"Smart" Contracts and External Financing

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First draft: October 2017; This draft: October 2018

Abstract

Hash-linked timestamping is the key feature behind blockchain. It enhances trust as it enables contracting parties to have common and reliable records of transactions and of their timing. I develop a model of raising external financing where, due to this new technology, some traditional contracting frictions are not present. However, there are informational frictions whereby borrowers learn from data frequently, which affects their effort incentives. I identify conditions under which contracts benefit from being "smart", i.e., self-adjusting based on timestamps. I further show that increasing the frequency of learning makes traditional assets, e.g., debt and equity, costlier and more restrictive.

Keywords: blockchain, smart contracts, FinTech, contract design, Bayesian learning, profit-sharing, equity, debt, dynamic moral hazard.

JEL codes: D82, D86, G23, G32, G35


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1 Introduction

An important criterion for assessing the relevance of new technologies related to Finance (sometimes called "FinTech") is whether these technologies make access to external financing easier for firms and individuals who lack internal resources and/or who find it difficult to pledge future cash flows in return for the funds needed for an investment. Blockchain is seen as one of these new promising technologies for raising financing, as, quoting Casey and Vigna in MIT Technology Review May/June 2018, blockchain "is all about creating one priceless asset: trust". "Lack of trust" is well known in Finance and Economics to be the prime reason why external financing can only be accessed by those with enough own assets (see e.g., Tirole 2006). It is also the main reason why scarce and arguably expensive venture capital funding has been the best source of innovation financing (see e.g., Lerner et. al. 2012). More fundamentally, lack of trust is at the root of the fundamental frictions that make financial instruments necessary in the first place (see e.g., Kiyotaki and Moore 2002).

Blockchain technology could indeed enhance trust in the context of external financing by overcoming some traditional financing frictions that exist due to lack of cash flow pledgeability and commitment.1 The key distinguishing feature of blockchain is hash-linked timestamping, the technical details of which are further discussed in Appendix A. In essence, blockchain is a secure ledger/database which maintains a reliable, shareable and time-stamped record of transactions, ownership and rights, enabling contracting parties to have a common and verifiable history of events. Blockchain further helps contract enforcement as it can verifiably record and facilitate the execution of actions that are predetermined by contracting agents (via a pre-agreed, recorded and shared computer code), and it is an enabler for the development of “smart contracts”, i.e. contracts which can automatically self-adjust and execute pre-determined actions based on incoming data.2

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1Number of these benefits have also been pointed out by academics and practitioners (see e.g., Catalini and Gans 2017; Yermack 2015, Harvey 2016, PwC 2016, Deloitte 2016, Dhar and Bose 2016).

2There are real world examples of this: notable examples of platforms that incorporate smart contract functionality are Ethereum and the Hyperledger project. Ethereum is based on a permissionless and decentralized verification mechanism in the spirit of the blockchain technology behind bitcoin. Hyperledger is an open-source Linux based community that advocates permissioned (members-only) verification mechanisms and is supported by technology firms such as IBM and Cisco, as well as financial services providers such as JP Morgan, SWIFT and ABN Amro among others. While there are trade-offs in terms of costs and cryptographic security, both approaches enable similar core benefits.
While the broad definition of "smart contracts" includes the possibility of executing a standard traditional financing contracts, e.g., a debt contract, automatically, it would not be a fundamentally different contract. For this reason, I will call a contract "smart" in this paper if it relies on timestamps and chronological order at which cash flows arrive: that is, the payoffs are different depending on the sequence at which cash flows arrive, even if the total cash flows generated are the same. One on the main goals of this paper is to identify conditions under which such "smart" contract makes access to external financing easier.

I develop a model of raising external financing in an environment where blockchain enables the borrower (an entrepreneur or individual) to credibly pledge the future cash-flows and to pre-commit to any, possibly "smart" financing contract. While blockchain based financing contracts can rely on richer state space compared to traditional contracts, which depend on cash flows generated over a relatively longer period of time, such blockchain-based contracts would nevertheless not be complete. This is because generating cash flows still requires human effort that is neither contractible nor verifiable.

Furthermore, I explore the effect of more frequent learning from cash flows on optimal contracts as well as on standard traditional assets. Indeed, there is a trend towards better data analysis capabilities that enable firms to learn about their future prospects from incoming cash flows more frequently. Blockchain based record keeping could enhance such frequent learning. Learning from data in turn affects the effort incentives of entrepreneurs, and more accurate information does not necessarily make contracting easier (see e.g., Hirshleifer 1971 in the context of insurance, and Kaplan 2006 and Dang et. al. 2017 in the context of banking).

To be specific, I consider a setting where an entrepreneur (the borrower) can develop a project that will enable her/him to sell new products or services (called "widgets" in this paper) to potentially interested customers. The project requires a fixed investment and the borrower may need to or prefer to use external financing to cover this cost. There is no information asymmetry between

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3 This timing factor differentiates blockchain records from audited accounting records, which aim to ensure that the aggregate total in- and outflows are correctly recorded over a relatively long period of time. This makes contracting on project specific cash flows and their arrival times not feasible in traditional environments. This is further complicated when firms strategically time their earnings reporting and information disclosure (see e.g., Bartov, 1993 and Healy and Palepu, 2000).
lenders and borrower at the time on signing the contract, but whether or not the borrower made efforts is his private information. I also assume that raising financing is product (project) specific rather than firm specific, i.e., the entrepreneur pledges cash flows related to developing a specific product (e.g., as in the case of many forms of crowdfunding). This guarantees that incoming cash flows, effectively the sales records, is something possible to record on blockchain and to contract on.

A key feature of my model is that the borrower cannot influence the preferences of his target consumers and takes the distribution of potential sales opportunities as given: these are determined by the fundamental characteristics of the target market and the widget. For example, entrepreneurs, especially start-up firms, often target a market of limited size and face uncertainty about whether their target consumers value their product, and how easily they can find consumers who value their product. When the prime source of uncertainty is about the firm not knowing the preferences of target consumers likely to have similar taste, then successful past sales tend to raise the borrower’s expectations about future sales prospects. If instead the prime source of uncertainty is about how quickly the firm will find its target in a limited market, then successful past sales tend to lower the borrower’s expectations about future sales prospects. In my main setting I do not impose the most usual restrictions on the joint distribution of potential sales opportunities, and I explore how the core properties of potential sales distribution determine the desirable financing contracts in the described environment. These features also distinguish my model from other contracting papers with dynamic moral hazard. I also allow effort cost to follow any dynamics.

I show that it is crucial to distinguish an environment where potential sales are stochastically affiliated (log-supermodular)\(^4\) from an environment where they are not. As stochastic affiliation implies that successful past sales lead the entrepreneur to Bayesian update his beliefs about future sales upward, I call such environment a "success raises prospects" (SRP) environment. In contrast, a "success lowers prospects" (SLP) environment is one where sales are log-submodular, and past sales success leads the entrepreneur to Bayesian update his beliefs about future sales downward.

\(^4\)Stochastic affiliation implies monotone likelihood ratio property (see e.g., Milgrom, 1981 and Milgrom and Weber, 1982), which in turn often plays a central role in principal agent models.
As discussed above, one determinant of whether the firm operated in SRP or SLP environment is a type of uncertainty it faces. Furthermore, firms that produce products that have positive network effects (e.g., a platform business) are more likely to operate in a SRP market, while those producing items that their target consumers value because of their rarity and uniqueness (e.g., design items) are more likely to operate in a SLP environment. While the economics and finance literature most often focuses on environments with independent cash flow processes, and somewhat less frequently on affiliated ones, an environment where selling can become more difficult after early success is equally plausible.

I find that in all these environments the optimal contract can be represented in a relatively simple form where each successful sale is split between the lender and the borrower according to a dynamically adjusting splitting rule on incoming cash flows/sales revenues that depend on the history sales up to that point. In fact, the splitting rule is determined by variables that are predictable and can be easily and dynamically calculated by an econometric algorithm embedded in the code that implements a "smart contract": such as the expected value of next sales conditional on the history of past sales up to that point, and such as the expected value of sales at some point in the future if no successful sale happens in between (i.e., a "worst case scenario analysis"). Furthermore, the optimal contract typically benefits from being "smart" unless the following restrictive conditions hold: effort cost is constant, potential sales are stochastically affiliated and exchangeable random variables.\(^5\) Even when these conditions hold, the optimal contract is not-standard as the borrowers payoff is concave in total sales. This is opposite to debt contracts or convertible notes where the borrower’s payoff is convex in total sales. In a very special case where effort cost is constant, and potential sales are independently and identically distributed over time, the optimal contract is simple equity. In such case there is no learning from past sales.

I further show that there are a number of important asymmetries between SRP and SLP environments. In SRP environment the optimal contract resembles progressive taxation whereby higher past sales lead the borrower to have a lower claim on future profits. In order to undertake a break-

\(^5\) Exchangeability requires that the joint distribution of potential sales is unaffected by reordering the realizations. It would be violated if the chances of meeting interested and uninterested consumers is not random, and if there is some cyclicality is sales prospects, for example.
even investment project, the borrower would need to invest some own funds. The optimal contract needs to maintain effort incentives in states where the borrower is relatively pessimistic about his future prospects. This creates a manipulation possibility whereby the borrowed strategically withholds the effort anticipating better contractual terms, similar to what is called "information rents" in some contract theory papers discussed below. Own investment is essentially needed to offset these information rents. In contracts, all positive NPV projects can be financed without the need for the entrepreneur’s own funds in a SLP environment. This is because a contract based on timestamped records can perfectly offset any distortions due to learning. The optimal contract in a SLP environment resembles a merit based scheme whereby past success should give the entrepreneur a right to a higher share of the next sale, and vice versa, to offset the fact that past success indicates greater difficulties to achieve further success.

This paper shows that under blockchain technology, the economic outcomes are closer to those in the frictionless market of Modigliani and Miller (1958, 1963) in the sense that the expected profit of a borrower who obtains external financing is as high as it would be in a frictionless market, and using own funds is not cheaper than external financing for most borrowers. Furthermore, borrowers can finance all profitable investment projects with minimal own funds, and even zero own funds in some informational environments. However, in contrast to Modigliani and Miller (1958, 1963) all financing contracts are not equivalent. I show that the reason why debt is suboptimal in this environment is that debt gives poor incentives to continue making efforts following unlucky outcomes (similar to debt overhang). Anticipating no effort in some states will make the investors more reluctant to lend. I also show that equity is suboptimal whenever learning from past sales reveals information about future sales prospects. With simple equity, the borrower would need to have high enough own share of revenues to maintain effort incentives in the "worst case scenario", while he would prefer to accept a contract that gives him a lower share in the "best case scenario" in return for the benefit of accessing external financing more easily ex-ante. Furthermore, I show that both debt and equity become increasingly expensive when the borrower learns and adjusts his effort more frequently. For this reason I argue that the "smart contracts" facilitated by blockchain technology should not just be seen as a way to make the execution of financing contracts cheaper,
but could be seen as a counter force to mitigate the negative forces generated by more frequent learning.

The early literature on credit rationing and financial contracting (see e.g., Tirole, 2016 and Allen and Winton, 1995, for reviews) has often found debt or debt-like contracts to be the "second best" optimal contracts in settings with realistic contracting frictions in traditional environments. This reinforces the idea that the new technologies considered in this paper have the potential to fundamentally change the dominant financing frictions and consequently lead to different ways to raise external funds. One of the prominent theoretical arguments for the optimality of debt stems from the costly state verification literature (see e.g., Townsend, 1979, Diamond, 1984, Gale and Hellwig, 1985, Mookherjee and Png, 1989.), which highlights that debt contracts enable efficient monitoring and minimize verification costs. As argued above, the key promise of blockchain technology is to eliminate (or greatly reduce) these costs by guaranteeing easily verifiable and commonly shared records of transactions. Debt also features as optimal in static models of moral hazard where the aggregate cash flows are assumed to be verifiable, such as in Innes (1990), and in settings building on this. The crucial difference with my paper is that Innes (1990) does not consider the effect of frequent learning and effort decisions and assumes that the borrower makes his effort choice once and for all, just after signing the contract. As argued above, entrepreneurs are likely to learn from incoming data and can adjust their effort frequently after signing the contracts, which is the prime reason that makes debt contracts increasingly costly in my setting. Indeed, Chiesa (1992) extends Innes’s setting to allows the borrower to observe information before making an one-off effort decision and shows that debt combined with warrants does better than simple debt for a similar reason as in my paper: debt contracts give bad incentives to make an effort after bad news. Other important arguments for debt emphasize the robustness of these contracts to renegotiations (see e.g., Hermalin and Katz, 1991, and Dewatripont et. al., 2003) and enforcement (see e.g., Hart and Moore, 1998) - both of these frictions could arguably be eliminated by smart contracts, or at least be of second order importance for an individual small borrower that uses blockchain-based smart contracts.

This paper further relates to the literature on dynamic moral hazard, and the associated lit-
erature on principal-agent problems. My paper emphasizes that to fully benefit from blockchain technology, the borrowers and lenders would need to write contracts that depend not just on sales that happened, but also on when they happened. In this context it is important to clarify the connection of my paper with Holmström and Milgrom (1987), often cited for justifying the optimality of linear contracts. However, the more general insight in their paper is that in many principal agent settings, the optimal dynamic scheme should only depend on the number of times a particular outcome occurs, and not on the order in which these outcomes occur, i.e., a sales history \(\{0, 1, 1\}\) should give the same payoff as a sales history \(\{1, 1, 0\}\). If that would be the case in my setting, time-stamping would not matter. The crucial difference is that the potential sales distribution is realistically not under the full control of the entrepreneur: she/he cannot dictate the preferences of her/his target consumers regardless of her/his effort, and can only decide whether to explore the opportunity or not. This is the main reason why distributional properties of sales process matter.

Furthermore, the results of Holmström and Milgrom (1987) are derived in a setting where incentive compatibility constraints bind. When "success lowers prospects", I show that the borrower’s effort is easy to incentivise, because incentive compatibility constraints do not bind, and the binding constraint is the participation constraint.

In my model the possibility to commit a flexible repayment schedule enables the borrower to optimally manage the risks associated with learning by enabling him to trade off a lower share of profits in states he is more optimistic about the future prospects with a higher share of profits in the states that he is more pessimistic about future prospects. This finding relates to Palfey and Spatt (1985) who highlights the risk-sharing benefits of long-term contracting compared to a sequence of short-term contracts in an insurance market where contracting parties can learn about underlying risks. Dunn and Spatt (1988) further the prevalence of long-term contracting in the context of mortgage contracts. While mortgage and other loan contracts contracts sometimes allow some flexibility such as the possibility of early prepayment and adjustable rates, these contracts are typically not fully contingent on borrower income or cash flows. While the contracts in my setting are contingent on realized cash flows, the contracts are not complete as verifying and contracting on effort is not possible.
More recent advances in dynamic moral hazard and financing contracts also highlight the benefits of more nuanced and history dependent financing contracts, but tend to also assume greater control of agents over the cash flow process compared to the setting considered in this paper and more restrictive cash flow processes conditional on the agent’s actions. For example, DeMarzo and Fishman (2007) and Biais et. al. (2007, 2011) find that financing contracts that combine equity, debt, and credit line or well managed cash reserves are optimal in a dynamic setting where cash flow process conditional on effort is i.i.d. across time. DeMarzo and Sannikov (2016) consider a setting where cash flow process is governed by a Brownian motion, which is a parametric example of a SRP environment. In their paper, as in my paper, there are "information rents" and equity contract is optimal under some very specific circumstances. Similar information rents also feature in He et. al (2010) and Prat and Jovanovic (2014) that also use Brownian motion. These settings generally do not explore the SLP environment and the role of frequency of learning. It seems plausible that the combination of standard assets that constitutes the optimal contract in these papers could be also expressed as a dynamically adjusting profit sharing contract. At the same time traditional assets, that do not adjust based on each cash flow, cannot replicate the self-adjusting profit sharing contracts in all the informational environments considered in this paper. The latter is because statistical dependencies between the firm’s target market and target consumer preferences can be too complex to maintain effort incentive under a limited set of assets, while the effect of learning on incentives is nevertheless predictable.

