Macro-Finance Models of Interest Rates and the Economy

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Overview of this lecture

1. Macro in a finance model
2. Bond pricing in a macro model
3. AFNS model
Overview of this lecture

Two different views of the term structure: finance and macro
What is the synergy from combining approaches and models?
What are the advantages from cross-fertilization?

Three broad research directions:
1. Add macro to the finance NA model
   — Application (1): Great moderation
   — Application (2): Bond "conundrum"
2. Bond pricing in a DSGE model
3. AFNS model
   — Application (1): Estimating inflation risk premiums
   — Applications (2) & (3): Liquidity and Stochastic volatility
Macro and finance views of the term structure

Modeling short-term nominal interest rates ($i_t$ or $r_t$):
- Finance view: short rate is sum of latent factors
- Macro view: short rate is set by central bank

Modeling long-term nominal interest rates ($i_t(\tau)$ or $y_t(\tau)$):
- Macro view: long rates are determined by expectations of future short rates; i.e., the expectations hypothesis of the term structure
- Finance view: movements in long rates largely reflect changes in risk premiums
Observed state variables $X_t$ drive economy and yields:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t$$

Short-term interest rate is set according to monetary policy rule:

$$i_t = \delta_0 + \delta_t X_t$$

Longer-term yields are determined by expectations hypothesis:

$$y_t(n) = \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}$$
Latent (unobserved) state variables $X_t$ drive yields:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t$$

Short-term interest rate is linear function of state variables:

$$i_t = \delta_0 + \delta'_t X_t$$

Price of risk $\lambda_t$ is linear function of state variables $X_t$:

$$\lambda_t = \lambda_0 + \lambda_1 X_t$$

Longer-term zero-coupon yields are linear in $X_t$:

$$y_t(n) = \frac{1}{n}(a_n + b'_n X_t)$$

$$a_{n+1} = \delta_0 + a_n + b'_n(\mu - \Sigma \lambda_0) - b'_n \Sigma \Sigma' b_n$$

$$b_{n+1} = \delta_1 + (\Phi - \Sigma \lambda_1)' b_n$$
Key questions for modeling yields


- How should yield factors be constructed? (observable, latent, macro)
- How much macro structure should be employed? (VAR, New Keynesian, DSGE)
- What links between macro variables and yield curve factors? (unidirectional, bidirectional)
- How useful are the no-arbitrage modeling restrictions?
- What is the appropriate specification of the term premium?

Features of this analysis:
- Three yield curve factors (level, slope, and curvature from Nelson-Siegel fitted curve)
- Three macro variables (capacity utilization, price inflation, fed funds rate)
- A six-variable VAR with no macro or finance structure
- Finds significant bidirectional links between macroeconomic variables and the yield curve factors.
Three broad research directions

1. Add macro to the finance model
Add macroeconomic variables and structure to the canonical finance affine no-arbitrage term structure model.

2. Bond pricing in a DSGE model
Examine bond pricing in a macroeconomic dynamic, stochastic general equilibrium (DSGE) model. Stochastic discount factor is derived from explicit utility optimization.

3. AFNS model
Impose Nelson-Siegel factor loading structure on affine no-arbitrage term structure model. Resulting AFNS model is easy to estimate and a useful framework for macro-finance extensions.
1. Add macro to the finance model

Research estimating macro-finance models:


Rudebusch-Wu macro-finance model

Short rate is sum of two latent term structure factors:

\[ i_t = \delta_0 + L_t + S_t, \]

Monetary policy—through a Taylor rule—connects these yield curve factors to capacity utilization \((y_t)\) and inflation \((\pi_t)\).

**Level** is perceived inflation objective of the central bank.

\[ L_t = \rho_L L_{t-1} + (1 - \rho_L) \pi_t + \varepsilon_{L,t} \]

**Slope** is set by central bank to achieve output and inflation stabilization:

\[ S_t = g_y y_t + g_\pi (\pi_t - L_t) + u_{S,t} \]

\[ u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}, \]
New Keynesian structure drives macro variables (adjusted for monthly data and term structure elements):

\[ \pi_t = \mu_\pi L_t + \left(1 - \mu_\pi\right)\left[\alpha_{\pi 1} \pi_{t-1} + \alpha_{\pi 2} \pi_{t-2}\right] + \alpha_y y_{t-1} + \varepsilon_{\pi, t} \]

\[ y_t = \mu_y E_t y_{t+1} + \left(1 - \mu_y\right)\left[\beta_{y 1} y_{t-1} + \beta_{y 2} y_{t-2}\right] - \beta_r (i_{t-1} - L_{t-1}) + \varepsilon_{y, t} \]

Standard no-arbitrage formulation of yield curve:

\[ \Lambda_t = \lambda_0 + \lambda_1 X_t; \quad y_t(n) = \frac{1}{n} (a_n + b'_n X_t) \]

Estimation sample: January 1988 to December 2000

Using data on 1-, 3-, 12-, 36-, and 60-month U.S. Treasury yields, capacity utilization, and price inflation
RW model: Impulse responses to output shock

![Graph showing impulse responses to output shock. The top graph plots output over months, with a dashed line indicating a downward trend. The bottom graph plots inflation, 1-month rate, 12-month rate, and 5-year rate over months.]
RW model: Impulse responses to level shock

Great Moderation in macroeconomic volatility since 1980s.

