Longevity Risk and Retirement Savings

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Abstract

Over the last couple of decades there have been unprecedented, and to some extent unexpected, increases in life expectancy which have raised important questions for retirement savings. We study optimal consumption and saving choices in a life-cycle model, in which we allow for changes in the distribution of survival probabilities according to the Lee-Carter (1992) model. We use historical empirical evidence and actuary’s projections on longevity to parameterize the model. We show that when agents use official period life tables, which do not allow for future improvements in life expectancy, to make their savings decisions, the effects of longevity improvements on individual welfare can be significant. This is particularly so in the context of declining payouts of defined benefit pensions, which are correlated with improvements in life expectancy.

Next we study the role of longevity bonds in allowing households to hedge this risk. We first show that, if households know the exact parameters of the stochastic process for mortality rates, then the benefits from investing in this asset are very small. In fact, a negative risk premium of -10 basis points (in line with recent estimates) is enough to deter investors from holding these bonds almost completely. However, we also show that, small misperceptions about future improvements in mortality rates are enough to induce very large welfare gains from investing in longevity bonds: between 1% and 1.5% of annual consumption. This indicates that investors might not actually buy these bonds even if they actually stand to gain significantly from investing in them.
1 Introduction

Over the last few decades there has been an unprecedented increase in life expectancy. For example, in 1970 a 65-year-old United States male individual had a life expectancy of 13.04 years.\(^1\) Three decades later, a 65-year-old male had a life expectancy of 16.26 years. This represents an increase of 1.12 years per decade. To understand what such increase implies in terms of the savings needed to finance retirement consumption, consider a fairly-priced annuity that pays $1 real per year, and assume that the real interest rate is 2 percent. The price of such annuity for a 65 year old male would have been $10.52 in 1970, but it would have increased to $12.89 by 2000. This is an increase of roughly 23 percent. Or in other words, a 65 year old male in 2000 would have needed 23 percent more wealth to finance a given stream of real retirement consumption than a 65 year old male in 1970.

These large increases in life expectancy were, to a large extent, unexpected and as a result they have often been underestimated by actuaries and insurers. This is hardly surprising given the historical evidence on life expectancy. From 1970 to 2000 the average increase in the life expectancy of a 65 year old male was 1.12 years/decade, but over the previous decade the corresponding increase had only been 0.15 years. In the United Kingdom, a country for which a longer-time series of data on mortality is available, the average increase in the life expectancy of a 65 year old male was 1.23 years/decade from 1970 to 2000, but only 0.17 years/decade from 1870 to 1970. These unprecedented longevity increases are to a large extent responsible for the underfunding of pay as you go state pensions,\(^2\) and of defined-benefit company sponsored pension plans. For individuals who are not covered by such defined-benefit schemes, and who have failed to anticipate the observed increases in life expectancy, a longer live span implies a lower average level of retirement consumption.

The response of governments has been to decrease the benefits of state pensions, and to give tax and other incentives for individuals to save privately, through defined contribution pension schemes. Likewise, many companies have closed company sponsored defined benefit plans to new members, and instead offer to contribute towards personal pensions that tend to be

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\(^1\)The data in this paper on life-expectancy was obtained from the Human Mortality Database.

\(^2\)The decrease in birth rates that has occurred over this period has also contributed to the underfunding.
defined contribution in nature. There has been a considerable amount of research comparing defined benefit and defined contribution pension plans (Campbell and Feldstein, 2001). Some have argued that individuals may be able to replicate the risk/return characteristics of defined benefit plans, within defined contribution plans, through an appropriate choice of financial assets. Investment risk, or the risk associated with the returns on the portfolio of invested assets, may be reduced or even eliminated by investing retirement savings in long-term inflation indexed bonds.

Longevity risk, or the risk that the individual might live longer than average, may be reduced by the purchase of annuities at retirement age. However, whereas the purchase of annuities at retirement age provides insurance against longevity risk as of this age, a young individual saving for retirement faces substantial uncertainty as to the level of aggregate life expectancy, and consequently annuity prices, that she will face when she retires. Furthermore, markets may be incomplete in the sense that they may lack the financial assets that would allow individuals to insure against this risk. This paper studies how much are individuals affected by longevity risk, and the potential role of financial instruments that allow them to hedge this exposure.

We use this empirical analysis to parameterize a standard life-cycle model of consumption and saving choices. The novelty of the model is that the survival probabilities are stochastic and evolve according to the Lee-Carter model (1992), which is the leading statistical model of mortality in the demographic literature. We study how the individual’s consumption, savings, and welfare are affected by longevity risk. We find that agents respond to longevity improvements by increasing their savings, and in this way are able to at least partially self insure against longevity shocks. We say partially because when agents guide their decisions using official period life tables, which do not take into account future improvements in longevity, the effects of longevity risk on welfare can be substantial, particularly as of retirement age. More generally, we show that longevity risk can have significant welfare implications when agents underestimate the probability of future mortality improvements. This result is suggestive of the importance of household financial literacy, a point emphasized by Lusardi and Mitchell (2006).

These welfare losses are substantially higher when the payouts of defined benefit pension plans are negatively correlated with aggregate survival rates. In this case, which is motivated by recent
events, when longevity increases and households need more wealth to finance their retirement consumption, they are more likely to receive a lower pension. Furthermore, markets may be incomplete in the sense that they may lack the financial assets that would allow individuals to insure against this risk.

There have been recent attempts to address this market incompleteness, and offer economic agents financial instruments that would allow them to hedge against longevity risk. In December 2003, Swiss Re. issued a $400m three-year life catastrophe bond. This was a direct attempt by Swiss Re. to insure itself against a catastrophic mortality deterioration (e.g. a pandemic). This bond offered an opposite hedge to pension funds and other annuity providers, since when aggregate mortality rates increase their liabilities decrease. This issue was well received and fully subscribed, and followed by a second bond placement in April 2005. The second bond had a five-year maturity, and a total principal amount of $362m. This second issue was also fully subscribed. In the meantime, in November 2004, The European Investment Bank (EIB) announced the issuance of a 25-year £540m longevity bond. Despite receiving substantial public attention, the issue was not well perceived by market participants, and was eventually withdrawn.

With these financial innovations in mind, we extend the set of available assets in the model, and allow the agent to invest in financial assets whose returns are correlated with longevity shocks, which can therefore be used to hedge longevity risk. Such investments could be made directly or indirectly, by owning shares in mutual funds that buy them. We study the potential benefits from these investments, and draw implications for security design.\(^5\,6\)

\(^3\)Designing financial instruments for hedging longevity risk was first proposed by Blake and Burrows (2001), who recommend that governments should issue “survival bonds”, to allow the private sector to hedge this source of risk.

\(^4\)The bond was supposed to be issued by the EIB, with Partner Re. acting as the longevity reinsurer and BNP Paribas structuring, managing, and marketing the placement.

\(^5\)There is a growing literature that studies the optimal pricing of longevity bonds and related instruments (see, for example, Dahl (2004) and Carins, Blake and Dowd (2006)). Here we instead take bond prices as given and discuss their potential role in household portfolios.

