Practical DSGE models

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Introduction

• Policymakers somewhat dissatisfied with their models. Lack of flexibility? Lack of forecasting power? Lack of appropriate storytelling?


• Agent-based /new economic thinking models failed so far to deliver tractable frameworks to be used in policy institutions. What is the effect of a 25 basis point decline in the nominal interest rate?

• Future: Less structured models? HANKs?

• Alternative: "Practical DSGEs": use current models but more data for estimation and analysis, exploit institutional information, use time series blocks to account for missing features.
History of macro modeling


- Constructed from national account identities.
- Specify demand relationships with delay adjustments.
- No supply side, no capital stock, no expectations.
- Focus on multiplier effects of exogenous fiscal-monetary changes.

- Add supply side considerations (production function).

- Dynamics describe adjustments of today’s value to long run targets.

- Introduce the notions of gaps and of inflation dynamics.

- Introduce (adaptive) expectations; financial assets (Tobin’s Q, etc.)


• Construct the steady state from a static optimization problem; calibrate it to the data; add dynamics (reverse process relative to 2G).

• Equations functions of "deep parameters". Use policy rules.

• Focus analysis on the effects of shocks (rather than changes in exogenous variables) on gaps.


- Use dynamic optimization to develop decision rules. Use frictions to create slow adjustments to shocks.

- Sectorial. Open economy. Some heterogeneities (Calvo, ROT).

- General equilibrium (stock-flow consistency). Central role for Euler equation and Phillips curves.

- Long run and short fluctuations jointly considered.

- Tight corset. Good for story telling; difficult short run forecasting performance; no role for institutional knowledge or non-rational behavior.
5G: Large heterogeneity of views.


- Stripped down version of 4G models with non-structural or ad-hoc features tagged along: COMPASS (2013); MAJA (2019).

Practical DSGEs

• Core DSGE structure.
• Add data, non-core features, institutional information, etc. via measurement equations.
• Use large information set for estimation and forecasting (macro, micro, institutional, etc.)
• Flexibility; more precise parameter and latent variable estimates; smaller forecast uncertainty; more robust inference.
Basic idea

• DSGE core (4G Model):

\[
\begin{align*}
x_t &= A(\theta)x_{t-1} + B(\theta)e_t \\
y_t &= C(\theta)x_{t-1} + D(\theta)e_t
\end{align*}
\]

(1)

• There could be latent objects (potential output, NAIRU, R*) in \((x_t, y_t)\). Use \(z_t = S[x'_t, y'_t]' = F(\theta)z_{t-1} + G(\theta)e_t, n \times 1\) matrix; \(S=\) selection matrix in estimation.

• \(e_t\) could include permanent disturbances (generating balance growth) and transitory disturbances (generating deviation from balance growth).
• Measurement equations:

\[ w_t = G(\lambda)z_t + F(\psi, q_t) + u_t \]  \hspace{1cm} (2)

where \( w_t \) is a vector observables, which may include counterparts/proxy of model variables or additional variables not in the model, \( u_t \) is measurement error vector, \( F(\psi, q_t) \) is a non-DSGE (time series) component.

• (1)-(2) is an extended state space system.

• Jointly estimate \((\theta, \Sigma_e)\) and \((\psi, \lambda, \Sigma_u, \Sigma_q)\) with standard techniques.

• Can use size of \( \text{var}(F(\psi, q_t))/\text{var}(w_t) \) as model fit/comparison device across models with different DSGE features.
Advantages

• Core DSGE structure: can use standard narrative.

• Flexible overcoat: can be adjusted to the needs and the questions.

• Use as much information as available for parameter estimation; can examine dynamics to shocks, historical decompositions for variables not modeled in the DSGE (say, $w_{2t}$).

• Inference about latent variables (potential output, natural rates, etc.) uses model structure and all available information.
Potential disadvantages

- Overparameterization.

- Numerical difficulties (nasty objective functions, underidentification of parameters).

- Not distinguishing core vs. non-core for policy analysis/narratives. In a successful model: **core DSGE should explain large portion of observables fluctuations.**
What is in $w_t$?

