Exchange rates and monetary spillovers

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December 23, 2015

Abstract

When does the combination of flexible exchange rates and domestic inflation-oriented monetary policy guarantee insulation from global financial conditions? We examine a dynamic global game model of international investment flows where, for some combination of parameters, the unique equilibrium exhibits the observed empirical feature of prolonged episodes of capital inflows and appreciation of the domestic currency, followed by abrupt reversals where capital outflows go hand-in-hand with currency depreciation.

JEL codes:  F32, F33, F34
Keywords: Currency appreciation, capital flows, global games

*The views expressed here are those of the authors and not necessarily those of the Bank for International Settlements. We thank Patrick Bolton, Nicolas Coeurdacier, Doug Diamond, Andy Filardo, Itay Goldstein, Urban Jermann, Emanuel Kohlscheen, Karen Lewis, Ady Pauzner, Lasse Pedersen, Richard Portes, Ricardo Reis, Helene Rey, Martin Schneider, and James Yetman for their comments on this and earlier drafts. Plantin acknowledges support from a European Research Council Starting Grant (No 263673 – RIFIFI).
1 Introduction

Milton Friedman’s essay on the case for flexible exchange rates (Friedman (1953)) is a classic statement of the role of floating exchange rates in allowing an economy to adjust to external shocks. In the same spirit, the flexible exchange rate version of the Mundell-Fleming model (Fleming (1962), Mundell (1963)) lays out the case for how flexible exchange rates allow monetary authorities to pursue domestic macroeconomic objectives in a world of free capital flows.

Post-crisis discussions of monetary spillovers have revisited these classic propositions. The BIS report on global liquidity (BIS (2011)) is a recent exposition of cross-border monetary spillovers and it has been followed by an active literature which has examined the extent to which floating exchange rates are sufficient to insulate monetary policy from external developments (see, for instance, Agrippino and Rey (2014), Rey (2013, 2014), Bruno and Shin (2015a, 2015b)).

We shed further light on the economic mechanisms behind monetary spillovers in a regime of floating exchange rates. We explore a dynamic global game model of international investment flows where global investors interact with each other and with a central bank that pursues monetary policy aimed at domestic price developments. Our focus is on the exchange rate channel of monetary policy; we ask how currency appreciation is related to domestic interest rates and credit growth in an environment with flexible exchange rates.

Figure 1 overleaf illustrates the type of empirical regularity that motivates the questions in our paper. Figure 1, taken from Hofmann, Shim and Shin (2015), depicts the returns on emerging market sovereign bonds in diversified bond mutual funds, where returns are measured both in local currency and US dollar terms. The blue scatter is the local currency return (in percent) against local currency yield changes (in percentage points), while the red scatter is the US dollar return plotted against the same yield change. The right hand panel is a magnified version of the scatter chart for one of the funds.
In all the panels in Figure 1, the red and blue scatter plots diverge, with the red line steeper than the blue line. The right hand half of each panel is the region where yields have risen, and so returns are negative, but dollar returns are lower than local currency returns, implying that the local currency has depreciated against the dollar.

Conversely, the left hand side of the panels corresponds to states where domestic interest rates fall. In these states, the local currency tends to appreciate against the dollar.\(^1\) In either case, exchange rate changes are correlated with shifts in financial conditions as measured by local interest rates; currency appreciation is associated with more permissive financial conditions.

The combination of rapid credit growth and an appreciating currency has been a good

\(^1\)Hofmann, Shim and Shin (2015) show in a panel investigation of emerging market economies that the relationship between exchange rates and financial conditions shown in Figure 1 holds more generally for sovereign yields, CDS spreads and as well as for forward rate adjusted spreads from the swapped return spread defined by Du and Schreger (2015).
early warning indicator of booms in emerging markets that precede crises. Gourinchas and Obstfeld (2012) conduct an empirical study using data from 1973 to 2010 and find that two factors emerge consistently as the most robust and significant predictors of financial crises, namely a rapid increase in leverage and a sharp real appreciation of the currency.

We offer a parsimonious model that sheds light on a possible mechanism through which global financial conditions exert influence on credit and asset prices in economies with flexible exchange rates. We do so in two steps. In the first step, we examine a perfect foresight model in which global investors have access to both the dollar bond market and the local currency bond market of a small open economy. Global investors form portfolios in anticipation of monetary policy pursued by the central bank, which employs a monetary policy rule that takes account of the overall domestic price developments.

We show that for reasonable parameters of the model, there are two stable steady-state solutions – one associated with capital inflows and the other with capital outflows. The steady state with capital inflows is associated with relatively lower domestic interest rates and appreciation of the domestic currency, while the steady state with capital outflows is associated with higher domestic interest rates and currency depreciation.

In the second step of our analysis, we build a dynamic model based on ingredients from the steady state analysis. We introduce exogenous shocks to the dollar interest rate and use the global game technique of Burdzy, Frankel, and Pauzner (2001) to refine the outcome of the model to a unique solution. We show that the state space can be partitioned into two regions – a region where all investors pile into the local currency bond, and one in which they all pile out. The transition between the two regions can be triggered by small fluctuations in the US dollar interest rate and the endogenous changes in domestic financial conditions. The unique equilibrium features dynamics that are reminiscent of boom-bust cycles in which prolonged episodes of credit growth and capital inflows, benign domestic financial conditions, and real appreciation of the currency follow an easing of global monetary conditions. Subsequent small increases in the US rate do not immediately reverse the up-phase of the cycle but will abruptly reverse it when a “tantrum” boundary is reached. Hitting the “tantrum” boundary triggers a large currency depreciation, capital outflows, and a crash in the domestic bond
Related Literature

The relationship between exchange rates and leverage is our point of contact with the literature on the determinants of vulnerability to financial crises. In addition to Gourinchas and Obstfeld (2012) mentioned above, Schularick and Taylor (2012) similarly highlight the role of leverage in financial vulnerability, especially that associated with the banking sector. There is a large and growing literature on the determinants of emerging market crises that emphasise the association between exchange rate appreciation and over-extension of the financial system (Kaminsky and Reinhart (1999), Borio and Disyatat (2011), among others).

Our results complement the recent work on the risk-taking channel of currency appreciation, introduced by Bruno and Shin (2015a, 2015b) in the context of cross-border banking, whereby currency mismatches on borrowers’ balance sheets lead to credit supply effects of exchange rate fluctuations. The risk-taking channel relies on Value-at-Risk (VaR)-induced behaviour that is sensitive to tail risks of credit portfolios. Hofmann, Shim and Shin (2015) apply the risk-taking channel to domestic currency sovereign yields through shifts in the tail risk of diversified local currency sovereign bond portfolios. Whereas existing models of the risk-taking channel are static, our global game model solves for the dynamic path of the key macro variables. The trade-off is that our model is a more parsimonious model of decentralised portfolio choice.

