Flight to Liquidity and Systemic Bank Runs

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Abstract

I present a novel, general equilibrium, monetary model of banking with multiple equilibria. In the good equilibrium, all banks are solvent. In the bad equilibrium, 1) many banks are insolvent and subject to runs; 2) depositors fly to liquidity; 3) the velocity of money drops and the economy experiences deflation. Some central bank interventions are more effective than others at eliminating the bad equilibrium. In some circumstances, interventions that do not eliminate the bad equilibrium reduce further nominal prices and money velocity, increase the losses of depositors, and amplify the flight to liquidity.

JEL Codes: E44, E51, G21

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1 Introduction

“The attempted liquidation of assets to acquire [...] money drove down their prices and rendered insolvent banks that would otherwise have been entirely solvent.” Friedman and Schwartz, *A Monetary History of the United States*, “Chapter 7: The Great Contraction, 1929-33.”

During both the Great Depression and the 2008 US financial crisis, several financial institutions became insolvent and were subject to runs. More than one-fifth of the commercial banks in the United States suspended operations during the Great Depression (Friedman and Schwartz, 1963). In 2008, the collapse of Lehman Brothers was followed by a run on the repo market (Gorton and Metrick, 2012a,b), and on other institutions not covered by deposit insurance such as money market mutual funds (Duygan-Bump et al., 2013).

Another peculiar event during these crises was a dramatic increase in the private sector’s willingness to hold liquid assets, a “flight to liquidity.” Friedman and Schwartz (1963) argue that the absence of an adequate response to bank runs and to the flight to liquidity by the Federal Reserve generated deep deflation and transformed a modest or deep recession into the Great Depression. In contrast, the Federal Reserve reacted aggressively in 2008 and the price level decreased only slightly: the Consumer Price Index dropped 3.4% from September to December 2008.

Despite the interactions between bank runs, flight to liquidity, and monetary policy interventions, very few models analyze the interconnections between these phenomena. Most of the literature on banking crises assumes that banks operate in environments with only one real good, without fiat money. While this approach is useful for many purposes, in practice banks use fiat money and nominal contracts, giving rise to non-negligible interactions with monetary policy choices.\(^1\)

To fill this gap, this paper provides a general equilibrium model of banking with nominal contracts and a specific role for money, which features multiple equilibria.\(^2\) In the good equilibrium, all banks are solvent and there are no runs. In the bad equilibrium, several banks are insolvent and subject to runs; thus the model generates a *systemic* crisis. Furthermore, the bad equilibrium is associated with a flight to liquidity, deflation, and other developments typical of banking crises such as a decline in asset prices, decrease in money velocity and the money multiplier, and the sale of assets from banks to other agents.

A second contribution of this paper is to apply the model to analyze some of the monetary policies used during the recent US financial crisis. In the model, a central bank can inject money into the economy by either 1) buying assets on the market (*asset purchases*), or 2) setting up

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\(^1\) Few papers deal with this observation; I review this literature in the next Section.

\(^2\) Multiple equilibria arise in some regions of the parameter space. Other regions of the parameter space have a good equilibrium only or a bad equilibrium only, depending on the fundamentals of the economy.
liquidity facilities (in order to provide loans to banks). Large monetary injections eliminate the bad equilibrium. This result is consistent with the Friedman-Schwartz hypothesis regarding the Great Depression. However, if a monetary injection does not eliminate the bad equilibrium, in some circumstances it reduces further nominal prices and money velocity, increases the losses of depositors, and amplifies the flight to liquidity.

For some parameterizations, and with some monetary injections, a bad equilibrium with very little deflation exists. Therefore, the model is relevant to analyze not only the Great Depression but also the 2008 US crisis.

I first use a simple environment in which all results are proven analytically. I then relax some assumptions, using a specification that generates a richer feedback between agents’ choices and policy. Using numerical simulations, I show that the same forces are at work in the richer model, although some results are quantitatively different.

Overview of the model. The model is populated by banks, households, and a central bank. Following Diamond and Dybvig (1983) and much of the literature on bank runs, I use a three-period model \( t = 0, 1, 2 \), with preference shocks realized at \( t = 1 \) to motivate the need for liquidity. However, several other assumptions are crucially different, and some are motivated by an infinite-horizon formulation. As a result, the channel that gives rise to multiple equilibria is different from that of Diamond and Dybvig. Although multiplicity is based on a coordination failure, as it is typical in models of multiple equilibria, the failure in my model arises in the decision to fly to liquidity and propagates through prices, rather than in the decision to run vs. not to run modeled by Diamond and Dybvig.

With respect to technology, there is a fixed supply of a productive asset (capital) that produces consumption goods at \( t = 1 \) and at \( t = 2 \). Capital has an endogenously determined price, or, equivalently, an endogenous return. This is similar to simple dynamic macroeconomics or asset pricing models but different from Diamond and Dybvig (1983) and many other models of bank runs, in which the productive asset has an endogenously determined size and a fixed exogenous return. Due to the exogenous return, Diamond and Dybvig is often interpreted as a partial equilibrium model of one bank, rather than a model of the entire banking system such as the one in this paper.

With respect to endowments, banks are endowed at \( t = 0 \) with both assets (capital and fiat money) and liabilities, denominated in terms of money (pre-existing deposits). This also differs from the standard bank runs literature, in which banks have typically no endowments or only an endowment of assets.\(^3\) Moreover, banks are heterogeneous in their endowment of capital and have private information about this endowment at \( t = 0 \). Therefore, at \( t = 0 \), depositors cannot

\(^3\)There are some other models where banks have pre-existing deposits, e.g. Faria-e Castro et al. (2015).
distinguish between “strong” and “weak” banks. This informational asymmetry is resolved at 
\( t = 1 \) when information about banks’ endowment becomes common knowledge.

The last set of assumptions are related to the role of money. There is an exogenous supply 
of money, which is fixed in the baseline model without policy intervention. Due to a cash-in-advance constraint, households need money to finance consumption expenditure at \( t = 1 \) after the 
realization of preference shocks. Holding money is costly, however, due to the opportunity cost 
represented by the return on capital. Money can be converted into consumption goods at \( t = 2 \) at 
an exogenous rate, motivated by an infinite-horizon formulation in which money can be carried to 
the next period.

The logic of the good equilibrium is similar to Diamond and Dybvig (1983). At \( t = 0 \), house-
holds hold demand-deposits issued by banks, but no money. At \( t = 1 \), households are hit by 
preference shocks and those with high utility from consumption withdraw money from banks and 
buy consumption goods. Therefore, banks insure households against the liquidity risk arising from 
preference shocks.

The channel that gives rise to the bad equilibrium is very different from Diamond and Dybvig 
(1983). Consider the net worth of banks at \( t = 0 \):

\[
\begin{pmatrix}
\text{net worth,} \\
\text{\( t = 0 \) }
\end{pmatrix}
= \begin{pmatrix}
\text{endowment} \\
\text{of capital}
\end{pmatrix}
\begin{pmatrix}
\text{nominal price} \\
\text{of capital,} \\
\text{\( t = 0 \)}
\end{pmatrix}
+ \begin{pmatrix}
\text{endowment} \\
\text{of money}
\end{pmatrix}
- \begin{pmatrix}
\text{endowment} \\
\text{of deposits}
\end{pmatrix}
\]

and make the guess (to be verified later) that the price of capital in the bad equilibrium is lower 
than in the good equilibrium. As a result, the net worth of banks decreases. Banks with large 
endowments of capital remain solvent (net worth \( \geq 0 \)), but banks with low endowments of capital 
become insolvent (net worth \( < 0 \)). However, due to asymmetric information at \( t = 0 \), depositors 
cannot distinguish between solvent and insolvent banks. The informational asymmetry is then 
resolved at \( t = 1 \), when households learn the identity of insolvent banks and run on these banks. 
At \( t = 0 \), households have rational beliefs and anticipate that some banks will be subject to runs. 

Runs are problematic because some households with high utility at \( t = 1 \) will be “last in line” 
in a run, and thus unable to withdraw the money needed to finance consumption expenditure. 
Therefore, at \( t = 0 \) households fly to liquidity, increasing their demand for money. Scrambling for 
money, everybody tries to sell the only other asset in the economy, capital. But capital is in fixed 
supply and someone has to hold it all, therefore the nominal price of capital at \( t = 0 \) must decrease

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4This is motivated by the observation of e.g. Gorton (2008) who emphasizes the uncertainty regarding the identities 
of the weakest financial institutions, i.e., those that incurred significant losses associated with the housing market 
during the Great Recession.

5Money in the model can be reinterpreted, more generally, as liquid assets that facilitate transactions. For instance, 
Krishnamurthy and Vissing-Jorgensen (2012) show that US Treasuries provide liquidity services.

6Households also reduce their holdings of deposits. However, banks invest part of the deposits in capital and 
therefore a reduction of deposits forces banks to sell capital.
to equalize supply and demand. This corresponds to the epigraph from Friedman and Schwartz (1963), and is the essence of the general equilibrium channel in the model. The decrease in the price of capital was the initial guess proposed to explain the bad equilibrium. The guess is thus verified and this is indeed an equilibrium.

The bad equilibrium is also associated with deflation. At $t = 1$, some money is held by households with low realized preference shocks, implying little or no utility from consumption. Money held by these agents is not spent and is “stored under the mattress,” therefore less money is in used for transactions at $t = 1$ than in the good equilibrium. As a result, the price level drops. (The price level is proportional to money used for transactions, due to the cash-in-advance constraint.) Using the equation of exchange $\text{money} \times \text{velocity} = \text{price} \times \text{output}$, the drop in the price level is equivalent to a drop in velocity, a fact that characterizes both the Great Depression and the 2008 financial crisis.

The combination of deflation and nominal endowment of deposits creates debt-deflation, as in Fisher (1933). Deflation increases the real value of nominal deposits, creating distress in the banking system because deposits are liabilities for banks. However, debt-deflation is not a crucial element in the channel that gives rise to multiple equilibria. In a companion paper (Robatto, 2015a), I present a model with no money and no debt-deflation, in which a similar flight to liquidity gives rise to multiple equilibria; in that paper, banks’ distress originates from a fire-sale of assets at a depressed real price. The flight to liquidity is therefore the key element that produces multiple equilibria in the models, as long as it is coupled with some general-equilibrium channel that feeds back into banks’ balance sheets. Whether the feedback is due to debt-deflation or to fire-sales is irrelevant; under both modeling assumptions, multiple equilibria arise and the bad equilibrium is associated with several stylized facts of financial crises.

Within the category of bad outcomes there are up to two bad equilibria, depending on parameters. The two bad equilibria look qualitatively identical, but they have some crucial difference that is reflected in the effects of monetary injections.

**Overview of monetary policy analysis.** I first analyze the effects of a small increase in money supply that is temporary, so that the money supply reverts to the initial level in the last period. The direct effect of monetary injections is to increase prices. However, an additional general equilibrium effect reduces interest rates, including those paid by banks on deposits. This force tends to reduce further deposits, exacerbating the flight to liquidity, the drop of money velocity, and the decrease of nominal prices. Therefore, contrary to the conventional wisdom that reducing

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7 Alternatively, debt-deflation can be viewed as a force that transfers wealth from debtors (banks) to creditors (households).

8 Although the distinction between debt-deflation and fire-sales is not important for the existence of multiple equilibria, the distinction may be important for policy analysis.
interest rates (or spreads) always helps to ameliorate a crisis, my results show a negative effect of reducing interest rates. The net effect on prices varies across the bad equilibria. In one of the bad equilibria, prices increase; in the other, prices decrease. The difference between the two equilibria depends on how monetary injections interact with the strategic complementarity that gives rise to multiple equilibria. Therefore, the implications of policy analysis might differ from models in which the flight to liquidity is caused by exogenous, policy-invariant shocks (see next Section).

Sufficiently large monetary injections eliminate the bad equilibria. Whether loans to banks or asset purchases are more effective depends on the specification. Asset purchases are more effective in the baseline model, whereas loans to banks are more effective in the numerical simulation of the richer model.

1.1 Comparison with the literature

This paper differs from Diamond and Dybvig (1983) and most of the bank runs literature by incorporating nominal banking contracts and fiat money. The few other papers that do so differ from this paper in that they use exogenous shocks rather than coordination failures to model crises. Diamond and Rajan (2006) briefly analyze the case in which a shock forces banks to sell assets to acquire money, depressing prices. In Allen et al. (2013), the central bank can print money to accommodate an increase in demand due to an exogenous shock, achieving the first-best and increasing the price level. While this increase in the price level is consistent with some episodes of banking crises, it is not consistent with major crises such as the one in 2008. In 2008, the price level slightly decreased despite massive monetary injections, a fact that my model replicates. Bianchi and Bigio (2014) analyze a “bank run shock” that increases the demand of liquidity by banks.

An important assumption in my model is asymmetric information about the balance sheets of banks. Several studies analyze banks in environments with asymmetric information (see e.g. Freixas and Rochet, 2008). Bigio (2012) introduces asymmetric information in a general equilibrium model of banking. Although there are some differences between his model and the one in this paper, asymmetric information has a similar effect in preventing resources from being intermediated.

Brunnermeier and Sannikov (2011), Carapella (2012) and Cooper and Corbae (2002) present monetary models with banks; Carapella (2012) and Cooper and Corbae (2002) also have multiple equilibria. However, their focus is on banking intermediation rather than insurance against liquidity risk and runs. Moreover, debt-deflation is crucial in Brunnermeier and Sannikov (2011) and

Deposit insurance also eliminates multiple equilibria, as in many other models of panic-based runs. However, I abstract from this. As a result, banks in the model are unregulated institutions that perform maturity transformation without deposit insurance, similar to commercial banks in the 1930s and shadow banks in recent years.

An exception is a recent paper by Andolfatto et al. (2015), who combines Diamond-Dybvig runs with a monetary model à la Lagos and Wright (2005)
Carapella (2012), whereas I show in a companion paper (Robatto, 2015a) that a flight to liquidity similar to the one in this paper gives rise to multiple equilibria in a model with fire-sales instead of debt-deflation. In Cooper and Corbae (2002), multiplicity is related to increasing returns to scale in intermediation, and monetary policy eliminates the bad equilibrium by increasing the growth rate rather than the level of money.

Some results of the baseline model with no policy interventions, such as the combination of bank runs and drop in asset prices, are similar to other papers in the bank runs literature (see e.g. the interaction of bank runs and cash-in-the-market in the general equilibrium models of Allen and Gale, 2004 and Allen et al., 2009). However, a better comparison requires looking at policy analysis, which better reflects the microfundation as in any modern economic model, rather than looking at the “static” results without policy intervention. The results of my policy analysis are different and novel compared to the literature, reflecting a different and novel microfundation at modeling bank runs in general equilibrium.

From an empirical standpoint, the debate about the quantitative importance of runs in the 2008 financial crisis is still open, in particular in the repo market (see Krishnamurthy et al., 2014). Adding to this debate, I provide an example in Section 5.2 showing that the model is consistent with the possibility that a “small run” generates sizable general equilibrium effects.

2 Model

Time is discrete and there are three periods indexed by \( t \in \{0, 1, 2\} \). The economy is populated by a unit mass of banks indexed by \( b \in B \equiv [0, 1] \), a double continuum of households indexed by \( h \in \mathbb{H} = [0, 1] \times [0, 1] \), and a central bank. Superscripts \( h \) and \( b \) refer to household \( h \) and bank \( b \), respectively.

2.1 Preferences

Household \( h \in \mathbb{H} \) derives utility from consumption at \( t = 1 \) and \( t = 2 \), denoted by \( C_1^h \) and \( C_2^h \):

\[
\mathbb{E} [\bar{u} (C_1^h)] + \beta C_2^h
\]

(1)

where \( 0 < \beta < 1 \). The functional form of \( \bar{u} (\cdot) \) depends on the realization of a preference shock, realized at the beginning of \( t = 1 \):

\[
\bar{u} (\cdot) = \begin{cases} 
\bar{u} (\cdot) & \text{ (impatient) with probability } \kappa \\
0 & \text{ (patient) with probability } 1 - \kappa
\end{cases}
\]

(2)
where \( \bar{u}(\cdot) \) is piecewise-linear:

\[
\bar{u}(C) = \begin{cases} 
\theta C & \text{if } C < \overline{C} \\
\theta \overline{C} + (C - \overline{C}) & \text{if } C \geq \overline{C}
\end{cases} \quad \theta > 1. \tag{3}
\]

and \( \overline{C} > 0 \). The functional form of \( \bar{u}(C) \) produces risk aversion, although with a specification that is almost everywhere linear and thus allows to prove all the results analytically. The preference shock is i.i.d. across households and the law of large numbers holds for each subset of \( H \) with a continuum of households.\(^{11}\) The preference shock is also private information of household \( h \).

Banks act competitively and have linear utility in consumption \( C^b_2 \) at time \( t = 2 \).