Interestingly, the finding that the optimal contract could take the form of a time-varying share contract features in Bergemann and Hege (1998). Their paper considers dynamic moral hazard in a different context, where moral hazard is not associated with an effort to generate sales, but with the possibility to divert investor’s funds instead of investing these in the project development before the known sales outcome is generated. These result are complementary to those in my paper. An important difference is that despite the fact that a product development process resembles a "success raises prospects" environment, a pre-sales product development requires continued investments which make the optimal contract in their paper to be more similar to the one applicable to "success lowers prospects" environment.
There is also an emerging literature on blockchain in economics and finance, which focuses on different aspects of blockchain. Kroll et. al (2013), Biais et. al. (2018), Cong et. al. (2018), Easley et. al (2018) and Saleh (2008) focus on the viability of decentralized verification mechanisms. Malinova and Park (2017) explore different market designs of blockchain in the context of asset trading. Yermack (2017) and Catalini and Gans (2017) discuss possible applications of blockchain technology more generally. Cong and He (2018) focus on the role of smart contracts in the context of tacit collusion and industrial organization, rather than a financial contracting perspective. In particular, they highlight the consideration that blockchain records could reveal information to competitors. This problem would be avoided in a permissioned verification system which does not need sensitive details of the contract publicly available (see further on different types of blockchains in Appendix A).

The mechanisms developed in this paper have practical relevance in the context of recently emerged forms of fund-raising that either rely on blockchain technology directly or use platforms that incorporate blockchain technology to facilitate contracting and borrower-lender interactions. A noticeable example is a subset of initial coin offerings (ICOs) which involve issuing tokens that give the participants rights on the firm’s revenues, profits or shares. Another example is a set of emerging FinTech platforms that act as a new type of intermediaries that facilitate specific financing contracts, and typically use blockchain technology as part of their business model. These new forms of fund-raising are growing fast and are often based on equity or profit sharing contracts. While a full implementation of blockchain based financing contracts may require wider adoption of this technology, hybrid models that benefit from some aspects of this technology, while relying on

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6 According to www.icodata.io, the overall funds raised via ICOs stood at $6 billion in 2017 (in comparison to $90 million in 2016), exceeding the early stage venture capital investment volume (see e.g., See Oscar Williams-Grut at Business Insider, January 31, 2018), and is already around $3.9 billion in January-April 2018. According to a recent paper byAdhami, Giudici and Martinazzi (forthcoming) tokens offering profit rights are already quite common and constitute 26.1% of ICOs surveyed, and they find that offering profit (or service) tokens, increases the probability of the success of fund-raising via an ICO. Other common types of tokens include governance rights or the right to access the platform service, latter of which resembles reward-based crowdfunding (see e.g., Li and Mann (2018)).

7To name a few that have attracted attention: Funderbeam (https://www.funderbeam.com) enables equity investments in the spirit of venture capital investments combined with blockchain-based secondary market trading, Corl (https://corl.io) proposes revenue sharing contracts where some of the contractual terms, such as the term, adjust depending on realized revenues. There are also platforms that facilitate debt contracts, such as SALT (https://www.saltlending.com), which facilitates blockchain based lending and enables the use of crypto-currencies and -tokens as collateral.
centralized parties for up-to-date data, or contract enforcement in other aspects, can also deliver at least some of the benefits my model highlights.

2 The model of external financing with frequently observed cash flows

This section sets up and finds the optimal contract in a frequent learning environment where after signing the contract the borrower (an entrepreneur or an individual) observes each widget sale, regularly updates his beliefs about its target market, and decides whether to make efforts to generate further sales. I assume that there is a blockchain-based ledger\(^8\), which guarantees that there is a reliable, verifiable and timestamped record of each widget sale. The optimal contract can therefore be any, possibly "complex", function of each realized widget sale and on when these sales happened. Solving this contracting problem answers whether and when "smart" contracts that benefit from timestamps enable better access to financing compared to contracts that rely on the total cash flows generated by the project only.

Solving this problem is also a necessary step for understanding the role of more frequent learning and decision making on accessibility to external financing under different contracts. In Section 3, I will consider a comparable setting where the borrower observes sales and makes effort decisions less frequently. I will then contrast the impact of more frequent learning under the optimal contract with those that emerge if the borrower was limited to rely on raising funds using traditional assets, such as debt and equity. I will also highlight the role of prior uncertainty in driving the differences between "smart" contracts and traditional assets.

I will assume throughout the analysis that contracting parties pre-commit to pre-agreed contractual terms. This is facilitated by blockchain (and timestamped records in particular), as the code implementing the terms of financing contracts can be verifiably recorded as well, and it can be made sufficiently costly and difficult to change the contractual terms ex-post.

\(^8\)The immediate applications involve products that need to be, or benefit from being recorded on blockchain, such as goods sold for crypto-currencies, or benefit from being recorded on blockchain due to certification. It has been further imagined that blockchain could allow the creation of a World Wide Ledger that would record all transactions and ownership data (see e.g., Harvey 2016, Shackelford and Myers 2017, Swan 2015 and Tapscott and Tapscott 2016).
2.1 The setting

A would-be borrower (an entrepreneur or an individual) has a project idea that requires fixed investment $I$ at date 0. The borrower is risk-neutral and has own funds $A$, which he may choose to invest in the project or not. Assume that lenders operate in a competitive market and there is a representative risk-neutral lender.

If the project is pursued, the borrower can at each date $t = 1, ..., T$, make a potential sale $s_t = \{0, 1\}$. The joint distribution of potential sales, $p(s_1, ..., s_T)$, is known at date 0 to both parties, and the only restriction on this joint distribution imposed throughout the analysis is that all possible sequences of potential sales occur with strictly positive probability, i.e., $p(s_1, ..., s_T) > 0$ for any sequence $s_1, ..., s_T$. While the distribution of the potential sales is determined by the nature of the borrower’s project and the environment in which he operates, the borrower needs to make efforts. The effort decision at $t = 0, ..., T - 1$ is denoted with an indicator function $1_t = \{1, 0\}$, where $1_t = 1$ if the borrower makes an effort, $1_t = 0$ otherwise, and the effort cost is $e_t \geq 0$ in monetary equivalent units. If the borrower chooses $1_t = 0$, then the sales at $t + 1$ are zero with probability 1. The realized sales are denoted with $\hat{s}_t = 1_t s_t$.

Because the effort is neither observable nor contractible, the contract between the borrower and the lender is a function of realized sales, $\hat{s}_t$, rather than the potential sales, $s_t$. The discount rate is normalized to 1, and I assume that $e_t$ is small enough to guarantee that making an effort is always optimal, i.e., it holds that $e_t \leq \mathbb{E}[s_{t+1}|1_0 s_1, ..., 1_{t-1} s_t, 1_0, ..., 1_{t-1}]$ for any $t$.

The financing contract is signed at date 0, and I assume that it is the borrower who proposes the contract to the lenders. The contract specifies the borrower’s reward, a function $w(\hat{s}_1, ..., \hat{s}_T)$ which depends on realized sales $\hat{s}_t$ for $t = 1, ..., T$, and the lender’s reward which is the difference between total sales and the borrower’s reward. The borrower’s preferences depend on his expected returns and his own investment in the project. I assume lexicographic preferences, such that $w(\hat{s}_1, ..., \hat{s}_T) \succ (\succ) w(\hat{s}'_1, ..., \hat{s}'_T)$ if either "$\mathbb{E}[\pi] > \mathbb{E}[\pi']$" or "$\mathbb{E}[\pi] = \mathbb{E}[\pi']$ and $A_0 \leq (\leq) A'_0$".

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Footnote: Discrete time captures the discrete nature of blockchain records. It also takes time to confirm a transaction (the number of transactions confirmed per second peaks at 7 in Bitcoin network, 15 in Ethereum and 3500 in Hyperledger Fabric).
$A_0 = [0, A]$ is the borrower’s own investment in the project under the contract $w(s_1, ..., s_T)$, and $E[\pi']$ and $A'_0$ are the same variables under an alternative contract $w'(s'_1, ..., s'_T)$. The date $T$ value of the project for the borrower is

$$\pi = w(s_1, ..., s_T) - A_0 - \sum_{t=0}^{T-1} 1_t e_t. \quad (1)$$

A feasible contract must satisfy the lender’s break even constraint

$$I - A_0 = E \left[ \sum_{t=1}^{T} \hat{s}_t - w(s_1, ..., s_T) \right] \quad (2)$$

and the borrower makes optimal effort decisions at each date, i.e., for every $t = 0, ..., T - 1$, it holds that $1_t = 1$ if, and only if

$$E[\pi|\hat{s}_1, ..., \hat{s}_t, 1_0, ..., 1_{t-1}, 1_t = 1] \geq E[\pi|\hat{s}_1, ..., \hat{s}_t, 1_0, ..., 1_{t-1}, 1_t = 0]. \quad (3)$$

I also assume that there is limited liability, i.e., $w(s_1, ..., s_T) \geq 0$.

It is worthwhile to discuss some of the features of this setting. Lexicographic preferences cover the standard case where the borrower just maximizes his expected returns from his project. However, as the blockchain is assumed to eliminate a number of financing frictions, we may need more structure to compare contracts that give the same expected profit. It is then natural to consider that the borrower would prefer to risk losing as little of his own funds as possible, while still trying out his project idea. An additional advantage is that the optimal contract derived under lexicographic preferences is one that maximizes the availability of outside financing to borrowers with little own assets, and thus minimizes credit rationing (see further in Section 2.2).

The incentive compatibility constraint (3) highlights that when making an effort decision at date $t$, the borrower has additional information compared to date 0 - he knows whether he made an effort in past, and what the sales were at dates when he tried to actively sell. Because the effort is not verifiable, the contract cannot distinguish between zero sales due to bad luck or due to lack of effort, and the borrower has superior information when making his optimal effort decisions. As $\pi$ in (1) depends on all efforts between date 0 and $T$, it is also possible that when making the the effort decision at $t$, the borrower expects this decision to affects his future effort decisions.
2.1.1 Key distributional properties and related examples of target markets

While the main setting imposes minimal restrictions on the joint distribution of potential sales, \( p(s_1, ..., s_T) \), it is useful to recall the definitions of some useful properties of random variables to structure the analysis, and to consider some examples that highlight economic forces that generate these distributional properties.

**Definition 1** Given \( n \) random variables jointly distributed in \( \mathbb{R}^n \) according to distribution function \( f \), let \( x = (x_1, ..., x_n) \) and \( y = (y_1, ..., y_n) \) be two points in \( \mathbb{R}^n \). The random variables are stochastically affiliated if, and only if for every \( x \) and \( y \)

\[
    f(x \wedge y) f(x \lor y) \geq f(x) f(y),
\]

where

\[
    x \wedge y = (\min(x_1, y_1), \ldots, \min(x_n, y_n))
\]

\[
    x \lor y = (\max(x_1, y_1), \ldots, \max(x_n, y_n))
\]

i.e., the joint distribution is log-supermodular. If random variables have this property I will refer to the environment as a "success raises prospects (SRP)" environment. If the inequality in (4) is reversed, then the joint distribution is log-submodular and I will refer to the environment as a "success lowers prospects (SLP)" environment.

Stochastic affiliation is a strong form of positive correlation (which implies the monotone likelihood ratio property). It can be called a "success raises prospects (SRP)" environment, because a successful sale at date \( t \) makes the borrower to revise upwards his beliefs about the probability of making successful sales in the subsequent periods. In the opposite case of stochastic affiliation, the log-submodular potential sales process captures a market where early sales success indicates a greater difficulty to find future sales opportunities and can therefore be called a "success lowers prospects (SLP)" environment.

Start-up projects often involve a new niche product that targets a market with a finite size, and there is likely to be some uncertainty regarding target consumer preferences as well as how easy it is to find potentially interested consumers. As an example of a framework that incorporates
both features, consider that after investing the entrepreneur will aim to sell his widget in a market with \( N \) consumers, whose type and preferences are not perfectly known to the entrepreneur (and the lenders). Suppose that consumers in this market arrive in a random sequence and that the consumer who arrives at date \( t \) is of type \( \tau_t = \{NI, PI\} \), where the type \( \tau_t = NI \) consumer is not interested in the widget with probability one, and the type \( \tau_t = PI \) consumer is interested in buying the widget with some unknown probability \( \theta \in (0, 1] \). There are \( K \leq N \) type "PI" consumers in this market and the preferences of type "PI" consumers are drawn from a beta distribution \( \theta \sim Be(\lambda \theta_0, \lambda (1 - \theta_0)) \), where \( \theta_0 = \mathbb{E}[\theta] \) is the mean and the parameter \( \lambda \) measures the degree of uncertainty regarding the unobservable preference parameter \( \theta \) (lower \( \lambda \) meaning higher uncertainty about \( \theta \) as \( Var[\theta] = \frac{\theta_0 (1 - \theta_0)}{1 + \lambda} \)). Preferences of type "PI" consumers are conditionally independent and \( \Pr(s_t = 1|\theta, \tau_t = PI) = \theta \).

In the described environment sales are statistically dependent and a successful early sale can both raise or lower future prospects. Which effect dominates, depends on the core parameters, \( K, N, \theta_0 \) and \( \lambda \), which are determined by the characteristics of the target market, and which can be assessed based on market analysis. Appendix B.1 provides further intuition regarding these parameters by fully characterizing the statistical dependencies between the first two potential sales. It shows that a firm is more likely to operate in a SRP environment when its total market is large (high \( N \)), when the firm meets potentially interested (type "PI") consumers frequently (high \( K/N \)), when it is uncertain about type "PI" consumers preferences (low \( \lambda \)), and when it has low prior expectations about their interest (low \( \theta_0 \)). A SLP environment is more likely when the firms faces a limited target (low \( N \)) with potentially very interested consumers (high \( \theta_0 \) and \( \lambda \)) who are difficult to find (low \( K/N \)). Start-up developing innovative consumer products may be operating in either type of environment. The key difference is whether the prime source of uncertainty has to do with not knowing whether the firm’s potential consumers with correlated preferences value the product (SRP), or how difficult it is for the firm to find all its potential consumers (SLP). For example, when there are three consumers (\( N = 3 \)), out of which two may be interested (\( K = 2 \) and each

---

\(^{10}\)Beta distribution captures a wide set of prior beliefs: be it uniform \( Be(1, 1) \), hump-shaped, U-shaped, and right- or left-skewed.
potentially interested consumer is interested with 50% probability \( \theta_0 = \mathbb{E} [\theta] = 0.5 \), then there is a cut-off value \( \lambda = 2 \) such that \( \lambda < 2 \) implies a SRP environment and \( \lambda > 2 \) a SLP environment.\(^{11}\)

If the firm sells one widget at date 1, it must have met a type "PI" consumer. On one hand this sale lowers future prospects because the probability of meeting a type "NI" consumer next is now higher. On the other hand this sale increases future prospects, as the consumer’s decision to buy is a positive signal about the other "PI" consumer’s interest to buy the product as well. The second effect is stronger when there is greater prior uncertainty about type "PI" consumer preferences (lower \( \lambda \)).

Some examples used later on benefit from the two limit cases that have a convenient parametric representation. One case limit case is where the firm has perfectly identified the set of potentially interested consumers (type "PI"), i.e., \( K = N \), but is unsure about the preferences of these consumers, i.e., \( \theta_0 \in [0,1] \) and \( \lambda \) is finite. This environment is always SRP. Another limit case where there is uncertainty about when the firm will find its potentially interested consumers, i.e., \( K < N \), while the preferences of potentially interested consumers are not correlated, i.e., \( \lambda \to \infty \).\(^{12}\) This environment is always SLP.

There are other target market specific characteristics that push towards either SRP or SLP environments. For example, SRP is more likely when the firm’s product exhibits positive network effects (e.g., it has a platform business), and SLP is more likely when there are negative externalities (e.g., the firm produces design items that consumers value as rare or limited edition products). A further factor that can push towards SLP is possible retaliation (e.g., an advertising war) by dominant other firms producing imperfect substitutes to the firm’s product if the firm’s product proves successful.