\(^6\)Menoncin (2007) also introduces longevity bonds in an optimal savings and portfolio choice problem. However, in his model there is no labor income or retirement.
We first show that, if households know the exact parameters of the stochastic process for mortality rates, then the benefits of being able to trade longevity bonds are quite small. If the risk premium on these bonds is zero, although they will invest a significant fraction of their wealth in this asset, the welfare gain from doing so is relatively small. In fact, a negative risk premium of 10 basis points is enough to deter investors from holding these bonds almost completely, depending on the exact calibration. Unfortunately, since longevity bonds are a fairly recent financial development, it is hard to compare this number with empirical estimates. Using the CCAPM and mortality rates forecasts (also based on the Lee-Carter model) Friedberg and Webb (2005) obtain an estimate of -0.02% with a risk aversion coefficient of 10.

However, small misperceptions about future improvements in mortality rates are enough to induce very large welfare gains from investing in longevity bonds. More precisely, if households guide their decisions using official period life tables, which do not take into account future improvements in longevity, the true benefit of investing in this asset can be as large as 2% of annual consumption. Naturally, from the household’s perspective, her ex-ante estimate of this gain is very small, since she believes period life tables allow for future mortality improvements. This difference is extremely important because it suggests that investors might not buy these bonds even if they actually stand to gain significantly from investing in them.

The paper is organized as follows. In section 2 we use long term data for a cross section of countries to document the existing empirical evidence on longevity. In sections 3 and 4 we setup and parameterize a life cycle model of the optimal consumption and saving choices of an individual who faces longevity risk. The results of the model are discussed in section 5. In section 6 we expand the set of assets that the agent has available to include longevity bonds. The final section concludes and discusses extensions for future research.
2 Empirical Evidence on Longevity

In this section we consider the existing empirical evidence on longevity. The data is from the Human Mortality Database, from the University of California at Berkeley. At present the database contains survival data for a collection of 28 countries, obtained using a uniform method for calculating such data. The database is limited to countries where death and census data are virtually complete, which means that the countries included are relatively developed.

We focus our analysis on period life expectancies. These life expectancies are calculated using the age-specific mortality rates for a given year, with no allowance for future changes in mortality rates. For example, period life expectancy at age 65 in 2006 would be calculated using the mortality rate for age 65 in 2006, for age 66 in 2006, for age 67 in 2006, and so on. Period life expectancies are a useful measure of mortality rates actually experienced over a given period and, for past years, provide an objective means of comparison of the trends in mortality over time. Official life tables are generally period life tables for these reasons. It is important to note that period life tables are sometimes mistakenly interpreted by users as allowing for subsequent mortality changes. This is an important issue when we analyze, in the context of our model, the welfare costs associated with an underestimation of future mortality improvements.

We focus our analysis on life expectancy at ages 30 and 65. Over the years there have been very significant increases in life expectancy at younger ages. For example, in 1960 the probability that a male newborn would die before his first birthday was as high as 3 percent, whereas in 2000 that probability was only 0.8 percent. In England, and in 1850, the life expectancy for a male newborn was 42 years, but by 1960 the life expectancy for the same individual had increased to 69 years. Our focus on life expectancy at ages 30 and 65 is due to the fact that we are interested on the relation between longevity risk and saving for retirement. Furthermore, the increases in life expectancy that have occurred during the last few decades have been due to increases in life expectancy in old age. This is illustrated in Figure 1, which plots life expectancy for a male individual for the United States and England over time, at birth, age 30, and at age 65.

Table 1 reports average annual increases in life expectancy for a 65 year old male for selected
countries included in the database and for different time periods. It is important to note that the sample period available differs across countries. This table shows that there have been large increases in life expectancy since 1970, and these have not been confined to the US or England. Furthermore, except for Japan, there does not seem to be evidence that the increases in life are becoming smaller over time: in the US, and in the 1970s, the average annual increase was 0.13 years, whereas in the 1990s it was 0.11 years. In England, the corresponding values are 0.07 and 0.15. These increases in life expectancy have been attributed to changes in lifestyle, smoking habits, diet, and improvements in health care, including the discovery of new drugs.

Figure 2 shows the conditional probability of death for the a male US individual, for different years, and for ages 30 to 110. This figure shows, for each age, the probability that the individual will die before his next birthday. As it can be seen from this figure the probability of death has decreased substantially from 1970 to 2000, mainly after ages 65. This confirms the results in figure 1, that the increases in life expectancy that have occurred over the past few decades have been due to decreases in mortality at old age.

The uncertainty as to future increases in life expectancy in old age is reflected in the projected life expectancies for a 65 year old UK male individual released by the Government Actuary’s Department (GAD) in October 2005, and shown in Figure 3. Importantly, this figure plots cohort life expectancies, and not period life expectancies. Cohort life expectancies are calculated using age-specific mortality rates which allow for known or projected changes in mortality in future years. If mortality rates at a given age and above are projected to decrease in future years, the cohort life expectancy at that age will be greater than the period life expectancy at the same age.

This figure plots both a principal projection, and a high and low variants, which allow for high and low increases in life expectancy, respectively. In the low variant, the future increases in life expectancy are assumed to go to zero. For the cohort life expectancies, which capture the uncertainty in future increases in life expectancy, and by year 2037, the high variant is 26.7 years whereas the low variant is considerable lower and equal to 19 years. We will use these projections in order to parameterize the model.
3 A Model of Longevity Risk

3.1 Survival Probabilities

We solve a life-cycle model of consumption and savings, similar to Carroll (1997) and Gourinchas and Parker (2001), but in which survival probabilities are stochastic. We let \( t \) denote age, and assume that the individual lives for a maximum of \( T \) periods. Obviously, \( T \) can be made sufficiently large, to allow for increases in life expectancy in very old age. We use the Lee-Carter (1992) model to describe survival probabilities. This is the leading statistical mortality model in the demographic literature, and it has been shown to fit the data relatively well. In addition, it has the advantage of being a relatively simple model. Mortality rates are given by:

\[
\ln(m_{t,x}) = a_t + b_t \times k_x
\]  

where \( m_{t,x} \) is the death rate for age \( t \) in period \( x \). The \( a_t \) coefficients describe the average shape of the \( \ln(m_{t,x}) \) surface over time. The \( b_t \) coefficients tell us which rates decline rapidly and which rates decline slowly in response to changes in the index \( k_x \). The \( b_t \) are normalized to sum to one, so that they are a relative measure. The index \( k_x \) describes the general changes in mortality over time. If \( k_x \) falls then mortality rates decline, and if \( k_x \) rises then mortality worsens. When \( k_x \) is linear in time, mortality at each age changes at its own constant exponential rate.

Lee and Carter (1992) show that a random walk with drift describes the evolution of \( k_x \) over time well. That is:

\[
k_x = \mu^k + k_{x-1} + \epsilon^k_x
\]  

where \( \mu^k \) is the drift parameter and \( \epsilon^k_x \) is normally distributed with mean zero and standard deviation \( \sigma^k \). This model can be used to make stochastic mortality projections. The drift parameter \( \mu^k \) captures the average annual change in \( k \), and drives the forecasts of long-run change in mortality. A negative drift parameter indicates an improvement in mortality over time.
3.2 Preferences

Let $p_t$ denote the probability that the individual is alive at age $t+1$, conditional on being alive at age $t$, so that $p_t = 1 - m_t$. For a given individual age and time are perfectly co-linear, so that in order to simplify the exposition from now on we include only age indices. We assume that the individual’s preferences are described by the time-separable power utility function:

$$E_1 \sum_{t=1}^{T} \delta^{t-1} \left( \prod_{j=0}^{t-2} p_{j} \right) \left\{ p_{t-1} \frac{C_t^{1-\theta}}{1-\theta} + b (1 - p_{t-1}) \frac{D_t^{1-\theta}}{1-\theta} \right\},$$

where $\delta$ is the discount factor, $C_t$ is the level of age/date $t$ consumption, $\theta$ is the coefficient of relative risk aversion, and $D_t$ is the amount of wealth the individual bequeaths to his descendants at death. The parameter $b$ controls the intensity of the bequest motive.

