Incentive constrained risk-sharing, endogenous segmentation and asset pricing

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Risk sharing and collateral

Financial markets: agents invest in/hold risky assets + share risk

Relatively risk tolerant agents insure more risk averse ones: sell CDS, puts, futures, etc.

If agent $i$ sold insurance against state $\omega$, must pay if $\omega$ occurs

If agent $i$ had no resource in that state: counterparty default

To avoid this, agent $i$ must hold assets generating payoff in state $\omega$

These assets back the promise made by $i \rightarrow$ collateral
Endogenous collateral value

- Bank, holding portfolio of loans, sells CDS
  - portfolio of loans = collateral
  - collateral value depends on bank’s monitoring effort
- Venture capitalist issues claims to investors
  - cash flow from ventures = collateral
  - collateral value depends on venture capitalist’s effort
- Broker dealer holds securities, and sells puts
  - securities = collateral
  - deposited with custodian
  - collateral less valuable if custodian unreliable/fraud

No effort $\rightarrow$ collateral less valuable for agent who bought insurance, but no cost of effort/private benefit from shirking for agent who sold insurance
Key economic mechanism

If agent $i$ sold lots of insurance against $\omega$

Tempted to shirk, divert value from collateral $\rightarrow$ reduces collateral value for counterparty

$\rightarrow$ Incentive compatibility condition: state $\omega$ contingent liability not too large, so that no incentive to divert/shirk

$\rightarrow$ Limits risk sharing: market endogenously incomplete
Asset pricing implications

Endogenous segmentation
→ different assets held by different agents
→ different pricing kernels (≠ law of one price, but no arbitrage)
→ comovement among assets held by same class of agents

Equilibrium expected excess return \( (E(R_j) - R_f) \) reflect two premia

1. risk premium: positive if \( R_j \) large when pricing kernel \( M \) low
   \( (M \) does not mirror aggregate consumption, or even individual consumption, due to IC constraints)  
2. divertibility premium: positive if \( R_j \) large when IC binds:
   inverse U shaped with \( \beta \) (SML flat at top)
Introduction Model Equilibrium Two agents’ types Conclusion

Literature (1): GE asset pricing with limited commitment

Similarity:
As in Kehoe Levine (1993), Alvarez Jermann (2000) full set of AD securities + possible strategic default on corresponding liabilities

Difference:
- Literature: non-tradeable asset (human capital) generates fully divertible payoff (labor income) while other assets tradeable, seizable, payoff can’t be diverted
- This paper: tradeable assets with partially divertible payoff → results different from literature: divertibility discount, segmentation

(Literature: cost of default = future exclusion ≠ here: cost of default: diversion partial & inefficient)
**Literature (2): Corporate finance**

At time 0 raise funds to buy asset $\rightarrow$ output at time 1

The incentive problem on which we focus is: cash diversion, as in DeMarzo Fishman (2007), in line with Bolton Scharfstein (1990), and also Townsend (1979)

- $\equiv$ ex-post moral hazard
- $\equiv$ ex-ante moral hazard, Holmstrom Tirole (1997, 98, 2001)

$\rightarrow$ only fraction of output can be credibly promised (pledgeable income, Holmstrom Tirole) $\rightarrow$ IC constraint

Different focus:

- we study risk sharing in an endowment economy (Townsend 1979) $\neq$ corporate finance models study production economy, often risk neutral
- asset pricing and asset allocation (which we can study since assets tradeable)
Assets, markets and agents

Two dates: 0 and 1. State $\omega$ realized at date 1, with proba $\pi(\omega)$

Assets (trees): $j \in [0, 1]$ with payoff (fruits): $d_j(\omega)$

- tree supply $\tilde{N}$ positive measure on $[0, 1]$
  - can be discrete, continuous or both

$I$ types, each in measure 1: type $i$, endowed with $\bar{n}_i$ shares of market portfolio

Concave utility over date-1 consumption $U_i = \sum_\omega \pi(\omega)u_i(c_i(\omega))$

At date 0, can trade trees and complete set of state–$\omega$ contingent Arrow Debreu securities $\rightarrow$ potential for risk–sharing
Investor i’s program

Agent i chooses tree holdings: $N_i$ positive measure over $[0, 1]$ and Arrow securities holdings: $a_i(\omega)$, to max $U_i$ s.t.,
t = 1 bc: consumption = fruits of trees + payoff AD security

\[ c_i(\omega) = \int_j d_j(\omega) dN_{ij} + a_i(\omega) \]

t = 0 bc: initial endowment ≥ portfolio held (trees + AD)

\[ \int_j p_j d\tilde{N}_{ij} \geq \int_j p_j dN_{ij} + \sum_\omega q(\omega) a_i(\omega) \]

