hold higher equity proportion in his liquid asset portfolio relative to the baseline case. The rapidly increasing home equity position also leads to a declining homeowner’s networth equity proportion over his lifetime. This is in contrast to the baseline case in which the equity proportion in networth exhibits a slight hump-shape.

### 4.5. Comparative Static Analysis

In this section we provide comparative static analysis to demonstrate how different features of the model affect investors’ housing and asset allocation decisions. Specifically, we examine five alternative specifications including an expected utility with constant relative risk aversion (\( \zeta = 1/\gamma \)),\(^{22}\) zero per period stock investment cost (\( \varrho = 0 \)), zero mortgage refinancing charges (\( \psi = 0.0 \)), a collateral borrowing constraint that is applicable both at loan initiation and for ongoing loans (\( l_t \leq \delta \)), and a model without home ownership (\( D^p_t = 0 \)). Our discussions focus on simulated household wealth accumulation, home and stock ownership, and portfolio choices for various model specifications.

Table 3 reports the average household wealth–income ratio before receiving labor income, the percentage of home ownership, and the percentage of stock ownership across different ages for the baseline model and the alternative specifications.\(^{23}\) Overall, the wealth accumulated in all models are hump-shaped and peaking just before the investor’s retirement. Home ownership increases and

---

\(^{22}\)When the elasticity of intertemporal substitution is the reciprocal of the risk aversion coefficient, the recursive Epstein-Zin preference is reduced to the constant relative risk aversion preference.

\(^{23}\)The home ownership in the \( D^p_t = 0 \) model is zero. For this case, we only report household wealth-income ratio and the percentage stock ownership.
peaks at 100 percent except for the last few years when investors approach the terminal date. The stock ownership also exhibits a hump-shape peaking at 100 percent. The only exception is the case with zero per period stock investment cost.

Compared to the baseline case with Epstein-Zin preferences, investors with time-additive CRRA preferences accumulate more wealth due to lower intertemporal rate of substitution ($\zeta = 0.2$). They are less inclined to become homeowners earlier. Their stock ownership increases much faster and reaches 100% much earlier than under the baseline case. This is consistent with the higher wealth accumulation and lower home ownership under the CRRA preferences.

Omitting per period stock investment cost, as in Cocco (2005) and Yao and Zhang (2005), increases wealth accumulation. It also drastically increases stock ownership. Interestingly, young investors postpone home ownership slightly relative to the baseline case in order to enter stock market earlier and invest in stock heavily. Furthermore, with the stock market entry cost only, the model cannot generate exit from the stock market.

Eliminating mortgage refinancing charges reduces the costs to access liquid home equity and increases an investor’s incentive to purchase a house early. Seventy-four percent of investors own their homes at age 30, higher than 62 percent in the baseline case. Stock ownership is also higher than the baseline case at young ages. This can be attributed to the larger fraction of wealth in liquid form without refinancing changes.

The presence of a minimum home equity requirement, an assumption made in Cocco (2005) and Yao and Zhang (2005), severely discourages home ownership by eliminating mortgage default option and forcing investors to inject liquid wealth to maintain home equity position in a housing market down turn (the collateral constraint becomes a maintenance margin requirement). Young investors wait much longer to accumulate more wealth before becoming a homeowner. Interestingly, relative to the baseline case, stock market entry starts earlier and increases at a faster rate at younger ages as renters invest their liquid wealth accumulated for house down payment.
Finally, when the investor can only rent housing services, as implicitly assumed in models without housing and mortgage decisions, the stock ownership is much higher among young investors. For example, at age 30, more than one third of investors are stockholders, relative to the baseline case of 11%. This reflects that a renter’s entire wealth is in liquid form and can be used for stock investment.

Table 4 presents the average liquid financial assets–networth ratio, the average equity proportion in liquid financial assets and in networth for homeowners participating in stock market for the baseline and five alternative models. Liquid financial asset holdings are hump-shaped and peaking between age 45 (under the expected utility with CRRA preferences) and age 60 (in the model without refinance charge).

Under the time-additive CRRA preferences, young homeowners accumulate much more liquid financial assets in networth than in the baseline case. For instance, at age 30, homeowners on average hold 67 percent of networth in liquid financial assets under the CRRA utility preferences compared to 42 percent in the baseline case. The difference, however, gradually reduces as the investor ages. Conditional on participating in the stock market, the investor holds a higher equity proportion in his networth before age 45 but a lower equity proportion afterwards. The equity proportion in liquid financial assets is slightly lower than in the baseline case.

