COMPETITION, R&D, AND 
THE COST OF INNOVATION

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Abstract.

This paper proposes a model in the spirit of Aghion et al. (2005) that encompasses the magnitude of the impact of competition on R&D according to the cost of the innovation. The effect of competition on R&D is an inverted U-shape. However, the shape is flatter and competition policy is therefore less relevant for innovation when innovations are relatively costly. Intuitively, if innovations are costly for a firm, competitive shocks have to be significant to alter its innovation decisions. Empirical investigations using a unique panel dataset from the Banque de France show that an inverted U-shaped relationship can be clearly evidenced for the largest firms, but the curve becomes flatter when the relative cost of R&D increases. For large costs, the relationship even vanishes. Consequently, in sectors in which innovations are costly, policy changes have to be on a very large scale for an impact to be expected; at the extreme end, in certain sectors, the curve is so flat that competition policy is not an appropriate tool for boosting the research effort of firms.

Keywords: Competition, R&D, innovation.
JEL classification: L51, O31.

Résumé.

Nous proposons dans cet article un modèle qui rend compte de l’importance des effets de la concurrence sur les efforts de R&D relativement au coût de l’innovation. La relation entre concurrence et R&D a une forme de U inversé. Néanmoins, lorsque les innovations sont coûteuses, cette relation s’aplatisit et les politiques de concurrence en faveur de l’innovation sont moins pertinentes. Un travail empirique mené sur un panel unique de données d’entreprises issu de la Banque de France confirme cette relation en U inversé. Mais la courbure de cette relation diminue lorsque le coût relatif de la R&D augmente, pour disparaître complètement pour des coûts d’innovation très importants. En conséquence, les changements de politiques économiques dans les secteurs où les innovations sont coûteuses doivent être importants afin d’en attendre un effet; dans certains secteurs, la courbe est même si plate que la politique de concurrences ne semble pas être un outil approprié pour amplifier les efforts d’innovation de ces entreprises.

Mots-Clés : concurrence, recherche et développement, innovation.
Classification JEL : L51, O31.
Non technical summary.

In this paper we investigate the inverted-U curve which connects competition and innovation, taking into account the effects of size and cost related to innovation on this shape. Based on a model that encompasses these features, we show that whatever the predominant effect (escape from competition or disincentive effect), when the size of incremental innovation of a sector increases, or the size of the firm decreases as compared to the cost of innovation, the slope of the inverted-U shape declines; thus the relationship becomes flatter.

These theoretical predictions are corroborated by estimations based on French firms panel data over the 1991-2004 period from FiBen database, held at the Companies Observatory of the Banque de France. A first set of results shows that, excluding interactions related to innovation cost or firm size, only largest firms evolve significantly on an inverted-U shape relationship. A second run of econometric findings including such interactions points out significant evidence of the inverted-U shape over the whole sample of firms.

We conclude that effects on innovation from fostering competition policies could have a limited impact in sectors where innovation costs are relatively high, or in sectors where firms are relatively small.

Résumé non technique.

Nous nous intéressons dans ce papier à la forme en U inversé qui relie la concurrence et l’innovation, en tenant compte des impacts que pourrait avoir sur cette forme le coût relatif à la taille de l’entreprise de ces innovations. Sur la base d’un modèle théorique qui rend compte de ces spécifications, nous montrons que quel que soit l’effet qui prédomine (fuite de la concurrence ou effet désincitatif), lorsque la taille de l’innovation incrémentale d’un secteur augmente, ou que la taille de l’entreprise relative au coût de l’innovation diminue, la pente de la courbe en U inversé tend à diminuer en valeur absolue, ce qui signifie que la courbe s’aplatit.

Ces prédictions théoriques sont corroborées par des estimations effectuées à partir d’un panel d’entreprises françaises sur la période 1991-2004, extrait de la base Fiben et de la Centrale de Bilans, toutes deux gérées par la Banque de France. Un premier jeu de résultats montre que sans tenir compte des interactions relatives au coût de l’innovation ou à la taille de l’entreprise, la relation en U inversé n’est validée statistiquement que sur un panel restreint aux firmes les plus grosses. En revanche, dans un deuxième temps, la prise en compte des termes d’interactions permet de confirmer l’existence d’une courbe en U inversé pour l’ensemble de l’échantillon.
Nous en concluons que l’impact sur l’innovation des mesures en faveur de la concurrence pourrait avoir un effet limité dans les secteurs où les coûts de l’innovation sont relativement élevés et dans les secteurs où les entreprises sont relativement petites.
1 Introduction

The debate about the impact of market structure, and thus competition policy on innovation is still vigorous. The classic opposition is between Schumpeter (1942), Arrow (1962), Dasgupta and Stiglitz (1980), etc. Schumpeter (1942) argues that a firm in a monopoly position has an incentive to innovate to prevent entry; a monopoly position may also ensure long-term objectives and can prompt making some risky investments in R&D. Some empirical evidence seems to support the Schumpeterian view. For example, recently, Blundell et al. (1999) have found a positive correlation between individual *ex ante* market share and innovation.

However, following Arrow (1962), under perfect *ex post* appropriation, the profit margins may be larger in an *ex ante* competitive industry than under monopoly. Aghion et al. (2002, 2005) attempt to reconcile the two approaches in a model that captures both mechanisms. Their model exhibits an inverted U-shape relationship between innovation and competition. In this model, competition may increase the innovation profit margin for firms close to the technological frontier (since they escape competition) but strong competition could also reduce incentives to innovate for laggards (disincentive effect).

The goal of our paper is to extend this combined approach by studying the magnitude of the impact of competition on innovation behavior. We propose a model that takes into account firm size and/or the size of the innovation in the sector. Intuitively, if innovations are large-scale and costly in the firm’s sector, competitive shocks have to be large to change its innovation choices. Therefore, the inverted U-shape is flatter and competition policy is therefore less relevant for innovation when innovations are costly (or firm size is small).

