Bank Recapitalization and
the Information Value of a Stress Test in a Crisis

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Abstract
During the recent financial crisis, uncertainties about banks’ solvency paralyzed financial markets and persuaded regulators to reveal an unprecedented amount of information about banks. We examine the trade-off faced by a welfare-maximizing regulator who can choose whether to disclose banks’ capital shortfall in crisis times. Disclosure forces banks to reduce their risk of default, but leads them to downsize unless the regulator is able to recapitalize the banks that do not replenish their shortfall. We show that a regulator who cannot recapitalize banks will prefer less information to be disclosed if the costs of downsizing are greater than expected default costs. In the opposite case, or in case the regulator is able to recapitalize banks, we demonstrate that banks’ capital shortfall will be fully revealed. Our model explains why the market’s reaction to stress tests was favorable in the U.S. and negligible in Europe. Our results also have implications for bank regulation. Among them, we highlight that recapitalizing banks through the European Stability Mechanism would make bank stress tests more effective in Europe.

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I. Introduction

Regulators responded to the recent financial crisis with extraordinary measures, including bank stress tests. Stress tests are simultaneous and forward-looking bank examinations that aim to assess the value of banks’ capital in a hypothetical adverse scenario. Unlike ordinary bank examinations, the results of bank stress tests in the U.S. and Europe were publicly disclosed. The rationale behind this unprecedented information disclosure rests on the view that banks’ opacity contributed to the recent financial crisis. According to this view, banks’ opacity impaired the ability of investors to assess banks’ solvency after the subprime crisis exploded in August 2007. Investors became reluctant to lend fearing information asymmetries. This caused the worsening of banks’ funding conditions over the course of the recent financial crisis.

The conventional wisdom is that the disclosure of stress test results would enhance the transparency of banks, as stress tests provide more thorough information than available to the market. Transparency would revive financial markets enabling market participants to sort out the good from the bad banks. A proper analysis of information disclosure, however, should take into account the nature of the information disclosed by regulators. Stress tests reveal banks’ capital shortfall. Banks’ undercapitalization is socially costly, especially in times of crisis. First, the lower bank’s capital, the more likely bank default. A bank default generates social costs, as the turmoil that followed Lehman Brothers’ bankruptcy has demonstrated. Second, governments often inject taxpayers’ money into banks in times of crisis. The costs of rescuing banks became evident during the recent crisis, with southern European countries and Ireland caught in a bank-sovereign downward spiral. Third, the lack of capital impairs the ability of banks to supply credit. The reduction in the credit supply is one of the channels through which a shock to banks turned into a severe economic crisis. All in all, other frictions besides information asymmetries exist in times of crisis. Hence it is unclear from a theoretical point of view whether information disclosure is optimal in times of crisis.

This paper examines whether the disclosure of information about banks maximizes welfare in times of crisis. We are interested in the interaction among the disclosure of banks’ capital shortfall, regulator’s ability to recapitalize banks, and the choice of banks to replenish their capital. The disclosure of banks’ capital shortfall leads banks to replenish their capital and hence to reduce their probability of default. However, replenishing capital in times of crisis requires either state recapitalization or a reduction in banks’ size. This

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Heider et al. (2009) describe the functioning of interbank markets during the recent financial crisis, and provide a theoretical explanation based on adverse selection. Gorton (2008), Dudley (2009), and Lewis, ed (2008) argue that banks’ opacity contributed to the 2007-2009 financial crisis.
leads to the following trade-off: Information disclosure leads to a reduction in banks’ risk of default, but
costs either taxpayers’ money or a lower supply of credit.

We provide a positive analysis of bank stress tests, and abstract from socially optimal regulatory poli-
cies. We take as our starting point that regulators adopted bank stress tests as a response to the recent
financial crisis, and examine the effectiveness of bank stress tests in times of crisis. Our crisis environment
includes some key features of financial crises: Asymmetric information about banks’ capital shortfall, banks'
reluctance to raise capital in the market\(^2\), costs of recapitalization, default\(^3\), and downsizing. We consider a
regulator who discloses information about banks’ capital shortfall and can recapitalize banks. The regulator
can manage information disclosure choosing the accuracy of the stress test. We interpret accuracy as the
amount of information the stress test is based on, and the effort in extracting this information from banks.
The higher accuracy, the greater the probability that a stress test identifies a bank with a capital shortfall
out of the pool of banks. We prefer this interpretation to one where regulators lie about banks because reg-
ulators usually disclose data about banks’ risk exposure and the stress testing methodology\(^4\). This enables
the market to verify the results of stress tests.

We show that a welfare-maximizing regulator fully reveals banks’ capital shortfall if it can recapitalize
banks. In case it cannot, we demonstrate that partial disclosure might be optimal. The logic of our argument
is that market discipline forces banks to replenish their capital. Banks will raise capital in the market only
if the regulator has easy access to funds and hence can credibly threaten recapitalization. In this case, the
regulator will fully reveal banks’ capital shortfall because banks fill their capital shortfall and invest. If the
regulator cannot recapitalize banks, banks will downsize and a trade-off will arise. Downsizing reduces the
probability of default, but at the cost of foregoing valuable investment opportunities. Optimal information
disclosure will depend on the relative magnitude of these two costs. In case the foregone returns are greater
than expected default costs, the regulator will prefer less information to be disclosed. However, the regulator
will have incentives to reveal at least part of banks’ capital shortfall to prevent the remaining banks from
downsizing. In spite of not being able to recapitalize banks, the regulator will fully reveal banks’ capital

\(^2\) Banks’ reluctance to raise capital in the market is consistent with the empirical evidence in Adrian and Shin (2011), who
show that adjustments in banks’ leverage occur through changes in banks’ asset size

\(^3\) Banks’ default destroys the value of bank-borrower relationships (see Slavin et al. (1999)), and generates contagion (see Allen and Gale (2000), Freixas et al. (2000)).

\(^4\) This was the case for the U.S. SCAP program and the 2011 EU-wide stress test. The 2010 EU-wide stress test did not
require the disclosure of risk exposures, but most of banks voluntarily disclosed them. The 2009 EU-wide stress test did not
require the disclosure of any information.
shortfall if expected default costs are greater than the foregone returns on assets.

Our results explain some empirical and anecdotal evidence about stress tests in the US and Europe. First, our model links the favorable market’s response to the U.S. stress test (see Greenlaw et al. (2012), Peristiani et al. (2010), Hirtle et al. (2009), and Schuermann (2012)) to the implementation of the Capital Assistance Plan (CAP). As the U.S. could raise funds at relatively low costs, the CAP was a credible backstop mechanism. Indeed, U.S. banks filled the $75 bn capital shortfall raising capital in the market. Second, our model relates the skeptical market’s reaction to the European stress tests (Greenlaw et al. (2012), Hirtle et al. (2009), and Schuermann (2012)) to the lack of a credible EU-wide backstop mechanism and to the different incentives of EU’s members. Countries in EU’s periphery were reluctant to borrow from the European Financial Stability Facility (EFSF) fearing that this would precipitate a bank-sovereign downward spiral. Using taxpayers’ money to save banks would have been politically costly for core EU countries. Regulators had little powers to prevent European banks failing the stress test from downsizing. Downsizing was presumably more costly in Germany, where the economy was performing well, than in Spain, which was in the midst of a crisis originated by the burst of a property bubble. According to our model, and consistent with anecdotal evidence, the German regulator should have preferred less information to be disclosed than the Spanish. As the different incentives of EU’s members had presumably to be considered, European stress tests ended up being not as effective as in the U.S. Few banks failed the European stress tests, and all of them were either restructured or acquired by other banks.

Our model implies that the recapitalization of banks is crucial for the effectiveness of bank stress tests in times of crisis. The reason is that only second best equilibria arise if banks failing the stress test do not raise capital in the market. In equilibrium, either some banks default and/or downsize. We highlight two policy implications following from this result. First, directly injecting capital into struggling banks through the European Stability Mechanism (ESM) would make stress tests more effective in Europe. The costs of recapitalization would fall on the ESM and not on the countries. Second, banks should be required to maintain a certain absolute level of capital rather than a capital to assets ratio, as argued by the supporters of the macroprudential approach to banking regulation (see Greenlaw et al. (2012) for example). Banks would downsize independently of regulator’s funding costs if capital requirements are expressed in ratio terms.

