Risk-Adjusted Capital Allocation and Misallocation

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Summer 2018
Introduction

In an ideal world, all capital should be deployed to its most productive use!

- But, $\sigma^2_{MPK} \gg 0 \rightarrow$ capital misallocation, large losses in TFP, output
- Why doesn’t capital flow to the most productive firms?
- Sources of dispersion still unclear, e.g., sizable persistent component

Our analysis: with aggregate risk

$$1 = \mathbb{E}_t [M_{t+1} (MPK_{it+1} + 1 - \delta)] \Rightarrow \mathbb{E}_t [MPK_{it+1}] = \alpha_t + \beta_i \lambda_t$$

- Expected discounted $MPK$’s should be equalized
- $MPK_{it}$ depends on exposure to agg risk factors $\rightarrow$ CS return predictability: persistent deviations
- $\sigma^2_{MPK}$ depends on dispersion in risk exposures ($\sigma^2_{\beta}$) and price of risk ($\lambda_t$) $\rightarrow$ TS return predictability: countercyclical variation

$\Rightarrow$ Novel link between nature of agg shocks, asset pricing and resource allocations
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  $\rightarrow$ TS return predictability: countercyclical variation

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This paper

Neoclassical growth model with idiosyncratic and aggregate risk

- Firms with greater exposure to agg risk offer higher MPK
- MPK dispersion increases when price of risk is high (i.e., is countercyclical)

→ Verify predictions for US firms

Quantitative analysis: how much MPK dispersion from risk premium effects?

- More structure: connect agg risk to technology shocks
- Empirical strategy: use link with stock market returns

→ Key result: $\sigma^2_{E[mpk]} \propto \sigma^2_{E[r]}$

Findings: important role for risk considerations

- Accounts for 40% of $\sigma^2_{MPK}$ → reduces long-run TFP 8%
- Adds countercyclical component to $\sigma^2_{MPK}$ → amplifies agg shocks
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Cues from the basic neoclassical growth model

Continuum of firms, only capital, idio. + agg. risk

- Production: \( Y_{it} = X_t^{\beta_{it}} Z_{it} K_{it}^{\theta} \), \( X_t \) correlated with SDF \( M_t \)
- \( K \) accumulation: \( K_{it+1} = I_{it} + (1 - \delta) K_{it} \)
- Dividends: \( D_{it} = Y_{it} - I_{it} \)

Firm problem:
\[
V (X_t, Z_{it}, K_{it}) = \max_{K_{it+1}} X_t^{\beta_{it}} Z_{it} K_{it}^{\theta} - K_{it+1} + (1 - \delta) K_{it} + E_t [M_{t+1} V (X_{t+1}, Z_{it+1}, K_{it+1})]
\]

Optimality: \( 1 = E_t [M_{t+1} (MPK_{it+1} + 1 - \delta)], \quad MPK_{it+1} = \theta \frac{Y_{it+1}}{K_{it+1}} \)

\[
\Rightarrow \quad E_t [MPK_{it+1}] = \alpha_t + \beta_{it} \lambda_t, \quad \sigma^2_{E_t[MPK_{it+1}]} = \sigma^2_{\beta} \lambda_t^2
\]

where \( \beta_{it} = -\frac{\text{cov}_t(\text{MPK}_{it+1}, M_{t+1})}{\text{var}_t(M_{t+1})} \), \( \lambda_t = \frac{\text{var}_t(M_{t+1})}{E_t[M_{t+1}]} \)
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\]

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\[
\Rightarrow \quad \mathbb{E}_t [MPK_{it+1}] = \alpha_t + \beta_{it} \lambda_t, \quad \sigma_{\mathbb{E}_t[MPK_{t+1}]}^2 = \sigma_\beta^2 \lambda_t^2
\]

where \( \beta_{it} = -\frac{\text{cov}_t(MPK_{it+1}, M_{t+1})}{\text{var}_t(M_{t+1})} \), \( \lambda_t = \frac{\text{var}_t(M_{t+1})}{\mathbb{E}_t[M_{t+1}]} \)
Examples