Empirical investigations using a unique panel dataset from the Banque de France confirm a clear inverted U-shaped relationship for the largest French firms. This result is consistent with Aghion et al. (2005)’s findings on UK listed firms. But the curve flattens when the relative cost of R&D increases: when the sectoral cost of innovation relative to the firm’s size is large, changes in the competitive position of the firm does not seem robustly associated with changes in R&D intensity. For large costs, the relationship even vanishes: competition does not seem to impact firm R&D behavior.

These results are also related to the literature on innovation decisions that stresses the role of firm size (e.g. Cohen and Klepper, 1996). It may be easier to finance R&D in large firms because they may have a reputation and enjoy deeper relations with external investors or bank lenders. They may also gain through a “too big to fail” mechanism. Finally, because of sunk costs
associated with innovation investment, large firms have more incentive to engage in innovative activities. Empirical evidence seems to support this view. For example, Savignac (2006) shows that French firms with plans to innovate face financial constraints that reduce the likelihood of their embarking on such projects, and that these constraints decrease with firm size.

The paper is organized as follows. Section 2 is devoted to detail the model and the theoretical predictions. Econometric strategies and data are presented in section 3, that also provides the key empirical findings. Some perspectives are given in a last section.

2 Theoretical framework

2.1 Basic elements of the model

We present in this section a simple model that can encompass the role of the cost of innovation and the market’s—or similarly the firm’s—size as regards the impact of competition on R&D. This model is an extension of the standard Aghion et al. (2005)’s framework.

Households and final goods

We assume that a unit mass of homogenous households supplying labor inelastically seeks to maximize discounted sum of logarithmic instantaneous utility flows with a constant rate $r$. The argument of these utility functions is the consumption good $y$ which is produced according to the following production function $\ln(y_t) = \int_0^1 \Phi_j \ln(x_{jt})dj$ with $\Phi_j > 0 \ \forall j$ and $\int_0^1 \Phi_jdj = 1$, where $\Phi_j$ represents the weight in the utility function of intermediate input $x_j$. Intermediate goods $j$ are yielded by duopolists $A$ and $B$ that combine their perfectly substitutable production according to the relation $x_j = x_{Aj} + x_{Bj}$. The assumptions upon which the model is built allow us for choosing the numeraire for the prices of intermediate goods in each sector by normalizing the households’ current expenditure in good $j$ proportionally to its weight in the utility function, such that $p_jx_j = \Phi_j$.

Intermediate production

Intermediate firms produce goods from labor with constant returns to scale taking the wage rate as given, which leads to independent of quantities produced unit cost of production for each duopolist. Contrary to Aghion et al. (2005), we can not normalize labor to one because $\Phi_j$’s are heterogeneous. We assume that one unit of labor employed by each intermediate duopolist

1Note that Aghion et al. (2005) take a particular form for which $\Phi_j = 1 \ \forall j$.

2In the remaining part of the paper, sectoral subscript $j$ is omitted as long as it does not create confusion.
generates outflow equal to:
\[
\Gamma_i = \gamma^{k_i}, \quad i = \{A, B\},
\] (1)

where \( k_i \) is the technology level of duopoly firm \( i \) in a certain sector, and \( \gamma > 1 \) is the size of the leading-edge innovation. The total output of a duopolist is
\[
L_i \Gamma_i = L_i \gamma^{k_i}, \quad i = \{A, B\},
\] (2)

where \( L_i \) is the amount of labor devoted to production by the firm. By the same way, we define \( \bar{\pi}_m \) and \( \bar{\pi}_{-m} \) to be the equilibrium profit flow by employee of a firm \( m \) steps ahead, respectively behind, its rival. Hence, the economy is composed of two types of sector: either leveled (neck-and-neck) type, where there is no technology gap, or unleveled where a leader and a follower coexist.

**R&D**

We assume a R&D cost function \( \psi(n) = \beta n^2 / 2 \), where \( \beta \) is an increasing function of the size of innovation \( \gamma \) with \( \lim_{\gamma \to 1} \beta = 0 \). \( \psi(n) \) defines the total cost that a leading firm have to spend to gain one technological step according to a Poisson process of parameter \( n \). This cost can also be spent by a following firm to move a step forward with hazard rate \( n + h \), where \( h \) represents the opportunity gain to copy the leader, even if no R&D efforts are made. From now, \( n_0 \) denotes the R&D efforts of each firm in a neck-and-neck sector and \( n_m \) (resp. \( n_{-m} \)) those of a leader (resp. follower) firm in an unleveled industry.\(^3\)

**Product market competition**

To complete the model, we describe the profit flow for each type of intermediate firms. In each unleveled sector, all profits are kept by the leader firm, say \( A \) for instance, so that it receives the difference between its revenue \( p_A x_A \) and the total cost of production \( c_A x_A \), where \( p_A \) and \( c_A \) are respectively the price and the production cost of one unit produced by the leader firm. The market structure à la Bertrand inside the sector implies that the leader’s price equals production cost of the follower. Because of perfect substitution among intermediate goods and the Bertrand competition, the follower does not produce and makes no profit. But the firm still exists and may for example conduct research in order to reach the technological frontier. Based on our normalization, we have thus \( c_A x_A = c_A \Phi / p_A = c_A \Phi / c_B \). Since one unit of labor can produce \( \gamma^{k_i} \) units of goods for \( i \in \{A, B\} \), firms need \( x_i / \gamma^{k_i} \) units of labor to produce \( x_i \). So the unit cost of production is \( c_i = \omega / \gamma^{k_i} \), where \( \omega \) is the wage rate assumed to be the same among the firms.

\(^3\)It is worthwhile to bear in mind that the catch-up process prevents the leader firm to innovate, so that its R&D efforts in the case where the maximum sustainable gap is \( m = 1, n_1 \), are zero.
Hence, we have \( c_A x_A = \Phi \gamma^{-k_A} = \Phi \gamma^{-m} \). As a result, profits of the leader firm are given by \( \tilde{\pi}_m = \pi_m \Phi \) with \( \pi_m = 1 - \gamma^{-m} \). On the contrary, follower firm makes no profit so that \( \tilde{\pi}_{-m} = 0 \).