- Standard macro variables.
- Proxies (IP for GDP), flash estimates of standard variables ($\pi_t, \Delta Y_t$).
- Multiple indicators of model variables (e.g. hours in JPT, 2013, AEJM).
- Sectoral data, foreign data, conjunctural data (Gelfer, 2019, RED).
- Nowcasts, forecasts, news, expectation, sentiment and survey data, uncertainty measures.
• Micro data : intermediate step before HANK.

• Can use mixed frequency data (e.g. Foroni and Marcellino, 2014, JOE).

  What is \( F(\psi, q_t) \)?

• Missing idiosyncratic trends (Canova, 2014, JME).

• Missing dynamics.
Proxies

- Let $z_t^j$ be hours and $z_t^j'$ be GDP. Only $w_{1t}^j$ employment and $w_{1t}^j'$ industrial production are available. Then:

$$w_{1t}^j = z_t^j + u_t^j$$

$$w_{1t}^j' = z_t^j' + u_t^j'$$

(3) Flash estimates

- Let $z_t^k$ be output growth and $w_{2t}^k$ be l=1,2,3...month(s) ahead flash estimates of quarterly output growth. Then:

$$w_{2t}^k = z_{t+l}^k + u_t^k$$

(4)
Multiple indicators

- Let $z_t^i$ = price inflation; $w_{3t}^i$ = CPI inflation, PPI inflation, GDP deflator inflation, etc.

- Measurement equations:

$$
\begin{bmatrix}
w_{3t}^1 \\
w_{3t}^2 \\
\vdots \\
w_{3t}^N
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
\lambda_2 \\
\vdots \\
\lambda_N
\end{bmatrix}
\begin{bmatrix}
z_t^i \\
u_t
\end{bmatrix}
$$

(5)

- $u_t$ vector could be serially correlated as long as $N$ is large. Could restrict the cross sectional variance of $u_t$.

- $\lambda_j$ interpreted as informational content of $w_{3t}^j$ relative to $w_{3t}^1$ (with $\lambda_1$ normalized to 1).
Other interesting variables

- $w_{4t}=$ housing, labor market, financial market; sectorial or foreign variables, commodity prices, forecasts, nowcasts, surveys, forward looking indicators (US: Light vehicle sales; price of tomatoes in Munich).

Measurement equations:

\begin{align}
    w_{4t} &= \Lambda x x_t + u_t \\
    &= \Lambda z z_t + u_t
\end{align}

- $\Lambda$ is a $M \times k$ matrix. Need careful choices of what is in $w_{4t}$ to avoid overparameterization.

- Need $u_t$ to be cross sectionally uncorrelated.
Micro data

• Same framework can be used to add information about micro distributions (consumption, wealth, income). Let \( w_{5t}^r \) the \( r = 1, \ldots, 10 \) decile of the distribution

• Measurement equations (linking data to the states):

\[
\begin{bmatrix}
w_{5t}^1 \\
w_{5t}^2 \\
\vdots \\
w_{5t}^{10}
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
\omega_2 \\
\vdots \\
\omega_{10}
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
x_t \\
\omega_{10}
\end{bmatrix} + u_t
\]

(8)
Mixed frequencies

• Let the DSGE be specified at monthly data, but have $w_{1t}$ at monthly data and $w_{2t}$ at quarterly data.

• Measurement equations (average data):

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} = \begin{bmatrix} 1/3(z_{2t}^m + z_{2t-1}^m + z_{2t-2}^m) \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

(9)

or (point in time data):

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} = \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

(10)

some $\bar{t}$. 
Missing trends

• Models typically have balanced growth, e.g., unit root in TFP or in investment specific disturbances or both. Data does not.
• If disregard missing trends or use filtered data, parameter estimates twisted (see Canova, 2014). Parametrize $F(\psi, q_t)$ (‘The bridge’), to capture missing features e.g. 

\[ q_t = \alpha_t + \rho_1 q_{t-1} + v_{1t} \quad (11) \]
\[ \alpha_t = \rho_2 \alpha_{t-1} + v_{2t} \quad (12) \]

where $v_{2t} \sim (0, \Sigma_{v2})$, $\Sigma_{v2} = \kappa \Sigma_{v1}$.

• $1 + \kappa$ related to $\lambda_{HP}$.

• Up to 2 unit roots or deterministic trends are possible. Can be used for selected variables.

• Possibility to generate missing peaks at certain frequencies if $\rho_1, \rho_2 \neq 1$.