Without per period stock market participation cost, young homeowners accumulate more liquid wealth than in the baseline case. They hold slightly more stock as a fraction of his total wealth to take advantage of better investment opportunities in equity than in the baseline case. The conditional equity proportions are quite similar to the baseline case after age 30.

In the absence of mortgage refinancing costs, an investor will always cash out excess equity beyond the minimum requirement. By doing so they increase the fraction of liquid wealth as well as the value of default option. Thus, liquid wealth accounts for a higher proportion of the investor’s total wealth than in the baseline case. The smaller illiquid home equity also leads to a higher equity proportion in wealth. The equity proportion in liquid financial assets, however, decreases much faster than in

---

24Since there is no home ownership in the $D^2$ model, we report the equity proportions for renters. Also note that the equity proportion in liquid financial assets is the same as the equity proportion in networth in this case.
the baseline case as the investor ages, reflecting the larger liquid wealth available for investment in financial assets than under the baseline case.

When the collateral constraint becomes a maintenance margin requirement, young home owners have a slightly larger fraction of total networth in liquid form than in the baseline case. This is because investors buy a smaller home in order to meet future home equity requirement. The fraction of homeowners’ networth allocated to equity is also slightly higher at this stage than in the baseline model. However, investors in the baseline model accumulate liquid wealth at a faster rate since they do not need to allocate liquid wealth to satisfy mortgage maintenance margin requirement. This leads to a higher networth equity proportion for most of a homeowner’s life time in the baseline model. The equity proportion in liquid financial assets in the baseline model is slightly lower, reflecting lower diversification benefit of home equity for equity holdings.

The impact of home ownership on stock ownership and portfolio choices is best illustrated by contrasting the results with and without home ownership. Without home equity tying up his wealth, the investor holds an all equity portfolio in their networth until age 40 conditional on stock ownership, while in the baseline case the equity proportion in networth ranges from 63 percent at age 28 to 77 at age 40. As the investor ages, the equity proportion decreases. The liquid asset equity proportion in the baseline model, however, is higher than in the case without home ownership. This reflects the diversification benefit from home ownership for stock investment in the presence of home ownership.

Overall, our comparative static analysis highlights two important effects. First, the transition to home ownership affects stock market participation by influencing the investor’s wealth accumulation while renting. Second, the illiquidity of home equity has an important effect on portfolio compositions conditional on stock market participation. On one hand, a larger home equity reduces networth equity proportion. On the other hand, it increases equity proportion in the investor’s liquid portfolio.

Compared with the empirically documented household housing and mortgage decisions, and asset allocations, we have the following observations. First, the simulated peak home and stock ownership rates are much higher than the observed counterparts. The model with the CRRA preference does
the best in generating lower home ownership. The baseline model with the Epstein-Zin preference does the best in matching the stock ownership, especially with a positive borrowing-lending interest rate spread.

Second, consistent with observed home equity–networth ratio, the simulated liquid wealth relative to total wealth implies a U-shaped home equity in wealth. However, the implied peak level home equity–wealth ratio is higher than the observed counterpart. The model does the best in matching the observed home equity–networth ratio when investors have to meet the home equity requirement every period or there is a positive borrowing and lending spread (figure not shown).

Third, the simulated equity proportion in networth resembles the observed counterpart in its hump-shape in age. It reaches the peak between ages 40 to 45, which are slightly earlier than the observed counterpart (reaching its peak between 45 to 55). The simulated equity proportion is however much higher than the observed counterpart. Again, the model with the collateral constraint as the maintenance margin does the best among all the specifications.

All model specifications fail to capture the observed liquid asset equity proportion. The simulated liquid asset equity proportion is much higher than the observed counterpart. It also decreases in age while the observed counterpart does not demonstrate a clear pattern.

5. Conclusion

We investigate household finance decisions (asset allocation among risk free assets, stocks, housing and mortgage) by extending existing studies on life-cycle dynamic portfolio choices in two important dimensions. First, we introduce costly refinancing of existing mortgage. This provides incentive for households to simultaneously hold both risk free assets and mortgage, and eliminates the undesirable counterfactual implication that investors do not hold bond or an all-equity portfolio until they completely pay off their mortgage. Second, we allow mortgage balance to fall below the initial down-payment requirement in subsequent periods so that households can choose to optimally default on
their mortgage. We identify that households’ liquid financial wealth plays an important role for both stock and housing decisions. While introducing costly refinancing reduces households’ liquid asset wealth, allowing mortgage balance to fall below the initial downpayment requirement and default option increases households’ liquid wealth.