As regards neck-and-neck industries, the profit flows depend on which extent the duopolists collude, according to the assumption of a competition à la Bertrand. As a result, the profit of a leveled firm is comprised between zero and the half of what a monopolist could earn, which leads to \( \tilde{\pi}_0 = \pi_0 \Phi = \varepsilon \pi_1 \Phi \), with \( 0 \leq \varepsilon \leq 1/2 \) and \( \nu = 1 - \varepsilon \) is a global measure of product market competition.

Furthermore, it is worth noting that the size of duopolists of sector \( j \) is directly related—more precisely proportional—to the magnitude of its nominal demand. Indeed, according to the model and assuming a constant wage rate \( \omega \), the workforce of firms in leveled sector is given by:

\[
L_{\text{neck-and-neck}} = \frac{\Phi}{2 \omega} (1 - 2 \pi_0) \tag{3}
\]

whereas the leader firm’s size can be written as

\[
L_{\text{leader}} = \frac{\Phi}{\omega} (1 - \pi_m). \tag{4}
\]

### 2.2 Equilibrium research efforts

**Bellman equations**

This subsection determines the equilibrium conditions in the model. Let \( V_{-m}, V_m, \) and \( V_0 \) denote respectively the steady state value of being currently a follower, a leader, and a neck-and-neck firm. We have standard Bellman asset equations:

\[
r V_m = \pi_m \Phi + n_m (V_{m+1} - V_m) + (n_{m-1} + h)(V_{m-1} - V_m) - \beta \frac{n_m^2}{2}, \tag{5}
\]

\[
r V_{-m} = \pi_{-m} \Phi + n_m (V_{-m+1} - V_{-m}) + (n_{-m} + h)(V_{-m-1} - V_{-m}) - \beta \frac{n_{-m}^2}{2}, \tag{6}
\]

\[
r V_0 = \pi_0 \Phi + n_0 (V_1 - V_0) + \bar{n}_0 (V_{-1} - V_0) - \beta \frac{n_0^2}{2}, \tag{7}
\]

where \textit{ex post} \( \bar{n}_0 = n_0 \) represents the R&D intensity by the other duopolist in a leveled sector, which is identical in the Nash equilibrium. The first order conditions give:

\[
V_{m+1} - V_m = \beta n_m, \tag{8}
\]

\[
V_{-m+1} - V_{-m} = \beta n_{-m}, \tag{9}
\]

\[
V_1 - V_0 = \beta n_0. \tag{10}
\]
The one-step case

As Aghion et al. (2005), we have to restrict \( m = 1 \) in order to be able to obtain a closed-form solution for the model. This means that the maximum sustainable gap is one: if a leader innovates, then the follower can imitate the leader’s past technology with no cost.

In the case where \( m = 1 \), we can rewrite the previous systems as follows:

\[
\begin{align*}
    rV_1 &= \pi_1 \Phi + (n_{-1} + h)(V_0 - V_1), \\
    rV_{-1} &= \pi_{-1} \Phi + (n_{-1} + h)(V_0 - V_{-1}) - \beta \frac{n_{-1}^2}{2}, \\
    rV_0 &= \pi_0 \Phi + n_0(V_1 - V_0) + n_0(V_{-1} - V_0) - \beta \frac{n_0^2}{2},
\end{align*}
\]

Then, the first order conditions give:

\[
\begin{align*}
    V_0 - V_{-1} &= \beta n_{-1}, \\
    V_1 - V_0 &= \beta n_0.
\end{align*}
\]

According to these first order conditions, as an increase in market competition diminishes profits of a leveled firm, and consequently its market value, \( V_0 \) decreases and one could expect an increase of \( n_0 \) and a decline in \( n_{-1} \), as schematically described by Figure 1. Therefore, these effects are less pronounced when the size, and \textit{a fortiori} the cost of innovation \( \beta \), increases.

![Figure 1: Impact of the competition on innovation according to the cost of innovation](image)

(a) Neck-and-neck sectors  
(b) Unleveled sectors

Validating these intuitions from a more rigorous viewpoint requires to write the solutions of the equilibrium in an analytical way. The following section presents such developments.
2.3 Analytical solutions

First order conditions combined with the Bellman equations imply the following reduced form R&D equations:

\[
\beta \frac{n_0^2}{2} + \beta (r + h)n_0 - (\pi_1 - \pi_0)\Phi = 0 \quad (16)
\]

\[
\beta \frac{n_{-1}^2}{2} + \beta (r + h + n_0)n_{-1} - n_0\Phi - \beta \frac{n_0^2}{2} = 0 \quad (17)
\]

Neck-and-Neck sector

The solution of equation (16) gives the equilibrium research intensity for the leveled firm:

\[
n_0 = -(r + h) + \sqrt{(r + h)^2 + \frac{2\nu\pi_1\Phi}{\beta}}
\]

Taking into account the extend of innovation, we have the following proposition:\footnote{This property includes the Aghion et al. (2005)' result that the R&D efforts in neck-and-neck sectors, \( n_0 \), is an increasing function in the competition index.}

**Proposition 1** Assuming a convex shape in the cost of innovation for \( \beta \), the slope of the relationship between innovation and competition in the neck-and-neck sectors is a non negative, decreasing function of the size of innovation \( \gamma \).

**Proof.** See Appendix A

The size of the demand for intermediate good could improve the relationship established in Proposition 1, as described in the following corollary:

**Corollary 1** For a given size of innovation, the lower the size of the demand \( \Phi \), the lower the effect of competition on research activities in the leveled sector.