1. No aggregate risk (or risk neutrality): $M_{t+1} = \rho$

$$E_t [MPK_{it+1}] = \frac{1}{\rho} - (1 - \delta) = r_f + \delta$$

2. CAPM: $M_{t+1} = a - br_{mt+1}$

$$E_t [MPK_{it+1}] = \alpha_t + \frac{cov_t (r_{mt+1}, MPK_{it+1})}{\text{var}_t (r_{mt+1})} \underbrace{E_t [r_{mt+1} - r_{ft+1}]}_{\beta_{it} \lambda_t}$$

3. CCAPM: $M_{t+1} = \rho \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}$

$$E_t [MPK_{it+1}] = \alpha_t + \frac{cov_t (\Delta c_{t+1}, MPK_{it+1})}{\text{var}_t (\Delta c_{t+1})} \underbrace{\gamma \text{var}_t (\Delta c_{t+1})}_{\beta_{it} \lambda_t}$$
Empirical Predictions

\[ \mathbb{E}_t [MPK_{it+1}] = \alpha_t + \beta_{it} \lambda_t, \quad \sigma^2_{\mathbb{E}_t[MPK_{t+1}]} = \sigma^2_\beta \lambda_t^2 \]

1. Predictions on expected MPK (Empk)
   - Cross-section: exposure to risk factors is a determinant of expected MPK (Empk)
   - Time-series: predictable variation in \( \lambda_t \) leads to predictable variation in Empk

2. Predictions on MPK dispersion
   - Cross-section: cross-sectional MPK dispersion is related to \( \beta \) dispersion
   - Time-series: MPK dispersion increases with \( \lambda_t \)

→ We find evidence supporting these predictions using data on US publicly traded firms
Prediction 1 - standard risk factors are determinants of Empk (1/2)

Relate mpk to expected stock market returns

Sort firms into 10 portfolios based on \( mpk_{it} = y_{it} - k_{it} \), calculate \( \mathbb{E}[r_{t+1}] \)

<table>
<thead>
<tr>
<th></th>
<th>Low ( mpk )</th>
<th>High ( mpk )</th>
<th>HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{E}[r_{t+1}] ), not industry-adjusted</td>
<td>6.84*</td>
<td>13.82***</td>
<td>6.98***</td>
</tr>
<tr>
<td></td>
<td>(1.94)</td>
<td>(3.43)</td>
<td>(3.10)</td>
</tr>
<tr>
<td>( \mathbb{E}[r_{t+1}] ), industry-adjusted</td>
<td>9.18*</td>
<td>13.48***</td>
<td>4.29***</td>
</tr>
<tr>
<td></td>
<td>(1.95)</td>
<td>(3.16)</td>
<td>(3.24)</td>
</tr>
</tbody>
</table>

⇒ High \( mpk \) firms offer high expected stock returns

Robust to using

- Contemporaneous or long-term (3 year) returns
- Unlevered returns
- Double sorting on \( mpk \) and other variables, e.g., size, book-to-market
Prediction 1 - standard risk factors are determinants of Empk (2/2)

Directly relate $mpk_{it+1}$ to measures of $\beta_{it}$

Compute $\beta_{mpk}$ and $\beta_r$ using CAPM, Fama French 3 factor model

Estimate regressions: $mpk_{it+1} = \psi_0 + \psi_1 \beta_{it} + controls + \zeta_{it+1}$

\[
\begin{array}{l}
\beta_{mpk,mkt} & 0.020^{***} & (6.74) \\
\beta_{mpk,ff} & 1.008^{***} & (9.62) \\
\beta_{r,mkt} & 0.0127^{***} & (4.14) \\
\beta_{r,ff} & 0.005^{***} & (5.59) \\
\end{array}
\]