Several possible reasons given in the literature:

- Good policy (especially monetary policy)
- Good luck (a run of smaller shocks—until recently?)
- Structural change (e.g., inventory management)

Is moderation in macro volatility also reflected in a less volatile term premium?

Can macro-finance model account for this shift?
Have interest rate risk dynamics also changed?

Model suggests lower risk on Fed’s inflation objective

“Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience ... For the moment, the broadly unanticipated behavior of world bond markets remains a *conundrum*.”

Fed Chairman Alan Greenspan, February 16, 2005
Was there a bond yield "conundrum"?

RW Model Residuals for 10-Year Yield

40-50 bp
conundrum
-60
-40
-20
0
20
40
60
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
basis points
Why was there a bond yield "conundrum"?

List of factors that may have held down long-term yields:

1. Increased demand by foreign central banks and pension funds
2. Minimal inflation risk; greater monetary policy transparency
3. Low economic growth volatility
4. Excess global savings
5. "Reaching for yield"

"The boom in subprime mortgage lending was only a part of a much broader credit boom characterized by an underpricing of risk, excessive leverage, and the creation of complex and opaque financial instruments that proved fragile under stress. The unwinding of these developments is the source of the severe financial strain and tight credit that now damp economic growth."

Fed Chairman Ben Bernanke, October 31, 2008
Three broad research directions

1. **Add macro to the finance model**
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2. **Bond pricing in a DSGE model**
Examine bond pricing in a macroeconomic dynamic, stochastic general equilibrium (DSGE) model. Stochastic discount factor is derived from explicit utility optimization.

3. **AFNS model**
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2. Bond pricing in a DSGE model

Derive risk premium from utility in a production economy.

Long memory habit utility:


Epstein-Zin utility:

Rudebusch, Glenn D., and Eric Swanson, 2008b, "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," manuscript.
Why study asset prices in a DSGE model?

Asset pricing is important:

- DSGE models increasingly used for policy analysis; total failure to explain asset prices may signal flaws in the model
- many empirical questions about asset prices require a structural DSGE model to provide reliable answers

Equity prices have received much attention in the literature. But bond prices are at least as interesting because they:

- apply to a larger amount of securities
- provide an additional perspective on the model
- test nominal rigidities in the model
- model short-term interest rate process, not dividends
Recent studies of the bond premium puzzle

Wachter, 2005)
- can resolve bond premium puzzle using Campbell-Cochrane preferences in endowment economy

Rudebusch and Swanson (2008a)
- the term premium is far too small in a standard New Keynesian model, even with Campbell-Cochrane habits
- similar finding by Jermann (1998, Lettau and Uhlig, 2000) for equity premium in an RBC model

Piazzesi and Schneider (2007)
- can resolve bond premium puzzle using Epstein-Zin preferences in endowment economy

Rudebusch and Swanson (2008b) examine if Piazzesi-Schneider results generalize to a DSGE model and production economy.
Rudebusch-Swanson (2008b) analysis

Incorporate Epstein-Zin preferences in standard DSGE model in which households and firms solve explicit optimization problems and form rational expectations in the face of fundamental shocks.

The model has three key ingredients:

1. Intrinsic nominal rigidities
   - makes bond pricing interesting

2. Epstein-Zin preferences
   - makes households risk averse

3. Long-run risk (productivity or inflation)
   - introduces a risk households cannot offset
   - makes bonds risky
Epstein-Zin preferences

Standard preferences:

\[ V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1} \]

Epstein-Zin preferences:

\[ V_t \equiv u(c_t, l_t) + \beta (E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)} \]

With Epstein-Zin preferences, risk aversion can be modeled independently from the intertemporal elasticity of substitution.

