On the merits of conventional vs unconventional fiscal policy*

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Abstract

Recent influential work argue that the announcement of gradual sales tax hikes can be stimulative in long-lived liquidity traps by boosting inflation expectations. Higher public investments (e.g. in infrastructure) should also be more expansive than in normal times by raising the potential interest rate, aggregate demand and inflation expectations. We analyze the relative merits of these policy instruments in a stylized sticky price model without private capital, and in a workhorse New Keynesian model with endogenous private capital and financial frictions. Our key finding is that the favourable effects of sales tax hikes are not robust across various model specifications, whereas the benefits of higher public infrastructure investments are more robust across model environments. We therefore conclude that fiscal policy should consider public investment opportunities and not merely rely on tax policies to stimulate growth in a liquidity trap.

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1. Introduction

Keynes argued in favor of aggressive fiscal expansion during the Great Depression on the grounds that the fiscal multiplier was likely to be much larger in a liquidity trap than in normal times, and the financing burden correspondingly smaller. In today's environment, in which growth in many advanced economies is expected to remain subdued in a foreseeable future, rates of price and wage inflation is low or even absent, and equilibrium real rates are close to or even at record-low levels, there is again a strong case to be made for fiscal stimulus when monetary policy is constrained by its effective lower bound (see e.g. the discussion in Gaspar et al., 2016).

However, the ability and political will to pursue fiscal stimulus have been held back by the elevated post-crisis debt levels. Given the existing high debt levels, and the continued headwinds on public finances due to subdued projected growth rates and unfavorable future demographic developments, any sizeable fiscal stimulus must be near or completely self-financed.

In this context, the recent academic literature has promoted a new type of tax-based policy which may stimulate growth while being self-financed. In order to distinguish it from the conventional fiscal policy advocated by Keynes that is spending-based, this strategy has been referred to as unconventional fiscal policy. It builds on the important work by Correia et al. (2013) and a key ingredient in it is a gradually higher path of the sales tax. A credible commitment to a higher future sales tax boosts domestic demand by reducing the wedge between the actual and the potential real rate; it increases the equilibrium real rate and lowers the actual real rate through higher inflation and inflation expectations. By the consumption Euler equation, this policy thus increases consumption of households today. Moreover, by boosting economic activity this strategy also increase tax revenues (through higher tax rates and expanding the tax bases), shrink the public deficit and reduce government debt as share of GDP.

Another strategy which has received considerable attention (see e.g. Bussiere et al., 2017, and Bouakez et al., 2017) is conventional fiscal policy in the form of higher public infrastructure spending. The beneficial premise of such a strategy is that it combines the benefits of providing higher demand when the economy is experiencing low growth and increases potential output (to the extent that higher public spending elevates the effective capital stock) when the economy recovers from the slump. Thus, a properly sized infrastructure spending bill could thus provide notable stimulus in both the near- and medium-term and be fully – or nearly – self-financed.

As the empirical evidence of the two policy options are scant in a liquidity trap, we investigate
the robustness of two strategies using New Keynesian DSGE models. Although we are completely sympathetic to examine the merits of these policies in more empirically oriented frameworks, we note that data limitations (lack of episodes) makes such an exercise problematic.\footnote{D’Acunto et al. (2016) examine the effects of the announced VAT hike in Germany 2007, and de Michelis and Iacoviello (2016) examines the one-time VAT hike in Japan. However, there are few or non-existing cases of credible gradual hikes in the sales tax during long-lived liquidity traps. Moreover, there are few episodes with large changes in public infrastructure spending in liquidity traps.} Our starting point is that the gains of policies that are pursued in practice should be robust across different models, and should not be sensitive to the specifics of a given model. In this vein, we begin our analysis in a variant of the simple benchmark NK model of Eggertsson and Woodford (2003) with a fixed private capital stock. We use this model to study the effects on output and government debt of gradual sales tax hikes and increases in public infrastructure investment. Following Leeper, Walker and Yang (2010), we assume that it takes 1-6 years to complete government investment projects and that the efficiency by which public capital adds to the overall capital stock is limited. Hence, our results are not driven by unrealistic assumptions of speed and size by which higher public investments add to the effective capital stock.

Next, we move on to examining the robustness of the results in a more empirically-realistic model. In particular, we utilize a model that is similar to the estimated models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), but also incorporates “Keynesian” hand-to-mouth agents and financial frictions. The inclusion of some hand-to-mouth households enables this model to explain the evidence of a substantial response of household spending to the temporary U.S. tax rebates of 2001 and 2008, documented by Johnson, Parker, and Souleles (2006) and Parker et al. (2011) using micro data from the Consumer Expenditure Survey. to capture the evidence by Johnson et al. (2006) and Parker et al. (2011). Debortoli and Galí (2017) argues that our two-agent New Keynesian (TANK henceforth) model captures some key aspects of the dynamics in more fully-fledged HANK models (see e.g. Kaplan et al., 2016). Moreover, as argued by Galí, López-Salido, and Vallés (2007), the inclusion of Keynesian households can help to account for the positive response of private consumption to a government spending shock documented in structural VAR studies by e.g., Blanchard and Perotti (2002) and Perotti (2007); more generally, “Keynesian” hand-to-mouth agents and financial frictions elevates the multiplier by amplifying the response of the potential real interest rate.

Our main findings are as follows. First, we find that the beneficial effect of a gradual increase in the sales tax is not robust across various model specifications unless labor income taxes are adjusted aggressively simultaneously to stabilize the budget surplus. This finding suggests that the benefits
of unconventional fiscal policy is contingent on a “grand bargain” involving adjusting several tax rates simultaneously. This is politically hard to achieve, and may hence be a risky strategy.

Second, conventional fiscal policy, in the form of higher public infrastructure spending (roads, public transportation, military spending etc.) has robustly benign effects across all variations of the models. The beneficial premise of such a strategy is that it combines the benefits of providing higher demand when the economy is in the liquidity trap, and increases potential output (to the extent that the higher public spending elevates the economy’s capital stock) when the economy recovers from the slump. Thus, a properly sized infrastructure spending bill could thus provide notable stimulus in both the near- and medium-term and be fully – or nearly – self-financed.

Such a push is particularly relevant in the current context, as infrastructure investment has declined to low levels in several euro area countries (Figure 1). In France and Germany, for example, infrastructure investment was above 3 percent of trend GDP in the 1970s and the 1980s (Panel a), while it has declined to 1.5 percent since the mid-1990s. On the contrary, in some southern European countries that have benefitted from EU fiscal transfers (Spain, Italy), overall infrastructure investment has remained above 3 percent during this period but government investment have declined notably following the European debt crisis (Panel b). Our conclusion is that fiscal reforms should therefore consider public investment opportunities and not merely rely on tax policies to stimulate growth, especially in economies with considerable resource slack and limited debt capacity.

Apart from the papers already mentioned, there is growing literature on the macroeconomic effects of fiscal reforms. A recent paper by Bussière et al. (2017) analyzes which fiscal reforms could be useful for stimulating growth in a high debt environment. They focus on budget-neutral reforms – which would correspond to our simulations with aggressive tax rules – and show that higher government investment, financed by hikes of the labour income and consumption taxes, would be more beneficial for output growth than a fiscal devaluation (cuts of labour and capital taxes financed by hikes of the consumption tax). Even so, they do not consider the case of unconventional fiscal policy. Bouakez et al. (2017) shows that time-to-build plays a key role for generating an elevated multiplier of government investment in a liquidity trap. While the disinflationary effect of this policy occurs after the liquidity trap has ended because of time-to-build, its positive impact on household wealth amplifies the increase of aggregate demand and the fall of the real interest rate during the trap. The recent literature has also emphasized the role of the timing of impulses to government investment in a liquidity trap. Le Moigne et al. (2016) show that when part of the
higher investment spending occurs after the ZLB incident has ended, the private capital stock is reduced and the positive impact of the push to public investment is accordingly lower.

The remainder of this paper is organized as follows. Section 2 develops and analyses a stylized New Keynesian model with variations in sales taxes and public capital in which labor income taxes are used to stabilize government debt. The results for this model are then discussed in Section 3. In Section 4, we examine the robustness of the results in the more empirically-realistic model with capital, hand-to-mouth households and financial frictions. Finally, Section 5 concludes.

2. A Stylized New Keynesian Model

As in Eggertsson and Woodford (2003), we use a standard log-linearized version of the New Keynesian model that imposes a zero bound constraint on interest rates. The model is very similar to the simple model with distortionary labor income taxes analyzed by Erceg and Lindé (2014) with fixed private capital, which is here extended to allow for sales taxes and public infrastructure investment.

2.1. The Model

We start out by characterizing the model without public capital and discuss the effects of changes in the sales tax. We then describe how we introduce public capital accumulation and examine the effects of infrastructure investments (in Section 3.2). The key equations of the model without public capital are:

\[ x_t = x_{t+1 \mid t} - \hat{\sigma}(i_t - \pi_{t+1 \mid t} - r_{t}^{\text{pot}}), \]  
\[ \pi_t = \beta \pi_{t+1 \mid t} + \kappa_{mc} \left[ \phi_{mc} x_t + \frac{1}{1 - \tau_N} \left( \tau_{N,t} - \tau_{N,t}^{\text{pot}} \right) \right], \]  
\[ i_t = \max \{-i, (1 - \gamma_i) (\gamma_i \pi_t + \gamma_x x_t) + \gamma_i i_{t-1}\}, \]  
\[ y_t^{\text{pot}} = \frac{1}{\phi_{mc} \hat{\sigma}} [g_y g_t + (1 - g_y) \nu_c \nu_t - \frac{\hat{\sigma}}{1 - \tau_N} \tau_{N,t}^{\text{pot}} - \frac{\hat{\sigma}}{1 + \tau_C} \tau_{C,t}], \]  
\[ r_t^{\text{pot}} = \frac{1}{\hat{\sigma}} E_t \Delta y_{t+1}^{\text{pot}} - \frac{g_y}{\hat{\sigma}} E_t \Delta g_{t+1} - \frac{1 - g_y}{\hat{\sigma}} \nu E_t \Delta \nu_{t+1} + \frac{1}{1 + \tau_c} E_t \Delta \tau_{C,t+1}. \]
where $\hat{\sigma}$, $\kappa_{mc}$, and $\phi_{mc}$ are composite parameters defined as:

\[
\hat{\sigma} = \sigma (1 - g_y) (1 - \nu_c), \quad (6)
\]

\[
\kappa_{mc} = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p}, \quad (7)
\]

\[
\phi_{mc} = \frac{\chi}{1 - \alpha} + \frac{\alpha}{\sigma} + \frac{\alpha}{1 - \alpha}. \quad (8)
\]

All variables are measured as percent or percentage point deviations from their steady state level.\(^2\)

Equation (1) expresses the “New Keynesian” IS curve in terms of the output and real interest rate gaps. Thus, the output gap $x_t$ depends inversely on the deviation of the real interest rate ($i_t - \pi_{t+1|t}$) from its potential rate $r_{t|t}^{pot}$, as well as on the expected output gap in the following period. The parameter $\hat{\sigma}$ determines the sensitivity of the output gap to the real interest rate; as indicated by (6), it depends on the household’s intertemporal elasticity of substitution in consumption $\sigma$, the steady state government spending share of output $g_y$ ($c_y$ is the steady state consumption share, so $1 = g_y + c_y$), and a (small) adjustment factor $\nu_c$ which scales the consumption taste shock $\nu_t$. The price-setting equation (2) specifies current inflation $\pi_t$ to depend on expected inflation, the output gap and the labor-income tax gap, where the sensitivity to the latter is determined by the composite parameter $\kappa_{mc}/(1 - \tau_N)$ and the sensitivity of the output gap is determined by $\kappa_{mc} \phi_{mc}$.