Our state space analysis on the optimal decision rules reveals that the model can qualitatively explain the documented empirical relations between household asset allocations and their housing and mortgage choices. In our model, given wealth and labor income, the investor with a lower housing value–net worth ratio and/or a higher loan–to–value ratio has more liquid wealth. He is thus in a better position to take advantage of opportunities offered by the stock market because the cost associated with stock market participation is outweighed by the benefit to the investor holding a large liquid wealth. Further, the investor with a lower housing value–net worth ratio and/or a higher loan–to–value ratio also invests a higher proportion of wealth in stock because with a large portion of his wealth in liquid assets he can better buffer negative shocks to his wealth and income.

Conversely, the investor with a high housing value–net worth ratio or a low loan–to–value ratio has a large portion of his wealth tied to illiquid home equity. Hence he reduces the fraction of net worth invested in stock when borrowing against stock or human capital is not possible. However, the equity proportion in an investor’s liquid wealth increases with the size of his illiquid home equity, reflecting the diversification benefit of home equity for stock. Further, an investor might trigger refinance charge in order to convert a fraction of his illiquid home equity into liquid form to supplement consumption or to initiate stock ownership. An investor who is holding an all-equity liquid portfolio but still underexposed to equity may also optimally choose to refinance his mortgage to increase his stock investment.

Our simulation analysis further reveals that young homeowners not only have relatively low networth, they also have most of their wealth in illiquid form. This explains the lack of stock market participation and the low networth equity proportion for investors in this age group. However both the predicted networth equity proportion and liquid wealth equity proportion are still higher.
than their observed counterparts. While various frictions on the housing market help explaining important empirical relations between stock investment and housing and mortgage decisions, it remains a challenge to generate empirically plausible equity proportions.
6. Appendix: Derivation of the Model and Solution Algorithm

The investor’s wealth process and intertemporal budget constraint depend on the investor’s home and stock ownership statuses at time $t - 1$ and his housing, mortgage, and stock market entry and participation choices at time $t$. Let $D^e_t$ take the value of 1 if the investor has already entered the stock market at $t$ or decides to enter for the first time at $t$, and 0 otherwise, $D^p_t$ take the value of 1 if the investor participates in the stock market at $t$ and 0 otherwise, $D^o_t$ take the value of 1 if the investor owns a house at time $t$ and 0 otherwise, $D^r_t$ take the value of 1 if the investor refinances his mortgage at time $t$ and 0 otherwise, $D^s_t$ take the value of 1 if the investor sells his house at time $t$ and 0 otherwise, and $D^d_t$ take the value of 1 if the investor defaults on his mortgage at time $t$ and 0 otherwise. If the investor is a renter at time $t - 1$, then his spendable resource evolves according to:

$$Q_t = B_{t-1}R_f + S_{t-1}\tilde{R}^S_t + P^Y_{t-1}\exp\{f(t)\}\nu_t\epsilon_t.$$  \hspace{1cm} (7)

If the investor is a homeowner at time $t - 1$, then his spendable resource also includes home equity (net of liquidation cost associated with either selling or default): 25

$$Q_t = B_{t-1}R_f + S_{t-1}\tilde{R}^S_t + P^Y_{t-1}\exp\{f(t)\}\nu_t\epsilon_t + P^H_{t-1}H_{t-1}\tilde{R}^H_t \max\{1 - \tilde{l}_t - \phi, -\kappa\}.$$  \hspace{1cm} (8)

Let $l_t = M_t/P^H_t H_t$ be the investor’s mortgage loan-to-value ratio after housing and mortgage adjustments. The investor’s intertemporal budget constraint can then be written as:

1. If the investor is a renter at time $t - 1$ ($D^o_{t-1} = 0$), or if he is a homeowner at time $t - 1$ but decides to liquidate his house ($D^p_{t-1} = 1$ and $D^s_t + D^d_t = 1$), and rent for the current period ($D^o_t = 0$),

$$Q_t = C_t + B_t + S_t + P^H_t H_t \alpha + (1 - D^o_{t-1})D^e_t \eta P^Y_t + D^p_t g P^Y_t.$$  \hspace{1cm} (9)