In terms of the theoretical tools, our approach is most closely related to models of financial instability which involve coordination problems and self-fulfilling speculative episodes, such as the early currency crisis models of Obstfeld (1996). In a similar spirit, Farhi and Tirole (2012) and Schneider and Tornell (2004) offer related models of “collective moral hazard” in which the government bails out speculators if their aggregate losses are sufficiently large, thereby inducing a coordination motive among speculators. We formalise the dynamic coordination game among investors using the dynamic extension of global-game methods developed by Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) to obtain a unique equilibrium outcome. We show that these global-game tools can be adapted to the situation
where coordination motives coexist with congestion effects. This is important because most financial models with coordination motives also feature congestion effects. In a model of bank run, Goldstein and Pauzner (2004) adapt static global-game techniques to the case in which strategic complementarities similarly fail to hold everywhere. In a model of sovereign-debt refinancing, He, Krishnamurthy, and Milbradt (2015) also apply global-game techniques in a context in which a large debt size comes at the benefit of smaller congestion effects but at the cost of a higher rollover risk.

Most closely related to our work, He and Xiong (2012) apply the equilibrium selection techniques developed by Burdzy, Frankel, and Pauzner (2001) in a dynamic financial context - the roll-over of short-term debt.

Our paper is related to the literature on portfolio choice in incomplete markets. Garleanu, Panageas and Yu (2015) present a model in which investors face costs to extend their participation in markets located on a circle for diversification purposes. Our result that the profitability of investment increases in the weight of others’ participation bears similarities with their finding that participation and leverage reinforce each other, possibly leading to multiple equilibria. In their setup, increased participation leads to an increase in the attainable Sharpe ratio, which triggers more leverage and thus higher gains from participation and diversification.

Finally, in a recent contribution, Gabaix and Maggiori (2015) introduce financial intermediaries that operate in incomplete global financial markets by intermediating gains from trade between countries. Financial constraints imply that these institutions supply liquidity inelastically, which generates risk premia on cross-currency positions. We also model global investors as financial institutions operating in incomplete markets. In our setup, however, these institutions source funds in one country and end up amplifying financial fluctuations in the destination country. Thus we reach very different conclusions regarding the relationship between limits to arbitrage and excess returns. In our setup, excess returns are self-fulfilling and hence depend on the coordination outcome. Second, whereas tighter financial constraints lead to larger excess returns in Gabaix and Maggiori (2015), the opposite holds in our setup. Less constrained traders exert greater influence; they have a freer hand at reaping higher
excess returns, and thereby generate greater market amplification by doing so.

2 Benchmark model

Time is discrete and is indexed by $t$. There are two types of agents, households populating a small open economy and global investors. There is a single tradable good that has a fixed unit price in US dollars.

2.1 Households

The households live in a small open economy. They use a domestic currency that trades at $S_t$ dollars per unit at date $t$, where the exchange rate $S_t$ will be determined in equilibrium.

At each date, a unit mass of households are born. Households live for two dates, consume when young and old, and work when old. Each household receives an initial endowment at birth with nominal value $P_tW \geq 0$, where $P_t$ is the domestic price level. The cohort that is born at date $t$ has quasi-linear preferences over bundles of consumption and labor $(C_t, C_{t+1}, N_{t+1})$

$$U(C_t, C_{t+1}, N_{t+1}) = \ln C_t + \frac{C_{t+1} - N_{t+1}^{1+\eta}}{R},$$

(1)

where $\eta > 0$ and $R$ is the subjective discount rate.

Domestic consumption services $C_t$ are produced combining the tradable good $C_t^T$ and two nontradable goods $C_t^{N1}$ and $C_t^{N2}$ according to the technology

$$C_t = \frac{(C_t^T)^{\alpha} (C_t^{N1})^{\beta} (C_t^{N2})^{\gamma}}{\alpha^{\alpha} \beta^{\beta} \gamma^{\gamma}},$$

(2)

where $\alpha, \beta, \gamma \in (0, 1)$ and $\alpha + \beta + \gamma = 1$. Domestic firms set by old households use labor input to produce. Due to quasi-linear preferences, our results do not depend on the specification of the firms’ production functions. All that is needed is that both nontradable goods are produced in finite, non-zero quantities at each date. Households collect labor income and the profits from their firms when old.

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2If the endowment of young households is zero, then the global investors introduced below cannot have an aggregate short position in domestic bonds. A strictly positive endowment plays no other role than allowing such short positions.
An important ingredient of the model is that the prices of the nontradable goods are less flexible than that of the tradable good.\footnote{This is consistent with evidence documented by Burstein, Eichenbaum, and Rebelo (2005).} We formalize this as follows. First, we posit that the tradable good has a flexible price $P_t^T$ in the domestic currency, and that the law of one price holds ("PPP at the docks"). Second, the first nontradable good $N_1$ also has a fully flexible price $P_t^{N_1}$. A linear technology enables the transformation of each date-$t$ unit of $N_1$ into $F$ units of the tradable good, where $F > 0$. The second nontradable good $N_2$ has a fixed price that we normalize to 1 without loss of generality. Denoting $P_t^T$, $P_t^{N_1}$, and $P_t^{N_2}$ the respective prices of these three goods, we have:

\begin{align*}
P_t^T S_t &= 1, \\ P_t^{N_1} &= F P_t^T, \\ P_t^{N_2} &= 1.
\end{align*}

Equation (3) is the statement of the law of one price. Equation (4) states that domestic households are indifferent between purchasing the tradable good or producing it out of $N_1$. Finally, (5) is the statement of the rigidity of $N_2$’s price.

The introduction of the nontradable good with flexible price $N_1$ allows us to decouple the “openness” of the economy as measured by $\alpha = 1 - \beta - \gamma$ from the flexibility of the overall price index to changes in the tradable goods price, as measured by $1 - \gamma$. The parameters $\beta$ and $\gamma$ are defined in (2). The price index is given by

\begin{align*}
P_t &= (P_t^T)^\alpha (P_t^{N_1})^\beta (P_t^{N_2})^\gamma \\
&= (P_t^T)^{1-\gamma} F^\beta,
\end{align*}

where (6) follows from (4) and (5).

Households have access to the domestic bond market, in which risk-free one-period bonds denominated in the domestic currency are available in zero net supply. The nominal interest rate on these bonds is set by the domestic central bank according to a rule to be described below.
2.2 Global investors

A unit mass of global investors have access to both the local-currency bond market and to US dollar-denominated bonds. The exogenous nominal return on US dollar bonds is denoted by $I^* > 0$. Global investors consume outside the local economy, and their utility is increasing in the consumption of the tradable good.\(^4\)

In forming their financial portfolios, global investors face limits on the size of their exposures in domestic bonds, reflecting leverage constraints or exposure caps imposed by internal risk limits. We assume that the position in domestic bonds of any investor must lie in the interval $[P_t L^-, P_t L^+]$, where these limits are denominated in the domestic currency and

$$L^- > -W,$$

which ensures that households always consume positively.\(^5\) We do not consider the microfoundations of these limits. As is well-known, they could result from example from agency problems within globally investing firms such as, for example, a cash-flow diversion problem.

The return to a global investor from investing in the local currency bond market relative to the return on dollar bonds is given by

$$\Theta_{t+1} = \frac{S_{t+1} I_{t+1}}{S_t I^*}$$

(7)

We may interpret $\Theta_{t+1}$ as the return to a carry trade position in which the investor borrows dollars at rate $I^*$ and then invests the proceeds in the local currency bond yielding $I_{t+1}$. Uncovered interest parity (UIP) holds when $\Theta_{t+1} = 1$.