Observations: 79495, 79010, 107203, 106509

Firms: 8530, 8490, 10141, 10094

$R^2$: 0.065, 0.065, 0.06, 0.06

⇒ High $mpk$ firms have high betas
Prediction 3 - \( mpk \) dispersion is related to \( \beta \) dispersion

Relate \( \sigma^2_{mpk} \) to \( \sigma^2_{\beta} \) and \( \sigma^2_{E[r]} \) across industries

By firm: estimate \( E_t [r_{t+1}] \) and \( \beta_t \)'s from Fama French 3 factor model

By industry: calculate \( \sigma (mpk_{t+1}) \) and \( \sigma (E_t [r_{t+1}]) \)

Regress: \( \sigma (mpk_{t+1})_j = \psi_0 + \psi_1 \sigma (E_{t-1} [r])_j + \zeta_{jt+1} \)

\[
\begin{array}{|c|c|c|}
\hline
 & (1) & (2) & (3) \\
\hline
\sigma(E[r]) & 2.54*** & & \\
 & (34.14) & & \\
\sigma(E[r_\beta]) & & 11.63*** & \\
 & & (31.43) & \\
\sigma(\beta_{MKT}) & & & 0.24*** \\
 & & & (12.22) \\
\sigma(\beta_{HML}) & & & 0.12*** \\
 & & & (10.63) \\
\sigma(\beta_{SMB}) & & & 0.12*** \\
 & & & (8.54) \\
\hline
Observations & 2721 & 2746 & 2734 \\
R^2 & 0.300 & 0.265 & 0.306 \\
\hline
\end{array}
\]

\( \Rightarrow \) Industries w high expected return/\( \beta \) dispersion have high \( mpk \) dispersion
Prediction 4 - *mpk* dispersion increases with $\lambda$

Relate $\sigma_{mpk}^2$, $r_{hml}$ to (lagged) indicators of price of risk

Estimate regressions:

\[
\begin{align*}
\sigma_{mpk,t+1} &= \psi_0 + \psi_1 x_t + \zeta_{t+1} \\
r_{hml,t+1} &= \psi_0 + \psi_1 x_t + \zeta_{t+1}
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$ (<em>mpk</em>)</th>
<th>HML Spread (%)</th>
</tr>
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<tbody>
<tr>
<td>PD Ratio</td>
<td>-0.002** (-3.01)</td>
<td>-0.046* (-1.70)</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>0.012*** (3.69)</td>
<td>0.384** (2.31)</td>
</tr>
<tr>
<td>EB Premium</td>
<td>0.027*** (4.41)</td>
<td>0.580** (2.36)</td>
</tr>
<tr>
<td>Observations</td>
<td>148 148 148 152 152 152</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.05 0.09 0.16 0.05 0.13 0.10</td>
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$\Rightarrow$ *mpk* dispersion increases when $\lambda_t$ is high (i.e., is countercyclical)
Quantitative model

1. Technology

- Production: \( Y_{it} = X_t^{\hat{\beta}_i} \hat{Z}_{it} K_{it}^{\theta_1} N_{it}^{\theta_2} \rightarrow \Pi_{it} = X_t^{\hat{\beta}_i} Z_{it} K_{it}^{\theta} \)

- Productivity components follow AR(1) (in logs):
  \[
  x_{t+1} = \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon})
  \]
  \[
  z_{it+1} = \rho z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim \mathcal{N}(0, \sigma^2_{\tilde{\varepsilon}})
  \]

- For now, no other frictions (later: adj. costs, other frictions/distortions)

2. Stochastic discount factor

- Constant risk-free rate: \( r_f = -\log \rho \)

- Countercyclical price of risk: \( \text{var}_t (m_{t+1}) = \gamma^2_t \sigma^2_{\varepsilon} \downarrow \text{in } x_t \)