### 2.2 Assets, technology, and markets

There are two assets with exogenous supply in the economy: capital and money. Capital is in fixed supply \( K.\)\(^{12}\) The supply of money is \( M(1 + \mu_t) \), where \( M \) is a constant and \( \mu_t \) is chosen by the central bank, as described in Section 2.7. In the baseline version of the model with no policy intervention, \( \mu_t = 0 \) for all \( t \). Money is the unit of account. Thus, unless noted otherwise, prices and contracts are expressed in terms of money.

There is also a third asset in the economy, deposits, which are endogenously supplied. A deposit issued by bank \( b \) is a claim that is redeemable on demand at bank \( b \).

#### Technology and markets.  

The timing is represented in Figure 1.

At \( t = 0 \), there is a Walrasian market in which households and banks can trade capital and money. The price of capital is \( Q_0 \).

At \( t = 1 \), each unit of capital produces \( A_1 \) units of consumption goods. Consumption goods are sold at price \( P_1 \), and consumption expenditures are subject to a cash-in-advance constraint.\(^{13}\) Capital is illiquid at \( t = 1 \), i.e., it cannot be traded.\(^{14}\)

At \( t = 2 \), each unit of money can be converted into \( 1/\overline{P}_2 \) units of consumption goods, and each unit of capital can be converted into \( \overline{Q}_2/\overline{P}_2 \) units of consumption good. \( \overline{Q}_2 \) and \( \overline{P}_2 \) are exogenous parameters, but are motivated by an infinite-horizon formulation, in which fiat money and capital can be carried over and used in the next period (see Appendix A).

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\(^{11}\)This assumption is consistent with the results of Al-Najjar (2004) about the law of large numbers in large economies.

\(^{12}\)Adding aggregate shocks to capital does not qualitatively affect the results of the paper.

\(^{13}\)Households cannot consume goods produced by their own stock of capital, as in standard models with a cash-in-advance constraint such as Lucas and Stokey (1987).

\(^{14}\)Similar to Jacklin (1987), trading restrictions are required to provide a role for banks. If households could trade capital at \( t = 1 \) in exchange for money, there would be no role for banks.
2.3 Endowments

Households. The endowment of household $h \in \mathbb{H}$ at $t = 0$ is $\{M^h_{-1}, K^h_{-1}, D^h_{-1}\}$ where $M^h_{-1}$ is money, $K^h_{-1}$ is capital, and $D^h_{-1}$ are pre-existing deposits at a bank $b(h)$.\footnote{Formally, $b(h)$ is an exogenous function that maps the set of households $\mathbb{H}$ into the set of banks $\mathbb{B}$, so that each household has initial deposits at one bank.} I impose the following assumption about $D^h_{-1}$.

Assumption 2.1. (Nominal deposits) Pre-existing deposits $D^h_{-1}$ are denominated in money, i.e., they are a promise by bank $b(h)$ to pay $D^h_{-1}$ units of money on demand to household $h$.

Endowments are identical across households, i.e., $\{M^h_{-1}, K^h_{-1}, D^h_{-1}\} = \{M^{h'}_{-1}, K^{h'}_{-1}, D^{h'}_{-1}\}$ for all $h, h' \in \mathbb{H}$.

Banks. The endowment of bank $b \in \mathbb{B}$ at $t = 0$ is $\{M^b_{-1}, K^b_{-1}, D^b_{-1}\}$, where $M^b_{-1}$ is money, $K^b_{-1}$ is capital, and $D^b_{-1}$ is deposits. For banks, deposits are an obligation to pay money on demand to a unit continuum of households denoted by $\mathbb{H}(b) \subset \mathbb{H}$. The set of initial depositors of bank $b \in \mathbb{B}$ is represented in Figure 2.

There is heterogeneity in the endowment of capital across banks:

$$K^b_{-1} = \begin{cases} K^L & \text{for a fraction } \alpha \text{ of banks} \\ K^H & \text{for a fraction } 1 - \alpha \text{ of banks} \end{cases}, \quad K^H > K^L > 0 ; \alpha \in (0, 1).$$

Moreover, there is an informational asymmetry about banks’ capital holdings.

Assumption 2.2. (Private information about banks’ capital endowment) At $t = 0$, the endowment of capital $K^b_{-1}$ of each bank is private information of the respective bank. Other agents in the...
Figure 2: Banks and Households

The square represents the set of households $H = [0, 1] \times [0, 1]$; the vertical line represents the set of initial depositors of bank $b \in B = [0, 1]$.

The economy only know:

$$\Pr(K_{-1}^b = K^L) = \alpha$$
$$\Pr(K_{-1}^b = K^H) = 1 - \alpha;$$

At $t = 1$, the value of $K_{-1}^b$ of each bank becomes common knowledge and the informational asymmetry is resolved.\(^{16}\)

There is no heterogeneity in the endowment of money and deposits across banks; that is, $M_{-1}^b = M_{-1}^{b'}$ and $D_{-1}^b = D_{-1}^{b'}$ for all $b, b' \in B$.

**Total endowments in the economy.** Endowments satisfy $\int K_{-1}^h dh + \int K_{-1}^b db = K$, $\int M_{-1}^h dh + \int M_{-1}^b db = M$, and $\int D_{-1}^h dh = \int D_{-1}^b db$.

### 2.4 Interactions between households and banks

This Section describes briefly the interaction between households and banks. The next Section (2.5) explains the role of each assumption described in the preceding sections. More details about banks’ and households’ budget sets and choices are provided in Section 2.6.

At $t = 0$, households decide their holdings of deposits $D_0^h$.\(^{17}\) The choice of deposits $D_0^h$ taken by household $h$ at $t = 0$ is a decision regarding rolling over preexisting deposits $D_{-1}^h$ (fully or partially) and/or increasing deposits. For instance, if $D_0^h = D_{-1}^h$, then the dollar value of household $h$’s bank account remains constant. I impose the restriction that households can hold deposits at

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\(^{16}\)Information acquisition at $t = 1$ is exogenous. This is different from Hellwig and Veldkamp (2009), who model information acquisition endogenously.

\(^{17}\)Deposits at $t = 0$ are denominated in units of money because I use money as a unit of account. It is irrelevant whether deposits at $t = 0$ are in nominal terms or contingent on prices. The crucial assumption about deposits is the denomination of pre-existing deposits $D_{-1}^h$ and $D_{-1}^b$ in terms of money.
Table 1: Balance sheet of a bank $b$ at the beginning of $t = 0$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of capital = $K_{-1}^b Q_0$</td>
<td>Deposits = $D_{-1}^b$</td>
</tr>
<tr>
<td>Money = $M_{-1}^b$</td>
<td>Net worth = $N_0^b$</td>
</tr>
</tbody>
</table>

no more than one bank. I assume that households continue banking with the same bank $b(h)$ that received their initial deposits $D_{-1}^h$ (without loss of generality, due to perfect competition in the banking sector).

Banks’ deposits at $t = 0$ are denoted by $D_0^b$. For bank $b$, the difference $D_0^b - D_{-1}^b$ is the net issuance of deposits. If $D_0^b > D_{-1}^b$, bank $b$ increases its deposits and thus receives new resources from households. Otherwise, bank $b$ reduces its amount of preexisting deposits and must pay back some resources to households.19

At $t = 1$, households withdraw money $W_1^h$ subject to a sequential service constraint.20

At $t = 2$, banks repay deposits that have not been withdrawn at $t = 1$, plus a return.21

2.5 Discussion of assumptions

The model departs from standard models of banking in three ways.

1. Banks have some endowment at $t = 0$ of assets (capital and money) and liabilities (nominal deposits). This is in contrast to standard models, where banks have typically either no endowments or only an endowment of assets. Moreover, there is asymmetric information about the amount of capital banks have.

2. Capital is in fixed supply $K$ and its price $Q_0$ is endogenous; thus, its return is endogenous too. In contrast, standard models of bank runs assume the opposite: the amount of capital is endogenous but its return is exogenous.

3. Money is required to purchase consumption goods at $t = 1$, due to the cash-in-advance

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18 This assumption can be justified by costs of maintaining banking relationships. Formally, the cost would be zero if household $h$ holds deposits at one bank, and infinite if household $h$ holds deposits at two or more banks.

19 To describe precisely the interaction between banks and depositors, I must specify what happens if many preexisting deposits are not rolled over at $t = 0$ and the bank does not have enough resources to repay them, i.e., the bank does not have enough endowment of money, $M_{-1}^b$, and of capital, $K_{-1}^b$. If such a circumstance occurs, the bank is shut down immediately and depositors get pro-rata repayments.

20 The sequential service constraint is imposed as a physical constraint as in Wallace (1988), rather than as a restriction on contracts.

21 For simplicity, I assume that no return is paid on deposit withdrawn at $t = 1$. This contractual arrangement is (weakly) optimal because households have linear utility from consumption at $t = 2$. 

11
To analyze the role of Items 1 and 2, Table 1 presents the balance sheet of a bank $b$ at the beginning of $t = 0$. Net worth is the difference between the value of assets and deposits:

$$N^b_0 = K^b_{-1} Q_0 + M^b_{-1} - D^b_{-1}.$$ (4)

Figure 3 plots the net worth of banks, $N^b_0$, as a function of the price of capital $Q_0$.

If the price of capital $Q_0$ is high enough, as will be the case in the good equilibrium, both banks with endowment of capital $K^L$ and $K^H$ are solvent, that is, $N^b_0 \geq 0$ for all $b \in \mathbb{B}$. In this case, asymmetric information is irrelevant because all banks are solvent.

If the price of capital $Q_0$ is low, as will be the case in the bad equilibrium, banks with endowment of capital $K^b_0 = K^L$ have negative net worth ($N^b_0 < 0$) and are therefore insolvent, whereas banks with endowment of capital $K^b_0 = K^H$ have positive net worth ($N^b_0 \geq 0$) and so are solvent. The possible insolvency of banks at $t = 0$ is a consequence of Items 1 and 2: banks are endowed with both assets and liabilities, and the value of their net worth depends on the price of capital $Q_0$.\footnote{In the infinite-horizon model of Gertler and Kiyotaki (2015), insolvency of banks arises similarly. However, their model does not have money and displays fire-sales in the bad equilibrium, and is therefore related to my companion paper (Robatto, 2015a).}

Moreover, due to asymmetric information about $K^b_{-1}$, households cannot distinguish between solvent and insolvent banks. Thus insolvent banks are active at $t = 0$.\footnote{Without asymmetric information about banks’ balance sheets, households would move all their deposits to solvent banks, and thus only solvent banks would be active. This would be equivalent to the good equilibrium. As such, eliminating asymmetric information increases welfare. See e.g. Alvarez and Barlevy (2014) and Parlatore (2015) for banking models in which eliminating informational asymmetries does not necessarily increase welfare.} The asymmetric information about banks is then resolved at $t = 1$, when households learn which banks are insolvent and run on these banks.

Item 3 provides a justification for the general equilibrium feedback that determines the price of capital $Q_0$ in equilibrium. In the good equilibrium, banks insure against preference shocks and provide money on demand at $t = 1$. In a bad equilibrium, instead, some banks are subject to runs at $t = 1$; therefore, households demand more money at $t = 0$ to make sure that they can finance some expenditure at $t = 1$ in the event of a run. Thus, the overall demand for money increases, and the demand for capital drops (a flight to liquidity). Because the supply of capital is fixed, the price of capital $Q_0$ in the bad equilibrium is lower than in the good equilibrium.
Figure 3: Net worth of banks as a function of the price of capital $Q_0$

Net worth
\[ N_0^b = K_{-1}^b Q_0 + M_{-1}^b - D_{-1}^b \]

Bank $b$ with endowment $K^H$

Bank $b$ with endowment $K^L$

High $Q_0$, $N_0^b \geq 0$ for all $b$
(all banks are solvent)

Low $Q_0$, $N_0^b \begin{cases} \geq 0 \text{ (solvent)} & \text{if } K_{-1}^b = K^H \\ < 0 \text{ (insolvent)} & \text{if } K_{-1}^b = K^L \end{cases}$

2.6 Budgets

$t = 0$: trading and deposits. At $t = 0$, households decide their holdings of money, deposits, and capital:

\[ \begin{align*}
M^h_0 + D^h_0 + Q_0 K^h_0 & \leq N^h_0 \\
\text{money} & \text{deposits} & \text{capital}
\end{align*} \]  

subject to the non-negativity constraints $D^h_0 \geq 0$, $M^h_0 \geq 0$, $K^h_0 \geq 0$, and where $N^h_0$ is the net worth of households, defined similarly to Equation (4) but with deposits entering positively:

\[ N^h_0 = K^h_{-1} Q_0 + M^h_{-1} + D^h_{-1}. \]  

At $t = 0$, bank $b$ can buy money $M^b_0$ and capital $K^b_0$, using its net worth $N^b_0$ defined in Equation (4), deposits $D^b_0$, and loans from the central bank $B^b_0$, if any:

\[ \begin{align*}
K^b_0 Q_0 + M^b_0 & \leq N^b_0 + D^b_0 + B^b_0 \\
\text{capital} & \text{money} & \text{net worth} & \text{deposits} & \text{loans from central bank}
\end{align*} \]  

subject to the non-negativity constraints $D^b_0 \geq 0$, $M^b_0 \geq 0$, $K^b_0 \geq 0$, and $B^b_0 \geq 0$.  

24 Central bank policies are explained in Section 2.7; the reader interested in the model without policy intervention can ignore the choice of loans from the central bank and set $B^b_0 = 0$.

25 The asset side of a bank’s balance sheet at $t = 0$ is also private information of bank $b$, similar to endowment $K_{-1}^b$. If this were not the case, households would be able to distinguish between banks with endowment $K^L$ and banks with $K^H$. 


\( t = 1: \text{withdrawals and consumption.} \) At \( t = 1 \), households’ preference shocks are realized and each household decides its amount of withdrawals \( W_1^h \). In the event of large withdrawals from a bank, the bank might not have enough cash to serve all households. Let \( l_1^h \in \{0, +\infty\} \) to be a limit on withdrawals determined by the position in the line (recall from Section 2.4 that withdrawals happens sequentially). Then:

\[
0 \leq W_1^b \leq \begin{cases} 
D_0^h & \text{if } l_1^h = +\infty \text{ (no run, or among those first in line in a run)} \\
0 & \text{if } l_1^h = 0 \text{ (among those last in line in a run)}
\end{cases}
\]

or, more compactly, \( 0 \leq W_1^h \leq \min \{ D_0^h, l_1^h \} \). If household \( h \) is served when the bank is out of money, then \( l_1^h = 0 \) and thus \( W_1^h = 0 \). If household \( h \) is served when the bank still has money, then \( l_1^h = +\infty \) and \( 0 \leq W_1^h \leq D_0^h \).

Bank \( b \) is subject to a run if the limit on withdrawals is \( l_1^b = 0 \) for some depositors of bank \( b \). If bank \( b \) is subject to a run, the bank is liquidated at \( t = 2 \). In the event of liquidation, assets that the bank has at \( t = 2 \) are used to repay deposits not withdrawn. If the value of assets is insufficient, depositors are repaid pro-rata; if the value of assets is greater than the value of deposits not withdrawn, banks can use the difference for consumption \( C_2^b \).

After making withdrawals, households choose consumption expenditure subject to a cash-in-advance constraint. Consumption expenditure \( P_1 C_1^h \) cannot exceed the sum of money \( M_0^b \) chosen at \( t = 0 \) plus withdrawals \( W_1^h \) chosen at \( t = 1 \):

\[
P_1 C_1^h \leq M_0^b + W_1^h.
\] (8)

Banks do not make any economic decisions at \( t = 1 \). The amount of money withdrawn by depositors of bank \( b \) is:

\[
W_1^b = \int_{\mathcal{E}(b)} W_1^h dh \leq M_0^b
\] (9)

where the inequality arises from the fact that, at \( t = 1 \), there is no market in which banks can sell capital in exchange for money.

\( t = 2: \text{return on deposits and consumption.} \) At \( t = 2 \), households are entitled to receive a return \( 1 + r_2^D \) on deposits that are not withdrawn at \( t = 1 \). I refer to \( r_2^D \) as the \textit{promised return on deposits}.\(^{26}\)

Banks might not have enough resources to pay the promised return \( r_2^D \). Define \( r_2^b \leq r_2^D \) to be

\(^{26}\) is a market price that is taken as given by both banks and households, and is known at \( t = 0 \). The results are unchanged if I allow each bank to post a bank-specific return.
the actual return on deposits at bank $b$ (defined below). Note that $r^b_2$ can be lower than the promised return; if that is the case, then the quantity $1 + r^b_2$ can be interpreted as the recovery rate.

Before describing consumption of households and banks, it is useful to define the return on capital $1 + r^K_2$:

$$1 + r^K_2 = \frac{Q_2 + A_1 P_1}{Q_0}.$$  \hspace{1cm} (10)

Household consumption at $t = 2$ is:

$$C^h_2 = \frac{1}{P_2} \left[ \frac{Q_0 K^h_0 (1 + r^K_2)}{P_2} + \left( D^h_0 - W^h_1 \right) \left( 1 + r^{b(h)}_2 \right) + \left( M^h_0 + W^h_1 - P_1 C^h_1 \right) \right] + T_2. \hspace{1cm} (11)$$

Households transform all their assets into the consumption goods, and they may receive lump-sum transfers $T_2$ from the central bank. The assets of households (expressed in monetary value) are: capital $Q_0 K^h_0$ bought at $t = 1$, plus the return $r^K_2$; deposits not withdrawn $D^h_0 - W^h_1$ plus the actual return $r^{b(h)}_2$ paid by bank $b (h)$ associated with household $h$; and unspent money holdings $M^h_0 + W^h_1 - P_1 C^h_1$, which are positive if the cash in advance constraint (8) holds with inequality, and zero if the cash-in-advance constraint holds with equality. Since the assets are expressed in terms of money, they are transformed into $1/P_2$ units of consumption.