\footnote{The CAP program would provide mandatory convertible shares (MCP) in case banks failing the stress test were unable to raise capital privately. MCP shares would convert to common equity if the condition of banks did not improve within a defined time period.}
A broader implication of our model is that the efficient resolution of banks’ distress matters for the effectiveness of bank stress tests in times of crisis. In a more general model than ours, bank distress could be resolved not only by injecting capital, but also by restructuring banks. Restructuring consists in renegotiating banks’ liabilities, as in debt-for-equity swaps, or both banks’ assets and liabilities, as in “good-bank/bad-bank” solutions. If restructuring gives banks enough capital to sustain the lending activity, and does not cost taxpayers’ money, there would be no social costs associated to the disclosure of banks’ capital shortfall. This would make bank stress tests effective policy instruments in times of crisis. In our model there is a gain from recapitalizing banks failing the stress test, as the net present value of their assets in place is positive. Recapitalization would not be worth in case financial distress is more severe, and banks cannot continue as going concerns. In this case, efficient bankruptcy procedures would lower the costs of disclosing banks’ capital shortfall and make bank stress tests effective policies instruments.

The resolution of banks’ distress and the disclosure of bank supervisory information have been considered in isolation by most of the existing literature. A strand of literature has examined the reasons why regulators might have incentives to forbear rather than resolve a weak bank. In our model, the regulator faces a similar choice having to decide whether to reveal the capital shortfall of a bank. Another strand of literature has addressed the question whether regulators should disclose the results of ordinary bank examinations, which are usually kept confidential. Jordan et al. (1999) argued in favor, showing that announcing formal enforcement actions enhanced market discipline during the U.S. “Savings and Loan” crisis. Berger et al. (2000), Berger and Davies (1998), and Flannery and Houston (1999) find evidence that bank examinations have an information value. However, Prescott (2008) provides a theoretical argument against disclosure pointing out that disclosure might reduce the incentives of regulators to collect information from banks. Goldstein and Sapra (2012) provide a survey of second best theoretical environments where information disclosure might not be socially optimal. They argue that disclosing aggregate stress test results could minimize the social costs of disclosure. Our main contribution is to bridge these two strands of literature and point out that the resolution of banks’ distress is crucial for regulators’ incentives to disclose information.

The closest paper to ours is Shapiro and Skeie (2012). The authors also relate the disclosure of stress test results to regulator’s ability to recapitalize banks. They demonstrate that a high funding cost regulator prefers transparency in times of crisis, that is when priors about the banking system are negative. In their

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model the regulator reveals bank’s type because asymmetric information leads investors to run good banks if priors are unfavorable. Bank’s type might be revealed by a stress test or might be signaled by an equity injection. A regulator will prefer to run a stress test rather than inject equity if its funding costs are high and priors are unfavorable. Our contribution is to model banks’ reaction to information disclosure, which Shapiro and Skeie (2012) do not consider. Modeling banks’ reaction allows us to consider the trade-offs between costs of recapitalization and costs of reducing the credit supply, and costs of reducing the credit supply and default costs. These trade-offs are crucial in the macroprudential approach to banking regulation.

Other theoretical papers about bank stress tests examine whether disclosure is optimal, but do not relate disclosure to bank resolution policies. Bouvard et al. (2012) demonstrate that transparency is optimal in times of crisis but not in normal times. The reason is that disclosing stress test results prevents a bank run on the whole banking system in bad times (when priors are unfavorable). In good times opacity is optimal because it prevents bank runs on weak banks. Gick and Pausch (2012) demonstrate that disclosing stress test results together with the stress test methodology is optimal in a context where regulators can affect investors’ beliefs about the banking system.

This paper is structured as follows. Section 2 outlines the model setup. Sections 3 and 4 describe the equilibrium of the game between investors, banks, and the regulator. Section 5 highlights the empirical predictions, while Section 6 outlines the policy implications. Section 7 concludes the paper.

II. The Model Setup

We consider a game with a regulator, a continuum of risk neutral and competitive investors, and a measure 1 of banks.

A. The Payoff of Banks

Banks start out with a predetermined stock of debt D, a measure 1 of assets, and an amount of cash c. Cash can be exogenous profits realized at the beginning of the period, or the liquidity previously stored by banks. The value of assets at time 1 differs across banks and is not known to investors. A measure 1 − β of banks, which we define bad (B) banks, has a capital shortfall \( D - A^b \) in the adverse state of the world (A). In the favorable state of the world, bad banks have positive equity as their assets are worth \( A^b + b \), and \( A^b > D \).
We assume that \( b \) is not verifiable and cannot be pledged to investors\(^7\). The complementary fraction \( \beta \) of banks, which we define good (G) banks, has positive equity in both states of the world. For simplicity, we assume that their assets are worth \( A_G \) (> \( D \)) independently of the state of the world. The state of the world realizes at time 1 and can be adverse or favorable with probability \( \alpha \) and \( 1 - \alpha \).

At time 0, banks need to rollover debt and refinance a fraction \( \rho \) of assets. We assume \( \rho = c \) for simplicity, so that banks can potentially use their cash to refinance the assets in place. Table 1 summarizes banks’ payoffs. If the bank downsizes, it foregoes the return at time 1 but keeps the cash. As in Holmstrom and Tirole (1998), refinancing might capture additional cash needs or operational expenditures of firms borrowing from the bank. If firms do not meet this cash needs, their investment will not succeed and the bank will not be repaid. As in the baseline version of Holmstrom and Tirole (1998), the bank can only choose whether to refinance assets or not. Partial refinancing is not allowed.

### Table 1
Payoff structure

<table>
<thead>
<tr>
<th>Good Banks</th>
<th>Bad Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A &amp; F ) State</td>
<td>( A ) State</td>
</tr>
<tr>
<td>Refinance (R)</td>
<td>( A_G )</td>
</tr>
<tr>
<td>Downsize (DS)</td>
<td>( c + (1 - c)A_G )</td>
</tr>
</tbody>
</table>

\(^7\)The return \( b \) might represent the return from opaque activities which is difficult to assess by outside investors. It can also be interpreted as the compensation that makes the bank manager willing to implement the efficient investment project (see, for example, Hart and Moore (1994) and Holmstrom and Tirole (1998)).
Assumption 2.

\[ \alpha A^A_B + (1 - \alpha) A^F_B < D. \]

Assumption 3.

\[ c + (1 - c) A^A_B > D. \]

Assumptions 1, 2, and 3 represent an environment where downsizing allows bad banks to regain access to market funding, but at the costs of giving up assets with a positive net present value. Investors rollover bank’s debt if the return from lending to the bank equals the return from keeping D and investing it at the risk free rate, which we assume to be zero. Assumptions 1 and 2 imply that there is a wedge between the return on banks’ assets and the return that can be pledged to investors. By Assumption 2, this wedge is such that investors would run bad banks if they do not increase their capital. For simplicity, I assume that investors recover \( \alpha A^A_B + (1 - \alpha) A^F_B \) in case they run bad banks\(^8\). By Assumption 3, downsizing allows banks to restore solvency.

Bad banks might avoid downsizing by raising equity in the market. In order to raise E, bad banks must promise investors a return on equity such that

\[ s_B(E)V_B(E) = (1 + \eta)E. \]

Bad banks must promise investors a return on equity at least as large as the outside option. We assume that \( \eta \) is greater than zero, meaning that investors require a higher compensation for investing in equity rather than debt\(^9\). The return on equity is a fraction \( s_B(E) \) of bank’s value after the equity injection E. This value, which we denote \( V_B(E) \), is given by

\[
V_B(E) = (1 - \alpha)(A^F_B + b + E - DR) + \alpha \max\{A^A_B + E - DR, 0\}
= \mathbb{E}[A_B] + (1 - \alpha)b - D + E.
\]

Bank’s value after the equity injection E is the sum of the new equity E and the value of the old shares. The

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\(^8\)Assuming the inefficient liquidation of banks’ assets is not necessary to have socially costly bank runs. In fact, bank runs cost the loss of the return \( b \) on all the assets in place, and the default cost C.

\(^9\)The higher compensation for investing in equity rather than debt depends on various reasons. For example, \( \eta \) might represent a shortcut for the premium required by risk adverse investors in order to invest in equity rather than debt. It might also be interpreted as the compensation for the higher level of expertise and sophistication required by an investment in equity.
value of the old shares equals the profit from refinancing the assets in place.

Bad banks choose how much equity to raise given investors’ optimal strategy. The optimal choice solves the following problem:

$$\max_E (1 - s_b(E)) V_a(E)$$

subject to $E \geq D - \mathbb{E}[A_b]$. 

The amount of equity that must be raised is at least such that the value of debt equals the value of assets that can be pledged to investors. As the new equity is invested in cash and has a marginal cost of $\eta$, the optimal choice is $E^* = D - \mathbb{E}[A_b]$. 

