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THE LENDER OF LAST RESORT

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Abstract: It has been argued in the literature that emergency liquidity injections should be conducted preferably in the form of open market operations. As we show in the present paper, this is not necessarily the case when liquidity may be alternatively used for speculative purposes during the crisis. In such a situation, non-discriminating operations may attract unfunded market participants that divert funding resources away from its best uses in the financial sector. As a consequence, targeted liquidity assistance may become strictly superior. The analysis might have a bearing on recent developments in the context of the subprime crisis.

Keywords: Liquidity, financial markets, lender of last resort.

JEL codes: G14, G18.

Résumé : La théorie économique suggère parfois que les injections de liquidité d’urgence doivent être effectuées par le biais d’opérations d’open market. Nous montrons dans ce papier que ce n’est pas forcément le cas lorsque la liquidité injectée par les banques centrales peut être utilisée dans un but spéculatif au moment d’une crise financière. Dans ce contexte, des opérations d’open market non discriminatoires peuvent attirer des acteurs de marché manquant certes de fonds, mais qui peuvent dévoyer la monnaie centrale et en priver les acteurs financiers qui en ont le plus besoin. Des opérations de fourniture de liquidité ciblées deviennent alors strictement préférables. Nos résultats ne sont pas sans lien avec les développements associés à la crise dite des crédits « subprime » de l’été 2007.

Mots-clés : Liquidité, marchés financiers, prêteur en dernier ressort.

Codes JEL : G14, G18.
Non-technical summary: The present paper studies the scenario of a liquidity crisis in a market for a potentially illiquid financial asset. We evaluate several policy alternatives for the lender of last resort, including open market operations and targeted liquidity assistance. While earlier studies have focused on the moral hazard dimension of emergency intervention, our analysis is concerned with the trade-off between exposure for the lender of last resort and efficiency of the risk allocation in the private sector. Our main result on the policy dimension is a ranking that puts targeted emergency lending above an open market operation.

Our formal framework is based on the standard model of investor fear that can be outlined as follows. There is a population of investors, each of whom owns a single unit of the financial asset. If an investor holds the asset until maturity, it renders a positive expected return. However, there is a probability that the asset must be liquidated at an interim stage. To avoid the risk, some or all investors will liquidate the asset at an early stage, avoiding the risk of forced liquidation. Thus, there is a “run” on the financial market. Into this model, we introduce a population of buyers, who stand ready to invest when prices are low. We show that a run occurs whenever the mass of funded buyers in the market is lower than the mass of sellers that are potentially affected by the crisis.

Strategic investor behavior during a liquidity crisis has direct implications for the optimal policy response. Specifically, we show that when the lender of last resort chooses to provide emergency liquidity assistance in the form of an open market operation, then there will be (unfunded) buyers that participate in the auction. In our model, this effect leads to a situation in which banks in distress and “greedy” investors compete for excess funding provided during the crisis. Our analysis thereby provides a theoretical argument for the position that an open market operation may not be optimal during a liquidity crisis.

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1Our analysis does not take a stance concerning the question who should serve as the lender of last resort.
Our results might have a bearing on the recent developments in the context of the subprime crisis. On August 9, 2007, the executive board of the ECB decided to inject EUR 95bn through a fine-tuning operation. In contrast, the Federal Reserve on August 17 has chosen to offer targeted liquidity assistance by lowering the interest charged on discount window lending. Our analysis suggests that as a lender of last resort, the Eurosystem is less risk averse than the Federal Reserve, which would be consistent with perceived differences in central bank independence.

Résumé non-technique : Ce papier étudie le scénario d’une crise de liquidité sur un marché financier potentiellement illiquide. Nous évaluons plusieurs politiques de prêteur en dernier ressort, dont les opérations d’open market et la fourniture ciblée de liquidité d’urgence. Des études antérieures ayant déjà mis l’accent sur la dimension de hasard moral associée à de telles injections de liquidité, nous centrons notre analyse sur le conflit entre le degré d’exposition du prêteur en dernier ressort et l’efficience de l’allocation du risque dans le secteur privé. Le résultat principal en termes de politique d’intervention suggère que la fourniture de liquidité ciblée est préférable aux opérations d’open market.

Le cadre théorique repose sur un modèle de panique d’investisseurs où chaque participant de marché possède une unité de l’actif financier. Un investisseur conservant l’actif à échéance obtient un rendement espéré positif. Cependant, il est probable que l’actif doive être liquidé à un stade intermédiaire. Dans ce contexte, certains, voire tous les investisseurs vont vendre leur actif par anticipation afin d’éliminer le risque d’une liquidation forcée, donnant ainsi lieu à une panique sur le marché financier. Nous introduisons dans ce modèle un population d’acheteurs potentiels, prêts à investir lorsque les prix d’actifs sont bas, et montrons que la panique persiste tant que les acheteurs solvables sont moins nombreux que les investisseurs potentiellement affectés par la crise.

Le comportement stratégique des investisseurs durant une crise de liquidité a des implications directes quant à la réaction optimale de la banque centrale.
En particulier, nous montrons que lorsque le prêteur en dernier ressort choisit d’intervenir par le biais d’une opération d’open market, des spéculateurs potentiels, jusqu’alors en dehors du marché, participeront à l’opération. Dans notre modèle, cette participation implique une « compétition pour la liquidité centrale » entre les banques en détresse et les investisseurs « gourmands ». Notre analyse fournit ainsi un argument théorique qui met en cause l’optimalité d’opérations d’open market pendant une crise de liquidité sur les marchés financiers.

Nos résultats ne sont pas sans lien avec les développements associés à la crise dite des crédits « subprime » de l’été 2007. Le 9 août 2007, la Banque Centrale Européenne a injecté 95 milliards d’euros sous forme d’opération de réglage fin (fine-tuning) sur le marché interbancaire, alors que la Réserve Fédérale a opté pour une réduction du taux d’escompte appliqué à sa facilité de refinancement. Notre analyse suggère que l’Eurosystème, dans son rôle de prêteur en dernier ressort, est moins averse au risque que la Réserve Fédérale, ce qui est cohérent avec la différence d’indépendance perçue entre les deux institutions.
1. Introduction

According to the classic studies by Thornton [17] and Bagehot [2], the lender of last resort should provide targeted emergency assistance to troubled banks, with the qualification that lending should be at a high rate, against good collateral, and only to solvent institutions. An alternative to this “banking” view has emerged some twenty years ago in particular through contributions by Goodfriend and King [11], Bordo [4], Kaufman [12], and Schwartz [12]. These opposing views, sometimes aggregated as the “monetary” view, say that once the financial system has obtained sufficient liquidity through an equitable open market operation, interbank markets for short-term credit should be sufficiently efficient to warrant the availability of liquidity for any bank that deserves it. Since then, a fruitful theoretical debate about the role and identity of the lender of last resort has begun.\(^2\)

One recent strand of the literature that has received particular attention by supporting the banking view is concerned with the conditions under which the working of the interbank market can be relied upon even during a crisis situation. Flannery [6] has argued that the problem of adverse selection may make the screening of loan applicants more difficult for banks in times of market distress.\(^3\) Rochet and Vives [14] identify a potential coordination problem when lenders in the secondary market are heterogeneously informed. In their framework, the unique equilibrium may have the feature that with positive probability, there is no market assistance for the troubled bank. Finally, Freixas, Parigi, and Rochet [7] consider a double moral hazard problem involving the tasks of screening loan applicants and monitoring ongoing credit relationships. There, the lender of last resort has a role if and only if missing incentives for screening are the main source of moral hazard.