Given the Calvo-Yun contract structure, equation (7) implies that $\kappa_{mc}$ varies inversely with the mean contract duration ($1/\bar{\tau_p}$). The sensitivity of marginal cost to the output gap $\phi_{mc}$, equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in (8), $\phi_{mc}$ varies inversely with the Frisch elasticity of labor supply $1/\chi$, the interest-sensitivity of aggregate demand $\hat{\sigma}$, and the labor share in production $(1 - \alpha)$. The policy rate $i_t$ follows a standard interest rate rule subject to the zero lower bound (equation 3).

Equation (4) indicates that potential output $y_{t|t}^{pot}$ depends on the sales tax $(\tau_{C,t})$ and the labor income tax $(\tau_{N,t})$ and varies directly with exogenous movements in consumption demand $\nu_t$ and government spending $g_t$. The two latter shocks are assumed to follow AR(1) processes with the same persistence parameter $\rho_{\nu}$, e.g., the taste shock follows:

\[
\nu_t = \rho_{\nu} \nu_{t-1} + \varepsilon_{\nu,t}, \quad (9)
\]

\(^2\) We use the notation $y_{t+j|t}$ to denote the conditional expectation of a variable $y$ at period $t + j$ based on information available at $t$, i.e., $y_{t+j|t} = \mathbb{E}_t y_{t+j}$. The superscript ‘pot’ denotes the level of a variable that would prevail under completely flexible prices, e.g., $y_{t|t}^{pot}$ is potential output. See Appendix A for the model derivation.
where $0 < \rho_\nu < 1$. Given the front-loaded nature of the shocks, equation (5) indicates that positive realizations of these shocks boosts the potential real interest rate (noting $\phi_{mc} > 1$); this reflects that each shock – if positive – raises the marginal utility of consumption associated with any given output level. The sales tax shock is allowed to follow a general AR(2) process, here written on error-correction form

$$\Delta \tau_{C,t} = \rho_{\tau,1} \Delta \tau_{C,t-1} - \rho_{\tau,2} \tau_{C,t-1} + \varepsilon_{C,t}. \hspace{1cm} (10)$$

We now turn to discussing how $N_{t}$ is determined. The government issues nominal debt as needed to finance budget deficits. Under the simplifying assumption that government debt is zero in steady state, the log-linearized government budget constraint is given by:

$$b_{G,t} = (1 + r)b_{G,t-1} + g_{y}g_{t} - c_{y} \left[ \tau_{C,t} + \frac{\tau_{C}}{c_{y}} (y_{t} - g_{y}g_{t}) \right] - s_{N} \left[ \tau_{N,t} + \tau_{N} (y_{t} + \phi_{mc}x_{t}) \right] - \tau_{t}, \hspace{1cm} (11)$$

where $b_{G,t}$ is end-of-period real annualized government debt as share of trend output, $(y_{t} + \phi_{mc}x_{t})$ equals real labor income, $\tau_{t}$ is a lump-sum tax, and $s_{N}$ is the steady state labor share.\(^3\) Labor income taxes adjust according to the reaction function:

$$\tau_{N,t} - \tau_{N} = \varphi_{b} b_{G,t-1} + \varphi_{bb} \tilde{\tau}_{N,t}. \hspace{1cm} (12)$$

This rule has the convenient property that it can be calibrated so it is not very aggressive by selecting a low value for $\varphi_{b}$ (and by setting $\varphi_{bb}$ equal to nil). However, by setting $\varphi_{b} = 0$ and $\varphi_{bb} = \frac{1}{s_{N}}$, and defining $\tilde{\tau}_{N,t}$ in the log-linearized government budget constraint (11) so that $b_{G,t} = 0$ for all possible states, i.e.

$$0 = (1 + r)b_{G,t-1} + g_{y}g_{t} - c_{y} \left[ \tau_{C,t} + \frac{\tau_{C}}{c_{y}} (y_{t} - g_{y}g_{t}) \right] - s_{N} \left[ \tau_{N,t} + \tau_{N} (y_{t} + \phi_{mc}x_{t}) \right] - \tau_{t}, \hspace{1cm} (13)$$

then the rule in (11) mimics an aggressive “balanced budget”, because it holds government debt constant (i.e. $b_{G,t} = 0 \ \forall t$). Finally, note that the complete model includes versions of eqs. (11) - (13) which holds in the notional economy with flexible prices, determining $b_{G,t}^{\text{pot}}, \tau_{N,t}^{\text{pot}}$, and $\tilde{\tau}_{N,t}^{\text{pot}}$, respectively.

\(^3\) In (11), real government debt $b_{G,t}$ and real transfers $\tau_{t}$ are defined as a share of steady state GDP and expressed as percentage point deviations from their steady state values. That is, $b_{G,t} = \left( \frac{b_{G,t}}{Y_{t}} \right) - b_{G}$, where $B_{G,t}$ is nominal government debt, $P_{t}$ is the price level, and $Y$ is real steady state output; and similarly, $\tau_{t} = \left( \frac{\tau_{t}}{Y_{t}} \right) - \tau$. Because of our simplifying assumption that $b_{G} = 0$, a time-varying real interest rate does not enter in eq. (11). In the full model analyzed in Section 4, we allow for positive steady state government debt, and hence a role for time-varying debt service costs.
2.2. Parameterization

Our benchmark calibration is fairly standard at a quarterly frequency; intended to be relevant for the U.S. and the euro area. We set the discount factor $\beta = 0.995$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .01$ (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (log utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\lambda} = 0.4$, and the scale parameter on the consumption taste shock $\nu_c = 0.01$. Following recent arguments and evidence in Blanchard, Erceg and Lindé (2016) we select $\zeta_p = .9$ so that the results are not contingent on counterfactually large movements in actual and expected inflation. With this choice, the Phillips Curve slope $\kappa_{mc} = .011$ and the sensitivity of inflation w.r.t. the output gap, $\kappa_{mc} \phi_{mc}$, equals 0.06.

We assume that monetary policy would follow a standard simple policy rule by setting $\gamma_i = 0.7$, $\gamma_\pi = 2.5$ and $\gamma_x = 0.25$. In A.2 we present alternative results when monetary policy completely stabilize inflation and the output gap in the absence of a zero bound constraint, which can be regarded as a limiting case in which the coefficients on inflation, $\gamma_\pi$, and the output gap, $\gamma_x$, in the interest rate reaction function become arbitrarily large.

The government share of steady state output $g_y = 0.23$ (roughly in line with total government spending in the euro area and the United States), and the sales tax $\tau_C = 0.10$ in the steady state (as a compromise between the zero federal sales tax in the United States and the 20 percent rate prevailing in many euro area economies). By making the simplifying assumption that the government debt ratio is nil in the state and that $\tau = -.06$ (so net transfers equals 6 percent of GDP), equation (11) implies that $\tau_N$ equals about 33 percent in the steady state.\footnote{We study in A.2 the effects of allowing for a steady state debt share of 100 percent of GDP in the model, i.e. when we set $b_G = 4$ (recalling that $b_G$ in eq. (11) is defined as share of quarterly output). To allow for a clean comparison with our baseline results, we reduce the net transfers $\tau$ somewhat in this case, so that the steady labor tax rate remains unchanged at 33 percent.} When we consider a non-aggressive tax rule (12), we set the parameter $\varphi_b$ equal to .01 and $\varphi_{bb} = 0$, which implies that the response of the labor income tax rate to changes in government debt is extremely small in the first couple of years following a shock (so that almost all variation in tax revenue comes from fluctuations in hours worked). For the balanced budget rule, we set $\varphi_{bb} = \frac{1}{\sigma_N}$ and $\varphi_b = 0$ as explained previously. Obviously, such a rule will feature larger movements in $\tau_{N,t}$ to various shocks to keep government debt unchanged. Finally, the consumption preference shock is assumed to follow an AR(1) process with persistence of $\rho_\nu = 0.9$ in equation (9).
3. Results with the Stylized Model

In this section, we report the results in the stylized model.

3.1. Impulse Responses to a Gradual Sales Tax Hike

In Figure 2, we show the effects of a hike in the sales tax $\tau_{C,t}$ in normal times and in a 10-quarter liquidity trap. A 10-quarter liquidity trap is roughly the current projection in financial markets of how long the European Central Bank (ECB) is expected to keep their key policy rate at zero, and is generated in the model by assuming that an adverse consumption demand shock eq. (9) hits the economy. Following the insights in Corriea et al. (2013), we assume that the sales tax $\tau_{C,t}$ is hiked gradually and peaks at about 1.3 percent after 12 quarters.\(^5\) With our calibration of the consumption-output ratio in the steady state, a 1.3 percent hike in $\tau_{C,t}$ is consistent with generating 1 percent higher sales taxes revenues as share of GDP if consumption (and output) would remain unchanged.

In the figure, the left column reports results when the labor income tax adjusts gradually, i.e. $\varphi_b = 0.01$ and $\varphi_{bb} = 0$ in eq. (12), whereas the right column report results under complete debt stabilization (i.e. $\varphi_b = 0$ and $\varphi_{bb} = 1/s_N$). As expected from Correia et al. (2013), we see from the left column that the sales tax hike stimulates economic activity in a long-lived liquidity trap by causing the actual real rate to fall and the potential real rate to rise. However, in normal times when monetary policy would react to the higher sales tax path by raising the policy rate, we see that the impact on economic activity is much more muted. As labor income taxes are assumed to respond very slowly, the higher tax rate and consumption profile implies that tax revenues increase considerably, and government debt falls by roughly 5 percent after 5 years.

The results are qualitatively similar when labor income taxes responds aggressively to keep government debt unchanged (as can be seen in the bottom right panel), but there are some important lessons which deserves to be highlighted. In a liquidity trap, the labor income tax rate has to be cut more aggressively compared to normal times to stabilize debt, and this causes output, depicted in the top right panel, to rise more when the labor tax rule is aggressive. The finding that output rises more when labor income taxes are aggressively cut counters the wisdom from Eggertsson (2011) and Christiano, Eichenbaum and Rebel (2011), who both argue that a hike in the labor tax rate stimulates output at the zero lower bound. However, these authors consider exogenous shifts in

\(^5\) This is achieved by setting $\rho_{\tau,1} = 0.75$ and $\rho_{\tau,2} = 0.001$ in equation (10).
the tax rule, and hence they do not allow for the effects of tax gaps on inflation, i.e. the term \( \frac{\tau_{N,t}}{1-\tau_N} \left( \tau_{N,t} - \tau_{N,t}^{\text{pot}} \right) \) in the Phillips curve (2), and the effects an endogenous tax rule has on the potential real rate \( \tau_t^{\text{pot}} \) through its effects on potential output \( y_t^{\text{pot}} \) in eq. (4). Now, the inflation channel is in fact somewhat muted under an aggressive tax rule as it induces a persistent negative tax-wedge \( \tau_{N,t} - \tau_{N,t}^{\text{pot}} \) which can be seen by comparing the inflation response for the non-aggressive and aggressive tax rule panels in Figure 2. So, what drives the elevated output response under the aggressive labor tax rule is the benign impact on expected output growth (compare black-dashed lines in the left and right panels for output), which in turn helps to elevate the path for \( \tau_t^{\text{pot}} \) according to equation (5).