We use standard NK utility kernel:

\[ u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \]
Bond pricing

Epstein-Zin stochastic discount factor (nominal):

\[ m_{t,t+1} \equiv \frac{\beta u_1 \mid_{(c_{t+1},l_{t+1})}}{u_1 \mid_{(c_t,l_t)}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^\alpha \frac{P_t}{P_{t+1}} \]

Zero-coupon nominal bond pricing:

\[ p_t^{(n)} = E_t[m_{t+1} p_t^{(n-1)}] \]

\[ y_t(n) = -\frac{1}{n} \log p_t^{(n)} \]

Perceived inflation objective ($\pi^*_t$) varies over time:

$$\pi^*_t = \rho^*_\pi \pi^*_{t-1} + (1 - \rho^*_\pi) \theta^* \pi^* (\pi_t - \pi^*_t) + \varepsilon^*_t$$

The dependence of the inflation target on inflation makes long-term bonds more risky.
Specifically, after a technology/supply shock, $\pi \uparrow$, $C \downarrow$, and $p^{(10)} \downarrow$; therefore, the bond loses value when consumption is low.

Can this model match the unconditional moments of bond prices and macroeconomic variables?
Definitions of unconditional moments matched

<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>Real consumption*</td>
</tr>
<tr>
<td>sd[L]</td>
<td>Labor, total hours worked*</td>
</tr>
<tr>
<td>sd[w']</td>
<td>Real wage*</td>
</tr>
<tr>
<td>sd[π]</td>
<td>Price inflation, Annualized quarterly rate</td>
</tr>
<tr>
<td>sd[i^{(10)}]</td>
<td>10-year zero-coupon nominal rate, annualized p.p.</td>
</tr>
<tr>
<td>mean[ψ^{(10)}]</td>
<td>Term premium on 10-year zero-coupon bond</td>
</tr>
<tr>
<td>sd[ψ^{(10)}]</td>
<td>(affine no-arbitrage estimates)</td>
</tr>
<tr>
<td>mean[i^{(10)} − i]</td>
<td>Yield curve slope</td>
</tr>
<tr>
<td>sd[i^{(10)} − i]</td>
<td>(long - short rate, annualized p.p.)</td>
</tr>
<tr>
<td>mean[x^{(10)}]</td>
<td>Quarterly excess holding period return</td>
</tr>
<tr>
<td>sd[x^{(10)}]</td>
<td>(10-year bond, annualized p.p.)</td>
</tr>
</tbody>
</table>

*deviations from HP trend in percentage points
Moments with long-run inflation risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data 1961-2007</th>
<th>EU Preferences</th>
<th>Best Fit EZ Prefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>1.19</td>
<td>1.92</td>
<td>1.86</td>
</tr>
<tr>
<td>sd[L]</td>
<td>1.71</td>
<td>3.33</td>
<td>1.73</td>
</tr>
<tr>
<td>sd[w']</td>
<td>0.82</td>
<td>2.55</td>
<td>1.45</td>
</tr>
<tr>
<td>sd[π]</td>
<td>2.52</td>
<td>5.00</td>
<td>3.22</td>
</tr>
<tr>
<td>sd[i]</td>
<td>2.71</td>
<td>4.74</td>
<td>2.99</td>
</tr>
<tr>
<td>sd[r]</td>
<td>2.30</td>
<td>2.61</td>
<td>1.48</td>
</tr>
<tr>
<td>sd[i^{(10)}]</td>
<td>2.41</td>
<td>3.32</td>
<td>1.94</td>
</tr>
<tr>
<td>mean[ψ^{(10)}]</td>
<td>1.06</td>
<td>.002</td>
<td>.748</td>
</tr>
<tr>
<td>sd[ψ^{(10)}]</td>
<td>0.54</td>
<td>.001</td>
<td>.431</td>
</tr>
<tr>
<td>mean[i^{(10)} − i]</td>
<td>1.43</td>
<td>−.062</td>
<td>.668</td>
</tr>
<tr>
<td>sd[i^{(10)} − i]</td>
<td>1.33</td>
<td>1.60</td>
<td>1.11</td>
</tr>
<tr>
<td>mean[x^{(10)}]</td>
<td>1.76</td>
<td>.003</td>
<td>.737</td>
</tr>
<tr>
<td>sd[x^{(10)}]</td>
<td>23.43</td>
<td>16.96</td>
<td>11.83</td>
</tr>
</tbody>
</table>

memo: quasi-CRRA 2 65
Overview of DSGE model results with long-run risk

1. Epstein-Zin preferences appear to solve bond premium puzzle in DSGE model, as in an endowment economy. Agents are risk-averse and cannot offset long-run real or nominal risks.

2. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments.

3. Unresolved issues:
   - Reliance on technology shocks, not $\pi^*$ shocks
   - Fitting more moments, estimation from data
   - Is quasi-CRRA appropriate measure of risk aversion?
   - Little feedback from asset prices to economy
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Arbitrage-Free Nelson-Siegel term structure model

Introduce the AFNS model:


Generalize AFNS model (à la Svensson extension):

Motivation for AFNS model

Two approaches to modeling the term structure of bond yields:

Affine arbitrage-free term structure model
- Very popular among finance researchers.
- Applies theoretical restriction prohibiting riskless arbitrage.
- Major shortcoming: Difficult to estimate and evaluate.