Similarly, consumption of banks at $t = 2$ is:

$$C^b_2 = \frac{1}{P_2} \max \left\{ 0; \frac{Q_0 K^b_0 (1 + r^K_2)}{P_2} - \left( D^b_0 - W^h_1 \right) \left( 1 + r^D_2 \right) - B^b_0 \left( 1 + r^{C^B} \right) \right\}. \hspace{1cm} (12)$$

Differently from households, deposits are liabilities for banks (thus, they enter consumption with a minus sign), and banks must also repay loans $B^b_0$ obtained from the central bank, if any (see Section 2.7). If the value of assets minus liabilities is negative, consumption $C^b_2$ is bounded below at zero.

The actual return on deposits $r^b_2$ is defined by:

$$1 + r^b_2 = \begin{cases} 1 + r^D_2 & \text{if } C^b_2 > 0 \\ \frac{Q_0 K^b_0 (1 + r^K_2)}{(D^b_0 - W^h_1) + B^b_0} & \text{if } C^b_2 = 0 \end{cases} \hspace{1cm} (13)$$
If the assets of bank \( b \) at \( t = 2 \) are greater than its liabilities so that \( C^b_2 > 0 \), the actual return on deposits \( r^b_2 \) is equal to the promised return \( r^D_2 \). Otherwise, \( C^b_2 = 0 \) and \( r^b_2 \) is the return paid at \( t = 2 \) using the assets of bank \( b \), \( Q_0 K^b_0 (1 + r^K_2) \). These resources are split across deposits not withdrawn \( D^b_0 - W^b_1 \) plus loans from the central bank \( B^b_0 \), if any.\(^{27}\)

### 2.7 Central bank

Banks and households are initially endowed with \( \overline{M} \) units of money. Recall that the money supply is \( \overline{M} (1 + \mu_t) \), where \( \mu_t \) is chosen by the central bank. If there is no policy intervention, then \( \mu_t = 0 \) for all \( t \) and money supply is constant at \( \overline{M} \). (The reader interested in the model without policy intervention can skip the rest of this section.)

If there is a policy intervention, the central bank changes the money supply in the event of a panic by choosing \( \mu_t \). I assume that the change in the money supply takes place at \( t = 0 \) (\( \mu_0 \neq 0 \)), that money supply does not change at \( t = 1 \) (\( \mu_1 = \mu_0 \)), and that the money supply reverts to the pre-crisis level \( \overline{M} \) when the crisis is over, at \( t = 2 \) (\( \mu_2 = 0 \)). The restriction about \( \mu_1 \) is not crucial, whereas the restriction about \( \mu_2 \) is motivated by the temporary nature of the policies implemented during the recent US financial crisis; see Appendix \( D \) for additional discussion. The value of \( \mu_0 \) is announced before the Walrasian market opens at \( t = 0 \), and the central bank fully commits to the policy.

Given the initial endowment of money \( \overline{M} \) and the money supply \( \overline{M} (1 + \mu_0) \) at time \( t = 0 \), the monetary injection at \( t = 0 \) is \( \mu_0 \overline{M} \). I distinguish between two methods to deliver the monetary injection: asset purchases (purchases of capital) \( K^{CB}_0 \) and loans to banks \( B^{CB}_0 \), which must satisfy the budget constraint:

\[
\frac{Q_0 K^{CB}_0}{\text{asset purchases}} + \frac{B^{CB}_0}{\text{loans to banks}} \leq \mu_0 \overline{M}. \tag{14}
\]

The newly printed money \( \mu_0 \overline{M} \) is used to offer loans to banks \( B^{CB}_0 \), or to buy capital \( K^{CB}_0 \) on the market at price \( Q_0 \). Loans \( B^{CB}_0 \) are repaid by banks at \( t = 2 \). If a private bank borrows one dollar from the central bank at \( t = 0 \), it has to repay back \( 1 + r^{CB}_2 \) dollars at \( t = 2 \), where \( r^{CB}_2 \) is chosen by the central bank. Given \( r^{CB}_2 \), each bank \( b \in \mathbb{B} \) decides its demand for loans from the central bank, \( B^b_0 \).\(^{28}\) In equilibrium, I require that:

\[
B^{CB}_0 = \int_{\mathbb{B}} B^b_0 db \tag{15}
\]

\(^{27}\)If \( C^b_2 = 0 \), the actual return \( r^b_2 \) can also be computed using Equation (12) by setting consumption equal to zero, replacing \( r^D_2 \) and \( r^{CB}_2 \) with \( r^b_2 \), and then solving for \( r^b_2 \).

\(^{28}\)The result is unchanged if I allow each bank to place a bid specifying an amount of funds and an interest rate, similarly to the Term Auction Facility (TAF) implemented by the Federal Reserve in 2007-08.
Since some banks may be insolvent, I must consider the ability of the central bank to recover, at \( t = 2 \), the loans made at \( t = 0 \) and the interest \( r^{CB}_2 \). I assume that loans from the central bank have the same seniority as deposits from households. Therefore, if bank \( b \) borrowed \( B^b_0 \) from the central bank and is insolvent, the return paid to the central bank at \( t = 2 \) is equal to the actual return on deposits \( r^b_2 \) introduced in the previous section.\(^{29}\)

Thus, at \( t = 2 \), the real profits (or losses) resulting from the monetary injection are:

\[
T_2 = \frac{1}{P_2} \left[ \frac{Q_0 K^{CB}_0}{\text{capital+return}} (1 + r^K_2) + \int B^b_0 \left( 1 + \max \left\{ r^{CB}_2, r^b_2 \right\} \right) db - \frac{\mu_0 \overline{M}}{\text{reduction in money supply}} \right]
\]

which are rebated to households as lump-sum transfers. The last term in Equation (16), \( \mu_0 \overline{M} \), arises from the central bank reducing the money supply back to the initial level \( \overline{M} \).

3 Equilibrium

The full details of the households’ utility maximization and banks’ profit maximization are postponed to Appendix B. In the following sections, I focus on the elements that are most useful to understand the results.

3.1 Limit on withdrawals

Recall the limit on withdrawals \( l^b_1 \in \{0, +\infty\} \). If all depositors of bank \( b \) attempt to withdraw all their deposits at \( t = 1 \), only a fraction of depositors are able to withdraw, i.e., \( l^b_1 = +\infty \) for a subset of depositors of bank \( b \). In the (relevant) case in which all depositors of bank \( b \) choose the same value of deposits:

\[
\Pr \left( l^b_1 = +\infty \right) = \frac{M^{b(h)}_0}{D^{b(h)}_0} \quad \text{and} \quad \Pr \left( l^b_1 = 0 \right) = 1 - \frac{M^{b(h)}_0}{D^{b(h)}_0}
\]

that is, the probability that \( l^b_1 = +\infty \) is equal to the ratio of money held by bank \( b(h) \), \( M^{b(h)}_0 \), and its deposits, \( D^{b(h)}_0 \).

\(^{29}\)Alternatively, loans from the central bank could have higher seniority than deposits; that is, the central bank would be able to recover the full value of loans and interests, and depositors would split the value of the assets of the bank after the central bank is repaid. However, the effect of loans to banks with higher seniority than deposits is equivalent to the effect of asset purchases; see Appendix D.
3.2 Households: beliefs and deposits

At time $t = 0$, households form beliefs about the limit on withdrawals $l_1^h$ (that is, beliefs about the chance of being last in line in a run) and beliefs about the actual return on deposits $r_2^b$ paid by banks at $t = 2$. Let $\Pr^h\left(l_1^h, r_2^b(h)\right)$ be the belief of household $h$. In equilibrium, I require this belief to be rational, that is, to be equal to the realized probability distribution over $l_1^h$ and $r_2^b(h)$ in the economy. Denoting the realized probability distribution as $\Pr\left(l_1^h, r_2^b(h)\right)$, then:

$$\Pr^h\left(l_1^h, r_2^b(h)\right) = \Pr\left(l_1^h, r_2^b(h)\right). \quad (18)$$

The probability distribution $\Pr\left(l_1^h, r_2^b(h)\right)$ can be obtained by combining (13) and (17) with the choices taken by banks in equilibrium.

To simplify the derivation of the results, I impose the following Assumption.

**Assumption 3.1.** If household $h \in \mathbb{H}$ is indifferent among several choices of $D_0^h$, the household selects the smallest $D_0^h$ that maximizes its utility.

If the return on deposits is equal to the return on capital, $r_D^b = r_K^b$, and if there are no runs and insolvencies in the banking sector - i.e., in the good equilibrium - households are indifferent between directly investing a fraction of their wealth directly in capital vs. depositing more and letting banks buy capital on their behalf. Assumption 3.1 implies that households use banks only to insure against liquidity risk, and invest all the wealth they want to carry to $t = 2$ directly in capital. The assumption above is irrelevant for the bad equilibrium because the optimal $D_0^h$ is unique, and does not affect prices and consumption allocations in the good equilibrium.

3.3 Market clearing conditions

The market clearing conditions are as follows:

Capital market:
$$\int_{\mathbb{B}} K_0^b db + \int_{\mathbb{H}} K_0^b dh + K_{CB}^C = K. \quad (19)$$

Money market:
$$\int_{\mathbb{B}} M_0^b db + \int_{\mathbb{H}} M_0^b dh = M (1 + \mu_0). \quad (20)$$

Deposits:
$$\int_{\mathbb{B}} D_0^b db = \int_{\mathbb{H}} D_0^b dh. \quad (21)$$

Goods market:
$$\int_{\mathbb{H}} C_1^h dh = A_1 K. \quad (22)$$

Note that if there is no monetary policy intervention, the amount of assets bought by the central bank, $K_{CB}^C$, in (19) and $\mu_0$ in (20) are zero.
3.4 Equilibrium definition

In equilibrium, all banks offer the same promised return on deposits $r^D_2$ by assumption. Thus, I am imposing a pooling equilibrium in the banking market, similar to Akerlof (1970).

These results are unchanged if I allow each bank to post a bank-specific promised return on deposits. In this case, the equilibrium that arises still features pooling, because bad banks want to imitate good banks to survive as long as possible.

The next definition formalizes the equilibrium concept.

Definition 3.2. Given a monetary injection $\mu_0$, an equilibrium is a collection of:

- prices $Q_0$ and $P_1$ and promised return on deposits $r^D_2$;
- for each household $h \in H$, beliefs $Pr^h(\cdot)$; choices $M_0^h, D_0^h, K_0^h, W^h_1, C_1^h, C_2^h$; and limit on withdrawals $l^h_1 \in \{0, +\infty\}$;
- for each bank $b \in B$, choices $D_0^b, M_0^b, K_0^b, B_0^b, C_2^b$; and actual return on deposits $r^b_2$;
- central bank’s asset purchases $K_0^{CB}$ and loans to banks $B_0^{CB}$;

such that:

- households maximize utility (1) subject to (5), (8), and (11); and households’ beliefs are rational, i.e., Equation (18) holds for all $h \in H$;
- banks maximize consumption at $t = 2$, Equation (12), subject to (7);
- for each $b \in [0, 1]$, the actual return on deposits is defined by Equation (13), and limits on withdrawals satisfy:

$$l^b_1 = 0 \text{ for depositor } h \text{ of bank } b \Rightarrow \int_{H(b)} W^h_1 \left( l^h_1 = +\infty \right) dh' > M^b_0;$$

- the budget constraint of the central bank, Equation (14), holds;
- the market clearing conditions (15) and (19)-(22) hold.

I consider symmetric equilibria in which banks with the same net worth make the same choices at $t = 0$.

Restriction on parameters Motivated by an infinite-horizon formulation (see Appendix A), I impose the following restrictions on $Q_2$ and $P_2$:

$$Q_2 = \frac{\beta}{1 - \beta} \frac{\bar{M}}{\bar{K}}, \quad P_2 = \frac{\bar{M}}{\bar{A}_1 \bar{K}}. \quad (23)$$

$^{30}$This condition says that if a household faces a limit on withdrawals $l^h_1 = 0$, then withdrawals chosen by depositors of bank $b(h)$ would be greater than the amount of money $M^b_0$ held by bank $b$ if the limit on withdrawals were $+\infty$ for all depositors.
Moreover, I impose the restriction on \( C \) and on output:

\[
C = \frac{A_1 K}{\kappa}.
\]  

(24)

4 Good equilibrium

Before analyzing the good equilibrium, the next Proposition describes the first-best allocation of consumption at \( t = 1 \) in the model.

**Proposition 4.1.** *(First-best) The first-best allocation satisfies, at \( t = 1 \):*

\[
C^h_1 = \begin{cases} 
C & \text{if impatient} \\
0 & \text{if patient.}
\end{cases}
\]

Optimality requires that all goods at \( t = 1 \) are consumed by impatient households. This is a consequence of the restriction imposed on \( C \), Equation (24). To prove this result, note that total consumption goods at \( t = 1 \) is given by \( A_1 K \); thus if all impatient households consume the same amount, feasibility implies that each of them can consume at most \( A_1 K / \kappa \), i.e., at most \( C \). Since marginal utility is higher for impatient up to \( C \), the result follows.

I now present the good equilibrium of the model; the proof is provided in Appendix C.

**Proposition 4.2.** Given \( \mu_0 = 0 \), a good equilibrium exists if and only if:

\[
K^b_1 \left( \frac{\beta}{1 - \beta} \frac{M}{K} \right) + M^b_{-1} - D^b_{-1} \geq 0
\]  

(25)

for all banks \( b \in B \). The good equilibrium is characterized by:

- **prices:**
  \[
  Q_0 = \frac{\beta}{1 - \beta} \frac{M}{K} \equiv Q^*, \quad P_1 = \frac{M}{A_1 K} \equiv P^*;
  \]  

(26)

- **\( t = 0 \):** deposits \( D^h_0 = D^b_0 = D^* \), where \( D^* \equiv M / \kappa \), and money holdings \( M^h_0 = 0 \) and \( M^b_0 = \kappa D^* = M \); capital is residually determined by the budget constraint;
- **\( t = 1 \):** limit on withdrawals \( l^h_1 = +\infty \) for all \( h \), withdrawals:
  \[
  W^h_1 = \begin{cases} 
  D^* & \text{if impatient} \\
  0 & \text{if patient}
  \end{cases}
  \]
  
  and the allocation of consumption is the same as in the first-best;
- **\( t = 2 \):** returns \( r^K_2 = r^D_2 = 1 / \beta - 1 \) and \( r^b_2 = r^D_2 \) for all \( b \).
Given the equilibrium price of capital \( Q_0 = \frac{\beta}{1-\beta} \frac{M}{K} \), condition (25) guarantees that the net worth of all banks in the economy is positive. Thus, in the good equilibrium, all banks are solvent and households expect no runs. Therefore, all households hold deposits \( D^* > 0 \) and no money; banks pool the liquidity risk of households, allowing impatient households to withdraw money at \( t = 1 \) and patient households to receive a return on deposits at \( t = 2 \).

Patient households have no utility from consumption at \( t = 1 \), and therefore it is optimal for them not to consume. Impatient households consume \( C^* \), and therefore from Equation (3) any extra unit of consumption yields marginal utility of one, which is equal to the marginal utility at \( t = 2 \).

Since the return on capital is \( 1 + r^K_2 = 1/\beta \), it is (weakly) optimal for impatient households to postpone any additional consumption to \( t = 2 \).

Banks invest a fraction \( \kappa \) of deposits into money in order to serve withdrawals by impatient households at \( t = 1 \), and invest the remaining fraction \( 1 - \kappa \) in capital. Since the return on capital is \( 1 + r^K_2 = 1/\beta \), each dollar of deposits yields a gross return \( (1 - \kappa) (1 + r^K_2) = (1 - \kappa)/\beta \) at \( t = 2 \). As the bank has to pay a gross return \( 1 + r^D_2 = 1/\beta \) on the fraction \( 1 - \kappa \) of deposits not withdrawn, banks’ profits for each dollar of deposits are \( (1 - \kappa) [(1 + r^K_2) - (1 + r^D_2)] = 0 \). Thus, banks are indifferent about the quantity of deposits \( D^b_0 \) that they hold. Note that if \( r^K_2 > r^D_2 \) or \( r^K_2 < r^D_2 \), banks would make, respectively, positive or negative profits on each dollar of deposit, and thus would choose, respectively, \( D^b_0 = +\infty \) or \( D^b_0 = 0 \). But this outcome would violate the market clearing condition for deposits, and therefore in equilibrium \( r^K_2 = r^D_2 \) must hold.