C. The Crisis Environment

We take as given two facts that characterize crisis times: Banks’ reluctance to raise capital and asymmetric information.

C.1 Banks’ Reluctance to Raise Capital

Examining the balance sheet of financial intermediaries, Adrian and Shin (2011) find that equity is constant over time, and deleveraging occurs mainly through adjustments in the size of assets. Their finding holds both in normal and crisis times. The fact that banks are reluctant to raise equity emerges also from anecdotal evidence. The following statement, by a bank board member from the eurozone’s periphery, is enlightening: “What you want to do in the current environment is shrink and lend less, not issue capital at a discount to lend more”\(^{10}\). The environment he was referring to was characterized by bank stocks trading at a price equal to 50-60% the book value of equity. Raising capital would have implied high dilution costs for existing shareholders.

Consistent with this evidence, we make the following assumptions.

Assumption 4. 

$$\eta(D - E[A_b]) > c(E[A_b] - 1)$$

Assumption 5. We rule out “money burning” signals.

\(^{10}\)From “Banking: That shrinking feeling”, FT May 3, 2012.
Assumption 4 states that the costs of raising bad banks’ optimal amount of equity is greater than the net present value of bad banks’ assets. Bad banks prefer to downsize rather than raise equity in the market. Assumption 5 means that banks can only raise the amount equity required by the investment opportunity\(^{11}\). In our model, Assumption 5 implies that bad banks can raise equity but good banks cannot. Bad banks have to raise equity in order to refinance assets and rollover their debt. By contrast, good banks do not need additional funds because they are solvent even if they reinvest cash. Good banks would be ”burning money” by raising equity, as they would make a loss \(\eta\) on each unit of capital they raise. As the empirical and anecdotal evidence shows that banks are reluctant to raise capital in the market, and in our model good banks make losses by raising equity, Assumption 5 seems to be plausible in a crisis situation.

**C.2 Asymmetric Information**

In line with the view linking the recent financial crisis to banks’ opacity, we consider a setup where investors cannot distinguish bank’s type. Investors know only the prior distribution of banks, and the actions of banks do not reveal their type. The model setup rules out all the possible separating equilibria. First, there exists no separating equilibrium where the bad bank refinances assets without raising capital. Investors would not rollover debt and the bank would make zero profits. Mimicking the good bank would always be more profitable for the bad bank. Second, there exists no separating equilibrium where the bad bank downsizes and the good bank refines assets. The bad bank could refinance its assets in place pretending to be good as investors would rollover the debt D.

**D. Bank Recapitalization and Information Disclosure**

The objective of the regulator is to maximize aggregate welfare. Aggregate welfare is the sum of banks’ and investors’ payoffs net of the cost C in case a bank defaults and \(\lambda E\) in case the regulator injects E in banks. Default costs capture the inefficiencies from disorderly bank liquidation, like fire sales or contagion, the direct costs of bank liquidation, and the loss of bank-borrower relationships. The parameter \(\lambda\) represents a shortcut for regulator’s access to funds. For example, a country experiencing a sovereign crisis will have a higher \(\lambda\) than a country with sound public finances.

The regulator chooses the accuracy of the stress test and discloses the results at time \(-t_1\). Stress tests

\(^{11}\)See Myers and Majluf (1984) and Noe (1988) for theoretical models making this assumption.
allow the regulator to ameliorate adverse selection. We model the stress test as a signaling technology detecting good banks with no error and bad banks only with probability $a$. The probability $a$ represents the accuracy of the stress test. We interpret accuracy as the choice of the quality and quantity of information to acquire and disclose. For example, a stress test might examine the banking book besides the trading book, dig into banks’ funding conditions, and exclude hybrids and sovereign support measures from the core tier 1 capital. Banks usually submit stress test disclosures to the regulator, which checks their consistency with the guidelines and with the results of other banks. If the regulator checks banks’ disclosures with more diligence, it will be able to detect banks’ misreports and improve the quality of disclosures. Accuracy determines the size and the quality of the pool of banks passing the stress test. This pool is composed of all good banks ($\beta$) and the fraction $(1 - \beta)(1 - a)$ of bad banks that are not detected. In line with our interpretation of the stress test accuracy, assuming one-sided errors captures the fact that bad banks might be more opaque or more likely to use accounting tricks. As a result, bad banks are more difficult to detect.

The regulator chooses whether to inject equity in banks failing the stress test at time $t_1$. We make the following assumption.

**Assumption 6.**

$$\min\{\lambda E_H, \lambda E_L + \alpha C\} < (1 - \alpha)b + C$$

where $E_H = D - A_B^A$ and $E_L = D - E[A_B]$. Assumption 6 states that the costs of injecting the amount of capital that prevents a run on bad banks is smaller than the social costs of a bank run. Therefore the regulator prefers the recapitalization of banks failing the stress test to a bank run. This assumption reduces the number of cases to consider, and allows us to focus on the trade-off between bank recapitalization and downsizing.

**E. Timing**

The structure and timing of the game is the following:

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12These are among the proposals that some commentators have made to improve the disappointing 2010 EU-wide stress test exercise. See “Building a better European stress test”, FT Alphaville December 07, 2010; “Doubts return on stress tests credibility”, FT 23 November, 2010.

13Commentators have found out a number of misreports by banks. For example, “UniCredit did not disclose market-by-market data relating to its operations in eastern European countries within the EU, appearing instead to lump it into its Austrian numbers”. Lloyds Banking Group suffered large losses from its Irish business and “revealed its European credit exposures only in the UK in this year’s tests, saying exposures in other countries fell below the EBA’s threshold for disclosure, at 5 per cent of total exposures”. See “Quality of stress test disclosures a mixed bag”, FT 17/07/2011.

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In \(-t_1\), the regulator chooses the accuracy of the stress test. Banks choose whether to refinance assets and raise capital at time 0. Banks play conditional on the results of the stress test. In \(t_1\) the regulator chooses whether to inject equity in bad banks unable to replenish their capital\(^{14}\). Investors decide whether to rollover D in \(t_2\). Their choice is conditional on observing banks’ choice. Finally, payoffs realize at time 1.

III. The Equilibrium in Period 1

At time 0 there are \(a(1 - \beta)\) banks that have failed and \(\beta + (1 - \beta)(1 - a)\) banks that have passed the stress test. These banks play simultaneously, but there is no interaction among them. We can split the analysis and consider two subgames. The first is between banks that have failed the stress test, the regulator and investors. The second game is between banks that have passed the stress test and investors. The regulator does not participate to this subgame. Policies in favor of banks deemed under capitalized usually accompany stress tests. Banks passing the stress test are not required to raise capital. We take this fact as given and abstract from the reasons why this happens and whether this is the optimal choice. This modeling choice allows us to focus on the issue of our interest.

Before proceeding with the analysis, it is useful to introduce the following definition.

\(^{14}\)We assume that the bank can refinance assets until \(t_1\). If assets could be refinanced only at time 0, the regulator will never find it optimal to inject equity as the investment opportunity would be lost.
Definition 1. Social value of banks.

\[ W_{DS}^G = (1 - c)A_G + c \]
\[ W_{DS}^B = (1 - c) [\mathbb{E}[A_B] + (1 - \alpha)b] + c \]
\[ W_G^R = A_G \]
\[ W_B^R = \mathbb{E}[A_B] + (1 - \alpha)b \]

Definition 2 defines the social value of good and bad banks in case they refinance assets (R) or downsize (DS). The social value of bank is the sum of the payoffs of new shareholders, old shareholders, and investors.

A. The Equilibrium Conditional on Failing the Stress Test

This game is between banks that have failed the stress test, the regulator, and investors. There is no asymmetric information among agents because the stress test perfectly identifies bad banks. Bad banks play anticipating the optimal choices by investors and the regulator. Investors force bad banks to increase their capital by threatening not to rollover the debt. By Assumption 4, bad banks prefer to downsize rather than raise capital in the market. The regulator might force banks to raise capital by threatening equity injections. While investors’ threat is always credible, regulator’s threat might not be. We will proceed by analyzing regulator’s optimal behavior, and then showing the equilibrium of the subgame.

A.1 Regulator’s Behavior

The regulator can choose whether to inject an amount of capital \( E_{Reg} \) in the banks that have failed the stress test. It requires a share \( s_{Reg} \) of their value in exchange of the equity injection. The regulator decides at time \( t_1 \) conditional on the choice of banks whether to downsize or raise capital in the market.