### 3.2. Impulse Responses to Higher Government Investment

In this subsection, we examine the dynamic effects of higher government investment. To begin with, we describe briefly how government investment builds capital in the model. Then, we turn to the results.

#### 3.2.1. Extending the model with public investment

In the simple model just analysed, we assumed there is no private investment and that the aggregate capital stock was fixed. We now relax this assumption and assume that

\[
Y_t = Z_t \left( K_t^{\text{tot}} \right)^\alpha N_t^{1-\alpha}, \tag{14}
\]

where

\[
K_t^{\text{tot}} = (K_P)^\vartheta (K_{G,t})^{1-\vartheta}. \tag{15}
\]

Eq. (15) implies that the effective capital stock, \( K_t^{\text{tot}} \), is affected by the government capital stock \( K_{G,t} \). Following Leeper, Walker and Yang (2010), we assume that the direct impact on \( Y_t \) of a one percent increase in \( K_{G,t} \) equals 5\%.\(^6\) Given our choice of \( \alpha (.3) \), we calibrate \( \vartheta \) to \( .833 \) in order to match this output elasticity \( ((1-\vartheta)\alpha = .05) \). The law of motion for public capital \( K_{G,t} \) is standard:

\[
K_{G,t} = (1-\delta_G)K_{G,t-1} + I_{G,t},
\]

where we set \( \delta_G = .02 \). In line with how the real world works, we assume building the public capital stock takes time so expenses on public capital in period \( t \), \( G_{I,t} \), only turns into effective investment

\(^6\)The value 0.05 is the lower value they choose for this elasticity, the other one being 0.10.
into the public capital stock $I_{G,t}$ with delay:

$$I_{G,t} = \frac{1}{6} (G_{I,t-4} + G_{I,t-8} + G_{I,t-12} + G_{I,t-16} + G_{I,t-20} + G_{I,t-24}).$$  \hspace{1cm} (16)

The specification in eq. (16) implies a uniform distribution of project completion duration between 1 and 6 years. Leeper, Walker and Yang (2010) assumed a three-year time to build. Obviously, we have in mind that some projects may be relatively fast to complete, like repairing or extending a bridge or building, whereas more major projects, for example building a new freeway, take longer time to complete. Since our choice is arbitrary, we examine the sensitivity of our specification of $I_{G,t}$ by considering faster and slower average completion times. In addition, we examine the sensitivity of our results w.r.t. the parameter $\psi$.

In the log-linearized version of the model, all the key equations (1)-(5) remain unaltered, except the equation for $y_{pot}$ which now becomes

$$y_{pot} = \frac{1}{\sigma} \left( \frac{g_y}{\sigma} g_t + \frac{1}{\sigma} (1 - g_y) \nu_c \nu_t - \frac{1}{1 - \tau_N} \tau_{N,t} - \frac{1}{1 + \tau_C} \tau_{C,t} + \frac{1 + \chi}{1 - \alpha} (z_t + \alpha(1 - \psi) k_{G,t}) \right).$$ \hspace{1cm} (17)

In eq. (17), it is important to recognize that total government spending (in log-linearized terms) now equals

$$g_t = g_C g_{C,t} + g_I g_{I,t},$$

where $g_{C,t}$ is government consumption (in percent deviation from steady state) and $g_C = G_C / G$ and $g_I = 1 - g_C$. Since $g_y = 0.23$, and Figure 1 suggests that public investment share of GDP equals about 3 percent in the long-run, we set $g_C = 0.87$, so $g_I = 0.13$ (so that $g_y \times g_I = 0.03$).

### 3.2.2. Results

In Figure 3, we show the effects of a hike in government investment $g_{I,t}$ in normal times and in a 10-quarter liquidity trap. We assume a path with a constant increase of 1 percent of baseline GDP during 8 quarters followed by a gradual phasing out with a root of 0.9. This path is motivated by the fact that more resources must be spent early on in projects, but as the projects become completed less and less resources need to be spent. Again, we compare the cases with non-aggressive (left column) and aggressive (right column) labor income tax rule.

Under a non-aggressive tax rule, higher public investments (e.g. in infrastructure) is more expansionary in a liquidity trap than in normal times by raising aggregate demand and inflation expectations. Given a nominal interest rate stuck at zero, higher inflation expectations imply that...
the actual real interest rate falls sharply, something which does not happen in normal times. On the other hand, while the actual rate falls during the stimulus period (i.e. the first two years), the potential real interest rate, $r_t^{\text{pot}}$, remains unchanged and does not start to rise until the phasing-out period (from quarter 9 and onwards). This happens because in a flexible world the gradual phasing out of government investments implies an increasing path of private consumption which compensates for the fall of government spending. The resulting negative gap between the actual real interest rate and its potential level boosts the output gap, by more than 2% in the short run (output shown in upper left panel rises roughly equally much, given the small response of potential output under a non-aggressive tax rule). We can also notice that the fiscal stimulus is self-financed when the tax rule is not aggressive; the labor income tax is almost unchanged and the additional tax revenues induce a persistent yet temporary decline in government debt by around 1% of GDP.

Turning to the results with the aggressive tax rule, we see that output increases around 1.5% in the short run in a liquidity trap. The smaller effect is related to the usage of tax receipts. With the aggressive tax rule, the government uses extra receipts to cut the labor income tax, and in a liquidity trap, these tax cuts create deflationary pressures which moderate the fall of the real interest rate and the surge in output. When monetary policy is unconstrained by the effective lower bound (red dotted lines), the initial effects on output are slightly negative before turning positive as the aggressive labor income tax hikes exert a more negative drag on the economy than the boost to demand. Only when enough projects have been completed and the public capital stock and potential output have risen sufficiently to enable labor income taxes to recede, we see that output turns positive and the effects approach those obtained under the non-aggressive rule.

In Figure 4, we examine the robustness of a hike in government investment under a non-aggressive tax rule for four alternative assumptions: (i) public investment is not productive at all; (ii) public investments adds more to the effective capital stock than in our baseline; (iii) all public infrastructure becomes productive after 1 to 2 years; (iv) public infrastructure becomes productive after 5 to 10 years. As shown in Bouakez et al. (2017), productive government spending has two effects on future marginal costs absent from the non-productive case: a positive demand-side effect coming from the increase of permanent income; a negative supply-side effect generated by the future increase of the marginal productivity of inputs. In a liquidity trap, if the demand (supply) effect dominates, inflation expectations will be stronger (resp. weaker), the real interest rate will fall more (less) and, hence, output will also increase more (less). Here, the positive demand-side effect generally dominates for the large set of assumptions that we examine: for our benchmark calibration
as well as for alternative cases (ii) and (iii), the output response is amplified compared to the non-productive case (i), while the output response is neither amplified nor dampened for the case (iv) of a very long time-to-build.

So in a liquidity trap, we find that higher public investments stimulate to the economy, even for a country which must run a balanced budget. Outside of a liquidity trap, the overall effect is less favorable, especially if the fiscal space to sustain a short-run deficit is limited. Next, we examine the robustness of these results in a model with endogenous private capital.

4. Analysis in a Workhorse Model with Keynesian Households and Financial Frictions

In this section, we examine how the results hold up in an empirically realistic framework with endogenous capital accumulation. The core of the model we use is a close variant of the models developed and estimated by Christiano, Eichenbaum and Evans (2005), CEE henceforth, and Smets and Wouters (2003, 2007), SW henceforth. CEE show that their model can account well for the dynamic effects of a monetary policy innovation during the post-war period. SW consider a much broader set of shocks, and argue that their model – which is estimated by Bayesian methods – is able to fit many key features of U.S. and euro area-business cycles.

However, we depart from the CEE/SW environment in two substantive ways. First, we assume that a fraction of the households are “Keynesian”, and simply consume their current after-tax income. Galí, López-Salido and Vallés (2007) show that the inclusion of non-Ricardian households helps account for structural VAR evidence indicating that private consumption rises in response to higher government spending, and also allows their model to generate a higher spending multiplier. Second, to capture financial frictions explicitly omitted from the CEE/SW models, we incorporate a financial accelerator following the basic approach of Bernanke, Gertler and Gilchrist (1999). In this framework, the corporate finance premium varies with the degree of leverage of the economy due to an agency problem in private lending markets.\footnote{Following Christiano, Motto and Rostagno (2008), we assume that the debt contract between the entrepreneurs and lenders (households) is written in nominal terms (rather than real terms as in BGG 1999).}

In terms of parameterization, we set the share of Keynesian households to optimizing households to 0.5, implying that the former comprise about 30 percent of aggregate consumption in the steady state, and calibrate the parameters affecting the financial accelerator as in BGG (1999). However, we also report some results from a CEE/SW-type specification to help gauge the sensitivity to
Given that most features are standard, we relegate many details about the model, solution method, and calibration to Appendix B. Even so, it is important to highlight two features. First, in the model’s fiscal block, government revenue is assumed to be derived from taxes on consumption, labor and capital. While the sales tax rate and public investment varies exogenously we will, following the analysis with the stylized model in Section 2, start out by assuming that the distortionary labor income tax rate reacts follows the rule:

\[ \tau_{N,t} - \tau_N = \varphi \left( \tau_{N,t-1} - \tau_N \right) + \left( 1 - \varphi \right) \left[ \varphi_b \left( \tilde{b}_{G,t-1} - \tilde{b}_G \right) + \varphi_{bb} \tilde{\tau}_{N,t} \right], \]

where \( \tilde{b}_{G,t} \) denotes debt as share of trend annualized GDP, i.e. \( \tilde{b}_{G,t} \equiv \frac{B_{G,t}}{4P_tY} \), as deviation from a positive steady state value \( \tilde{b}_G \). This rule has the convenient property that it can be calibrated so that it exhibits substantial inertia – and is not very aggressive even in the long-run by selecting a high value for \( \varphi \) and a relatively low value for \( \varphi_b \) (and by setting \( \varphi_{bb} \) equal to nil). However, by setting \( \varphi_b = 0 \) and a positive coefficient \( \varphi_{bb} = \frac{4}{s_N} \) where \( s_N \) denotes the effective labor share, and defining \( \tilde{\tau}_{N,t} \) in the log-linearized government budget constraint as

\[ 0 = \tilde{b}_G (i_{t-1} - \pi_t) + \frac{1}{1 + \pi_b} \tilde{b}_{G,t} + \frac{1}{4} \left\{ g_t - \tau_t - c_y (\tau_{C,t} + \tau_{G,t}) - s_N (\tilde{\tau}_{N,t} + \tau_N (\tilde{w}_t + l_t)) \right\}, \]

this rule is a balanced debt rule since it ensures that end-of-period debt remains (or jumps to) steady state, i.e. \( \tilde{b}_{G,t} = \tilde{b}_G \), in all possible states. Notice that we will also consider a rule which uses the capital income tax \( \tau_{K,t} \) instead of the labor tax rate to stabilize debt, and in this case \( \tau_{N,t} \) in eq. (18) is replaced by \( \tau_{K,t} \) and \( \varphi_{bb} = \frac{4}{s_K (\tau_{K,t} - \delta)} \).