**Proof.** See Appendix A

The intuition behind proposition 1 and corollary 1 relies on the fact that for industries where costly innovations take place, gains in terms of research efforts are mitigated by the limited impact of competition on firms’ decision to innovate. On the contrary, R&D activities are stimulated in sectors related to cheap innovations in order to escape from competition with less difficulties. Furthermore, these effects are more pronounced in large firms related to sectors for which the demand is large.
Unleveled sector

As for the neck-and-neck sectors, we find in a similar way the equilibrium research intensity for the laggard firm from equation (17):

\[ n_{-1} = -(r + h + n_0) + \sqrt{(r + h)^2 + n_0^2 + \frac{2\pi_1 \Phi}{\beta}} \]  \hspace{1cm} (19)

Once again, taking into account the cost of innovation, we have the following proposition:\(^5\)

**Proposition 2** The slope of the relationship between innovation and competition in the unleveled sectors is a non positive, increasing function of the size of innovation \( \gamma \).

**Proof.** See Appendix A □

This proposition comes together with the following corollary:

**Corollary 2** For an either small or large innovation, the lower the extend of the demand \( \Phi \), the lower the effect of competition on research activities in the unleveled sector.

**Proof.** Demonstration follows proof of corollary 1. □

Aggregate innovation

We now derive the aggregate flow of innovations \( I \) from \( \mu_1 \) and \( \mu_0 \) which represents the steady-state probability of being an unleveled and a neck-and-neck industry respectively, with \( \mu_1 + \mu_0 = 1 \). The steady-state probability that a sector moves from an unleveled to leveled state is \( \mu_1(n_{-1} + h) \). The reverse move appears with a steady-state probability of \( 2\mu_0 n_0 \). In the steady-state we have \( \mu_1(n_{-1} + h) = 2\mu_0 n_0 \). Hence, the aggregate flow of innovations is given by:

\[ I = \int_{\{\Phi\}} \frac{4n_0(n_{-1} + h)}{2n_0 + n_{-1} + h} \]  \hspace{1cm} (20)

Implicitly \( I \) is a function of \( \nu \) and \( \gamma \). For a given firm, the expectation of its flow of innovation is proportional to \( I(\nu, \gamma) \). This last equation (20) leads to the inverted-U relationship between competition and innovation as stated by Aghion et al. (2005). Nevertheless, based on the two previous propositions, the shape of this relationship is dependant of the overall cost of innovations that occur in the economy. Hence, the following theorem holds:

**Theorem 1** The more the cost of global innovation in a given economy, the flatter the inverted-U relationship between competition and research activities in this economy.

\(^5\)This property includes the Aghion et al. (2005)' result that the R&D efforts in unleveled sectors, \( n_{-1} \), is a decreasing function in the competition index.
Proof. The theorem is directly established according to both propositions 1 and 2. Figure 2 depicts the shape of the relationship when the size of innovation varies.\(^6\)

Figure 2: The inverted-U relationship with a varying size of innovation

3 Empirical findings

The goal of this section is to illustrate the main theoretical predictions on French firms panel data: (i) the relationship between firm innovation flow \( n \) and competition \( \nu \) is inverted-U shape and (ii) this curve is flatter for higher size of innovation \( \gamma \) and for lower magnitude of the demand \( \Phi \) or alternatively the firm workforce.\(^7\)

3.1 Data

We use two datasets: first, firm-panel data from the observatory of firms at the Banque de France; second, the French R&D survey from the French ministry of research.\(^8\)

\(^6\)One can derive the analytical expression for the maximum as it leads to solve a 3\(^{rd}\)-order polynomial equation. As these computations are cumbersome and could alter the clarity of our main purpose, we prefer to show numerical simulation.

\(^7\)According to equations (3) and (4), these two variables are proportional.

\(^8\)Systematic data cleaning was implemented: firms with negative Lerner index were deleted, and outliers were identified by Tukey method based on the interquartile gap of the variable (see Kremp, 1995).
R&D and competition measures

The two main variables of interest in the regressions, namely the R&D effort and the competition index, come from the Fiben and Centrale des Bilans databases (Banque de France Balance Sheet dataset). They are collected on a voluntary basis. Clerks in the different local establishments of the Bank of France contact firm to complete a survey. The Fiben database is based on firms tax forms and includes all businesses with more than 500 employees and a fraction of smaller firms. It covers about 57% of employment for manufacturing but less for service sectors. The advantage of this base is to include firms that have episodic R&D activities or novel firms. We focus on firms that have conducted observable R&D activities at least one year since their creation. Our sample includes about 16,000 firms from 1990 to 2004. But it is not balanced. The sectoral distribution of observations is given in appendix.

A Lerner index for each firm can be built using these data. We only observe sectoral price provided by the INSEE, but we have detailed information on costs. The Lerner index is supposed to measure the market power of the firm by the difference between price and marginal costs (which equals the negative inverse of demand elasticity). Since neither price nor marginal costs are available at the firm level, we compute the index using operating profits net of depreciation and provisions minus the financial cost of capital (cost of capital*capital stock over sales (in line with Aghion et al., 2005). The Fiben database contains very detailed balance sheet information that enables to compute these Lerner indicators.\(^9\) We show in appendix A.4 that in our model the Lerner index is decreasing with \(\nu\), the measure of competition.

Cost of innovation

In addition, we exploit the R&D survey from the French Ministry of research. It includes information about total R&D expenditure and the number of patents for around 3000 firms. The question about the number of patents exists since 1999 only. The survey targets firms that are likely to do research and development. One flaw of this survey is that it covers well known firms that do research on a continuous basis pretty well whereas its quality is much smaller regarding firms that do research on an occasional basis only. We assume that this survey provides relevant information about the average size of a patent for a given sector (3 first digits of the NAF 700).