We denote $L_t \in [L_-, L_+]$ the real net aggregate borrowing by young households from global investors at date $t$ (possibly negative). Since the economy is deterministic, optimal portfolio choice by global investors implies that $L_t$ must satisfy:

$$L_t \begin{cases} = L^+ & \text{if } \Theta_{t+1} > 1, \\ = L^- & \text{if } \Theta_{t+1} < 1, \\ \in (L^-, L^+) & \text{if } \Theta_{t+1} = 1. \end{cases}$$

(8)

\(^4\)Whether they also derive utility from consuming other goods, and the curvature of their utility function are immaterial. This is only true because the economy is deterministic, and will no longer be so in Section 3.

\(^5\)Setting lending limits in real terms simplifies the exposition but is not crucial. Nominal rigidities in trading limits would actually amplify our results.
In words, global investors choose corner portfolios unless they are indifferent between investing in US-dollar denominated assets or in domestic bonds.

2.3 Monetary policy rule

We suppose that the domestic monetary authority sets the nominal interest rate between \( t \) and \( t + 1 \), \( I_{t+1} \), following the interest-rate feedback rule:

\[
I_{t+1} = R \left( \frac{P_t}{P_{t-1}} \right)^{1+\Phi} \tag{9}
\]

where

\[
\Phi > 0 \tag{10}
\]

The interest rate rule (9) follows the Taylor principle from (10) in that the nominal interest rate reacts more than one-for-one to the price level change. The Taylor principle is necessary for a determinate solution in many classes of monetary models (Taylor, 1993; Woodford, 2001), and our model also shares this feature as we see below.

Given our quasi-linear preferences, households’ Euler equation can be written as

\[
I_{t+1} = \frac{P_{t+1}}{P_t} \frac{R}{(L_t + W)} \tag{11}
\]

2.4 Steady-state solution

We are now equipped to solve for the perfect-foresight steady-states of this economy. A steady state must be such that the domestic economy is in equilibrium and global investors form optimal portfolios at each date. Formally, a perfect-foresight steady-state is a solution to (3), (6), (8), (9), and (11).
We introduce the following notation.

\[ r = \ln R, \]
\[ \delta = \ln \left( \frac{R}{I^*} \right), \]
\[ \theta_t = \ln \Theta_t, \]
\[ i_t = \ln I_t, \]
\[ s_t = \ln S_t, \]
\[ l_t = \ln (L_t + W), \]
\[ \pi_{t+1} = \ln \left( \frac{P_{t+1}}{P_t} \right). \]

The Euler equation (11) and the interest-rate rule (9) can be gathered as follows:

\[ i_{t+1} = r - l_t + \pi_{t+1}, \quad (12) \]
\[ i_{t+1} = r + (1 + \Phi) \pi_t. \quad (13) \]

Together, they define a linear-difference equation for the path of inflation:

\[ \pi_t = \frac{\pi_{t+1} - l_t}{1 + \Phi} \quad (14) \]

which has a unique non-exploding solution:

\[ \pi_t = - \sum_{k \geq 0} \frac{l_{t+k}}{(1 + \Phi)^{k+1}} \quad (15) \]

Equation (15) shows that current inflation is affected by current and future capital inflows \((l_{t+k})_{k \geq 0}\). The Taylor principle ensures that \(\pi_t\) is well-defined, since \(l_{t+k}\) is bounded and \(\Phi > 0\). This expression for \(\pi_t\) shows that our model shares the generic feature of standard interest-rule based monetary models that inflation reflects anticipated future “shocks.” In our context, the “shocks” are not the usual exogenously assumed policy shocks, but rather are the consequence of optimal portfolio choice by global investors.

Using (6) and (3), we have:

\[ \pi_{t+1} = - (1 - \gamma) (s_{t+1} - s_t). \quad (16) \]
Equation (16) is our exchange rate pass-through equation that expresses inflation in terms of exchange rate depreciation. As a rule of thumb, the exchange rate pass-through is normally thought of lying between 0.1 and 0.3 (Klau and Mihaljek (2008) and Gopinath (2015)), which implies a value of $\gamma$ between 0.7 and 0.9. The magnitude of $\gamma$ will play a key role in our analysis.

Plugging (16) and (11) in (7), we have

$$
\theta_{t+1} = s_{t+1} - s_t + i_{t+1} - \ln I^*,
$$

$$
= \frac{1}{1-\gamma} \pi_{t+1} + \pi_{t+1} - l_t + \delta,
$$

$$
= \frac{\gamma}{1-\gamma} \sum_{k=0}^{\infty} \frac{l_{t+k+1}}{(1+\Phi)^{k+1}} - l_t + \delta.
$$

(17)

(18)

where (17) follows from the exchange rate pass-through equation (16) and the Euler equation, and (18) follows from the solution for $\pi_{t+1}$ given by (15).

We now determine the steady states in which the debt level $l$ is constant over time. We introduce

$$
l \equiv \ln(W + L^-),
$$

$$
\bar{l} \equiv \ln(W + L^+).
$$

(19)

(20)

**Proposition 1** Suppose there exists $l^* \in (l, \bar{l})$ such that

$$
\frac{\gamma - \Phi(1-\gamma)}{(1-\gamma)\Phi} l^* + \delta = 0.
$$

Then $l = l^*$ is a steady state in which uncovered interest parity (UIP) holds. If $\Phi(1-\gamma) > \gamma$, there is no other steady-state solution. However, if $\Phi(1-\gamma) < \gamma$, there are two further steady-state solutions; there is a steady state with maximum capital inflows ($l = \bar{l}$), and there is a steady state with maximum capital outflows ($l = l$).

Proposition 1 highlights the possibility of both self-fulfilling capital flow surges and outflows as possible steady-state solutions of our model. The parameter $\gamma$ that determines the exchange rate pass-through to domestic inflation plays a key role whether such large inflows or outflows are steady-state solutions.
We have noted already that rules of thumb put $\gamma$ between 0.7 and 0.8, while Taylor (1993) considered the Taylor rule coefficient $\Phi = 0.5$. Thus, $\Phi(1 - \gamma)$ takes values around 0.1 to 0.15, which is much smaller than $\gamma$ itself. Hence, for reasonable values of $\Phi$ and $\gamma$, we fall in the region where multiple steady-state solutions hold, and self-fulfilling surges and outflows are sustainable as steady-state solutions.

We prove Proposition 1. First, note that for $l$ fixed, the relative return to investing in the local currency bond given by (18) can be written as

$$\theta = \frac{\gamma - \Phi(1 - \gamma)}{(1 - \gamma)\Phi}l + \delta$$

Let $l^* \in (\bar{l}, \bar{l})$ be such that $\theta = 0$. For such an $l^*$ investors are indifferent between investing in the local currency bond or the dollar bond. Hence, $l^*$ is a steady-state solution of our model, corresponding to the mixed strategy equilibrium of the one-shot game between investors where proportion $(l^* - l) / (\bar{l} - \bar{l})$ invest in the local currency bond. If $\Phi(1 - \gamma) > \gamma$, this is the only steady-state solution, since $\theta$ is decreasing in $l$. This corresponds in particular to the fully flexible benchmark ($\gamma = 0$).

Now consider the case where $\gamma > \Phi(1 - \gamma)$. In this case, we have two further steady-state solutions corresponding to the corner solutions $l = \bar{l}$ and $l = \bar{l}$.