Second, our calibration of the monetary policy rule and the Calvo price and wage contract duration parameters – while within the range of empirical estimates – tilts in the direction of reducing the sensitivity of inflation to various shocks. In particular, the monetary rule that is followed when policy is unconstrained is a Taylor rule with a fairly aggressive long-run coefficient of 2.5 on inflation, of unity on the output gap, and 0.7 on the lagged interest rate. Our choice of a price contract duration parameter of \( \xi_p = .92 \) implies a Phillips Curve slope of about .007, which is on the low side of the median estimates reported in the empirical literature, even if well within

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8 In the models considered, we have worked with log-linearized equations, aside from imposing the zero lower bound on policy rates. Given that we examine model dynamics well away from the steady state, a useful extension of our work would be to solve the models nonlinearly.

9 Given a steady state government spending share of 23 percent and debt/GDP ratio of 100 percent, the steady state tax rate on labor income is 33 percent, capital income 25 percent, and consumption 10 percent.
reported confidence intervals; and wages exhibit a commensurate degree of stickiness. These parameter choices are aimed at capturing the resilience of core inflation, and measures of expected inflation, during the global recession, see e.g. Erceg and Lindé (2014) for further discussion.

4.1. Dynamic Effects of Sales Taxes

Figure 5 shows the effects of the same gradual increase of the sales tax as in Figure 2, but now when we simulate the full model with all frictions (wage stickiness, hand-to-mouth households, habit consumption, adjustment costs of investment and financial frictions). As before, the figure reports the simulation results with the non-aggressive tax rule in the left column, and the aggressive rule in the right column. As explained in further detail below, the expansionary effects of a gradual increase of the sales tax in a liquidity trap is not robust to the financing scheme. The strong amplification in a liquidity trap only holds up when the labor income tax adjusts aggressively. In normal times, the effects are small and similar for both tax rules in the near term, whereas the effects are somewhat more positive in the medium-term under an aggressive rule, basically reflecting that labor taxes are more distortionary than the sales tax.

Under the aggressive tax rule, we get results in a liquidity trap that are qualitatively similar to those obtained with the stylized model: the falling real interest rate gap boosts output and inflation in the short run. The main qualitative difference is that the output response is now hump-shaped because of all the frictions included in the full model and that inflation moves less (due to slow nominal wage adjustment). Under a non-aggressive tax rule, the results are very different: the real interest rate does not fall, inflation remains stable and output quickly converges to its potential level that becomes negative because of the increased distortions imposed by the higher sales tax.

Which frictions are responsible for the failure of this unconventional fiscal policy with such a non-aggressive tax rule? In order to adress this issue, we proceeded as follows. First, we checked that we could get results very close to those of the stylized model, when we parametrized the workhorse model to make private investment constant and cancel other features which differs (nominal wage rigidity and habit persistence and hand-to-mouth consumers). To remove endogenous capital (and financial frictions), we set the depreciation rate at a value arbitrarily close to $0.10^{-6}$, the corporate default rate at $0$ and the monitoring cost arbitrarily close to $0.10^{-9}$, so that for a given capital

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10 The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler, and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of $0.009 - 0.014$. Our specification of the steady-state wage markup and a wage contract duration parameter of $\xi_w = 0.85$ - along with a wage indexation parameter of $t_w = 0.9$ - implies that wage inflation is about as responsive to the wage markup as price inflation is to the price markup.
share ($\alpha$) the investment share becomes negligible. For suppressing consumption frictions, we set the share of hand-to-mouth households at $10^{-9}$ and the habit parameter at 0.01. Finally, we made nominal wages flexible by supressing the nominal wage stickiness. The only nominal friction we kept is price stickiness.

Next, we study the role of key mechanisms in isolation. Figure 6 summarizes the results of this analysis. The top row shows the results in the most simplified variant of the workhorse model. Qualitatively, the results in this variant closely mimics those obtained with the stylized model (see Figure 2). The effects in a liquidity trap (blue solid line), are quantitatively larger in the simplified workhorse model because we in this model allow for dynamic indexation to past inflation in the Phillips curve and this generates a more persistent rise in expected inflation (upper right panel in Figure 6) which boosts the output response (upper left panel) by lowering the actual real rate path (not shown). In the second row, however, we see that adding nominal wage stickiness dampen the inflation response dramatically even in a long lived liquidity trap and this dampen the output response from over 2 percent (with flexible wages) to about 0.5 percent. So accounting for nominal wage stickiness is crucial.

When adding habit formation in consumer preferences (third row in Figure 6), we see that the output response falls a bit on impact and becomes more persistent as expected. Finally, adding the hand-to-mouth households (so using a TANK model) and endogenous investment subject to financial frictions causes a further decline in the output response. The key difference quantitatively between the stylized model in Section 2 and the workhorse model analyzed in this section is thus the assumption of gradual nominal wage adjustment. When nominal wage adjustment is slow, a higher VAT means a direct cut in the purchasing power of both optimizing and hand-to-mouth households.

Hence, in the end, we find that unconventional fiscal policy in the form of a gradual sales tax is not necessarily an efficient tool to stimulate economic activity. In a more realistic model its favorable effects hinges importantly on a package of fiscal instruments. To be fair, Correia et al. (2013) emphazises that simultaneous cuts in the labor income tax are required. Our simulations clarify that this adjustment is critical and that the effects of a higher sales tax in insolation does not provide any meaningful stimulus. On the other hand, the output costs to reduce government debt are notably lower than for outright spending cuts, which may be self-defeating in the near term (see e.g. Erceg and Lindé, 2014) in a long-lived liquidity trap.

Finally, in Figure 7, we look at the sensitivity of our baseline results to the choice of fiscal
instrument used to stabilize debt. More precisely, we assess the impact of the same gradual increase of the sales tax when the government use aggressive cuts of the capital income tax to stabilise government debt. We do not report results in the case with a non-aggressive rule, as the effects in this case would be similar to those reported and already discussed for the non-aggressive labor income tax rule in Figure 5 in the short- and medium-term.

As is well known since Chamley and Judd, the capital income tax creates stronger distortions than the labour income or the consumption sales tax and in a basic RBC framework this implies that the optimal capital tax is zero. Because of this feature, capital income tax cuts have generally a stronger multiplier than other tax cuts (see for example Clerc et al., 2017). Our workhorse model has also this feature and we find a strong amplification of the expansionary effect thanks to these tax cuts. Still, one could expect that a strong downside of a higher sales tax financed by a reduction in the capital income tax may increase the inequality between poor (hand-to-mouth) and rich (optimizing) households who owns the capital stock, as the latter directly benefit from these cuts while the former do not. However, interestingly enough our simulations in Figure 7 show that hand-to-mouth agents expands their consumption notably more in the near- and medium-term than optimizing households when this policy is implemented in a long-lived liquidity trap (blue solid line). In this situation, their current income also increases a lot due to the expansionary impact of this policy. Optimizing households invests in the capital stock which enables them to increase their consumption relative to the hand-to-mouth consumers in the longer term. In a normal situation, the red dotted line shows that consumption of poor agents falls more than for the rich households, especially in the medium and long-term.

4.2. Dynamic Effects of Public Investment

We now turn to the effects of a hike in public investment. In Figure 8, we report the effects of an identical expansion of government infrastructure investment as in Figure 3. As in the stylized model in Section 3, Figure 8 shows that the expansionary impact of a stimulus on public investment is robust to the aggressiveness of the tax rule. This expansionary impact is also again robust to alternative assumptions about how quickly and forcefully public infrastructure investment contributes to the capital stock (Figure 9). Thus, relative to the stylized model, the added mechanisms in the workhorse model mainly modifies the shapes of responses, which now features some humps. Another interesting feature of these simulations concerns the dynamics of investment: we notice that responses of the private capital stock are negative in normal times, as well as in
a liquidity trap for the case of an aggressive tax rule after 5 years. This crowding out of private investment might at first seem surprising, as public capital acts as a technology shock and we expect crowding-in of private investment after technology shocks. However, as shown in Lindé (2009), when the maximum effect of a technology shock is anticipated to happen in the future, agents find it worthwhile to postpone investment expenditures and initially switch their resources toward consumption and leisure because they know that labor effort and capital will become even more productive in the future. Here, we get a similar crowding-out effect in the short run because of the time-to-build in the public capital stock, which makes public investment productive only with a delay of 1 to 6 years.

5. Conclusions

For an economy facing a deep recession and prolonged liquidity trap, there is a strong argument for increasing government spending on infrastructure projects a temporary basis. Such a policy would boost demand in the near-term which is useful, and potential output in the longer term when the economy is recovering. For countries in the Euro area with fiscal space there is still thus a strong case for fiscal stimulus. But our analysis highlights the importance of recognizing that the marginal benefits of such stimulus may drop substantially outside of a liquidity trap, and may likely require financing by higher taxes. Thus for a country like the United States, which now experiences more normalized business cycle conditions, the macroeconomic argument for stimulus via infrastructure spending is much weaker. On the other, our results also suggest that lower capital income taxes financed by higher sales taxes may pose important trade-offs between efficiency and redistribution outside of a long-lived liquidity trap. Overall output expands following such a reform but poor households loose while the capital owning households benefits from such a reform.

As emphasized by Canova and Pappa (2011), a major issue for future research is to assess whether conditions that have been identified as likely to make fiscal policy highly effective hold empirically.\textsuperscript{11} Many recent papers, including our own, have used calibrated models with a binding zero lower bound constraint to show that a sizeable response of inflation plays a crucial role in generating a large spending multiplier well above unity; this is also true in models in models in which the monetary policy regime is passive at least for some time.\textsuperscript{12} However, the resilience of

\textsuperscript{11} These authors provide empirical evidence suggesting that the conditions for a high spending multiplier did not appear to be satisfied during the global recession.

\textsuperscript{12} For example, Davig and Leeper (2011) show in a regime-switching model that the government spending multiplier under a passive monetary policy regime is around 1-1/2 after 10 quarters, roughly twice as high as under an active policy regime. The disparity mainly reflects a much larger and more persistent response of inflation under the passive
inflation in the aftermath of the global financial crisis gives reason to question whether inflation is as responsive to fiscal policy, and to macro shocks generally, as implied by existing models that are calibrated based on estimates derived from pre-crisis data. Even more directly, recent analysis by Canova and Pappa (2011) – using a structural VAR with sign restrictions – found that stimulative government spending shocks induce only a transient increase in inflation, rather than the persistent inflation rise required for a big spending multiplier. In future research, it will be important to draw on evidence from the global recession to further refine our empirical understanding of the role of different factors and policies in influencing the response of inflation to fiscal policy, including the characteristics of the monetary and fiscal policy regimes, the parameters of the price and wage Phillips Curve, and the nature of the shocks driving the economy into a liquidity trap.