In case bad banks downsize, the optimal equity injection \( E_{Reg} \) solves the following problem:

\[
\max_{E} W(E) = \begin{cases} 
  a(1 - \beta)(W_B^R - \lambda E) & \text{if } E \geq D - A_B^R \equiv E_H \\
  a(1 - \beta)(W_B^R - \alpha C - \lambda E) & \text{if } E \in [D - \mathbb{E}[A_B], D - A_B^R) \\
  a(1 - \beta)(W_B^{NR} - \lambda E) & \text{if } E < D - \mathbb{E}[A_B] \equiv E_L 
\end{cases}
\]
At this stage, regulator’s choice does not affect the banks that have passed the stress test. The welfare function only includes the value of banks that have failed the stress test. There are two discontinuities in the choice of equity. The bad bank refinances assets if \( E \geq E_L \). It becomes solvent in both states of the world if \( E \geq E_H \), whereas it defaults in the adverse state if \( E \in (E_L, E_H) \). The regulator bears the default cost \( C \) if this state realizes. The bad bank downsizes if \( E < E_L \) because investors would not rollover debt otherwise.

Assumption 6 states that the regulator prefers recapitalization to a bank run. Therefore the regulator injects either \( E_L \) or \( E_H \) in the banks failing the stress test that do not raise capital in the market and refinance assets.

In case banks failing the stress test raise \( E_L \) in the market, the regulator solves the following problem:

\[
\max_E W(E) = \begin{cases} 
    a(1-\beta)(W^R_B - \lambda E) & \text{if } E \geq E_H - E_L \\
    a(1-\beta)(W^R_B - \alpha C - \lambda E) & \text{if } E < E_H - E_L
\end{cases}
\]

The regulator can prevent banks’ default by injecting at least \( E_H - E_L \) and filling banks’ capital shortfall. If the regulator injects a lower amount of equity, banks failing the stress test will invest but default with probability \( \alpha \).

The following Lemma illustrates regulator’s optimal behavior.

**Lemma 1.** If banks failing the stress test do not raise capital and refinance assets, the optimal equity injection is:

\[
E^*_{\text{Reg}} = \begin{cases} 
    E_H & \text{if } \lambda E_H \leq \lambda E_L + \alpha C \\
    E_L & \text{if } \lambda E_H > \lambda E_L + \alpha C,
\end{cases}
\]

where \( E_H \equiv D - A^A_B \), and \( E_L \equiv D - \mathbb{E}[A_B] \).

If banks failing the stress test downsize, the optimal equity injection is:

\[
E^*_{\text{Reg}} = \begin{cases} 
    E_H & \text{if } \lambda E_H \leq \min\{c\text{NPV}_B, \lambda E_L + \alpha C\} \\
    E_L & \text{if } \lambda E_L + \alpha C \leq \min\{c\text{NPV}_B, \lambda E_H\} \\
    0 & \text{if } c\text{NPV}_B < \min\{\lambda E_H, \lambda E_L + \alpha C\},
\end{cases}
\]

where \( c\text{NPV}_B \equiv \mathbb{E}[A_B] + (1-\alpha)b - 1 \).
If banks failing the stress test raise $E_L$ in the market, the optimal equity injection is:

$$E^*_{Reg} = \begin{cases} 
E_H - E_L & \text{if } \lambda E_H \leq \lambda E_L + \alpha C \\
0 & \text{if } \lambda E_H > \lambda E_L + \alpha C.
\end{cases}$$

If banks failing the stress test raise $E_H$ in the market, the optimal equity injection is 0.

Proof. The optimal solution to regulator’s problem is either $E_H$, $E_L$ or 0 because welfare is strictly decreasing in $E$.

Investors would run banks failing the stress test if they do not raise capital and refinance assets. By Assumption 6, the regulator prefers bank recapitalization to a bank run. The optimal equity injection is either $E_H$ or $E_L$. The regulator injects the amount of capital that minimizes social costs.

In case banks failing the stress test downsize, it holds that:

- $E_H$ is optimal if
  $$W^R_B - \lambda E_H \geq \max\{W^R_B - \lambda E_L - \alpha C, W^{NR}_B\}.$$ 
  This inequality holds if
  $$\lambda E_H \leq \min\{cNPV_B, \lambda E_L + \alpha C\}.$$

- $E_L$ is optimal if
  $$W^R_B - \lambda E_L - \alpha C \geq \max\{W^R_B - \lambda E_H, W^{NR}_B\}.$$ 
  This inequality holds if
  $$\lambda E_L + \alpha C \leq \min\{cNPV_B, \lambda E_H\}.$$

- 0 is optimal if
  $$W^{NR}_B \geq \max\{W^R_B - \lambda E_H, W^R_B - \lambda E_L - \alpha C\}.$$ 
  This inequality holds if
  $$cNPV_B \leq \min\{\lambda E_L + \alpha C, \lambda E_H\}.$$

The result in the Lemma follows from rearranging these conditions.
Banks failing the stress test refinance assets and default with probability $\alpha$ if they raise $E_L$ in the market. By injecting $E_H - E_L$, the regulator prevents banks’ default. This is optimal if $\lambda(E_H - E_L) < \alpha C$. 

By Assumption 6, the regulator prefers bank recapitalization to a bank run. Therefore the regulator injects equity in banks failing the stress test that do not raise capital and refinance assets. The optimal equity injection is the one minimizing social costs. In case banks downsize, the regulator faces a trade off between the cost of equity injections and the costs of downsizing. Figure 2 shows the social costs of injecting equity (solid line) as a function of the equity injection. Notice that the optimal solution is the minimum

**Figure 2**
Optimal Equity Injection
value of either interval $E \in [0, E_L)$, $E \in [E_L, E_H)$, or $E \geq E_H$. Any greater amount of capital injected in banks has a marginal cost $\lambda$, but yields no marginal gain. Subfigure (a) illustrates the case where not injecting capital is optimal. The net present value of the assets to refinance is lower than the social cost of injecting either $E_H$ or $E_L$. In subfigure (b), the net present value of assets is greater than in subfigure (a). It is optimal to inject $E_L$ because it costs less than downsizing. In subfigure (c), regulator’s cost of funds is lower than in subfigures (a) and (b). Injecting $E_H$ is optimal because foregoing the return on assets, and letting bad banks default, would be more costly.

In case banks failing the stress test raise $E_L$ in the market, the regulator can prevent default by filling banks’ capital shortfall. This is optimal if the costs of injecting the additional equity $E_H - E_L$ are lower than expected default costs. The regulator injects no equity if banks failing the stress test raise $E_H$ in the market, as these banks refinance assets and do not default.

A.2 The Response of Banks That Have Failed the Stress Test

Banks that have failed the stress test anticipate regulator’s strategy. The following Lemma illustrates their optimal choice.

**Lemma 2.** Banks that have failed the stress test raise $E_{\text{Reg}}^*$ if the regulator can credibly threaten to inject $E_{\text{Reg}}^*$ and requires a share of bank’s value $s_{\text{Reg}} \geq s_B$.

Regulator’s strategy affects the outside option of banks that have failed the stress test. If banks failing the stress test do not raise capital in the market and refinance assets, regulator’s threat is credible by Assumption 6. Recapitalization at a cost higher than the market is enough to give these banks incentives to raise capital in the market. In case banks failing the stress test downsize or raise $E_L$ in the market, the threat of recapitalization is not always credible. If $E_{\text{Reg}}^* = 0$, banks that fail the stress test choose not to raise equity in the market because this is optimal by Assumption 4. Revealing bad banks is socially costly because they downsize. If the regulator finds it optimal to inject $E_{\text{Reg}}^* > 0$, banks that have failed the stress test face a credible threat. Raising equity in the market becomes optimal because the regulator would inject equity at a higher cost otherwise. If $E^* = E_L$, revealing bad banks is socially costly because they might default. There are no costs associated to information disclosure if $E^* = E_H$, as bad banks refinance assets and are solvent in both states of the world.
B. The Equilibrium Conditional on Passing the Stress Test

Among the banks that have passed the stress test, $\beta$ are good and $(1 - \beta)(1 - a)$ are bad. Investors and banks passing the stress test play a signaling game because investors choose conditionally on observable actions. As described in Section II.C.2, there exists no equilibrium where actions reveal bank’s type. The only possible equilibria are those where banks pool on the same actions. Two pooling equilibria may exist: One where banks refinance assets (R), and another where they downsize (DS).