\(^9\)\(\text{Lerner}=(\text{value added-depreciation-cost of capital.capital stock-provision})/\text{sales}\)

Using the standard mnemonics of French tax forms: \(\text{Lerner}=[\text{VA-(AQ+AS+AU+AW+AY-AQ-1-AS-1-AU-1-AW-1-AY-1)}-0.085.\text{capital-(DR-DR-1)}/\text{FL.}\)
3.2 Proxy for innovation flow

In order to take the model to the data, we first have to estimate the flow of innovation. This flow, $n$, and the amount of R&D are related to each other according to $n = \frac{\text{R&D}}{\kappa}$, where $\kappa$ is the estimated innovation unit cost. The innovation unit cost is specific to a given sector and can be computed from the R&D survey of the French Ministry of Finance for 142 sectors, with for each sector $s$:

$$\kappa_s = \frac{\text{R&D in sector } s}{\text{number of patents in sector } s}$$

Both variables of the right hand side are taken from the R&D survey. R&D and number of patents in sector $s$ are taken as an average over 1999-2002. The average cost of a patent is around 1 million euros and the cost in the median sector is 860 thousands euros. Among sectors with low patent costs (below 150,000 euros) we find old sectors like wood, fabrication involving clay, fabrication of tiles etc. Sectors with high patent costs are more recent sectors (data processing, fabrication of electronic devices processing image and sound etc.).

It is then possible to derive a potential flow of innovation $n_j$, for each firm $j$, by the ratio:

$$n_j = \frac{\text{R&D of firm } j}{\kappa_s}.$$  

R&D of firm $j$ is measured as investment in R&D taken from FIBEN (variable KC of a standard tax form).

Noticeably, this flow of patents is not directly observed from a patent database but estimated from the R&D effort. Indeed, what we really have in mind is less to measure the observed flow of patents than to get a proxy for the innovation which potentially stems from the R&D effort of the firm. The flow of innovation $n_j$ is therefore firm specific and time specific. It can be interpreted as the potential number of patents that the firm is likely to register given the amount she devotes to R&D and the sectoral patent unit cost. This flow of innovation is subsequently used in the econometric investigation.

3.3 Econometric strategy

To approximate the inverted-U shape and to take into account the interactions with both the size of innovation and the size of the firm, we use a quadratic form. Precisely we estimate the model:

\[ \text{Total expenditures in R&D are available only on a smaller subsample (Centrale des Bilans.)} \]
\[ I_{j,t} = n_{j,t} = \lambda_{j,t}^2 \{ -\alpha_1 + \alpha_2 \ln(\text{innovation costs}_j) - \alpha_3 \ln(\text{firm workforce}_{j,t}) \} + \cdots \]

\[ + \lambda_{j,t} \{ \delta_1 - \delta_2 \ln(\text{innovation costs}_j) + \delta_3 \ln(\text{firm workforce}_{j,t}) \} + c_j + c_t + \epsilon_{j,t}, \quad (E) \]

where, \(\lambda_{j,t}\) is the firm Lerner index, \(c_t\) is a year dummy in order to correct for the overall business cycle and \(c_j\) is a firm fixed effect that encompasses fixed firm characteristics (sector, average workforce, etc.). The cost of innovation is the patent unit cost \(\kappa_s\). Firm size is measured as the number of employees.

### 3.4 Results

#### 3.4.1 Inverted-U shape without cost effect

Table 1: The U-inverted shape excluding size effect - French firms, 1991-2004

<table>
<thead>
<tr>
<th></th>
<th>Dependant variable: potential flow of innovation, (n)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Lerner(_{t-1})</td>
<td>0.0033</td>
<td>-3.5160***</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(-3.63)</td>
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<tr>
<td>Lerner(_{t-1})</td>
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<td>2.1688**</td>
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<td></td>
<td>(-2.81)</td>
<td>(2.35)</td>
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<td>yes</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>100,092</td>
<td>1,866</td>
</tr>
<tr>
<td>Number of firms</td>
<td>15,593</td>
<td>359</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.58</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Robust \(t\)-stat in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%.

Recall that Aghion et al. (2005) found an inverted-U relationship among firms listed on the London Stock Exchange. Thus, before estimating the full model, we check if we find also a such inverted-U-shaped curve for France, i.e. without interaction terms (Table 1). Estimation on the full sample of innovative firms does not support the quadratic form, and consequently the inverted-U shape (column 1). Rather, there is a significant linear relation: more competition is positively associated with more R&D.

But, if we restrict the panel to the 2%-largest firms in terms of real value added (column 2, Table 1), which should be more similar to a panel of listed firms, the figure is dramatically
different: the quadratic relation becomes statistically significant, leading to an inverted-U shaped curve. The top of the curve is obtained for reasonable lerner of 0.31. The clear different findings between the 2 samples, constitute a first hint of the relevance of the cost/size effect, and consequently of the inclusion of interaction terms in the empirical estimations.

3.4.2 Including cost effect

Figure 3 shows the lerner index versus the innovation flows for two different classes of innovation cost per employee. The two quadrants suggest a decrease in convexity of the inverted-U shape as the cost of innovation increases. The estimations of equation (E) confirm this property.

![Figure 3: The inverted-U relationship with a varying size of innovation](image)

Table 2 provides estimations of the full econometric model including interaction terms. For all specifications, a quadratic function seems a correct specification of how innovation and competition relates to each other: $\alpha_1$ and $\delta_1$ are positive and significant. Therefore, if the innovation cost was extremely low or, to put it differently, innovations could be implemented

\footnote{Additional estimations on subsamples in order to check for outliers have been implemented: results are not affected.}
with extremely small magnitude in an incremental way, the interaction terms would be negligible and the inverted-U relationship would fully apply.