25For ease of discussion, we simply refer to $Q_t$ as the investor’s wealth.
2. If the investor is a renter at time $t - 1$ ($D_{t-1}^o = 0$), or if he is a homeowner at time $t - 1$ but decides to liquidate his house ($D_{t-1}^o = 1$ and $D_t^s + D_t^d = 1$), and own a house for the current period ($D_t^p = 1$),

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1}(1 - l_t + \psi + \varphi) + (1 - D_{t-1}^o) D_t^p \eta P_t^Y + D_t^p \rho P_t^Y.$$  \hspace{1cm} (10)

3. If the investor is a homeowner at time $t - 1$ ($D_{t-1}^o = 1$), and decides to stay in existing house without refinancing his mortgage ($D_t^o = 1$, $D_t^s = D_t^d = D_t^p = 0$),

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1} \left[ (\bar{l}_t - l_t + \psi) + \max\{1 - \phi - \bar{l}_t, -\kappa\} \right] + (1 - D_{t-1}^o) D_t^o \eta P_t^Y + D_t^o \rho P_t^Y,$$

with $l_t \leq \bar{l}_t \left[ 1 - \frac{r_m^N/(1+r_m^N)}{1-(1+r_m^N)t^{-T}} \right]$.

4. If the investor is a homeowner at time $t - 1$ ($D_{t-1}^o = 1$), and decides to stay in existing house and refinance his mortgage ($D_t^o = 1$, $D_t^s = D_t^d = 0$, and $D_t^p = 1$),

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1} \left[ (\bar{l}_t - l_t + \psi + \xi) + \max\{1 - \phi - \bar{l}_t, -\kappa\} \right] + (1 - D_{t-1}^o) D_t^o \eta P_t^Y + D_t^o \rho P_t^Y,$$

with $l_t \leq 1 - \delta$. Since the investor will refinance only when he has more home equity than required by collateral borrowing constraint, i.e. $\bar{l}_t < 1 - \delta$, under the assumption that $\delta - \phi > -\kappa$, we can further simplify the wealth process when refinancing to

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1}(1 - l_t + \psi + \xi - \phi) + (1 - D_{t-1}^o) D_t^o \eta P_t^Y + D_t^o \rho P_t^Y.$$ \hspace{1cm} (12)

The vector of sufficient state variables consists of the beginning-of-period stock ownership dummy, home ownership dummy, moving shock dummy, total wealth, permanent labor income, price per unit of housing services, size of the existing house, and outstanding mortgage balance. We represent the vector of state variables by $X_t \equiv \{ D_{t-1}^o, D_{t-1}^s, D_{t-1}^d, Q_t, P_t^Y, P_t^H, H_{t-1}, M_{t-1}(1+r_m) \}$. 

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We assume that the investor has the recursive Epstein-Zin preference (Epstein and Zin (1989)),
 generalized to incorporate mortality risks, bequest motives, and demographic effects. This specification allows the separation of risk aversion and elasticity of intertemporal substitution. The investor’s problem at time \( t \) then has the following iterative representation:

\[
V_t(X_t) = \max_{A(t)} \left\{ (1 - \beta) \left[ \lambda_t u(C_t, H_t; N_t)^{1-\gamma} + (1 - \lambda_t) b(Q_t, P_t^H; L)^{1-\gamma} \right] + \beta \lambda_t E_t \left[ V_{t+1}(X_{t+1})^{1-\gamma} \right]^{1-\gamma/\zeta} \right\},
\]

where \( \beta \) is the discount factor, \( \lambda_t \) is the survival probability at time \( t \) conditional on living up to time \( t - 1 \), \( \zeta \) is the elasticity of intertemporal substitution, \( \gamma \) is the risk aversion coefficient, \( u(C_t, H_t; N_t) \) is a function which aggregates numeraire good consumption and housing services into utility at time \( t \), and \( b(Q_t, P_t^H; L) \) is a bequest function, subject to labor income process (equation 1), mortgage collateral constraint and amortization schedule (equations 6 and 7), borrowing and short sale constraints, and intertemporal budget constraint (equations 10–15), given the initial stock market entry status \( D_e(-1) \), home ownership status \( D_o(-1) \), exogenous moving shock \( D_m^0 \), after labor income wealth (net of house liquidation cost if applicable) \( Q_0 \), permanent income \( P_Y^0 \), housing price \( P_0^H \), housing stock \( H(-1) \), and mortgage value \( M_{-1} \). Due to the presence of stock market entry cost and house liquidation cost, time \( t - 1 \) stock market entry decision \( (D_e^t-1) \) and housing choices \( (D_o^t-1 \text{ and } H_t-1) \), are also state variables at time \( t \). \( A(t) = \{D_e^t, D_o^t, D^s_t, D^d_t, C_t, H_t, l_t, S_t, B_t\} \) is the vector of choice variables at period \( t \).