Another useful property of random variables is exchangeability:

**Definition 2** Random variables \( X_1, \ldots, X_n \) are exchangeable if a sequence \((X_1, \ldots, X_n)\) is equal in distribution to \((X_{i_1}, \ldots, X_{i_n})\) for any \( n! \) permutation \( i_1, \ldots, i_n \) of integers \( 1, \ldots, n \).

\(^{11}\)Note that when \( \theta_0 = 0.5 \) the prior distribution is symmetric, and \( \lambda < 2 \) corresponds to U-shaped prior, \( \lambda = 2 \) corresponds to uniform prior; \( \lambda > 2 \) corresponds to humpshaped prior.

\(^{12}\)If additionally \( \theta = \theta_0 = 1 \), then the sales process is random sampling without replacement.
Exchangeability in this setting means that the order at which potential sales arrive does not affect the joint probability. For example, with three potential sales opportunities it must be the case that joint probability $p(0,1,1) = p(1,1,0) = p(1,0,1)$, etc. Intuitively, exchangeability means that from an ex-ante perspective, potential sales opportunities are equally likely to arrive at any $t$. Considering the above examples, exchangeability would be violated if the firm did not meet is potential consumers randomly, e.g., it could face a predictably higher probability of meeting a type $\tau_t = PI$ after a launch event that involved an advertising campaign. Exchangeability would also be violated if there was some cyclicality in sales prospects.

2.2 First best and outside financing capacity

For the first best benchmark, consider the social planner who lends the borrower $I$ and who chooses the borrower’s optimal effort. It turns out that any optimal contract must focus on the borrower’s secondary motive.

**Lemma 3** 1) Under the first best, the borrower sets $1_t = 1$ for every $t$, and the project is undertaken as long as $I \leq I^{FB} = \pi^{FB}$, where

$$\pi^{FB} = \sum_{t=0}^{T-1} \mathbb{E}[s_t] - \sum_{t=0}^{T-1} e_t.$$  

(5)

2) Any optimal financing contract must also set $1_t = 1$ for every $t$. As the borrower’s expected profit under the optimal contract is the same as under the first best, the optimal contract is the one that minimizes $A_0$.

**Proof.** See Appendix B.2. ■

The first part of Lemma 3 is intuitive as we assumed that the effort cost is small enough that effort is always worth undertaking. The second part of Lemma 3 highlights that any optimal contract in this setting must also guarantee the first best effort, as doing so maximizes the borrower’s expected profit, which is his primary objective. Furthermore, lowering effort cannot lead to smaller own investment by the borrower. Hence, under any optimal contract all borrowers with $A \geq A_0$ can raise financing to cover any $I \leq I^{FB}$ and have as high expected profit as under the first best ($\pi^{FB}$). If $A_0 > 0$, then firms with $A < A_0$, are cannot pursue their project. Inducing first best
effort is achievable in this setting because cash flows are pledgeable.\textsuperscript{13} At the same time inducing the first best effort is not trivial, as the optimal contract must satisfy the $2^T - 1$ dynamic incentive compatibility constraints given by (3).

Lemma 3 further enables to simplify the contracting problem, as we can restate the problem as one that minimizes $A_0$, such that (3) are satisfied for $1_t = 1$ for every $t$. It is first useful to define $\tilde{w}(s_1, \ldots, s_T)$ as a contract offered by a break-even borrower with $I = I^{FB}$.

Minimizing the break even investor’s own investment $A_0$ while keeping $\pi = \pi^{FB}$ fixed is equivalent to minimizing the break-even borrower’s reward

$$
\min_{\{\tilde{w}(s_1, \ldots, s_T) \geq 0\}} \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)], \tag{6}
$$

such that the break-even borrower is willing to pursue the project

$$
\mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t \geq 0 \tag{7}
$$

and to choose first-best effort at each date $t = 0, \ldots, T - 1$

$$
\mathbb{E}[\tilde{w}(s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T)] - \tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T)|s_1, \ldots, s_T| \geq e_t. \tag{8}
$$

The solution of this problem will give the contract and the minimum downpayment necessary to cover the break-even investment cost $I^{FB}$, which is given by $A_0 = \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t$. The difference $\mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t$ can be interpreted as the information rent the borrower needs to obtain to satisfy the incentive compatibility constraints given by (8) at each date $t$ and for any sales history up to $t$. The distribution of potential sales revenues determines whether positive information rents are needed or not.

Naturally, any borrower who has fixed investment cost $I < I^{FB}$ is easier to incentivise. From (2) and (5) we can find that such a borrower maximizes his preferences by choosing

$$
A_0 = \max \left[ 0, \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I) \right]. \tag{9}
$$

When $\mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I) \geq 0$, then $\tilde{w}(s_1, \ldots, s_T)$ and $A_0 = \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I)$ constitutites the optimal contract for a borrower with $I < I^{FB}$. If $\mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I) < 0$, then $\tilde{w}(s_1, \ldots, s_T)$ and $A_0 = \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t$ are not feasible contracts. It is typical reason why borrowers benefit and prefer to invest all their own funds in the project in costly state verification literature is that cash flows are not pledgeable and the optimal contract needs to induce truth-telling.

\textsuperscript{13}Indeed, the typical reason why borrowers benefit and prefer to invest all their own funds in the project in costly state verification literature is that cash flows are not pledgeable and the optimal contract needs to induce truth-telling.
\[ \sum_{t=0}^{T-1} e_t - (I^{FB} - I) < 0, \] then the borrower obtains an additional surplus. Any contract to capture this surplus that does not violate (3) is optimal.

2.3 Main results

2.3.1 Two date example

To develop the intuition behind the main results it is useful to first consider the case where \( T = 2 \). There are four possible sales outcomes \((s_1, s_2) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}\); the corresponding probabilities and borrower’s rewards are \( p(s_1, s_2) \) and \( \tilde{w}(s_1, s_2) \) respectively. Define the marginal probabilities as \( p_t(1) = \Pr(s_t = 1) \) and \( p_t(0) = \Pr(s_t = 0) \) for \( t = 1, 2 \). Note that \( p_1(1) = p_2(1) \) if potential sales are exchangeable random variables; these probabilities may differ otherwise. We can further expand the constraint on effort costs\(^{14}\)

\[
e_0 \leq \mathbb{E}[s_1] = p_1(1); \quad e_1 \leq \min \left[ \mathbb{E}[s_2|s_1 = 1], \mathbb{E}[s_2|s_1 = 0] \right] = \min \left[ \frac{p(1, 1)}{p(1, 1)}, \frac{p(0, 1)}{p(0, 1)} \right]. \tag{10}
\]

We can restate the problem (6)-(8) as

\[
\min \mathbb{E}[\tilde{w}(s_1, s_2)] \tag{11}
\]

subject to the participation constraint

\[
\mathbb{E}[\tilde{w}(s_1, s_2)] \geq e_0 + e_1 \tag{12}
\]

and to the three incentive compatibility constraints

\[
\mathbb{E}[\tilde{w}(s_1, s_2) - \tilde{w}(0, s_2)] \geq e_0 \tag{13}
\]

\[
\mathbb{E}[\tilde{w}(0, s_2) - \tilde{w}(0, 0)|s_1 = 0] \geq e_1
\]

\[
\mathbb{E}[\tilde{w}(1, s_2) - \tilde{w}(1, 0)|s_1 = 1] \geq e_1
\]

and the non-negativity constraint \( \tilde{w}(s_1, s_2) \geq 0 \).

The incentive compatibility constraints highlight that the borrower must always get a positive payoff from generating the next sale. It is immediate that a contract that keeps the borrower’s

\(^{14}\)Note that the condition \( e_1 \leq \mathbb{E}[s_2|1_1 s_1] \) requires that \( e_1 \leq \mathbb{E}[s_2|s_1 = 0], e_1 \leq \mathbb{E}[s_2|s_1 = 1] \) and \( e_1 \leq \mathbb{E}[s_2] \). However, the last constraint is redundant because the law of iterated expectations implies that \( E[s_2] = p_1(1)E[s_2|1] + p_1(0)E[s_2|0] \).
payoff constant in some states (e.g., debt contract) cannot guarantee effort in all states and be optimal.

The last two incentive compatibility constraints also highlight the role of learning, as these can be written as \( \bar{w}(s_1, 1) - \bar{w}(s_1, 0) \geq \frac{e_1}{\Pr(s_2 = 1 | s_1)} \), where \( \Pr(s_2 = 1 | s_1) \) generally changes depending on the realizations of \( s_1 \), which in turn are determined by whether the borrower operates in a SRP or SLP environment.

**Proposition 4** The optimal contract of a break-even borrower depends on the joint distribution as follows

1) In a "success raises prospects" (SRP) environment, i.e., \( p(1,1)p(0,0) \geq p(1,0)p(0,1) \), the borrower’s reward is

\[
\begin{align*}
\bar{w}(0,0) &= 0 \\
\bar{w}(0,1) &= e_1 \frac{p_1(0)}{p(0,1)} \\
\bar{w}(1,0) &= \frac{e_0 - e_1}{p_1(1)} + \bar{w}(0,1) \frac{p_2(1)}{p_1(1)} \\
\bar{w}(1,1) &= \bar{w}(1,0) + e_1 \frac{p_1(1)}{p(1,1)},
\end{align*}
\]

and he invests

\[
A_0 = e_1 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)}.
\]

The project is undertaken if \( A \geq A_0 \).

2) In a "success lowers prospects" (SLP) environment, i.e., \( p(1,1)p(0,0) \leq p(1,0)p(0,1) \), the borrower’s reward is

\[
\begin{align*}
\bar{w}(0,0) &= 0 \\
\bar{w}(0,1) &= e_1 \frac{p_1(0)}{p(0,1)} \\
\bar{w}(1,0) &= e_0 \frac{1}{p_1(1)} \\
\bar{w}(1,1) &= \bar{w}(1,0) + e_1 \frac{p_1(1)}{p(1,1)},
\end{align*}
\]

and he invests

\[
A_0 = 0.
\]
The project is always undertaken and is fully externally financed.

**Proof.** See Appendix B.3. ■

The proof of this proposition uses the Duality and Complementary Slackness Theorems: given the primal minimization problem above, there is a dual maximization problem. The Duality Theorem implies that if both problems are feasible, both have a solution and the minimized (maximized) value of the primal (dual) is the same. The main insights are that in a SRP environment all the incentive compatibility constraints (13) must be binding and the participation constraint (12) can be slack, while in a SLP environment the participation constraint (12) must be binding and incentive compatibility constraints can be slack (13).

**Corollary 5** The optimal contract can be expressed as

\[ \tilde{w}(s_1, s_2) = \alpha_1 s_1 + \alpha_2 (s_1) s_2, \]

where

\[ \alpha_2 (s_1) = \frac{e_1}{\mathbb{E}[s_2|s_1]} \]  

and

\[ \alpha_1 = \begin{cases} \frac{e_0 - e_1}{\mathbb{E}[s_1]} + \frac{\mathbb{E}[s_2|s_1]}{\mathbb{E}[s_1]} \alpha_2 (0) & \text{in SRP environment} \\ \frac{e_0}{\mathbb{E}[s_1]} & \text{in SLP environment} \end{cases} \]  

**Proof.** Straightforward from (14) and (16) given Bayes' rule and the definition of conditional expectations. ■

Corollary 5 shows how the optimal contract can be expressed as a self-adjusting profit sharing rule, where the borrower receives the share \( \alpha_1 \) and \( \alpha_2 (s_1) \) of each date 1 and 2 sales, respectively, and the lender receives the remainder \( 1 - \alpha_1 \) and \( 1 - \alpha_2 (s_1) \).\(^{15} \) It is a "smart" contract as long as \( \alpha_2 (0) \neq \alpha_2 (1) \).

Whether or not such a smart contract brings added value compared to contracting on aggregate cash flows, boils down to assessing whether \( \tilde{w} (0, 1) = \tilde{w} (1, 0) \) is consistent with the optimal contract. In a SRP environment, we can see from (14) that \( \tilde{w} (0, 1) = \tilde{w} (1, 0) \) generally requires both:

\(^{15}\)We can see that if the cost of effort is constant, i.e., \( e_0 = e_1 = \bar{e} \) and sales are are exchangeable random variables, then the borrower and lender can immediately split the revenue from each sale, as \( \alpha_1, \alpha_2 (s_1) \leq 1 \), (see (10)). More generally, the contract can always take advantage of an escrow account if \( \alpha_1 > 1 \) and temporarily withhold some funds until the date 2 outcome is realized as well.
sales revenues must be exchangeable random variables (in which case $p_1(1) = p_2(1)$) and effort

cost must be constant ($e_0 = e_1$). In a SLP environment, having $\bar{w}(0, 1) = \bar{w}(1, 0)$ is generally not

possible unless we happen to have a coincidence of very specific exogenous parameters.

Proposition 4 and Corollary 5 further highlight that the properties of joint distribution are

crucial for understanding how the borrower’s incentives evolve based on observing the outcome

from date 1 sale. In a SRP environment $E[s_2|s_1 = 1] \geq E[s_2] \geq E[s_2|s_1 = 0]$, and the borrower

who Bayesian updates his beliefs finds it harder to sell another item if he failed to make a sale at
date 1, and easier to sell another item if he succeeded. To maintain effort incentives for date 2, his

marginal reward from making an effort after failure must be higher than his marginal reward after

success at date 1, i.e., $\alpha_2(0) \geq \alpha_2(1)$. In a SLP environment $E[s_2|s_1 = 1] \leq E[s_2] \leq E[s_2|s_1 = 0]$, and

the opposite holds, the borrower needs to receive a higher marginal reward following a success,
i.e., $\alpha_2(1) \geq \alpha_2(0)$.

There is still a need for the break-even borrower’s own contribution in a SRP environment,

while all positive NPV projects can be undertaken in a SLP environment. The reason is that

when deciding whether to pay an effort cost $e_0$, the borrower has a manipulation possibility in a

SRP environments - he can withhold effort and save this cost while knowing that he will receive

a bigger share of date 2 sale following no realized sales at date 1, which would be interpreted

along the equilibrium path as "bad luck" rather than no effort. To eliminate this manipulation

incentive, the borrower must receive a large enough share of date 1 sale. This increases the expected

value of the minimum reward that the borrower must receive for the contract to be incentive

compatible and the borrower invests some own funds to cover information rents that arise due to

the aforementioned manipulation possibility, an amount given in (15). Consequently, even though

cash flows are pledgeable there are still be borrowers who are credit rationed, in the sense that they

have a positive NPV project, but cannot obtain funding without having enough own assets. This

manipulation possibility is not present in a SLP environment, because strategically withholding

effort would give the borrower an even smaller share of date 2 sales revenue. The same applies for

the case with independent potential sales as in such case $p(1, 1) p(0, 0) = p(1, 0) p(0, 1)$.

Finally, the contracts described by (14) and (16) identify the optimal contract for a break-even
borrower with investment cost \( I = I^{FB} \). When \( I < I^{FB} \), then the result is Section 2.2 and (2) and (5) imply that the expected value of the financing contract for the borrower must be

\[
E[w(s_1, s_2)] = E[\tilde{w}(s_1, s_2)] + (I^{FB} - I).
\]

In a SRP environment, the borrower’s preferences are maximized by first reducing own investment, i.e., the borrower would set \( A_0 = \max \left[ 0, e_1 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)} - (I^{FB} - I) \right] \), while offering the same contract as in (14). If this maximum is zero, then there is a surplus for the borrower, which he can extract in multiple ways, including asking for a lump sum transfer of \( (I^{FB} - I) \) at date \( T \) or proposing a contract that sets \( w(s_1, s_2) = \gamma \tilde{w}(s_1, s_2) \), where \( \gamma = 1 + \frac{(I^{FB} - I) - e_1 p(1,1) p(0,0) - e_1 p(1,0) p(0,1)}{e_0 + e_1 + e_1 p(1,1) p(0,0) - e_1 p(1,0) p(0,1)} \geq 1 \). As all positive NPV projects are externally financed in a SLP environment \( (A_0 = 0) \) there is always a surplus for the borrower, which he can again extract in multiple ways, including asking for a lump-sum payment \( (I^{FB} - I) \) at date \( T \), or proposing a \( w(s_1, s_2) = \gamma \tilde{w}(s_1, s_2) \), where \( \gamma = 1 + \frac{(I^{FB} - I)}{e_0 + e_1} \geq 1 \). One advantage of scaling up the payments by \( \gamma \geq 1 \) is that it enables to settle the sharing of sales revenues whenever these sales revenues arrive, thereby eliminating some considerations outside this model: such as the risk that the lender does not have \( (I^{FB} - I) \) to transfer at date \( 2 \), or the need for the lender to put \( (I^{FB} - I) \) on an escrow account at the time of contracting at date \( 0 \).