First, suppose that all investors choose to invest in the local currency bond. Then $l = \bar{l}$, so that $\theta > 0$, implying that investing in the local currency bond is strictly better than investing in the dollar bond. Hence, all investors invest in the local currency bond, thereby sustaining maximum inflows $l = \bar{l}$ as a steady-state solution. Conversely, suppose that all investors choose to invest in the dollar bond. Then $l = \bar{l}$, so that $\theta < 0$, implying that investing in the dollar bond is strictly better. Hence all investors invest in the dollar bond, sustaining $l = \bar{l}$ as a steady state.

The intuition behind the multiplicity of steady states is as follows. First, as is transparent from relation (15), capital inflows act exactly as local positive policy shocks and thus lead inflation to be below target. Second, since the prices of nontradable goods are relatively stickier than that of the tradable good, this deflationary impact must operate relatively more through the prices of tradable goods, and thus through a large appreciation of the nominal exchange rate, as formalized by (16). If these two effects are sufficiently important,
then capital inflows generate a sufficiently large exchange rate appreciation that this more
than compensates global investors for holding expensive local bonds. This yields abnormal
profits, and the anticipation of future large capital inflows in (or outflows out of) the small
open economy is self-fulfilling. The first effect is sufficiently important when monetary policy
is sufficiently passive ($\Phi$ sufficiently small). The latter effect of nominal rigidities corresponds
to a large $\gamma$ that makes small changes in inflation expectations consistent with large swings
in the nominal exchange rate.

**Remark on the role of the monetary rule.** As is clear from Proposition 1, a suf-
ficiently aggressive interest rule ($\Phi$ sufficiently large) would suffice to pick out the steady
state in which UIP holds. Alternatively, the commitment to respond directly to the impact
of capital flows on bond prices would also suffice to pick out the UIP steady state. To see
this, note that replacing rule (13) with a rule of the form:

$$i_{t+1} = r_{t+1} + (1 + \Phi) \pi_t$$

(22)

would eliminate the effect of inflows on the domestic CPI. Notice, however, that the steady
states with large capital flows described in Proposition 1 could still arise under such a rule
in a richer model of nominal rigidities. Suppose for example that the consumption boom
due to inflows results in inflation in the price of nontradable goods. If rule (22) maintains
the overall price index constant, then this inflationary pressure must be compensated with
deflation in the price of the tradable good. Such a deflation, if sufficiently large, will again
create room for self-fulfilling excess returns on carry trades through the associated increase
in the nominal exchange rate.

### 2.5 Properties of steady states

We now explore some key features of the steady states of our model with an eye on the
empirical implications on the co-movement between the nominal interest rate, the exchange
rate and credit driven by capital flows.
Consider first the solution for the nominal interest rate in our model. From (15) we have

\[ i_{t+1} = r + \pi_{t+1}, \]
\[ = r - \sum_{k \geq 0} \frac{l_{t+k}}{(1 + \Phi)^{k+1}}. \]  
(23)

The nominal interest rate \( i_{t+1} \) is therefore decreasing in current and future credit financed by capital inflows. Meanwhile, our solution for exchange rate appreciation can be obtained from the pass-through equation (16) and equation (15). We have

\[ s_{t+1} - s_t = -\frac{1}{1-\gamma} \pi_{t+1}, \]
\[ = \frac{1}{1-\gamma} \sum_{k \geq 1} \frac{l_{t+k}}{(1 + \Phi)^k}. \]  
(24)

In other words, future credit financed by capital inflows is associated with an appreciation of the domestic currency. The parameter \( 1 - \gamma \) determines the size of the appreciation - the lower it is, the sharper the currency appreciation.

By putting together (23) and (24) we can address the type of empirical regularity depicted in Figure 1 at the outset of our paper. Recall that in Figure 1, we pointed to the empirical regularity between returns on emerging market bond mutual funds and currency appreciation in which domestic currency appreciation against the dollar is associated with more permissive financial conditions in the form of a reduction in domestic interest rates. Conversely, currency depreciation against the dollar is associated with tighter financial conditions in the form of rising domestic interest rates. The two steady states where \( l_t \) takes the corner solutions of \( l_t = \bar{l} \) and \( l_t = \underline{l} \) are associated with the extremes of the correlations. We summarize this finding in terms of the following proposition.

**Proposition 2** In the steady state with maximum domestic borrowing \( (l = \bar{l}) \), the domestic currency appreciates and the interest is lower than that when \( l = l^* \). Uncovered interest parity fails, and dollar returns are higher than domestic currency returns. In the steady state with maximum capital outflows \( (l = \underline{l}) \), the domestic currency depreciates and the interest rates is higher than that when \( l = l^* \). Uncovered interest parity fails and dollar returns are lower than domestic currency returns.
Proposition 2 is consistent with the relationship between exchange rates and financial conditions depicted in the scatter charts in Figure 1. Uncovered interest parity fails in a strong sense. Not only does UIP fail to hold, the portfolio position of global investors are given by corner solutions - either the maximum capital inflows or maximum capital outflows.

So far, we have addressed the steady states only. The shortcoming of a steady-state analysis is that it cannot address the switch from one steady state to another, nor the transition between steady states in response to shocks and other forces buffetting the economy. We therefore turn to a stochastic version of our benchmark model and employ global-game techniques to tie down a unique dynamic solution. The goal of the analysis is to provide the theoretical foundations of the dynamics of an open economy in which surges of capital inflows can be explained alongside the sudden reversals that happen in practice.

3 Stochastic model

We will proceed to develop a stochastic version of our model, and then solve for the uniquely determined time paths by using perturbation methods that resemble global-game methods, but which are better suited for dynamic contexts.

Begin by assuming that time is continuous. The fixed integer dates of the previous section are replaced by the arrival times of a Poisson process with intensity 1. Namely, at each arrival time $T_n$, a new cohort of households are born, and die at the next arrival time $T_{n+1}$. They value consumption and leisure only at these two dates, with preferences that are the same as that in our benchmark set-up:

$$\ln C_{T_n} + \frac{1}{R} E_{T_n} \left[ C_{T_{n+1}} - N_{T_{n+1}}^{1+\eta} \right].$$

At each arrival date $T_n$, the central bank sets a nominal rate $I_{T_{n+1}}$ between $T_n$ and $T_{n+1}$ according to the rule:

$$I_{T_{n+1}} = R \left( \frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi} \quad (25)$$

The unconventional assumption that households discount consumption at a random future date at a fixed discount factor $R$ greatly simplifies the algebra but plays no other role. Accordingly, we will define the interest rate on US and local bonds as a fixed coupon between two arrival dates.

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6 The unconventional assumption that households discount consumption at a random future date at a fixed discount factor $R$ greatly simplifies the algebra but plays no other role. Accordingly, we will define the interest rate on US and local bonds as a fixed coupon between two arrival dates.
Replacing integer dates with dates that arrive at a constant rate is not essential, and our set-up is designed for tractability. It is designed to ensure that the global investors’ portfolio choice problem is time homogeneous.