There are also open questions about whether the traditional channels through which fiscal policy affects aggregate demand remain operative in a severe recession. The potency of the interest rate channel might be impaired to the extent that tight credit and heavy debt burdens reduce the interest-sensitivity of households and firms. As argued by Merten and Ravn (2010), the stimulative effects of government spending may also be muted if the source of recession is a self-fulfilling loss in confidence, reflecting that the higher spending is perceived as a negative signal about the state of the economy. Conversely, various types of fiscal interventions could have a heightened impact through easing collateral constraints on borrowers, reducing precautionary savings, or by affecting financial market risk premia. From a modeling perspective, addressing some of these questions will require a non-linear stochastic framework to capture key channels through which fiscal interventions may operate in the presence of uncertainty such as in recent work by Bi, Leeper, and Leith (2012).  

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13 These authors examine how the effects of fiscal consolidation vary with the state of the economy, including the level of government debt.
References


Debortoli, Davide and Jordi Galí (2017), “Monetary Policy with Heterogeneous Agents: Insights from TANK models”, manuscript, CREI and University of Pompeu Fabra.


Figure 1: Two Proxies of Infrastructure Investment for Selected Euro Area Countries (% of trend GDP)

a. Investment in energy, water, transport and communication

b. Government Investment
Figure 2. Impulses to Sales Taxes in Normal Times and in a 10 Quarter Liquidity Trap

Non-agressive Tax Rule

- Output
- Real Interest Rate (APR)
- Sales Tax
- Labor Income Tax
- Govt. Debt (share of trend GDP)
- Inflation (APR)

Agressive Tax Rule

- Output
- Real Interest Rate (APR)
- Sales Tax
- Labor Income Tax
- Govt. Debt (share of trend GDP)
- Inflation (APR)
Figure 3. Impulses to Public Invest. in Normal Times and in a 10 Quarter Liqu. Trap

Non-agressive Tax Rule

Output

Real Interest Rate (APR)

Public Infrast. Investment (GDP share)

Labor Income Tax

Govt. Debt (share of trend GDP)

Effective Capital Stock

Quarter

Agressive Tax Rule

Output

Real Interest Rate (APR)

Public Infrast. Investment (GDP share)

Labor Income Tax

Govt. Debt (share of trend GDP)

Effective Capital Stock

Quarter
Figure 4. Alternative Simulations of Impulses to Public Invest.

**Benchmark calibration**

- **Output**
  - **Public infrast. is not productive**
  - **Public infrast. is more productive**
  - **Public infrast. is sooner productive (1-2 years)**
  - **Public infrast. is later productive (5-10 years)**

- **Inflation (APR)**
  - **Normal Times**
  - **Liquidity Trap**
Figure 5. Impulses to Sales Taxes in Normal Times and in a 10 Quarter Liquidity Trap in the Full Model.
Figure 6. Simulations of Impulses to Sales Tax in Simplified and Full Models

**Sticky Price Model**

- Output
- Inflation (APR)

**Sticky Price - Sticky Wage Model**

- Output
- Inflation (APR)

**Sticky Price - Sticky Wage Model with Habit Consumption**

- Output
- Inflation (APR)

**Full Model**

- Output
- Inflation (APR)
Figure 7. Impulses to Sales Taxes with Aggr. Rule on Cap. Income Tax in the Full Model.

Output

Real Interest Rate (APR)

Total Consumption

HtM Consumption

Optimizing Households Consumption

Capital Income Tax (P.P.)

Government Debt (Trend Share)

Private Investment

Policy Rate (APR)

Inflation (APR)
Figure 8. Impulses to Public Invest. in Normal Times and in a 10 Quarter Liqu. Trap in the Full Model.

Non-agressive Tax Rule

Agressive Tax Rule
Figure 9. Alternative Simulations of Impulses to Public Invest. in the Full Model.

**Benchmark calibration**

- **Output**
  - Percent
  - Potential

- **Inflation (APR)**
  - Percent
  - Normal Times
  - Liquidity Trap

**Public infrast. is not productive**

- **Output**
  - Percent
  - Potential

- **Inflation (APR)**
  - Percent
  - Normal Times
  - Liquidity Trap

**Public infrast. is more productive**

- **Output**
  - Percent
  - Potential

- **Inflation (APR)**
  - Percent
  - Normal Times
  - Liquidity Trap

**Public infrast. is sooner productive (1-2 years)**

- **Output**
  - Percent
  - Potential

- **Inflation (APR)**
  - Percent
  - Normal Times
  - Liquidity Trap

**Public infrast. is later productive (5-10 years)**

- **Output**
  - Percent
  - Potential

- **Inflation (APR)**
  - Percent
  - Normal Times
  - Liquidity Trap
Appendix A. The Stylized New-Keynesian Model

Appendix A contains two parts. A.1 describes and derives the model used in Section 2, including both the benchmark model with sales taxes and distortionary labor income taxes, and the extended model with public investment. A.2 contains additional results referred to in the main text.

A.1. The Model

A.1.1. Households

The utility functional for the representative household is

\[
E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\gamma} \left( C_{t+j} - C \nu_{t+j} \right)^{1-\frac{1}{\gamma}} - \frac{N_{t+j}^{1+\chi}}{1+\chi} + \mu_0 F \left( \frac{MB_{t+j} + (h)}{P_{t+j}} \right) \right\}
\]  

(A.1)

where the discount factor \( \beta \) satisfies \( 0 < \beta < 1 \). The period utility function depends on the household’s current consumption \( C_t \) as deviation from a “reference level” \( C \nu_{t+j} \), where a positive taste shock \( \nu_t \) raises this reference level and thus the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked \( N_t \). Following Eggertsson and Woodford (2003), the subutility function over real balances, \( F \left( \frac{MB_{t+j} + (h)}{P_{t+j}} \right) \), is assumed to have a satiation point for \( MB/P \). Hence, inclusion of money - which is a zero nominal interest asset - provides a rationale for the zero lower bound on nominal interest rates. However, we maintain the assumptions that money is additive and that \( \mu_0 \) is arbitrarily small so that changes in real money balances have negligible implications for seignorage. Together, these assumptions imply that we can disregard the implications of money for government debt and output.

The household’s budget constraint in period \( t \) states that its expenditure on goods and net purchases of (zero-coupon) government bonds \( B_{G,t} \) must equal its disposable income:

\[
P_t (1 + \tau_{C,t}) C_t + B_{G,t} + MB_{t+1} = (1 - \tau_{N,t}) W_t N_t + (1 + i_{t-1}) B_{G,t-1} + MB_t - T_t + \Gamma_t \quad (A.2)
\]

Thus, the household purchases the final consumption good (at a price of \( P_t \)) and subject to a sales tax \( \tau_{C,t} \). Each household earns after-tax labor income \((1 - \tau_{N,t}) W_t N_t \) (\( \tau_{N,t} \) denotes the tax rate), pays a lump-sum tax \( T_t \) (this may be regarded as net of any transfers), and receives a proportional share of the profits \( \Gamma_t \) of all intermediate firms.
In every period $t$, the household maximizes the utility functional (B.8) with respect to its consumption, labor supply and bond holdings. Forming the Lagrangian and computing the first-order conditions w.r.t. $[ C_t \ N_t \ B_{G,t} ]$, we obtain

$$(C_t - C_{\nu t})^{-\frac{1}{2}} - \lambda_t P_t (1 + \tau_{C,t}) = 0,$$

$$-N_t^{\chi} + \lambda_t (1 - \tau_{N,t}) W_t = 0,$$

$$-\lambda_t + \beta (1 + i_t) E_t \lambda_{t+1} = 0,$$

and by defining $\Lambda_t \equiv \lambda_t P_t$ as the pre-tax cost of consumption in utility units, we can rewrite the first-order conditions as

$$\Lambda_t = \frac{(C_t - C_{\nu t})^{-\frac{1}{2}}}{(1 + \tau_{C,t})},$$

$$N_t^{\chi} = \Lambda_t (1 - \tau_{N,t}) \frac{W_t}{P_t},$$

$$\Lambda_t = \beta E_t \frac{(1 + i_t)}{1 + \pi_{t+1}} \Lambda_{t+1},$$

where we have introduced the notation $1 + \pi_{t+1} = P_{t+1}/P_t$.

By substituting out for $\Lambda_t$, we derive the consumption Euler equation

$$\frac{(C_t - C_{\nu t})^{-\frac{1}{2}}}{(1 + \tau_{C,t})} = \beta E_t \frac{(1 + i_t)}{1 + \pi_{t+1}} \frac{(C_{t+1} - C_{\nu t+1})^{-\frac{1}{2}}}{(1 + \tau_{C,t+1})},$$

and the following labor supply schedule

$$mrs_t \equiv \frac{N_t^{\chi}}{(C_t - C_{\nu t})^{-\frac{1}{2}}} = \frac{(1 - \tau_{N,t}) W_t}{(1 + \tau_{C,t}) P_t}.$$  

(A.3) and (A.4) are the key equations for the household side of the model.

### A.1.2. Firms

We assume a familiar setting with a continuum of monopolistically competitive firms to rationalize Calvo-style price stickiness. The framework in the stylized model is identical to that described below in the full model with capital (Appendix B.1.1), with two important exceptions. First, aggregate capital is assumed to be fixed, so that aggregate production is given by

$$Y_t = K_{t}^{\alpha} N_{t}^{1-\alpha}.$$  

(A.5)

Despite the fixed aggregate stock, shares of the aggregate capital stock can be freely allocated across the $f$ firms, implying that real marginal cost, $MC_t(f)/P_t$ is identical across firms and equal
to

\[
\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1 - \alpha) K^\alpha N_t^{-\alpha}}.
\]  

(A.6)

The second notable difference relative to the setup in the full model with capital is that here we do not allow for dynamic indexation to lagged inflation. Instead, all firms which are not allowed to reoptimize their prices in period \(t\) (which is the case with probability \(\xi_p\)), update their prices according to the following formula

\[
\tilde{P}_t = (1 + \pi) P_{t-1},
\]  

(A.7)

where \(\pi\) is the steady-state (net) inflation rate and \(\tilde{P}_t\) is the updated price.

Given Calvo-style pricing frictions, firm \(f\) that is allowed to reoptimize its price (\(P_{t,\text{opt}}(f)\)) solves the following problem

\[
\max_{P_{t,\text{opt}}(f)} \mathbb{E}_t \sum_{j=0}^{\infty} \xi_p \psi_{t,t+j} \left[ (1 + \pi)^j P_{t,\text{opt}}(f) - MC_{t+j} \right] Y_{t+j}(f)
\]

where \(\psi_{t,t+j}\) is the stochastic discount factor (the conditional value of future profits in utility units, i.e. \(\beta \mathbb{E}_t \lambda_{t+j}^{-1}\), recalling that the household is the owner of the firms), \(\theta_p\) the net markup and the demand for firm \(f\) is given by \(Y_{t+j}(f) = \left[ \frac{P_{t,\text{opt}}(f)}{P_t} \right]^{-(1+\theta_p)} Y_t\). The first-order condition is given by

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \xi_p \psi_{t,t+j} \left[ (1 + \pi)^j - \frac{1}{\theta_p} - \frac{(1 + \theta_p)}{\theta_p} \frac{1}{P_{t,\text{opt}}(f)} MC_{t+j} \right] Y_{t+j}(f) = 0,
\]

which after multiplying through by \(\frac{-1}{1+\theta_p} P_{t,\text{opt}}(f)\) can be rewritten as

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \xi_p \psi_{t,t+j} \left[ (1 + \pi)^j P_{t,\text{opt}}(f) - MC_{t+j} \right] Y_{t+j}(f) = 0
\]  

(A.8)

By implication of equations (B.2) and (A.7), the evolution of the final goods price is given by

\[
P_t = \left[ (1 - \xi_p) \left( P_{t,\text{opt}} \right) \frac{1}{\pi_p} + \xi_p ((1 + \pi) P_{t-1}) \frac{1}{\pi_p} \right]^{\theta_p}
\]  

(A.9)

where we have used the fact that all firms that reoptimize will set the same price (because they face the same costs for labor and capital), and that the updating price for the non-optimizing firms equals the past aggregate price (as we consider a continuum of firms which does not re-optimize).