2.3.2 The full model with \( T \) periods

The \( T \)-period contracting problem for a break-even borrower is a similar linear programming problem with \( 2^T \) possible states and variables to solve for, and \( 2^T - 1 \) incentive compatibility constraints given by (8) and the participation constraint (7). I assume in this section that potential sales are exchangeable and the borrower always operates in either a SRP or a SLP environment as defined in Section 2.1.1\(^{16}\). Let us start from considering the SRL environment.

\(^{16}\)If the potential sales distribution does not satisfy these assumptions, the linear programming problem is still solvable via the same approach, albeit the analytical expressions regarding the optimal profit sharing rule would need to be more complex. Regarding violations of exchangeability, we can expect the same intuition as in Section 2.3.1 to apply, i.e., the borrower should receive a higher share of sales in states that are less likely ex-ante. Further, under real world relevant scenarios (such as those described in Section 2.1), a possible regime switch from SRP to SLP is more likely that a regime switch from SLP to SRP. As I will show that under SLP the optimal contract only needs to compensate for effort, rather than information cost, also in a \( T \) period case, it is possible to construct profit sharing agreements that first exhibit properties relevant to SRP environment, notably the information rents, and then only compensate for the effort cost.
Proposition 6 In a SRP environment the optimal contract for a break-even borrower with \( I = I^{FB} \) is

\[
\tilde{w}(s_1, ..., s_T) = \alpha_1 s_1 + ... + \alpha_t (s_1, ..., s_{t-1}) s_t + ... + \alpha_T (s_1, ..., s_{T-1}) s_T,
\]

where

\[
\alpha_T (s_1, ..., s_{T-1}) = \frac{e_{T-1}}{\mathbb{E}[s_T|s_1, ..., s_{T-1}]}
\]

and for any \( t = 0, ..., T - 2 \), it holds that

\[
\alpha_{t+1} (s_1, ..., s_t) = \alpha_{t+2} (s_1, ..., s_t, s_{t+1} = 0) + \frac{e_t - e_{t+1}}{\mathbb{E}[s_{t+1}|s_1, ..., s_t]}. \tag{20}
\]

In order to complete the project, the borrower needs to invest own funds

\[
A_0 = \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}], \tag{21}
\]

where \( \Phi_t \) is information rent the borrower obtains at \( t = 0, ..., T - 2 \), defined as

\[
\Phi_t \equiv e_{t+1} \Delta_t (s_1, ..., s_t) + e_{t+2} \Delta_t (s_1, ..., s_t, s_{t+1} = 0) + ... + e_{T-1} \Delta_{T-2} (s_1, ..., s_t, s_{t+1} = 0, ..., s_{T-2} = 0), \tag{22}
\]

where

\[
\Delta_t (s_1, ..., s_t) \equiv \frac{1}{\mathbb{E}[s_{t+2}|s_1, ..., s_t, s_{t+1} = 0]} - \frac{1}{\mathbb{E}[s_{t+1}|s_1, ..., s_t]} \geq 0 \tag{23}
\]

The expected value of the borrower's rewards at date 0 is

\[
\mathbb{E}[\tilde{w}(s_1, ..., s_T)] = \sum_{t=0}^{T-1} e_t + \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}].
\]

Proof. See Appendix B.4.

Corollary 7 If \( I < I^{FB} \), then the borrower must invest

\[
A_0 = \max \left[ 0, \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}] - (I^{FB} - I) \right]. \tag{24}
\]

If \( I \) is low enough such that \( A_0 = 0 \), then the borrower obtains an surplus, which he can extract in multiple ways including setting \( w(s_1, ..., s_T) = \gamma \tilde{w}(s_1, ..., s_T) \), where \( \gamma = 1 + \frac{(I^{FB} - I) - \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}]}{\sum_{t=0}^{T-1} e_t + \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}]} \geq 1. \)
Proof. Follows from Proposition 6 and (9). □

Proposition 6 confirms the intuition from the two date case for the SRP environment. The optimal contract can be expressed as a dynamically adjusting profit sharing contract (19) that splits each successful sale between the borrower and the lender according to the sharing rule that depends on the sales history up to that point. The optimal "smart" contract takes an intuitive form when the effort cost is constant: the borrower gets a high share of his first sale whenever it happens (i.e., \( \alpha_1 = \alpha_2 (s_1 = 0) = \alpha_3 (s_1 = 0, s_2 = 0) \) etc.) and whenever there is a successful sale, the borrowers share of the next successful sale revenues falls (i.e., \( \alpha_2 (s_1 = 1) = \alpha_3 (s_1 = 0, s_2 = 1) < \alpha_1 \), etc).

The intuition is the same as in the two period case, the optimal contract must both maintain the borrower’s effort incentives following unlucky outcomes and avoid the manipulation whereby the borrower withholds effort to obtain a bigger share on future revenues. Consequently there are "information rents", which necessitate that the break-even borrower invests enough own funds in the project. The optimal contract also benefits from adjusting based on to the dynamics of effort cost.

Figure 1 illustrates the optimal contract in a SRP environment where \( I = I^{FB} = 5, T = 12 \), and the effort cost is constant \( e_t = \bar{e} = 1/12 \). It shows how the borrower’s expectations and \( \alpha_{t+1} (s_1, \ldots, s_t) \) evolve following a random path of successful sales history \( (0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1) \) (dots in panel A and B). The figure is constructed under the parametric example described in Section 2.1.1, where \( \theta_0 = 0.5, \lambda = 4 \), and \( N = K = T \). The borrower’s initial expectations of the probability of selling a widget are equal to the prior mean \( \Pr (s_t = 1) = 0.5 \). Whenever there is no sale success, the borrower Bayesian updates his beliefs about future sales prospects downward, and whenever there is a successful sale the he revises his beliefs downward (see Panel A). The optimal contract starts with the borrower having a claim of around 62% on the first successful sale in this example, and the borrower’s claim following each successful sale adjusts downwards. Information rents are high at early dates, and their importance diminishes over time (Panel C). This is because information rents are cumulative as shown in Proposition 6. Whenever the effort cost is not constant, the dynamics of the effort cost further affect the optimal profit sharing rule. For example, when the effort cost is decreasing over time, the borrower’s claims following unlucky
outcomes are lower and thereby mitigate the negative effect of learning in a SRP environment. Whenever the effort cost changes, the optimal contract cannot be represented as a function of total sales.

Furthermore, while under exchangeability and constant effort, the contract can be represented as a function of total sales, it does not resemble typical traditional assets, as the borrower’s reward is concave in total sales. Figure 2 illustrates this by plotting the contract as a function of total sales for different values of $\lambda$ the dashed is constructed under the same parameter assumptions as 1 ($\lambda = 4$), while the solid and dotted line vary $\lambda$, while maintaining the same assumptions about other parameters. The more uncertainty and learning there is (low $\lambda$), the further away optimal contract get from linear (i.e., equity contract).

Corollary 7 shows the optimal contract in the case where $I < I^{FB}$. The borrower with lower investment costs is easier to incentivise, and as long as he has $A \geq A_0$ defined in (9) he is able to obtain external financing to cover $I - A_0$, and his expected profits are the same as under the first best. In monetary terms, a borrower’s own funds are as cheap as external financing. Firms that have low enough investment costs to chose $A_0 = 0$, have not just the first best expected profit, but are exactly as well off as under the first best.

**Proposition 8** In a SLP environment the optimal contract for a break-even borrower with $I = I^{FB}$
Figure 2: Optimal contract as function of total sales in a SRP environment with constant effort cost, and exchangeable potential sales.

is

$$
\tilde{w}(s_1, \ldots, s_T) = \alpha_1 s_1 + \cdots + \alpha_t (s_1, \ldots, s_{t-1}) s_t + \cdots + \alpha_T (s_1, \ldots, s_{T-1}) s_T, \quad (25)
$$

and for any $t = 0, \ldots, T - 1$, it holds that

$$
\alpha_{t+1}(s_1, \ldots, s_t) = \frac{e_t}{\mathbb{E}[s_{t+1} | s_1, \ldots, s_t]} \quad (26)
$$

The borrower sets $A_0 = 0$.

**Proof.** See Appendix B.4.

**Corollary 9** If $I < I^{FB}$, then the borrower obtains a surplus, which he can extract in multiple ways including setting $w(s_1, \ldots, s_T) = \gamma \tilde{w}(s_1, \ldots, s_T)$, where $\gamma = 1 + \frac{(I^{FB} - I)}{\sum_{t=0}^{T-1} e_t}$.

**Proof.** Follows from Proposition 8, and (9).

Proposition 8 also confirms the intuition from the two date case in a SLP environment: all projects that are worth undertaking can be fully externally financed, and the borrower would optimally not invest any of his own funds in the project. If $I = I^{FB}$, then the optimal contract dynamically adjusts the share retained by the borrower (26) upward if $e_t$ is higher and the updated expectation $\mathbb{E}[s_{t+1} | s_1, \ldots, s_t]$ is lower.
Figure 3: Learning and optimal contract in a SLP environment

Panel A shows how the borrower’s expectations evolve (solid line) given a stream of realized sales (dots); Panel B shows how the borrower’s claims on next date sales $\alpha_t(.)$ evolve given the same stream of realized sales as in Panel A.

Figure 3 gives an example of an optimal contract in SLP environment when $I = I_{FB}$ and $T = 12$. It shows how the borrower’s expectations and $\alpha_{t+1}(s_1, ..., s_t)$ evolve following the same realized path of sales as in Figure 1. The figure is now constructed under the assumption that the potential sales process is a random sampling without replacement, which is a special case of the setting described in Section 2.1.1, where $N = 30$, $K = 15$, $\theta_0 = 1$. As before, assume that $I_{FB} = 5$ and effort cost is $e_t = \bar{e} = 1/12$. The borrower now revises his expectations downward following a successful sale and upward following no sale. Furthermore, in contrast to the SRP environment, the optimal contract in the SLP environment does not need to give the borrower’s information rents. Because of that, the borrower claim on the next sales revenues adjusts upward and downward in an opposite direction compared to the borrower’s expectations. The optimal contract in this case resembles a reward scheme that gives the borrower a higher share of the next sale following a success and a lower share following a failure.

Finally, we can see from Propositions 6 and 8 that the variables that determine the optimal splitting rule are simple conditional expectations $E[s_{t+1}|s_1, ..., s_t]$, and in the case of a SRP environment also a "conservative" estimates of sales at a future dates $s_{t+h}$, if no sales happen in between, $E[s_{t+h}|s_1, ..., s_t, s_{t+1} = 0, ..., s_{t+h-1} = 0]$. These variables have an intuitive interpretation and could be estimated based on the data. A "smart contract" could either be based on analyzing
these scenarios ex-ante or could estimate these variables on a running basis via an automated (an
preagreed) learning algorithm.

3 The role of frequent learning

3.1 Optimal contract under infrequent learning

As argued in the introduction, there is a trend towards more frequent learning and decision making.
To understand the effect of this on external financing, one needs to contrast the results under frequent learning analyzed in Section 2 to the case where learning and possibilities to adjust effort are less frequent.

Suppose that the contracting environment remains as described in Section 2.1, except that the borrower observes realized cash flows only once at date \( T = 2 \), and assume that \( T \geq 4 \) is an even number. Consequently, the borrower decides whether to make effort only at date \( 0 \) and at date \( T = 2 \). Define the cumulative potential sales as \( c_{T/2} = \sum_{t=1}^{T/2} s_t \) and \( c_T = \sum_{t=1}^T s_t \), and call the effort cost needed to generate \( c_{T/2} \) and \( c_T - c_{T/2} \) with \( \frac{T \tilde{e}_0}{2} \) and \( \frac{T \tilde{e}_{T/2}}{2} \), respectively.

**Proposition 10** The optimal contract for a break-even borrower is

\[
\bar{w} \left( c_{T/2}, c_T - c_{T/2} \right) = \alpha_{T/2} c_{T/2} + \alpha_T \left( c_{T/2} \right) \left( c_T - c_{T/2} \right),
\]

where

\[
\alpha_T \left( c_{T/2} \right) = \frac{T \tilde{e}_{T/2}}{2} \frac{1}{\mathbb{E} \left[ c_T - c_{T/2} | c_{T/2} \right]}
\]

and

\[
\alpha_{T/2} = \begin{cases} \frac{T \left( \tilde{e}_0 - \tilde{e}_{T/2} \right)}{2 \mathbb{E} \left[ c_{T/2} \right]} + \alpha_T \left( 0 \right) \frac{\mathbb{E} \left[ c_T - c_{T/2} \right]}{\mathbb{E} \left[ c_{T/2} \right]} & \text{in SRP environment} \\ \frac{T \tilde{e}_0}{2 \mathbb{E} \left[ c_{T/2} \right]} & \text{in SLP environment} \end{cases}
\]

Furthermore, the borrower chooses

\[
A_0 = \frac{T \tilde{e}_{T/2}}{2} \frac{\mathbb{E} \left[ c_T - c_{T/2} \right] - \mathbb{E} \left[ c_T - c_{T/2} | c_{T/2} = 0 \right]}{\mathbb{E} \left[ c_T - c_{T/2} | c_{T/2} = 0 \right]},
\]

(27)
in SRP environment and \( A_0 = 0 \) in SLP environment.

**Proof.** See Appendix B.5. \( \blacksquare \)
Proposition 10 confirms the results in Section 2. Furthermore, note the optimal contract cannot generally be expressed as a function of total sales $c_T$ only. That is, even if the borrower operates in SRP environment, the effort cost is constant (i.e., $\bar{e}_0 = \bar{e}_T/2$), and the potential sales distribution is exchangeable (i.e., $\mathbb{E}[c_T - c_{T/2}] = \mathbb{E}[c_{T/2}]$), the timing of sales matters. For example, if the borrower sells two widgets in total, he will get $2\alpha_T(0)$ if he sells both before $T/2$, but he will get $\alpha_T(0) + \alpha_T(1) < 2\alpha_T(0)$ if he sells one before and one after date $T/2$.

Proposition 10 is mostly needed for gaining additional insights regarding the effect of increasing frequency of learning on the optimal contracts and accessibility of external financing when keeping the total cost of effort constant. As by Propositions 8 and 10, the optimal contract in SLP environment enables all positive NPV projects to be externally financed regardless of the frequency, we only need to consider the SRP environment. The quantitative effects of faster learning is best understood under specific parametric setting, so I will consider the parametric case described in Section 3.1, where $N = K = T$, and $\lambda$ is finite. Further assume that the effort cost is constant (i.e., $e_t = \bar{e}$ in the frequent decisions case and $\bar{e}_0 = \bar{e}_T/2 = \bar{e}$ in the infrequent decisions case), such that the total effort cost is fixed and equal to $T\bar{e}$.

**Corollary 11** In SRP environment described above, the break-even borrower’s own investment under the optimal contract is

\[ A_0^{\text{FREQ}} = \bar{e} \left( \sum_{t=0}^{T-2} \frac{T - 1 - t}{\lambda + t} \right), \]

if he observes each sale and adjusts his effort frequently, and

\[ A_0^{\text{INFREQ}} = \bar{e} \frac{T^2}{4\lambda}, \]

if he observes sales infrequently and adjusts effort only at date $T/2$. Furthermore, there exists a threshold $\tilde{\lambda}$, such that $A_0^{\text{FREQ}} < A_0^{\text{INFREQ}}$ if $\lambda < \tilde{\lambda}$, and $A_0^{\text{FREQ}} > A_0^{\text{INFREQ}}$ if $\lambda > \tilde{\lambda}$.