We now are more specific than in the previous section about global investors’ preferences in order to characterise their portfolio choice. We follow Gabaix and Maggiori (2015) and model global investors as a unit-mass of financial institutions that can form zero-cost portfolios in bonds denominated in either currency at each date $T_n$ with size within $[P_{T_n}L^-, P_{T_n}L^+]$ (in units of the domestic currency). The date-$T_n$ trade is unwound at the next date $T_{n+1}$ and the realised profit or loss is paid to the old households at this date. Each firm maximises the expected value of future consumption paid to all future households discounted at the households’ subjective discount rate $R$ between two arrival dates.

The two following modifications to the benchmark model are key to generate equilibrium uniqueness.

First, we assume that the interest rate on US dollar-denominated bonds between two arrival dates $T_n$ and $T_{n+1}$ is given by

$$I^*_{T_{n+1}} = R(1 - w_{T_n}),$$

(26)

where $w_t$ is a Wiener process with volatility $\sigma$ and no drift. We could also add exogenous shocks to domestic monetary policy. All that matters is that there are exogenous shocks to the interest-rate differential.

Second, we assume that the capital supplied by global investors is slow-moving in the following sense. Each investor can revise his investment strategy only at switching dates that are generated by a Poisson process with intensity $\lambda$. These switching dates are independent across investors. In between two switching dates, each global investor commits to a strategy and thus to form a zero-cost portfolio with a position in domestic bonds within $[P_{T_n}L^-, P_{T_n}L^+]$ at each arrival date $T_n$ (if any).

This model of slow-moving capital has a key property that will yield equilibrium uniqueness: Every investor knows that some other investors will revise their trading strategy almost surely between his current switching date and the next one.

Remark. (Arbitrage versus good deal and the role of trading limits) It is
important to stress that the exogenous trading limits \([P_{T_n}L^-, P_{T_n}L^+]\) fulfill a very different role from that played in Section 2. In the previous section, it was necessary to impose such limits regardless of global investors’ preferences because carry trades were (possibly) textbook arbitrage opportunities given the deterministic environment. As is well-known, any agent with increasing utility over consumption has an infinite demand for an arbitrage opportunity absent any financial constraint. In this stochastic environment, we will see that carry-trade portfolios generate losses with a non-zero probability in equilibrium. Thus, any risk-averse agent would form finite portfolios. Trading limits here only play the role of a very tractable substitute for risk aversion that is commonplace in models in which agents’ attitude towards risk is not the main focus.\(^7\) An interesting extension consists in studying risk limits that vary with exchange rate movements, as is the case in practice (see Hofmann, Shim and Shin (2015)).

Local risk-neutrality implies that global investors choose corner portfolios. We deem “long” a global investor who committed to maximum lending \(L^+\) at his last switching date, and “short” one who committed to the minimum lending \(L^-\) (corresponding to borrowing from households if \(L^- < 0\)). Suppose that a global investor has a chance to revise his position at a date \(t\) such that

\[ T_{n-1} < t < T_n. \]

Denoting \(T_\lambda\) his next switching date, its expected unit return from the carry trade - the expected value from committing to lend one additional real unit to each future cohort until \(T_\lambda\) - is

\[ \Theta_t = E_t \left[ \sum_{m \geq 0} \frac{1\{T_\lambda > T_{n+m}\}}{R^m} \frac{P_{T_{n+m}}S_{T_{n+m}}}{P_{T_{n+m+1}}S_{T_{n+m+1}}} \left( \frac{S_{T_{n+m+1}}I_{T_{n+m+1}}}{S_{T_{n+m}}R} - 1 + w_{T_{n+m}} \right) \right] \]  

Expression (28) states that the global investor earns the carry-trade return associated with each arrival date until he gets a chance to revise his position.\(^8\)

\(^7\)In recent work, Albagli, Hellwig, and Tsyvinski (2015) use a similar assumption to generate a high tractability and thus new insights in a standard noisy REE asset-pricing model. The binary portfolio choice of risk-neutral carry traders is particularly suited to our iterated-dominance solution.

\(^8\)To arrive at (28), note that investing one real unit at arrival date \(T_j\) costs USD \(P_{T_j}S_{T_j}\). The net rate of return \((S_{T_{j+1}}/S_{T_j})I_{T_{j+1}} - I_{T_{j+1}}\) applies to this dollar amount. The resulting consumption for old households
We let $x_t$ denote the fraction of long global investors at date $t$. Note that the paths of the process $(x_t)_{t \in \mathbb{R}}$ must be Lipschitz continuous, with a Lipschitz constant smaller than $\lambda$. The aggregate real net lending $L_{T_n}$ taking place at an arrival date $T_n$ is then equal to

$$L_{T_n} = x_{T_n} L^+ + (1 - x_{T_n}) L^-.$$  \hfill (29)

The evolution of the economy is fully described by two state variables, the exogenous state variable $w_t$ and the endogenous state variable $x_t$. The exogenous state variable directly affects only the expected return on carry trade $\Theta_t$ while the endogenous one directly affects both the carry trade return and the equilibrium variables $(L_{T_n}, I_{T_n}, P_{T_n}, S_{T_n})$ of the domestic economy. We are now equipped to define an equilibrium.

An equilibrium is characterized by a process $x_t$ that is adapted to the filtration of $w_t$ and has Lipschitz-continuous paths such that:

$$L_{T_n} = x_{T_n} L^+ + (1 - x_{T_n}) L^-,$$  \hfill (30)

$$I_{T_{n+1}} = R \left( \frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi},$$  \hfill (31)

$$P_{T_n} = (P_{T_n}^T)^{1-\gamma} F^\beta,$$  \hfill (32)

$$E_{T_n} \left[ \frac{I_{T_{n+1}} P_{T_n}}{P_{T_{n+1}}} \right] = \frac{R}{L_{T_n} + W},$$  \hfill (33)

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } \Theta_t < 0, \\ \lambda(1 - x_t) & \text{if } \Theta_t > 0. \end{cases}$$  \hfill (35)

Equations (31) to (34) state that the domestic economy is in equilibrium given the paths of $x_t$. Equation (35) states that global investors make optimal individual decisions. They become long at switching dates at which the expected return on the carry trade is positive (or remain long if this was their previous positions), and short if this is negative (or remain short if this was their previous positions).

at date $T_{j+1}$ is then this USD profit divided by $P_{T_{j+1}} S_{T_{j+1}}$. Re-arranging and discounting these terms yields (28).
Notice that relations (31) to (33) are identical to their counterparts in the perfect foresight case except for the re-labelling of dates. They are in particular log-linear. Conversely, the Euler equation (34) now features an expectation over the inverse of inflation given the stochastic environment. We will assume for the remainder of the paper that $W + L^-$ and $W + L^+$ are sufficiently close to 1 that $l$ and $\bar{l}$ defined in (19) and (20) are sufficiently close to 0, so that we can approximate

$$\ln E_t \left[ \frac{P_{T_n}}{P_{T_{n+1}}} \right] \simeq -E_t \left[ \ln \frac{P_{T_{n+1}}}{P_{T_n}} \right].$$

(36)

This implies of course that we restrict the analysis to the impact of relatively small capital inflows. Up to this log-linearization, we have

**Proposition 3** Suppose that

$$\gamma > \Phi (1 - \gamma).$$

(37)

For $\lambda$ sufficiently small, there exists a unique equilibrium defined by a decreasing Lipschitz function $f$ such that

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f(x_t), \\ \lambda (1 - x_t) & \text{if } w_t > f(x_t). \end{cases}$$

(38)

The frontier $f$ divides the $(w, x)$-space into two regions. Proposition 3 states that in the unique equilibrium, any investor decides to be long when the system is to the right of the frontier $f$ at his switching date, and short when it is on the left of the frontier. Thus, net lending (and therefore the exchange rate) will tend to rise in the right-hand region, and tend to fall in the left-hand region, as indicated by the arrows in Figure 2.