A.1.3. Government

The evolution of nominal government debt is determined by the following equation

\[
B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau_c C_t P_t C_t - \tau_N W_t N_t - T_t - MB_{t+1} + MB_t
\]  

(A.10)
where $G_t$ denotes real government expenditures on the final good $Y_t$. Scaling with $1/(P_t Y)$, we obtain

$$\frac{B_{G,t}}{P_t Y} = \frac{(1 + \pi_t)}{(1 + \pi_t)} \frac{B_{G,t-1}}{P_{t-1} Y} + \frac{G_t}{Y} - \tau_{C,t} \frac{C_t}{Y} - \tau_{N,t} \frac{W_t N_t}{P_t Y} - \frac{T_t}{P_t Y} - \frac{MB_{t+1}}{P_t Y} + \frac{MB_t}{P_t Y}. \quad (A.11)$$

The government adjusts the labor-income tax rate to stabilize the evolution of government debt (as share of nominal trend GDP, $b_{G,t} \equiv \frac{B_{G,t}}{P_t Y}$) according to the rule (12).

Turning to the central bank, it is assumed to adhere to the non-linear Taylor-type policy rule (in log-linearized form) in equation (3), where $i$ denotes the steady-state (net) nominal interest rate, which is given by $r + \pi$ where $r \equiv 1/\beta - 1$.

### A.1.4. The Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let $Y_t^*$ denote the unweighted average (sum) of output for each firm $f$, i.e.

$$Y_t^* = \int_0^1 Y_t(f) df$$

Recalling that $Y_{t+j}(f) = \left[ \frac{P^*_t(f)}{P_t} \right]^{-\frac{(1+\theta_p)}{\theta_p}} Y_t$, it follows that

$$Y_t^* = \int_0^1 Y_t(f) df = \int_0^1 \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{(1+\theta_p)}{\theta_p}} Y_t df$$

$$= \left( \frac{1}{P_t} \right)^{-\frac{(1+\theta_p)}{\theta_p}} \left[ \left( \int_0^1 P_t(f)^{-\frac{(1+\theta_p)}{\theta_p}} df \right)^{-\frac{\theta_p}{(1+\theta_p)}} \right]^{-\frac{(1+\theta_p)}{\theta_p}} Y_t$$

$$= \left( \frac{P_t^*}{P_t} \right)^{-\frac{(1+\theta_p)}{\theta_p}} Y_t,$$

where $Y_t$ is aggregate output of the final good sector, as defined above, and $P_t^*$ is the indicated weighted average of the individual prices, defined as

$$P_t^* \equiv \left( \int_0^1 P_t(f)^{-\frac{(1+\theta_p)}{\theta_p}} df \right)^{-\frac{\theta_p}{(1+\theta_p)}}. \quad (A.12)$$

Notice how the weights for $P_t^*$ differ from what they are for the aggregate price level $P_t$ (see eq. B.2). Now, actual output is $Y_t$, and this is what is available to be divided into private consumption and government spending:

$$Y_t = C_t + G_t. \quad (A.13)$$
Using the definition of the production function (A.5), we can write the resource constraint in real terms as follows:

\[
\frac{C_t + G_t}{\equiv Y_t} \leq \left( \frac{P_t^*}{P_t} \right)^{(1+\theta_p)} K^{\alpha} N_t^{1-\alpha}. \tag{A.14}
\]

The sticky price distortion clearly introduces a wedge between input use and the output available for consumption (including by the government). Even so, this term vanishes in the log-linearized version of the model.

A.1.5. Equilibrium

We now collect the equilibrium relationships in the model and derive a log-linear approximation of the model.

Collecting the equations  First, we may regard the households equations (A.3) and (A.4) as determining \( C_t \) and \( N_t \), and marginal cost relation equation (A.6) as determining \( MC_t/P_t \), and the aggregate resource constraint (A.14) as determining the real wage \( W_t = P_t \). The Taylor-type policy rule determines the nominal interest rate \( i_t \), and the firms pricing equations (A.8) and (A.9) determines the evolution of the aggregate price level \( P_t \), whereas the (shadow) gross real interest rate \( 1 + r_t \) is determined by the Fisher relationship

\[
1 + r_t = E_t \frac{(1 + i_t)}{(1 + \pi_{t+1})}. \tag{A.15}
\]

Finally, the fiscal budget constraint (A.11) determines the evolution of government debt \( B_{G,t} \), and the final goods resource constraint (A.13) relate consumption and government spending to final output \( Y_t \). The other fiscal variables, \( G_t, \tau_{C,t}, \tau_{N,t} \) and \( \tau_t \), are exogenous or determined by policy rules.

Log-linear Approximation of Model  We will now derive the equations in Section 2 in turn. We start with the sticky price equilibrium conditions, and then discuss the flex-price equilibrium. In general, a log-linearized variable is denoted with lower case letters, and derived as

\[
x_t = \frac{dX_t}{X}, \tag{A.16}
\]

except in the special case \( X = 0 \) when the log-linearized variable is simply given by \( dX_t \) (e.g. government debt as share of nominal trend GDP, and the lump-sum tax rate). Moreover, for inflation and interest rates, we use the approximation that \( d(1 + x_t) \approx x_t \) because \( x_t \) is small.
Finally, notice that for distortionary tax rates, we use $d\tau_{X,t} \equiv \tau_{X,t}$ (thus, rather than introducing new notation, the tax rates are henceforth understood to be in deviations from their steady state level; this is also the case for the preference shock $\nu_t$).

Totally differentiating the government debt evolution equation (A.11), we obtain (dropping the seignorage term which is assumed to be arbitrarily small)

$$b_{G,t} = (1 + r) b_{G,t-1} + g_y g_t - c_y (\tau_c + \tau_c c_t) - \frac{1 - \alpha}{1 + \theta_p} (\tau_{N,t} + \tau_N \zeta_t + \tau_N n_t) - \tau_t + b_G (1 + r)(i_{t-1} - \pi_t),$$

(A.17)

where we have introduced the notation that $\zeta_t$ represents the real wage (as percent deviation from steady state, i.e. $d(W_t/P_t)/(W/P)$), defined $g_y \equiv G/Y$, and used that $\frac{W_N}{PY} = \frac{1 - \alpha}{1 + \theta_p} \equiv s_N$ and our simplifying assumption that $b_G = 0$. Assuming that the labor income tax is the only tax which balances the budget in steady state, it then follows that:

$$g_y = \frac{1 - \alpha}{1 + \theta_p} \tau_N,$$

(A.18)

implying that the log-linearized budget constraint in the benchmark model with lump-sum taxes can be written as (11) in Section 2.

To derive a log-linearized representation for real marginal cost, we work from the equation (A.6), which implies

$$m_c = \zeta_t - y_t + n_t = \zeta_t + \frac{\alpha}{1 - \alpha} y_t,$$

where the second equality follows from (A.5). By noting that real marginal cost is constant in the flex-price equilibrium, we have

$$\zeta_t^\text{pot} - y_t^\text{pot} + n_t^\text{pot} = \zeta_t^\text{pot} + \frac{\alpha}{1 - \alpha} y_t^\text{pot} = 0.$$

(A.19)

Accordingly, we can write (log-linearized) real marginal cost as

$$m_c = \left( \zeta_t - \zeta_t^\text{pot} \right) + \frac{\alpha}{1 - \alpha} \left( y_t - y_t^\text{pot} \right).$$

(A.20)

In order to write this equation solely in terms of the output gap,

$$x_t \equiv y_t - y_t^\text{pot},$$

(A.21)

we need to derive a log-linearized equation for the real wage. To obtain such a measure, we log-linearize equation (A.4) to obtain

$$\chi n_t + \frac{1}{\sigma (1 - \nu)} (c_t - \nu \nu_t) = \zeta_t - \frac{\tau_{N,t}}{1 - \tau_N} - \frac{\tau_{C,t}}{1 + \tau_C},$$

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again recalling that $\tau_{j,t}$ for $j = [N, C]$ and $\nu_t$ are to be interpreted as percentage point deviations. By log-linearizing and substituting the aggregate resource constraint in (A.13) into this expression, we obtain

$$\zeta_t = \chi n_t + \frac{1}{\sigma (1 - \nu)} \left( \frac{1}{1 - g_y} (y_t - g_y g_t) - \nu \nu_t \right) + \frac{\tau_{N,t}}{1 - \tau_N} + \frac{\tau_{C,t}}{1 + \tau_C},$$

and using (A.5), i.e. that $n_t = \frac{1}{1 - \alpha} y_t$, we finally derive the following expression for the log-linearized real wage:

$$\zeta_t = \left( \frac{\chi}{1 - \alpha} + \frac{1}{\sigma (1 - \nu)} (1 - g_y) \right) y_t - \frac{g_y}{\sigma (1 - \nu)} (1 - g_y) g_t - \frac{\nu}{\sigma (1 - \nu)} \nu_t + \frac{1}{1 - \tau_N} \tau_{N,t} + \frac{1}{1 + \tau_C} \tau_{C,t}.$$  

(A.22)

Next, we log-linearize the consumption Euler equation, (A.3), to get

$$-\frac{c_t - \nu \nu_t}{\sigma (1 - \nu)} = E_t \left[ \bar{\pi}_t - \bar{\pi}_{t+1} - \frac{1}{1 + \tau_c} \Delta \tau_{C,t+1} - \frac{c_{t+1} - \nu \nu_{t+1}}{\sigma (1 - \nu)} \right],$$

where we have used that

$$1 = \beta \frac{1 + \bar{i}}{1 + \bar{\pi}} = \beta (1 + r).$$

By substituting the log-linearized aggregate resource constraint (A.13) into this expression, and defining:

$$\hat{\sigma} \equiv \sigma (1 - \nu) (1 - g_y),$$

we obtain after some re-arranging:

$$y_t = E_t y_{t+1} - \hat{\sigma} (i_t - E_t \bar{\pi}_{t+1}) - g_y E_t \Delta g_{t+1} - (1 - g_y) \nu E_t \Delta \nu_{t+1} + \frac{\hat{\sigma}}{1 + \tau_c} E_t \Delta \tau_{C,t+1},$$  

(A.24)

which is the log-linearized IS curve equation. Using the labor supply equation (A.22) and labor demand equation (A.19) under flexible prices, we get

$$\left( \frac{\chi}{1 - \alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1 - \alpha} \right) y_t^{pot} = \left[ \frac{g_y}{\hat{\sigma}} (1 - g_y) g_t + \frac{\nu}{\sigma (1 - \nu)} \nu_t - \frac{1}{1 - \tau_N} \tau_{N,t}^{pot} - \frac{1}{1 + \tau_C} \tau_{C,t}^{pot} \right],$$

where we use the notation $z_t^{pot}$ for endogenous variables, and simply $z_t$ for exogenous variables. Notice that $\tau_{N,t}^{pot}$ for the moment is treated as an endogenous variable as it potentially depends on other endogenous variables via (12). Using the notation

$$\phi_{mc} \equiv \frac{\chi}{1 - \alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1 - \alpha},$$