- Maximum Sharpe ratio: \( SR_t = \sigma_{mt} = \gamma_t \sigma_{\varepsilon} \)
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- Productivity components follow AR(1) (in logs):
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  x_{t+1} = \rho_x x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N \left( 0, \sigma_\varepsilon^2 \right)
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\[
m_{t+1} \equiv \log M_{t+1} = \log \rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2} \gamma_t^2 \sigma_\varepsilon^2
\]
\[
\gamma_t = \gamma_0 + \gamma_1 x_t, \quad \gamma_1 \leq 0
\]
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Risk premia and ‘misallocation’

Optimal investment:

\[ k_{it+1} = \frac{1}{1 - \theta} \left( \tilde{\alpha} + \beta_i \rho x_t + \rho z z_{it} - \beta_i \gamma_t \sigma^2_{\varepsilon} \right) \]

- Conditional on (expected) fundamentals, high \( \beta \) firms choose lower \( k \)

Expected \( mpk \):

\[ \mathbb{E}_t [mpk_{it+1}] = \alpha + \beta_i \gamma_t \sigma^2_{\varepsilon} \Rightarrow \sigma^2_{\mathbb{E}_t[mpk]} = \sigma^2_{\beta} \left( \gamma_t \sigma^2_{\varepsilon} \right)^2 \]

- Dispersion in \( Empk \) depends on \( \sigma^2_{\beta} \), price of risk (so is countercyclical)

Aggregate TFP:

\[ a_{t+1} = a^*_{t+1} - \frac{1}{2} \frac{\theta_1 (1 - \theta_2)}{1 - \theta_1 + \theta_2} \sigma^2_{mpk,t+1} = a^*_{t+1} - \frac{1}{2} \frac{\theta_1 (1 - \theta_2)}{1 - \theta_1 + \theta_2} \sigma^2_{\beta} \left( \gamma_t \sigma^2_{\varepsilon} \right)^2 \]

- Heterogeneity in risk premia depresses TFP, amplifies underlying shocks
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$$\mathbb{E}_t [mpk_{it+1}] = \alpha + \beta_i \gamma_t \sigma_\epsilon^2 \quad \Rightarrow \quad \sigma^2_{\mathbb{E}_t [mpk]} = \sigma^2_\beta \left( \gamma_t \sigma_\epsilon^2 \right)^2$$

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Aggregate TFP:

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- Heterogeneity in risk premia depresses TFP, amplifies underlying shocks
**Adjustment costs**

Add capital adjustments costs: \( \Phi (l_{it}, K_{it}) = \frac{\xi}{2} \left( \frac{l_{it}}{K_{it}} - \delta \right)^2 K_{it} \)

Optimal investment (first order approx):

\[
k_{it+1} = \phi_0 + \phi_1 \beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it} - \phi_0 \beta_i
\]

- Key effects of \( \xi \): \( \phi_3 \) and \( \phi_{01} \) ↑ in \( \xi \)

No closed-forms for \( E_t [mpk_{it+1}] \), focus on \( E [mpk_{it+1}] \):

\[
E [mpk_{it+1}] = \alpha + \frac{1 - \theta}{1 - \phi_3} \phi_{01} \beta_i \quad \Rightarrow \quad \sigma_{E[mpk]}^2 = \left( \frac{1 - \theta}{1 - \phi_3} \right)^2 \phi_{01}^2 \sigma_\beta^2
\]

- On their own, adj. costs do not generate persistent dispersion in \( mpk \)
- But, they amplify persistent risk premium effects! (\( \phi_3 \) and \( \phi_{01} \) ↑ in \( \xi \))
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No closed-forms for \( \mathbb{E}_t [mpk_{it+1}] \), focus on \( \mathbb{E} [mpk_{it+1}] \):

\[
\mathbb{E} [mpk_{it+1}] = \alpha + \frac{1 - \theta}{1 - \phi_3} \phi_{01} \beta_i \quad \Rightarrow \quad \sigma^2_{\mathbb{E}[mpk]} = \left( \frac{1 - \theta}{1 - \phi_3} \right)^2 \phi_{01}^2 \sigma_\beta^2
\]