The expected return on the carry trade at date $t$ is zero if and only if $w_t = f(x_t)$. It is positive if $(w_t, x_t)$ is on the right of the frontier $f$ in the $(w, x)$-space and negative if it is on the left of $f$.

The dynamics of $x_t$ implied by the unique equilibrium are given by:

$$dx_t = \lambda \left( 1_{\{w_t > f(x_t)\}} - x_t \right) dt,$$

(39)

where $1_{\{\cdot\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations*, and are
studied in Bass and Burdzy (1999) and Burdzy et al. (1998). These mathematics papers establish in particular that for almost every sample path of $w_t$, there exists a unique Lipschitz solution $x_t$ to the differential equation (39) defining the price dynamics for $f$ Lipschitz decreasing.

The main features of these dynamics can be seen from Figure 2. Starting on the frontier, a positive shock on $w$ will pull the system on the right of it. Unless the path of $w_t$ is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that lending grows for a while so that $x_t$ becomes close to 1, in which case $\frac{dx_t}{dt}$ becomes close to 0. If cumulative negative shocks on $w$ eventually lead the system back to the left of the frontier, then there are large outflows

$$\frac{dx_t}{dt} \approx -\lambda.$$
These dynamics therefore correspond to prolonged episodes of appreciation of the domestic currency, large cumulated capital inflows, and benign domestic financial conditions following a negative shock on the US interest rate. Subsequent small increases in the US interest rate do not reverse these dynamics until a tipping point is reached. This point triggers a large currency depreciation, important capital outflows, and a crash in the domestic bond market.

Condition (37) is the same as the one that generates multiple steady states in the perfect-foresight case. It is worthwhile commenting on the additional condition that capital move sufficiently slowly ($\lambda$ sufficiently small). This condition guarantees that the frontier $f$ is decreasing, and thus that carry trades are destabilizing. To better grasp its role, notice that if a carry trader expects other carry traders to become long in the future, then he expects the exchange rate to appreciate. This implies that on one hand, the currency will be expensive when he will purchase it to lend. On the other hand, it will keep appreciating over the duration of the loan, thereby generating a positive return. The former effect is akin to a congestion effect. Other traders make the trade more expensive and thus less desirable. Conversely, the latter effect is destabilizing as future carry trades make becoming long more appealing. That $\lambda$ be sufficiently small ensures that this latter effect offsets the former congestion effect because aggregate lending does not converge too quickly to its maximum value. Thus a carry trader with a current switching date is more likely to have a chance to lend before the currency becomes too expensive, and its upside potential too small. This congestion effect is the salient difference between our setup and that studied by Burdzy, Frankel, and Pauzner.

**Proof of Proposition 3**

The proof of Proposition 3 essentially extends to this stochastic environment the logic leading to the perfect-foresight results in Proposition 1. In a first step, we derive an expression of the expected return on carry trades (28) as a function of the future flows $(l_{t+s})_{s\geq 0}$ and of the interest-rate differential $w_t$ that is the stochastic counterpart of equation (18). Second, we use this expression to solve for a Lipschitz process that satisfies (35). This latter step is the
equivalent of the one that consisted in solving for feasible steady states given the expected return for carry traders (18) under perfect foresight.

More precisely, the first step consists in using relations (31) to (34) to express the nominal exchange rate and interest rate as functions of the expected future paths of capital inflows $L_t$. This yields in turn a relatively simple expression for the expected return on the carry trade $\Theta_t$ as a function of these expected capital inflows:

**Lemma 4** At first-order, the expected return on the carry trade is

$$
\Theta(w_t, x_t) = \int_0^{\infty} \left( \left( \frac{\chi \omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi \omega}{\omega - \rho - \lambda} e^{-\omega v} \right) E_t[l_{t+v}] dv
$$

$$
l_t = \ln(L_t + W) \simeq x_t \bar{L} + (1 - x_t) L, \tag{41}
$$

$$
\rho = 1 - \frac{1}{R}, \tag{42}
$$

$$
\omega = \frac{\Phi}{1 + \Phi}, \tag{43}
$$

$$
\chi = \frac{\gamma}{(1 - \gamma) \Phi}. \tag{44}
$$

The proof of Lemma 4 is given in the Appendix.

The factor that discounts future capital inflows in (40):

$$
\left( \frac{\chi \omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi \omega}{\omega - \rho - \lambda} e^{-\omega v}, \tag{45}
$$

is first negative, then positive as $v$ spans $[0, +\infty)$. This formalizes the above comment that future long investors create congestion effect for the current trader. The earliest inflows have a negative impact on $\Theta$ because they make the domestic currency expensive. The more remote inflows are desirable as the current investor is more likely to have lent before they occur. The following lemma establishes conditions under which the congestion effect is not too important.

---

9Notice that this is so regardless of the sign of $\omega - \rho - \lambda$. 23
Lemma 5 Suppose that $\chi > 1$. There exists $\overline{\lambda}$ such that for all $\lambda \leq \overline{\lambda}$, the following is true. Suppose that two processes $x^1_t$ and $x^2_t$ satisfy

\begin{align*}
0 < x^1_0 \leq x^2_0 < 1, \\
&\text{For } i = 1, 2, dx^i_t = \lambda \left(1_{\{w_t > f^i(x^i_t)\}} - x^i_t\right) dt,
\end{align*}

where $f^i$ is decreasing Lipschitz and $f^2 \leq f^1$. Then

$$
\Theta(w_t, x^2_0) \geq \Theta(w_t, x^1_0). 
$$

The inequality is strict if $f^1 \neq f^2$ and/or $x^1_0 \neq x^2_0$.

The proof of lemma 5 is in the Appendix.

Lemma 5 states that if (37) holds and $\lambda$ is sufficiently small, then future inflows make current carry trades more attractive because the reinforcing effect overcomes the congestion effect. In the balance of the paper, we suppose that the conditions in Lemma 5 are satisfied. We now show that there is in this case a unique Lipschitz process $x_t$ that satisfies the equilibrium conditions.

First, the proof of Lemma 5 also shows that the case in which $x_t$ obeys $\frac{dx_t}{dt} = -\lambda x_t$ for all $u \geq 0$ corresponds to a lower bound on the expected carry-trade return. When $x_t$ obeys such dynamics, there exists a frontier $f_0$ such that

$$
w_t = f_0(x_t) \implies \Theta(x_t, w_t) = 0.
$$

The frontier $f_0$ is decreasing from Lemma 5 (with $f^1 = f^2 = +\infty$) and is clearly affine and thus Lipschitz.$^{10}$ Thus an admissible equilibrium process must be such that investors who have a chance to switch when the system is on the right of $f_0$ become long.