Finally, the price of consumption goods \( P_1 \) is determined by equating consumption expenditures \( P_1 \left( \int_{\mathbb{E}} C^h_1 \, dh \right) \) to total money spent. Consumption expenditure can be rewritten as \( P_1 \left( A_1 \bar{K} \right) \) using the market clearing for goods (22). Therefore, money spent is equal to the supply of money \( \bar{M} \), because households hold \( M^b_0 = 0 \) at \( t = 1 \), and only impatient households withdraw at \( t = 1 \) to buy consumption goods. Thus, \( P_1 = \bar{M} / (A_1 \bar{K}) \).

5 Bad equilibria without policy intervention

In order to analyze the bad equilibria, I impose some restrictions on endowments of banks, as if they where generated by an infinite-horizon economy in which a good equilibrium is realized before \( t = 0 \). These restriction simplify the analysis of the bad equilibria, without altering qualitatively the results.

**Assumption 5.1.** Let \(-1 < \psi^L < 0 < \psi^H \). The endowments of banks satisfy:

\[
K^L = \bar{K} \left(1 + \psi^L\right) \frac{1 - \kappa (1 - \beta)}{\kappa \beta} \\
K^H = \bar{K} \left(1 + \psi^H\right) \frac{1 - \kappa (1 - \beta)}{\kappa \beta}
\]
\[ M^b_{-1} = \frac{1 - \kappa (1 - \beta)}{\beta} \text{ for all } b \in B \]
\[ D^b_{-1} = \frac{1 - \kappa (1 - \beta)}{\beta} \text{ for all } b \in B \]

Before turning to the analysis of bad equilibria, the next Corollary derives some implications of Assumption 5.1 for the existence of the good equilibrium, and for banks’ and households’ capital holdings at \( t = 0 \).

**Corollary 5.2.** Assume Assumption 5.1 holds. Then a good equilibrium (described by Proposition 4.2) exists if and only if \( \psi^L \geq 0 \), and if such equilibrium exists it is characterized by:

\[ K^b_0 = K^b_{-1} \text{ for all } b \in B, \quad K^h_0 = K^h_{-1} \text{ for all } h \in H. \]

### 5.1 Bad equilibria

Under some restrictions on parameters, at least one bad equilibrium exists.\(^{31}\)

**Proposition 5.3.** Depending on the value of parameters, there exists either one, two, or no bad equilibria. If a bad equilibrium exists, it is characterized by:

- \( t = 0 \): flight to liquidity by households \( (M^h_0 > 0 \text{ and } D^h_0 < D^*) \), insolvency of banks with endowment of capital \( K^b_{-1} = L^L \):
  \[ N^b_0 < 0 \text{ if } K^b_{-1} = L^L, \quad N^b_0 > 0 \text{ if } K^b_{-1} = H^h, \]
  and sales of capital by banks to households:
  \[ K^b_0 < K^b_{-1} \text{ for all } b \in B, \quad K^h_0 > K^h_{-1} \text{ for all } h \in H. \]
- \( t = 1 \): runs on insolvent banks, i.e., \( l^h_1 = +\infty \) for a fraction \( 1 - \kappa \) of depositors of bank \( b \) (\( h \));
- \( t = 2 \): returns \( r^D_2 = r^K_2 > 1/\beta - 1 \) and actual return on deposits:
  \[ r^b_2 \in (-1, 0) \text{ if } K^b_{-1} = L^L, \quad r^b_2 = r^K_2 \text{ if } K^b_{-1} = H^h. \]

\(^{31}\)I mostly focus on regions of the parameter space in which multiple equilibria exist. There are, however, regions of the parameter space in which banks’ fundamentals are good enough so that no bad equilibrium exists, and regions of the parameter space in which banks’ fundamentals are bad enough so that no good equilibrium exists. Moreover, under some conditions, an additional third bad equilibrium exists in which all bank are insolvent \( (N^b_0 < 0 \text{ for all } b, \text{ rather than just for a fraction of banks}) \). As all banks are insolvent, asymmetric information is irrelevant in this equilibrium. More importantly, the existence of this equilibrium is sensitive to some timing assumptions related to how new banks enter the banking market; for more discussion, see Appendix D in Robatto (2015b).
The following paragraphs explain the results, while Section 5.2 presents a numerical example and compares it to the Great Depression and to the 2008 U.S. crisis. Section 5.3 discusses the proof; the complete proof and the exact parameter restrictions are in Appendix C.

At \( t = 0 \), households believe that banks with low endowments of capital, \( K^L \), will be subject to runs. However, due to asymmetric information, households do not know whether their own bank has high or low holdings of capital, and therefore they assign positive probability to a run on their own bank. Therefore at \( t = 0 \), households fly to liquidity by choosing \( M^b_h > 0 \) in order to self-insure against the risk of being among those last in line during a run, and reduce deposits compared to the good equilibrium, \( D^*_h < D^* \).

As a result of the flight to liquidity, and since there are only two assets in the economy, an increase in the demand for money implies a reduction in the demand for capital, resulting in a drop of the price of capital \( Q_0 \) because capital is in fixed supply. Due to the low \( Q_0 \) and the fixed value of endowments of deposits, which are liabilities for banks, a bank \( b \) with endowment \( K^b_{-1} = K^L \) is insolvent, i.e., \( N^b_0 < 0 \), as discussed in Section 2.5 and represented in Figure 3.

I claim that banks with endowment \( K^L \) will not be able to pay the promised return on deposits \( r^D_2 \) at \( t = 2 \) for two reasons. First, these banks are insolvent at \( t = 0 \), i.e., \( N^b_0 < 0 \), as discussed above. Second, the promised return on deposits \( r^D_2 \) is equal to the return on capital \( r^K_2 \), and therefore banks make zero profits on deposits. Thus, insolvency at \( t = 0 \) and zero profits on deposits imply that insolvent banks will not have enough resources at \( t = 2 \) and will pay an actual return on deposits \( r^b_2 < r^D_2 \).

At \( t = 1 \), impatient households want to withdraw money from their own banks in order to finance consumption expenditure. Moreover, for banks with \( r^b_2 < 0 \) (that is, banks with endowment \( K^b_{-1} = K^L \) and \( N^b_0 < 0 \)), impatient households want to “run” to withdraw as well. If these households manage to be among those first in line, they can recover the full value of deposits; if they do not run, or if they run and are among those last in line, they will incur losses on their deposits because the actual return on deposits \( r^b_2 \) is negative. Note that the belief at \( t = 0 \) that some banks in the economy will be subject to run is then verified at \( t = 1 \), confirming that this is indeed an equilibrium.

What happens to the price of consumption goods \( P_1 \)? At \( t = 0 \), all households fly to liquidity and hold \( M^b_0 > 0 \). At \( t = 1 \), patient households do not spend money (and they also do not spend the money withdrawn, if they run on their bank). Thus, a fraction of the money supply \( \overline{M} \) is

\[ \text{For solvent banks, the logic is the same as discussed after Proposition 4.2; for insolvent banks, see Appendix B.1.} \]
unspent. As a result:

\[ P_1 \int \mathcal{H} C^h dh = \left( \frac{\text{money spent}}{M} \right) \Rightarrow P_1 < \frac{M}{\int \mathcal{H} C^h dh} = \frac{M}{\frac{A \bar{K}}{P^*}} \]

where the last equality uses the market clearing condition for goods (22). Moreover, recall from Proposition 4.2 that the price level in the good equilibrium is \( P^* = \frac{M}{A \bar{K}} \). Thus, in a bad equilibrium the price level is lower, \( P_1 < P^* \); that is, the economy experiences deflation.\(^{33}\)

As stated by Proposition 5.3, banks reduce their holdings of capital compared to their initial endowment and thus, by market clearing, households increase their holdings of capital. There are two forces that contribute to this result. First, households hold fewer deposits compared to the good equilibrium, \( D^h_0 < D^*_h \), and in the good equilibrium banks’ capital holdings are equal to their endowment, \( K^b_0 = K^b_{-1} \). Therefore, since banks invest a fraction \( 1 - \kappa \) of deposits in capital (see discussion after Proposition 4.2) and deposits drop, capital held by banks declines as well. Second, there is a general equilibrium effect that transfers wealth from banks to households, as discussed in the next paragraph, thereby reducing the net worth of banks \( N^b_0 \). Using the budget constraint Equation (7), capital holdings by banks are \( K^b_0 = \left[ (D^b_0 - M^b_0) + N^b_0 \right] / Q_0 \), and thus a reduction of net worth contributes further to the reduction of \( K^b_0 \).

The reason for the transfer of wealth from banks to households is the denomination of the endowment of deposits \( D^b_{-1} \) and \( D^h_{-1} \) in terms of money. Since these deposits are fixed in nominal terms, a reduction in the price level generates debt-deflation, as described by Fisher (1933). Debt-deflation occurs when nominal prices drop and thus the real value of a debt denominated in nominal terms increases, transferring wealth from debtors to creditors. In the model, debt is represented by endowment of deposits \( D^b_{-1} \) and \( D^h_{-1} \), banks are debtors, and households are creditors.

### 5.2 Example

If two bad equilibria exist, they look qualitatively identical, as shown by an example in Table 2. In one of the equilibria, the flight to liquidity is more pronounced, the drop in prices is larger, and the losses of depositors at insolvent banks are larger (the return on deposits at insolvent banks is lower). Although the two equilibria are qualitatively identical, they have a difference related to the force that gives rise to them, namely, a strategic complementarity across depositors’ choices, as discussed in Section 5.3.

The model replicates qualitatively the evolution of monetary aggregates during the Great Depression in the US, depicted in Figure 4. Between 1931 and 1934, currency held by the public

\(^{33}\)More generally, in the infinite-horizon formulation, and if the bad equilibrium is unanticipated, the economy experiences lower-than-anticipated inflation.
Table 2: Equilibria comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Good Equilibrium</th>
<th>First Bad Equilibrium</th>
<th>Second Bad Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money, households</td>
<td>$M^h_0$</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>Deposits</td>
<td>$D^h_0$</td>
<td>2</td>
<td>1.41</td>
</tr>
<tr>
<td>M1</td>
<td>$M^h_0 + D^h_0$</td>
<td>2</td>
<td>1.71</td>
</tr>
<tr>
<td>Price of capital</td>
<td>$Q_0$</td>
<td>80</td>
<td>66.92</td>
</tr>
<tr>
<td>Price level</td>
<td>$P_1$</td>
<td>1</td>
<td>0.79</td>
</tr>
<tr>
<td>Return deposits, $b$ insolvent</td>
<td>$r^b_2$ s.t. $N^b_0 &lt; 0$</td>
<td>(n.a.)</td>
<td>-0.07</td>
</tr>
<tr>
<td>Capital held by households</td>
<td>$K^h_0/K$</td>
<td>0.977</td>
<td>0.982</td>
</tr>
<tr>
<td>Capital held by banks</td>
<td>$K^b_0/K$</td>
<td>0.023</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Parameter values: $\beta = 0.9877$, $\kappa = 0.5$, $\alpha = 0.17$, $\theta = 2.25$, $\psi^L = 0$, $\psi^H = 1$, $\bar{M} = 1$, $\bar{K} = 1$, $A_1 = 1$.

(i.e., money held by households) is high, and M1 and deposits are low, corresponding to a bad equilibrium in the model. Before 1931 and after 1934, currency held by the public is low, and M1 and deposits are high, corresponding to a good equilibrium.

Note that the drop in M1 with constant money supply is equivalent to a drop in the money multiplier, or, equivalently, a drop in money velocity.\(^{34}\) The drop in the money multiplier and money velocity are important monetary facts that characterize both the Great Depression (documented by Friedman and Schwartz, 1963) and the 2008-09 U.S. financial crisis.\(^{35}\) Notably, velocity drops in the model even if the central bank injects money into the economy, as long as monetary injections do not eliminate the bad equilibrium (see Section 7). Therefore, the model is consistent not only with the Great Depression, but also with the 2008-09 financial crisis.

Table 2 shows also that capital held by banks decreases during a crisis, as discussed in the previous Section. This result is consistent with the empirical evidence provided by He et al. (2010) about the 2008 U.S. financial crisis.\(^{36}\)

Moreover, the results about capital holdings contribute to the debate about the quantitative

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\(^{34}\)In this context, the money multiplier is the ratio of M1 over the money supply $\bar{M}$ and money velocity is defined by the equation of exchange $\bar{M} \times \text{velocity} = P_1 \times \text{output}$, where $\text{output} = A\bar{K}$.

\(^{35}\)In 2008-09 in the US, money velocity dropped dramatically, from about 17 in the second quarter of 2008, to about 8 in the first and second quarter of 2009 (calculations based on FRED St. Louis Fed Economic Data; velocity is computed as nominal GDP divided by the monetary base).

\(^{36}\)He et al. (2010) find that securitized assets shifted from sectors dependent on repo financing to commercial banks. If the banking sector in the model is interpreted as the shadow banking system, and households in the model are interpreted as including both the US non-financial sector and commercial banks, the model is consistent with the evidence of He et al. (2010).
importance of runs in the 2008 US financial crisis. Krishnamurthy et al. (2014) document that, as a result of the “run on repo,” the contraction in an important part of the repo market backed by private ABS (asset-backed securities) was small relative to the stock of private ABS. Based on this fact, Krishnamurthy et al. (2014) conclude that the run on repo could have not played a central role in the crisis, unless other considerations are added. However, the results of my model overturn the conclusions of Krishnamurthy et al. (2014). To see this, capital in the model must be interpreted as ABS and deposits must be interpreted as repo transactions. Table 2 shows that the stock of capital held by banks in the good equilibrium (i.e., ABS backing repo) is small relative to the total supply of capital (i.e., relative to the stock of private ABS), approximately 2%. As a result, in the event of a crisis, the decline in capital held by banks (i.e., ABS backing repo) cannot be large relative to the total supply of capital (i.e., relative to the stock of private ABS). Nonetheless, sizable general equilibrium effects arise in the bad equilibrium.

The motivation for the small fraction of capital in the hands of banks is related to two observations. First, for a reasonable choice of the discount factor $\beta$ (close to one), the value of consumption expenditure is small compared to the overall value of capital in the economy; second, households use banks only to insure against the need to finance their consumption expenditure (see Assumption 3.1).

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37 It is irrelevant whether banks in the model hold capital directly or hold claims on the cash-flow produced by capital. Since ABS are claims on the cash-flow produced by the securities backing the asset, there is an analogy between capital in the model and ABS in the data.

38 In an infinite-horizon formulation, the price of capital depends on the infinite stream of payoff derived from holding it, discounted at rate $\beta$. Thus, the price is proportional to $\frac{1}{1-\beta}$. Therefore, if $\beta$ is close to one, the overall value of capital in the economy is large, while the value of consumption expenditure is small.
5.3 Strategic complementarity and multiple bad equilibria

The flight to liquidity arises from a strategic complementarity in households’ decisions. If all other households are flying to liquidity and holding few deposits, household $h$’s best response is to fly to liquidity and hold few deposits as well, and vice-versa. I exploit this strategic complementarity to show the existence of the bad equilibria. For a given level of deposits, one can compute a best response whose fixed points are the equilibria of the model. However, given the structure of the model, it is more convenient to work with a similar “best response” for the actual return on deposits $r^b_2$, rather than for deposits. This “best response” is represented in the left panel of Figure 5.

Thus, the proof of Proposition 5.3 works as follows.

1. Fix a value $\tau^L$ for the actual return on deposits paid by insolvent banks, $r^b_2 = \tau^L$ for all banks $b$ endowed with low level of capital, $K^b_{-1} = K^L$ (horizontal axis in Figure 5);
2. Solve for the remaining endogenous variables using all the equations of the model except the one that defines $r^b_2$, Equation (13). Given $\tau^L$, the remaining endogenous variables solve a linear system of equations with a unique solution;
3. Use the values of the endogenous variables computed in step 2 to solve for $r^b_2|_{K^b_{-1}=K^L}$, using Equation (13).

This procedure yields a continuous mapping between $\tau^L$ and $r^b_2$, depicted in the left panel of Figure 5. Evaluating this mapping at $r^b_2 = \tau^L$ delivers a cubic polynomial in $\tau^L$, which can solve to find the fixed points. One of the roots of the polynomial is greater than zero and therefore does not correspond to an equilibrium.\(^{39}\) The other two roots are in the interval $(-1, 0)$, under some param-

\(^{39}\)The actual return on deposits $r^b_2$ is bounded below by -1 (if $r^b_2 = -1$, depositors lose 100% of their deposits, and
eter restrictions. Therefore, there are at most two bad equilibria. Under some weaker parameter restrictions, only one of the roots is in the interval \((-1, 0)\), and thus only one bad equilibrium exists. An example of this case is provided in the right panel of Figure 5.

Using this framework, the effects of the monetary injections can be analyzed by studying how a change in money supply affects the best response depicted in Figure 5. This is the focus of the next Section.

6 Monetary policy

Recall from Section 2.7 that the central bank chooses the money supply at time \(t = 0\), \(\bar{M}(1 + \mu_0)\). While \(\mu_0 = 0\) in the baseline model, this Section considers the case in which the central bank varies \(\mu_0\) and thereby changes the money supply.