As to intratemporal preferences, we assume that the investor aggregates housing and numeraire consumptions using the Cobb-Douglas function:

\[
u(C_t, H_t; N_t) = N_t \left( \left( \frac{C_t}{N_t} \right)^{1-\omega} \left( \frac{H_t}{N_t} \right)^{\omega} \right)^{1-1/\zeta} \right]^{1-1/\zeta} = N_t^{1/\zeta} C_t^{1-\omega} H_t^\omega,
\]

where \( \omega \) measures the relative importance of housing services versus numeraire good consumption, and \( N_t \) measures the effective size of the family, capturing the economies of scales in household
consumption. We assume that the proceeds from the bequest are equally divided among \( L \) beneficiaries and used to pay for the beneficiaries’s numeraire good consumption and housing rental costs. Further, the beneficiary’s numeraire good and housing–service consumption is set at the fixed proportion of \((1 - \omega)/\omega\), the optimal proportion when the beneficiaries have the Cobb-Douglas utility function when renting.\(^{26}\)

Specifically, the investor’s bequest function is represented by

\[
b(Q_t, P^H_t; L) = \left[ N_t \left( \frac{Q_t}{N_t} (1 - \omega) \left( \frac{\omega}{\alpha P^H_t} \right)^\omega \right)^{1 - 1/\xi} \right]^{1/1 - \xi} = L^{\xi - 1} Q_t (1 - \omega)^{1 - \omega} \left( \frac{\omega}{\alpha P^H_t} \right)^\omega. \tag{15} \]

We simplify the investor’s optimization problem by normalizing the investor’s continuous consumption and asset allocation choices by his wealth \( Q_t \). Let \( c_t = C_t/Q_t \) be the consumption–wealth ratio; \( h_t = P^H_t H_t/Q_t \) the house value–wealth ratio; \( b_t = B_t/Q_t \) the fraction of the investor’s wealth invested in bonds after trading; and \( s_t = S_t/Q_t \) the fraction of wealth allocated to stocks after trading. By assuming a Cobb-Douglas intratemporal utility function, proportional housing purchase, maintenance and liquidation costs, proportional mortgage down payment, initiation, refinancing and default costs, and proportional stock market entry and per period participation costs, we ensure that the numeraire good consumption, housing services, mortgage, and portfolio rules are independent of the investor’s wealth level, \( Q_t \). With the above normalization, the relevant state variables for the investor’s problem can be written as \( x_t = \{ D^e_t, D^p_t, D^m_t, q_t, h_t, l_t \} \), where \( q_t = Q_t/P^Y_t \) is the investor’s wealth–labor-income ratio, \( h_t = P^H_t H_t/Q_t \) is the homeowner’s beginning–of–period house value–wealth ratio and \( l_t = M_t - 1/(1 + r_m) P^H_t H_{t-1} \) is the homeowner’s beginning–of–period loan–to–value ratio. Depending on the investor’s home ownership status and the realization of the moving shock at the beginning of the period, we can further specify the state and choice variables as follows. If the investor rents his housing services in the previous period \((D^e_{t-1} = 0)\), or if he is a homeowner who experiences a moving shock \((D^o_{t-1} = D^m_t = 1)\), then

\[
x_t \equiv \{ D^e_t, q_t \} \quad \text{and} \quad a_t \equiv \{ D^e_t, D^p_t, D^m_t, c_t, h_t, l_t, s_t, b_t \}.
\]

\(^{26}\)This specification of bequest function preserves homogeneity needed for normalization to reduce the dimension of the state space.
If the investor owns his residence in the previous period and does not have to move for exogenous reasons \((D_{t-1}^{o} = 1\) and \(D_{t}^{p} = 0\)), then
\[
x_t \equiv \{D_{t-1}^e, q_t, \rho_t, l_t\} \quad \text{and} \quad a_t \equiv \{D_t^e, D_t^p, D_t^o, D_t^d, c_t, h_t, l_t, s_t, b_t\}.
\]