3.3 Labor Income

During working life age-$t$ labor income, $Y_t$, is exogenously given by:

$$\log(Y_t) = f(t, Z_t) + v_t + \varepsilon_t \text{ for } t \leq K,$$

where $f(t, Z_t)$ is a deterministic function of age and of a vector of other individual characteristics, $Z_t$, $\varepsilon_t$ is an idiosyncratic temporary shock distributed as $N(0, \sigma^2_\varepsilon)$, and $v_t$ is a permanent income shock, with $v_t = v_{t-1} + u_t$, where $u_t$ is distributed as $N(0, \sigma^2_u)$ and is uncorrelated with $\varepsilon_t$. Thus before retirement, log income is the sum of a deterministic component that can be calibrated to capture the hump shape of earnings over the life cycle, and two random components, one transitory and one persistent.

The individual retires at age $t_R$, and after this age income is modeled as a constant fraction $\lambda$ of permanent labor income in the last working-year:

$$\log(Y_t) = \log(\lambda) + f(K, Z_K) + v_K \text{ for } t > K,$$

The parameter $\lambda$ measures the extent to which the individual has defined benefit pensions, which implicitly provide insurance against longevity risk. In the current version of our model we
assume away labor supply flexibility, but we plan to consider this possibility later. Retiring later in life may be an additional natural mechanism to insure against increases in life expectancy.

### 3.4 The Optimization Problem

In the first version of the model we assume that there is a riskless asset in which the individual can invest with interest rate $R$, but in later sections we will expand the set of available financial assets. In each period the timing of the events is as follows. The individual starts the period with wealth $W_t$. Then labor income and the shock to survival probabilities are realized. Following Deaton (1991) we denote cash-on-hand in period $t$ by $X_t = W_t + Y_t$. We will also refer to $X_t$ as wealth: it is understood that this includes labor income earned in period $t$. Then the individual must decide how much to consume, $C_t$. The wealth in the next period is then given by the budget constraint:

$$W_{t+1} = (1 + R)(W_t + Y_t - C_t). \tag{6}$$

The problem the investor faces is to maximize utility subject to the constraints. The control variable is consumption/savings. The state variables are age, cash-on-hand, and the current survival probabilities. In our setup the value function is homogeneous with respect to permanent labor income, which therefore is not a state variable.

### 4 Calibration

#### 4.1 Time and preference parameters

The initial age in our model is 30, and the individual lives up to a maximum of 110 years of age. That is $T$ is equal to 110. Retirement age, $K$, is set equal to 65, which is the typical retirement age. We assume a discount factor, $\delta$, equal to 0.98, and a coefficient of relative risk aversion, $\theta$, equal to three. In the baseline model we assume that there is no bequest motive.
4.2 Survival probabilities

Undoubtedly, the calibration of the parameters for the mortality process are likely to be the most controversial. This is in itself a sign that there is a great deal of uncertainty with respect to what one can reasonably expect about future increases in life expectancy. In order to parameterize the stochastic process for survival probabilities we do two things. First, we estimate the parameters of the Lee-Carter model using historical data. Second, we try to determine which are the parameters of such model that match the projected increases in life expectancy shown in Figure 3, made by the UK government actuaries department. The latter projections are forward looking measures that reflect historical data, other information, and expectations of future improvements in mortality.

For the estimation of the Lee-Carter model using historical data, we use US data from 1959 to 2002, which is the data period available in the Human Mortality Database, and estimate:

\[ \ln(m_{t,x}) = a_t + b_t \times k_x + \varepsilon_{t,x} \]  

where \( \varepsilon_{t,x} \) is an error term with mean zero and variance \( \sigma^2_\varepsilon \), which reflects particular age-specific historical influences not captured by the model. This model is undetermined: \( k_x \) is determined only to a linear transformation, \( b_t \) is determined only up to a multiplicative constant, and \( a_t \) is determined only up to an additive constant. Following Lee and Carter (1992) we normalize the \( b_t \) to sum to unity and the \( k_x \) to sum to zero, which implies that the \( a_{t,x} \) are the simple averages over time of the \( \ln(m_{t,x}) \). This model cannot be fit by ordinary regression methods, because there are no given regressors. On the right side of the equation there are only parameters to be estimated and the unknown index \( k_x \). We apply the singular value decomposition method to the logarithms of the mortality rates after the averages over time of the log age-specific rates have been subtracted to find a least squares solution.

Figure 4 shows the actual and estimated mortality rates for two different years, namely 1959 and 2002, the first and the last year in our sample. From this figure we see that the model fits the data relatively well. In Figure 5 we plot the evolution over time of the \( k_x \) parameter. This figure confirms, from a different perspective, the data shown in Table 1. The parameter \( k_x \) was relatively constant during the 1960’s, a period during which there were not mortality
improvements. However, after 1970 there have been a decrease in $k_x$ reflecting the significant decreases in mortality rates that have taken place since then.

We use the time series data of $k_x$ to estimate the parameters of the random walk. The estimated drift parameter $\mu^k$ is $-0.7095$ and the standard deviation of the shocks $\sigma^k$ is $1.299$ (both of these parameters are reported in Table 2). In order to understand what such estimated parameters imply in terms of future improvements in life expectancy, we use them to make projections. More precisely, we ask the following question: consider an individual who is 30 years old in 2002 (the starting age in our model and the latest year for which we have data from HMD database respectively). How does life expectancy at age 65 in 2002 compare to life expectancy at age 65 when the individual reaches such age (i.e. in year 2037)? In other words, we ask which is the increase in the life expectancy of a 65 year old individual that the model forecasts to take place over the next 35 years.

Obviously, such increase will be stochastic as it will depend on the realization of the shocks to survival probabilities that will take place over the next 35 years. Therefore, in Table 3 we report increases/decreases in life expectancy for several percentiles of the distribution of the shocks to life expectancy (10, 25, 50, 75, and 90). The first row of Table 3 shows that for the values of $\mu^k$ and $\sigma^k$ estimated using historical data, the median increase in the life-expectancy of a 65 year old over a such a period is 2.57 years. The 10th and 90th percentiles are 1.51 and 3.59 years, respectively.

The calculations in the first row of the second panel of Table 3 are based on a statistical model that extrapolates for the future based on the history of mortality improvements that has taken place between 1959 and 2002, without conditioning on any other information.