As an additional “banking” argument for why targeted lending may be superior to open market operations, the present paper considers speculative

\(^2\)For an overview of the policy discussion and for further references, see Goodhart and Huang [10] and Santos [15].

\(^3\)It has been noted that also the lender of last resort will face an asymmetric distribution of information and that the public sector may not necessarily be better informed than the private sector.
motives by commercial banks and their affiliated securities houses.\textsuperscript{4} Indeed, as we argue, with asset prices being depressed during the crisis, liquidity is attractive not only for commercial banks in trouble but also for commercial banks that seek to exploit the liquidity shock.\textsuperscript{5} An open market operation at the conditions of the current monetary stance is unable to discriminate and thereby makes these groups of banks compete for liquidity. The effect is that some of the troubled banks will have to liquidate their balance sheets, while speculators might gain. To the extent that such liquidations are not socially desirable, the effect will make targeted liquidity assistance more appropriate than an open market operation in our framework.

The rest of this paper is structured as follows. Section 2 outlines the formal framework, and describes the equilibrium in the financial market. Section 3 considers three policy alternatives, emergency lending, open market operation, and outright intervention in the asset market, and investigates the impact of these policies on the trade-off between market efficiency and central bank exposure. Section 4 offers extensions and discusses the robustness of our findings. The conclusions are collected in Section 5. Appendix A gives a formal account of the equilibrium concept. Proofs have been relegated to Appendix B.

2. The model

We envisage a financial market in which some investors face the risk of having to liquidate their positions at prices below the fair value, while others stand ready to exploit the temporary illiquidity of the market. To capture

\textsuperscript{4}To understand how our results would apply in the institutional context of the money market, it is important to note that what matters for survival is control about liquidity rather than liquidity itself. For instance, extending a credit in a crisis situation to a non-bank such as a security house weakens the liquidity position of a bank not because high-powered money would leave the bank, but because the bank loses control over those reserves.

\textsuperscript{5}This may happen by trading on own accounts or by extending loans to third parties. Garcia [8] reports that during the 1987 stock market crash, there were NYSE specialists seeking funds to increase their portfolio positions.
this scenario, we adapt the convenient model of financial market runs (cf. Bernardo and Welch [3]).

The model of investor fear can be outlined as follows. There is a population of investors, each of whom owns a single unit of a financial asset. If an investor holds the asset until maturity, it renders a positive expected return. However, there is a probability that the asset must be liquidated at an interim stage. To avoid the risk, some or all investors will liquidate the asset at an early stage, avoiding the risk of forced liquidation. Thus, there is a “run” on the financial market.

Into this model, we introduce a population of buyers, who stand ready to invest when prices are low. Formally, we consider a financial market for a single risky asset (“the asset”) over three dates, where trade is feasible at dates 0 and 1, and the value of the asset \( \tilde{v} \) is revealed and paid out to the holder of the asset at date 2. Before date 2, the value of the asset is uncertain, and known to be distributed normally with mean \( v \) and variance \( \sigma^2 \). Both trade and payment occur in terms of a riskless asset (“cash”), whose return is normalized to zero.

Three types of traders are in the market. First, there is a continuum of risk-neutral traders referred to as the sellers, that hold the asset but no cash, and that may be forced to liquidate the asset at date 1. The size of the population of sellers is normalized to one. Second, there is a continuum of risk-neutral traders, referred to as the buyers, who do not hold the asset. Buyers can be either funded or unfunded. Funded buyers have a cash endowment equivalent to the asset’s fair value \( v \), while unfunded buyers have no cash endowment. Denote by \( \beta^f \geq 0 \) the size of the population of funded buyers, and by \( \beta^u \geq 0 \) the size of the population of unfunded buyers. Finally, there is a perfectly competitive risk-averse market making sector that clears the market at dates 0 and 1.

At date 1, there is a probability \( s < 1 \) that the bad state \( S \) (for shock) of the world realizes, in which the entire seller population is forced to liquidate.

\(^{6}\)The assumption of normal returns is made for convenience.
individual positions. Otherwise and with probability $1 - s$, the state of the world is $N$ (for no shock), and no trader is forced to liquidate. The realization of the state of nature becomes public information immediately before trading takes place at date 1.

Apart from the forced liquidations, sellers and funded buyers have full discretion concerning the dates at which they place their market orders.\(^7\) More specifically, a seller may choose to either sell at date 0, or to sell at date 1, or else to hold on. If the seller sells at either date 0 or at date 1, she receives the respective market price prevailing at that date.\(^8\) If the seller does not sell, she realizes the fundamental value $\tilde{v}$ of the asset at date 2. A funded buyer may either buy at date 0 or at date 1, or not at all. If a funded buyer invests at date 0, she may either hold the asset until maturity or sell it again at date 1 at the prevalent market price.\(^9\) If a buyer invests at date 1, she pays the market price at that date and holds the asset until maturity. The profit for a potential buyer of not trading at all is normalized to zero. An unfunded buyer may choose to invest at date 1, but only after having obtained the necessary funding.

The market making sector is modeled as in Bernardo and Welch [3]. That is, market orders are generally submitted without limit. Moreover, the market making sector is equipped with an initial cash endowment of $x_0$, and sets the price at each point in time competitively while maximizing a utility function with absolute risk aversion $\gamma > 0$. As will become clear, these assumptions imply an elastic demand for the risky asset and positive autocorrelation of the price process, which are the essential ingredients to construct the market equilibrium.

Next, we determine market prices at dates 0 and 1 as a function of aggregate order volumes. Denote by $\alpha_0$ and $\beta_0$ the mass of the sellers and funded

\(^7\)The model is deliberately kept simple by assuming that funded buyers can either trade the asset or not trade, irrespective of the price level. The strategy space of the buyers will be further enlarged in Section 4.