- On their own, adj. costs do not generate persistent dispersion in \( mpk \)
- But, they amplify persistent risk premium effects! (\( \phi_3 \) and \( \phi_{01} \uparrow \) in \( \xi \))
MPK dispersion and stock market returns

How to quantify...? Our approach: use link with stock market returns

To a first-order approximation:

$$\mathbb{E}_t [r_{it+1}^e] \equiv \log \mathbb{E}_t [R_{it+1}^e] = \psi \beta_i \gamma_t \sigma^2_{\varepsilon} \quad \Rightarrow \quad \sigma^2_{\mathbb{E}_t[r]} = \psi^2 \sigma^2_{\beta} \left( \gamma_t \sigma^2_{\varepsilon} \right)^2$$

- $\sigma^2_{\mathbb{E}_t[mpk]} \propto \sigma^2_{\mathbb{E}_t[r]}$
- $\psi$ depends on prod. parameters, ind. of $\gamma_0$, decreasing in $\gamma_1$

Suggests empirical strategy:

- Market portfolio (perfectly diversified):

  $$SR_{mt} = \gamma_t \sigma_{\varepsilon} \quad \Rightarrow \quad \mathbb{E} [SR_{mt}] = \gamma_0 \sigma_{\varepsilon}$$
  $$\mathbb{E}_t [r_{mt+1}^e] = \psi \beta \gamma_t \sigma^2_{\varepsilon} \quad \Rightarrow \quad \mathbb{E} [r_{mt+1}^e] = \psi \beta \gamma_0 \sigma^2_{\varepsilon}$$

  $\rightarrow$ Estimates of $\gamma_0$ and $\gamma_1$

- $\sigma^2_{\mathbb{E}_t[r]} \rightarrow$ estimate of $\sigma^2_{\beta}$ (compute $\mathbb{E}_t [r_{it+1}^e]$ using FF 3 factor model)

  $\sigma^2_{\mathbb{E}_t[r],FF} = 0.13^2 \rightarrow \sigma^2_{\beta} = 5$
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$$
E_t [r_{it+1}^e] \equiv \log E_t [R_{it+1}^e] = \psi \beta_i \gamma_t \sigma^2 \varepsilon \quad \Rightarrow \quad \sigma^2_{E_t[r]} = \psi^2 \sigma^2_\beta \left( \gamma_t \sigma^2 \varepsilon \right)^2
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  $$

  \rightarrow Estimates of $$\gamma_0$$ and $$\gamma_1$$

- $$\sigma^2_{E_t[r]} \rightarrow estimate of \sigma^2_\beta$$ (compute $$E_t [r_{it+1}^e]$$ using FF 3 factor model)

  $$\sigma^2_{E_t[r], FF} = 0.13^2 \rightarrow \sigma_\beta = 5$$
Main results - risk premia and misallocation

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Only Risk</th>
<th>Constant Risk</th>
<th>Only Constant Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MPK ) Implications</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E \sigma^2_{\text{Empk}} )</td>
<td>0.20</td>
<td>0.07</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>( \frac{E \sigma^2_{\text{Empk}}}{\sigma^2_{\text{mpk}}} )</td>
<td>0.44</td>
<td>0.15</td>
<td>0.40</td>
<td>0.15</td>
</tr>
<tr>
<td>( \frac{E \sigma^2_{\text{Empk}}}{\sigma^2_{\text{mpk}}} )</td>
<td>0.67</td>
<td>0.23</td>
<td>0.60</td>
<td>0.22</td>
</tr>
<tr>
<td>( \Delta \bar{a} )</td>
<td>0.08</td>
<td>0.03</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>( \text{corr}(\sigma^2_{\text{Empk},t}, x_t) )</td>
<td>-0.25</td>
<td>-0.98</td>
<td>0.48</td>
<td>0.08</td>
</tr>
</tbody>
</table>