We carry out a second calibration exercise in which we choose parameters for the Lee-Carter model such that when simulate the model we are able to generate increases in life expectancy that roughly match the forward looking projections of the UK government actuaries department shown in Figure 3. At the bottom of table 3, and based on the same data that we have used in figure 3, we report the UK GAD projected increases in life expectancy of a 65 year old over a 35 year period (from 2002 to 2037). We report the projected increase for the low, principal, and high variants.
These principal, high and low variant projections are not carried out in the context of a model, but they agreed upon by the government actuaries. They are calculated by assuming annual rates of mortality improvement of 1, 0.5 and zero percent at all ages for the high, principal and low life-expectancy variants, respectively. Therefore, the GAD does not assign probabilities to these different variants. They report that “these (high and low variants) are intended as plausible alternative assumptions and do not represent lower and upper limits.” (GAD report no. 8, page 28)

It is not clear what “plausible alternative assumptions” means exactly, but we think that it is reasonable, and conservative, to compare the high and low variant projections to the 10th and to the 90th percentiles of the distribution of improvements in life expectancy that is generated by the model. We argue that this is a conservative assumption because this means that the probability of life improvements predicted by the model being higher (lower) than those predicted by the GAD under the high (low) variant (which are described as “plausible alternatives”) is only 10%.

Comparing the mortality improvements projected using the estimated historical parameters of the Lee-Carter model (shown in the first row of Table 3), to the projections of the GAD we see that the former lead to lower median mortality improvements, and to lower dispersion in the forecasts. This suggests that the historically estimated parameters of the Lee-Carter model do not reflect the forward-looking views of the GAD. To motivate this dispersion the GAD reports that “it could be argued that uncertainty over long-term mortality levels is higher than ever given current research in areas such as mapping the human genome and gene therapy.” ((GAD report no. 8, page 25)

We have experimented with different drift and volatility parameters for the Lee-Carter model, and projected simulated mortality improvements under such alternative parameterizations. In the second row of table 3 we report values for the drift and volatility parameters that generate mortality improvements that roughly match those projected by the GAD. These parameters involve a smaller (more negative) drift parameter and a much higher volatility than those.

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7The projections made by the GAD are for the UK, whereas we have estimated the Lee-Carter model for the US, but Deaton and Paxson have shown that the UK and the US have similar histories of mortality improvements.
estimated from historical data. Thus the GAD projections entail higher risk and higher average rates of mortality improvement than those that have taken place between 1959 and 2002. We solve our model both for the parameters estimated using historical data and for this alternative parameterization based on GAD projections.

Figure 6 plots the probability distribution of life expectancy at age 65 predicted by the model, for both the historical estimated parameters of the Lee-Carter model and for the parameters chosen based on the GAD projections.

4.3 Labor Income and Asset Parameters

To calibrate the labor income process, we use the parameters estimated by Cocco, Gomes and Maenhout (2005) for individuals with a high school degree, which are also reported in Table 2. Deaton and Paxson (2001) have investigated the correlation between aggregate labor income and mortality improvements using UK and US data and concluded that the two aggregate series do not seem to be correlated. Therefore we assume zero correlation between labor income shocks and shocks to longevity. We assume that the real interest rate is equal to 1.5 percent.

One important possibility that we also consider, is that the retirement replacement ratio is correlated with improvements in life expectancy. This is motivated by recent events: the large improvements in life expectancy that have occurred over the last decades have led governments to reduce the benefits of pay as you go state pensions. Therefore we will consider a parameterization in which the replacement ratio is reduced when there is an improvement in life expectancy, such that the present value of the retirement benefits that the individual receives is unchanged. Figure 7 shows the probability distribution that we obtain for the retirement replacement ratio, at age 65, for both the historical parameters and for those based on GAD projections.
4.4 Solution Technique

The model was solved using backward induction. In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards.

To avoid numerical convergence problems and in particular the danger of choosing local optima we optimized over the space of the decision variables using standard grid search. The sets of admissible values for the decision variables were discretized using equally spaced grids. The state-space was also discretized and, following Tauchen and Hussey (1991), approximated the density function for labor income shocks using Gaussian quadrature methods, to perform the necessary numerical integration.

In order to evaluate the value function corresponding to values of cash-on-hand that do not lie in the chosen grid we used a cubic spline interpolation in the log of the state variable. This interpolation has the advantage of being continuously differentiable and having a non-zero third derivative, thus preserving the prudence feature of the utility function. The support for labor income realizations is bounded away from zero due to the quadrature approximation. Given this and the non-negativity constraint on savings, the lower bound on the grid for cash-on-hand is also strictly positive and hence the value function at each grid point is also bounded below. This fact makes the spline interpolation work well given a sufficiently fine discretization of the state-space.
5 Results

5.1 Life-Cycle Profiles

We use the optimal policy functions to simulated the consumption and savings profiles of thirty thousand agents over the life-cycle. In Figure 8 we plot the average simulated income, wealth and consumption profiles. We see that households are liquidity constrained during, roughly, the first five years. Consumption tracks income very closely and a small level of savings is accumulated to use as insurance-cushion against negative labor income shock. As labor income increases, and its profile becomes less steep, agents start accumulating wealth for retirement. The consumption profile ceases to be increasing as agents get older, reflecting the fact that the liquidity constraint becomes less binding. Finally, during retirement effective impatience increases due to mortality risk and the consumption path slopes down, while wealth is depleted at a fast rate. The standard hump-shaped consumption profile emerges.

In order to better understand the effects of longevity improvements on individual choices we simulate the profiles for two different individuals who face exactly the same labor income shocks over the life-cycle, but differ in terms of the shocks to life expectancy. For one agent the realization of the survival probabilities shocks is negative in each period, so that in each period there is an improvement in life expectancy (this is the agent labeled as ‘increase’ in Figure 9). For the other agent we set the realizations of shocks to survival probabilities as positive in each period.

In figure 9 we see that the agent who faces an improvement in life expectancy decides, from age 40 onwards, to save more than the individual who faces no improvement. Just before retirement there is a large difference in accumulated savings between the two. Thus, the optimal response of agents to an increase in life-expectancy is to save more in order to self-insure against the fact that they expect to live longer, and need to finance more retirement consumption out of accumulated savings. Because longevity risk is realized slowly over the life-cycle, the agent can, through his consumption and saving decisions self insure against these shocks. In the next sections we try quantify both the extent to which she is able to insure herself completely or not, and the total costs of obtaining insurance and of not being able to fully insure.
5.2 The costs of underestimating future mortality improvements

When agents are at each point in time aware of the increases in longevity, and rationally take into account any expected future increases, they can through their consumption and saving decisions partially self-insure against longevity risk. This is in spite of the fact that the agent in our model does not have at his disposal financial assets or securities that would allow him to hedge such risk.

We now investigate the welfare losses suffered by an agent who uses official period life tables to make his consumption and saving decisions, without allowing for future mortality improvements. That is, we investigate the consumption and saving choices of an agent who in each period looks up mortality rates in official period life tables, and in this way is informed about survival rates, but who fails to recognize that such survival rates are likely to improve in the future. This is a common mistake made by users of life tables, who think these official life tables allow for future mortality improvements. In terms of our model this means that the agent thinks that $\mu_k$ is zero when making his consumption/saving decisions.

We calculate welfare losses under the form of consumption equivalent variations. That is for each scenario (with and without mistakes) we compute the constant consumption stream that makes the individual as well-off in expected utility terms as the consumption stream that can be financed by his decision. Relative utility losses are then obtained by measuring the percentage difference in these equivalent consumption streams.