Table 2: The cost of innovation and the magnitude of the U-inverted shape - French firms, 1991-2004

<table>
<thead>
<tr>
<th>Dependant variable: potential flow of innovation, n</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lerner(_{t-1}^2)</td>
<td>-0.5268***</td>
<td>0.0749**</td>
<td>-0.9277***</td>
<td>-0.5644***</td>
</tr>
<tr>
<td></td>
<td>(-3.64)</td>
<td>(2.48)</td>
<td>(-4.40)</td>
<td>(-2.48)</td>
</tr>
<tr>
<td>Lerner(_{t-1}^2.ln(\kappa_s))</td>
<td>0.0802***</td>
<td>—</td>
<td>0.1525***</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(3.65)</td>
<td>—</td>
<td>(4.59)</td>
<td>—</td>
</tr>
<tr>
<td>Lerner(_{t-1}^2.ln(workforce))</td>
<td>—</td>
<td>-0.0837**</td>
<td>-0.0955**</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(-2.28)</td>
<td>(-2.50)</td>
<td>—</td>
</tr>
<tr>
<td>Lerner(_{t-1}^2.ln(\kappa_s/workforce))</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.0982***</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(2.60)</td>
</tr>
<tr>
<td>Lerner(_{t-1})</td>
<td>0.5797***</td>
<td>-0.1866***</td>
<td>0.7226***</td>
<td>0.4510***</td>
</tr>
<tr>
<td></td>
<td>(4.29)</td>
<td>(-3.89)</td>
<td>(4.33)</td>
<td>(2.98)</td>
</tr>
<tr>
<td>Lerner(_{t-1}.ln(\kappa_s))</td>
<td>-0.0967***</td>
<td>—</td>
<td>-0.1380***</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(-4.99)</td>
<td>—</td>
<td>(-5.89)</td>
<td>—</td>
</tr>
<tr>
<td>Lerner(_{t-1}.ln(workforce))</td>
<td>—</td>
<td>0.0859***</td>
<td>0.0938***</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(3.58)</td>
<td>(3.79)</td>
<td>—</td>
</tr>
<tr>
<td>Lerner(_{t-1}.ln(\kappa_s/workforce))</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-0.0982***</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(-4.20)</td>
</tr>
<tr>
<td>Years</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Number of obs. 100,041 99,556 99,556 99,556  
Number of firms 15,586 15,527 15,527 15,527  
\(R^2\) 0.58 0.57 0.57 0.57  

Robust \(t\)-stat in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%.

Column 1 of table 2 displays the regression coefficients of the estimated model when \(\kappa\) is taken as the patent unit cost. Column 2 shows the results when introducing firm size only as a parameter of the curve. In column 3, both innovation cost and firm workforce are introduced to parameterize the link between innovation and competition. What clearly emerges from the estimation results is first that inverted-U shape relationship between innovation and competition seems to strongly hold when controlling for firm and innovation sizes. In column 1, the coefficients are correctly signed (\(\alpha\)'s and \(\delta\)'s are positive), although the relation is almost flat for the majority of firms: given the distribution of \(\ln(\kappa)\) (ranging from 4 to 9.5, see table 3), the
coefficients of $\lambda^2$ and $\lambda$ are very close to zero. Working with firm size in column 2 gives a clear inverted-U shaped curve for all firms. Introducing both types of size simultaneously supports theorem 1 and corollaries 1 and 2 of section 2.3. The relationship between innovation and competition gets looser and looser as firm size decreases (corollaries 1 and 2) and as innovation size increases (theorem 1).

In a last stage (table 2, column 4), we work directly with the innovation cost per employee by restricting the two coefficients of innovation and firm size to be equal. Again, the curve gets flatter as the innovation cost per employee increases: for the median firm (for which the unit cost of patent is around 25,000 euros per employee), the maximum of the curve is obtained for reasonable value of the Lerner index (around 0.27). The estimated impact of competition on innovation is economically significant. Decreasing the lerner index from 0.35 to 0.27, i.e. half of a standard deviation, should increase the innovation flow of median firm by about a quarter. But again, in sectors with more costly innovations the curve becomes flat. The impact of competition vanishes.

4 Perspectives

Both theoretical predictions and empirical illustrations support the mechanism that competition impacts firm decisions less when the cost of innovation in its sector is high in absolute terms or relative to its value added.

These results may have significant policy implications. The inverted U-shape already suggests that competition policy should be adjusted to take account of the state of the industry (Aghion et al., 2005). A second aspect of differentiation should be the nature of innovation in the industry. If innovations are costly, policy changes have to be on a very large scale for an impact to be expected; at the extreme end, in such sectors, the shape is so flat that competition policy is not an appropriate tool for boosting the research effort of firms.

Because this conclusion is strong, more in-depth research may be worthwhile. On the theoretical side, the size of innovations or firm size are partly endogenous to the competitive environment. Endogenizing them may alter our arguments. An interesting avenue would also be to explore the effects of credit constraints, which particularly affect small innovative firms. Additional surveys by the Banque de France concerning these constraints could help to directly test the associated predictions.
In addition, recent papers have highlighted the joint effects of product market and labor market regulation on economic performances (see OECD, 2006, for a review); therefore the model’s predictions also need to be tested for countries with more or less restrictive labor market regulations than France.

References


A Appendix: Proofs

A.1 Proof of proposition 1

Let’s note $\sigma_0$ and $\sigma_{-1}$ the first derivatives of the R&D intensity with respect to product market competition respectively in the leveled and unleveled sectors. As regards neck-and-neck sectors, we have:

$$\sigma_0 = \frac{\partial n_0}{\partial \nu} = \frac{\chi}{2\sqrt{(r + h)^2 + \nu \chi}}$$

with $\chi \equiv 2\pi_1 \Phi / \beta$. The first derivative of $\sigma_0$ with respect to $\gamma$ gives:

$$\frac{\partial \sigma_0}{\partial \gamma} = \frac{\partial \chi}{\partial \gamma} \frac{2(r + h)^2 + \nu \chi}{4((r + h)^2 + \nu \chi)^{3/2}}$$

which is of the sign of $\frac{\partial \chi}{\partial \gamma}$.