\(^8\)For simplicity, we exclude the possibility of re-investment by early sellers.

\(^9\)We assume that funded buyers are never forced to sell.
buyers, respectively, that trade at date 0. It follows from our assumptions that the market maker sets a price $p_0$ such that the certainty equivalent of the market maker’s material payoff is not affected through the execution of the orders. As Lemma 1 below shows, this determines the price at date 0 as a function of $\alpha_0$ and $\beta_0$. The market price at date 1 depends on the realization of the liquidity shock. If the shock occurs, then all those sellers who have not sold at date 0 will be forced to liquidate. Thus, in this case the entire population of size $\alpha_1^S = 1 - \alpha_0$ of remaining sellers will sell at date 1. In the absence of a liquidity shock, however, an endogenous subpopulation of size $\alpha_1^N \leq 1 - \alpha_0$ of market participants sells at date 1. In addition to those sellers, there may be early buyers that liquidate at date 1. We denote by $N_1^{\alpha}$ the mass of buyers disinvesting at date 1 in state $\omega$. On the demand side, there is a population of size $\leq \beta^f - \beta_0$ of funded buyers that has not invested at date 0 and may therefore decide to buy at date 1. In addition, there may be demand by a subpopulation of size $\leq \beta^u$ of unfunded buyers, that become funded at date 1. We denote by $\beta_1^S$ and $\beta_1^N$ the total mass of buyers that demand the asset in state $S$ and $N$, respectively.

**Lemma 1.** The market price at date 0 is given by

$$p_0 = p_0(\alpha_0, \beta_0) = v - \frac{\gamma \sigma^2}{2} (\alpha_0 - \beta_0).$$  \hfill (1)

The price $p_1^\omega$ in state $\omega$ is given by

$$p_1^\omega = p_1^\omega(\alpha_0, \beta_0, \alpha_1^\omega, \alpha_1^{\alpha}, \beta_1^\omega)$$

$$= v - \frac{\gamma \sigma^2}{2} (2(\alpha_0 - \beta_0) + \alpha_1^\omega + \alpha_1^{\alpha} - \beta_1^\omega).$$  \hfill (2)

Thus, as in Grossman and Miller [9], the market price reflects the limited risk-taking capacity of the market makers, which implies a liquidity premium for one side of the market. For example, for $\alpha_0 > \beta_0$, there are more sellers than buyers in the short term, depressing the market price relative to the fundamental long-term value of the asset. We will see below that this is the only possible deviation of the asset price, i.e., the equilibrium price will never exceed the asset’s fair value. In fact, as pointed out by Bernardo and
Welch [3], the price at date 0 will typically fall below $v$ because some sellers decide to sell already at date 0 in anticipation of the possibility of a forced liquidation at date 1.

The equilibrium concept employed in the formal analysis (cf. Appendix A) reflects strategic considerations on the part of both buyers and sellers, and is illustrated in the subsequent example. At each date, a market participant will trade with certainty when the transaction price anticipated for a delayed transaction is strictly less attractive. The market participant will not trade if the opposite development for the market price is anticipated. If the price process presents itself to the market participant as a simple martingale, she may either trade or not trade. The development of the market price is bound to the decision of individual traders regarding the date at which to place their orders.

**Example 1.** Consider a set-up with the following exogenous parameters:

$$s = \frac{1}{4}, \beta^f = \frac{1}{3}, \gamma = 2, \sigma^2 = 9, v = 10$$

Then the parameter values

$$\alpha_0 = \frac{2}{9}, \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = \beta_1^S = \frac{1}{3}$$

describe an equilibrium, which generates a price sequence

$$p_0 = 8, p_1^N = 9, p_1^S = 2.$$ 

In general, the price path determined by rational trading behavior is uniquely determined, and involves inefficient precautionary liquidations unless $\beta^f \geq 1$.

**Proposition 1.** The unique equilibrium price path $(p_0, p_1^S, p_1^N)$ in the intertemporal trading game satisfies

$$sp_1^S + (1 - s)v \leq p_0 \leq v$$  \hspace{1cm} (3)$$

and $p_1^S \leq p_1^N \leq v$. Moreover, when $s > 0$ and $\beta^f < 1$, we have $\alpha_0 > 0$ and the equilibrium is inefficient.
Thus, provided that the mass of funded buyers in the market is less than what would be needed to make up for the mass of potentially forced liquidations, there will be a market impact of investor fear. Proposition 1 suggests thereby that investor fear as identified by Bernardo and Welch [3] should be expected even in situations where the buyers with ready money are in the market.

The full characterization of the equilibrium can be found in Appendix B. There are three scenarios (cf. also Figure 1). For high values of $s$ satisfying $s \geq 1/(2 - \beta_f)$, the equilibrium predicts that all sellers will liquidate early. The endowment change in the market making sector causes a further drop in prices, which is avoided by the sellers, but attracts the buyers. No price effect in state N is predicted for very low values $s < \beta_f/(2 - \beta_f)$. For intermediate values of $s$ satisfying

$$\beta_f \leq s(2 - \beta_f) \leq 1,$$

some but not all sellers will liquidate early. For the rest of the paper, we will confine ourselves to this most interesting case where (4) is satisfied. In this area of the parameter space, we have $0 < \alpha_0 < 1$ and $p_1^N < v$, as in Example 1. Moreover,

$$p_1^S = v - \frac{\gamma \sigma^2}{2} \frac{1}{1 - s} (1 - \beta_f).$$

The reader will note that in the case where (4) holds, there is nobody except the market makers who is willing to buy the asset at date 0, despite its price being below the fundamental value. To see why this happens, consider Example 1 again. In fact, at date 0, the buyers’ expectation of the price at date 1 is

$$E^B[p_1] = \frac{3}{4} p_1^N + \frac{1}{4} p_1^S = 7\frac{1}{4} < 8.$$  

In contrast, the sellers’ expectation of realized value is

$$E^S[p_1] = \frac{3}{4} v + \frac{1}{4} p_1^S = 8.$$ 

The precautionary selling creates a temporary downwards price trend which is anticipated and exploited by rational buyers.\(^\text{10}\) Thus, the market may

\(^{10}\)The possibility of short-selling is considered in Section 4.
not be able to fully resolve the temporary illiquidity of an asset, even in the presence of risk-neutral buyers.

In the absence of intervention by the lender of last resort, the strategic timing of individual market orders may cause a nontrivial social cost. One can check that in the example given above, the welfare loss, i.e., the loss of aggregated utilities of buyers and sellers compared to the second best in which liquidations take place only at date 1 amounts to \( \Delta = \frac{1}{12} \).