The results for different parameterizations are shown in Table 4. The first row of table 4 shows the welfare losses associated with such mistakes for the parameters for the $k_x$ process calibrated to the projections of the GAD. It shows that, as of age 30, the individual looses the equivalent of 0.028 percent of his annual consumption for not recognizing future mortality improvements in his decisions. The reason for such relatively low welfare loss is in part due to the fact that we are calculating welfare losses as of age 30, and the losses associated with an underestimation of future mortality improvements are incurred late in life, and are small when discounted to age 30. Therefore, in the following column of table 4 we calculate welfare losses as of age 65. We see that as of this age the agent looses 1.30 percent of his annual consumption for having failed to recognize future mortality improvements.
This reason for this relatively large welfare loss as of age 65 is due to the fact that the agent by failing to anticipate future mortality improvements saves significantly less. The last column of Table 4 reports the percentage difference in financial wealth accumulated at age 65 between agents who correctly anticipate future improvements in life expectancy, and those who fail to do so. This percentage difference is as high as 8.78 percent.

In the second row of Table 4 we consider the case of an agent who is informed about the current survival probabilities (from period life tables), starts his life thinking that $\mu^k$ is zero, but who updates this value based on what has happened during his life. More precisely, we assume that agent thinks that the value of $\mu^k$ is given by:

$$\mu_{t,Agent}^k = \omega_t \mu_{t-1,Agent}^k + (1 - \omega_t) \sum_{j=1}^{t} \frac{\mu_j^k}{t}$$

Thus in each period $t$, the expected value of $\mu_{t,Agent}^k$ is a weighted average of the initial prior probability and the observed average increase in life expectancy during his life. We set $\omega_t = t/100$, so that each annual observation has weight of one percent.

The motivation for this scenario is to model the situation of an individual who looks at the past one hundred years of data, and concludes that the probability of an improvement in life expectancy in old age is relatively small. As time passes, and there are large improvements to life expectancy, the agent revises his initial probability assessment. The second row of Table 4 shows the welfare results. The losses are still very close to the ones previously obtained, even though the agent learns over time the value of the probability of an increase in life expectancy.

The second panel of Table 4 also shows the welfare losses associated with mistakes for the values of $\mu^k$ and $\sigma^k$ estimated from historical data. As expected, the welfare losses are smaller than for the parameters that better approximate the GAD projections, but they are still significant, particularly as of retirement age. Obviously, there is a mapping between welfare losses and financial savings as of retirement age: the larger are welfare losses, the larger the percentage difference in such savings.
5.3 Comparative statics

In the recent years there has been a trend away from defined benefit pensions, and towards pensions that are defined contribution in nature. Faced with the severe projected underfunding of pay as you go state pensions systems, many governments have reduced the level of benefits of such schemes, or are planning to do so. In addition, many companies have closed their defined benefit schemes to new employees. This means that in the future the level of benefits that individuals will derive from defined benefit schemes are likely to be smaller than the one that we have estimated using historical data. This is important since defined benefit pension plans, because of their nature, provide insurance against longevity risk. Thus one might argue that for a high level of defined benefit pensions longevity risk does not matter much, but given the expected reductions in the level of such benefits, longevity risk will become more important.

We use our model to investigate the extent to which that is likely to be the case. More precisely, we carry out two different exercises. In the first we decrease the replacement ratio from the baseline value of 0.68 to 0.50, and investigate the effects of such lower replacement ration on welfare. The results are shown in table 4. The welfare costs associated with the mistakes are considerably higher.

In the second, and probably more realistic scenario, we allow for negative correlation between the replacement ratio and and longevity shocks. In this case, when there are improvements in mortality rates the level of retirement benefits is decreased. This is motivated by recent events: the providers of defined benefit pension plans have in recent years reduced the benefits that are to be paid out, also through and increase in the retirement age, as a response to the large increases in life expectancy that have occured over the last decades. When the replacement ratio is correlated with longevity shocks the welfare losses of underestimating future mortality improvements are very substantial, and as high as 3.51 percent of annual consumption as of age 65, reflecting the fact that the agent has saved 15 percent less than if the mortality improvements had been correctly forecast.

A final comparative statics exercise is to allow for a bequest motive, with the parameter $b$ set equal to one. From table 4 we see that as of age 30 the welfare losses associated with mistakes regarding the rate of mortality improvement are lower. This suggests that individuals with a
bequest motive may be in a better position to provide insurance against longevity risk to those who do not have a bequest motive.

6 Financial Innovation: Longevity Bonds

We now extend the baseline model by introducing an additional financial asset whose returns are correlated with shocks to life expectancy in addition to the risk-free asset.

6.1 Asset Markets

We augment the set of financial assets, and allow the agent in our model to invest in longevity bonds, whose returns are perfectly negatively correlated with the mortality rate innovations, thus providing the investor with a perfect hedge against this risk. In reality the returns to longevity bonds are correlated with aggregate mortality rates, but there is not a perfect (or even linear) relationship between the two. We discuss this issue in more detail below. More precisely, the return on the longevity bond ($R^L_t$) follows a two-state process, and is given by:

$$R^L_t = \mu^L + \frac{\sigma^L}{\sigma^k}\varepsilon^k_t$$

where $\mu^L$ and $\sigma^L$ correspond to the mean and standard deviation, respectively.

Returns on longevity bonds are linked to an aggregate mortality index. The bonds issued by Swiss Re. pay a quarterly fixed coupon equal to 3-month US dollar LIBOR plus 135 basis points. The principal is then repaid in full if the mortality index does not exceed a given threshold, but the payment decreases linearly with the index if that threshold is reached. The EIB bond would have floating coupons that were directly linked to a mortality index. Unfortunately, since these bonds are a fairly recent financial development, it is hard to estimate their risk premia and volatility empirically. Using mortality rate forecasts, also based on the Lee-Carter model (Lee and Carter (1992)), Friedberg and Webb (2005) compute the hypothetical returns for the EIB longevity bond, if the issue had indeed occurred. They obtain a return volatility of 3%, which we use as the baseline value in our calibration ($\sigma^L = 3\%$). They also find that the returns are
negatively correlated with aggregate consumption and, in a CCAPM framework with relative risk aversion of 10, the implied risk premium would be $-0.02\%$. Therefore, we set our baseline risk premium to zero ($\mu^L = 1.5\%$), but will also consider alternative values.

### 6.2 A 3-period model with longevity bonds

We start by presenting a 3-period model which is better suited to understand the intuition behind our results, and the determinants of the demand for longevity bonds.

#### 6.2.1 Model set-up

The agent’s objective is

$$MaxE[U] = E \left[ \frac{C_1^{1-\theta}}{1-\theta} + p_1 \delta \frac{C_2^{1-\theta}}{1-\theta} + p_1 p_2 \delta \frac{C_3^{1-\theta}}{1-\theta} \right]$$

where $p_1$ and $p_2$ denote the conditional survival probabilities, as before.