(A.25)

the solution for potential output can be written

$$y_t^{pot} = \frac{1}{\phi_{mc} \hat{\sigma}} \left[ g_y g_t + (1 - g_y) \nu \nu_t - \frac{\hat{\sigma}}{1 - \tau_N} \tau_{N,t}^{pot} - \frac{\hat{\sigma}}{1 + \tau_C} \tau_{C,t}^{pot} \right].$$  

(A.26)
To get a tractable solution for the potential real interest rate, we use the definition in (A.23) to rearrange \( (A.24) \) as:

\[
    r_t^{pot} = \frac{1}{\sigma} E_t \Delta y_t^{pot} - \frac{g_y}{\sigma} E_t \Delta g_{t+1} - \frac{1 - g_y}{\sigma} \nu E_t \Delta \nu_{t+1} + \frac{1}{1 + \tau_c} E_t \Delta \tau_{C,t+1},
\]

and by substituting the expression for \( y_t^{pot} \) in (A.26) into this equation, we obtain

\[
    r_t^{pot} = \frac{1}{\sigma} E_t \left[ \frac{g_y}{1 - \tau_N} \Delta g_{t+1} + \frac{1 - g_y}{1 + \tau_c} \Delta \tau_{C,t+1} \right] - \frac{g_y}{\sigma} E_t \Delta g_{t+1} - \frac{1 - g_y}{\sigma} \nu E_t \Delta \nu_{t+1} + \frac{1}{1 + \tau_c} E_t \Delta \tau_{C,t+1},
\]

which can be rearranged as

\[
    r_t^{pot} = \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma \phi_{mc}} \right) E_t \left[ \frac{g_y}{1 - \tau_N} \Delta g_{t+1} - \frac{1 - g_y}{1 + \tau_c} \Delta \tau_{C,t+1} \right] + \frac{1}{1 + \tau_c} E_t \Delta \tau_{C,t+1},
\]

which is the general solution for the potential real interest rate.

**The Benchmark Model With Lump-sum Taxes** From the equations above, it is an easy task to derive the benchmark model with lump-sum taxes. In this version of the model, \( \tau_{N,t} = \tau_{C,t} = 0 \) for all \( t \). Accordingly, equation (5) follows from (A.27), and (4) follows from (A.26). The IS-curve (1) obtains from (A.24) which holds for actual and potential output, so that:

\[
    y_t - y_t^{pot} = \left( E_t y_{t+1} - \hat{\sigma} (i_t - E_t \pi_{t+1}) - g_y E_t \Delta g_{t+1} - (1 - g_y) \nu E_t \Delta \nu_{t+1} \right) - \left( E_t y_t^{pot} - \hat{\sigma} r_t^{pot} - g_y E_t \Delta g_{t+1} - (1 - g_y) \nu E_t \Delta \nu_{t+1} \right),
\]

which can be written as equation (1) by using the definitions (A.21) and (A.23).

As is well-known, log-linearization of (A.8) and (A.9) around the inflation target \( \pi \) results in the following Phillips curve

\[
    \pi_t = \beta E_t \pi_{t+1} = \frac{1 - \xi_p}{\xi_p} \left( 1 - \beta \xi_p \right) m_{c,t}. \tag{A.28}
\]

To write the model in terms of the output gap \( x_t \) instead of \( m_{c,t} \) as in the text, we use (A.20) and (A.22), which in the model with time-varying lump-sum taxes simplifies to

\[
    m_{c,t} = (\xi_t - \zeta_t^{pot}) + \frac{\alpha}{1 - \alpha} \left( y_t - y_t^{pot} \right) \tag{A.29}
\]

\[
    = \left( \frac{\chi}{1 - \alpha} + \frac{1}{\sigma (1 - \nu) (1 - g_y)} \right) \left( y_t - y_t^{pot} \right) + \frac{\alpha}{1 - \alpha} (y_t - y_t^{pot}) \tag{A.29}
\]

\[
    = \phi_{mc} x_t,
\]

where \( x_t \) is defined accordingly with (A.21) and \( \phi_{mc} \) is defined as in (A.25). Using this in (A.28), we obtain (2) with \( \kappa_p \) defined as in (7).
As mentioned previously, (11) obtains from (A.17) by using $\tau_C = 0$ and (A.18). Apart from the equations stated in the main text, we use (A.26) to compute $y_t^{pot}$, which enables us to compute actual output as $y_t = x_t + y_t^{pot}$. To get hours worked and real wage in (11), we use (A.22) and $n_t = \frac{1}{1-\alpha} y_t$.

**The Model With Exogenous Distortionary Labor-Income Taxes** In this variant of the model we have that $\tau_{N,t}$ varies *exogenously*, but we still assume that $\tau_{C,t} = \tau_C = 0$ for all $t$. Since labor income taxes are exogenous, all the model equations are identical, except that the labor income tax affects $r_t^{pot}$ according to (A.27) and $y_t^{pot}$ according to (A.26). In this version of the model, the IS-curve (1) is identical to the benchmark model. Finally, $\tau_{N,t}$ enters the government budget constraint (A.17).

**The Model With Endogenous Distortionary Labor-Income Taxes** In this variant of the model $\tau_{N,t}$ varies endogenously according to the rule given by equation (12 in Section 2.5, but we still assume $\tau_{C,t} = \tau_C = 0$ for all $t$. Also in this version of the model, the IS-curve (1) is identical to the benchmark model. However, the expression for marginal costs changes, because it follows from (A.22) that the following wedge between the actual and potential labor tax-rate will enter into marginal costs:

$$mc_t = \left( \zeta_t - \zeta_t^{pot} \right) + \frac{\alpha}{1-\alpha} \left( y_t - y_t^{pot} \right)$$

$$= \left( \frac{\chi}{1-\alpha} + \frac{1}{\sigma (1-\nu)(1-g_y)} \right) \left( y_t - y_t^{pot} \right) + \frac{1}{1-\tau_N} \left( \tau_{N,t} - \tau_{N,t}^{pot} \right) + \frac{\alpha}{1-\alpha} \left( y_t - y_t^{pot} \right)$$

$$= \phi_{mc} x_t + \frac{1}{1-\tau_N} \left( \tau_{N,t} - \tau_{N,t}^{pot} \right),$$

implying that a negative gap between the actual and potential labor income tax rate will put downward pressure on marginal costs and hence inflation. In all other aspects, this variant is identical to the model with exogenous $\tau_{N,t}$, with the exception that the exogenous process for $\tau_{N,t}$ is replaced with the rule (12).

**A.2. Additional Results in Stylized Model**

Below, we report and discuss briefly some additional results referred to in Section 3 of the main text.
Appendix B. The New-Keynesian Model with Keynesian Agents and Financial Frictions

This appendix contains two parts. Section B.1 describes the model used in Section 4. Section B.2 discusses some additional results referred to in the main text, including the construction of the baseline for our simulations, and a decomposition of the sources of improvement in the debt/GDP ratio that underlie a "fiscal free lunch."

B.1. The Model

The model is essentially a variant of the CEE/SW model augmented with “Keynesian” households, as in Erceg, Guerrieri and Gust (2006), and financial frictions, following Bernanke, Gertler and Gilchrist (1999). As such, our model incorporates nominal rigidities by assuming that labor and product markets exhibit monopolistic competition, and that wages and prices are determined by staggered nominal contracts of random duration (following Calvo (1983) and Yun (1996)). In addition, the model includes an array of real rigidities, including habit persistence in consumption, and costs of changing the rate of investment. Monetary policy follows a Taylor rule, and fiscal policy specifies that taxes respond to government debt.

B.1.1. Firms and Price Setting

*Final Goods Production* We assume that a single final output good $Y_t$ is produced using a continuum of differentiated intermediate goods $Y_t(f)$. The technology for transforming these intermediate goods into the final output good is constant returns to scale, and is of the Dixit-Stiglitz form:

$$Y_t = \left[ \int_0^1 Y_t(f)^{1+\theta_p} \, df \right]^{1+\theta_p}$$  \hspace{1cm} (B.1)

where $\theta_p > 0$.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$ that can be interpreted as the aggregate price index:

$$P_t = \left[ \int_0^1 P_t(f)^{\frac{1}{\theta_p}} \, df \right]^{-\theta_p}$$  \hspace{1cm} (B.2)
Intermediate Goods Production

A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand function for its output good that varies inversely with its output price $P_t(f)$, and directly with aggregate demand $Y_t$:

$$Y_t(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\theta_p} Y_t \quad (B.3)$$

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^\alpha L_t(f)^{1-\alpha} \quad (B.4)$$

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{Kt}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun style staggered nominal contracts. In each period, each firm $f$ faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, we follow Christiano, Eichenbaum and Evans (2005) by assuming that it adjusts its price by a weighted combination of the lagged and steady state rate of inflation, i.e.,

$$P_t(f) = \pi_{t-1}^{\tau_p} \pi_{t-\tau_p}^{1-\tau_p} P_{t-1}(f)$$

where $0 \leq \tau_p \leq 1$. A positive value of $\tau_p$ introduces structural inertia into the inflation process.

B.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household’s labor services $N_t(h)$, $h \in [0, 1]$, as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The
labor index $L_t$ has the Dixit-Stiglitz form:

$$L_t = \left[ \int_0^1 N_t(h) \frac{1}{1 + \theta_w} \, dh \right]^{1 + \theta_w} \quad (B.5)$$

where $\theta_w > 0$. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost $W_t$:

$$W_t = \left[ \int_0^1 W_t(h) \frac{1}{1 + \theta_w} \, dh \right]^{-\theta_w} \quad (B.6)$$

It is natural to interpret $W_t$ as the aggregate wage index. The aggregator’s demand for the labor hours of household $h$ – or equivalently, the total demand for this household’s labor by all goods-producing firms – is given by

$$N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\frac{1 + \theta_w}{\theta_w}} L_t \quad (B.7)$$

The utility functional of a typical member of household $h$ is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1 - \sigma} C_{t,j}(h) - \kappa C_{t,j-1} - \nu_c \nu_t \right\}^{1 - \sigma} - \frac{\lambda^0}{1 + \lambda} N_{t+j}(h)^{1+\lambda} \right\} \quad (B.8)$$

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$’s current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for the possibility of external habit persistence (Smets and Wouters 2003). As in the simple model considered in the previous section, a positive taste shock $\nu_t$ raises the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked $N_t(h)$.