A welfare loss comes about as a consequence of inefficient allocation of risks in the economy. Indeed, on an individual level, the sellers do not take into account the effect that selling has on the development of the price path. Early liquidation, when chosen by a non-negligible subpopulation of the sellers, leads to a socially undesirable allocation of risks even when the shock eventually does not realize. The inefficiency could be remedied if arbitrageurs were to buy early for prices just below the asset’s long-term valuation. Our analysis shows, however, that for \( \beta^f < 1 \), buyers have an interest to delay their orders, which does not help to resolve the inefficiency.

3. Policy options

How can the lender of last resort react?\(^{11}\) One theoretical possibility is the implementation of efficient price levels through outright intervention (OI) in the asset market. However, this strategy exposes the lender of last resort to significant market risk and is therefore never optimal in our context. Less risky policy options include the conduct of an open market operation in the money market (MM), and targeted assistance (TA), e.g., through the discount window. These operations imply credit risk, correlated to market performance, for the lender of last resort. In the sequel, we will analyze the consequences of all three policy alternatives.

\(^{11}\)Bernardo and Welch [3] offer an extension in which market makers obtain more liquidity, which deepens the market at date 1, and thereby reduces the price impact of the liquidations. An alternative modeling approach, in which the market making sector is assumed to be credit constrained, is studied by Brunnermeier and Pedersen [5]. There, the provision of emergency funds to the market making sector helps to mitigate the crisis.
The reader will note that there is a wide flexibility in evaluating policy options. We do not take a stance on the question which institution should serve as the lender of last resort.\footnote{For a political economy perspective on emergency liquidity provision, see Repullo [13].} Policy objectives pursued by the institution in charge may include fiscal concerns, price stability, market efficiency, the discouragement of moral hazard, the exposure to financial risks, and others. The present analysis focuses on the trade-off between efficiency and exposure for the lender of last resort.

Risks resulting from involvement in emergency lending should be expected to be evaluated very carefully. The lender of last resort may be subject, in particular, to both market and credit risks. The subsequent analysis applies to a wide class of risk metrics, including value-at-risk and expected loss measures. We will call a characteristic \( \psi(\cdot) \) of random variables a monotonic risk measure if for two random variables \( X \) and \( Y \) satisfying \( X \leq Y \) in any state realization, we have \( \psi(Y) \leq \psi(X) \).\footnote{Cf., e.g., Artzner et al. [1].} This definition is broad enough to encompass most risk metrics used in practice, such as value-at-risk, expected loss, expected shortfall, and many others. In particular, it will be noted that the lender of last resort may be risk-averse, risk-neutral, or risk-seeking in our model.

**Assumption A.** The lender of last resort evaluates the risk dimension using a monotonic risk measure \( \psi(\cdot) \).

In the sequel, we will refer to \( \psi(\cdot) \) as the exposure. Our second assumption says that making profits is not a primary goal of the lender of last resort.

**Assumption B.** In the evaluation of policy options, the lender of last resort does not trade off potential gains against potential losses.

Technically, Assumption B says that when \( \pi \) denotes the financial return to the lender of last resort from the chosen intervention strategy, then any potential profits \( \pi > 0 \) will be evaluated as if \( \pi = 0 \). Thus, realizing trading gains is not part of the policy objective. We deem Assumption B as plausible.
While a risky strategy might indeed offer potential rewards, especially during a crisis, it is very unlikely that, even if they should realize, such rewards would be assessed as an accomplishment for the lender of last resort. To the contrary, it would be more natural to see the speculative strategy being publicly discussed after an unfortunate outcome.

Our final assumption concerns the size of the credit facility that needs to be extended by the lender of last resort to salvage a troubled institution.

**Assumption C.** The amount of credit $c > 0$ needed to avert the forced liquidation of a representative seller is strictly less than the market price of the risky asset under market strain, i.e., $c < p^S_1$.

Such an assumption would be reasonable when the availability of credit from the lender of last resort induces other stakeholders to support the troubled institutions, for instance, by injecting additional funds or by interpreting contractual obligations in a less restrictive way.

We will now compare the three policy options. It is, however, not obvious which metrics to apply. In principle, what we would like to do is to maximize efficiency subject to a constraint on a given level of exposure to market risk. It turns out that it is more convenient to study the dual problem which is to minimize exposure subject to a given level of efficiency. As a proxy for efficiency, we shall use the price impact in the crisis. Specifically, we will consider the consequences, in terms of exposure, for the lender of last resort of securing a price level of $v - \varepsilon$ in the bad state. Thus, in the sequel, we fix $\varepsilon$ and minimize exposure subject to the constraint $p^S_1 \geq v - \varepsilon$.\(^{14}\)

**Outright intervention in asset markets.** In principle, the lender of last resort could actively trade the asset to reduce the inefficient risk allocation. Most obviously, the lender of last resort could buy the asset outright in state $S$ at date 1, when the market price is prone to go below fundamentals. Consider a scenario in which the lender of last resort buys $q > 0$ assets to

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\(^{14}\)For simplicity, we will also assume that $\beta_f + \beta_u = 1$. 
stabilize prices in the critical state. The equilibrium price in this state would be
\[ p_1^S = v - \frac{\gamma \sigma^2}{2} \frac{1}{1-s} (1 - \beta^f - q). \]

Keeping the market price at some limit \( v - \varepsilon \), for \( \varepsilon > 0 \) small, below the fundamental value would require buying a quantity
\[ q^* = 1 - \beta^f - \frac{1-s}{\lambda} \varepsilon \]

of the asset, where \( \lambda = \gamma \sigma^2 / 2 \). The uncertain return of such a strategy to the lender of last resort at date 2 would be
\[ \pi^{OI} = (\bar{v} - (v - \varepsilon))q^*. \]

We will compare this uncertain return with the return resulting from the other two strategies.

**Open market operation.** With an open market operation, the lender of last resort (usually the central bank) offers additional credit to any counterparty eligible to take part in the operation. Note that collateral requirements do not exclude any market participants from the operation. Sellers in distress, for instance, are in possession of the risky asset which can be used as collateral. But also unfunded buyer could obtain liquidity, provided that they pledge the asset to be acquired with the help of the credit to the central bank. Thus, the liquidity offered by the central bank in the open market would be available to all market participants. However, provided that \( \varepsilon > 0 \), which is realistic in our view, we would expect that also buyers take part in the operations. This is because the seller’s gain from averting an imminent liquidation corresponds to the difference between market prices at dates 1 and 2, but this is just what the gain would be for a buyer to invest at date 1 and to harvest the return of her investment at date 2!