In period 1 she consumes ($C_1$) and allocates her savings between riskless bonds (as before) and longevity bonds:

$$W_2 = (W_1 - C_1)[\alpha(1 + R^L_2) + (1 - \alpha)(1 + R)] + Y_2$$

where $\alpha$ denotes the fraction of wealth invested in longevity bonds. Her decisions are a function of the current level of wealth ($W_1$), expected future labor income ($Y_2$ and $Y_3$), and expected future survival probabilities ($p_1$ and $p_2$). Although she already knows $p_1$, she does not yet know $p_2$ for sure. Longevity bonds allow her to insure against this uncertainty. In period 2 she again chooses her optimal consumption ($C_2$), but now based on the exact realization of the survival probability $p_2$, while in period 3 she simply consumes all available wealth ($C_3 = W_3$). We are interested in understand the behavior of $\alpha$ for different assumptions about labor income, which will mimic the different stages of the life cycle.
6.2.2 Demand for longevity bonds

In Figure 10 we plot the optimal share invested in longevity bonds when labor income is riskless, $Y_2 = Y_3 (= 10)$, and the bonds earn a zero risk premium. This figure captures some features of the retirement period in our life-cycle model. Intuitively, the policy function is decreasing in financial wealth. With constant relative risk aversion households always want to insure a constant fraction of their total wealth. As financial wealth increases, relative to their future labor income, this is achieved by decreasing the fraction invested in longevity bonds. It is important to note that the optimal fraction is not 100%. Longevity bonds are not riskless annuities. When agents invest in longevity bonds they know these will yield a low return if $p_2$ happens to be lower than expected. From the perspective of period 1, this implies that the agent will anticipate having less total resources in period 2 in such scenario. Since she cannot off-set this by borrowing against her future labor income then she must trade-off the increase in consumption risk in period 2, versus the decrease in conditional consumption risk in period 3.

This trade-off is clearly illustrated in Figure 11, where we also plot the policy function for case of $Y_3 = 2 \times Y_2$, for comparison. In this second case the investor is much less willing to transfer additional resources to period 3, and thus the demand for longevity bonds decreases significantly, except for very high values of wealth.\footnote{The share invested in longevity bonds is higher for very high values of wealth, because there is a lower level of savings (and thus a lower ratio of financial wealth to future labor income).} Riskless bonds allow the agent to transfer the desired level of wealth from period 1 to period 2 without risk. Investing in longevity bonds would introduce risk. The compensation for this risk, which is potential for higher savings for period 3 if $p_2$ happens to be higher than expected, is almost worthless since the agent doesn’t want to save for period 3 anyway ($Y_3$ is much higher than $Y_2$). These policy functions will allow us to explain the behavior of the portfolio allocation during retirement.

In our third experiment we try to capture the behavior during working life and thus consider a case with labor income risk ($Y_2$ is either equal to 1 or 19, with equal probabilities). The results are shown in Figure 12. For very low levels of cash-on-hand there is no demand for longevity bonds: labor income risk is much more important. From the perspective of hedging this risk,
riskless bonds clearly dominate. Only as wealth increases does the household start to invest in longevity bonds.

6.3 Life-Cycle Model

We now introduce the longevity bond in the life-cycle model discussed in section 3.

6.3.1 Baseline model

Figure 13 plots the share of wealth invested in the longevity bond over the life cycle. As expected from our discussion of the 3-period model, early in life the demand for this asset is crowded-out by labor income risk. Young households haven’t accumulated much wealth and therefore they are significantly exposed to labor income shocks. Naturally the riskless asset is better suited to hedge this risk, since it has no volatility. However, as households accumulate more wealth and start saving for retirement, the allocation to longevity bonds quickly converges to 100%. This exposure decreases again as they approach retirement since wealth accumulation is increasing rapidly and, as previously discussed, the portfolio rule is decreasing in wealth (see figure 12).

During retirement the shape of the allocation to longevity bonds first increases, then decreases and finally increases again. The intuition for the first two effects is explained by the second policy function in figure 11 (with $Y_3 = 2Y_2$). Initially, households have a significant amount of accumulated wealth and therefore they are (mostly) on the decreasing part of the policy function. As they get older, financial wealth decreases at a faster rate than future labor income, and therefore the portfolio allocation to longevity bonds is increasing in age. Eventually, however, as wealth is significantly reduced and since the discount rate for future consumption is very high, households effectively become liquidity constrained. They would like to be able borrow against their future retirement transfers but are unable to do so. Therefore, any significant transfer of resources for future years is suboptimal: they are now in the increasing part of the policy function. As a result the optimal portfolio allocation is now decreasing in age: as wealth keeps decreasing they become more and more constrained over time. To explain the final increasing
pattern, during the last years of retirement, before financial wealth is completely depleted, we now need to consider both policy functions in figure 11. At this stage of the life cycle, the ratio of the agent’s illiquid future labor income versus her next-period’s income is decreasing fast which, in the language of the 3-period model, is equivalent to $Y_3$ becoming closer to $Y_2$. Therefore, the policy function quickly converges to the one with $Y_3 = Y_2$, and the increasing pattern for low levels of wealth almost disappears.

### 6.3.2 Alternative return calibrations

As previously discussed, the empirical evidence on the returns of longevity bonds is very limited. Therefore, in this section, we consider alternative values for both its expected return and volatility. Figure 14 plots the optimal portfolio share invested in longevity bonds for three different calibrations of its expected return ($\mu^L$): our baseline value, 1.5% and two cases with a small negative risk premium, 1.475% and 1.45%. The choice of these exact alternative numbers is motivated by the results. Naturally, as we decrease their expected return, the demand for longevity bonds falls. Interestingly, the results in Figure 14 show that a 2.5 basis points reduction is enough to decrease the demand by more than half, and a 5 basis points negative risk premium is enough to deter almost all investors from buying those bonds.

Next we increase the volatility of longevity bond’s return ($\sigma^L$) to 10%. The results are shown in Figure 15. Since the returns are perfectly correlated with the mortality shocks, a higher volatility effectively increases the investor’s hedging position for a given portfolio allocation. Therefore, when we increase $\sigma^L$, the share invested in longevity bonds is naturally lower at every age. In a frictionless world this would be a simple re-scaling effect, and nothing else should change. Therefore, for comparison purposes, we also plot in Figure 16 the scaled down baseline portfolio allocation. Comparing this with the actual allocation for the $\sigma^L = 10\%$ case, we find that there is something else going on in our model. Since the investor is facing short-selling constraints this result no longer holds. The higher volatility case allows her to achieve levels of hedging that were previously unfeasible. Therefore, we do not have a simple proportional shift. When the short-selling constraints were binding in the baseline case, we observe a much smaller difference between the two optimal allocations. This shows that, under the previous
calibration, the investor would have liked to be able to invest more than 100% in the longevity bonds at those ages. We could have increased $\sigma^L$ even further, but this would not have made any meaningful difference, since the short-selling constraints are no longer binding for almost any agent in the simulations.

Interestingly, early in life we have the opposite result, as the portfolio allocation for the $\sigma^L = 10\%$ case is actually slightly lower than the simple scaled-down version of $\sigma^L = 3\%$ case. This, is due to the borrowing constraints and labor income risk. As previously discussed, in this stage of the life cycle, households are less willing to buy longevity bonds because they constitute a poor hedge against labor income risk. As we increase their return volatility this result becomes stronger. We believe that these are important conclusions to keep in mind for the optimal design of longevity bonds. In general, if the correlation with aggregate mortality risk is kept constant, then it is better to develop bonds with significant return volatility. However, excessive levels of volatility might deter young households from buying this asset.