Define \(G_{t+1} = q_{t+1}/q_t\) as the real (gross) growth rate of the investor’s wealth. If the investor is a renter at time \(t - 1\), then
\[
G_{t+1} = b_t R_f + s_t \tilde{R}_{t+1}^S + \exp\{f(t + 1)\} \nu_{t+1} \epsilon_{t+1}/q_t.
\]

If he is a homeowner at time \(t - 1\), then
\[
G_{t+1} = b_t R_f + s_t \tilde{R}_{t+1}^S + \exp\{f(t + 1)\} \nu_{t+1} \epsilon_{t+1}/q_t + h_t \tilde{R}_{t+1}^H \max\{1 - l_{t+1} - \phi, -\kappa\}.
\]

Similarly, the budget constraint in equation 9–12 can be normalized as:
\[
\begin{align*}
1 &= c_t + b_t + s_t + h_t \alpha + (1 - D_{t-1}^e)D_t^e \eta + D_t^p \vartheta, \\
1 &= c_t + b_t + s_t + h_t (1 - l_t + \psi + \varphi) + (1 - D_{t-1}^e)D_t^e \eta + D_t^p \vartheta, \\
1 &= c_t + b_t + s_t + \rho_t [(l_t - l_t + \psi) + \max\{1 - \phi - l_t, -\kappa\}] + (1 - D_{t-1}^e)D_t^e \eta + D_t^p \vartheta, \\
1 &= c_t + b_t + s_t + \rho_t (1 - l_t + \psi + \xi - \phi) + (1 - D_{t-1}^e)D_t^e \eta + D_t^p \vartheta.
\end{align*}
\]

Define \(v_t(x_t) = V_t(X_t)P_t^{H\omega}/Q_t\) to be the normalized value function. The investor’s problem now can be restated as follows:
\[
v_t(x_t) = \max_{a(t)} \left\{ (1 - \beta) \left[ \lambda_t \left( N_t^c c_t^{-1} h_t^\omega \right)^{1-1/\zeta} + (1 - \lambda_t) \left( L_t^c \left( 1 - \omega \right)^{1-1/\zeta} \left( \omega/\alpha \right)^{\omega} \right)^{1-1/\zeta} \right] \right. \\
+ \left. \beta \lambda_t E_t \left[ \left( v_{t+1}(x_{t+1}) G_{t+1} / \left( \tilde{R}_{t+1}^H \right)^{\omega} \right)^{1-\gamma} \right] \right\}^{1-1/\zeta}.
\]
The above problem can be solved numerically using backward recursion. We first discretize the wealth–labor-income ratio, \( q_t = Q_t/P_t^Y \), into a grid of 320 over the interval \([0.1, 100]\) equally spaced in the logarithm of the ratio, the house value–wealth ratio, \( \bar{h}_t = P_t^{H_{t-1}}/Q_t \), into an equally spaced grid of 160 over the interval \([0, 16]\), and the loan-to-value ratio, \( \bar{l}_t = M_{t-1}(1+r_m)/P_t^{H_{t-1}} \), into an equally spaced grid of 160 over the interval \([0, 2.0]\), respectively. At the terminal date \( T \), \( \lambda_T = 0 \), and the investor’s value function is a constant:

\[
v_T(x_T) = \left\{ (1 - \beta) \left[ L^{\frac{1}{\zeta}} (1 - \omega)^{1-\omega} \left( \frac{\omega}{\alpha} \right)^{1-1/\zeta} \right] \right\}^{\frac{1}{1-1/\zeta}} = (1 - \beta)^{\frac{1}{1-1/\zeta}} L^{\frac{1}{\zeta}} (1 - \omega)^{1-\omega} \left( \frac{\omega}{\alpha} \right)^{\omega} \tag{24}
\]

at all points in the state space. The value function at date \( T \) is then used to solve for the optimal decision rules for all points on the state space grid at date \( T - 1 \). A three-dimension B-spline interpolation is used to approximate the value function for points that lie between grid points in the state space. The procedure is repeated recursively for each time period until the solution for date \( t = 0 \) is found.
References