Since we have:

$$\frac{\partial \chi}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left( 2\frac{(1 - \frac{1}{\gamma}) \Phi}{\beta(\gamma)} \right) = \frac{2\Phi}{\beta^2(\gamma)} \left( \frac{1}{\gamma^2} \beta(\gamma) - (1 - \frac{1}{\gamma}) \beta'(\gamma) \right)$$

hence $\text{sign}(\frac{\partial \chi}{\partial \gamma}) = \text{sign}(\frac{1}{\gamma^2} \beta(\gamma) - (1 - \frac{1}{\gamma}) \beta'(\gamma))$. Introducing $\Psi$ defined as:

$$\Psi(\gamma) = \frac{1}{\gamma^2} \beta(\gamma) - (1 - \frac{1}{\gamma}) \beta'(\gamma)$$

we have $\Psi'(\gamma) = -\frac{2\beta'(\gamma)}{\gamma^3} - \beta''(\gamma)(1 - \frac{1}{\gamma})$, which is non positive for each $\gamma$ if $\beta$ is convex. Since we have $\Psi(1) = 0$, $\Psi$, and consequently $\frac{\partial \chi}{\partial \gamma}$ is negative for each $\gamma > 1$. As a result, $\sigma_0$ is a decreasing function of the size of innovation. This establishes Proposition 1 related to neck-and-neck sectors.

A.2 Proof of corollary 1

Since we have $\frac{\partial \sigma_0}{\partial \chi} > 0$ according to the notation from the previous section, and as $\chi$ is increasing with respect to $\Phi$, the slope $\sigma_0$ is an increasing function of the firm’s size. This establishes Corollary 1.

A.3 Proof of proposition 2

We first prove the following cases: assuming that the R&D cost function is convex and such as $\beta(\gamma) = o(1 - 1/\gamma)$, for large or costly innovations ($\gamma \gg 1$) as well as for small or cheap innovations ($\gamma \sim 1$), the slope of the relationship between innovation and competition in the unleveled sectors is a non positive, increasing function of the size of innovation $\gamma$. 

20
For the size of innovation being in the neighborhood of 1, which means that innovations are incremental, and under the assumption that \( \beta(\gamma) = \frac{2(1 - \frac{1}{2})\Phi}{\beta(\gamma)} \gg 1 \), we have:

\[
\chi = \frac{2(1 - \frac{1}{2})\Phi}{\beta(\gamma)} \gg 1.
\]

So we find an equivalent of \( n_0 \) for large \( \chi \)’s:

\[
n_0 \sim (\nu \chi)^2,
\]

and so the research intensity in leveled sectors:

\[
n_{-1} \sim (\sqrt{1 + \nu} - \sqrt{\nu})\chi^2,
\]

As a consequence, we have the slope of the relationship between competition and R&D intensity for small \( \chi \)’s:

\[
\sigma_{-1} = \frac{\partial n_{-1}}{\partial \nu} \approx \frac{\sqrt{\nu} - \sqrt{1 + \nu}}{2\sqrt{1 + \nu} \sqrt{\nu}} \chi^2
\]

which is a non positive, increasing function of \( \gamma \).

In a similar way, for costly innovations, \( i.e. \) large \( \gamma \), we have:

\[
\chi = \frac{2(1 - \frac{1}{2})\Phi}{\beta(\gamma)} \ll 1.
\]

Hence, the first order approximation of \( n_0 \) for \( \chi \) sufficiently small gives:

\[
n_0 = \frac{\nu \chi}{2(r + h)} + o(\chi).
\]

It follows that the linear approximation of \( n_{-1} \) is:

\[
n_{-1} = \frac{(1 - \nu)\chi}{2(r + h)} + o(\chi).
\]

As a consequence, we have the slope of the relationship between competition and R&D intensity for small \( \chi \)’s:

\[
\sigma_{-1} = \frac{\partial n_{-1}}{\partial \nu} \approx \frac{\chi}{2(r + h)}
\]

which is a non positive, increasing function of \( \gamma \).

The system of equations giving \( n_0 \) and \( n_{-1} \) is quasi-homogenous in \( r + h \). It can be thus rewritten as a non-parametric system as following.

Let \( m_0 = n_0/(r + h) \), \( m_{-1} = n_{-1}/(r + h) \) and \( X = \pi_1 \Phi/[\beta(r + h)]^2 \). The system becomes

\[
m_0 = -1 + \sqrt{1 + \nu X}, \tag{24}
\]
\[ m_{-1} = -1 - m_0 + \sqrt{1 + m_0^2}X. \] (25)

Proving the proposition (i.e. showing that \( \frac{\partial^2 n_{-1}}{\partial \nu^2} > 0 \) or \( \frac{\partial^2 n_{-1}}{\partial L \partial \nu} < 0 \)) is then equivalent to show that \( \frac{\partial^2 n_{-1}}{\partial X \partial \nu} < 0 \), with \( \nu \in ]0; 1[ \) and \( X \in ]0; +\infty[ \).

It is clearly true for large or small value of \( X \). But formal calculus does not allow proving this property for all \( X \). Therefore, we use a numerical representation (see figure 4) of \( \frac{\partial^2 n_{-1}}{\partial X \partial \nu} \) on the field \( (\nu, X) \in ]0; 1[ \times ]0; +\infty[ \). This completes the proof of Proposition 2.

Figure 4: \( \frac{\partial^2 n_{-1}}{\partial X \partial \nu} \) on the field \( (\nu, X) \in ]0; 1[ \times ]0; +\infty[ \)
A.4 Theoretical counterpart of Lerner index at the firm level

Based on this model, the theoretical counterpart of our empirical measure of competition, the Lerner index, is given by

\[
\begin{align*}
\lambda_0 &= (1 - \nu)(1 - \gamma^{-1}) \\
\lambda_{-1} &= 0 \\
\lambda_1 &= 1 - \gamma^{-1},
\end{align*}
\]

where \(\lambda_0, \lambda_{-1},\) and \(\lambda_1\) are related to leveled, follower, and leader firm respectively. Hence, the Lerner index decreases with competitive pressure for neck-and-neck firms, whereas the Lerner index depends positively on the size of innovation. For a given firm, the expected Lerner index is then given by

\[
E(\lambda) = \mu_0 \lambda_0 + \frac{\mu_1}{2} \lambda_{-1} + \frac{\mu_1}{2} \lambda_1 = (1 - \mu_1)(1 - \nu)(1 - \gamma^{-1}) + \frac{\mu_1}{2}(1 - \gamma^{-1})
\]

The marginal effect of a change in the Lerner index on innovation is decreasing with \(\gamma\).