6.3.3 Welfare gains

Model with a fixed replacement ratio. In this section we compute the welfare gains from having access to the longevity bond. Table 5 reports the age-30 utility gains, measured in certainty equivalent consumption units. For our baseline case ($\mu^L = 1.5\%$ and $\sigma^L = 3\%$) we find very low welfare gains: 0.032% of annual consumption. On one hand, this could be seen as a striking result, based on the evidence in figure 13, which shows that the investor will optimally allocate a large fraction of her wealth to this asset. However, on the other hand, this is consistent with the evidence in figure 14, which shows that a small change in the bond’s expected return is enough to drive the demand close to zero. As also expected from the results in figure 15, when we compute the welfare gains with a negative risk premium of -0.025%, they are indeed almost identical to zero (0.005%). Households only invest a very small fraction of their wealth in longevity bonds and therefore, from their perspective, this asset is not worth very much. On the other hand, if we increase the volatility of longevity bond’s returns, the household is now better off: the welfare gain increases to 0.038%. As previously explained, due to the existence of short-selling constraints, a higher return volatility allows the investor
to increase its hedging position for a given portfolio allocation. In the previous section we have also found a negative effect for young households, those concerned about labor income risk, but these results show that, for these levels of volatility, the benefits are more important.

Model with a stochastic retirement replacement ratio. In this section we consider the version of the model with a stochastic retirement replacement ratio. More precisely, we now consider the likely impact of demographic changes (via changes in mortality rates) on households’ retirement income, as described in sections 4.3 and 5.3. Table 6 reports the corresponding welfare gains. In this case, the benefits from having access to longevity bonds are almost an order of magnitude higher than before. For the zero risk premium case we obtain increases of 0.15% to 0.24% of annual consumption, depending on the level of volatility (respectively 3% or 10%). In fact, in the previous case a negative risk premium of 5 basis points was enough to drive the demand for longevity bonds down to essentially zero, while now this is no longer the case, and there is still a welfare gain of 0.054%. Naturally, if we decrease the risk premium further, eventually there is no demand for longevity bonds and no welfare gain. In this case a negative risk premium of −15 basis points will deliver that result.

Model with mis-perceptions. We now consider the set-up of section 5.2. That is: we investigate the welfare gains for an agent who uses official period life tables to make his consumption and saving decisions, without allowing for future mortality improvements. More precisely, we consider an agent who in each period looks up mortality rates in official period life tables, understands that they are stochastic, but does not acknowledging that they are likely to improve in the future (i.e. assumes that the drift for $k_t$ is zero). As previously discussed, this is a common mistake made by users of life tables.

Table 7 reports the welfare gains from having access to longevity bonds in this case. It is important to mention that these welfare gains are computed using the objective probability measure, and thus reflect the real welfare gains that the typical investor will realize. Naturally, from the investor’s own ex-ante perspective, the welfare gains are still the ones reported in the previous subsection (Tables 5 and 6), since she thinks that her expectations about future survival probabilities are correct. The results in Table 7 show that the real welfare gains are
now very large and significantly higher than the previous ones. Even for the case with a fixed replacement ratio the welfare gain is now 1.20% of annual consumption, and if we take into account for the likely effects of the demographic changes on retirement income this number increases to 2.04%. It is important to note that these are welfare gains as of age 30.

This combination of results, very large real welfare gains and very small perceived ex-ante welfare gains, is extremely important. The results in the previous section show that household demand for longevity bonds is likely to be very small unless these assets earn a zero (or positive) risk premium. Since the very limited existing empirical evidence (Friedberg and Webb (2005)) suggests that they earn a negative risk premium, even though a very small one, such demand is indeed likely to be close to zero. However, we have now shown that, unless investors are actually able to make a good assessment of the expected future path of survival probabilities, the real gains from investing in those bonds are actually extremely large.

7 Conclusion and Future Research

The objective of this paper was two fold. First, to document that existing evidence on life expectancy. Second to solve a life cycle model with longevity risk, and investigate how much such risk affects the consumption and saving decisions, and the welfare of an individual saving for retirement.

We have found that when the agent is informed of the current survival probabilities and correctly anticipates the probability of a future increase in life expectancy, the agent can through his consumption and saving decisions self insure against longevity shocks. Since longevity risk is realized slowly over the life-cycle, the agent optimally saves more in response to an improvement in longevity. This is the case even if the agent does not have access to financial assets that allow him to insure against longevity risk.

However, we also show that when agents are informed about life expectancy, but make an incorrect assessment of the probability of future improvements in life expectancy, the effects of longevity risk on individual welfare can be substantial. This is a common mistake since official life tables usually are period life tables which do not allow for future mortality improvements.
We have shown that the welfare losses associated with such mistakes are particularly large in the context of declining payouts of defined benefit pensions, especially when such declining payouts are correlated with longevity improvements, as suggested by recent events. In this case the agents have substantial welfare gains from investing in longevity bonds.

There are several extensions that are worth exploring in future research. Perhaps the most important ones are to consider labor supply flexibility, and in particular endogeneize retirement age, and to allow the agent to invest in annuities.
References


Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany).


Table 1: Average annual increases in life expectancy in number of years for a 65 year old male

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>United States</th>
<th>Canada</th>
<th>England</th>
<th>Sweden</th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Sample</td>
<td>0.08</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>Pre 1970</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>1970 - 2000</td>
<td>0.11</td>
<td>0.09</td>
<td>0.12</td>
<td>0.09</td>
<td>0.13</td>
<td>0.13</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>1960 - 1969</td>
<td>-0.01</td>
<td>0.03</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.08</td>
<td>-0.03</td>
<td>-0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>1970 - 1979</td>
<td>0.13</td>
<td>0.09</td>
<td>0.07</td>
<td>0.04</td>
<td>0.12</td>
<td>0.14</td>
<td>0.08</td>
<td>0.22</td>
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<tr>
<td>1980 - 1989</td>
<td>0.08</td>
<td>0.08</td>
<td>0.12</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>1990 - 1999</td>
<td>0.11</td>
<td>0.11</td>
<td>0.15</td>
<td>0.11</td>
<td>0.14</td>
<td>0.11</td>
<td>0.12</td>
<td>0.08</td>
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</table>

Note to Table 1: This table shows average annual increases in life expectancy for a 65 year old male over time and for different countries. The data is from the Human Mortality Database.
Table 2: Parameters of the model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survival probabilities: Historical</td>
<td>$\mu_k$</td>
<td>-0.7095</td>
</tr>
<tr>
<td></td>
<td>$\sigma_k$</td>
<td>1.2992</td>
</tr>
<tr>
<td>Survival probabilities: GAD Projections</td>
<td>$\mu_k$</td>
<td>-1.1095</td>
</tr>
<tr>
<td></td>
<td>$\sigma_k$</td>
<td>3.8976</td>
</tr>
<tr>
<td>Time Parameters</td>
<td>$t_0$</td>
<td>30</td>
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<tr>
<td></td>
<td>$t_R$</td>
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<tr>
<td></td>
<td>$T$</td>
<td>110</td>
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<tr>
<td>Preference Parameters</td>
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<td></td>
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<tr>
<td></td>
<td>$b$</td>
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<td>Labor Income</td>
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<tr>
<td></td>
<td>$\sigma_u$</td>
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<tr>
<td></td>
<td>$\lambda$</td>
<td>0.68212</td>
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<tr>
<td>Asset Returns</td>
<td>$R_f$</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Note to Table 2: This table reports the parameters of the model.
Table 3: Increase in life expectancy at age 65 between 2002 and 2037 at different percentiles of the distribution and GAD projections