The pooling equilibrium where banks raise capital and refinance assets is ruled out by Assumption 5. Assumption 5 simplifies the analysis because it avoids multiple equilibria. As the pooling where banks refinance assets, the pooling where banks raise capital requires a low adverse selection premium in order for the incentive constraint of good banks to be satisfied. Ruling out multiple equilibria simplifies the analysis and allows us to get sharper insights on regulators’ incentives to disclose information. The non existence of a pooling where banks passing the stress test raise capital is consistent with banks’ reluctance to raise capital.

B.1 Banks that Pass the Stress Test Refinance Assets

Investors face default risk if the bad banks that have passed the stress test refinance assets and do not raise capital. They choose to rollover the debt of banks that have passed the stress test if

$$p_B \left[ (1 - \alpha) \min\{A^B_A, DR\} + \alpha \min\{A^A_B, DR\} \right] + (1 - p_B) \min\{A_G, DR\} = D.$$  \hfill (1)

Equation 1) states that investors must be indifferent between lending and investing in the outside option. Lenders’ outside option yields a zero net return. The return from lending equals the expected repayment from banks and depends on the fraction of bad banks passing the stress test ($p_B$). Investors rollover debt at the interest rates

$$R = \begin{cases} \frac{D - \alpha p_B A^A_B (1 - \alpha p_B)}{(1 - \alpha p_B) D} & \text{if } p_B \leq \frac{A_G - D}{\alpha (A_G - A^A_B)} = \overline{p}_1, \\ \frac{D - \alpha p_B A^A_B (1 - p_B) A_G}{(1 - \alpha p_B) D} & \text{if } p_B \leq \frac{A_G - D}{A_G - E[A_B]} = \overline{p}_2. \end{cases}$$

Investors do not rollover debt if $p > \overline{p}_2$.

A pooling equilibrium where banks passing the stress test refinance assets exists if no bank has incentives to deviate. The incentive to deviate depends on the specification of out of equilibrium beliefs. We assume
that investors attribute the choice not to refinance assets to type B and G with probability \( p_B \) and \( 1 - p_B \). The incentive constraints of good and bad banks are the following:

\[
A_g - \frac{D - \alpha p_B A^A_g}{1 - \alpha p_B} \geq (1 - c)A_g + c - D
\]  
\[\text{ (2) } \]

\[
(1 - \alpha)(A^F_B + b - DR) \geq (1 - c)(E[A_g] + (1 - \alpha)b) + c - D.
\]  
\[\text{ (3) } \]

Inequalities 2) and 3) state that the equilibrium payoff must be greater than the payoff from not refinancing assets for good and bad banks. The equilibrium payoff of the bad bank is positive if \( p_B \leq \bar{p}_2 \), whereas that of the good bank is positive only if \( p_B \leq \bar{p}_1 \left( < \bar{p}_2 \right) \). Adverse selection makes the interest rate so high that the good bank defaults if \( p_B > \bar{p}_1 \). Downsizing is the best deviation for bad banks by Assumption 4, and is the only possible deviation for good banks by Assumption 5.

The existence of a pooling equilibrium might depend exclusively on the specification of out of equilibrium beliefs. The Cho-Kreps intuitive criterion is typically used to impose additional structure on out of equilibrium beliefs. The intuitive criterion suggests that investors should believe \( \mu(B|DS) = 0 \) if the deviation DS is equilibrium dominated for the bad bank. The out of equilibrium beliefs \( \mu(G|DS) = 1 - p_B \) and \( \mu(B|DS) = p_B \) would be unreasonable in this case. The deviation DS is equilibrium dominated if

\[
(1 - \alpha)\left( A^F_B + b - DR \right) > (1 - c)(E[A_g] + (1 - \alpha)b) + c - D.
\]  
\[\text{ (4) } \]

Inequality 4) states that the equilibrium payoff is greater than the payoff from deviating to DS given investors attribute the deviation to good banks. Note that investors’ beliefs do not affect the payoff from deviating as investors rollover the debt of both banks at the risk free rate in case they downsize. In case inequality 4) holds, and the deviation DS is not equilibrium dominated for good banks, the pooling equilibrium where banks that pass the stress test refinance assets does not exist. The deviation DS is not equilibrium dominated if

\[
A_g - \frac{D - \alpha p_B A^A_g}{1 - \alpha p_B} < (1 - c)A_g + c - D.
\]  
\[\text{ (5) } \]

Inequality 5) states that the equilibrium payoff is lower than the payoff from deviating to DS for the good bank. The good bank has incentives to deviate from the equilibrium strategy if inequality 5) holds.

The following Lemma states the conditions under which a pooling equilibrium where banks refinance
Lemma 3. There exists a pooling equilibrium where banks refinance assets if \( p_B \leq \bar{p} \).

Proof. The incentive constraint of the good bank is satisfied if
\[
p_B \leq \frac{c (A_G - 1)}{\alpha (c (A_G - 1) + D - A_B)} = \bar{p}.
\]
The incentive constraint of the bad bank is always satisfied because
\[
D - \alpha A_B^A - (1 - \alpha) \frac{D - \alpha p_B A_B^A}{(1 - \alpha p_B)} \geq -c (E[A_B] + (1 - \alpha) b - 1),
\]
which is always true since the left hand side is positive and the right hand side is negative. Note that it is sufficient to check the incentive constraint with the interest rate \( \frac{D - \alpha p_B A_B^A}{(1 - \alpha p_B)} \) because the good bank would make zero profits and have no incentives to pool.

Both incentive constraints are satisfied if \( p_B \leq \bar{p} \). Note that the good bank can separate by deviating to (DS), but finds it worth if \( p_B > \bar{p} \). There is no pooling equilibrium for this range of \( p_B \). Therefore the pooling equilibrium exists if \( p_B \leq \bar{p} \).

A pooling equilibrium where banks refinance assets exists when the fraction of bad banks that pass the stress test is lower than the threshold \( \bar{p} \). Intuitively, good banks can borrow at a low adverse selection premium if \( p_B \) is small. Deviating to downsizing is not attractive for the good bank because the equilibrium profit is high enough if \( p_B \leq \bar{p} \). Bad banks have no incentives to deviate because borrowing and refinancing assets represent a subsidy for them. There exist a range of \( p_B \) for which deviating to downsizing is equilibrium dominated for the bad bank. The good bank would have incentives to deviate from the equilibrium strategy if \( p_B > \bar{p} \), but the incentive constraint is not satisfied for this values of \( p_B \). The pooling equilibrium where banks refinance assets survives the Cho-Kreps intuitive criterion.

B.2 Banks that Pass the Stress Test Downsize

All banks that have passed the stress test will be solvent in both states of the world if they downsize. Investors rollover the debt at the risk free rate because they face no default risk. A pooling equilibrium
where banks that have passed the stress test downsize exists if no bank has incentives to deviate. Assuming the same out of equilibrium beliefs as in the previous section, the incentive constraints of bad and good banks are:

\[
(1 - c)(\mathbb{E}[A_B] + (1 - \alpha)b + c - D) \geq \max\{ (1 - \alpha)(A_B^p + b - DR), \mathbb{E}[A_B] + (1 - \alpha)b - D - \eta E^\ast \} \tag{6}
\]

\[
(1 - c)A_G + c - D \geq A_G - DR. \tag{7}
\]

Inequalities 6) and 7) state that the equilibrium payoff must be greater than the payoff from the most profitable deviation. The equilibrium payoffs do not depend on adverse selection as investors rollover debt at the risk free rate. Bad banks can make two possible deviations, that is refinancing assets with (C,R) and without (R) raising capital. Good banks can deviate to R. The payoff from deviating to (R) equals the equilibrium payoff in equations 2) and 3). The amount of capital bad banks would deviate to is

\[
E^\ast = \arg \max_E (1 - s_B(E))V_G(E).
\]

The out of equilibrium belief \( \mu(B|R) = p_B \) is reasonable as

\[
(1 - c)(\mathbb{E}[A_B] + (1 - \alpha)b + c - D) < (1 - \alpha)(A_B^p + b - D).
\]

The equilibrium payoff of the bad bank is lower than the payoff from deviating to R given investors attribute the deviation to the good bank. The pooling equilibrium where banks do not refinance assets exists as long as the incentive constraints of good and bad banks hold. The following Lemma illustrates the conditions under which this is the case.