Assume that the lender of last resort offers liquidity \( l > 0 \) (i.e., short-term credit) in the money market in state \( S \). Let \( r^{MM} \geq 0 \) denote the interest rate to be paid by market participants for the liquidity obtained in the open
market operation. We assume that $r^{\text{MM}}$ is not to high. Specifically, we require

$$\varepsilon > r^{\text{MM}}(v - \varepsilon),$$

so that funds are attractive both for unfunded buyers and sellers.

Each seller seeks $c$, while unfunded buyers demand $p_1^S > c$. Thus, total demand in the open market operation would be equal to $c + (1 - \beta^f)p_1^S$. Hence, after rationing incoming requests for credit, an individual market participant (either seller or buyer) would obtain funding with probability

$$\rho = \min\{1; \frac{l}{c + (1 - \beta^f)p_1^S}\}.$$ 

As a consequence of the additional buying and selling in state $S$, the price $p_1^S$ in the market under distress at date 1 would satisfy

$$p_1^S = v - \frac{\gamma \sigma^2}{2} \frac{1}{1 - \delta}(1 - \beta^f - \rho(2 - \beta^f)).$$

Thus, for $p_1^S = v - \varepsilon$, we obtain the allotment ratio

$$\rho = \frac{q^*}{2 - \beta^f} < 1,$$

and consequently

$$l = \left(\frac{1}{2 - \beta^f}c + \frac{1 - \beta^f}{2 - \beta^f}(v - \varepsilon)\right)q^*.$$

so that in particular $\rho < 1$. Any seller that received funds in the open market operation generates a net return to the lender of last resort of $-c$ if $\tilde{v} \leq 0$, a return of $\tilde{v} - c$ if $0 < \tilde{v} < c(1 + r^{\text{MM}})$, and a return of $c r^{\text{MM}}$ if $\tilde{v} \geq c(1 + r^{\text{MM}})$. Similarly, each unfunded buyer that obtained funding in the operation causes a net return of $-(v - \varepsilon)$ if $\tilde{v} \leq 0$, a return of $\tilde{v} - (v - \varepsilon)$ if $0 < \tilde{v} < (v - \varepsilon)(1 + r^{\text{MM}})$, and a return of $(v - \varepsilon)r^{\text{MM}}$ if $\tilde{v} \geq (v - \varepsilon)(1 + r^{\text{MM}})$.

The return from the open market operation would consequently be given by $\pi^{\text{MM}}$, where

$$\frac{\pi^{\text{MM}}}{q^*} = \frac{1}{2 - \beta^f}(\max\{\min\{\tilde{v}; c(1 + r^{\text{MM}}); 0\} - c\} + \frac{1 - \beta^f}{2 - \beta^f}(\max\{\min\{\tilde{v}; (v - \varepsilon)(1 + r^{\text{MM}}); 0\} - (v - \varepsilon)\}).$$
Emergency lending. If the lender of last resort provides liquidity assistance to a subpopulation $q^*$ of the sellers in distress at an interest rate $r^{TA}$, then funds in the dimension of $cq^*$ would be necessary. Any salvaged seller would keep the asset until date 2, and then reimburse $\min\{\bar{v}; (1 + r^{TA})c\}$ to the lender of last resort. The lender of last resort would end up with a return of

$$\pi^{TA} = (\max\{\min\{\bar{v}; (1 + r^{TA})c\}; 0\} - c)q^*$$

at date 2.

For the following ranking of policy alternatives, we have to assume that interest rates are not too high, because otherwise neither buyers nor sellers would find it sufficiently attractive to take up the credit from the lender of last resort. But then, as illustrated in Figure 2, we obtain a clear ranking of the three policies in terms of exposure for the lender of last resort.

**Proposition 2.** Assume that condition (4) holds and that Assumptions A through C are satisfied. Then for any sufficiently modest interest rates $r^{TA} \geq 0$ and $r^{MM} \geq 0$, the truncated return profiles for the lender of last resort satisfy

$$\min\{\pi^{TA}; 0\} \geq \min\{\pi^{MM}; 0\} \geq \min\{\pi^{OI}; 0\},$$

where the inequalities are strict with positive probability.

Proposition 2 suggests a theoretical argument of why, under certain circumstances, it may be preferable to assist institutions in trouble directly than to conduct an anonymous market operation. The reason is the strategic behavior of potential buyers. Just like the distressed sellers, they have a motive for seeking funds when the market price falls significantly under fundamentals. In the model, this incentive is strict provided that $\varepsilon > 0$.

4. Extensions and robustness

This section treats a number of extensions.
Short-selling. Short-selling can indeed be considered in the model, provided that $\beta^f < s$ (not too many buyers in the market, which is not much more restrictive than our assumptions so far). As before we focus on a situation where the perceived risks are high and there is little willingness to invest. Short-selling would mean that a funded buyer repoes the risky asset from a seller (or from a market maker) against cash between date 0 to date 1. The risky asset would be sold in the market at date 0 and bought back by the buyer at date 1. In case that the buyer wishes to buy another unit at date 1, the buyer may submit two market offers (there should be unlimited credit for buyers and sellers within the trading date to make this work).

Short selling has several effects: The effect on precautionary liquidation is always negative in the model. In fact, precautionary liquidations are substituted by short-sales. The selling pressure in the liquidation state $S$ is increased due to a higher population of sellers that have not taken precautionary measures, at the same time the selling pressure is weakened by the buy-backs by the buyers that resolve their short positions. The overall effect balances out in the model. The buying pressure is higher in the non-liquidation state $N$, because here, the sellers wait (until date 2), while the buyers (who are short) buy, as in state $S$, twice as much as without short-selling. Welfare is higher with short-selling (should be), because risks are allocated more efficiently.

Moral hazard. We have abstracted so far from the incentive effects of policy regimes for the lender of last resort on commercial bank risk taking. It is feasible to adapt to model to capture also moral hazard concerning liquidity risk-taking (this requires considering a perfectly divisible asset). Intuitively, commercial banks may decide first about the quantity of liquidity to invest. When a shock occurs, then some banks will be affected, while others are not affected. Banks that are affected would have to liquidate part of their portfolio, while banks that are not affected have funds available for investment in the crisis. In such a scenario, moral hazard is caused likewise by targeted assistance and money market intervention. The reason why money market intervention may cause moral hazard is because of the
speculative motive of commercial banks. When it is anticipated that funds will be offered to all market participants, then there is a reduced incentive for prospective speculators to hold transaction liquidity.