6.1 The effects of “small” monetary injections

The next Proposition describes the effect of a monetary injection that marginally increases the money supply. The results apply both to asset purchases and loans to banks, and hold for any initial \(\mu_0\).

**Proposition 6.1.** (Small monetary injection) If two bad equilibria exist, then, for the bad equilibrium with \(r_{2b}^{b} = K_L\) “close” to \(-1\):

\[
\frac{d}{d\mu_0} \left( \frac{r_{2b}^{b}}{K_{1b} = K_L} \right), \quad \frac{dQ_0}{d\mu_0}, \quad \frac{dP_1}{d\mu_0} > 0, \quad \frac{dD_0}{d\mu_0}, \quad \frac{d(velocity)}{d\mu_0} \gg 0 \text{ (depending on parameters)}
\]

they cannot loose more than that and must be less than zero (otherwise patient households would not run on insolvent banks at \(t = 1\), see Appendix B.2).

Some general equilibrium analyses focus only on the “stable” equilibrium, that is, the one for which the best response crosses the 45 degrees line with a slope less than one, motivated by a tâtonnement stability argument. The general equilibrium literature has however moved beyond the analysis of tâtonnement stability, by focusing on a more detailed modeling of what happens while the equilibrium has not been reached yet, see e.g. Scarf (1960) and Fisher (2011). Nonetheless, even by focusing on the tâtonnement-stable bad equilibrium, multiple equilibria still exist (one good equilibrium and one bad equilibrium), and the counterintuitive effects of monetary injections (prices decrease, flight to liquidity is amplified, and losses of depositors of insolvent banks increase) arise in the tâtonnement-stable bad equilibrium.

That is, the results apply not only when comparing small injections to the case with no policy intervention, but also when studying further injections starting from a baseline scenario in which the central bank has already increased the money supply.
and for the other bad equilibrium, with $r^b_2|_{K_{b-1}=K^L}$ “close” to zero:

$$\frac{d\left(r^b_2|_{K_{b-1}=K^L}\right)}{d\mu_0}, \frac{dQ_0}{d\mu_0}, \frac{dP_1}{d\mu_0}, \frac{dD^b_0}{d\mu_0}, \frac{d\left(\text{velocity}\right)}{d\mu_0} < 0$$

where “velocity” is the velocity of money in the equation of exchange.

Two results stand out in Proposition 6.1. First, monetary injections reduce money velocity and deposits in at least one bad equilibrium; that is, they amplify the flight to liquidity. Second, monetary injections reduce nominal prices in one bad equilibrium. To understand these results, recall that money velocity is implicitly defined by the equation of exchange $\overline{M}(1 + \mu_0) \times \text{velocity} (\mu_0) = P_1 (\mu_0) \times \text{output}$, where I have emphasized that both $P_1$ and velocity are (possibly) functions of the monetary injection $\mu_0$; output $= A_1K$ is instead constant in the model.\footnote{The results of Proposition 6.1 can alternatively be explained by defining a “money velocity” with respect to $Q_0$ and the quantity of capital, rather than $P_1$ and the quantity of output.} If velocity were constant, as in several monetary models such as Lucas and Stokey (1987), an increase in $\mu_0$ would translate one-to-one in an increase in $P_1$. However, money velocity in the model is endogenous, and therefore a monetary injection does not necessarily translate into an increase of nominal prices.

The detailed proof of Proposition 6.1 is provided in Appendix C. The proof decomposes the effects of a monetary injection into two steps. Step 1 analyzes the (downward) shift of the “best response” depicted in the left panel of Figure 6 as $\mu_0$ increases, for a given $\tau^L$, showing that:

$$\frac{dD^b_0}{d\mu_0}\bigg|_{\tau^L \text{ fixed}} < 0, \quad \frac{d\left(\text{velocity}\right)}{d\mu_0}\bigg|_{\tau^L \text{ fixed}} < 0,$$
\[
\frac{dQ_0}{d\mu_0} \bigg|_{\tau^L \text{ fixed}} = 0, \quad \frac{dP_0}{d\mu_0} \bigg|_{\tau^L \text{ fixed}} = 0, \quad \frac{d r^b_2}{d\mu_0} \bigg|_{K^b_{1} = K^L} \bigg|_{\tau^L \text{ fixed}} < 0 .
\]

To understand this first step, fix \(\tau^L\). If velocity were constant, a monetary injection would increase nominal prices \(Q_0\) and \(P_1\). The higher \(Q_0\) would imply a drop in the return on capital \(r^K_2\), see Equation (10). Solvent banks pay the promised return on deposits \(r^D_2 = r^K_2\) because banks invest a fraction of deposits into capital (see Proposition 5.3). Therefore, the return \(r^D_2\) would decline with the monetary injection as well. This effect reduces the demand for deposits by households. Therefore, the flight to money and away from deposits is amplified. As a result of a larger flight to liquidity, more money will be unspent at \(t = 1\), compared to the case with no policy intervention (due to the larger flight to liquidity, at \(t = 1\) there is more money in the pockets of patient households, who consume \(C^h_1 = 0\)). More unspent money means less money in circulation, i.e., a reduction in velocity.

Thus, there are two forces at play. First, an increase in money supply tends to increase prices, but also to amplify the flight to liquidity \((D^h_0 \downarrow, M^h_0 \uparrow)\) and thereby to reduce velocity. Second, the reduction in velocity resulting from the first effect tends to reduce prices. The effects on prices offset each other and, for a given \(\tau^L\), \(P_1\) and \(Q_0\) are constant. Deposits decrease, however, worsening the strategic complementarity and shifting downward the “best response” in the left panel of Figure 6.

The second step of the proof recognizes that a monetary injection actually changes the values of \(\tau^L\) that correspond to fixed points in the left-panel of Figure 6. The actual return on deposits increases in one bad equilibrium, and decreases in the other. Thus, the second step of the proof analyzes how \(Q_0\), \(P_0\), and \(D^h_0\) change when moving along the new “best response” toward the new fixed point, showing that:

\[\quad \frac{dD^h_0}{d\tau^L} \bigg|_{\mu_0 \text{ fixed}} > 0, \quad \frac{d(\text{velocity})}{d\tau^L} \bigg|_{\mu_0 \text{ fixed}} > 0, \quad \frac{dQ_0}{d\tau^L} \bigg|_{\mu_0 \text{ fixed}} > 0, \quad \frac{dP_0}{d\tau^L} \bigg|_{\mu_0 \text{ fixed}} > 0 .\]

In the equilibrium for which the fixed point is achieved by a higher \(\tau^L\), the actual return on deposits paid by insolvent banks is higher, encouraging households to hold more deposits. This increase in deposits put upward pressure on velocity (for the same reason explained before, although with the opposite sign), translating into higher prices and explaining the first set of results in Proposition 6.1. Deposits are affected by two forces: the decrease that occurs when fixing \(\tau^L\), and the increase arising when allowing \(\tau^L\) to change to the new fixed point. The total effect on deposits is therefore

\[43\text{In an infinite-horizon version of the model, it is crucial that the monetary injection is temporary so that the price of capital in the following period (corresponding to } Q^*_2 \text{ in the three-period model) is not affected by the monetary injection.}\]
ambiguous, and depends on the value of parameters. Similarly, the total effect on velocity is ambiguous as well.

For the other bad equilibrium, for which the fixed point is achieved by a lower \( \tau^L \), the actual return on deposits paid by insolvent banks is lower. As a result, households reduce deposits even further (compared to the case with fixed \( \tau^L \)), driving down velocity and thus prices. The total effect on deposits and velocity is therefore unambiguously negative in this case.

Note that there is no violation of standard “monetary neutrality” results here. If I compare two identical economies with different \( \overline{M} \), real quantities in the two economies are the same even in the bad equilibria. This exercise, however, requires changing money supply not only in a crisis, but also before the crisis (by changing \( \overline{M} \), endowments change) and after the crisis (by changing \( \overline{M} \), and using Equation (23), \( \overline{Q}_2 \) and \( \overline{P}_2 \) change).

On a more general note about the class of “small” interventions, effective policies seem to be those that increase the interest rate on deposits. This is in contrast to the commonly held notion that policy interventions should aim at reducing interest rates (or spreads) during a crisis. In an infinite-horizon version of the model, an increase of interest rates can be achieved by a monetary injection that is permanent (\( \mu_2 \neq 0 \)), because such a policy would raise the future price of capital (i.e., \( \overline{Q}_2 \)) and thereby put upward pressure on interest rates. Permanently higher money supply, though, may come at the cost of future inflation.

### 6.2 Asset purchases vs. loans to banks

The sign of the results in Proposition 6.1 is independent of whether the central bank injects money using asset purchases or loans to banks. However, the magnitude of the effects of a monetary injection is different depending on whether the central bank uses asset purchases or loans to banks, as stated by the next Proposition.

**Proposition 6.2.** (Small monetary injections: asset purchases vs. loans to banks) Assume a bad equilibrium exists. Then:

\[
\left. \frac{d \left( r^L_2 \right)_{K^b_1-\bar{K}^L}}{d\mu} \right|_{\text{asset purchases}} > \left. \frac{d \left( r^L_2 \right)_{K^b_1-\bar{K}^L}}{d\mu} \right|_{\text{loans to banks}}.
\]

Thus, a monetary injection of a given size has larger effects if it is implemented by purchasing capital on the market, rather than by offering loans to banks. As explained above, for a given \( \tau^L \), monetary injections shift the “best response” downward. However, when the central bank offers loans to banks, the central bank incurs losses on loans made to insolvent banks (see Section 2.7). Therefore, the central bank shares the losses of insolvent banks with households. This indirect
effect increases the demand for deposits, counteracting the negative effects analyzed in Proposition 6.1. As a result, the best response in the right panel of Figure 6 does not drop as much as in the case of asset purchases, in which the central bank does not bear any loss. However, the force arising from loans to banks is not strong enough, and the “best response” shifts downward both under asset purchases and loans to banks. Formally, the result of Proposition 6.2 arises from the fact that loans to banks $B_{0}^{b}$ enters directly in the definition of $r_{2}^{b}$ whereas asset purchases don’t; see Equation (13).

### 6.3 The effects of “large” monetary injections

As discussed above, a monetary injection shifts the “best response” downward as represented in the left panel Figure 6. If the monetary injection is large enough, the downward shift is large as well and implies that the “best response” does not intersect with the 45-degree line, and therefore no bad equilibrium exists. The result follows as a Corollary of Proposition 6.1 and Proposition 6.2.

**Corollary 6.3.** (Large monetary injections) If two bad equilibria exist with $\mu_{0} = 0$, then there exists a $\hat{\mu} \in (0, +\infty)$ such that, if $\mu_{0} = \hat{\mu}$, no bad equilibria exist.

Thus, the central bank eliminates the bad equilibria by shifting down the “best response,” i.e., paradoxically, by increasing the strategic complementarity across depositors. In this scenario with two bad equilibria, asset purchases are more effective than loans to banks, in the sense that the size of the monetary injection required to eliminate the bad equilibria is smaller if the money is injected using asset purchases rather than loans to banks. This result follows from the fact that the downward shift of the “best response” is larger with asset purchases.

### 7 Robustness

The results above are derived under the assumption of a piecewise-linear utility of consumption $C_{1}^{h}$ for impatient households, Equation (3). In this Section, I relax this assumption and study the robustness of the results by modifying preferences of impatient households as follows:

$$
\bar{u}(C) = \begin{cases} 
\overline{C} \log C & \text{if } C < \overline{C} \\
\overline{C} \log \overline{C} + (C - \overline{C}) & \text{if } C \geq \overline{C}.
\end{cases}
$$

(27)

44 More precisely, a commitment by the central bank to perform a large monetary injection in the event a bad equilibrium arises is sufficient to prevent the bad equilibrium.  
45 Other analysis of “large” monetary injections can be performed in the model (such as studying the case in which only one bad equilibrium exists if $\mu_{0} = 0$, or comparing the results with parameter values such that a good equilibrium exists vs. parameter values for which only the bad equilibrium exists). However, they are all consistent with the result that a sufficiently large $\mu_{0}$ eliminates all bad equilibria. Results are available upon request.
The green dotted line represents the good equilibrium (without monetary intervention); the blue solid lines represent the two bad equilibria, as a function of the monetary intervention $\mu_0$. Parameter values: $\beta = 0.9877$, $\kappa = 0.869$, $\alpha = 0.1$, $\psi^L = 0$, $\psi^H = 1$, $\overline{M} = 1$, $\overline{K} = 1$, $A_1 = 1$.

Under this parameterization, the marginal utility $\bar{u}'(C)$ for $C < \overline{C}$ is endogenous. Thus, there is a richer feedback between policy choices and households’ choices than in the previous sections.

Under this new parameterization of preferences, the model is more complicated to solve and I have to rely on numerical simulation to compute the bad equilibria and to perform the policy analysis. However, I argue that all the forces discussed in the simpler version are still at work, although some results are quantitatively different.

**Numerical example.** A numerical example of bad equilibria and of monetary injections using asset purchases is represented in Figure 7. I choose parameters such that a good equilibrium exists, and such that a bank with endowment of capital $K^b_{-1} = K^L$ has $N^b_0 = 0$ in the good equilibrium. As in to the model with piecewise-linear utility, two bad equilibria exist. The bad equilibrium

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46 Preferences are also smooth, because the marginal utility approaches one as $C \uparrow \overline{C}$.

47 The preferences in (27) are still consistent with an infinite-horizon formulation; see Appendix A. In a previous version of the paper, similar numerical results were obtained using preferences of the form $\log C$ for all $C > 0$.

48 While it is possible to follow an approach similar to the proof of Proposition 5.3 to show the existence of bad equilibria, finding the fixed points as described in Section 5.3 requires solving a complicated quartic (fourth order) polynomial, making the proof de facto intractable.
The green dotted line represents the good equilibrium (without monetary intervention). The blue solid lines represent the two bad equilibria, as a function of the monetary intervention $\mu_0$. Parameter values: $\beta = 0.9877$, $\kappa = 0.869$, $\alpha = 0.1$, $\psi^L = 0$, $\psi^H = 1$, $M = 1$, $\overline{K} = 1$, $A_1 = 1$.

without policy intervention corresponds to $\mu_0 = 0.49$.

In this example, velocity and deposits drop in one bad equilibrium, and are approximately constant in the other, similarly to the result of Proposition 6.1. However, the effects of monetary injections on prices and actual return on deposits $r^{h}_{1|K^h_{1} = K^L}$ are quantitatively different from Proposition 6.1, although I argue that the logic behind the results is the same.

Recall from Proposition 6.1 that the drop in velocity offsets the direct effect of the monetary injection. Therefore, prices $Q_0$ and $P_1$ decrease in one of the bad equilibria. In Figure 7, however, the drop in velocity (in one of the bad equilibria) is not enough to offset the direct effect of the monetary injection on prices. Therefore, $Q_0$ and $P_1$ are increasing in $\mu_0$ in both bad equilibria. When $Q_0$ and $P_1$ are pushed to the good equilibrium level, the bad equilibria are eliminated (see Figure 7, top left and top middle panels).

The actual return on deposits (Figure 7, bottom right panel) is increasing in one bad equilibrium, and approximately constant in the other. Therefore, under the preferences in Equation (27),

49The model is solved in Mathematica using the command NSolve, that computes the numerical Gröbner bases associated with the system of polynomial equations that define the equilibrium. See Kubler and Schmedders (2010) for an introduction to Gröbner bases applied to the computation of equilibria in economic models.
monetary policy does not simply shift the “best response” downward as in the left panel of Figure 6, but it rather bends it in a more complicated way.

The results are quite different if the central bank uses loans to banks, depicted in Figure 8. These results can be understood by analyzing the bottom right panel, the actual return on deposits paid by insolvent banks $r_2^b|K_{b_1}^b=K_L$. Monetary injections implemented as loans to banks push the actual return $r_2^b|K_{b_1}^b=K_L$ either above zero or below $-1$, depending on the bad equilibrium under analysis (the actual return on deposits $r_2^b|K_{b_1}^b=K_L$ must be between $-1$ and zero to have a bad equilibrium, see Section 5.3).

In the model with piecewise-linear utility, recall from Proposition 6.2 that loans to banks partially offset the downward shift of the “best response.” That result follows from the fact that the central bank shares the losses of insolvent banks with households, and so households increase their demand for deposits. The same force is at work under the parameterization of preferences in Equation (27), but in this case loans to banks more than offset the downward shift of the “best response.” Therefore the “best response” shifts upward, reducing $r_2^b|K_{b_1}^b=K_L$ for the bad equilibrium with $r_2^b|K_{b_1}^b=K_L$ close to $-1$, and increasing $r_2^b|K_{b_1}^b=K_L$ for the other bad equilibrium.