**Proof.** See Appendix B.6

Corollary (11) shows that in an SRP environment, greater uncertainty about sales prospects (lower $\lambda$) makes raising external financing more difficult: both $A_0^{\text{FREQ}}$ and $A_0^{\text{INFREQ}}$ are higher when $\lambda$ is lower. When there is more uncertainty there is also more learning which generates

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greater "information rents". Interestingly, the last part of the Corollary (11) shows that when \( \lambda \) is particularly low and raising external financing is most difficult, then the own investment required is relatively lower under more frequent learning. This suggests that using an optimal "smart" contract can partially mitigate these particularly negative effects of more frequent learning.

In the next section, I will contrast the effect of increasing frequency of learning under optimal contracts to those that emerge if the borrowers where to be limited to use inflexible traditional assets, such as equity or debt. I will also contrast the overall magnitudes of own investment under these contracts at a given frequency of learning.

### 3.2 Smart-contract vs. traditional assets

To keep things comparable, I maintain here assumption that these standard contracts also benefit from the positive features of blockchain, i.e., verification is costless and borrowers commit to the contractual terms in date 0.\(^{17}\) I will focus in greater detail on debt and equity and discuss convertible securities at the end of this section.

As payoffs under standard debt and equity depend on total sales, let us define total potential sales and realized sales respectively as

\[
c_T = \sum_{t=0}^{T-1} s_{t+1} \quad \text{and} \quad \tilde{c}_T = \sum_{t=0}^{T-1} \tilde{s}_{t+1} = \sum_{t=0}^{T-1} 1_t s_{t+1}
\]

The standard equity contract specifies the borrower’s reward as

\[
w_E (\tilde{c}_T) = \alpha_E \tilde{c}_T,
\]

where \( \alpha_E \) is fixed, and the standard debt contract specifies the borrower’s reward as

\[
w_D (\tilde{c}_T) = \begin{cases} \tilde{c}_T - d, & \text{if } \tilde{c}_T \geq d \\ 0, & \text{otherwise} \end{cases}
\]

For the sake of concreteness, I focus here on the parametric examples for both SRP and SLP environments following the setting based on Section 2.1.1. As an example of SRP environment I consider exactly the same case as in Section 3.1, where the borrower is not uncertain about who the potentially interested customers are, but is uncertain about their correlated preferences

\[^{17}\]If (realistically) traditional assets are associated with further frictions due to positive verification and enforcement costs, then both debt and equity contracts cannot be less expensive than what is assumed here.
drawn from a beta distribution. In such a case $c_T$ has beta-binomial distribution with parameters $(T, \theta_0, \lambda (1 - \theta_0))$. As an example of SLP environment, I consider the case where the borrower knows that each potentially interested consumer will buy the product, but meets interested and non interested consumers randomly, i.e., using the notation from section 2.1.1 $T < K < N$ and $\theta_0 = 1$. In such a case $c_T$ has a hypergeometric distribution with parameters $(T, N, kN)$, where $k = \frac{K}{N}$ denotes the prior mean of this distribution. Notice that the limit with $\lambda \rightarrow \infty$ in the beta-binomial distribution case and $N \rightarrow \infty$ in the hypergeometric distribution case corresponds to independent sales, where $c_T$ has binomial distribution $Bin (T, \theta_0)$ and $Bin (T, k)$ respectively. Assume further that $e_t = e_0 = e_T = \bar{e}$.

Let us start by considering a break-even borrower, who has sufficient own funds to cover investment cost $I = I^{FB}$. This benchmark is helpful as we know that the borrower’s utility from the project is maximized when he makes an effort at all dates and assuming he has enough own funds simply guarantees that raising some external financing is possible under all contracts.

Consider then an equity contract. All incentive compatibility constraints (8) need to be satisfied for the borrower to make an effort at all dates. As equity contract is linear in sales, this requires that $\alpha E \mathbb{E} [s_t | c_t] \geq \bar{e}$ for any $t$ in the frequent learning case and $\alpha E \mathbb{E} [c_{T/2}] \geq \bar{e}$ and $\alpha E \mathbb{E} [c_T - c_{T/2} | c_{T/2}] \geq \bar{e}$ in the infrequent learning case. The least costly equity contract is the one where the incentive compatibility constraint binds in the state where the borrower’s beliefs about the prospects of making a next sale are at their lowest. In a SRP environment these beliefs are at their lowest when the borrower has not made any sales until the last time he chooses effort. This implies that the borrower’s equity share is $\alpha^{FREQ}_{E,SRP} = \frac{\bar{e}(T - 1 + \lambda)}{\lambda \theta_0}$ and $\alpha^{INFREQ}_{E,SRP} = \frac{\bar{e}(T + \lambda)}{\lambda \theta_0}$ in the case of frequent and infrequent learning, respectively. Using this we find that the break-even borrower’s own investment required is

$$A^{FREQ}_{E,SRP} = \bar{e} \frac{(T - 1) T}{\lambda} > A^{INFREQ}_{E,SRP} = \bar{e} \left( \frac{T^2}{4 \lambda} \right)$$

under frequent and infrequent learning, respectively. We can see that equity requires noticeably greater own investment from the borrower that the optimal smart contract, e.g., twice as high in
the case of infrequent decisions. Furthermore, as $A_{E,SRP}^{FREQ} - A_{E,SRP}^{INFREQ} = \frac{e}{2}T(T-2)$ is positive and decreasing in $\lambda$ for any $T > 2$ and finite $\lambda$, more frequent learning makes raising equity financing always more costly, and more so if there is more uncertainty. Recall that this is in contrast to the optimal "smart" contract, where more frequent learning leads to a relatively lower requirement of own financing with a sufficiently low $\lambda$.

In a SLP environment, the borrowers’ expectations about future sales prospects are at their lowest when the borrower has made successful sales every period until he makes his last contract relevant effort decision. This implies that $\alpha_{E,SLP}^{FREQ} = \frac{e}{k}N - (T - 1)$ and $\alpha_{E,SLP}^{INFREQ} = \frac{e}{k}N - \frac{T}{2}$, and the break-even borrower’s own investment required is

\[
A_{E,SLP}^{FREQ} = \frac{e}{k}N - (T - 1) > 0
\]
\[
A_{E,SLP}^{INFREQ} = \frac{e}{k}N - \frac{T}{2} > 0
\]

Obtaining external financing is more difficult than under the optimal "smart" contract also in the SLP environment. In particular, all positive NPV projects cannot be equity financed without the borrowers own funds. Furthermore, the more informative past sales are about future prospect (lower $N$), the more own funds are needed, and the more costly raising external financing becomes when frequency of learning and effort decisions increases (as $A_{E,SLP}^{FREQ} - A_{E,SLP}^{INFREQ} = \frac{e}{k}(1-k)N(T-2)}{(kN-(T-1))(kN-\frac{T}{2})} > 0$ and decreasing in $N$).

At the limit $\lambda \to \infty$ or $N \to \infty$, the sales are independent over time. Because effort cost is constant, the equity contract is the optimal contract (with $\alpha_E = \frac{e}{k}$ or $\alpha_E = \frac{e}{k}$) in the special case of i.i.d sales. This highlights that what makes equity suboptimal in this setting is learning from past sales. An equity contract that induces effort in all periods requires the borrower to maintain a high share of all sales revenues in all states. This is costly because the borrower would instead benefit from signing a "smart" contract that gives him a lower share in states where he considers the prospect of making further sales relatively easier in order to get financing at better terms ex-ante.

Satisfying all incentive compatibility constraints (8) under a simple debt contract is even harder. The borrower owes a fixed amount, which implies that his effort incentives are at their lowest when he has not managed to sell any widgets until the last date he makes an effort decision. The
consequences of this are particularly stark, in the frequent learning and decisions case as no debt contract with a face value \( d \geq 1 \) can incentivize the borrower to make an effort at date \( T - 1 \) when \( c_{T-1} = 0 \). This implies that the probability of making just one sale at the last date must be sufficient to cover \( d \), and the maximum face value of debt must therefore satisfy

\[
\Pr (s_T - d \geq 0 | c_{T-1} = 0) \mathbb{E} [s_T - d | c_{T-1} = 0, s_T - d \geq 0] = \bar{\epsilon}
\]

As \( s_T - d \geq 0 \) only if \( s_T = 1 \), this simplifies to

\[
d = 1 - \frac{\bar{\epsilon}}{\Pr (s_T = 1 | c_{T-1} = 0)}.
\]

We obtain the required borrower’s own investment from the lender’s break-even constraint as

\[
A_D = I^{FB} - d (1 - \Pr (c_T = 0)),
\]

where \( I^{FB} = \mathbb{E} [c_T] - T \bar{\epsilon} \). As the term \( d (1 - \Pr (c_T = 0)) \) is small, it follows that a break even investor would need to largely self-finance the break-even project to be able to undertake it.

Using the specified distributional assumptions, we can also find that \( d \) is given by

\[
d_{SRP}^{FREQ} = 1 - \bar{\epsilon} \frac{\lambda + T - 1}{\lambda \theta_0}, \\
d_{SLP}^{FREQ} = 1 - \bar{\epsilon} \frac{N - (T - 1)}{kN},
\]

in a SRP and SLP environment, respectively. The firm’s ability to raise debt financing is limited in both cases, as well as in the case of independent sales (when \( \lambda \to \infty \), or \( N \to \infty \)). When there is more uncertainty, the borrower can raise even less debt in a SRP environment as \( d_{SRP}^{FREQ} \) is increasing in \( \lambda \). Not only does the borrower need to have a low enough debt burden to have an incentive to continue after not selling, his expectations about the prospects of selling a widget at the last data are particularly low because of learning. There is an opposite learning effect in a SLP environment and \( d_{SLP}^{FREQ} \) is higher when there is more learning (lower \( N \)).

Figure 4 compares the own funds the break-even investor needs to contribute to pursue his project under different financing contracts, and the frequencies of learning and decision making. It considers the same parameter values as in Section 2.3.1.\(^{18}\) As \( T = 12 \), we can refer to the full

\(^{18}\)That is \( T = 12, I^{FB} = 5, \bar{\epsilon} = 1/12 \). The SLP environment further assumes \( \theta_0 = 0.5, \lambda = 4 \) and \( N = K = 12 \). The i.i.d. environment assumes probability of selling at any period is always 0.5, and SLP environment assumes \( \theta_0 = 1, N = 30 \) and \( K = 15 \), such that \( k = 0.5 \).
period as "a year", so that there is one selling opportunity each month. The frequent learning and effort decision are the "monthly" decisions, and infrequent decisions take place in at the start of "January" and "July" only.

We can see that traditional assets indeed require noticeably higher own investment by the borrower compared to the optimal contract. This gap widens when the frequency increases (black bars reflect frequent learning and grey bars reflect infrequent learning). Furthermore, debt is more expensive than equity. These main observations are not specific to the choice of parameters.

It was assumed up to now that the borrower has enough own funds and simply prefers not to use these (because of lexicographic preferences). When $I < I^{FB}$, then the above results also indicate the maximum investment a borrower with no own funds can undertake while still obtaining the the same expected profits as under the first best. Such maximum investment cost can be calculated by subtracting the values reported on Figure 4 from $I^{FB} = 5$. For example, in a SRP environment with frequent decisions, investment costs up to $5 - 0.63 = 4.37$ can be always covered via the optimal contract, while investment costs up to only $5 - 2.75 = 2.25$ and $5 - 4.64 = 0.36$ can be covered via equity and debt contract, respectively.

Borrowers that have higher investment costs can still get financing via debt or equity, but financing in such a case must always be costlier for them, which in turn reduces their expected utility from the project. Consider the example of a debt contract. If a debt contract with face
value $d \geq 1$ is feasible, then the borrower and the lender both expect that the borrower will quit making an effort after observing some sufficiently bad sales history. Because the realized sales $\tilde{c}_T$ is a sum that is increasing in effort decisions, it follows that the distribution on realized sales $\tilde{c}_T$ would be first order stochastically dominated by the distribution of $c_T$, hence the expected value of the project would be lower. It is then immediate from the lender’s break even constraint (2) that at any given wealth of the borrower and investment cost, the reward that the borrower receives is also lower, which reduces the borrower’s expected returns. It then follows that there exists a threshold investment level, lower than the one under the optimal contract, above which debt financing is impossible. A similar argument applies to equity contracts.

![Figure 5: Distribution of realized sales $\tilde{c}_T$, under a debt contract and endogenous effort choices at different frequencies.](image)

To illustrate it further, consider a borrower with no own funds, i.e., $A = 0$, $T = 12$ and $\bar{e} = 1/12$ and with a project that can generate i.i.d. potential sales, each with probability 0.5. Figure 5 plots the distribution of total realized sales, $\tilde{c}_T$, under the highest investment cost that can be debt financed. If there is just one effort choice at the beginning of "January", then Modigliani and Miller results remain applicable and the highest first best investment $I = 5$ can be debt financed (the resulting realized sales distribution is binomial and represented by the light grey bars). When the borrower can adjust his effort in "July", the highest investment cost that can be debt financed is $I = 4.96$. Under this contract the borrower will optimally stop making efforts if he has not sold
at least one widget during the first half of the year. The resulting realized sales distribution is
stochastically worse as the low outcomes are more likely (darker grey bars). When the borrower
makes effort decisions frequently then the realized sales distribution becomes ever more shifted
towards the low outcomes (black bars), and the highest investment cost that can be covered is
$I = 4.42$. Due to more frequent learning and effort decisions, the borrower is more likely to quit
and stop making efforts if he does not achieve the following benchmarks: 1) at least one successful
sale by date 4; 2) at least two successful sales by date 6; 3) at least three successful sales by date
8 and 4) at least four successful sales by date 10. Furthermore, a discount bond that covers the
investment cost under these three cases has an increasingly high discount rate when the frequency
increases: 9.0%, 9.9% and 23.8% under "January" only, "January" and "July", and "monthly"
decision frequencies.

Overall these results highlight that more frequent of learning and decisions makes financing via
traditional assets more difficult for the borrowers. As the relative benefits of the "smart" contracts
derived in this paper become higher when the learning and effort decisions are more frequent, it
suggests that an important benefit of the new technological possibility enabled by hashlinked and
timestamped record keeping and the associated possibility to write frequently adjusting "smart"
contracts is that these contracts mitigate the negative effects that the trend towards better and
faster learning technologies has on the accessibility of external financing. This is particularly
important when past sales data provide more information about future prospects.

Finally, the contract under infrequent decision making could indeed be viewed as a convertible
asset. Indeed start-ups sometimes also raise financing via convertible assets (such as preferred
shares used by venture capitalist). Nevertheless, in a SRP environment the optimal convertible
asset in this setting adjusts in an opposite direction compared the preferred shares used in venture
capital context. This is not surprising as venture capitalists are argued to use preferred shares
primarily to safeguard their investment against the entrepreneur misusing or diverting the initial
funds invested, e.g., by selling the firm too early (see e.g., Lerner et. al. 2012). This friction is not
present in my setting as it could at least theoretically be eliminated via the so called "conditional
payments" that can be built in "smart" contracts based on the same technology. Such code can
delegate the agreed investment to an escrow account and be pre-programmed to release these only "conditional" on a proof of pre-agreed use. For example, it could effectively disallow the initial owner to sell the firm or product line before a given moment in time by making the new owner’s claims on future sales equal to zero. While a typical venture capital contract would adjust is the "right" direction in SLP environment, it may be less relevant in a venture capital context if one considers that these firms often target firms with large potential for positive network effects, which as discussed in Section 2.1.1 is a characteristic of a SLP environment.

The two period case aside, commonly used convertible securities do not convert arbitrarily frequently. With high enough $T$ and rich enough statistical dependencies between potential sales, one cannot express the "smart" contract derived as a combination of traditional securities.

4 Conclusion

This paper explored how the blockchain environment could change the borrower-lender relationship. Blockchain brings efficiency gains such as costless verification and automatic enforcement, and all possible financing contracts would benefit from these gains. At the same time, the blockchain technology is emerging in an environment where agents can learn from their cash flow data (due to for example advances in computer power and big data analytics), update their beliefs about future prospects and make decisions faster. If anything, blockchain is likely to facilitate such faster learning and decision making.

I show that an ideal borrowing contract in this environment would be an automatically adjusting profit sharing agreement, and for this reason reliable timestamps of records is an important feature of blockchain records. How the optimal contract should adjust depends on the fundamental characteristics of the market where the firm is selling its products. Dynamics differ depending on whether the firm operates in a market where better than expected sales outcomes are associated with expected higher future demand or lower future demand. With such a contract, agents are able to test out their ideas with minimal need for own funds, and the economic outcomes are closer to those of a frictionless market.