• $F(\psi, q_t)$ corresponds to $1 - R^2$. If large, model is poor.
To be remembered

- The core model is the same. We only add information for estimation and policy analysis.

- If the core is poor, the analysis is likely to be poor.

- What is in $w_t$ an be adapted to the questions of interest:
  - Why inflation expectations and inflation diverge?
  - Why employment low during the latest recovery?
  - Why is the exchange rate weak?
Example: Gelfer, 2019

- Data-rich estimation and forecasting with SWFF. Add 40 sectoral variables in estimation.
- Posterior distributions.
- Impulse responses.
- Estimates of the state variables.
- Forecasts.
Illustration

- Use basic SW model to analyze drivers of the output gap. Four versions:

- Model quarterly; quarterly data on (ΔY, ΔC, ΔI, N, ΔW, CPIINF, FFR)

Measurement equations:

\[
\begin{align*}
\Delta Y_t &= 400 \times (y_t - y_{t-1}) + c_1 \\
\Delta C_t &= 400 \times (c_t - c_{t-1}) + c_2 \\
\Delta I_t &= 400 \times (i_t - i_{t-1}) + c_3 \\
H_t &= h_t + c_4 \\
\Delta W_t &= 400 \times (w_t - w_{t-1}) + c_5 \\
\Delta P_t &= inf_t + c_6 \\
FFR_t &= r + c_7
\end{align*}
\]

Note \(c_2 \neq c_2 \neq c_3 \neq c_5\).
• Model monthly, mixed frequencies: $CPIINF, FFR$ monthly.

Measurement equations:

\[
\begin{align*}
\Delta Y_t &= 400 \ast (y_t - y_{t-3}) + c_1 \\
\Delta C_t &= 400 \ast (c_t - c_{t-3}) + c_2 \\
\Delta I_t &= 400 \ast (i_t - i_{t-3}) + c_3 \\
H_t &= (h_t + h_{t-1} + h_{t-2}) + c_4 \\
\Delta W_t &= 400 \ast (w_t - w_{t-3}) + c_5 \\
\Delta P_t &= inf_t + c_6 \\
FFR_t &= r_t + c_7
\end{align*}
\]
• Model monthly, mixed frequencies, and bridge in FFR.

• Why bridge in FFR?
- Measurements equations: (20)-(25) are the same. (26) substituted with

\[
FFR_t = Nr_t + r_t + c_7 \quad (27)
\]
\[
Nr_t = Nr_{t-1} + wr_{t-1} \quad (28)
\]
\[
w_r = \rho wr_{t-1} + w_t \quad (29)
\]
• Model monthly, mixed frequency, bridge in FFR, other in house-data.

- Measurement equations: (20)-(25)-(27)-(29) plus

\[
\begin{align*}
\Delta GDP_{NOW,t} &= 400 \times (y_{t+3} - y_t) + u_{1t} \\
\Delta GDP_{pot,t} &= 400 \times (y^f_t - y^f_{t-3}) + u_{2t} \\
\Delta YN_t &= 400 \times (y_{t+3} - h_{t+3} - y_t + h_t) + u_{3t} \\
inf_t^e &= inf_{t+12} + u_{4t} \\
inf_t^{com} &= 1/3 \times (inf_t + inf_{t+1} + inf_{t+2}) + u_{5t} \\
BaaSpread_t &= (rk_t - r_t) + c_7 + u_{6t}
\end{align*}
\]
• Quarterly model, quarterly data
• Monthly model; mixed frequencies.
• Monthly model, mixed frequencies, FFR bridge.
• Monthly model, mixed frequencies, FFR bridge, inflation expectations.
• Monthly model, mixed frequencies, FFR bridge, inflation expectations, spread.
• Monthly model, mixed frequencies, FFR bridge, inflation expectations, spread, nowcast
- Monthly model, mixed frequencies, FFR bridge, inflation expectations, spread, nowcast, and potential
- Monthly model, mixed frequencies, FFR bridge, inflation expectations, spread, nowcast, potential, output per worker, commodity prices
Conclusions

• Considerable uncertainty about the time series features of the output gap and its drivers, even with the same observables.

• Adding inflation expectations, spread makes gaps estimates in last 10 years less severe.

• Adding output-per-woker, commodity prices, potential measures do not help with estimation or inference.

• No evaluation of what the best specification is.
References:


