Monetary Policy with Heterogeneous Agents: Insights from Tank Models

Davide Debortoli    Jordi Galí

October 2017
Motivation

- Heterogeneity in Monetary Models
  - idiosyncratic shocks
  - incomplete markets
  - borrowing constraints
  - supply side as in conventional NK models → HANK

Main lessons:
- heterogeneity in MPCs → role of redistribution
- direct vs. indirect effects

Drawbacks
- computational complexity
- limited use in classroom or as a policy input

**Wanted:** a simple tractable framework that is a good approximation
Present Paper

- An Organizing Framework
  - heterogeneity between constrained and unconstrained households
  - heterogeneity within unconstrained households

- Baseline Two-Agent New Keynesian model (TANK)
  - constant share of hand-to-mouth households
  - redistribution through fiscal rule

- Question: How well does a TANK model capture the main predictions of HANK models regarding the aggregate economy’s response to aggregate shocks?
Main Findings

- A baseline TANK model with a suitably calibrated transfer rule provides a good approximation to a prototypical HANK model
  - effects of monetary policy and other shocks
  - role of fiscal policy

- Isomorphic to a modified RANK model

- **Application**: Optimal monetary policy in TANK
  - heterogeneity as a source of a policy tradeoff
  - calibrated model: price stability nearly optimal (as in RANK!)
Some References


An Organizing Framework

- Continuum of heterogeneous households, indexed by $s \in [0, 1]$.

- Preferences: $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^s, N_t^s; Z_t)$ where

$$U(C, N; Z) \equiv \left( \frac{C^{1-\sigma} - 1}{1 - \sigma} - \frac{N^{1+\varphi}}{1 + \varphi} \right) Z$$

- $\Theta_t \subset [0, 1]$ : set of unconstrained ("Ricardian") households in period $t$
  - measure $1 - \lambda_t$
  - access to one-period bonds with riskless return $R_t$.

- **Goal**: derive a (generalized) Euler equation for aggregate consumption
  - in the spirit of Werning (2015), but different emphasis
Individual Euler equation: For all $s \in \Theta_t$

$$Z_t(C_t^s)^{-\sigma} = \beta R_t \mathbb{E}_t \{ Z_{t+1}(C_{t+1}^s)^{-\sigma} \}$$
An Organizing Framework

- Averaging over all \( s \in \Theta_t \) and letting \( C_t^R \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} C_t^s \, ds \) yields:

\[
Z_t(C_t^R)^{-\sigma} = \beta R_t E_t \left\{ Z_{t+1}(C_{t+1}^R)^{-\sigma} V_{t+1} \right\}
\]

where

\[
V_{t+1} \equiv \left( \frac{C_{t+1|t}^R}{C_{t+1}^R} \right)^{-\sigma} \left[ \frac{2 + \sigma(1 + \sigma) \text{var}_{s|t} \{c_{t+1}^s\}}{2 + \sigma(1 + \sigma) \text{var}_{s|t} \{c_t^s\}} \right]
\]

with

\[
C_{t+1|t}^R \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} C_{t+1}^s \, ds
\]

\[
\text{var}_{s|t} \{c_{t+k}^s\} \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} (c_{t+k}^s - c_{t+k}^R)^2 \, ds
\]

- \( V_t \): index of heterogeneity \textit{within} unconstrained households
An Organizing Framework

- Recall:
  \[ Z_t(C_t^R)^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ Z_{t+1}(C_{t+1}^R)^{-\sigma} V_{t+1} \right\} \]

- Letting \( C_t \equiv \int_0^1 C_t^s \, ds \):

  \[ Z_t H_t^{-\sigma}(C_t)^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ Z_{t+1} H_{t+1}^{-\sigma}(C_{t+1})^{-\sigma} V_{t+1} \right\} \]

where

\[ H_t \equiv \frac{C_t^R}{C_t} \]

- \( H_t \): index of heterogeneity *between* constrained and unconstrained households
An Organizing Framework

- Imposing market clearing and log-linearizing:
  \[ y_t = \mathbb{E}_t \{ y_{t+1} \} - \frac{1}{\sigma} \hat{r}_t - \frac{1}{\sigma} \mathbb{E}_t \{ \Delta z_{t+1} \} + \mathbb{E}_t \{ \Delta h_{t+1} \} - \frac{1}{\sigma} \mathbb{E}_t \{ \hat{v}_{t+1} \} \]

- Representation in levels:
  \[ \hat{y}_t = -\frac{1}{\sigma} \left( \hat{r}_t^L - z_t \right) - \hat{h}_t - \hat{f}_t, \]
  \[ \hat{r}_t^L = \mathbb{E}_t \{ \hat{r}_{t+1}^L \} + \hat{r}_t \]
  \[ \hat{f}_t = \mathbb{E}_t \{ \hat{f}_{t+1} \} + \frac{1}{\sigma} \mathbb{E}_t \{ \hat{v}_{t+1} \} \]

- Comparison to RANK
- How important are fluctuations in \( \hat{h}_t \) and \( \hat{v}_t \) (and \( \hat{f}_t \))? Open question
- Next: A baseline TANK model (\( \hat{f}_t = \hat{v}_t = 0 \)
Two types of households: $s \in \{K, R\}$

**Ricardian:** measure $1 - \lambda$

$$C_t^R + \frac{B_t^R}{P_t} + Q_t S_t^R = \frac{B_{t-1}^R (1 + i_{t-1})}{P_t} + W_t N_t^R + (D_t + Q_t) S_{t-1}^R + T_t^R$$

**Keynesian:** measure $\lambda$

$$C_t^K = W_t N_t^K + T_t^K$$
A Baseline TANK Model: Policy

- Fiscal Policy
  - transfer rule:
    \[ T^K_t = \tau_0 D + \tau_d (D_t - D) \]
  - balanced budget:
    \[ \lambda T^K_t + (1 - \lambda) T^R_t = 0. \]

Special cases:
(i) laissez-faire \((\tau_0 = \tau_d = 0)\)
(ii) full steady state redistribution \((\tau_0 = 1, \tau_d = 0)\)
(iii) full redistribution period by period \((\tau_0 = 1, \tau_d = 1) \Rightarrow \text{RANK}\)

- Monetary Policy: Taylor-type rule
Wage equation

\[ W_t = \mathcal{M}^w C_t^\sigma \mathcal{N}_t^\varphi \]

with \( N_t^R = N_t^K = N_t \), and \( \mathcal{M}^w > 1 \).

Technology

\[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]

Inflation equation (Rotemberg pricing):

\[ \Pi_t(\Pi_t - 1) = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} (\Pi_{t+1} - 1) \right\} \]

\[ + \frac{\epsilon}{\zeta} \left( \frac{1}{\mathcal{M}_t^p} - \frac{1}{\mathcal{M}^p} \right) \]

where \( \mathcal{M}_t^p \equiv (1-\alpha) A_t N_t^{-\alpha} / W_t \) and \( \mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1} \)
Goods market clearing

\[ C_t = Y_t \Delta^p(\Pi_t) \]

where \( \Delta^p(\Pi_t) \equiv 1 - (\xi/2)(\Pi_t - 1)^2 \) and \( C_t \equiv (1 - \lambda)C_t^R + \lambda C_t^K \)
A Baseline TANK Model

- Aggregate Euler equation

\[ 1 = \beta (1 + i_t) \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{H_{t+1}}{H_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \Pi_{t+1}^{-1} \right\} \]

where

\[ H_t \equiv \frac{C_t^R}{C_t} = \frac{1}{\Delta^p(\Pi_t)} \left( \frac{W_t N_t}{Y_t} + \frac{1}{1 - \lambda} \frac{D_t}{Y_t} + \frac{T_t^R}{Y_t} \right) \]

\[ = 1 + \frac{1}{\Delta^p(\Pi_t)} \left[ \frac{\lambda (1 - \tau_d)}{1 - \lambda} \left( \frac{\Delta^p(\Pi_t)}{\Pi^p_t} - \frac{1 - \alpha}{\mathcal{M}_t^p} \right) - \frac{\lambda (\tau_0 - \tau_d)}{1 - \lambda} \left( 1 - \frac{1 - \alpha}{\mathcal{M}^p_t} \right) \frac{Y}{Y_t} \right] \]
A Baseline TANK Model

- Log-linearized equilibrium conditions

\[
\begin{align*}
\pi_t &= \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \\
\hat{y}_t &= \mathbb{E}_t \{ \hat{y}_{t+1} \} - \frac{1}{\sigma} (\hat{i}_t - \mathbb{E}_t \{ \pi_{t+1} \}) + \mathbb{E}_t \{ \Delta \hat{h}_{t+1} \} + \frac{1}{\sigma} (1 - \rho_z) z_t \\
\hat{h}_t &= \chi_y \tilde{y}_t + \chi_n \tilde{y}_t^n \\
\hat{i}_t &= \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t
\end{align*}
\]

where \( \tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n \), and \( \hat{y}_t^n \equiv \frac{1 + \varphi}{\sigma(1 - \alpha) + \alpha + \varphi} a_t \)

\[
\begin{align*}
\chi_n &\equiv \frac{\lambda (\tau_0 - \tau_d)}{(1 - \lambda) H} \left( 1 - \frac{1 - \alpha}{\mathcal{M}_p} \right) \\
\chi_y &\equiv \chi_n - \frac{\lambda (1 - \tau_d)(1 - \alpha)}{(1 - \lambda) H \mathcal{M}_p} \left( \sigma + \frac{\alpha + \varphi}{1 - \alpha} \right)
\end{align*}
\]
A Baseline TANK Model

Three-equation representation

\[ \pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

\[ \tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma (1 + \chi_y)} (\hat{i}_t - \mathbb{E}_t \{ \pi_{t+1} \} - \hat{r}_t^n) \]

\[ \hat{i}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]

where \( \hat{r}_t^n \equiv (1 - \rho_z) z_t + \sigma (1 + \chi_n) \mathbb{E}_t \{ \Delta \tilde{y}_{t+1} \} \)

⇒ isomorphic to a modified RANK model

<table>
<thead>
<tr>
<th>( \chi_y )</th>
<th>( \partial y / \partial r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>dampened</td>
</tr>
<tr>
<td>zero</td>
<td>unchanged</td>
</tr>
<tr>
<td>negative</td>
<td>amplified</td>
</tr>
</tbody>
</table>
A Baseline TANK Model: Calibration and Simulations

- Preferences: $\beta = 0.9925$, $\sigma = \varphi = 1$, $\epsilon = 9$
- Technology: $\alpha = 0.25$, $\zeta = 372$
- Financial frictions: $\lambda = 0.21$
- Policy rule: $\phi_\pi = 1.5$, $\phi_y = 0.125$

- Monetary policy shocks
  - the role of fiscal policy
  - the role of financial frictions
Monetary Policy Shocks and Transfer Rules
Monetary Policy Shocks and the Share of Constrained Households
A Baseline HANK Model

- Continuum of heterogeneous households, indexed by $s \in [0, 1]$
- Labor income: $W_t N_t \exp\{e_t^s\}$, where $e_t^s = 0.966 e_t^s + 0.017 \varepsilon_t^s$
- Assets: one-period nominal bond, in zero net supply.
- Borrowing limit: $B_t^s \geq -\psi Y$
- Alternative calibrations: $\psi = \{0.5, 1, 2\}$, implying $\lambda = \{0.36, 0.21, 0.11\}$
- Fiscal policy: government takes all profits and redistributes lump-sum, same transfer rule as in TANK
HANK vs. TANK Comparison

- After solving HANK model (Reiter (2010)), TANK calibrated to match
  \[ \lambda : \text{fraction of constrained households} \]
  \[ \beta : \text{steady state real rate} \]

- Simulations
## Interest Rate Elasticity of Output

### Type I Fiscal Policy

<table>
<thead>
<tr>
<th>$\tau_0$, $\tau_d$, $\psi$</th>
<th>Transfer Regime</th>
<th></th>
<th>Borrowing Limit</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(1,0,1)$</td>
<td>$(1,0.5,1)$</td>
<td>$(1,1,1)$</td>
<td>$(1,0,0.5)$</td>
</tr>
<tr>
<td>RANK</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>TANK</td>
<td>1.64</td>
<td>1.24</td>
<td>1.00</td>
<td>5.74</td>
</tr>
<tr>
<td>HANK</td>
<td>1.70</td>
<td>1.23</td>
<td>0.97</td>
<td>5.30</td>
</tr>
<tr>
<td>Implied $\lambda$</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.36</td>
</tr>
</tbody>
</table>
The Effects of a Monetary Policy Shock
The Case of Rigid Prices
The Effects of a Monetary Policy Shock
The Effects of a Preference Shock

![Graphs showing the effects of a preference shock on various economic indicators such as Output, Real Rate, h-index, v-index, Inflation, and Pref. Shock. The graphs compare HANK, RANK, and TANK scenarios.](image-url)
Welfare losses (second order approximation)

\[ L_t \approx \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \alpha_y \tilde{y}_t^2 + \alpha_h \tilde{h}_t^2 \right\} \]

where \( \alpha_y = \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) \frac{1}{\xi} \) and \( \alpha_h \equiv \frac{\sigma}{\xi} \left( \frac{1 - \lambda}{\lambda} \right) \)

Optimal monetary policy problem

\[ \min \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \alpha_y \tilde{y}_t^2 + \alpha_h \tilde{h}_t^2 \right\} \]

subject to

\[ \pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

\[ \tilde{h}_t = \chi_y \tilde{y}_t + \chi_n \tilde{y}_t^n \]

\[ \lambda (\tau_0 - \tau_d) \neq 0 \Rightarrow \chi_n \neq 0 \Rightarrow \text{policy tradeoff} \]
Representation of the optimal policy:

$$\hat{p}_t \equiv p_t - p_{-1} = -\frac{1}{\kappa} (\alpha_y \tilde{y}_t + \chi_y \alpha_h \hat{h}_t)$$

Impulse responses
Optimal Monetary Policy in TANK
Conclusions

- A baseline TANK model with a suitably calibrated transfer rule provides a good approximation to a prototypical HANK model
  - effects of monetary policy and other shocks
  - role of fiscal policy
- Isomorphic to a modified RANK model
- **Application**: Optimal monetary policy in TANK
  - heterogeneity as a source of a policy tradeoff
  - price stability nearly optimal (as in RANK!)