A.5 Description of the sample

Table 3: Distribution of observations by main sectors (NES 16) in %

<table>
<thead>
<tr>
<th>Sector</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, hunting, forestry, and fishing</td>
<td>1.1</td>
</tr>
<tr>
<td>Food and agricultural manufacturing</td>
<td>5.4</td>
</tr>
<tr>
<td>Consumer goods manufacturing</td>
<td>8.2</td>
</tr>
<tr>
<td>Car manufacturing</td>
<td>1.7</td>
</tr>
<tr>
<td>Equipment good manufacturing</td>
<td>17.0</td>
</tr>
<tr>
<td>Intermediary good manufacturing</td>
<td>20.7</td>
</tr>
<tr>
<td>Energy</td>
<td>0.5</td>
</tr>
<tr>
<td>Construction</td>
<td>4.7</td>
</tr>
<tr>
<td>Gross and retail trade</td>
<td>21.3</td>
</tr>
<tr>
<td>Transport</td>
<td>3.0</td>
</tr>
<tr>
<td>Real estate</td>
<td>0.7</td>
</tr>
<tr>
<td>Business sectors</td>
<td>14.0</td>
</tr>
<tr>
<td>Services</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Table 4: Descriptive statistics for Patent unit cost $\kappa_s$

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>93.35125</td>
</tr>
<tr>
<td>5%</td>
<td>149.7383</td>
</tr>
<tr>
<td>10%</td>
<td>244.1933</td>
</tr>
<tr>
<td>25%</td>
<td>490.0326</td>
</tr>
<tr>
<td>50%</td>
<td>822.2892</td>
</tr>
<tr>
<td>75%</td>
<td>1959.291</td>
</tr>
<tr>
<td>90%</td>
<td>4199.069</td>
</tr>
<tr>
<td>95%</td>
<td>6593.991</td>
</tr>
<tr>
<td>99%</td>
<td>10772.08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Largest</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs 142</td>
<td>Sum of Wgt. 142</td>
</tr>
</tbody>
</table>

Mean 1695.942

Table 5: Descriptive statistics for the patent unit cost per employee $\kappa_s/L_j$

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.4770227</td>
</tr>
<tr>
<td>5%</td>
<td>1.761845</td>
</tr>
<tr>
<td>10%</td>
<td>3.35403</td>
</tr>
<tr>
<td>25%</td>
<td>9.245318</td>
</tr>
<tr>
<td>50%</td>
<td>24.36968</td>
</tr>
<tr>
<td>75%</td>
<td>65.34879</td>
</tr>
<tr>
<td>90%</td>
<td>153.8309</td>
</tr>
<tr>
<td>95%</td>
<td>268.2161</td>
</tr>
<tr>
<td>99%</td>
<td>755.3947</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Largest</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs 99557</td>
<td>Sum of Wgt. 99557</td>
</tr>
</tbody>
</table>

Mean 72.32101

Table 6: Descriptive statistics for ln($\kappa$) at the firm level

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>4.536369</td>
</tr>
<tr>
<td>5%</td>
<td>5.210752</td>
</tr>
<tr>
<td>10%</td>
<td>5.399779</td>
</tr>
<tr>
<td>25%</td>
<td>6.183435</td>
</tr>
<tr>
<td>50%</td>
<td>6.582801</td>
</tr>
<tr>
<td>75%</td>
<td>7.224261</td>
</tr>
<tr>
<td>90%</td>
<td>7.912662</td>
</tr>
<tr>
<td>95%</td>
<td>8.342618</td>
</tr>
<tr>
<td>99%</td>
<td>9.227607</td>
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</table>

<table>
<thead>
<tr>
<th>Largest</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs 100042</td>
<td>Sum of Wgt. 100042</td>
</tr>
</tbody>
</table>

Mean 6.680216

24
Table 7: Descriptive statistics for the lerner (lagged)

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
<th>Largest</th>
<th>Obs</th>
<th>Sum of Wgt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.0227513</td>
<td>0.0000224</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>0.0729581</td>
<td>0.000109</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0.1170003</td>
<td>0.0001328</td>
<td>100042</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.2003858</td>
<td>0.0001342</td>
<td></td>
<td>100042</td>
</tr>
<tr>
<td>50%</td>
<td>0.304963</td>
<td></td>
<td></td>
<td>Mean .3208857</td>
</tr>
<tr>
<td>75%</td>
<td>0.4187701</td>
<td>3.722385</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td>0.5395694</td>
<td>4.85848</td>
<td></td>
<td>Variance .0298848</td>
</tr>
<tr>
<td>95%</td>
<td>0.629789</td>
<td>5.023373</td>
<td></td>
<td>Skewness 2.011237</td>
</tr>
<tr>
<td>99%</td>
<td>0.7852377</td>
<td>6.87043</td>
<td></td>
<td>Kurtosis 41.19343</td>
</tr>
</tbody>
</table>

Table 8: Descriptive statistics for $n_j$

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
<th>Largest</th>
<th>Obs</th>
<th>Sum of Wgt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0</td>
<td>0</td>
<td>100041</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0</td>
<td>0</td>
<td></td>
<td>100041</td>
</tr>
<tr>
<td>50%</td>
<td>0</td>
<td></td>
<td></td>
<td>Mean .0570318</td>
</tr>
<tr>
<td>75%</td>
<td>0</td>
<td>34.76926</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td>0.0658991</td>
<td>40.7429</td>
<td></td>
<td>Variance .2957216</td>
</tr>
<tr>
<td>95%</td>
<td>0.198134</td>
<td>41.85245</td>
<td></td>
<td>Skewness 40.37153</td>
</tr>
<tr>
<td>99%</td>
<td>1.015539</td>
<td>50.05535</td>
<td></td>
<td>Kurtosis 2418.018</td>
</tr>
</tbody>
</table>

Figure 5: Distribution of $\ln(\kappa)$


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