→ intertemporal/consolidated bc:

\[ \int_j p_j d\tilde{N}_{ij} + \sum_\omega q(\omega) \int_j d_j(\omega) dN_{ij} \geq \int_j p_j dN_{ij} + \sum_\omega q(\omega) c_i(\omega). \]

and IC !!
Incentive compatibility constraint

Instead of holding promises, agent can strategically default and abscond with fraction $\delta$ of trees’ payoff

IC: $c_i(\omega)$ if hold promises $\geq$ if diversion-counterparty default

$$c_i(\omega) \geq \delta \int_j d_j(\omega) dN_{ij} \iff (1 - \delta) \int_j d_j(\omega) dN_{ij} \geq -a_i(\omega).$$

Pledgeable state-$\omega$ fruits $\geq$ state $\omega$–contingent liability

- If $a_i(\omega) \geq 0$: IC slack
- If $a_i(\omega) < 0$: debt overhang $\rightarrow$ strategic default tempting
Interpreting IC in terms of collateral and pledgeable income

Assets held by agent = collateral for AD securities issued → not fully pledgeable ($\delta > 0$)

Payment promised by agent $i$ in state $\omega \leq$ pledgeable income from $i$’s collateral

Not mandated by regulation, requested by market counterparties

Implemented by CCP (aggregate collateral requirement for entire portfolio // portfolio margining)
Equilibrium

Consumption plans $c_i(\omega)$ and tree holdings $N_i$

Prices for Arrow securities $q(\omega)$ and trees $p_j$

s.t.

Agent maximize given price and budget and IC constraint

Markets clear

$$\sum_{i \in l} c_i(\omega) = \sum_{i \in l} \int_j d_j(\omega) dN_{ij}$$

$$\sum_i a_i(\omega) = 0$$

$$\sum_i N_i = \bar{N}$$
Welfare theorem and existence

Constrained Pareto optimality:
  no price in IC $\implies$ standard proof of welfare theorem

Existence: follows by variation of Negishi proof
  planner's solution, implementable without transfer

Uniqueness: obtains with two CRRA types with $\gamma \leq 1$
First order condition with respect to consumption

\[ \pi(\omega) u'_i(c_i(\omega)) - \lambda_i q(\omega) + \mu_i(\omega) = 0, \text{ if } c_i(\omega) > 0 \]

where \( \lambda_i = \) multiplier of BC and \( \mu_i(\omega) = \) multiplier of IC

If IC slack, MRS equal across agents // pricing kernel \( M \)

\[ \frac{u'_i(c_i(\omega_1))}{u'_i(c_i(\omega_2))} = \left( \frac{q(\omega_1)}{\pi(\omega_1)} \right) / \left( \frac{q(\omega_2)}{\pi(\omega_2)} \right) = \frac{M(\omega_1)}{M(\omega_2)} \]

If IC binds: \( (\mu_i(\omega) > 0) \) wedge between agents MRS: imperfect risk–sharing due to IC constraint \( \rightarrow \) AD securities pricing kernel reflects agent’s marginal utility \( u'_i(c_i(\omega)) \) and shadow cost of IC

\[ M(\omega) = \left[ \frac{1}{\lambda_i} \frac{\partial u_i(c_i(\omega))}{\partial c_i(\omega)} + A_i(\omega) \right], \text{ if } c_i(\omega) > 0, \]

where \( A_i(\omega) = \frac{\mu_i(\omega)}{\lambda_i \pi(\omega)} \) is the shadow cost of IC
Imperfect risk-sharing

Agent 2 wants to buy insurance from Agent 1 against state $\omega$

But constrained by IC ($\mu_1(\omega) > 0$): It is not IC to transfer too much consumption from Agent 1 to Agent 2

$\implies c_1(\omega)$ higher and $c_2(\omega)$ lower than under perfect risk-sharing

First order condition with respect to holdings of trees

If tree $j$ held by agent $i$

$$p_j = E \left[ M(\omega) d_j(\omega) - A_i(\omega) \delta d_j(\omega) \right]$$

$1^{st}$ term: asset's cash flows, valued at pricing kernel $M(\omega)$

$2^{nd}$ term: shadow cost of IC when buying asset $j // A_i(\omega)$

→ Divertibility discount: $p_j < E \left[ M(\omega) d_j(\omega) \right]$
→ AD securities & trees priced by $\neq$ pricing kernels
→ $\neq$ trees held by $\neq$ agents $\rightarrow$ priced by $\neq$ kernels

Not arb opportunity: Arb $\rightarrow$ buy the asset $\rightarrow$ hit IC constraint