Household $h$’s budget constraint in period $t$ states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_t C_t(h) + P_t I_t(h) + \frac{1}{2} \psi_t P_t \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)} +$$

$$P_{B,t} B_{G,t+1} - B_{G,t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \quad (B.9)$$

$$= (1 - \tau_N) W_t(h) N_t(h) + (1 - \tau_K) R_{K,t} K_t(h) + \delta \tau_K P_t K_t(h) + \Gamma_t(h) - T_t(h)$$

Thus, the household purchases the final output good (at a price of $P_t$), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. The total cost of investment to each household
$h$ is assumed to depend on how rapidly the household changes its rate of investment (as well as on the purchase price). Our specification of investment adjustment costs as depending on the square of the change in the household’s gross investment rate follows Christiano, Eichenbaum, and Evans (2005). Investment in physical capital augments the household’s (end-of-period) capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1 - \delta)K_t(h) + I_t(h)$$  \hspace{1cm} (B.10)

In addition to accumulating physical capital, households may augment their financial assets through increasing their government bond holdings $(P_{B,t}B_{G,t+1} - B_{G,t})$, and through the net acquisition of state-contingent bonds. We assume that agents can engage in frictionless trading of a complete set of contingent claims. The term $\int \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time $t$. Each member of household $h$ earns after-tax labor income $(1 - \tau_{N,t})W_t(h)N_t(h)$, after-tax capital rental income of $(1 - \tau_K)R_{K,t}K_t(h)$, and a depreciation allowance of $\delta\tau_{R}P_tK_t(h)$. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (this may be regarded as taxes net of any transfers).

In every period $t$, each member of household $h$ maximizes the utility functional (B.8) with respect to its consumption, investment, (end-of-period) capital stock, bond holdings, and holdings of contingent claims, subject to its labor demand function (B.7), budget constraint (B.9), and transition equation for capital (B.10). Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described above. Thus, the probability that a household receives a signal to reoptimize its wage contract in a given period is denoted by $1 - \xi_w$. In addition, we specify a dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize, i.e., $W_t(h) = \omega_{t-1}^{\text{w}}\pi^{1-\text{w}}W_{t-1}(h)$, where $\omega_{t-1}$ is gross nominal wage inflation in period $t - 1$. Dynamic indexation of this form introduces some structural persistence into the wage-setting process.

### B.1.3. Fiscal and Monetary Policy and the Aggregate Resource Constraint

Government purchases $G_t$ are assumed to follow an exogenous AR(1) process with a persistence coefficient of 0.9. Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. Government expenditures are financed by a combination of labor, capital, and lump-sum taxes. The government does not need to balance
its budget each period, and issues nominal debt to finance budget deficits according to:

\[ P_{B,t}B_{G,t+1} - B_{G,t} = P_I G_t - T_t - \tau_{C,t} P_I C_t - \tau_{N,t} W_t L_t - \tau_{K,t} (R_{K,t} - \delta P_I) K_t. \]  

(B.11)

In eq. (B.11), all quantity variables are aggregated across households, so that \( B_{G,t} \) is the aggregate stock of government bonds and \( K_t \) is the aggregate capital stock, and \( T_t = (\int_0^1 T_t(h) \, dh) \) aggregate lump-sum taxes. In our benchmark specification, the lump-sum and capital tax rate is held fixed, and labor-income taxes adjust endogenously according to a tax rate reaction function that allows taxes to respond to debt subject to smoothing. In log-linearized form:

\[ \tau_{N,t} - \tau_N = \varphi_T \left( \tau_{N,t-1} - \tau_N \right) + (1 - \varphi_T) \varphi_b \left( \tilde{b}_{G,t} - \bar{b}_G \right), \]  

(B.12)

where \( \tilde{b}_{G,t} \equiv \frac{B_{G,t}}{YY} \). As the difference between lump-sum and distortionary tax financing can potentially be important in long-lived liquidity traps, we choose to work with distortionary tax financing – which we think is more empirically plausible – in this paper.

Monetary policy is assumed to be given by a Taylor-style interest rate reaction function similar to equation (3) except allowing for a smoothing coefficient \( \gamma_t \):

\[ i_t = \{ \max (-i, (1 - \gamma_t) (\gamma_T \pi_t + \gamma_x x_t) + \gamma_t i_{t-1}) \} \]  

(B.13)

Finally, total output of the service sector is subject to the resource constraint:

\[ Y_t = C_t + I_t + G_t + \psi_{I,t} \]  

(B.14)

where \( \psi_{I,t} \) is the adjustment cost on investment aggregated across all households (from eq. B.9, \( \psi_{I,t} \equiv \frac{1}{2} (I_t(h) - I_{t-1}(h))^2 \)).

**B.1.4. Keynesian Households**

In the full with non-Ricardian households, we assume that a fraction \( s_{kh} \) of the population consists of “Keynesian” households whose members consume their current after-tax income each period, and set their wage equal to the average wage of the optimizing households. Because all households face the same labor demand schedule, each Keynesian household works the same number of hours as the average optimizing household. Thus, the consumption of Keynesian households \( C^K_t(h) \) is simply determined as

\[ P_I C^K_t(h) = (1 - \tau_{N,t}) W_t(h) N_t(h) - T_t, \]

where \( T_t \) denotes (net) lump-sum taxes. Consumption of the non-Keynesian households is given the consumption Euler equation derived by maximizing (B.8) subject to (B.9).
B.1.5. Production of capital services

We build on the model described above by incorporating a financial accelerator mechanism following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, the intermediate goods producers rent capital services from entrepreneurs (at the price $R_{Kt}$) rather than directly from households. Entrepreneurs purchase capital from competitive capital goods producers, with the latter employing the same technology to transform investment goods into finished capital goods as described by equations B.10) and B.9). To finance the acquisition of physical capital, each entrepreneur combines his net worth with a loan from a bank, for which the entrepreneur must pay an external finance premium (over the risk-free interest rate set by the central bank) due to an agency problem. We follow Christiano, Motto and Rostagno (2008) by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist, 1999). Banks obtain funds to lend to the entrepreneurs by issuing deposits to households at the interest rate set by the central bank. By assuming perfect competition and free entry among banks and that all bank portfolios are well diversified (i.e., that each bank lends out to a continuum of entrepreneurs, whose default risk is independently distributed), it follows that banks make zero profits in each state of the economy and that there is no credit risk to households associated with bank deposits.\footnote{We refer to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2008) for further details. An excellent exposition is also provided in Christiano, Trabandt and Walentin (2007).}

B.1.6. Solution and Calibration

To analyze the behavior of the model, we log-linearize the model’s equations around the non-stochastic steady state. Nominal variables, such as the contract price and wage, are rendered stationary by suitable transformations. To solve the unconstrained version of the model, we compute the reduced-form solution of the model for a given set of parameters using the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980).

When we solve the model subject to the non-linear monetary policy rule (B.13), we use the techniques described in Hebden, Lindé and Svensson (2009). An important feature of the Hebden, Lindé and Svensson algorithm is that the duration of the liquidity trap is endogenous, and is affected by the size of the fiscal impetus. Their algorithm consists of adding a sequence of current and future innovations to the linear component of the policy rule to guarantee that the zero bound
constraint is satisfied given the economy’s state vector. The sequence of innovations is assumed to be correctly anticipated by private agents at each date. This solution method is easy to use, and well-suited to examine the implications of the zero bound constraint in models with large dimensional state spaces; moreover, it yields identical results to the method of Jung, Terinishi, and Watanabe (2005).

As in Section 2, we set the discount factor $\beta = 0.995$, and steady state (net) inflation $\pi = .005$, implying a steady state nominal interest rate of $i = .01$ at a quarterly rate. The subutility function over consumption is logarithmic, so that $\sigma = 1$, and the parameter determining the degree of habit persistence in consumption $\chi$ is set at 0.6 (similar to the empirical estimate of Smets and Wouters 2003). The Frisch elasticity of labor supply $\frac{1}{\chi}$ of 0.4 is well within the range of most estimates from the empirical labor supply literature (see e.g. Domeij and Flodén, 2006).

The capital share parameter $\alpha$ is set to 0.35. The quarterly depreciation rate of the capital stock $\delta = 0.025$, implying an annual depreciation rate of 10 percent. We set the cost of adjusting investment parameter $\psi_I = 3$, which is somewhat smaller than the value estimated by Christiano, Eichenbaum, and Evans (2005) using a limited information approach; however, the analysis of Erceg, Guerrieri, and Gust (2006) suggests that a lower value may be better able to capture the unconditional volatility of investment.

We maintain the assumption of a flat Phillips curve by setting the price contract duration parameter $\xi_p = 0.93$. As in Christiano, Eichenbaum and Evans (2005), we also allow for a fair amount of intrinsic persistence by setting the price indexation parameter $\iota_p = 0.9$. It bears emphasizing that our choice of $\xi_p$ does not necessarily imply an average price contract duration of 14 quarters. Altig et al. (2011) show in a model very similar to ours that a low slope of the Phillips curve can be consistent with frequent price reoptimization if capital is firm-specific, at least provided that the steady-state markup is not too high, and it is costly to vary capital utilization; both of these conditions are satisfied in our model, as the steady state markup is 10 percent ($\theta_p = .10$), and capital utilization is fixed. Specifically, our choice of $\xi_p$ implies a Phillips curve slope of about 0.007. Given strategic complementarities in wage-setting across households, the wage markup influences the slope of the wage Phillips curve. Our choices of a wage markup of $\theta_W = 1/3$ and a wage contract duration parameter of $\xi_w = 0.88$—along with a wage indexation parameter of $\iota_w = 0.9$—imply that wage inflation is about as responsive to the wage markup as price inflation is to the price markup.

The parameters of the monetary policy rule are set as $\gamma_i = 0.7$, $\gamma_{\pi} = 3$ and $\gamma_x = 0.25$. 

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These parameter choices are supported by simple regression analysis using instrumental variables over the 1993:Q1-2008:Q4 period. This analysis suggests that the response of the policy rate to inflation and the output gap has increased in recent years, which helps account for somewhat higher response coefficients than typically estimated when using sample periods which include the 1970s and 1980s. Overall, as noted in the main text, our calibration of the monetary policy rule and the Phillips Curve slope parameters tilts in the direction of reducing the sensitivity of inflation to macroeconomic shocks.

We set the population share of the Keynesian households to optimizing households, $s_{kh}$, to 0.47, which implies that the Keynesian households’ share of total consumption is about 1/3. This calibration perhaps overstates the role of non-Ricardian households in affecting consumption behavior, but seems useful to help put plausible bounds on how the multiplier may vary with the degree of non-Ricardian behavior in consumption (recognizing that the CEE/SW workhorse model is a special case in which $s_{kh} = 0$ and there are no financial frictions). Our calibration of the parameters affecting the financial accelerator follow BGG (1999). Thus, the monitoring cost, $\mu$, expressed as a proportion of entrepreneurs’ total gross revenue, is 0.12. The default rate of entrepreneurs is 3 percent per year, and the variance of the idiosyncratic productivity to entrepreneurs is 0.28.

The share of government spending of total expenditure is set equal to 20 percent. The government debt to GDP ratio is 0.5, close to the total estimated U.S. federal government debt to output ratio at end-2009. The steady state capital income tax rate, $\tau_K$, is set to 0.2, while the lump-sum tax revenue to GDP ratio is set to 0.02. For simplicity, we set the depreciation allowance $\delta \tau_K = 0$. Given these choices, the government’s intertemporal budget constraint implies that labor income tax rate $\tau_N$ equals 0.27 in steady state. The parameters in the fiscal policy rule in equation (B.12) are set to $\varphi_r = 0.92$, $\varphi_b = 0.1$ following the evidence in Traum and Yang (2011), implying that the tax rule is not very aggressive. Importantly, given the low share of government revenue accounted for by lump-sum taxes, most of the variation in the government budget deficit reflects fluctuations in revenue from the capital and labor income tax (due to variations in the tax base), and the service cost of debt.