**Lemma 4.** There exists a pooling equilibrium where banks downsize if \( p_B \geq p_2 \).

**Proof.** The optimal amount of capital a bad bank would raise is \( E = D - E[A_B] \). Assumptions 4 implies that the payoff from not refinancing assets is greater than raising \( E = D - E[A_B] \) for the bad bank, so that the incentive constraint always holds. When the most profitable deviation is (R), i.e. when \( p_B < p_2 \), the incentive constraint is not satisfied because

\[
(1 - c)(E[A_B] + (1 - \alpha)b) + c - D \geq (1 - \alpha)(A_B^p + b - D - \alpha p_B A_B^p - (1 - p_B)A_G) \]

\[
\rightarrow -c(E[A_B] + (1 - \alpha)b - 1) \geq (1 - \alpha)(1 - p_B)(A_G - D)
\]
The good bank has incentives not to deviate to (R) if \( p_B > \overline{p}^F \). Both incentive constraints are satisfied if \( p_B \geq \overline{p}_2 \) because \( \overline{p}^F < \overline{p}_2 \).

The pooling where banks downsize is the most inefficient and least profitable for banks. It exists because the fraction of bad banks that pass the stress test is so large (\( p_B \geq \overline{p}_2 \)) that investors do not rollover the debt of banks passing the stress test. Raising capital and refinancing assets is not an attractive deviation for bad banks because investors require too high a compensation for capital. Good banks cannot separate by refinancing assets because the bad bank would always mimic this deviation.

\[ B.3 \quad \text{Graphical Illustration} \]

Before proceeding with the illustration, it is useful to introduce the following definition.

**Definition 2. Thresholds in terms of accuracy**

\[ p_B \leq \overline{p}^F \iff a \geq \frac{\alpha(D - A^A_B)(1 - \beta) - (1 - (1 - \beta)\alpha)c(A_G - 1)}{(1 - \beta)(\alpha(D - A^A_B) - (1 - \alpha)c(A_G - 1))} = \underline{a}_H \]

\[ p_B \geq \overline{p}_2 \iff a \leq \frac{D - \beta A_H - (1 - \beta)E[A_G]}{(1 - \beta)(D - E[A_G])} = \underline{a}_L \]

Recall that \( p_B \) is the probability of a bank being of bad type given it has passed the stress test. As \( p_B = \frac{(1 - \beta)(1 - a)}{\beta + (1 - \beta)(1 - a)} \), it is possible to rewrite the thresholds defining the pooling equilibria in terms of accuracy. Note that \( \underline{a}_H > \underline{a}_L \) because \( \overline{p}_2 > \overline{p}^F \) and accuracy is inversely related to \( p_B \). It holds that \( \underline{a}_H > \underline{a}_L \).

Figure 3 illustrates the social costs due to banks that have passed the stress test as a function of accuracy.

The slope of the function (solid line) is negative, because the greater accuracy, the fewer the banks that pass the stress test. Only good banks pass the stress test if accuracy takes the maximum value. The function is piecewise linear. If accuracy is greater than \( \underline{a}_H \), banks that have passed the stress test refinance assets. The social costs are due to the fact that the bad banks default with probability \( \alpha \). As good banks are solvent in both states of the world, social costs are nil if \( a = 1 \). If accuracy is smaller than \( \underline{a}_L \), banks that have passed the stress test downsize because investors require too high an adverse selection premium. There is an equilibrium in mixed strategies for intermediate levels of accuracy. Social costs are an average of those in
the other accuracy intervals. The function is discontinuous because banks’ behavior, and the ensuing social costs, vary over the three regimes.

The convexity of the function depends on how default costs compare to downsizing costs. Since banks downsize when accuracy is too low, the marginal cost of decreasing accuracy is greater when downsizing is less efficient than default. This implies that the function is convex. By contrast, the function is concave if default is less efficient than downsizing.

IV. The Equilibrium in the First Stage

The regulator plays in the first stage of the game taking into account the optimal equity injection choice and the optimal strategies of banks that pass and fail the stress test. The regulator chooses the accuracy of the stress test with the objective to maximize aggregate welfare. Aggregate welfare is piecewise linear in accuracy and depends on the amount of capital the regulator can inject. Conditional on \(E_{Reg}^* \in \{0, E_L, E_H \}\),
the welfare function is

\[ W(0) = \begin{cases} 
\beta W_D^G + (1 - \beta)W_D^B & \text{if } a \leq a_L \\
\beta W_D^G + (1 - \beta)(1 - a)(W_R^B - \alpha C) & \text{if } a \geq a_H 
\end{cases} \]

\[ W(E_L) = \begin{cases} 
(a - \beta)(W_R^B - \alpha C) + \beta W_D^G + (1 - \beta)(1 - a)W_D^B & \text{if } a \leq a_L \\
(a - \beta)(W_R^B - \alpha C) + \beta W_R^G + (1 - \beta)(1 - a)(W_R^B - \alpha C) & \text{if } a \geq a_H 
\end{cases} \]

\[ W(E_H) = \begin{cases} 
a(1 - \beta)(W_R^B - \alpha C) + \beta W_D^G + (1 - \beta)(1 - a)W_D^B & \text{if } a \leq a_L \\
(a - \beta)(W_R^B - \alpha C) + \beta W_R^G + (1 - \beta)(1 - a)(W_R^B - \alpha C) & \text{if } a \geq a_H 
\end{cases} \]

Banks that pass the stress test play a mixed strategy in case \( a \in (a_L, a_H) \). Aggregate welfare is an average of the welfare from the strategies in the support of the mixed strategy. Independently of the optimal equity injection in banks that fail the stress test, banks that pass the stress test refinance assets if \( a \geq a_H \) but do not if \( a \leq a_L \). The optimal choice of banks that fail the stress test depends on the optimal equity injection. If the regulator can credibly threaten to inject equity \( E_{Reg}^* > 0 \), banks that fail the stress test raise \( E_{Reg}^* \). These banks default only if \( E_{Reg}^* = E_L \). Banks that fail the stress test downsize if the regulator cannot credibly threaten an equity injection.

The trade off arising from this setup is that a higher effort allows banks that pass the stress test to refinance assets, but implies failing more bad banks. Failing bad banks is socially costly in terms of equity injections or foregone investment opportunities. The following Proposition illustrates the optimal choice by the regulator.

**Proposition 1.** The optimal accuracy choice is

\[ a^* = \begin{cases} 
1 & \text{if } E_{Reg}^* = E_H \\
1 & \text{if } E_{Reg}^* = 0 \text{ and } \alpha C > cNPV_B \\
 a_H & \text{if } E_{Reg}^* = 0 \text{ and } \alpha C < cNPV_B 
\end{cases} \]

whereas \( a^* \in [a_H, 1] \) if \( E_{Reg}^* = E_L \).
Proof. If \( E_{\text{Reg}}^* = E_H \), maximization of \( W_{ST}(E_H) \) with respect to \( a \) yields the following first order condition:

\[
(1 - \beta)\alpha C - \delta_1 + \delta_2 = 0 \quad \text{if} \quad a \geq a_H
\]

\[
(1 - \beta)cNPV_B - \delta_3 + \delta_4 = 0 \quad \text{if} \quad a \leq a_L,
\]

where \( \delta_1, \delta_2, \delta_3, \) and \( \delta_4 \) are the multipliers of the constraints \( a \leq 1, \ a \geq a_H, \ a \leq a_L \) and \( a \geq 0 \). The first order condition if \( a \in (a_L, a_H) \) is the average of the previous two because banks that pass the stress test play a mixed strategy. The solution of the two first order conditions implies that accuracy is the maximum in each interval. The optimal choice is \( a^* = 1 \) because

\[-\beta c(A_G - 1) - (1 - \beta)(1 - a_L)cNPV_u < 0.\]

If \( E_{\text{Reg}}^* = E_L \), maximization of \( W_{ST}(E_L) \) with respect to \( a \) yields the following first order condition:

\[-\theta_1 + \theta_2 = 0 \quad \text{if} \quad a \geq a_H
\]

\[
(1 - \beta)(cNPV_B - \alpha C) - \theta_3 + \theta_4 = 0 \quad \text{if} \quad a \leq a_L,
\]

where \( \theta_1, \theta_2, \theta_3, \) and \( \theta_4 \) are the multipliers of the constraints \( a \leq 1, \ a \geq a_H, \ a \leq a_L \) and \( a \geq 0 \). The first order condition if \( a \in (a_L, a_H) \) is the average of the previous two because banks that pass the stress test play a mixed strategy. The first condition implies that any accuracy choice \( a \in [a_H, 1] \) is optimal. The second condition implies that \( a^* = a_L \) because it is optimal to inject \( E_L \) when \( cNPV_B > \alpha C + \lambda E_L > \alpha C \). The optimal choice is any accuracy \( a \in [a_H, 1] \) because

\[-\beta c(A_G - 1) - (1 - \beta)(1 - a_L)cNPV_B - \alpha C < 0.\]