<table>
<thead>
<tr>
<th>Model</th>
<th>Percentile of the distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Historical: ( \sigma_k = 1.299, \mu_k = -0.7095 )</td>
<td>1.51</td>
</tr>
<tr>
<td>GAD Proj.: ( \sigma_k = 3.8976, \mu_k = -1.1095 )</td>
<td>0.58</td>
</tr>
<tr>
<td>GAD Projections</td>
<td>Low Var</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
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</table>

Note to table 3: This table shows the increases in life expectancy at age 65 predicted by the model over a 35 year period for different percentiles of the distribution and for different model parameters. The second part of the table shows the projected increase in cohort life expectancy at age 65 between 2002 and 2037 by the UK Government Actuary’s Department for the low, principal and high variants.
Table 4: Welfare Gains in The Form of Consumption Equivalent Variations (Percent)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Welfare loss At age 30</th>
<th>Wealth diff At age 65</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAD: $\sigma_k = 3.8976$, $\mu_k = -1.1095$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{Agent}^k = 0.0$</td>
<td>-0.028</td>
<td>-1.296</td>
</tr>
<tr>
<td>$\mu_{Agent}^k = 0.0$ with updating</td>
<td>-0.017</td>
<td>-0.974</td>
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<tr>
<td>$\mu_{Agent}^k = -0.7796$</td>
<td>-0.014</td>
<td>-0.942</td>
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<tr>
<td>$\mu_{Agent}^k = 0.7796$</td>
<td>-0.016</td>
<td>0.959</td>
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<tr>
<td>Replacement Ratio = 0.5</td>
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<tr>
<td>$\mu_{Agent}^k = 0.0$</td>
<td>-0.064</td>
<td>-1.934</td>
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<td>$\mu_{Agent}^k = 0.0$ with updating</td>
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<td>Bequest motive $b = 1$</td>
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<tr>
<td>$\mu_{Agent}^k = 0.0$</td>
<td>-0.024</td>
<td>-1.126</td>
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<tr>
<td>$\mu_{Agent}^k = 0.0$ with updating</td>
<td>-0.015</td>
<td>-0.833</td>
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<tr>
<td>Correlation: replacement ratio and longevity shocks</td>
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<td>$\mu_{Agent}^k = 0.0$</td>
<td>-0.184</td>
<td>-3.512</td>
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<tr>
<td>$\mu_{Agent}^k = 0.0$ with updating</td>
<td>-0.115</td>
<td>-2.587</td>
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<tr>
<td>Historical: $\sigma_k = 1.299$, $\mu_k = -0.7095$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{Agent}^k = 0.0$</td>
<td>-0.012</td>
<td>-0.740</td>
</tr>
<tr>
<td>$\mu_{Agent}^k = 0.0$ with updating</td>
<td>-0.008</td>
<td>-0.559</td>
</tr>
</tbody>
</table>

Note to table 4: This table reports the welfare gains at ages 30 and 65 for different parameters of the model. The last column reports the percentage difference in wealth accumulation at retirement age.
Table 5: Welfare gains from investing in the longevity bond (percent).

<table>
<thead>
<tr>
<th>$\mu_L$</th>
<th>1.5</th>
<th>1.5</th>
<th>1.475</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_L$</td>
<td>3</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$CE$</td>
<td>0.032</td>
<td>0.038</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Note to Table 5: The gains are measured as a percentage increase in annualized certainty equivalent (CE) consumption levels.

Table 6: Welfare gains from investing in the longevity bond with stochastic replacement ratio (percent).

<table>
<thead>
<tr>
<th>$\mu_L$</th>
<th>1.5</th>
<th>1.5</th>
<th>1.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_L$</td>
<td>3</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$CE$</td>
<td>0.151</td>
<td>0.242</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Note to Table 6: The gains are measured as a percentage increase in annualized certainty equivalent (CE) consumption levels.

Table 7: Welfare gains from investing in the longevity bond with mis-perceptions (percent).

<table>
<thead>
<tr>
<th></th>
<th>fixed $\lambda$</th>
<th>stochastic $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE$</td>
<td>1.201</td>
<td>2.042</td>
</tr>
</tbody>
</table>

Note to Table 7: The gains are measured as a percentage increase in annualized certainty equivalent (CE) consumption levels. The agent assumes that the expected drift in mortality rates is equal to zero. Results are shown for the case both with and without a stochastic replacement ratio (respectively fixed $\lambda$ and stochastic $\lambda$).
Figure 1: Life expectancy in the United States and England for a male individual at selected ages

Note to Figure 1: This figure shows period life expectancy over time and at selected ages (birth, age 30, and age 65) for the United States and for England. The data is from the Human Mortality Database. The data for the United States is from 1959 to 2002, and for England is from 1841 to 2003.
Figure 2: Conditional probability of death for a male US individual

Note to Figure 2: This figure shows the conditional probability of death over the life-cycle for selected years (1960, 1970, 1980, 1990, and 2000) and for a male United States and for England. The data is from the Human Mortality Database.
Figure 3: Projected cohort life expectancy for a 65 year old United Kingdom male individual

Note to Figure 3: This figure plots projected cohort life expectancy over time for a 65 year old male United Kingdom individual. This figure plots a principal, a high and a low variant. The projections were done by the UK government actuaries department and are available at www.gad.gov.uk.
Note to Figure 4: This figure shows the data and the estimated conditional probabilities of death for a United States male individual at selected years. The data used in the estimation is from the Human Mortality Database from 1959 to 2002. The estimation is done using the Lee-Carter model.
Figure 5: Estimated $k(x)$ parameter in the Lee-Carter model

Note to Figure 5: This figure shows the estimated $k(x)$ parameter in the Lee-Carter model. The data used in the estimation is for a male US individual, from the Human Mortality Database from 1959 to 2002.
Figure 6: Model implied age 65 life-expectancy for different parameters of the $k(x)$ stochastic process

Note to Figure 6: This figure plots the model implied life expectancy at age 65 for different parameters of the stochastic process for $k(t)$ and the probability that such life expectancy will occur.
Note to Figure 7: This figure plots the replacement ratio at age 65 for different parameters of the stochastic process for $k(x)$ and its probability.
Figure 8: Simulated Consumption, Income and Wealth in the Baseline Model

Note to Figure 8: This figure plots the simulated consumption, wealth, and income in the baseline model. The figure plots an average across 30,000 simulated profiles.
Figure 9: Simulated Consumption, Income and Wealth for two different individuals

Note to Figure 9: This figure plots the simulated consumption, wealth, and income in the baseline model for two different individuals who differ in the shocks to longevity.
Figure 13 - Portfolio Allocation in the Baseline Case

Figure 14 - Portfolio Allocation for Different Values of the Expected Return

Figure 15 - Portfolio Allocation for Different Values of Return Volatility