The Nelson-Siegel yield curve
- Popular among practitioners and central banks.
- Intuitive, three factors: level, slope, curvature.
- Easy to estimate. Good fit and forecasting performance.
- Major shortcoming: NS does not prohibit arbitrage.

Can we get the best of both worlds?
Severe estimation difficulties in implementation.

Key problem: Multiple likelihood maxima with close to identical likelihood values but very different yield decompositions.

Consequence: Two researchers may arrive at different estimation results despite using *same* data, *same* model and *same* estimation method. See Kim and Orphanides (2005).

Duffee (2008): "Overcoming these difficulties requires a fairly elaborate hands-on estimation procedure."
The Nelson-Siegel yield curve assumes that zero-coupon bond yields take the following functional form:

\[
y(\tau) = \beta_1 + \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \beta_2 + \left[ \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right] \beta_3.
\]

In Nelson and Siegel (1987) \(\beta_1, \beta_2, \beta_3\) and \(\lambda\) are constants fitted to the cross-section of yields on a single day.

In Diebold and Li (2006), the three \(\beta\)'s are given a dynamic interpretation:

\[
y_t(\tau) = L_t + \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) S_t + \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right) C_t
\]

as level, slope, and curvature factors.
The arbitrage-free Nelson-Siegel (AFNS) model

**Proposition:** If the risk-free rate is defined by

\[ r_t = X_t^1 + X_t^2 \]

and the \( Q \)-dynamics of \((X_t^1, X_t^2, X_t^3)\) are given by

\[
\begin{pmatrix}
    dX_t^1 \\
    dX_t^2 \\
    dX_t^3
\end{pmatrix} =
\begin{pmatrix}
    0 & 0 & 0 \\
    0 & \lambda & -\lambda \\
    0 & 0 & \lambda
\end{pmatrix}
\left[
\begin{pmatrix}
    \theta_1^Q \\
    \theta_2^Q \\
    \theta_3^Q
\end{pmatrix} -
\begin{pmatrix}
    X_t^1 \\
    X_t^2 \\
    X_t^3
\end{pmatrix}
\right] dt + \sum dW_t^Q,
\]

then zero-coupon yields are given by

\[
y_t(\tau) = X_t^1 + \frac{1 - e^{-\lambda\tau}}{\lambda\tau} X_t^2 + \left[ \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right] X_t^3 - \frac{A(\tau)}{\tau}.
\]
Add essentially affine risk premium specification, Duffee (2002):

\[ \Gamma_t = \gamma^0 + \gamma^1 X_t. \]

Results in real-world yield curve $P$-dynamics given by

\[ dX_t = K^P(\theta^P - X_t)dt + \Sigma dW^P_t. \]

Maximally flexible AFNS model has 19 parameters (3 restrictions).

The AFNS model provides the best of both worlds:

- theoretical consistency
- easy to estimate
- good fit
- good forecast performance
Christensen, Jens, Jose Lopez, and Glenn D. Rudebusch, 2008a, "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," manuscript.

Prices of nominal and real bonds must satisfy

\[ P_t^N(\tau) = E_t^P \left[ \frac{M_{t+\tau}^N}{M_t^N} \right] \quad \text{and} \quad P_t^R(\tau) = E_t^P \left[ \frac{M_{t+\tau}^R}{M_t^R} \right]. \]

Price level is ratio of nominal and real stochastic discount factors:

\[ Q_t = M_t^R / M_t^N. \]

Breakeven inflation (BEI) rate is difference between nominal and real yields and sum of expected inflation and inflation risk premium:

\[ BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau). \]
AFNS model-implied 5-Year expected inflation

Time
Rate
Model-implied BEI rate
Model-implied expected inflation
Survey-based inflation forecast

Graph showing the time series of inflation rates with different data points and lines representing model-implied and survey-based forecasts from 2003 to 2008.
Christensen, Jens, Jose A. Lopez, and Glenn D. Rudebusch, 2008b, Do Central Bank Liquidity Facilities Affect Interbank Lending Rates?, manuscript.

Christensen, Jens, Jose A. Lopez, and Glenn D. Rudebusch, 2008c, Stochastic Volatility in the Affine Arbitrage-Free Nelson-Seigel Term Structure Model, manuscript.
Summary: What have we learned?

Important gains to trade across macro and finance research. Important synergies strategies, models, questions, and answers.

Three fruitful research directions:

1. Add macro to the finance NA model
2. Bond pricing in a DSGE model
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Further macro-finance research (current credit squeeze?)