\( \Rightarrow \) Risk premium effects lead to substantial \( mpk \) dispersion

- 44% of observed \( \sigma^2_{\text{mpk}} \), two-thirds of perm. component
  \( \rightarrow \) Reduces long-run TFP 8%

- Adds countercyclical element to \( \sigma^2_{\text{mpk}} \)
  \( \rightarrow \) E.g., -1% shock to \( x_t \) generates 1.2% fall in \( a_t \)
Other Forms of Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>Adj. Costs</th>
<th>Large Adj. Costs</th>
<th>$\theta$</th>
<th>$\delta$</th>
<th>$\rho_z$</th>
<th>$\sigma^2_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min. $Er$</strong></td>
<td>0.036</td>
<td>0.038</td>
<td>0.029</td>
<td>0.031</td>
<td>0.032</td>
<td>0.023</td>
</tr>
<tr>
<td><strong>Mean $Er$</strong></td>
<td>0.037</td>
<td>0.039</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
</tr>
<tr>
<td><strong>Max. $Er$</strong></td>
<td>0.038</td>
<td>0.040</td>
<td>0.056</td>
<td>0.042</td>
<td>0.042</td>
<td>0.040</td>
</tr>
<tr>
<td><strong>Spread</strong></td>
<td>0.002</td>
<td>0.002</td>
<td>0.027</td>
<td>0.012</td>
<td>0.010</td>
<td>0.017</td>
</tr>
</tbody>
</table>

⇒ Unobserved variation in technological parameters unlikely to account for large spreads in expected returns observed in the data
Robustness/extensions

1. Other distortions (e.g., markups, financial frictions, policies, etc.)
   - Add idiosyncratic output ‘tax’ \(1 - \tau_{it}, \tau_{it} = -\nu z_{it} - \eta_{it+1}\)

   \[
   mpk_{it+1} = \alpha + \beta_i \varepsilon_{t+1} + \varepsilon_{it+1} + \nu \rho z_{it} + \eta_{it+1} + \beta_i \gamma_t \sigma^2_{\varepsilon}
   \]

   → Risk premium, expected stock returns unaffected, analysis unchanged!

2. Are we picking up adj. costs, other forms of heterogeneity?
   - Set \(\beta_i = \bar{\beta}\), compute \(\sigma^2_{E[r]}\) with adj. costs, firm-specific parameters (e.g., \(\theta\))
   → Do not generate large dispersion in expected returns

3. Alternative measure of \(\beta\)’s using production-side data
   - Estimate: \(\beta_i \Delta x_t + \Delta z_{it} = \kappa_i + \Delta x_t + \zeta_{it}\)
   → Gives value of \(\sigma^2_{\beta}\) similar to baseline
Conclusion

Theory connecting misallocation to exposure to agg risk

- Persistent firm-level \( mpk \) deviations and countercyclical dispersion
- Accounts for substantial portion of measured \( mpk \) dispersion
- New link between CS asset pricing, agg fluctuations and long-run outcomes

Next steps

- Other countries - developing economies more volatile
- Sources of \( \beta \) dispersion...
- Private firms and idiosyncratic risk
Variation in $\beta$ - example 1
**Variation in $\beta$ - example 2**

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**Target Corporation Stock Comparison**

**TGT $61.2789' 4.6589 \uparrow 8.23\%**

*Delayed - data as of Nov. 29, 2017 12:43 ET - Find a broker to begin trading TGT now

---

This company is compared to stocks in the same industry. It is sorted by largest to the smallest market value. Feel free to change the stock symbol and select the compare button.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>WMT</th>
<th>COST</th>
<th>DG</th>
<th>DLTR</th>
<th>TGT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
<td>Wal-Mart Stores, Inc.</td>
<td>Costco Wholesale Corporation</td>
<td>Dollar General Corporation</td>
<td>Dollar Tree, Inc.</td>
<td>Target Corporation</td>
</tr>
<tr>
<td>Market Value</td>
<td>$291M</td>
<td>$77M</td>
<td>$24M</td>
<td>$24M</td>
<td>$33M</td>
</tr>
</tbody>
</table>