**Lending at a penalty rate.** We have assumed in Proposition 2 that money market intervention happens at moderate interest rates. To see what happens when liquidity is offered at penalty rates, assume that

\[ r_{\text{MM}}^c \varepsilon < r_{\text{MM}}(v - \varepsilon). \]

Under this condition, only sellers in distress would find it in their interest to participate in the open market operation. This suggests an alternative policy option which would be an open market operation in which the interest rate is chosen so that the credit is valuable for the troubled investors, but not for speculators. Given Assumption C, such an interest rate will always exist. This provides a theoretical argument that is different from the traditional focus on moral hazard of why a penalty should be imposed on emergency funds, namely to make such funds unattractive for speculators.\(^{15}\)

5. Conclusion

When a population of investors fears a future need for liquidation, then it will be rational for some or even for all sellers to liquidate their positions before the actual arrival of the crisis. We have shown that this finding is robust with respect to the introduction of a population of funded buyers. Precautionary liquidations occur unless the mass of funded buyers in the population at least outweighs the mass of potentially distressed sellers. The rationale behind this finding is the strategic behavior of buyers. An asymmetry between sellers and buyers is caused by the fact that sellers will hold on in the good state while buyers have to use their only remaining opportunity. This effect renders the ex ante valuation of the asset by the sellers to be higher than the buyers’ valuation, which motivates the speculative trading on the part of the buyers.

\(^{15}\)Repullo [13] shows that penalty rates can also improve the incentives for the lender of last resort.
The extended model allowed us to rank three commonly perceived policy options for the lender of last resort in a situation of market distress. Specifically, we showed that outright intervention in the asset market is an inferior strategy to the provision of short-term credit. Moreover, among the methods of providing short-term funding to a banking system in distress, targeted lending is more desirable for the lender of last resort, at least under the assumptions made in the analysis.

Our results might have a bearing on the recent developments in the context of the subprime crisis. On August 9, 2007, the executive board of the ECB decided to inject EUR 95bn through a fine-tuning operation. In contrast, the Federal Reserve on August 17 has chosen to offer targeted liquidity assistance by lowering the interest charged on discount window lending. Our analysis suggests that as a lender of last resort, the Eurosystem is less risk averse than the Federal Reserve, which would be consistent with perceived differences in central bank independence.

Appendix A. Equilibrium conditions

The trading model allows strategic decisions on dates 0 and 1, where two states (\(N\) and \(S\)) are feasible at date 1. In the sequel, we describe the equilibrium conditions resulting from individual profit maximization for the case \(\beta^0 = 0\). The adaptations to the general case are straightforward. As sellers have no discretion in case of a liquidity shock (i.e., \(\alpha^S_1 = 1 - \alpha_0\)), an equilibrium can be formally described by a vector

\[
(\alpha_0, \alpha^N_1, \alpha^N_{1,\beta}, \alpha^S_1, \beta_0, \beta^N_1, \beta^S_1).
\]

To constitute an equilibrium, a number of conditions must be satisfied.

First, and most obviously, there are nonnegativity constraints

\[
\alpha_0 \geq 0, \beta_0 \geq 0
\]

\[
\alpha^\omega_1 \geq 0, \beta^\omega_1 \geq 0, \alpha^\omega_{1,\beta} \geq 0 \quad \text{for } \omega = N, S,
\]
as well as population accounting constraints

\[
\alpha_0 + \alpha^N_1 \leq 1
\]
\[
\beta_0 + \beta^\omega_1 \leq \beta^f \quad \text{for } \omega = N, S
\]
\[
\alpha^\omega_{1,\beta} \leq \beta_0 \quad \text{for } \omega = N, S.
\]

In addition, there are several restrictions from incentive compatibility. For the sellers at date 0, we obtain

\[
\text{if } p_0 < sp^S_1 + (1 - s) \max\{p^N_1, v\} \text{ then } \alpha_0 = 0
\]
\[
\text{if } p_0 > sp^S_1 + (1 - s) \max\{p^N_1, v\} \text{ then } \alpha_0 = 1.
\]

Similarly, for the funded buyers at date 0,

\[
\text{if } p_0 > s \min\{p^S_1, v\} + (1 - s) \min\{p^N_1, v\} \text{ then } \beta_0 = 0
\]
\[
\text{if } p_0 < s \min\{p^S_1, v\} + (1 - s) \min\{p^N_1, v\} \text{ then } \beta_0 = \beta^f.
\]

Incentive compatibility at date 1 is tantamount to

\[
\text{if } p^N_1 < v \text{ then } \alpha^N_1 = 0 \text{ and } \beta^N_1 = \beta^f - \beta_0
\]
\[
\text{if } p^N_1 > v \text{ then } \alpha^N_1 = 1 - \alpha_0 \text{ and } \beta^N_1 = 0
\]
\[
\text{if } p^S_1 < v \text{ then } \beta^S_1 = \beta^f - \beta_0
\]
\[
\text{if } p^S_1 > v \text{ then } \beta^S_1 = 0.
\]

Disinvesting at date 1 is governed by

\[
\text{if } p^\omega_1 < v \text{ then } \alpha^\omega_{1,\beta} = 0, \text{ and}
\]
\[
\text{if } p^\omega_1 > v \text{ then } \alpha^\omega_{1,\beta} = \beta_0
\]

for \( \omega = N, S \). Finally, prices at dates 0 and 1 are given by Lemma 1.

Appendix B. Proofs

**Proof of Lemma 1.** Assume that a subpopulation of size \( \alpha_0 \) of the sellers and a subpopulation of size \( \beta_0 \) of the funded buyers decides to trade at date
Then the market maker sets a price $p_0$ such that expected utility remains unchanged, i.e.,

$$E[-\exp\{-\gamma x_0\}] = E[-\exp\{-\gamma(x_0 + (\alpha_0 - \beta_0)(\overline{v} - p_0))\}].$$

(5)

In the CARA-normal framework, equation (5) is equivalent to

$$x_0 = E[x_0 + (1 - \beta^f)(\overline{v} - p_0)] - \frac{\gamma}{2} V[x_0 + (\alpha_0 - \beta_0)(\overline{v} - p_1)],$$

where $V[.]$ denotes the variance. Re-arranging yields the first assertion of the lemma. Similarly, the price $p_1^v$ at date 1 is implicitly given by

$$E[-\exp\{-\gamma(x_0 + (\alpha_0 - \beta_0)(\overline{v} - p_0) + (\alpha_1^v + \alpha_{1,\beta}^v - \beta_1^v)(\overline{v} - p_1))\}] = E[-\exp\{-\gamma(x_0 + (\alpha_0 - \beta_0)(\overline{v} - p_0))\}]$$

(6)

Combining (6) with (5) and subsequently applying the rules for the CARA-normal model yields the second assertion, and thereby the lemma. □