An environment with frequent decisions is particularly bad for standard debt contracts because
it gives poor incentives for the borrower to make effort after unlucky outcomes. If one were to be restricted to use only debt and simple equity contracts, then equity would dominate debt contracts. When compared to the optimal contract, the equity contract is almost as good when sales do not reveal a lot of new information, and noticeably worse if sales data are very informative about future prospects. Equity contracts also do not optimally adjust to the dynamics of effort cost.

It can be argued that even without blockchain, debt contracts could become worse in the future, given that there is a trend towards shorter contracts, and improving capabilities for faster data analysis and decision making. If the negative effects of frequent decision making are foreseen by lenders, it can make debt contracts more expensive for borrowers, and if not foreseen, it could increase the frequency of defaults. Perhaps the most compelling reason for using debt contracts has been the costs associated with verifying cash flow and optimizing monitoring efforts. As verification costs are becoming smaller, it could encourage the creation of new types of financing contracts that are "equity-like" and ideally more flexible than equity; the recent emergence and success of equity crowdfunding and initial coin offerings seems to be a movement in this direction.

My setting assumes that there is no information asymmetry at the time of contracting and the potential sales distribution is known to both parties. One would obtain the same results if the borrower and lender agreed on how the borrower's incentives are likely to change following realized sales, even if the prior joint probability distribution is subjective. It is also plausible that the borrower could have superior information about his target market at the time of contracting. While this could lead to additional difficulties, there are other contemporary developments that would enable the contracting parties to mitigate this information asymmetry before contracting, e.g., reward-based crowdfunding, which can be used to test the market and produce public information about the preferences of target consumers (see e.g., Chemla and Tinn 2017).

More broadly this paper contributes to the financial contracting literature by highlighting the effects of frequent learning and decision making on incentives, and characterizes how these effects depend on general distributional properties. I showed that in a realistic environment where borrowers' effort choices are dynamic, the sequence of cash flow arrivals is important and this explains why my findings are in contrast to some influential existing papers on this topic.
My analysis suggests that the more important benefit of blockchain and "smart" financing contracts could be the possibility to design new, more flexible, types of contracts rather than the possibility to manage and seize collateral more easily, an argument often emphasized in this context. My model shows that there is less need for own investment and collateral under the optimal self-adjusting contract, which makes external financing noticeably more accessible.
Appendix

A Further background on blockchain technology

A blockchain, or more precisely a hash-linked and timestamped ledger, is a database where digital data is recorded in chronological timestamped blocks that are cryptographically linked via a hash function. Hash functions are one way functions that take an input (numerical and/or textual data) and return a fixed-size alphanumeric string. The main feature of any good cryptographic hash function is that it is easy to calculate the correct hash from the given data and computationally extremely difficult to reconstruct the original input from a given hash. A hash-linked and timestamped ledger further imposes a chronological structure, where the hash reflecting the state of agreed records at some point in time is included to the next record. For cryptographic integrity, the main benefit of this structure is that it becomes increasingly difficult to alter historical records. When considering financial contracting, data recorded on such ledger could be detailed accounting data, ownership data, terms of a financing contract, as well as a code for implementing pre-agreed contractual terms.

While the precise origin and definition of the term "blockchain" itself is subject to some disagreements, hash-linked timestamping is a common feature behind all forms of blockchain technology. The idea that hash-linked timestamping can be used to create secure digital records which provide a reliable digital proof when these records were generated was first proposed in computer science by Haber and Stornetta (1991, 1997). This idea has lead to both further research and prominent real world applications. It is well known that Nakamoto (2008) combined this idea with a proposal of a proof-of-work verification system to propose Bitcoin, digital money created outside the control of traditional public and private institutions. It is perhaps less known that in 2007 Guardtime built on hash-linked timestamping to launch a formally verifiable security system for the Estonian Government, which also eliminates third parties, trusted insiders or cryptographic keys in the verification of the integrity of government records, networks and systems. Guardtime was built on research by Buldas, Laud, Lipmaa, and Villemson (1998), and Buldas and Saarepera (2004) who provided
a formal proof of the idea of time-stamping proposed by Haber and Stornetta. Nowadays there are many forms of blockchain initiatives that broadly fall under two categories: those that rely on permissionless verification systems as Bitcoin (most notable of those is the Ethereum project that enables smart contract functionality based on a Ethereum coin), and those that rely on permissioned verifications systems in the spirit of Guardtime (most notable of those is the Hyperledger project - a consortium of well-known technology platform companies, financial services and Business Software companies). While verification systems differ, all blockchains rely on hash-linked timestamping. The particular verification mechanism is not of primary importance for the mechanisms in this paper. Permissioned systems do not require for example costly mining to record a block of transactions, are more scaleable and can maintain privacy more easily, while permissionless systems are more decentralized as any member of public can participate in validating new blocks. The extent of decentralization in permissioned systems depends upon the number of peers, the expected number of bad nodes in the network, and the type of consensus mechanism the members agree to.

The term "smart contracts" was highlighted by Szabo (1994, 1997) as a tool that enables the automatic execution of contracts, lowers enforcement costs and minimizes the need for intermediaries. Werback and Cornell (2017) emphasise that from a legal perspective the distinct feature of "smart contracts" is that the burden of proof is reversed - a pre-agreed contract would first be automatically executed and disputes about the fairness of the contracts could happen later. In my setting, this effectively safeguards the lender by eliminating enforcement costs and makes it easier/cheaper to contract for the borrower (see also Rius (2018) for the analyses of Ethereum based "smart contracts" from a computer science perspective).

19 Buldas and Saarepera are further associated with a number of patents assigned to Guardtime See e.g., patents granted to Guardtime US8719576B2 (2003) "Document verification with distributed calendar infrastructure" that refers to timestamping; US9853819B2 (2013) "Blockchain-supported, node ID-augmented digital record signature method" that refers to timestamping and blockchain. The company now also provides blockchain based digital security services for DARPA, and Erisson, among others.

20 Including technology firms (Cisco, Fujitsu, Hitachi, IBM, Intel, NEC, NTT DATA, Red Hat, VMware), financial services firms (ABN AMRO, ANZ Bank, BNY Mellon, CLS Group, CME Group, the Depository Trust & Clearing Corporation (DTCC), Deutsche Börse Group, J.P. Morgan, State Street, SWIFT, Wells Fargo), Business Software companies like SAP, Systems integrators and others such as: (Accenture, Calastone, Wipro, Credits, Guardtime, IntellectEU, Nxt Foundation, Symbiont).

21 For example, Blockchain-based records and smart contracts are often referred to as immutable, because the records are verifiable and a code implementing a smart contract can be recorded on the blockchain as well. Immutability is
B Proofs

B.1 Parametric example of SRP vs. SLP in two period case

Consider a parametric setting described in Section 2.1.1 and two first sales at \( t = 1, 2 \). By the law of total expectations, the joint distribution

\[
p(s_1, s_2) = \Pr(\tau_1 = NI, \tau_2 = NI)p(s_1, s_2|\tau_1 = NI, \tau_2 = NI) \\
+ \Pr(\tau_1 = NI, \tau_2 = PI)p(s_1, s_2|\tau_1 = NI, \tau_2 = PI) \\
+ \Pr(\tau_1 = PI, \tau_2 = NI)p(s_1, s_2|\tau_1 = PI, \tau_2 = NI) \\
+ \Pr(\tau_1 = PI, \tau_2 = PI)p(s_1, s_2|\tau_1 = PI, \tau_2 = PI),
\]

where the probabilities of meeting different consumer types are \( \Pr(\tau_1 = NI, \tau_2 = NI) = \frac{N-K}{N} \cdot \frac{N-1-K}{N-1} \), \( \Pr(\tau_1 = NI, \tau_2 = PI) = \Pr(\tau_1 = PI, \tau_2 = NI) = \frac{N-K}{N} \cdot \frac{K}{N-1} \), and \( \Pr(\tau_1 = PI, \tau_2 = PI) = \frac{K}{N} \cdot \frac{K-1}{N-1} \).

Note also that the probability of making a positive sale to a type "NI" consumer is impossible, hence for example \( p(s_1, s_2|\tau_1 = NI, \tau_2 = NI) = 0 \) whenever \( s_1 = 1 \) or \( s_2 = 1 \),

\[
p(s_1 = 0, s_2 = 1|\tau_1 = PI, \tau_2 = NI) = 0,
\]

etc. Furthermore, meeting a type "NI" reveals no information about \( \theta \), and thus for example \( p(s_1 = 0, s_2 = 1|\tau_1 = NI, \tau_2 = PI) = p(s_2 = 1|\tau_2 = PI) = \mathbb{E}[p(s_2 = 1|\theta, \tau_2 = PI)|\tau_2 = PI] = \mathbb{E}[\theta] = \theta_0. \)

At the same time, meeting two type "PI" consumers reveals information about \( \theta \), as by the law of iterated expectations and the expression for p.d.f. of beta distribution, we find that

\[
p(s_1, s_2|\tau_1 = PI, \tau_2 = PI) = \mathbb{E}[s_1, s_2|\theta, \tau_1 = PI, \tau_2 = PI] \\
= \mathbb{E}\left[\theta^{s_1+s_2}(1-\theta)^{1-(s_1+s_2)}\right] = \int_0^1 \theta^{s_1+s_2}(1-\theta)^{2-(s_1+s_2)} \theta^{\lambda\theta_0-1}(1-\theta)^{\lambda(1-\theta_0)-1} d\theta \\
= \frac{B(\lambda\theta_0 + s_1 + s_2, \lambda(1-\theta_0) + 2 - (s_1 + s_2))}{B(\lambda\theta_0, \lambda(1-\theta_0))},
\]

where \( B(x, y) \) is a beta function for Re \( x > 0 \) and Re \( y > 0 \). Using further that beta function can be expressed via gamma functions \( \Gamma(.) \) as \( B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \), and gamma function satisfies the
recurrence relation $\Gamma (x + 1) = x\Gamma (x)$, we find that

\[
p (s_1 = 0, s_2 = 0 | \tau_1 = P1, \tau_2 = P1) = \frac{\lambda (1 - \theta_0) (\lambda (1 - \theta_0) + 1)}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 0, s_2 = 1 | \tau_1 = P1, \tau_2 = P1) = \frac{\lambda \theta_0 \lambda (1 - \theta_0)}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 1, s_2 = 0 | \tau_1 = P1, \tau_2 = P1) = \frac{\lambda \theta_0 (1 - \theta_0)}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 1, s_2 = 1 | \tau_1 = P1, \tau_2 = P1) = \frac{\lambda \theta_0 (\lambda \theta_0 + 1)}{\lambda (\lambda + 1)}
\]

From all this, we can specify the joint distribution of sales as

\[
p (s_1 = 0, s_2 = 0) = \frac{N - K}{N} \frac{N - K - 1}{N - 1} + 2 \frac{N - K}{N} \frac{K}{N - 1} (1 - \theta_0)
\]

\[
+ \frac{K K - 1}{N} \frac{1}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 0, s_2 = 1) = \frac{N - K}{N} \frac{K}{N - 1} \theta_0 + \frac{K K - 1}{N} \frac{\lambda \theta_0 \lambda (1 - \theta_0)}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 1, s_2 = 0) = \frac{N - K}{N} \frac{K}{N - 1} \theta_0 + \frac{K K - 1}{N} \frac{\lambda \theta_0 \lambda (1 - \theta_0)}{\lambda (\lambda + 1)}
\]

\[
p (s_1 = 1, s_2 = 1) = \frac{K K - 1}{N} \frac{\lambda \theta_0 (\lambda \theta_0 + 1)}{\lambda (\lambda + 1)}
\]

In the two period case stochastic affiliation is equivalent to comparing conditional probabilities as

\[
p (s_1 = 0, s_2 = 0) p (s_1 = 1, s_2 = 1) \geq p (s_1 = 0, s_2 = 1) p (s_1 = 1, s_2 = 0) \iff
\]

\[
\frac{p (s_1 = 0, s_2 = 0)}{p (s_1 = 1, s_2 = 1)} \geq \frac{p (s_1 = 1, s_2 = 0)}{p (s_1 = 1, s_2 = 1)} \iff
\]

\[
1 + \frac{p (s_1 = 0, s_2 = 0)}{p (s_1 = 1, s_2 = 1)} \geq 1 + \frac{p (s_1 = 1, s_2 = 0)}{p (s_1 = 1, s_2 = 1)} \iff
\]

\[
\frac{p (s_1 = 0)}{p (s_1 = 1, s_2 = 1)} \geq \frac{p (s_1 = 1)}{p (s_1 = 1, s_2 = 1)} \iff
\]

\[
\Pr (s_2 = 1 | s_1 = 1) \geq \Pr (s_2 = 1 | s_1 = 0).
\]

Using the expressions above, we find that $p (s_1 = 0) = \frac{N - K}{N} + \frac{K}{N} (1 - \theta_0)$, $p (s_1 = 1) = \frac{K}{N} \theta_0$, and

\[
\Pr (s_2 = 1 | s_1 = 1) = \frac{K K - 1}{N - 1} \frac{\theta_0 + 1}{\lambda + 1}
\]

\[
\Pr (s_2 = 1 | s_1 = 0) = \frac{N - K}{N - 1} \frac{K \theta_0}{N - K + K (1 - \theta_0)} + \frac{K K - 1}{N - 1} \frac{\lambda \theta_0 (1 - \theta_0)}{\lambda (\lambda + 1) (N - K + K (1 - \theta_0))}
\]

From here we obtain that $\Pr (s_2 = 1 | s_1 = 1) \geq \Pr (s_2 = 1 | s_1 = 0)$ if, and only if,

\[
\lambda \theta_0 (N - K) \leq N (K - 1) - \theta_0 K (N - 1).
\]
First, notice that this inequality always holds when $K = N$, as left hand side becomes zero and the right hand side becomes $N(N - 1)(1 - \theta_0) \geq 0$. Second, provided that $N > K$, we obtain that this inequality never holds if $\theta_0 > \frac{N(K - 1)}{K(N - 1)}$. Note that such values for prior mean always exist as $\frac{N(K - 1)}{K(N - 1)} < 1$. Denoting the share of potentially interested consumers with $k \equiv \frac{K}{N}$, we can write $\frac{N(K - 1)}{K(N - 1)} = \frac{Nk - 1}{k(N - 1)}$, which is increasing in $N$ and $k$. Hence the condition $\theta_0 > \frac{N(K - 1)}{K(N - 1)}$ is less likely to hold when $\theta_0$ is lower, and $N$ and $k$ are higher. Third, when $N > K$ and $\theta_0 \leq \frac{N(K - 1)}{K(N - 1)}$, then there exists a cutoff value of $\lambda$, such that sales are stochastically affiliated if, and only if,

$$\lambda \leq \frac{N(K - 1) - \theta_0 K(N - 1)}{\theta_0 (N - K)} = \frac{(Nk - 1) - \theta_0 k(N - 1)}{\theta_0 (1 - k)}.$$

As $\frac{\partial \lambda}{\partial \theta_0} < 0$, $\frac{\partial \lambda}{\partial N} > 0$ and $\frac{\partial \lambda}{\partial k} > 0$, also this condition is more likely to hold when $\theta_0$ is lower, and $N$ and $k$ are higher. As with two periods the random variable can only be affiliated of not, higher $\lambda$ and $\theta_0$, and lower $N$ and $k$ imply a SLP environment.