Define now $f_1$ such that

$$
w_t = f_1(x_t) \implies \Theta(x_t, w_t) = 0
$$

if for all $u \geq 0$,

---

$^{10}$ The frontier simply obtains from writing $E_t[l_{t+e}] = l + (I - I) x_t e^{-\lambda v}$ in (40).
That is, $f_1$ is such that an investor is indifferent between being long or short when the system is on $f_1$ at his switching date if he believes that other investors become long if and only if they are on the right of $f_0$. This function $f_1$ must be decreasing. Suppose otherwise that two points $(w, x)$ and $(w', x')$ on $f_1$ satisfy

\[ x' > x, \]
\[ w' \geq w. \]

Then applying Lemma 5 with $f^2 = f_0$, $f^1 = f_0 + w' - w$ contradicts that both points generate the same expected carry-trade return. We also show in the appendix that $f_1$ is Lipschitz, with a Lipschitz constant smaller than that of $f_0$.

By iterating this process, we obtain a limit $f_\infty$ of the sequence of frontiers $(f_n)_{n \geq 0}$ that is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The process

\[
\frac{dx_t}{dt} = \begin{cases} 
-\lambda x_t & \text{if } w_t < f_\infty(x_t), \\
\lambda(1 - x_t) & \text{if } w_t > f_\infty(x_t) 
\end{cases}
\] (50)

is an admissible equilibrium since by construction, if all investors switch to being short to the left of $f_\infty$ and to being long to the right, the indifference point for an investor also lies on $f_\infty$. We now show that this is the only equilibrium process.

Consider a translation to the left of the graph of $f_\infty$ in $(w, x)$ so that the whole of the curve lies in a region where $w_t$ is sufficiently small that being short is dominant regardless of the dynamics of $x_t$. Call this translation $f'_0$. To the left of $f'_0$, going short is dominant. Then construct $f'_1$ as the rightmost translation of $f'_0$ such that an investor must choose to be short to the left of $f'_1$ if he believes that other investors will play according to $f'_0$. By iterating this process, we obtain a sequence of translations to the right of $f'_0$. Denote by $f'_\infty$ the limit of the sequence. Refer to Figure 3.

The boundary $f'_\infty$ does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of $f'_0$. However, we know that if all others were to play according
to the boundary $f'_\infty$, then there is at least one point $A$ on $f'_\infty$ where the investor is indifferent. If there were no such point as $A$, this would imply that $f'_\infty$ is not the rightmost translation, as required in the definition.

We claim that $f'_\infty$ and $f_\infty$ coincide exactly. The argument is by contradiction. Suppose that we have a gap between $f'_\infty$ and $f_\infty$. Then, choose point $B$ on $f_\infty$ such that $A$ and $B$ have the same height - i.e. correspond to the same $x$. But then, since the shape of the boundaries of $f'_\infty$ and $f_\infty$ and the values of $x$ are identical, the paths starting from $A$ must have the same distribution as the paths starting from $B$ up to the constant difference in the initial values of $w$. This contradicts the hypothesis that an investor is indifferent between the two actions both at $A$ and at $B$. If he were indifferent at $A$, he would strictly prefer being long at $B$, and if he is indifferent at $B$, he would strictly prefer being short when in $A$. But we constructed $A$ and $B$ so that investors are indifferent in both $A$ and $B$. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no “gap”, and we must have $f'_\infty = f_\infty$. $\blacksquare$
Proposition 3 shows that adding exogenous shocks $w_t$ to the carry return eliminates the indeterminacy of the perfect-foresight case. More precisely, equilibrium uniqueness stems from the interplay of these shocks with the fact that each investor, when he receives a switching opportunity, needs to form beliefs about the decisions of the investors that will have an opportunity to switch between now and his next switching date. Suppose that $(w_t, x_t)$ is close to a dominance region in which investors would prefer a course of action for sure, but just outside it. If $w_t$ was fixed, it may be possible to construct an equilibrium for both actions, but when $w_t$ moves around stochastically, it will wander into the dominance region between now and the next opportunity that the trader gets to switch with some probability. This gives the investor some reason to hedge his bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on.

**Remark 1. (Bounded shocks to the US rate)** We model the interest-rate differential as a Brownian motion for expositional simplicity. It is easy to see that we could write it as $d(w_t)$, where $w_t$ is a standard Brownian motion, and $d$ a Lipschitz increasing function, possibly bounded as long as there are still dominant actions for $w_t$ sufficiently large or small.

**Remark 2. (Transitory shocks to the US rate)** While a strong persistence in shocks to the US rate is undoubtedly realistic, extensions of this framework can also accommodate for various forms of mean-reversion (Burdzy, Frankel, and Pauzner, 2001, or Frankel and Burdzy, 2005).

**Remark 3. (Illiquidity and slow-moving capital)** The condition that $\lambda$ be sufficiently small seems particularly relevant for the carry trades that involved many retail investors, such as those targeting New Zealand dollar or Icelandic krona. The glacier bonds denominated in Icelandic krona or the uridashi bonds used by Japanese investors to invest in New Zealand had a typical maturity of 1 to 5 years, and were principally purchased by retail investors. More generally, Bacchetta and van Wincoop (2009) claim an average two-year rebalancing frequency to be plausible in FX markets in general, and assume it in order to quantitatively explain the forward discount bias. Also, well-documented price pressure and illiquidity in currency markets, especially for small currencies, may force professional FX
speculators to build-up or unwind large positions more gradually than they would like to.

**The case of small shocks**

The limiting case in which the volatility $\sigma$ of the interest-rate differential tends to zero yields useful insights. It is possible to characterize the shape of the frontier $f$ in this case.

In this section we denote the frontier $f_\sigma$ to emphasize its dependence on $\sigma$. Suppose the economy is in the state $(f_\sigma(x_t), x_t)$ at date $t$. That is, it is on the equilibrium frontier. For some arbitrarily small $\varepsilon > 0$, introduce the stopping times

$$T_1 = \inf_{u \geq 0} \{ x_{t+u} \notin (\varepsilon, 1 - \varepsilon) \},$$
$$T_0 = \sup_{0 \leq u < T_1} \{ w_{t+u} \neq f_\sigma(x_{t+u}) \}.$$

In words, $T_1$ is the first date at which $x_t$ gets close to 0 or 1, and $T_0$ is the last date at which $x_t$ crosses the frontier before $T_1$. If $T_0$ is small in distribution, it means that the economy is prone to bifurcations. That is, it never stays around the frontier for long. Upon hitting it, it quickly heads towards extreme values of $x$. The next proposition shows that this is actually the most likely scenario when $\sigma$ is small. This, in turn, yields a simple explicit determination of the frontier.

**Proposition 6**

1. As $\sigma \to 0$, $T_0$ converges to 0 in distribution, and the probability that $\frac{dx_t}{dt} > 0$ (respectively $\frac{dx_t}{dt} < 0$) over $[T_0, T_1]$ converges to $1 - x_t$ ($x_t$ respectively).

2. As $\sigma \to 0$, the frontier $f_\sigma$ tends to an affine function. For $\lambda$ sufficiently small, the slope of this function is increasing in $\Phi$ and decreasing in $\gamma$.

The proof of Proposition 6 is in the Appendix.