If \( E_{\text{Reg}}^* = 0 \), maximization of \( W_{ST}(0) \) with respect to \( a \) yields the following first order condition:

\[
(1 - \beta)(\alpha C - cNPV_B) - \gamma_1 + \gamma_2 = 0 \quad \text{if} \quad a \geq a_H
\]

\[- \gamma_3 + \gamma_4 = 0 \quad \text{if} \quad a \leq a_L,
\]

where \( \gamma_1, \gamma_2, \gamma_3, \) and \( \gamma_4 \) are the multipliers of the constraints \( a \leq 1, \ a \geq a_H, \ a \leq a_L \) and \( a \geq 0 \). The first
order condition if \( a \in (\underline{a}_L, \overline{a}_H) \) is the average of the previous two because banks that pass the stress test play a mixed strategy. The first condition implies that \( a^* = 1 \) if \( \alpha C > cNPV_b \) and \( a^* = \overline{a}_H \) otherwise. The second condition implies that any accuracy \( a \in [0, \underline{a}_L] \) is optimal. If \( \alpha C > cNPV_b \), the optimal choice is \( a^* = 1 \) because

\[-\beta c(A_G - 1) < 0.\]

If \( \alpha C < cNPV_b \), the optimal choice is \( a^* = \overline{a}_H \) because

\[-\beta c(A_G - 1) + (1 - \beta)(1 - \overline{a}_H)(\alpha C - cNPV_b) < 0.\]

\( \square \)

The choice of accuracy depends on the amount of equity the regulator finds it optimal to inject. If the regulator can credibly threaten to inject \( E_H \), it will choose the maximum accuracy. The reason is that banks failing the stress test will raise \( E_H \) and become solvent in both states of the world. The regulator can implement the first best allocation choosing the maximum accuracy.

In case injecting no equity is optimal, the regulator will choose \( a^* = 1 \) or \( a^* = \overline{a}_H \) depending on whether \( \alpha C \) is greater or smaller than \( cNPV_b \). Banks that fail the stress test downsize, whereas bad banks that pass the stress test refinance assets and default with probability \( \alpha \). Figures 4 illustrates the case where \( cNPV_b > \alpha C \). The higher accuracy, the greater the social costs from downzizing \( (SC^F(0) \text{ function in subfigure } (a)) \), but the lower the social costs due to banks passing the stress test \( (SC^P(0) \text{ function}) \). The level of accuracy minimizing total social costs \( \text{(subfigure (b))} \) is \( \overline{a}_H \). For lower accuracy levels, even the good banks would downsize, at least with some probability. For greater accuracy levels, banks that have passed the stress test refinance assets, whereas those that have failed the stress test downsize and become solvent. Since default costs are lower than downsizing costs, it is optimal to choose \( \overline{a}_H \), as this level of accuracy minimizes the social costs from downsizing. Figures 5 illustrates the opposite case where \( cNPV_b < \alpha C \). Since default is less efficient than downzizing, downsizing is optimal because banks become solvent. The optimal choice is \( a = 1 \), as this level of accuracy minimizes default costs.

In case the optimal equity injection is \( E_L \), the regulator finds any accuracy \( a^* \in [\underline{a}_H, 1] \) optimal. Figure 6 illustrates this case. Banks that fail the stress test raise \( E_L \) and refinance assets in equilibrium. Investors
rollover debt but require a risk premium because banks will default with probability $\alpha$. The marginal cost of accuracy equals the expected default cost of bad banks. The marginal gain equals the reduction in the social costs due to banks passing the stress test. These social costs are a convex function of accuracy (Figure 3.b) because, by Lemma 1, the net present value of assets is greater than the expected bankruptcy costs in
case injecting $E_L$ is optimal. The regulator will choose at least $a = a_H$ in order to make sure that the good banks passing the stress test choose to refinance assets. Any effort $a \in [a_H, 1]$ will be optimal because bad banks refinance assets and default independently of whether they pass or fail the stress test.

V. Empirical Predictions

We highlight four empirical predictions:

1. banks failing the stress test raise capital in the market only if the regulator can credibly threaten recapitalization;

2. a stress test will be fully informative if the regulator can credibly threaten to fill the equity gap of bad banks;

3. a stress test will be partially informative if the regulator cannot inject equity in banks that fail the stress test and expected default costs are smaller than the net present value of assets;

4. a stress test will be fully informative if the regulator cannot inject equity in banks that fail the stress test and expected default costs are greater than the net present value of assets.
No clear prediction can be made in case the regulator finds it optimal to inject $E_L$ as $a^* \in [a_H, 1]$. However, under the reasonable assumptions that effort is costly in terms of time and resources, the prediction would be that the stress test will be partially informative.

The empirical and anecdotal evidence on banks failing the stress tests in the U.S. and Europe is consistent with these predictions. The U.S. could raise funds at a relatively low cost and implemented the CAP program. The CAP program, which accompanied the stress test in the U.S., would provide equity to the banks unable to replenish their capital privately. The U.S. stress test identified an aggregate capital shortfall of $\$75$ bn among 10 of the 19 participating banks. Consistent with prediction 1), the 10 banks deemed undercapitalized raised $\$77$ bn of equity in the 6 months following the stress test. None of them needed to draw on CAP funds. Prediction 2) suggests that the U.S. should have implemented a fully informative stress test. In line with this prediction, Peristiani et al. (2010) find that markets used the information from U.S. stress tests to revalue banks. Greenlaw et al. (2012) point out that U.S. banks have seen a remarkable decline in CDS prices and a consistent surge in equity prices in the three months following the stress test.

The costs of state recapitalization were much higher in Europe. Sovereign debt problems limited the firepower of peripheral EU countries. Core EU countries would have presumably faced political costs from using taxpayers’ money to save banks. For example, Germany wound the national bail out fund down in December 2010, and since then state help was no longer available for German banks. State recapitalization was an idle threat in Europe. Consistent with prediction 1), none of the few banks that failed the EU-wide stress test exercises raised capital in the market. Among these banks, the Spanish ones either merged with or were acquired by other banks, whereas the Greek ones, the German Hypo Real Estate, and the Austrian OeVAG underwent a restructuring process.

The 2011 EU-wide stress test also revealed 16 banks with a core tier 1 capital slightly above the 5% passing threshold. The EBA conducted the EBA capital exercise with the aim to encourage these banks to raise capital. The EBA identified an aggregate capital shortfall of €115 bn and required banks to fill the gap by June 2012. In October 2012, the EBA reviewed the fulfillment of the recapitalization plans of 27 banks, which had a total capital shortfall of €76 bn. These banks have raised €115.7 bn. From the results that EBA published, it emerges that only €46 bn consist of core tier 1 capital. The rest includes ongoing backstops and mainly measures affecting risk weighted assets. The €46 bn include the issuance of new ordinary shares, the scheduled conversion of hybrid bonds, but also measures, like retained earnings,
that some commentators believe not to be fully credible. Commentators’ concerns are justified also by the fact that “People familiar with its thinking insist that even if the EBA executive believes some banks capital plans to be aggressive and unachievable, it will seek to resolve the issues quietly, behind the scenes, and may ultimately have to back down in some cases if national regulators are determined that their banks are healthy. Even if the EBA is sceptical of an individual banks plan, it does not have direct authority to order a change”.

The concerns about banks’ recapitalization plans are consistent with a more nuanced prediction than ours: The regulator will accept not fully credible recapitalization plans if it cannot provide any credible guarantee of bank recapitalization. The intuition behind this alternative prediction is the same as ours. It does not arise from our model because we only let banks choose whether to raise capital or not.

According to predictions 2) and 3), the information value of the European stress tests should depend on the relative size of the default and downsizing costs. EU members differed in this respect. Peripheral EU countries were in the midst of an economic and sovereign crisis, whereas core EU countries showed a better economic performance. Expected default costs were presumably greater than the costs of foregoing investment opportunities in peripheral EU countries. Core EU countries were likely to be in the opposite situation. Consistent with prediction 4), Spain applied an extra level of stress to its banks and encouraged the disclosure of sovereign risk exposure by banks. The fraction of Spanish banks subject to stress test was the largest in Europe, and most of the banks that failed the stress tests were from Spain. In line with prediction 3), “some bankers, analysts and officials are pointing the finger at German regulators and lenders, claiming they have led efforts to push for weaker testing standards and less transparency in the results.”