Asset's payoff large when asset holder IC binds $\rightarrow$ large discount
Discount versus premium


No contradiction, different benchmarks

Collateral premium: Asset price > value for agent if she consumed all its cash flows: \( E[u'_i(c_i(\omega))d_j(\omega)] \)

Divertibility discount: Asset price < price of replicating portfolio of AD securities \( E[M(\omega)d_j(\omega)] \)

Substituting FOC wrt consumption into FOC wrt holdings (which implied “divertibility discount”), we get “collateral premium”

\[
p_j = \frac{1}{\lambda_i} E \left[ \frac{1}{\lambda_i} u'_i(c_i(\omega))d_j(\omega) + A_i(\omega)(1 - \delta)d_j(\omega) \right]
\]
Equilibrium holdings

Marginal valuation of agent $i$ for asset $j$

$$v_{ij} = E [M(\omega)d_j(\omega) - A_i(\omega)\delta d_j(\omega)]$$

First term: $E [M(\omega)d_j(\omega)] = "common value"$

Second term: $-E [A_i(\omega)\delta d_j(\omega)] = "endogenous private value"$

$v_{ij} = p_j$ iff agent $i$ holds tree $j$, $v_{ij} < p_j$ iff $i$ does not hold $j$

Agents who hold asset are those who value it the most, because they have the lowest shadow price of holding it
Equilibrium expected excess returns

If agent $i$ holds asset $j$, FOC wrt holdings

$$p_j = E \left[ M(\omega)d_j(\omega) - A_i(\omega)\delta d_j(\omega) \right]$$

Define risky return: $R_j(\omega) = \frac{d_j(\omega)}{p_j}$, risk-free return: $R_f = \frac{1}{E[M(\omega)]}$

Equilibrium excess return

$$E \left[ R_j(\omega) \right] - R_f = -R_f \text{Cov}(M(\omega), R_j(\omega)) + R_f E[A_i(\omega)\delta R_j(\omega)]$$

1$^{st}$ term: risk premium, positive if $R_j(\omega)$ large when $M(\omega)$ low

2$^{nd}$ term: divertibility premium, positive if divertible income $\delta R_j(\omega)$ large when IC binds (for agents holding the asset)
Interpreting risk premium

Equilibrium expected excess return

\[ E[R_j(\omega)] - R_f = -R_f \text{Cov}(M(\omega), R_j(\omega)) + R_f E[A_i(\omega)\delta R_j(\omega)] \]

looks like standard risk-premium obtained in frictionless market

\[ E[R_j(\omega)] - R_f = -R_f \text{Cov}(M(\omega), R_j(\omega)) \]

(see, e.g., Huang and Litzenberger (1988) equation 6.2.8)

Unlike frictionless CCAPM, \( M(\omega) \) does not mirror aggregate consumption (not even individual consumption):

IC prevents perfect risk-sharing \( \rightarrow \) endogenous incompleteness
Interpreting divertibility premium

2\textsuperscript{nd} “factor” in equilibrium expected excess return, when \( n_{ij} > 0 \)

\[
E[R_j(\omega)] - R_f = -R_f \text{Cov}(M(\omega), R_j(\omega)) + R_f E[A_i(\omega)\delta R_j(\omega)]
\]

Varies across assets because \( \neq \) assets held by \( \neq \) agents:

Endogenous segmentation reflecting \( \neq \) risk-aversion, inducing \( \neq \) IC constraints

// “intermediary asset pricing”: pricing reflects characteristics of institution holding assets (// He, Kelly, Manela (2016) “financial intermediaries are marginal investors in many markets”)

But differences:

- \textit{endogenous} segmentation
- price \( \neq \) marginal utility of holder
- \( \neq \) assets held by \( \neq \) institutions \( \implies \) shocks to different institutions affect different assets differently
More explicit results in simpler case

So far: general model → economic intuitions, but pricing intertwined with allocation of assets
In general case, hard to characterize explicitly and simply

- 2 states: aggregate dividend larger in state $\omega_2$ than in $\omega_1$
- 2 agent’s types: Type-1 less risk averse than 2 (in any unconstrained Pareto optimum, consumption share of 2 decreasing with aggregate dividend, e.g., CRRA $\gamma_1 < \gamma_2$)

$\Rightarrow$ In first best, type-1 would insure 2 against risk of bad state 1

$\Rightarrow$ IC$_1$ binds in $\omega_1$ ($A_1(\omega_1) > 0$), IC$_2$ in $\omega_2$ ($A_2(\omega_2) > 0$)
Consumption beta

Continuum of trees indexed by \( j \in [0, 1] \)