Differently from the case of asset purchases in Figure 7, the central bank does not need to push $Q_0$ and $P_1$ up to the good equilibrium level in Figure 8 to eliminate the bad equilibria, because it acts directly on the strategic complementarity across depositors. Therefore, in this example, the monetary injection required to eliminate the bad equilibrium using loans to banks is smaller than the monetary injection required to eliminate the bad equilibrium using asset purchases.\(^{50}\)

8 Conclusions

I have presented a new framework to analyze bank runs in a general equilibrium monetary model of banking, and I have used the framework to study unconventional monetary policy during financial crises. The direct positive effect of monetary injections is offset by a general equilibrium feedback that reduces money velocity and interest rates. Contrary to the commonly held notion that reducing interest rates (or spreads) always helps to ameliorate a crisis, I have shown an opposite result: lower interest rates decrease demand for deposits, amplifying the flight to liquidity that is the source of the crisis. The interaction between monetary injections and the strategic complementarity that gives rise to multiple equilibria shapes the overall effects of policy interventions.

This paper opens up several directions for future research. First, on the theoretical side, more

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\(^{50}\) In a richer model in which solvent banks are leverage constrained, from a partial equilibrium perspective a bank might be unable to get a loan from the central bank without increasing leverage. Yet, general equilibrium effects reduce leverage by increasing the price of capital $Q_0$ (thereby increasing assets) and reducing deposits (thereby decreasing liabilities). In the numerical example of Figure 8, the general equilibrium effects prevail and thus loans to banks reduce leverage in equilibrium.
research is needed to identify the frictions that justify some of the assumptions that I have used, such as the nominal deposit contract. Second, the framework I have presented can be used to analyze other policies such as capital requirements and equity injections; I pursue this topic in my companion paper (Robatto, 2015a). Third, on the empirical side, a richer dynamic version of the model can be employed in quantitative analyses to assess how panic-based runs, fundamental shocks, and other financial frictions contribute to financial crises.

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Appendix

A Infinite-horizon formulation

The infinite-horizon formulation that generalizes the models of Section 2 and Section 7 is identical to the one presented in the previous working paper version of this paper (Robatto, 2015b), with one difference.

In the infinite-horizon formulation in Robatto (2015b), impatient households have utility function \( \log C \) for all \( C > 0 \). The optimization problem of households can be represented recursively using a Bellman equation. Due to log utility, it is possible to guess and verify that the corresponding household value function is also logarithmic, in household’s wealth.

Instead, in the infinite-horizon formulation that corresponds to the models of Section 2 and Section 7, the utility function of impatient households has the same functional form as in (2) and (3). Therefore, the utility function is locally linear and it is possible to guess and verify that the corresponding value function is linear in wealth, as well (this follows the approach used by Lagos and Wright, 2005).\(^{51}\) As a result, when facing the utility maximization problem at time \( t \), an household’s continuation value is linear in wealth in \( t + 1 \).

Since the continuation value of the infinite-horizon formulation is linear, the problem of an household collapses to the one presented in the 3-period model. The \( t + 1 \) continuation value of

\(^{51}\) More precisely, one need to assume that \( \overline{C} < \frac{A_K}{\pi} \), rather than holding with equality as in Equation (24). This implies that, in the good equilibrium in which impatient households consume \( \frac{A_K}{\pi} \), the utility function is locally differentiable, rather than at a kink. However, \( \overline{C} \) can be arbitrary close to \( \frac{A_K}{\pi} \), therefore Equation (24) can be interpreted as holding in the limiting economy in which \( \overline{C} \uparrow \frac{A_K}{\pi} \).
the infinite-horizon model corresponds to period-2 utility in the 3-period model, which is linear as well. Thus, all the results of households’ utility maximization of the infinite horizon model are equal to those of the 3-period model.\footnote{In the infinite-horizon model, an household subject to a long sequence of “impatient” preference shocks will be driven to a region with negative net worth. To avoid this problem, households can be grouped together in large families, as in Gertler and Kiyotaki (2015), therefore the representative family has a constant wealth over time.}

B Banks and households: individual problem

B.1 Banks: profit maximization

Bank $b$ chooses deposits $D^b_0$, money $M^b_0$, capital $K^b_0$, and loans from the central bank $B^b_0$ in order to maximize consumption at $t = 2$, defined by Equation (12), subject to the budget constraint in Equation (7) and the non-negativity constraints $K^b_0 \geq 0$, $M^b_0 \geq 0$, $D^b_0 \geq 0$, and $B^b_0 \geq 0$, where $N^b_0$ is defined by Equation (4). The next Proposition provides the solution to the bank problem.

**Proposition B.1.** Given $N^b_0$ and prices $Q_0$, $r^D_2$, and $r^K_2 \geq 0$, the optimal choice of bank $b$ is:

1. deposits:
   
   $D^b_0 = \begin{cases} 
   0 & \text{if } r^D_2 > r^K_2 \\
   \text{any amount } \geq 0 & \text{if } r^D_2 = r^K_2 \\
   +\infty & \text{if } r^D_2 < r^K_2; 
   \end{cases}$

2. loans from the central bank:
   
   $B^b_0 = \begin{cases} 
   0 & \text{if } r^{CB}_2 > r^K_2 \\
   \text{any amount } \geq 0 & \text{if } r^{CB}_2 = r^K_2 \\
   +\infty & \text{if } r^{CB}_2 < r^K_2; 
   \end{cases}$

3. money holdings $M^b_0 = \kappa D^b_0$ and capital holdings $K^b_0 = \frac{N^b_0 + D^b_0 - M^b_0 + B^b_0}{Q_0}$.

provided that the non-negativity constraint $K^b_0 \geq 0$ is not binding.

For a bank with positive net worth, $N^b_0 \geq 0$, the explanation is the same as the one provided after Proposition 4.2, in Section 4. For a bank with negative net worth, $N^b_0 < 0$, I explain here only the relevant equilibrium case $r^D_2 = r^K_2$ and $r^{CB}_2 = r^K_2$. In this case, banks earn no profits on deposits or on loans from the central bank. Therefore, since the bank is insolvent at $t = 0$ ($N^b_0 < 0$) and makes zero profits on deposits, it will continue to be insolvent at $t = 2$. Due to limited liability, the bank’s payoff will always be zero, and thus the bank is indifferent among any
choice. Therefore, making the same choices as a solvent bank is (weakly) optimal in the sense that no profitable deviation exists.\textsuperscript{53}

Proof of Proposition B.1. First, focus on the case \( N_0^b \geq 0 \) and guess that \( C_2^b > 0 \) (or \( C_2^b = 0 \) with assets equal liabilities in Equation (12)). Then, from Equation (13), the bank will pay the promised return on deposits \( r_2^D \). As a result, only impatient households will withdraw at \( t = 1 \), and a fraction \( \kappa \) of depositors are impatient. Therefore, bank \( b \) chooses \( M_0^b = \kappa D_0^b \) in order to have enough money to repay depositors at \( t = 1 \). Withdrawals are given by \( W_1^b = \kappa D_0^b \), and they satisfy the feasibility constraint (9).

Therefore, banks invest a fraction \( 1 - \kappa \) of the deposits they receive in capital. Since the return on capital is \( 1 + r_2^K \), each dollar of deposit yields a gross return \( (1 - \kappa) (1 + r_2^K) \) at \( t = 2 \). As the bank has to pay a gross return \( 1 + r_2^D \) on the fraction \( 1 - \kappa \) of deposits not withdrawn, banks’ profits for each dollar of deposits are
\[
(1 - \kappa) [(1 + r_2^D) - (1 + r_2^K)] = (1 - \kappa)(r_2^K - r_2^D).
\]
Thus, if \( r_2^D > r_2^K \), banks make negative profits on deposits, thus they are willing to hold \( D_0^b = 0 \); if \( r_2^D = r_2^K \) they make zero profits on deposits, therefore they are willing to hold any amount; and if \( r_2^D < r_2^K \), they make positive profits, thus they are willing to hold an infinite amount of deposits.

The result about loans from the central bank \( B_0^b \) can be proven in the same way. Note that because of non-negative profits on deposits and loans from the central bank, and from \( N_0^b \geq 0 \), the initial guess about \( C_2^b \) is verified, using Equation (12).

Next, consider the case in which \( N_0^b < 0 \). If \( r_2^D < r_2^K \) or \( r_2^{CB} < r_2^K \) (or both), the bank can achieve \( C_2^b > 0 \) by issuing \( D_0^b = +\infty \) and/or \( B_0^b = +\infty \). Therefore, guessing \( C_2^b > 0 \) and following the same steps as before allows us to verify the guess.

To complete the proof, I have to analyze the case \( N_0^b < 0, r_2^D \geq r_2^K, \) and \( r_2^{CB} \geq r_2^K \). Intuitively, banks cannot make positive profits on deposits or on loans from the central bank, therefore their consumption at \( t = 2 \) is zero due to limited liability. The next Lemma formalizes this idea.

Lemma B.2. If \( N_0^b < 0, r_2^D \geq r_2^K \geq 0, \) and \( r_2^{CB} \geq r_2^K \geq 0, \) then \( C_2^b = 0 \) and \( r_2^b < r_2^D \) for any feasible choice \( \{D_0^b, B_0^b, M_0^b, K_0^b\} \).

\textbf{Proof.} Using Equation (12):
\[
C_2^b = \frac{1}{P_2} \max \left\{ 0; Q_0 K_0^b (1 + r_2^K) - (D_0^b - W_1^b) (1 + r_2^D) - B_0^b (1 + r_2^{CB}) \right\}
= \frac{1}{P_2} \max \left\{ 0; N_0^b (1 + r_2^K) + (D_0^b + B_0^b - M_0^b) (1 + r_2^K) - (D_0^b - W_1^b) (1 + r_2^D) - B_0^b (1 + r_2^{CB}) \right\}
\]
\textsuperscript{53}Note also that a bank with \( N_0^b < 0 \) does not have enough resources to invest all its deposits into money. If \( K_0^b = 0 \) (and \( B_0^b = 0 \)), then \( M_0^b = D_0^b + N_0^b \) from the budget constraint, and it follows that \( M_0^b < D_0^b \) because \( N_0^b < 0 \).
\[
= \frac{1}{P_2} \max \left\{ 0; N_0^b \left(1 + r_2^K\right) + r_2^K \left(D_0^b - M_0^b\right) - r_2^D \left(D_0^b - W_1^b\right) + B_0^b \left(r_2^K - r_2^{CB}\right) \right\}
\]

where the second line uses the budget constraint (7) and the third line rearranges. Since feasibility (9) requires \( W_1^b \leq M_0^b \), then \( D_0^b - M_0^b \leq D_0^b - W_1^b \). Note that \( D_0^b - M_0^b > 0 \) using the budget constraint (7) and the assumption \( N_0^b < 0 \). Therefore, using the assumption \( 0 \leq r_1^K \leq r_1^D \) and \( r_2^{CB} \geq r_2^K \geq 0 \) of the Lemma, it follows that \( r_2^K \left(D_0^b - M_0^b\right) - r_2^D \left(D_0^b - W_1^b\right) \leq 0 \) and \( B_0^b \left(r_2^K - r_2^{CB}\right) < 0 \). Furthermore, using \( N_0^b < 0 \) it also follows \( N_0^b \left(1 + r_2^K\right) < 0 \). Thus, consumption is constraint at zero, and using Equation (13), the result \( r_2^b < r_2^D \) follows.

As a result of the previous Lemma, it is (weakly) optimal to take the choices described by Proposition B.1, in the sense that no deviation from those choices is profitable.

**B.2 Households: individual behavior**

I now turn to the analysis of the behavior of households, starting at \( t = 1 \) and proceeding backwards (at \( t = 2 \), the consumption is defined by Equation (11) and households make no choices).

At \( t = 1 \), all uncertainties are resolved for household \( h \). Household \( h \) observes her preference shock, the limit on withdrawals \( l_1^h \), and the actual return on deposits paid by her bank \( r_{b(h)}^2 \); she then decides withdrawals \( W_1^h \) and purchases of consumption goods \( C_1^h \). The following proposition summarizes the choices of households at \( t = 1 \), in the relevant case in which the return on promised return on deposits is not “too high” so that impatient households prefer to withdraw at \( t = 1 \).

**Proposition B.3.** Assume Assumption 3.1 holds, \( r_2^D \geq 0 \), and \( \frac{1}{P_1} \geq \frac{\beta (1 + r_2^D)}{P_2} \). Then, at \( t = 1 \), households choose withdrawals:

\[
W_1^h = \begin{cases} 
D_0^b & \text{if } l_1^h = +\infty; \text{ and if } h \text{ is impatient, or } r_{b(h)}^2 < 0, \text{ or both} \\
0 & \text{otherwise}
\end{cases}
\]

and consumption:

\[
C_1^h = \begin{cases} 
\frac{M_0^b + W_1^h}{P_1} & \text{if impatient} \\
0 & \text{if patient}.
\end{cases}
\]

**Proof.** Consumption choices follow directly from the utility function, Equation (1) and Equation (3), and from the cash-in-advance constraint Equation (8). In particular, an impatient household who, at \( t = 1 \), consumes more than \( C \), compares:

\[
\frac{1}{P_1} \geq \beta \left(1 + r_2^D\right) \quad \text{and} \quad \frac{1}{P_2}
\]

\( \text{marginal utility of a dollar } t=1 \)

\( \text{return paid at } t=2 \)

\( \text{marginal utility of a dollar } t=2 \)

(29)
which holds by assumption; thus she prefers to withdraw all her deposits at \( t = 1 \) rather than wait until \( t = 2 \). If the impatient household consumes less than \( \overline{C} \), her marginal utility of a dollar at \( t = 1 \) is \( \theta/P_1 > 1/P_1 \) because \( \theta > 1 \), thus using Equation (29) the household prefers to withdraw at \( t = 1 \) as well.

If \( r_2^{b(h)} = r_2^D \), and since \( r_2^D \geq 0 \), a patient household is better off by waiting until \( t = 2 \) and getting a positive return on deposits, rather than withdrawing at \( t = 0 \). If instead \( r_2^{b(h)} < 0 \), a patient household is better off by running on the bank and withdrawing; otherwise she would take a loss; see Equation (11). In any case withdrawals are possible provided that the limit is \( l_1^h = +\infty \).

At \( t = 0 \), households solve:

\[
\max_{M_0^h, D_0^h, K_0^h} \mathbb{E} \left[ \tilde{u}(C_1^h) + \beta C_2^h \right]
\]  

subject to the budget constraint, Equation (7). The expectation in (30) is taken with respect to the preference shock, \( l_1^h \), and \( r_2^{b(h)} \). While the probability distribution over the preference shock is given by (2), the household forms beliefs about \( l_1^h \) and \( r_2^{b(h)} \), denoted by \( \Pr^h \left( l_1^h, r_2^{b(h)} \right) \) (see Section 3.2).

To formalize the problem at \( t = 0 \), consider the three cases that can happen at \( t = 1 \):

1. \( r_2^{b(h)} = r_2^D \) and \( l_1^h = +\infty \): this is the case if the bank pays the promised return on deposits, and there is no run on the bank;
2. \( r_2^{b(h)} < 0 \) and \( l_1^h = +\infty \): this is the case if the actual return on deposits is negative and there is a run on the bank, but household \( h \) is among those first in line, and thus able to withdraw without any limit;
3. \( r_2^{b(h)} < 0 \) and \( l_1^h = 0 \): this is the case if the actual return on deposits is negative and there is a run on the bank, and household \( h \) is among those last in line, thus unable to withdraw.

No matter which case is realized, the value invested in capital, \( Q_0 K_0^h \), gives a return \( 1 + r_2^K \) at \( t = 2 \) and can be transformed into consumption goods at rate \( 1/P_2 \). Thus, the problem of household at \( t = 0 \) includes these three possibilities:

\[
\max_{M_0^h, D_0^h, K_0^h} \beta \frac{Q_0 K_0^h (1 + r_2^K)}{P_2} \left[ \Pr^h \left( l_1^h = +\infty, r_2^{b(h)} = r_2^D \right) \right]
\]

\[
\begin{align*}
\text{Case 1:} & \quad \kappa \left( \frac{M_0^h + D_0^h}{P_1} \right) + (1 - \kappa) \beta \frac{D_0^h (1 + r_2^D) + M_0^h}{P_2} \\
\text{patient:} & \quad \text{impatient:}
\end{align*}
\]
If the bank pays the promised return and there are no runs (Case 1), using Proposition B.3, the household withdraws $W^h_1 = D^h_0$ and spends $M^h_0 + D^h_0$ only if she is impatient (i.e., with probability $\kappa$), and withdraws and spends zero if patient (i.e., with probability $1 - \kappa$). If $r^h_2 < 0$ and $l^h_1 = +\infty$ (Case 2), using Proposition B.3, the household withdraws $W^h_1 = D^h_0$ no matter what her preference shock is, and spends $M^h_0 + D^h_0$ only if she will be patient (i.e., with probability $\kappa$), and zero otherwise (i.e., with probability $1 - \kappa$). Finally, if $r^h_2 < 0$ and $l^h_1 = 0$ (Case 3), using once more Proposition B.3, household $h$ can only use her money $M^h_0$ to buy consumption goods if she is impatient (i.e., with probability $\kappa$); regardless of whether she will be patient or impatient, she will get back her deposits plus return $D^h_0 \left(1 + r^h_2 \right) < D^h_0$ at $t = 2$ and therefore take a loss on deposits. Note that in the last case, I am guessing that the household is not holding much money, thus the marginal utility if patient must be $\theta > 1$; and in the first two cases, I guess that the household holds enough money and deposits such that her marginal utility if patient is one. These guesses will then be verified when showing the existence of equilibria.