**Proof of Proposition 1.** Uniqueness of the price path is a consequence of Lemmas B.1 through B.3 below. The inequalities concerning the price process follows from Lemmas B.4 through B.7 below. Assume now that $s > 0$ and $\beta^f < 1$. We wish to show that $\alpha_0 > 0$. To provoke a contradiction, assume that $\alpha_0 = 0$. Then Lemma 1 delivers $p_0 \geq v$, and Lemma B.7 implies that $p_0 = v$, so that by another application of Lemma 1, we find that $\beta_0 = \alpha_{1,\beta}^v = 0$. Moreover, $\alpha_1^S = 1 - \alpha_0 = 1$, and therefore, by Lemma 1,

$$p_1^S = v - \frac{\gamma \sigma^2}{2} (1 - \beta_1^S) < p_0.$$

In particular, since $p_1^N \leq v$ by Lemma B.6 then

$$E[\max\{p_1^v, v\}] < v = p_0,$$

when $s > 0$. But then $\alpha_0 = 1$, which is the desired contradiction. To see why the equilibrium is inefficient, assume $\beta^f < 1$, and that (4) is satisfied. In the second-best allocation, risk-averse market makers hold a zero position in the asset. Only if a liquidity shock occurs, sellers will liquidate. As a mass of $\beta^f$
funded buyers stands ready to buy the asset conditional on a crisis, prices at
dates 0 and 1 would be $p_0 = p_1^N = v$ and

$$p_1^s = v - \frac{\gamma \sigma^2}{2} (1 - \beta^f).$$

Thus, in the second best, without loss of generality,

$$\alpha_0 = 0, \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = 0, \beta_1^S = \beta^f.$$  

By our assumption on zero-rent market making, the market making sector
can be left out of the welfare analysis. The expected loss in utility for the
sellers caused by inefficient selling would be $-s\lambda (1 - \beta^f)$, where $\lambda = \gamma \sigma^2/2$. For each funded buyer, there is a countervailing utility gain of the same absolute size. As the mass of sellers that transfer their asset to the market making sector is just $1 - \beta^f$, on aggregate, we obtain a welfare of $W^{SB} = -s\lambda (1 - \beta^f)^2$. When a run on the financial market occurs, however, a mass of

$$\alpha_0 = \frac{s}{1 - s} (1 - \beta^f)$$

sellers liquidates early, and no market participant buys early, leading to an
outcome

$$\alpha_0 = \frac{s}{1 - s} (1 - \beta^f), \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = \beta^f, \beta_1^S = \beta^f.$$  

This creates a disutility for an early seller of

$$-\lambda \frac{s}{1 - s} (1 - \beta^f). \quad (7)$$

As sellers are indifferent between selling early and selling late, the aggregate
utility of the sellers is also given by (7). The buyers all buy at date 1, with
a probability of $s$ at price $p_1^s$ and with probability $1 - s$ at price $p_1^N$. This
yields an expected utility for a buyer of

$$\lambda \left\{ \frac{s}{1 - s} (1 - \beta^f) + 2s - (1 + s)\beta^f \right\}.$$  

Aggregating over all buyers and including the sellers yields the third-best
welfare

$$W^{TB} = -\lambda \left\{ \frac{s}{1 - s} (1 - \beta^f)^2 - 2s\beta^f + (1 + s)(\beta^f)^2 \right\}.$$
Compared to the second-best allocation, the loss in welfare therefore amounts to

$$\Delta = W^{TB} - W^{SB}$$
$$= -\lambda \left\{ \frac{s}{1-s} (1 - \beta^f)^2 + (\beta^f)^2 - s \right\} < 0.$$ 

The other two cases are proved analogously. For $\beta^f = 1$, Lemma B.1 below says that the market price for the asset at dates 0 and 1 cannot fall below $v$.

**Lemma B.1.** When $\beta^f \geq 1$, then $p_0 = p_1^S = p_1^N = v$. This is an equilibrium outcome.

**Proof.** From Lemma B.5 below, we know that

$$p_0 = sp_1^S + (1-s)v. \tag{8}$$

But then

$$\alpha_0 - \beta_0 = s(1 + \alpha_0 - 2\beta_0 - \beta_1^S).$$

Rewriting yields

$$(\alpha_0 - \beta_0)(1-s) = s(1 - \beta_0 - \beta_1^S).$$

From Proposition 1, we know that $p_1^S \leq v$. But then, if we had $p_1^S < v$, then $\beta_1^S = \beta^f - \beta_0$ and therefore $1 - \beta_0 - \beta_1^S \leq 0$. But then also $\alpha_0 - \beta_0 \leq 0$. Thus, $p_0 \geq v$. From Lemma B.7, this can only be true, however, when $p_0 = v$, which contradicts $p_1^S < v$ because of (8). Thus, $p_1^S = v$. Hence, also $p_0 = p_1^N = v$.

The equilibrium set is described by $\alpha_0 = \beta_0 \in [0; 1]$ and by $\alpha_1^N = \beta_1^N = 1 - \alpha_0$ for $\omega = N, S$. □

**Lemma B.2.** When $p_0 > sp_1^S + (1-s)v$, then

$$\alpha_0 = 1, \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = \beta_1^S = \beta^f. \tag{9}$$

The tuple (9) is an equilibrium provided that $s(2 - \beta^f) > 1$. 

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**Proof.** Assume that (3) holds strictly. Then, as \( p_1^N \leq v \) all sellers have a strict incentive to liquidate at date 0. Thus, \( \alpha_0 = 1 \) and \( \alpha_1^N = \alpha_1^S = 0 \). Moreover, from

\[
p_0 > sp_1^S + (1-s)v \geq sp_1^S + (1-s)p_1^N ,
\]

all buyers have a strict incentive to trade only at date 1, if at all. Hence, in equilibrium, \( p_0 = v - \gamma \sigma^2/2 \) and

\[
p_1^S = v - \frac{\gamma \sigma^2}{2} (2 - \beta_1^S).
\]

The strict version of (3) implies \( \beta_1^S = \beta^f \) and therefore also \( s(2 - \beta^f) > 1 \). Conversely, when \( s(2 - \beta^f) > 1 \), then it is straightforward to check that (9) describes an equilibrium. \( \square \)

**Lemma B.3.** Assume \( \beta^f < 1 \). When \( p_0 = sp_1^S + (1-s)v \), then

\[
\alpha_0 - \beta_0 = \frac{s}{1-s} (1 - \beta^f).
\]

This behavior is part of an equilibrium provided that \( s(2 - \beta^f) \leq 1 \). Moreover, \( \beta_0 = 0 \) provided that \( s(2 - \beta^f) \leq \beta^f \).