### B.2 Proof of Lemma 3

**Proof of Part 1) of Lemma 3.** The value of the project

$$\pi = \sum_{t=0}^{T-1} (1_t s_{t+1} - 1_t e_t) - I,$$

(28)

Let us establish that setting $1_t = 1$ for all $t$ is optimal, and thus $\pi^{FB} = \sum_{t=0}^{T-1} (s_{t+1} - e_t) - I$. Suppose instead that the borrower makes no effort at some date $\tau$. Then value of the project under such deviating strategy is

$$\pi' = \sum_{t=0}^{\tau-1} (s_{t+1} - e_t) + \sum_{t=\tau+1}^{T-1} (s_{t+1} - e_t) - I,$$

From here $\pi^{FB} - \pi' = s_{\tau+1} - e_\tau$, and $\mathbb{E}[\pi^{FB} - \pi'] = \mathbb{E}[s_{\tau+1}] - e_\tau$. As the effort cost was assumed to be small, i.e., $e_\tau \leq \mathbb{E}[s_{\tau+1}1_{0}s_1, \ldots, 1_{t-1}s_t, 1_0, \ldots, 1_{t-1}]$, it follows that $\mathbb{E}[s_{\tau+1}] \geq e_\tau$, and such deviation is not profitable. Under $1_t = 1$,

$$\mathbb{E}[\pi^{FB}] = \sum_{t=0}^{T-1} \mathbb{E}[s_{t+1}] - \sum_{t=0}^{T-1} e_t - I.$$

**Proof of Part 2) of Lemma 3.** Consider then the contracting problem (1)-(3). Replacing the lender’s break even constraint (2) in the borrower’s utility (1), we obtain that the borrower’s
expected utility from the project at date 0 is

$$\mathbb{E}[\pi] = \mathbb{E} \left[ \sum_{t=0}^{T-1} 1_t s_{t+1} - 1_t e_t \right] - I,$$

(29)

which cannot be higher than $\mathbb{E}[\pi^FB]$ above, and is achieved by setting $1_t = 1$ for all $t$. Notice that $A_0$ does not enter in (29), and thus under $1_t = 1$, all contracts are equivalent and maximize the borrower’s primary objective. By the borrower’s secondary objective, the optimal contract must then set own investment, $A_0$, to a lowest feasible value. Furthermore, setting $1_t = 0$ for some $t$ cannot lead to a lower $A_0$. To see this, suppose that there exists a feasible contract with $A_0'$ and $w'(\hat{s}_1, ..., \hat{s}_T)$, which has $1_t = 0$ for some $t$, and gives expected profit $\mathbb{E}[\pi^FB]$. To give the borrower expected profit $\mathbb{E}[\pi^FB]$, by (1) it must be the case that

$$A_0' = \mathbb{E}[w'(\hat{s}_1, ..., \hat{s}_T)] - \mathbb{E}[\pi^FB] - \sum_{t=0}^{T-1} 1_t e_t =$$

$$= \mathbb{E}[w'(\hat{s}_1, ..., \hat{s}_T)] + I - \mathbb{E} \left[ \sum_{t=0}^{T-1} s_{t+1} \right] - \sum_{t=0}^{T-1} (1_t - 1) e_t$$

Any feasible contract must guarantee that the lender breaks even or makes a profit, hence the own investment from the lender’s perspective must have

$$A_0 \geq I - \mathbb{E} \left[ \sum_{t=0}^{T-1} 1_t s_{t+1} \right] + \mathbb{E}[w'(\hat{s}_1, ..., \hat{s}_T)]$$

We then obtain

$$A_0 \geq A_0' + \sum_{t=0}^{T-1} (1 - 1_t) (\mathbb{E}[s_{t+1}] - e_t)$$

where $\mathbb{E}[s_{t+1}] \geq e_t$ implies that the term $\sum_{t=0}^{T-1} (1 - 1_t) (\mathbb{E}[s_{t+1}] - e_t) \geq 0$ (and can be positive) when $1_t = 0$ for some $t$, and is zero if $1_t = 1$ for some $t$.

**B.3 Proof of Proposition 4**

Let $W' = [ \tilde{w}(0, 0) \; \tilde{w}(0, 1) \; \tilde{w}(1, 0) \; \tilde{w}(1, 1) ]$ be the vector of the borrower’s payoffs, $P' = [ p(0, 0) \; p(0, 1) \; p(1, 0) \; p(1, 1) ]$ be the vector of associated probabilities, and $b' = [ e_0 + e_1 \; e_0 \; e_1 \; e_1 ]$ be the right hand side of constraints. Defining also a matrix of
coefficients

\[
G = \begin{bmatrix}
  p(0,0) & -p(1,0) & -\frac{p(0,1)}{p(0,1)} & 0 \\
  p(0,1) & -p(1,1) & \frac{p(0,1)}{p(0,1)} & 0 \\
  p(1,0) & p(1,0) & 0 & -\frac{p(1,1)}{p(1,1)} \\
  p(1,1) & p(1,1) & 0 & \frac{p(1,1)}{p(1,1)} 
\end{bmatrix}
\]

we can express the primal problem (11)-(13) as

\[
\min \mathbb{E}[\tilde{w}(s_1, s_2)] = WP, \text{ subject to } WG \geq b, W \geq 0 \tag{30}
\]

Defining the vector \( X' = [x_p \ x_0 \ x_1^0 \ x_1^1] \) as corresponding shadow prices of constraints (12)-(13), i.e., \( x_p \) corresponds to the participation constraint, \( x_0 \) to date 0 effort incentive compatibility constraint and \( x_1^1 \) to date 1 effort incentive compatibility constraint when date 1 sale outcome is \( s_1 \), there is a dual maximum problem

\[
\max V = b'X, \text{ subject to } GX \leq P, X \geq 0. \tag{31}
\]

The primal and dual are linked via the Duality Theorem (Theorem 5.1 in Chvátal, 1983) and if one has a solution, then so does the other one, and values are the same.

Let us solve the dual maximum problem. The dual maximum problem simplifies to

\[
\max V = e_0(x_p + x_0) + e_1(x_1^0 + x_1^1 + x_p), \tag{32}
\]

subject to

\[
x_p \leq 1; \tag{33}
\]

\[
x_p + x_0 \leq 1;
\]

\[
x_1^0 \leq p_1(0)(1 - x_p) + \frac{p(1,1)p_0(0)}{p(0,1)}x_0; \tag{34}
\]

\[
x_1^1 \leq p_1(1)(1 - x_p - x_0);
\]

\[
x_p, x_0, x_1^0, x_1^1 \geq 0
\]

Because the objective function (32) is increasing in \( x_1^0 \) and \( x_1^1 \), the third and the fourth inequalities in (33) must be binding. Using this in (32), the problem simplifies to

\[
\max V = \ e_0(x_p + x_0) + e_1 \left( x_p + (1 - x_p) + x_0p(0,1) \left( \frac{p(1,1)}{p(0,1)} - \frac{p(0,1)}{p(0,1)} \right) \right) \tag{34}
\]

\[
= e_0(x_p + x_0) + e_1x_0 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)}.
\]

47
subject to \( x_p \leq 1, \ x_p + x_0 \leq 1, \ x_p, x_0 \geq 0 \). It then follows that the sign of \( p(1, 1) p(0, 0) - p(1, 0) p(0, 1) \) determines the maximizing values of \( x_0 \) and \( x_p \), and hence also the maximizing values of \( x_1^0 \) and \( x_1^1 \).

If \( p(1, 1) p(0, 0) > p(0, 1) p(1, 0) \) then from (34) the solution of the dual maximum problem is \( x_p = 0, \ x_0 = 1, \ x_1^0 = p_1(0) \left( 1 + \frac{p(1, 1)}{p(0, 1)} \right) \), \( x_1^1 = 0 \), and the maximized value is

\[
V = e_0 + e_1 + e_1 \frac{p(1, 1) p(0, 0) - p(0, 1) p(1, 0)}{p(0, 1)}
\]

Given this solution, complementary slackness (see e.g., Theorem 5.3 in Chvátal, 1983) then implies that \( \bar{w}(0, 0) = 0 \), the participation constraint need not be binding and the first two incentive compatibility constraints in (3) must be binding. This, and the observation that the objective function of primal problem (30) is minimized when \( \bar{w}(1, 1) \) is at its lowest value, implies that all incentive compatibility constraints are binding under the optimal solution. Solving that system gives (14). Plugging this solution in the objective function of the primal minimization problem (30) verifies that \( E[\bar{w}(s_1, s_2)] = V \), as implied by the Duality Theorem.

If \( p(1, 1) p(0, 0) < p(0, 1) p(1, 0) \) then from (34) \( x_p = 1, \ x_0 = 0, \ x_1^0 = 0, \) and \( x_1^1 = 0 \), and the value

\[
V = e_0 + e_1
\]

Complementary slackness now implies that \( \bar{w}(0, 0) = 0 \), the participation constraint must be binding, and incentive compatibility constraints do not need to be binding. The Duality Theorem in that case further implies that \( E[\bar{w}(s_1, s_2)] = V \), which can be confirmed by plugging the solution (16) in the primal minimization problem. Note that there can be other optimal solutions of the primal maximum problem, but the presented solution is the only one that holds for any values of \( e_0, e_1 \geq 0 \), which satisfy 10.

If \( p(1, 1) p(0, 0) \neq p(0, 1) p(1, 0) \), then the two contracts are equivalent.\(^{22}\) This proves the optimality of the solution in Proposition 4.

\(^{22}\) It is sufficient to show that if \( p(1, 1) p(0, 0) = p(0, 1) p(1, 0) \), then \( \bar{w}(1, 0) \) in Part 1) and 2) of Proposition 4. Indeed, we obtain that

\[
\begin{align*}
\frac{e_0 - e_1}{p(1, 1)} + e_1 \frac{p_1(0) p_2(1)}{p(0, 1)} &= \frac{e_0}{p(1, 1)} + e_1 \frac{p_1(0) p_2(1) - 1}{p(0, 1)} = \frac{e_0}{p(1, 1)} + e_1 \frac{p_1(0) p_2(1) - 1}{p(0, 1)} \\
&= \frac{e_0}{p(1, 1)} + \frac{e_1}{p(1, 1)} \left( \frac{p_1(0) p_2(1)}{p(0, 1)} - \frac{p_2(0)}{p(1, 1)} \right) = \frac{e_0}{p(1, 1)} + \frac{e_1}{p(1, 1)} \left( \frac{p_1(0) p_2(1)}{p(0, 1)} - \frac{p_2(0)}{p(1, 1)} \right)
\end{align*}
\]
B.4 Proof of Propositions 6 and 8

The solution of the two date problem, suggests that solution of the primal minimization problem, given by (7) and (3), takes the form described in the propositions 6 and 8. In order to prove optimality, we again benefit from the duality and complementary slackness theorems. Denote $x_p$ the shadow price of the participation constraint, and $x_{s_{t-1}^t}^{s_{1}^{t-1}}$ the shadow price of the incentive compatibility constraint following realization history $s_1, ..., s_t$. The dual maximization problem is

$$\max V = x_p \sum_{t=0}^{T-1} e_t + e_0 x_0 + ... + e_t \sum_{s_1=0}^{1} ... \sum_{s_t=0}^{1} x_{s_{1}^{t-1}}^{s_{t}^{t-1}} + ... + e_{T-1} \sum_{s_1=0}^{1} ... \sum_{s_{T-1}=0}^{1} x_{s_{1}^{T-1}}^{s_{T}^{T-1}}$$

subject to

$$p (s_1, ..., s_T) x_p + (-1)^{1-s_1} p (1, s_2, ..., s_T) x_0 + ... + \frac{(-1)^{1-s_t} p (s_1, ..., s_{t-1}, 1, s_{t+1}, ..., s_T)}{p (s_1, ..., s_{t-1})} x_{s_{t-1}^t}^{s_{t-1}^t} + ... + \frac{(-1)^{1-s_T} p (s_1, ..., s_{T-1}, 1)}{p (s_1, ..., s_{T-1})} x_{s_{T-1}^{T-1}}^{s_{T-1}^{T-1}} \leq p (s_1, ..., s_T)$$

and the non-negativity constraints $x_{s_{1}^{t-1}}^{s_{t}^{t-1}} \geq 0$ for every $t = 1, ..., T - 1$.

By Theorem 5.1-5.3 in Chvátal (1983) the proof that the solutions in Propositions 6 and 8 are optimal requires checking that these solutions are feasible, using then complementary slackness and confirming that the implied solution of the dual maximum is feasible as well.

**Proof of Proposition 6.**

*Step 1) Proof that the primal guess is feasible.* First, we can prove by induction that all incentive compatibility constraints hold with equality under the guessed solution. It is clear that this is true if $t = T - 1$: using (19) in (8) gives

$$\mathbb{E} \left[ \frac{e_{T-1}^{s_{T-1}^{T-1}}}{e_{s_{T}}^{s_{T}^{s_{T-1}^{T-1}}}} | s_1, ..., s_{T-1} \right] = e_{T-1}. \text{ Suppose then that the incentive compatibility constraint holds with equality at some period } t + 1, \text{ i.e.,}$$

$$\mathbb{E} \left[ \tilde{w} (s_1, ..., s_{t+1}, 0, s_{t+3}, ..., s_T) - \tilde{w} (s_1, ..., s_{t+1}, 0, s_{t+3}, ..., s_T) | s_1, ..., s_{t+1} \right] = e_{t+1}. \text{ (36)}$$
It then follows that at \( t \)

\[
\mathbb{E}[\tilde{w}(s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) | s_1, \ldots, s_t] \\
= \mathbb{E}[\mathbb{E}[\tilde{w}(s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) | s_1, \ldots, s_t] | s_1, \ldots, s_t] \\
= \mathbb{E}[\tilde{w}(s_1, \ldots, s_{t+1}, 0, s_{t+3}, \ldots, s_T) | s_1, \ldots, s_t] + \epsilon_{t+1} - \mathbb{E}[\tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) | s_1, \ldots, s_t] \\
= \alpha_{t+1}(s_1, \ldots, s_t) \mathbb{E}[s_{t+1}|s_1, \ldots, s_t] - \alpha_{t+2}(s_1, \ldots, s_t, 0) \mathbb{E}[s_{t+2}|s_1, \ldots, s_t] + \epsilon_{t+1} + \\
\mathbb{E}[(\alpha_{t+3}(s_1, \ldots, s_t, s_{t+1}, 0) - \alpha_{t+2}(s_1, \ldots, s_t, 0, s_{t+2})) s_{t+3}|s_1, \ldots, s_t] + \ldots + \\
\mathbb{E}[(\alpha_T(s_1, \ldots, s_{t+1}, 0, s_{t+3}, \ldots, s_{T-1}) - \alpha_T(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_{T-1})) s_T|s_1, \ldots, s_t] \\
= \alpha_{t+1}(s_1, \ldots, s_t) \mathbb{E}[s_{t+1}|s_1, \ldots, s_t] - \alpha_{t+2}(s_1, \ldots, s_t, 0) \mathbb{E}[s_{t+2}|s_1, \ldots, s_t] + \epsilon_{t+1} = \epsilon_t 
\]

where the first equality follows from the law of iterated expectations, the second equality uses (36), the third equality uses (19), the fourth follows from exchangeability, which implies that \( \alpha_{t+h}(s_1, \ldots, s_t, s_{t+1}, 0, s_{t+3}, \ldots, s_{t+h-1}) = \alpha_{t+h}(s_1, \ldots, s_t, 0, s_{t+2}, s_{t+3}, \ldots, s_{t+h-1}) \) for any \( h = 3, \ldots, T - t - 1 \), and the fifth follows from (20) and exchangeability, which implies that \( \mathbb{E}[s_{t+1}|s_1, \ldots, s_t] = \mathbb{E}[s_{t+2}|s_1, \ldots, s_t] \). This proves that all incentive compatibility constraints are binding under the guessed solution of the primal minimization problem.

In order to confirm that the participation constraint (7) holds, note that \( \Delta_t(s_1, \ldots, s_t) \), defined in (23), is non-negative as \( \Delta_t(s_1, \ldots, s_t) \geq 0 \) if \( \mathbb{E}[s_{t+2}|s_1, \ldots, s_t] \geq \mathbb{E}[s_{t+2}|s_1, \ldots, s_t, 0] \Leftrightarrow \mathbb{E}[s_{t+2}|s_1, \ldots, s_t, 0] \leq \mathbb{E}[s_{t+2}|s_1, \ldots, s_t, 1] \). The latter holds because

\[
\frac{p(s_1, \ldots, s_t, 0, 1)}{p(s_1, \ldots, s_t, 0)} \leq \frac{p(s_1, \ldots, s_t, 1, 1)}{p(s_1, \ldots, s_t, 1)} \Leftrightarrow \\
p(s_1, \ldots, s_t, 0, 0) p(s_1, \ldots, s_t, 1, 1) \geq p(s_1, \ldots, s_t, 1, 0) p(s_1, \ldots, s_t, 0, 1)
\]

and the SRP environment assumes stochastic affiliation. Furthermore, the inequality is strict unless potential sales are i.i.d. It then follows from (19), (20) and the definitions (22) and (23) that the participation constraint holds, and is not-binding (unless potential sales are i.i.d).