The FOCs of the problem, together with the budget constraint Equation (5), can be used to find the solution.

C Proofs

Proof of Proposition 4.2. Given the price of capital $Q_0$ and Equation (25), the definition of net worth, Equation (4), implies that all banks have $N^b_0 \geq 0$ and so are solvent. The choices of banks follow from Proposition B.1. Using the definition of $C^b_2$, Equation (12), then $C^b_2 \geq 0$ for all $b$ (and, if $C^b_2 = 0$, the zero lower limit on consumption is not binding), and thus $r^b_2 = r^D$ for all $b$ from Equation (13).

Since households beliefs must be rational, then $Pr^h \left(l^h_1 = +\infty, r^h_2 = r^D_2 \right) = 1$ and the household problem (31) becomes:

$$\max_{M^h_0 \geq 0, D^h_0 \geq 0, K^h_0 \geq 0} \beta \frac{Q_0 K^h_0 (1 + r^K_2)}{P_2} + \left[ \kappa \left( \frac{M^h_0 + D^h_0}{P_1} \right) + (1 - \kappa) \frac{D^h_0 (1 + r^D_2) + M^h_0}{P_2} \right].$$
subject to the budget constraint Equation (5). Since \( 1 + r_2^D = 1/\beta > 1 \), then the non-negativity constraint on money is binding, thus \( M_0^h = 0 \). The FOCs with respect to capital and deposits can be combined, yielding:

\[
\kappa \frac{1}{P_1} + (1 - \kappa) \beta \frac{1 + r_2^D}{P_2} = \beta \frac{1 + r_2^K}{P_2}
\]

Using \( r_2^D = r_2^K \), the price level \( P_1 = \frac{M}{A_1K} \), the restriction on \( P_2 \) in (23), and rearranging:

\[
1 + r_2^K = 1/\beta.
\]

Thus, the household is indifferent among any amount of deposits, and \( D_0^h \) is determined by Assumption 3.1 as the minimum amount required to finance consumption at \( t = 1 \). Capital \( K_0^h \) is determined residually by the budget constraint. Also, the assumption \( \frac{1}{P_1} \geq \frac{\beta (1 + r_2^D)}{P_2} \) in Proposition B.3 holds because \( P_1 = \frac{M}{A_1K} \) and thus \( P_1 = P_2 \) using Equation (23), and because \( r_2^D = r_2^K \) and \( 1 + r_2^K = 1/\beta \). The guess that the marginal utility of consumption in Case 1 in problem (31) is equal to one follows from the fact that all agents consume \( \overline{C} \), given the utility function in Equation (3).

The market clearing condition for money holds because

\[
\int_B M_0^h db + \int_H M_0^h dh = \int_B \kappa D_0^h db = M
\]

where I have used the result from the profit maximization of banks (see Proposition B.1). The market clearing condition for goods at \( t = 1 \) holds because \( \int_H C_1^h dh = \kappa \frac{A_1K}{K} = A_1K \) (where the first equality uses that \( C_1^h = 0 \) for patient households and \( C_1^h = \frac{A_1K}{K} \) for impatient households, and there is a mass \( \kappa \) of impatient households). The market clearing condition for capital holds by Walras’ Law.

Finally, to prove that Equation (25) is necessary for the existence of a bad equilibrium, assume by contradiction that a good equilibrium exists but Equation (25) does not hold. Thus, some banks in the economy have negative net worth, and from Lemma B.2 \( r_2^b < r_2^D \) for these banks and the beliefs of households must reflect that, leading to different choices than those analyzed before. This is a contradiction. \( \square \)

**Proof of Proposition 5.3.** Before proving the result, I list the restrictions on parameters required to prove the result:

\[
1 - 2\alpha > \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa], \quad (32)
\]

\[
\kappa M < (A_1K) \frac{P_2K (1 - 2\alpha - (\theta - 1) \alpha [\alpha (1 - \kappa) + \kappa])}{\beta (1 - 2\alpha) \kappa} < M (1 - \alpha (1 - \kappa)), \quad (33)
\]
\[ \Delta \equiv \left( \kappa^2 \left( \beta + \alpha^2 \left( \beta + 2(\beta - 1)\psi^L - 2 \right) + \alpha (-2\beta + \theta - 2(\beta - 1)\psi^L + 1) \right) \right) \\
= -(\alpha - 1)\kappa(\alpha(\beta + (3\beta - 2)\psi^L - 2) - 1) + (\alpha - 1)\alpha\beta\psi^L \right)^2 \\
+ \left[ 4\alpha(\alpha - 1)^2(\kappa - 1)\kappa(\alpha(\beta - 1)(\kappa - 1)(\psi^L + 1) - \beta\kappa) \right] \\
\times \left[ (\alpha(-\theta) + \alpha + \kappa(\alpha(\theta - 1) + (\beta - 1)(-\psi^L) + 1) + \beta\psi^L - 1) \right] > 0 \quad (34) \]

\( \psi^H \) sufficiently large, and, defining:

\[ \tau^L \left( \sqrt{\Delta} \right) = \frac{\kappa / \alpha \beta}{\kappa((\theta - 1)\kappa + 1) - 1 + \sqrt{\Delta} - (\alpha - 1)\beta(\kappa(\alpha(k - 1) - \kappa) + \alpha(\kappa - 1)\psi^L)} \times \left( \beta + 2\alpha^4(\beta - 1)(\theta - 1)(\kappa - 1)^2(\psi^L + 1) + \alpha(\kappa^2(\beta - \beta\theta) + \kappa(-5\beta - 2(\beta - 1)\psi^L + 2)) \right) \\
- 2\alpha^3(\kappa - 1)((\theta - 1)\kappa(3\beta - 2(\beta - 1)\psi^L - 2) - (\beta - 1)(\theta - 3)(\psi^L + 1)) \\
+ 2\alpha^2(\beta(2(\theta - 1)\kappa^2 - 2(\theta - 3)\kappa - 3) - (\kappa - 1)((\theta - 1)\kappa + 3) + (\beta - 1)(\kappa - 1)((\theta - 1)\kappa + 3)\psi^L) \\
- (2\alpha - 1)\beta(\sqrt{\Delta} - (\alpha - 1)\beta(\kappa(\alpha(k - 1) - \kappa) + \alpha(\kappa - 1)\psi^L)) + \alpha(2(\beta - 1)(\psi^L + 1)) \right] \]

either \( \tau^L \left( \sqrt{\Delta} \right) \in (-1, 0) \) or \( \tau^L \left( -\sqrt{\Delta} \right) \in (-1, 0) \) (only one bad equilibrium exists), or both \( \tau^L \left( \sqrt{\Delta} \right) \in (-1, 0) \) and \( \tau^L \left( -\sqrt{\Delta} \right) \in (-1, 0) \) (two bad equilibria exists). While these conditions are difficult to interpret, their implications are simple to understand, due to the steps in which they are used in the proof. Equations (32) and (33) guarantee that prices, money holdings, and deposits are positive; Equation (34) guarantees that the strategic complementarity, described in Section 5.3, is strong enough; and the restrictions on \( \tau^L \left( \sqrt{\Delta} \right) \) and \( \tau^L \left( -\sqrt{\Delta} \right) \) guarantee that the actual return on deposits paid by insolvent banks is between -1 and 0, as explained in Section 5.3.

I can now prove the result. Households’ beliefs are given by

\[ \Pr^h \left( l^h_1 = +\infty, \ r^b_{2(h)} = r^D_2 \right) = 1 - \alpha \]
\[ \Pr^h \left( l^h_1 = +\infty, \ r^b_{2(h)} < 0 \right) = \alpha \kappa \]
\[ \Pr^h \left( l^h_1 = +\infty, \ r^b_{2(h)} < 0 \right) = \alpha (1 - \kappa) \]

because a fraction \( 1 - \alpha \) of banks are solvent (those with endowment of capital \( K^H \)) and a fraction \( \alpha \) are insolvent (those with endowment \( K^L \)). Moreover, the probability that \( l^h_1 = +\infty \) at an insolvent bank is equal to \( \kappa \) and follows from plugging the choices of banks in Proposition B.1 into Equation (17).

The rest of the proof proceeds as described in Section 5.3. The first step is to fix \( \tau^L \in (-1, 0) \)
and to set \( r_2^b = \bar{\tau}^L \) for banks \( b \) such that \( K_{c1}^b = K^L \) (i.e., insolvent banks).\(^{54}\) The second step is to solve for the equilibrium value of all other endogenous variables, for a fixed \( \tau^L \). The third step is to compute \( r_2^b \) using Equation (13), obtaining a mapping \( \tau^L \to r_2^b (\tau^L) \), and finding the fixed points of the mapping.

Given \( \tau^L \in (-1, 0) \), I solve the other endogenous equation by looking first at the FOCs of the household problem, (31), combined with the beliefs (36)-(38). The FOCs with respect to \( M_{0h}^b \) and \( D_{0h}^b \) (after plugging in the value of the Lagrange multiplier of the budget constraint using the FOC of the household problem with respect to \( K_{0h}^b \), and using \( r_2^D = r_2^K \)) are a linear system in \( \frac{1}{P_1} \) and \( r_2^K \), for a given value of \( r_2^b = \tau^L \):

\[
\begin{bmatrix}
\kappa [1 - \alpha + \alpha \kappa + \alpha (1 - \kappa) \theta] & -\beta \frac{P_2}{P^2} \\
\kappa [1 - \alpha (1 - \kappa)] & -\beta \frac{L_1}{P^2}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{P_1} \\
r_2^K
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\beta \kappa}{P_2} \\
\beta \kappa + \alpha (1 - \kappa) \left(-\tau^L - \kappa\right)
\end{bmatrix}
\]

The determinant of the system is:

\[
\frac{(1 - \kappa) \kappa \beta [\alpha (\tau - 1) (\alpha (1 - \kappa) + \kappa) - (2 \alpha - 1)]}{P^2} < 0
\]

because \( \theta > 1, 0 < \kappa < 1 \), and, from Equation (32), \( 0 < \alpha < 1 / 2 \). Thus, the solution of the system exists and is unique. Let \( P_1 (\tau^L) \) and \( r_2^K (\tau^L) \) be the solution:

\[
P_1 (\tau^L) = \frac{P_2 \kappa (1 - 2 \alpha - \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa])}{\beta \left(1 - 2 \alpha - \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa]\right)} \tag{39}
\]

\[
r_2^K (\tau^L) = \frac{\alpha (\theta - 1) [\alpha (1 - \kappa) (-\tau^L - \kappa) + \kappa] - \alpha \tau^L}{1 - 2 \alpha - \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa]} \tag{40}
\]

Note that \( P_1 (\tau^L) > 0 \) due to Assumption (32); \( r_2^K (\tau^L) > 0 \) too, because the denominator is strictly positive due to Equation (32), and the numerator is continuous and monotone in \( \tau^L \) and converges to \( \alpha (\theta - 1) \kappa [1 - \alpha (1 - \kappa)] > 0 \) as \( \tau^L \to 0 \), and it converges to \( \alpha (\theta - 1) \left[\alpha (1 - \kappa)^2 + \kappa\right] + \alpha > 0 \) as \( \tau^L \to -1 \). As a result, the price level \( Q_0 (\tau^L) \) implicitly defined by:

\[
1 + r_2^K (\tau^L) = \frac{Q_2 + A_1 P_1 (\tau^L)}{Q_0 (\tau^L)} \tag{41}
\]

is strictly positive.

Next, consider the two other equations that are required to solve for the equilibrium.

\(^{54}\)See Section 5.3 for an explanation of the restriction \( \tau^b \in (-1, 0) \).
The first one is the market clearing condition for goods, (22), multiplied by $P_1$:

$$ P_1 \left( A_1 K \right) = \int_{H} P_1 C_t^h dh $$

$$ = \kappa \left[ (1 - \alpha) \left( M_0^h + D_0^h \right) + \alpha \kappa \left( M_0^h + D_0^h \right) + \alpha \left( 1 - \kappa \right) M_0^h \right] \tag{42} $$

where the second line is derived as follows. First, only a fraction $\kappa$ of households (impatient households) spend money at $t = 1$. Second, consumption expenditure $P_1 C_t^h$ for impatient households who have their deposits with the fraction $1 - \alpha$ of banks that are solvent is, from Proposition B.3 $M_0^h + W_1^h = M_0^h + D_0^h$, where $W_1^h = D_0^h$ because these banks are not subject to runs. Third, consumption expenditure for impatient households who have their deposits with the fraction $\kappa$ of banks that are insolvent is, from Proposition B.3, $M_0^h + W_1^h$, where $W_1^h = D_0^h$ because the fraction $\kappa$ of households that are among those first in line (the probability that $l_1^h = +\infty$ at an insolvent bank is equal to $\kappa$ and follows from plugging the choices of banks in Proposition B.1 into Equation (17)), and $W_1^h = 0$ for the fraction $1 - \kappa$ of households that are among those last in line.

The second equation is the market clearing condition for money at $t = 0$, Equation (20), with $\mu_0 = 0$ (no monetary injection), rearranged as follows:

$$ \bar{M} = \int_{\mathbb{B}} M_0^b db + \int_{\mathbb{H}} M_0^h dh $$

$$ = \int_{\mathbb{B}} \kappa D_0^b db + \int_{\mathbb{H}} M_0^h dh $$

$$ = \int_{\mathbb{H}} \left[ \kappa D_0^h + M_0^h \right] dh \tag{43} $$

where the second line uses the choice of banks $M_0^b = \kappa D_0^b$ in Proposition B.1, and the last line uses the market clearing for deposits, $D_0^b = D_0^h$.

Therefore, given $P_1 (\bar{\tau}^L)$, Equations (42) and (43) evaluated at $M_0^h = M_0^h (\bar{\tau}^L)$ and $D_0^h = D_0^h (\bar{\tau}^L)$ form a system of two linear equations in $M_0^h (\bar{\tau}^L)$ and $D_0^h (\bar{\tau}^L)$ (the integral in Equation (43) can be ignored because all households are identical). The system has a unique solution (the determinant is positive), given by:

$$ M_0^h (\bar{\tau}^L) = \frac{\bar{M} (1 - \alpha (1 - \kappa)) - \left( A_1 K \right) P_1 (\bar{\tau}^L)}{(1 - \kappa) \kappa (1 - \alpha)} \tag{44} $$

$$ D_0^h (\bar{\tau}^L) = \frac{\left( A_1 K \right) P_1 (\bar{\tau}^L) - \kappa \bar{M}}{\kappa (1 - \kappa) (1 - \alpha)} \tag{45} $$

which are positive using Equation (39), Assumption (33) and $-1 < \bar{\tau}^L < 0$.

Next, consider the expression for the actual return on deposits $r_2^h$ for insolvent banks (banks
with endowment $K_{-1}^b = K^L$, i.e., the second case in Equation (13). Using $B_0^b = 0$ (no monetary injections), $K_0^b = \frac{N_0^b + D_0^b - M_0^b}{Q_0}$ from the budget constraint of banks Equation (7), $W_1^b = M_0^b = \kappa D_0^b$ from the optimal choices of banks in Proposition B.1, the definition of $N_0^b$ in Equation (4), and the endowment of capital $K_{-1}^b = K^L$, and rearranging:

$$(1 + r_2^b) = (1 + r_2^K) \left(1 + \frac{Q_0 K^L + M_0^{b-1} - D_0^{b-1}}{D_0^{b-1} (1 - \kappa)}\right).$$

Evaluating this expression at $D_0^b (\bar{\tau}^L)$, $Q_0^b (\bar{\tau}^L)$, $r_2^K (\bar{\tau}^L)$ delivers a continuous function $\bar{\tau}^L \to r_2^b (\bar{\tau}^L)$:

$$[1 + r_2^b (\bar{\tau}^L)] = [1 + r_2^K (\bar{\tau}^L)] \left(1 + \frac{Q_0 (\bar{\tau}^L) K^L + M_0^{b-1} - D_0^{b-1}}{D_0^b (\bar{\tau}^L) (1 - \kappa)}\right)$$  \hspace{1cm} (46)

Equation (46) evaluated at $r_2^b (\bar{\tau}^L) = \bar{\tau}^L$ is a third-order polynomial in $\bar{\tau}^L$ because $r_2^K (\bar{\tau}^L)$ is (affine) linear in $\bar{\tau}^L$ from (40), $D_0^b (\bar{\tau}^L)$ is (affine) linear in $P_1 (\bar{\tau}^L)$ from (45), and $1/P_1 (\bar{\tau}^L)$ is (affine) linear in $\bar{\tau}^L$ from (39). For a third order polynomial with real coefficients, one solution is always real (no matter what the value of parameters is) and is given by:

$$\bar{\tau}^L = \frac{\kappa}{\alpha \beta^2} \left[1 - 2\alpha - \alpha (1 - \beta) (\theta - 1) (\alpha (1 - \kappa) + \kappa)\right].$$

However, this solution is strict positive due to the assumption in Equation (32) and due to $0 < \beta < 1$. Therefore, it does not correspond to an equilibrium (the actual return on deposits must lie in the interval $(0, 1)$ to have a bad equilibrium, as explained in Section 5.3). The other two solutions of the third order polynomial are real if the discriminant is non-negative; the discriminant is given by $\Delta$, defined in Equation (34) and thus, by Assumption, it is nonnegative. The two solutions are given by $\bar{\tau}^L \left(\sqrt{\Delta}\right)$ and $\bar{\tau}^L \left(-\sqrt{\Delta}\right)$, defined by Equation (35). Thus, depending on whether one, both, or none of the solutions are in the interval $(-1, 0)$, then one, two, or zero bad equilibria exist.