## Table 1
Summary Statistics for Housing and Portfolio Choices from the Survey of Consumer Finances (SCF), 1989–2001

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Table 1 reports percentage of home ownership (OWNHOUSE), average of house value–asset ratio (PH/ASSET), house value–net worth ratio (PH/NW), percentage of homeowners owning mortgage (OWEMORT), average households’ mortgage–total debt ratio (MORT/DEBT), mortgage loan-to-value ratio (LTV), percentage of stock ownership (OWNSTOCK), average equity proportion in total asset (S/ASSET), net worth (S/NW) and liquid financial assets (S/SB), across age groups and various years. Households with negative value for net worth and income, and households in the top and bottom 1% of the sample for the ratio variables are dropped to limit the bias of outliers.
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Table 3
Summary Statistics for Wealth Accumulation, Home and Stock Owningships from Simulations

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Table 3 reports average wealth-income ratio before labor income ($Q_t/P_{t-1} - 1$), and the rate of home and stock ownerships from simulations under various parameters.
Table 4
Summary Statistics for Homeowners’ Portfolio Choices from Simulations

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<tr>
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<td>0.46</td>
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<tr>
<td>80</td>
<td>0.40</td>
<td>0.42</td>
<td>0.41</td>
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Table 4 reports the homeowners’ average liquid wealth–networth ratio \(\frac{S_{t+B_t}}{S_{t+B_t}+P_{H_t-M_t}}\), networth equity proportion \(\frac{S_{t+B_t}}{S_{t+B_t}+P_{H_t-M_t}}\), and the liquid wealth equity proportion \(\frac{S_{t+B_t}}{S_{t+B_t}}\) for homeowners who participate in the stock market.
Figure 1. Panel a plots the optimal housing and stock market participation decision of an investor coming into the period as a renter without stocks, as a function of wealth–labor-income ratio. Panel b plots the optimal housing and mortgage decision of an investor coming into the period as a homeowner who previously have entered the stock market, as a function of the beginning-of-period house value–wealth ratio and loan–to–value ratio for an investor at age 50. The investor’s networth–labor-income ratio is fixed at $Q_t/P_t = 1.75$ in panel b.
Figure 2. The figure plots the optimal liquid investment as a function of the beginning-of-period housing value–wealth ratio and loan–to–value ratio for an investor at age 50. The investor’s wealth worth–labor-income ratio is fixed at $Q_t/P_t\gamma = 7.0$. The investor’s house value–wealth ratio is fixed at $P_t^H H_{t-1}/Q_t = 0.8$ for panel b.
Figure 3. The figure plots the optimal equity proportion in total networth as a function of the beginning-of-period housing value–wealth ratio and loan–to–value ratio for an investor at age 50. The investor’s wealth worth–labor-income ratio is fixed at $Q_t/P_t^{Y} = 7.0$. The investor’s house value–wealth ratio is fixed at $P_t^{H}H_{t−1}/Q_t = 0.8$ for panel b.
Figure 4. The figure plots the optimal equity proportion in liquid wealth as a function of the beginning-of-period housing value–wealth ratio and loan–to–value ratio for an investor at age 50. The investor’s wealth worth–labor-income ratio is fixed at $Q_t / P_t^y = 7.0$. The investor’s house value–wealth ratio is fixed at $P_t^H H_{t-1} / Q_t = 0.8$ for panel b.
Figure 5. The figure plots the percentage of homeownership (panel a), percentage of households refinancing their mortgage (panel b), percentage of households defaulting on their mortgage (panel c), and the average loan-to-value ratio for refinanced and defaulted mortgages (panel d) as a function of age. The results are calculated based on 50,000 simulations using the baseline parameter values.
Figure 6. The figure plots the average households’ numeraire good consumption and labor incomes (panel a), the average house value–income ratio and networth–income ratio (panel b), the average loan-to-value ratio (panel c), and the average home equity–income ratio and liquid wealth–income ratio (panel d) as a function of age. The results are calculated based on 50,000 simulations using the baseline parameter values.
Figure 7. The figure plots the percentage of households owning stocks (panel a), the average equity proportion in liquid wealth (panel b), the average stock investment in networth (panel c), and the average equity investment in dollars for stockholders (panel d). The results are calculated based on 50,000 simulations for the baseline parameter values.
Figure 8. The figure plots the percentage of households owning homes (panel a), the average loan-to-value ratios (panel b), the average stock investment in liquid wealth (panel c), and the average stock investment in networth for stockholders (panel d), when there is a 1 percent credit spread between borrowing and lending rate. The results are calculated based on 50,000 simulations.