Macro factors and sovereign bond spreads: a quadratic no-arbitrage model

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Discussion:

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Euro-area no-arbitrage term-structure models

- Many papers studying euro-area spreads using regression techniques.
- Far less papers exploiting the cross-sections of yields (maturities + countries) in a no-arbitrage framework; though this allows to:
  - compute risk premia and derive physical PDs,
  - improve the estimation of the relationship between factors and yields.
Euro-area no-arbitrage term-structure models

Non-linearities in spreads

Geyer, Kossmeier and Pichler, 2004
Monfort and Renne, 2012

Hördahl and Tristani, 2012

Macro-factors in spreads

Borgy et al., 2011
The paper in a nutshell

- Estimation of the joint dynamics of 5 term structures of spreads vs Germany (France, Greece, Italy, Portugal and Spain)

- Main results:
  - The model-implied proba. of default (PDs) are related to
    - macroeconomic factors: growth rates and debt/GDP ratios.
    - a common latent factor.
  - Probabilities of default depend non-linearly on Debt/GDP.
  - Risk premia account for the main part of spreads
    - Risk premia are predominantly driven by the common factor.

- Technical achievements:
  - The model captures salient features of spread volatilities.
  - Good forecasting performances.
  - Interesting time-varying impulse responses.
Model

- The factors: \( X^i_t = [C^i_t, Y^i_t] \)
  - Unobserved common factor: \( C^i_t \)
  - Observed country-specific factors \( Y^i_t = [g^i_t, d^i_t] \).

- Dynamics of the factors:

\[
X_t = \begin{bmatrix}
\phi_{C,C} & 0 & 0 \\
0 & \phi_{g,g} & 0 \\
\phi_{d,C} & \phi_{d,g} & \phi_{d,d}
\end{bmatrix} X_{t-1} + \begin{bmatrix}
\varepsilon_{C,t} \\
\varepsilon_{g,t} \\
\varepsilon_{d,t}
\end{bmatrix}
\]
Default process ($D_{i,t} = 1$: default; $D_{i,t} = 0$: no default):

$$P(D_{i,t} = 1 | D_{i,t} = 0, X_t) = 1 - \exp(-\Lambda_{i,t})$$

or

$$P(D_{i,t} = 1 | D_{i,t} = 0, X_t) \approx \Lambda_{i,t}$$

conditional default proba.
Model

- Default process \((D_{i,t} = 1: \text{default}; D_{i,t} = 0: \text{no default})\):

\[
P(D_{i,t} = 1 \mid D_{i,t} = 0, X_t) = 1 - \exp(-\Lambda_{i,t})
\]

or

\[
P(D_{i,t} = 1 \mid D_{i,t} = 0, X_t) \approx \Lambda_{i,t}
\]

conditional default proba.

- Default intensity:

\[
\Lambda_{i,t} = \lambda_0^{i} + \lambda_C^{i} C_t + \lambda_g^{i} g_t^{i} + \lambda_d^{i} d_t^{i} + \ldots
\]

\[
...\lambda_{dd}^{i} \times (d_t^{i})^2
\]
Model

- A stochastic discount factor is specified, exponential affine in $X_t$, such that under $\mathbb{Q}$:

$$X_t = \begin{bmatrix} \phi_{c,c}^* & 0 & 0 \\ 0 & \phi_{g,g}^* & 0 \\ \phi_{d,c}^* & \phi_{d,g}^* & \phi_{d,d}^* \end{bmatrix} X_{t-1} + \begin{bmatrix} \varepsilon_{c,t}^* \\ \varepsilon_{g,t}^* \\ \varepsilon_{d,t}^* \end{bmatrix}$$

- In that context:

$$s_{t+n}^t = -\frac{1}{n} \log E^{\mathbb{Q}} \left( \exp(-\Lambda_{t+1} - \ldots - \Lambda_{t+n}) \right) = a_n + b_n X_t + X_t c_n X_t$$

(Leippold and Wu, 2002, Ang et al., 2008, Gourieroux and Sufana, 2011)

- Estimation technique:
  - Part of $X_t$ is latent (factor $C_t$).
  - Non-linear filtering required (a) to compute the likelihood function and (b) estimate $C_t$. 
Why not including the common factor $C_t$ in the quadratic intensities?