Notice further that the two guesses related to the household problem and used in Appendix B.2 are verified. First, the guess $\frac{1}{P_1} \geq \frac{\beta (1 + r_2^D)}{P_2}$ in Proposition B.3 is verified using $r_2^D = r_2^K$ from Proposition B.1, plugging in Equations (39) and (40), and rearranging. Second, when writing down the problem of households (31), I guessed that households who are able to withdraw (i.e., with $l_1^b = +\infty$, Cases 1 and 2) have marginal utility one, while households “last in line” in a run (i.e., with $l_1^b = 0$, Case 3) have marginal utility $\theta$. This must be the case because the former set of households spend $M_0^b + D_0^b$, while the latter set of households spend only $M_0^b$. Since $M_0^b + D_0^b > M_0^b$, and in the good equilibrium each impatient household was consuming $\bar{C}$, then in the bad equilibrium the first set of households must consume more than $\bar{C}$ (thus, their marginal utility equal to one, see Equation (3)) and the second one must consume less than $\bar{C}$ (thus, their marginal utility equal to $\theta$, see Equation (3)) to make sure that the goods market clearing condition
holds.

To conclude the proof of existence of bad equilibria, I still need to check two conditions. First, the non-negativity constraint on capital held by banks, $K^b_0 \geq 0$ must not be binding for all $b \in B$. This is satisfied if the actual return on deposits $r^b_t \geq -1$. To see this, for banks with endowment $K^b_0 = K^L$, the actual return on deposits is given by the second entry on the right-hand side of Equation (13). If $r^b_t \geq -1$, the left-hand side is non-negative; since all objects on the right-hand side other than $K^b_0$ are non-negative, then $K^b_0$ must be non-negative too. For banks with $K^b_1 = K^H$, the constraint $K^b_0 \geq 0$ is satisfied because $K^H > K^L$; since the constraint is not violated for banks with endowment $K^L$, then it cannot be violated for banks with endowment $K^H > K^L$.

Second, banks with a large endowment of capital, $K^b_{21} = K^H$ must be solvent, which is the case because $\psi^H$ is large enough and thus $K^H$ is large enough by Assumption 5.1.

Finally, I show that the inequalities $D^b_0 < D^*$ and $K^b_0 < K^b_{11}$, $K^b_0 > K^b_{11}$ hold for all $h \in \mathbb{H}$ and for all $b \in B$. Since Equation (43) implies $\bar{M} = \kappa D^0_0 + M^b_0$ for all $h$ in the bad equilibrium, and $\bar{M} = \kappa D^*$ in the good equilibrium, and since $M^b_0 > 0$ in the bad equilibrium due to Equation (44), then $D^b_0 < D^*$. The result $K^b_0 < K^b_{11}$ follows from the fact that $K^b_0 = K^b_{11}$ in the good equilibrium (see Corollary 5.2) and that, in the bad equilibrium, banks have fewer deposits to invest in capital. Formally, solving for $K^b_0$ the budget constraint of banks, Equation (7) evaluated at $B^b_0 = 0$ (no monetary injection):

$$K^b_0 = \frac{N^b_0 + D^b_0 - M^b_0}{Q_0}$$
$$= \frac{N^b_0 + D^b_0 (1 - \kappa)}{Q_0}$$
$$= \frac{K^b_{11} Q_0 + M^b_{11} - D^b_{11} + D^b_0 (1 - \kappa)}{Q_0}$$
$$< \frac{K^b_{11} Q_0 + \left(\frac{1 - \kappa}{\kappa} \frac{1 - \beta}{\beta}\right) - \left(\frac{1 - \kappa}{\kappa} \frac{1}{\beta}\right) + \left(\frac{1 - \kappa}{\kappa} \frac{1}{\kappa}\right)}{Q_0}$$
$$= K^b_{11}$$

where the second line uses $M^b_0 = D^b_0 (1 - \kappa)$ from Proposition B.1; the third line uses the definition of $N^b_0$ in Equation (4); the fourth line uses $D^b_0 < D^*$ as shown before, Assumption 5.1, $D^* = \bar{M}/\kappa$ from Proposition 4.2; and the last line rearranges. The result $K^b_0 > K^b_{11}$ follows from $K^b_0 < K^b_{11}$ and the market clearing condition for capital, Equation (19).

Proof of Proposition 6.1. To prove the result, it is useful to show first the following Lemma.

Lemma C.1. Given $\tau^L \in (-1, 0)$, there exists a continuous mapping $\tau^L \rightarrow r^b_2 (\tau^L, \mu_0)$ such that
1 + r^b_2 (\tau^L, \mu_0 = 0) = 1 + r^b_2 (\tau^L) \text{ defined in Equation (46) and } \frac{\partial r^b_2 (\tau^L, \mu_0)}{\partial \mu_0} < 0; \text{ moreover, if two bad equilibria exist, the mapping } \tau^L \rightarrow r^b_2 (\tau^L, \mu_0) \text{ has two fixed points } r^b_2 (\tau^L, \mu_0) = \tau^L \text{ in } (-1, 0).

Proof. The function \( r^b_2 (\tau^L, \mu_0) \) can be derived as in the proof of Proposition 5.3, by replacing the left-hand side of Equation (43) with \( \overline{M} (1 + \mu_0) \). Thus, continuity in \( \mu_0 \), the statement that \( 1 + r^b_2 (\tau^L, \mu_0 = 0) \) is the same as \( 1 + r^b_2 (\tau^L) \) defined in Equation (46), and the existence of two fixed points (if two bad equilibria exist) follow.

Since \( r^b_2 (\tau^L, \mu_0) \) can be derived as in the proof of Proposition 5.3, \( D^h_0 (\tau^L; \mu_0) \) is defined similarly to Equation (45), replacing \( \overline{M} \) with \( \overline{M} (1 + \mu_0) \):

\[
D^h_0 (\tau^L; \mu_0) = \left( A_1 \overline{K} \right) \frac{P_1 (\tau^L) - \kappa \overline{M} (1 + \mu_0)}{\kappa (1 - \kappa) (1 - \alpha)}. \tag{47}
\]

Thus, similar to Equation (46) (but allowing for the possibility of loans from the central bank \( B^h_0 (\mu_0) \), in which I emphasize their dependence on \( \mu_0 \)), \( 1 + r^b_2 (\tau^L, \mu_0) \) is given by:

\[
1 + r^b_2 (\tau^L, \mu_0) = \left[ 1 + r^K_2 (\tau^L) \right] \left( 1 + \frac{Q_0 (\tau^L) K^L + M^b_{-1} - D^b_{-1}}{D^h_0 (\tau^L; \mu_0) (1 - \kappa) + B^h_0 (\mu_0)} \right).
\]

Differentiating with respect to \( \mu_0 \):

\[
\frac{\partial \left[ 1 + r^L_2 (\tau^L) \right]}{\partial \mu_0} = \left[ 1 + r^K_2 (\tau^L) \right] \left( - \frac{Q_0 (\tau^L) K^L + M^b_{-1} - D^b_{-1}}{D^h_0 (\tau^L; \mu_0) (1 - \kappa) + B^h_0 (\mu_0)} \right)^2 \times \left( (1 - \kappa) \frac{\partial D^h_0 (\tau^L; \mu_0)}{\partial \mu_0} + \frac{\partial B^h_0 (\mu_0)}{\partial \mu_0} \right) < 0 \tag{48}
\]

The term in the first parenthesis on the right-hand side is the return on capital, which is positive. The second term is the ratio between the net worth of insolvent banks \( Q_0 (\tau^L) K^L + M^b_{-1} - D^b_{-1} < 0 \) (which is negative because these banks are insolvent) and a squared term in the denominator; thus the term in the second parenthesis is positive, due to the minus sign. To show the sign of the term in the third parenthesis on the right-hand side, I distinguish between the case of asset purchases and loans to banks. In the former case, \( \frac{\partial B^h_0 (\mu_0)}{\partial \mu_0} = 0 \) and differentiating Equation (47) with respect
to \( \mu_0 \) implies:

\[
(1 - \kappa) \frac{\partial D_0^{b} (\tau^L, \mu_0)}{\partial \mu_0} = -\frac{M}{(1 - \alpha)} < 0 . \tag{49}
\]

In the case of loans to banks, differentiating Equation (14) with respect to \( \mu_0 \) (after setting \( K_0^{CB} = 0 \)) gives \( \frac{\partial B_0^{b}(\tau^L)}{\partial \mu_0} = M \). Thus:

\[
(1 - \kappa) \frac{\partial D_0^{b} (\tau^L, \mu_0)}{\partial \mu_0} + \frac{\partial B_0^{b} (\mu_0)}{\partial \mu_0} = -\frac{M}{1 - \alpha} + M = -\frac{\alpha M}{1 - \alpha} < 0 . \tag{50}
\]

I can now prove the main result. Since by assumption there exists two bad equilibria, using the previous Lemma, the mapping \( \tau^L \rightarrow r_2^b (\tau^L, \mu_0) \) is well-defined and has two fixed points in \((-1, 0)\). Moreover, using once more the previous Lemma, a marginal increase in \( \mu_0 \) generates a downward shift in \( r_2^b (\tau^L, \mu_0) \), thus \( \frac{d (r_2^b)^{-1} (\theta, \mu_0)}{d \mu_0} > 0 \) for the bad equilibrium in which \( r_2^b (\tau^L, \mu_0) \) intersect the 45-degree line with a slope > 1 (see Figure 6, left panel), and \( \frac{d (r_2^b)^{-1} (\theta, \mu_0)}{d \mu_0} < 0 \) for the other bad equilibrium in which \( r_2^b (\tau^L, \mu_0) \) intersect the 45-degree line with a slope < 1.

To show the results for the other endogenous variables, differentiate Equations (39) and (40) with respect to \( r^L \):

\[
\frac{\partial P_1 (\tau^L)}{\partial \tau^L} = \frac{P_2 (1 - 2\alpha - \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa])}{\beta ((1 - 2\alpha) \kappa - \alpha \tau^L)^2} \alpha > 0
\]

\[
\frac{\partial r_2^K (\tau^L)}{\partial \tau^L} = -\frac{\alpha^2 (\theta - 1) (1 - \kappa) - \alpha}{1 - 2\alpha - \alpha (\theta - 1) [\alpha (1 - \kappa) + \kappa]} < 0
\]

where the results follow from the restriction on parameters in Equation (32) that are used to prove Proposition 5.3. Then, using Equation (41) and the results just derived:

\[
\frac{\partial Q_0 (\tau^L)}{\partial \tau^L} = \frac{A_1}{1 + r_2^K (\tau^L)} \left( \frac{\partial P_1 (\tau^L)}{\partial \tau^L} \right) - \frac{Q_2 + A_1 P_1 (\tau^L)}{[1 + r_2^K (\tau^L)]^2} \left( \frac{\partial r_2^K (\tau^L)}{\partial \tau^L} \right) > 0 .
\]

Thus, the results about \( \frac{d P_0}{d \mu_0} \) and \( \frac{d Q_0}{d \mu_0} \) follow from evaluating the previous expression at \( \tau^L = r_2^b (\tau^L, \mu_0) \), and by recalling that the fixed point \( \tau^L = r_2^b (\tau^L, \mu_0) \) increases in one bad equilibrium and decreases in the other as \( \mu_0 \) changes.
Finally, using Equation (47) to totally differentiate $D_0^h(\bar{\tau}^L, \mu_0)$:

$$\frac{dD_0^h(\tau^L; \mu_0)}{d\mu_0} = \frac{1}{(1 - \kappa)(1 - \alpha)} \left( \frac{1}{\kappa} \left( A_1 \bar{K} \right) \frac{\partial P_1(\tau^L)}{\partial \tau^L} \frac{d\tau^L}{d\mu_0} - M \right)$$

Thus, for the bad equilibrium characterized by $\frac{d}{d\mu_0} \left( \frac{v^L}{\mu^L} \right)_{\mu^L=0} < 0$, it follows

$$\frac{d\tau^L}{d\mu_0} \bigg|_{\tau^L = \frac{v^L}{\mu^L}} < 0$$

and thus $\frac{dD_0^h}{d\mu_0} < 0$. For the other bad equilibrium, instead, the sign of $\frac{dD_0^h(\tau^L; \mu_0)}{d\mu_0}$ is ambiguous. The result about money velocity can be proven similarly, because:

$$\frac{d}{d\mu_0} \left( \frac{velocity(\tau^L; \mu_0)}{\mu} \right) = \frac{d}{d\mu_0} \left( \frac{P_1(\tau^L)A_1 \bar{K}}{M(1 + \mu_0)} \right) = \frac{A_1 \bar{K}}{M(1 + \mu_0)} \frac{\partial P_1(\tau^L)}{\partial \tau^L} \frac{d\tau^L}{d\mu_0} - \frac{P_1(\tau^L)A_1 \bar{K}}{M(1 + \mu_0)^2}.$$  

Proof of Proposition 6.2. The result follows as a Corollary of Equations (48)-(50). The magnitude of the shifts of the best response in Figure 6 depends on the magnitude of the term in the last parenthesis of Equation (48). Comparing Equation (49) and Equation (50), the magnitude is larger with asset purchases.

D Monetary injections: discussion

In this Appendix, I discuss the restrictions imposed on monetary injections, in Section 2.7.

The restriction that $\mu_1 = \mu_0$ is not crucial. If the monetary injection is implemented with asset purchases, it can be implemented only at $t = 0$, because, by assumption, capital cannot be traded at $t = 1$. If the monetary injection is implemented with loans to banks, then the central bank (like any other agent in the model) is able to identify insolvent banks at $t = 1$, and I assume here that it does not provide loans to such insolvent banks.\[^{55}\] Solvent banks, instead, would receive money at $t = 1$. Since the policy is announced at $t = 0$ before the asset market opens, then solvent banks are aware that they will receive money at $t = 1$. Recall, from Proposition B.1, that in the baseline

\[^{55}\]Central banks are typically not allowed to make loans to an insolvent bank due to the losses that would result from this operation.
model banks invest a fraction $\kappa$ of deposits into money and the remainder $1 - \kappa$ into capital. The overall investment of banks into money is thus $\kappa D^b_0$. Differently, under the policy discussed here, banks anticipate that they will receive money at $t = 1$. Therefore, at $t = 0$, banks would invest a lower fraction of deposits into money, and a larger fraction into capital. If the monetary injection is less or equal than $\kappa D^b_0$, it is therefore irrelevant whether the loans are extended at $t = 0$ or at $t = 1$. If the loans are extended at $t = 0$, they are invested in capital; if the loans are extended at $t = 1$, a larger fraction of deposits at $t = 0$ is invested in capital. In any case, the amount of resources invested by banks in capital increases. If the monetary injection is greater than $\kappa D^b_0$, any dollar in excess of $\kappa D^b_0$ has no effect on the economy, because solvent bank do not need more than $\kappa D^b_0$ money at $t = 1$.

If the policy intervention is announced at $t = 1$, rather than at $t = 0$, and loans are offered only to solvent banks, there are no effects at all of the monetary injection. This is because solvent banks have already set aside money to pay withdrawals at $t = 1$, and thus they do not need any additional money.

The restriction that the changes in the money supply are temporary, $\mu_2 = 0$, is important, and is motivated by the policies implemented during the recent US financial crisis. In testimony before the Committee on Financial Services of the U.S. House of Representatives, Bernanke (2010) suggests that the monetary expansions of the Federal Reserve are temporary: “In due course [...] as the expansion matures the Federal Reserve will need to begin to tighten monetary conditions.”

Permanent injections ($\mu_2 \neq 0$) can also be analyzed, but requires to make $P_2$ and $Q_2$ a function of the permanent monetary injection, in order to avoid the possibility that the central bank increases consumption at $t = 2$ just by printing money.

**Loans to banks with higher seniority than deposits.** As discussed in the previous working paper version of this paper (Robatto, 2015b), loans to banks with higher seniority than deposits are equivalent to asset purchases in terms of effects on key equilibrium variables (prices, returns, households’ choices, banks’ choices of money holding). Formally, given a policy $\mu_0 > 0$, $\mu_1 = \mu_0$ and $\mu_2 = 0$ implemented with asset purchases, if there exists a bad equilibrium, then the same equilibrium exists in an economy in which the same policy is implemented using loans to banks with higher seniority than deposits. The proof of this result is identical to the one presented in Robatto (2015b).