As the European supervisory authority had presumably to take into account the heterogeneity among EU countries, European stress tests were less informative than in the U.S. The reaction of EU banks’ CDS and stock prices was weak after the stress tests (Greenlaw et al. (2012)). Skepticism around European stress tests was widespread among commentators and practitioners (Hirtle et al. (2009), Schuermann (2012)). The capital shortfall of European banks amounted to €3.5 bn in 2010 and €2.5 in 2011, way below market expectations. Only 8 banks in 2011, and 7 banks in 2010, failed the EU-wide stress tests. Allied Irish Banks did not need any additional capital according to the 2010 EU-wide stress test, but received a €3 bn

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15 Some commentators showed concern for the too optimistic earning expectations. See ”EBA set to opt for pragmatism over publicity”, FT February 6, 2012.
16See ”Stress test results "underwhelming””, FT July 26, 2010.
17From ”EU Defends Stress Tests as Standards Draw Doubts”, WSJ March 10, 2011.
18See ”Banks: Again under strain”, FT July 7, 2011.
bailout after a few months. Three months after passing the 2011 EU-wide stress test, Dexia underwent a restructuring process involving state guarantees and the creation of a "bad bank".

VI. Discussion and Policy Implications

We have deliberately used a simple model to sharpen the analysis of regulator’s incentives to disclose information in a crisis. We have taken as a starting point an environment similar to the recent financial crisis. Investors face uncertainty about banks’ risk exposures and the market for funds gets tighter. Banks are reluctant to raise capital in the market because its cost is too high. Regulators react by disclosing stress test results. Since we focus on a crisis situation, we have taken capital shortfalls as given and not modeled bank’s risk taking behavior. We have considered a setup where the bad banks have a capital shortfall, but can continue operating without renegotiating pre-existing contracts. The capital shortfall is such that bad banks cannot borrow against all their assets in place, but become solvent by downsizing. Downsizing is an inefficient way to increase equity because there is a wedge between bank’s value and the value that can be pledged to investors. We do not endogenize this wedge, which we interpret as the returns on opaque assets or as the compensation for managers not to shirk. Assuming a run by investors and the inefficient liquidation of bank’s assets would deliver similar results. However, banks would not be able to raise capital if the value of assets is lower than the value of debt. Our modeling choice gives banks the possibility to replenish the capital shortfall either through raising capital in the market or downsizing.

Our model relates the effectiveness of bank stress tests in times of crisis to the costs of default, recapitalization and downsizing. If the regulator has easy access to funds and hence can recapitalize banks, the disclosure of stress test results will lead to the first best allocation. The reason is that banks fill their capital shortfall by raising capital in the market and invest. By contrast, only second best equilibria arise if the regulator cannot recapitalize banks. There are inefficiencies either in terms of downsizing or default. These results imply a link between bank stress tests and a number of well-debated issues in banking regulation.

First, directly recapitalizing weak banks through the European Stability Mechanism would make stress tests more informative in Europe. The costs of recapitalization would be beared by the ESM and not added to countries’ sovereign debt. Countries, especially in the EU’s periphery, would face lower funding costs. In the context of our model, recapitalization through the ESM would imply a reduction in $\lambda$. The reduction
in $\lambda$ could shift the optimal equity injection from zero to $E_H$, as in Figures (2.a) and (2.c), and give the regulator incentives to resolve information asymmetries.

Second, improving bankruptcy procedures, for example by requiring banks to write their "living wills"\textsuperscript{19}, would have a twofold effect. On the one hand, it would make regulators more reluctant to reveal capital shortfalls in countries like the EU’s periphery, where raising money to recapitalize banks is costly. On the other hand, it would reduce the social costs of injecting $E_L$ in banks. The first effect can be illustrated by Figures 4 and 5. In Figure 5 the regulator chooses the maximum accuracy because injecting equity in banks is not optimal and expected default costs are greater than downsizing costs. A reduction in default costs could shift the optimal accuracy choice to $a_H$, as in Figure 4. The intuition is that the regulator will minimize downsizing if expected default costs are lower than downsizing costs. The second effect can be illustrated as a downward shift of the $\lambda E + \alpha C$ schedule in Figure 2. A consistent reduction in default costs might make injecting $E_L$ the optimal choice for the regulator. This would increase the equilibrium accuracy choice to any value $a \geq a_H$.

Third, regulators should enforce capital requirements in absolute terms. Bank supervisors typically require banks to maintain a certain capital to assets ratio. Supporters of a macroprudential view of bank supervision argue that such requirement gives banks incentives to replenish their capital by shrinking assets rather than raising equity in the market. In our model, banks would prefer to downsize independently of regulator’s funding costs if capital requirements are expressed in ratio terms.

\textbf{VII. Conclusions}

This paper has examined the incentives of regulators to reveal information in crisis times. We have provided a positive analysis of information disclosure. We have taken as given the fact that regulators have conducted stress tests as a response to the recent financial crisis. Our analysis builds on the evidence that, in crisis times, banks are reluctant to raise capital in the market and information asymmetries get more severe.

Regulator’s incentives to disclose information are crucial for market discipline. In our model, information disclosure ameliorates adverse selection, and prevents good banks from inefficiently downsizing. In a model where banks choose risk, information disclosure would also prevent excessive risk taking by making banks pay

\textsuperscript{19}“Living wills” are guidelines for unwinding banks in case of default. The 2010 Dodd Frank Act required more than 100 large financial firms to submit "living wills" to the Federal Reserve and the Federal Deposit Insurance Corporation. In November 2011, the leaders of the G-20 nations agreed to require the 29 largest banks worldwide to submit "living wills".
for the risk they take. Less risk taking would imply lower capital shortfalls, and would reinforce regulator’s incentives to disclose information.

We have shown that, in our crisis environment, regulators will prefer less information to be disclosed if banks react by inefficiently downsizing. Unless the regulator can credibly threaten recapitalization at a dilution cost higher than the market, a bank will prefer to downsize rather than raise capital in order to fill the capital shortfall revealed by the stress test. Downsizing makes the bank solvent at the cost of a lower level of investment. If the foregone returns are greater than expected default costs, the regulator will minimize information disclosure. The regulator will reduce adverse selection to the point where the remaining banks prefer to keep investing rather than downsizing. By contrast, the regulator will have incentives to reveal information if its funding costs are low. Low funding costs make the recapitalization threat credible. Banks will prefer to raise capital and invest. The regulator will have incentives to reveal information also in case expected default costs are greater than the costs of downsizing. Downsizing is optimal given the regulator cannot credibly threaten recapitalization.

Our model links the favorable market reaction to the U.S. stress test to the implementation of a backstop mechanism (CAP) for weak banks. The lack of a EU-wide backstop mechanism, and the costs of downsizing for core EU members, are consistent with the skeptical market reaction to European stress tests.

Our model implies that the efficient resolution of banks’ distress is crucial for the effectiveness of bank stress tests in times of crisis. This links our analysis to a number of well-debated issues in banking regulation. First, directly recapitalizing weak banks through the European Stability Mechanism would make stress tests more informative in Europe. The reason is that the costs of recapitalization would be beared by the ESM and not be added to the country’s sovereign debt. Second, regulators should enforce capital requirements in absolute terms. If capital requirements are expressed in ratio terms, banks would prefer to downsize independently of regulator’s funding costs.

In our model, downsizing allows banks to replenish their capital shortfall. This might not always be the case in reality. If banks’ financial distress is more severe, restructuring banks’ assets and/or liabilities might be necessary. Our model suggests that splitting ailing banks into a "good" and a "bad" bank, or swapping debt for equity, would give regulators stronger incentives to reveal capital shortfalls. The "good-bank/bad-bank" solution consists of taking bad assets and senior debt off the balance sheet of ailing banks,
and transferring them to a "bad" bank. The "bad" bank would own the "good" bank\textsuperscript{20}. Debt-for-equity swaps represent a form of debt renegotiation through which bank creditors accept to become equity holders. If bank's equity after restructuring is sufficiently large, these policies will guarantee solvency and the supply of credit at no cost for taxpayers. As creditors have incentives to free ride, renegotiating debt might be difficult. A better solution would be to require banks to hold contingent convertible bonds that would convert into equity once a contract defined trigger event occurs\textsuperscript{21}.

\textsuperscript{20}The "good-bank/bad-bank" solution has been adopted in Ireland in 2009 and in Sweden in 1991.

\textsuperscript{21}Switzerland will require its two largest banks a 19\% capital requirement, of which 9\% may be held in the form of contingent convertible debt. Contingent capital proposals are also currently under discussion within the Basel Committee, the Financial Stability Board (FSB), and the European Union.
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