---

**Price Information**

| Current Last Sale (CLS) | $97.68 | $175.6 | $88.715 | $103.73 | $61.2789 |
| Net Change / % from close | ▲ 0.91 / 0.94\% | ▲ 1.95 / 1.12\% | ▲ 0.985 / 1.12\% | ▲ 2.79 / 2.76\% | ▲ 4.6589 / 8.23\% |
| Target Price (TP) / CLS' % of TP | $60 / 163\% | $69 / 254\% | $34 / 261\% | $53 / 196\% | $61 / 100\% |
| 52 Week High / % Change | $100.13 / -2\% | $183.18 / -4\% | $86.13 / 1\% | $101.09 / 3\% | $78.69 / -22\% |
| 52 Week Low / % Change | $85.28 / 50\% | $100 / 17\% | $65.97 / 34\% | $65.83 / 58\% | $48.56 / 26\% |

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**Trade Information**

| Beta | 1.25 | 0.9 | 0.54 | 1.09 | 0.68 |
| Volume | 7,010,517 | 2,266,041 | 2,404,127 | 1,431,442 | 9,493,155 |
| Avg Daily Volume | 8,876,488 | 3,038,384 | 2,586,315 | 2,136,582 | 6,862,157 |
| Short Interest | N/A | 15,127,524 | N/A | 4,558,081 | N/A |
| % of shares outstanding | 0\% | 3449\% | 0\% | 1925\% | 0\% |
| Days to cover | N/A | 8.751549 | N/A | 2.368876 | N/A |
Variation in $\beta$ - example 3

American Airlines Group, Inc. Stock Comparison

$49.77^* 0.55 \uparrow 1.12\%$

*Delayed - data as of Nov. 29, 2017 12:26 ET - Find a broker to begin trading AAL now

This company is compared to stocks in the same industry. It is sorted by largest to the smallest market value. Feel free to change the stock symbol and select the compare button.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>FDX</th>
<th>DAL</th>
<th>LUV</th>
<th>RYAAY</th>
<th>AAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Value</td>
<td>$650M</td>
<td>$37M</td>
<td>$35M</td>
<td>$29M</td>
<td>$24M</td>
</tr>
</tbody>
</table>

Price Information

| Current Last Sale (CLS) | $224.92 | $52.059 | $59.14 | $124.89 | $49.77 |
| Net Change / % from close | ▲ 6.41 / 2.93% | ▲ 1.475 / 2.92% | ▲ 2.97 / 5.29% | ▲ 2.69 / 2.2% | ▲ 0.55 / 1.12% |
| Target Price (TP) / CLS % of TP | $99.55 / 228% | $17 / 306% | $15.25 / 388% | $15.25 / 819% | $15.25 / 326% |
| 52 Week High / % Change | $231.35 / -3% | $55.75 / -7% | $64.39 / -4% | $122.675 / 2% | $54.48 / -9% |
| 52 Week Low / % Change | $182.89 / 23% | $43.81 / 19% | $44.70 / 32% | $78.345 / 50% | $39.21 / 27% |

Trade Information

| Beta | 1.09 | 2.2 | 1.18 | 1.04 | 1.71 |
| Volume | 757,035 | 4,796,117 | 4,325,795 | 566,375 | 3,251,288 |
| Avg Daily Volume | 1,178,018 | 7,170,076 | 5,280,399 | 404,833 | 4,622,932 |
| Short Interest | N/A | N/A | N/A | 847,901 | 26,356,901 |
| % of shares outstanding | 0% | 0% | 0% | 358% | 5412% |
| Days to cover | N/A | N/A | N/A | 2.017913 | 7.676002 |