**Step 2)** **Proof of optimality.** Given the results in step 1, complementary slackness implies that \( x_p = 0 \) (as the participation constraint is not binding) and constraints (35) must hold with equality.
whenever at least one \( s_t \neq 0 \) (as \( \bar{w}(s_1, ..., s_T) > 0 \) for all sequences of \( s_1, ..., s_T \) except the sequence of zeros only).

To prove optimality, we need to that the solution of the dual maximization problem under these constraints is non-negative. To shorten the notation, define \( z_t(s_1, ..., s_t) \equiv \frac{x^{s_1,...,s_t}}{p(s_1,...,s_t)} \), which has the same sign as \( x^{s_1,...,s_t} \). Hence it needs to be proven that \( z_t(s_1, ..., s_t) \geq 0 \) for any \( t = 0, ..., T - 1 \). We already know from Proposition 4 that it \( T = 2 \) then there is a feasible solution of the dual problem, and we can extend the proof to \( T > 2 \) by induction.

Suppose that there exists a feasible solution \( z_0, z_1(s_1), ..., z_{\tau-1}(s_1, ..., s_{\tau-1}) \geq 0 \) when \( T = \tau \), which satisfies the complementary slackness conditions, i.e., for all \( t = 0, ..., \tau - 1 \) it holds that

\[
( -1 )^{1-s_1} p (1, s_2, ..., s_\tau) z_0 + ... + \\
( -1 )^{1-s_t} p (s_1, ..., s_{t-1}, 1, s_{t+1}, ..., s_\tau) z_{t-1}(s_1, ..., s_{t-1}) + ... + \\
( -1 )^{1-s_\tau} p (s_1, s_2, ..., s_{\tau-1}, 1) z_{\tau-1}(s_1, ..., s_{\tau-1}) \leq ( = ) \ p (s_1, s_2, ..., s_\tau),
\]

where the weak inequality holds for any \( s_t \) and the equality holds whenever at least one \( s_t \neq 0 \).

To prove optimality, it must then follow that there also exists a feasible solution \( z_\tau(s_1, ..., s_\tau) \geq 0 \) when \( T = \tau + 1 \). As by complementary slackness, it then follows that the equivalent to (37) holds when \( T = \tau + 1 \) and as \( s_{\tau+1} = \{0, 1\} \), we must show that

\[
( -1 )^{1-s_1} p (1, s_2, ..., s_\tau, 0) z_0 + ... + \\
( -1 )^{1-s_t} p (s_1, s_2, ..., s_{\tau-1}, 1, 0) z_{\tau-1}(s_1, ..., s_{\tau-1}) + \\
( -1 ) p (s_1, s_2, ..., s_{\tau-1}, s_\tau, 1) z_{\tau}(s_1, ..., s_\tau) \leq ( = ) \ p (s_1, s_2, ..., s_\tau, 0),
\]

where equality holds as long as at least one \( s_t \neq 0 \) in set \( \{s_1, ..., s_\tau\} \), and

\[
( -1 )^{1-s_1} p (1, s_2, ..., s_\tau, 1) z_0 + ... + \\
( -1 )^{1-s_t} p (s_1, s_2, ..., s_{\tau-1}, 1, 1) z_{\tau-1}(s_1, ..., s_{\tau-1}) + \\
( +1 ) p (s_1, s_2, ..., s_{\tau-1}, s_\tau, 1) z_{\tau}(s_1, ..., s_\tau) = p (s_1, s_2, ..., s_\tau, 1)
\]

for any realizations of \( s_t \in \{s_1, ..., s_\tau\} \). Notice that adding up (38) and (39) gives (37). This implies that the solutions \( z_t(s_1, ..., s_t) \) for \( t = 0, .., \tau - 1 \) do not change when we increase \( T \) from \( \tau \) to \( \tau + 1 \).

What is left to be show is that \( z_{\tau}(s_1, ..., s_\tau) \geq 0 \) for any sequence \( s_1, ..., s_\tau \).
First, suppose that $s_\tau = 0$. If $s_t = 0$ also for any $t = 1, \ldots, s_\tau - 1$, then (39) implies that
\[
z_\tau (0, \ldots, 0) = 1 + \frac{p(1,0,\ldots,0,1)}{p(0,0,\ldots,0,1)} z_0 + \ldots + \frac{p(0,\ldots,0,1,1)}{p(0,0,\ldots,0,1)} z_{s_\tau - 1} (0, \ldots, 0) > 0, \tag{40}
\]
As $(-1)^{1-0} = -1$, (38) is satisfied as well, and does not need to be binding.

If $s_\tau = 0$ and at least one $s_t \neq 0$ in set $\{s_1, \ldots, s_\tau - 1\}$ then (38) must hold with equality. By the assumption of existence of a feasible solution for $T = \tau$, exchangeability (which implies that
\[
(-1)^{1-0} \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)} = -1 \text{ and } (38),\]
follows that
\[
z_\tau (s_1, \ldots, s_{\tau-1}, 0) = (-1)^{1-s_1} \left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) z_0 + \ldots + (-1)^{1-s_{\tau-1}} \left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) z_{\tau-2} (s_1, \ldots, s_{\tau-2}) \]
Notice that exchangeability implies that
\[
(-1)^{1-s_t} \left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 1, s_t + \ldots + s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, s_t + \ldots + s_{\tau-1}, 0, 1)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 1, s_t + \ldots + s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 1)} \right) = 0 \text{ whenever } s_t = 0. \]
We can then simplify
\[
z_\tau (s_1, \ldots, s_{\tau-1}, 0) = \left[ \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right] (s_1 z_0 + \ldots s_{\tau-1} z_{\tau-1} (s_1, \ldots, s_{\tau-1} - 1) - 1).
\]
As by the solution of $T = 2$ and (40) $z_0 \geq 1$, $z_1 (0) \geq 1$, $z_2 (0, 0) \geq 1$ etc., it holds that $z_1 z_0 + \ldots s_{\tau-1} z_{\tau-1} (s_1, \ldots, s_{\tau-1} - 1) - 1 \geq 0$. Furthermore, "success raises prospects"/stochastic affiliation implies that
\[
\frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)} \geq \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \iff
p(s_1, s_2, \ldots, s_{\tau-1}, 1, 1) p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0) \geq p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0) p(s_1, s_2, \ldots, s_{\tau-1}, 0, 1)
\]
This proves that $z_\tau (s_1, \ldots, s_{\tau-1}, 0) \geq 0$.

If $s_\tau = 1$, then (39) and (37), imply that
\[
z_\tau (s_1, \ldots, s_{s_\tau - 1}, 1) = (-1)^{1-s_1} \left( \frac{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1, 1)}{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1, 1)} - \frac{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1)}{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1, 1)} \right) z_0 + \ldots + (-1)^{1-s_{s_\tau - 1}} \left( \frac{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 0, 1)}{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 0, 1)} - \frac{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1)}{p(s_1, s_2, \ldots, s_{s_\tau - 1}, 1, 1)} \right) z_{s_\tau - 1} (s_1, \ldots, s_{s_\tau - 1}),
\]
52
which is non-negative as $(-1)^{1-s_t}\left(\frac{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)} - \frac{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}\right) = 0$ if $s_t = 1$, and $(-1)^{1-s_t}\left(\frac{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)} - \frac{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}{p(s_1,\ldots,s_{t-1},s_{t+1},\ldots,s_T)}\right) \geq 0$ if $s_t = 0$ due to stochastic affiliation. This proves that $z_T(s_1,\ldots,s_{T-1},1) \geq 0$, which completes the proof of optimality of the primal guess.

The value of the primal and the dual, and the implied information rents and own investment needed for $IFB$ follow from (19) and (20).

**Proof of Proposition 8.** Using the law of iterated expectations, (25) and (26), it is easy to confirm that the participations constraint (7) holds and is binding. The guessed solution is feasible as

$$
E[\tilde{w}(s_1,\ldots,s_t,s_{t+1},s_{t+2},\ldots,s_T) - \tilde{w}(s_1,\ldots,s_t,0,s_{t+2},\ldots,s_T)|s_1,\ldots,s_t] = 
$$

$$
e_t + e_{t+1}E\left[\frac{h_{t+2}}{E[s_{t+2}|s_{t+1},\ldots,s_T]} - \frac{h_{t+2}}{E[s_{t+2}|s_{t+1},0]}|s_1,\ldots,s_t\right] +
$$

$$+ \ldots + e_{T-1}E\left[\frac{h_T}{E[s_T|s_1,\ldots,s_T]} - \frac{h_T}{E[s_T|s_1,0,0,s_{T+2},\ldots,s_T]}|s_1,\ldots,s_T\right]
$$

which satisfies (3) as by the law of total expectations it holds for any $h = 2,\ldots,T - t$ that

$$E\left[\frac{s_{t+h}}{E[s_{t+h}|s_{t},\ldots,s_{t+h-1}]} - \frac{s_{t+h}}{E[s_{t+h}|s_{t},0]}|s_1,\ldots,s_T\right] =
$$

$$= E\left[\frac{s_{t+h}}{E[s_{t+h}|s_{t},s_{t+1},\ldots,s_{t+h-1}]} - \frac{s_{t+h}}{E[s_{t+h}|s_{t},0,s_{t+2},\ldots,s_{t+h-1}]}|s_1,\ldots,s_T\right]
$$

$$= E\left[\frac{s_{t+h}}{E[s_{t+h}|s_{t},s_{t+1},\ldots,s_{t+h-1}]} - \frac{s_{t+h}}{E[s_{t+h}|s_{t},s_{t+1},\ldots,s_{t+h-1}]}|s_1,\ldots,s_T\right],
$$

$E[s_{t+2}|s_1,\ldots,s_t,s_{t+1}+\ldots+s_{t+h}] = E[s_{t+2}|s_1,\ldots,s_t,0,s_{t+2}+\ldots+s_{t+h}]$ if $s_{t+1} = 0$, and by the definition of a "success lowers prospects" environment in (1)

$E[s_{t+2}|s_1,\ldots,s_t,1,s_{t+2}+\ldots+s_{t+h}] \leq E[s_{t+2}|s_1,\ldots,s_t,0,s_{t+2}+\ldots+s_{t+h}]$. This proves that the guessed solution (25) and (26) is feasible. The proof of optimality is trivial as $x_p = 1$ and $x_0 = \ldots = x^{s_1,\ldots,s_T} = 0$ for any $t = 1,\ldots,T - 1$ clearly satisfies (35) and the primal and the dual have the same value.

**B.5 Proof of Proposition 10.**

The observation that the optimal contract can be expressed as an adjusting profit sharing rule follows from the results in Section 2. Given this, we can express the break-even borrower’s problem

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23 The profit sharing rule in this case corresponds to two consecutive equity contracts, as there are no frictions within date intervals $1$ to $T/2$ and $T/2+1$ to $T$, Modigliani and Miller results would imply that many other equivalent within date interval contracts exist as well.
as choosing \( \alpha_{T/2} \) and \( \alpha_T (c_{T/2}) \) for every \( c_{T/2} \) to

\[
\min \alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] + \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) \right]
\]

subject to the participation constraint and IC constraints

\[
\begin{align*}
\alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] + \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) \right] & \geq \frac{T (\bar{e}_0 + \bar{c}_{T/2})}{2} \\
\alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] + \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) \right] - \alpha_T (0) \mathbb{E} \left[ c_T - c_{T/2} \right] & \geq \frac{T \bar{e}_0}{2} \\
\alpha_T (c_{T/2}) \mathbb{E} \left[ \left( c_T - c_{T/2} \right) | c_{T/2} \right] & \geq \frac{T \bar{c}_{T/2}}{2} \text{ for every } c_{T/2}.
\end{align*}
\]

as well as non-negativity constraints \( \alpha_{T/2}, \alpha_T (c_{T/2}) \geq 0 \). For the same reasons as before, the date \( T/2 \) constraints must be binding (the objective function could only be higher otherwise). This implies that \( \alpha_T (c_{T/2}) = \frac{T \bar{e}_{T/2}}{2} \mathbb{E} \left[ (c_T - c_{T/2}) | c_{T/2} \right] \). Furthermore, by the law of iterated expectations it then follows that \( \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) \right] = \mathbb{E} \left[ \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) | c_{T/2} \right] \right] = \frac{T \bar{e}_{T/2}}{2} \). Using this, the above system simplifies to

\[
\begin{align*}
\min \alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] + \frac{T \bar{e}_{T/2}}{2} \text{ st.} \\
\alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] & \geq \frac{T \bar{e}_0}{2} \\
\alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] & \geq \frac{T \bar{e}_0}{2} + \alpha_T (0) \mathbb{E} \left[ c_T - c_{T/2} \right] = \frac{T \bar{e}_0}{2} + \frac{T \bar{e}_{T/2}}{2} \mathbb{E} \left[ c_T - c_{T/2} \right] = \frac{T \bar{e}_{T/2}}{2} \mathbb{E} \left[ c_T - c_{T/2} | c_{T/2} = 0 \right]
\end{align*}
\]

Proposition 10 then follows from comparing the right hand side of the two inequalities above to identify the conditions under which one of the other is more restrictive, and noticing that in this case \( A_0 = \alpha_{T/2} \mathbb{E} \left[ c_{T/2} \right] + \mathbb{E} \left[ \alpha_T (c_{T/2}) \left( c_T - c_{T/2} \right) \right] - \frac{T (\bar{e}_0 + \bar{e}_{T/2})}{2} \).

**B.6 Proof of Corollary 11**

The expressions for \( A_0 \) follow from using the assumed distributions in (21) and (27). In particular notice that cumulative sales \( c_t \equiv \sum_{h=1}^{t} s_h \) in this case has beta-binomial distribution with parameters \((t, \lambda \theta_0, \lambda (1 - \theta_0))\), which implies that \( \mathbb{E} \left[ s_{t+h} | c_t \right] = \frac{\lambda \theta_0 + c_t}{\lambda + t} \) for any \( h \geq 1 \). In order to derive \( A_0^{\text{FREQ}} \), further notice that from (22) and (23), it holds that \( \Delta_t (c_t) = \frac{1}{\lambda \theta_0 + c_t} \) and \( \Phi_t = \bar{e}_T \frac{1-t}{\lambda \theta_0 + c_t} \). As \( \mathbb{E} \left[ s_{t+1} | c_t \right] = \frac{\lambda \theta_0 + c_t}{\lambda + t} \), we obtain \( \mathbb{E} \left[ \Phi_t s_{t+1} \right] = \mathbb{E} \left[ \mathbb{E} \left[ \Phi_t s_{t+1} | c_t \right] \right] = \mathbb{E} \left[ \Phi_t \mathbb{E} \left[ s_{t+1} | c_t \right] \right] = \bar{e}_T \frac{1-t}{\lambda + t} \). For the last part, notice that

\[
\frac{A_0^{\text{FREQ}}}{A_0^{\text{INFREQ}}} = \frac{4}{T^2} \sum_{t=0}^{T-2} \frac{\lambda}{\lambda + t} \left( T - 1 - t \right)
\]
As $\frac{\partial \lambda}{\partial \lambda} = \frac{t}{(\lambda+t)T}$, $A_{0}^{FREQ}$ is monotonically increasing in $\lambda$, $\lim_{\lambda \to 0} A_{0}^{FREQ} = 0$ and $\lim_{\lambda \to \infty} A_{0}^{FREQ} = \frac{4}{\pi^2} \sum_{t=0}^{T-2} (T - 1 - t) = \frac{2(T-1)}{T} > 1$ for any $T > 2$, it follows that there exists $\tilde{\lambda}$ such that $\frac{A_{0}^{FREQ}}{A_{0}^{INFREQ}} < (> ) 1$ if $\lambda > (< ) \tilde{\lambda}$.

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