**Proof.** Assume that

\[
p_0 = sp_1^S + (1-s)v.
\]

Combining this with (1) and (2), and subsequently applying \( \beta^f < 1 \) yields \( p_1^S < p_0 < v \). Equation (11) implies

\[
\alpha_0 - \beta_0 = s(1 + \alpha_0 - 2\beta_0 - \beta_1^S).
\]

Because of \( \beta_1^S = \beta^f - \beta_0 \), we have proved (10). There are now two cases, according to whether \( p_1^N < v \) or \( p_1^N = v \). Assume first \( p_1^N < v \). The endogenous parameters characterizing such an equilibrium would satisfy the conditions

\[
\alpha_0 \geq 0, \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = \beta^f, \beta_1^S = \beta^f
\]

(12)
Moreover, prices are given by

\[ p_0 = v - \frac{\gamma \sigma^2}{2} \alpha_0 \]  
\[ p_1^N = v - \frac{\gamma \sigma^2}{2} (2\alpha_0 - \beta f) \]  
\[ p_1^S = v - \frac{\gamma \sigma^2}{2} (1 + \alpha_0 - \beta f). \]

Plugging the explicit expressions for the prices in the no-arbitrage condition (11) and re-arranging yields (10) provided that \( s(2 - \beta f) \leq 1 \). Plugging (10) back into the price formulas (13), (14), and (15) delivers

\[ p_0 = v - \frac{\gamma \sigma^2}{2} \frac{s}{1-s} (1 - \beta f) \]  
\[ p_1^N = v - \frac{\gamma \sigma^2}{2} \left\{ \frac{2s}{1-s} - \frac{1+s}{1-s} \beta f \right\} \]  
\[ p_1^S = v - \frac{\gamma \sigma^2}{2} \frac{1}{1-s} (1 - \beta f). \]

Note that \( p_1^N \leq v \) is tantamount to \( (2 - \beta f)s \geq \beta f \). It is now straightforward to check that

\[ \alpha_0 = \frac{s}{1-s} (1 - \beta f), \beta_0 = 0, \alpha_1^N = 0, \beta_1^N = \beta_1^S = \beta f. \]

forms an equilibrium. To treat the second case, assume that \( p_1^N = v \). Lemma 1 implies

\[ 2(\alpha_0 - \beta_0) + \alpha_1^N - \beta_1^N = 0. \]  

(19)

This proves the assertion. \( \square \)

**Lemma B.4.** \( p_1^S \leq p_1^N \).

**Proof.** Assume to the contrary that \( p_1^N < p_1^S \). Then, buyers’ rationality implies \( \beta_1^N \geq \beta_1^S \) and \( \alpha_1^S \leq \alpha_1^S \). Moreover, \( \alpha_1^N \leq \alpha_1^N \). Lemma 1 implies that \( p_1^N \geq p_1^S \). The contradiction shows that indeed \( p_1^N \geq p_1^S \). \( \square \)

**Lemma B.5.** \( p_0 \geq sp_1^S + (1-s)v \).

**Proof.** Assume to the contrary that

\[ sp_1^S + (1-s)v > p_0. \]  

(20)
Then there would be no precautionary selling, i.e., $\alpha_0 = 0$. But then, by Lemma 1, $p_0 \geq v$. Using (20), this implies $p_1^S > v$. But then, by the buyers’ rationality, $\alpha_{1,\beta}^S = \beta_0$, and $\beta_1^S = 0$. From Lemma 1, we obtain

$$p_1^S = v - \frac{\gamma \sigma^2}{2} (1 - \beta_0) < v - \frac{\gamma \sigma^2}{2} (-\beta_0) = p_0,$$

a contradiction to (20) as $v < p_0$. This proves the lemma. □

**Lemma B.6.** $p_1^N \leq v$.

**Proof.** Assume to the contrary that $p_1^N > v$. Then, sellers’ rationality implies $\alpha_{1,\beta}^N = \beta_0$ and $\alpha_1^N = 1 - \alpha_0$. Buyers’ rationality implies $\beta_1^N = 0$. Hence, by Lemma 1,

$$p_1^N = v - \frac{\gamma \sigma^2}{2} (1 + \alpha_0 - \beta_0) < p_0. \quad (21)$$

Using Lemma B.4, $p_0$ is strictly larger than all future prices. Hence, $\beta_0 = 0$, and (21) implies $p_1^N < v$, a contradiction. Hence, the assertion. □

**Lemma B.7.** $p_0 \leq v$.

**Proof.** Assume to the contrary that $p_0 > v$. Then, by Lemma 1, there must be more buying than selling at date 0, i.e., $\beta_0 > \alpha_0$. In particular, $\beta_0 > 0$. But early buying is rational only when market participants expect to be able to realize a weakly higher price from date 1 onwards, i.e., when

$$E[\max\{p_1^\omega, v\}] \geq p_0 > v. \quad (22)$$

But then, for at least one state $\omega$, we must have that $p_1^\omega > v$, which is impossible in view of Lemmas B.4 and B.6. □

**Proof of Proposition 2.** There are three cases. Assume first that $\tilde{v} \leq 0$.

Then

$$\pi^{TA}(\tilde{v}) = -c > -(\frac{1}{2 - \beta f} c + \frac{1 - \beta f}{2 - \beta f} (v - \varepsilon)) = \pi^{MM}(\tilde{v})$$

and

$$\pi^{MM}(\tilde{v}) > -(v - \varepsilon) \geq \tilde{v} - (v - \varepsilon) = \pi^{OI}(\tilde{v}).$$
Assume now that $0 < \tilde{v} \leq c$. Then

$$\pi^{TA}(\tilde{v}) = \tilde{v} - c > \tilde{v} - \left( \frac{1}{2 - \beta f} c + \frac{1 - \beta f}{2 - \beta f} (v - \varepsilon) \right) = \pi^{MM}(\tilde{v})$$

and

$$\pi^{MM}(\tilde{v}) > \tilde{v} - (v - \varepsilon) = \pi^{OI}(\tilde{v}).$$

For $\tilde{v} > c$, we have $\pi^{TA}(\tilde{v}) \geq 0$, so that it suffices to show that

$$\pi^{MM}(\tilde{v}) \geq \pi^{OI}(\tilde{v})$$

for all realizations of $\tilde{v} > c$ such that $\pi^{MM}(\tilde{v}) < 0$. Without loss of generality, we may assume that $r^{MM} = 0$ (otherwise, the net return from the open market operation can only be higher). But then $\pi^{MM}(\tilde{v}) < 0$ if and only if $\tilde{v} < (v - \varepsilon)$. But for values $\tilde{v} \in [c; v - \varepsilon]$, the slope of $\pi^{MM}(\tilde{v})$ is only $(1 - \beta f)/(2 - \beta f) < 1$, while the slope of $\pi^{OI}(\tilde{v})$ is 1. Hence the assertion. □
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Figure 1: Full characterization of the equilibrium

Figure 2: Policy ranking
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