If tree \( j \) such that \( d_j(\omega_2) \) large relative to \( d_j(\omega_1) \)

\[ \rightarrow \text{payoff large in good state } \omega_2 \text{ relative to bad state } \omega_1 \]

\[ \rightarrow \text{large consumption } \beta \]
Equilibrium segmentation

\[ \exists B, \text{ s.t.} \]

- risk-tolerant type 1 hold high consumption $\beta$ trees: \( \frac{d_j(\omega_2)}{d_j(\omega_1)} > B \)
- risk-averse type 2 hold low consumption $\beta$ trees: \( \frac{d_j(\omega_2)}{d_j(\omega_1)} < B \)

First best:

- risk tolerant agent 1 insures agent 2 against bad state $\omega_1$ → agent 2 consumes relatively more than 1 in bad state $\omega_1$
- risk sharing can be engineered by asset holdings and by AD security trading → holdings indeterminate

With IC constraint:

- if risk tolerant agent 1 held assets paying a lot in bad state, tempted to divert
- to minimize diversion temptation, allocate assets with low consumption $\beta$ to risk averse agent 2
Equilibrium asset prices in simple case

Asset $j > k$, held by risk-tolerant agent 1 → divertibility discount
reflects shadow price of agent 1’s IC (binds in bad state $\omega_1$)

$$p_j = E \left[ M(\omega) d_j(\omega) \right] - A_1(\omega_1) \delta d_j(\omega_1)$$

Asset $j < k$, held by risk-averse agent 2 → divertibility discount
reflects shadow price of agent 2’s (binds in good state $\omega_2$)

$$p_j = E \left[ M(\omega) d_j(\omega) \right] - A_2(\omega_2) \delta d_j(\omega_2)$$
Divertibility discount and beta

Consumption $\beta$ increases as $d_j(\omega_2)$ increases & $d_j(\omega_1)$ decreases

Among assets held by more risk-tolerant agent 1 (which, to start with, tend to have high $\beta$)

large $\beta$ (low dividend in state in which IC1 binds, $d_j(\omega_1)$) → low discount $A_1(\omega_1)\delta d_j(\omega_1)$

Similarly, among assets held by agent 2 (which, to start with, tend to have low $\beta$)

large $\beta$ (high $d_j(\omega_2)$) → high discount $A_2(\omega_2)\delta d_j(\omega_2)$

→ divertibility discount inverse U-shaped with (consumption) $\beta$: smallest for very low $\beta$ and very high $\beta$, largest for intermediary $\beta$
In terms of expected returns

In terms of prices: low divertibility discount for high $\beta$ assets

→ In terms of expected returns: low divertibility *premium* for high $\beta$ assets

→ SML flat at top (Black, 1972)

Also low divertibility premium for low $\beta$ assets, divertibility premium inverse-U shaped with $\beta$

Consistent with Hong and Sraer (2016), Frazzini Pedersen (2010)
Suppose that, for asset $j$ held by agent $i$, $\delta$ increased by $\epsilon$:

Impact on segmentation and holdings: $IC_i$ tightens $\rightarrow$ set of assets held by $i$ shrinks

Impact on pricing:

- $j$’s divertibility discount increases
- prices of other assets held by $i$ decrease relative to prices of assets held by $-i$ (because $i$’s shadow cost of $IC$ increases more than $-i$’s)

Shock to intermediaries $i$ affects equilibrium prices (He, Kelly, Manela, 2016), especially those of assets held by $i$
Wealth effects

Consider what happens as agent 1’s initial endowment ($\bar{N}_1$) varies relative to that of agent 2.

If agent 1 endowed with all the wealth $\rightarrow$ consumes everything both in unconstrained and constrained equilibrium.

By continuity FB still implementable when agent 1’s initial endowment large relative to agent 2’s.

Symmetrically, FB implementable when agent 2 very rich.

Incentive constraints bind in the middle, when agent 1 neither very poor nor very rich.

Divertibility discount inverse U shaped in agent 1’s initial wealth.

(Agent 1, who sells insurance, can be interpreted as financial intermediary.)
Conclusion

Simple one period general equilibrium asset pricing model + standard corporate finance friction (cash-diversion) →

- Endogenous market incompleteness
- Endogenous market segmentation → comovement
- Equilibrium expected excess returns $(E(R_j) - R_f)$ reflect two premia:
  - Risk premium: positive if $R_j$ large when pricing kernel low (pricing kernel does not reflect aggregate consumption, or even individual consumption, due to IC constraints)
  - Divertibility premium: positive if divertible income $\delta R_j$ large when IC binds
- Divertibility premium U shaped with $\beta$ (SML flat at top)