If $C_t$ is the manifestation of self-fulfilling market dynamics, it would make sense to have the intensity depending on it in a non-linear way.

Other potential explanations for $C_t$:

- changes in the prospects for global economic activity,
- changes in the expected recovery rates,
- political uncertainty,
- liquidity-pricing effects.
Liquidity effects in spreads
Comments/Questions

Liquidity effects in spreads
In the model:

\[
\begin{aligned}
(2 \times N) + 1 & \quad \text{sources of risk} \\
(3 \times N) & \quad \text{prices of risk}
\end{aligned}
\]

- The prices of risk associated with \( C_t \) differ across countries.
- The risk associated with \( C_t \) is not priced in the same way by the creditors of the different countries.
- Consistent with the view of completely segmented euro-area sovereign-bond markets.
Comments/Questions

Complexities associated with the quadratic framework

- **Quadratic framework:**
  - Makes it possible to model non-linearities
  - Could be used to impose positiveness of yields/spreads (e.g. Kim and Singleton, 2011)

- **Decomposing yields/spreads is not straightforward:**

\[ s_{t+n} = a_n + \begin{bmatrix} b_c & b_g & b_d \end{bmatrix} X_t + X_t \begin{bmatrix} c_{c,c} & 0 & c_{c,d} \\ 0 & c_{g,g} & c_{g,d} \\ c_{c,d} & c_{g,d} & c_{d,d} \end{bmatrix} X_t \]

- Interaction terms can be sizable, how to split these onto the different components?
- why not using the shocks (implicitly identified here) to compute the different components?

- Potential caveat of the quadratic framework: the relationship between the intensity and the factors is not monotonous.
Non-monotonous dependencies

\[ \frac{\Delta \Lambda}{\Delta d} < 0 \quad \text{and} \quad \frac{\Delta \Lambda}{\Delta d} > 0 \]
Non-monotonous dependencies

Intensity $\Lambda$

$\Delta\Lambda/\Delta d < 0$  $\Delta\Lambda/\Delta d > 0$

95% confidence interval of Debt/GDP ratio
Comments/Questions
Non-monotonous dependencies

\[ \frac{\Delta \Lambda}{\Delta d} < 0 \quad \frac{\Delta \Lambda}{\Delta d} > 0 \]

95% confidence interval of Debt/GDP ratio
Risk premia

- Risk premia (in terms of yields) are...
  - ... negative if bond returns tend to be positive in “bad states of the world” (insurance)
  - ... positive if bond returns tend to be negative in “bad states of the world” (risk exposure)

- Negative risk premia over the first part of the sample, ...
  ... meaning that before 2008, agents would reckon that decreases in sovereign PDs (positive bond returns) correspond to bad states of the world.

- Alternative (econometric) explanation: the unconditional average of the default intensity is too large.
  - Small sample: the unconditional average is imprecisely estimated.
  - Could be constrained to imply lower PDs before 2008 (in the early 2000’s, 10-year PD for Spain: 5%, for Greece: 18%).
Comments/Questions
Forecasting ability of the model

\[ \text{Risk Premium} = (\text{Observ. spread}) - (\text{spread if investors were risk-neutral}) \]
\[ = (\text{Q-expected } \Lambda_t \text{'s}) - (\text{P-expected } \Lambda_t \text{'s}) \]

- Assuming investor beliefs ~ survey-based forecasts (CF), model-implied forecasts and Consensus Forecasts should coincide.
- If not, it notably implies that the model-implied risk-neutral PDs would not coincide with the ones of the professional forecasters.
- Two potential objectives that do not necessarily coincide:
  - to have a model that is good at forecasting,
  - to have a model that is able to reproduce agents’ expectations (and hence to compute risk premia).
Including Greece in the dataset is bold...

... but challenging for this kind of model over the whole estimation period.

The hypothesis of an (expected) constant recovery rate is debatable in the Greek case.
The fiscal variables feature missing values.

Pre-filtering using the Kalman filter and a simple autoregressive law of motion for the fiscal variable.

Why not dealing with that simultaneously with the main estimation? Filtering techniques can easily handle missing values.
To sum up

- The paper nicely illustrates the potential of the quadratic framework to handle non-linearities in sovereign-bond pricing.
- In spite of a deep technical content, the paper remains clear.
- Different modeling choices and interpretations should be investigated/discussed further.
- A very nice piece of work.