First, Proposition 6 clears the concern that in equilibrium, $x$ would only exhibit small fluctuations around a fixed value because Brownian paths cross the frontier too often. As $\sigma$ becomes smaller, the system exhibits more frequent bifurcations towards extremal values of $x$. When the system reaches the frontier, it is all the more likely to bifurcate towards capital
outflows when cumulative inflows have been large \((x \text{ large})\). Thus the model does generate “destabilizing carry trades,” whereby global investors generate durable self-justified excess returns on the carry trade followed by large reversals.

The second point in Proposition 6 relates the slope of the frontier \(f_{\sigma}\) to the monetary parameters of the model \(\Phi\) and \(\gamma\) in this case of small shocks. The slope of the frontier affects the dynamics of capital inflows and in turn the exchange-rate dynamics. If the graph of the frontier is closer to being horizontal in the \((w, x)\) plane, then the system should cross the frontier less often, and thus do so only for more extreme values of \(x\). Carry-trade returns should in this case exhibit more serial correlation and fatter tails. Point 2 states that, at least for \(\lambda\) sufficiently small, the frontier is flatter when \(\Phi\) is smaller, and \(\gamma\) larger. In other words, if monetary policy does not respond much to capital inflows, then carry trade returns should exhibit more skewness.

4 Further analytical results

Our model generates a rich set of qualitative features, of which many have empirical implications. As a by-product of our analysis, we provide in particular a novel explanation for the seemingly high Sharpe ratio generated by carry-trade strategies.

Profitability of FX carry trades

The equilibrium expected return on the carry trade \(\Theta(w, x)\) increases with respect to \(w\) and \(x\), it is positive on the right of the frontier \(f\) in the \((w, x)\) plane and negative on the left. On the other hand, the interest-rate differential increases in \(w\) and decreases in \(x\). We have indeed:

**Lemma 7** At first-order, the interest-rate differential at a given arrival date \(T_n\) is given by

\[
R \left( w_{T_n} - l_{T_n} - \frac{1}{1 + \Phi} \int_0^{+\infty} e^{-\omega s} E_{T_n} [l_{T_n+s}] \, ds \right).
\]  

The proof of lemma 7 is in the Appendix.

The interest-rate differential increases w.r.t. \(w\) but decreases w.r.t. \(l\) (and thus \(x\)) because the current domestic real rate is lower and future deflation more likely when \(l\) is large. Thus
the expected return on the carry trade is not unambiguously increasing in the interest-rate differential. Yet, when the interest-rate differential is sufficiently large in absolute terms, it must be that the system is on the right (left) of the frontier when the differential is positive (negative). Thus, we have:

A positive (negative) interest-rate differential predicts a positive (negative) return on the carry-trade for sufficiently large absolute differentials.

In particular, for \( l, l \) sufficiently small, most of the interest-rate differential is due to the exogenous component \( w \). The threshold above which a positive (negative) interest-rate differential is associated with a positive (negative) excess return on the carry trade is arbitrarily small. This rationalization of carry-trade returns as self-fulfilling genuine excess returns contrasts with existing theories that seek to explain the return on carry trades as a compensation for (possibly mismeasured) risk. Farhi and Gabaix (2015) thoroughly survey this existing literature. We do not deny that a significant fraction of carry-trade returns may reflect risk premia, and view our theory as a complement to such risk-based considerations rather than a competing alternative.

**Peso problem**

A large literature argues that the return on the carry trade partly reflects a risk premium for rare and extreme events that may not show in finite samples (see, e.g., Farhi and Gabaix, 2015, or Lewis, 2007, and the references therein). We closely connect to this literature as follows. Fix \( \epsilon > 0 \) small. The expected return on the carry trade is 0 starting both from \((f(\epsilon), \epsilon)\) and \((f(1-\epsilon), 1-\epsilon)\) in the \((w, x)\) plane. Yet from Proposition 6, as \( \sigma \) becomes small, most paths starting from \((f(\epsilon), \epsilon)\) will exhibit long periods of appreciation of the domestic currency ended with rare (and large) depreciations, while paths starting from \((f(1-\epsilon), 1-\epsilon)\) will feature a symmetric prolonged depreciation. The interest-rate differential is positive in the former case and negative in the latter. Thus, due to rare reversals, finite samples should yield that a positive interest-rate differential predicts a positive excess return on the carry trade even when the true expected return is zero.
Profitability of FX momentum strategies

Proposition 6 shows that as $\sigma \to 0$, the system often bifurcates in one direction. This implies that, at least at a sufficiently short horizon, returns are positively autocorrelated, so that momentum strategies in FX markets should generate a positive excess return.

It is important to stress that the profitability of momentum strategies is not a mechanical consequence of the assumption of slow-moving capital ($\lambda$ sufficiently small). Returns on the carry trade are still positively auto-correlated if the system bifurcates quickly towards extreme values of lending $x$. The key economic force behind this profitability of momentum strategies is that once carry traders coordinate on a course of action, they stick to it until a sufficiently large reversal of the interest-rate differential leads them to switch to a different strategy. Such a rationalization of momentum returns with coordination motives is novel to our knowledge.

Monetary policy and carry-trade returns

In addition to relating to the above existing empirical findings, the model also generates a new range of predictions on the relationship between the stance of monetary policy and the distribution of the returns on carry-trade strategies. Proposition 6 suggests that the frontier is flatter when $\Phi$ is smaller and $\gamma$ larger. Otherwise stated, if an economy is such that the CPI is not too sensitive to the exchange rate, and/or the central bank not too aggressive, then this economy should be more prone to large fluctuations in carry-trade activity because it will experience more prolonged bifurcations. Thus the returns on carry-trade and momentum strategies should have fatter tails. These predictions are novel, to our knowledge.

5 Concluding remarks

The independence of monetary policy under liberalised capital flows and floating exchange rates has been a benchmark principle in international finance. In our paper, we have explored a parsimonious model of global investors facing each other in a dynamic global game and found that for reasonable parameters of the model, the model generates boom bust cycles
associated with coordinated capital inflows and outflows. In such a setting, monetary conditions depend on the coordination outcome of investors who have access to the domestic bond market, as well as on the economic fundamentals. In this sense, we can reject the strict version of the claim that a floating exchange rate guarantees monetary autonomy.

We see our paper as a useful benchmark in the debate. We have shown in our dynamic global game model of international investment flows that, for reasonable combination of parameters, the unique equilibrium exhibits the observed empirical feature that currency appreciation goes hand-in-hand with looser domestic financial conditions in the form of lower domestic interest rates and higher credit growth fuelled by capital inflows. When reversed, tighter monetary conditions go hand-in-hand with capital outflows and currency depreciation.

We offer these findings as the first steps towards a more normative analysis. They come at a time when they may be of some use in making sense of recent macro developments in emerging economies.
A Appendix

A.1 Proof of Lemma 4

Using the first-order approximation (36) in (34), relations (31) and (34) yield domestic inflation as a function of future expected inflows as in the perfect-foresight case:

\[
\ln \frac{P_{T_n}}{P_{T_n-1}} = -\sum_{k \geq 0} \frac{ET_n [l_{T_{n+k}}]}{(1 + \Phi)^{k+1}},
\]

where \( l_t = \ln(L_t + W) \). As in the perfect-foresight case, (32) and (33) yield in turn:

\[
ET_n \left[ \ln \frac{S_{T_{n+1}} I_{T_{n+1}}}{RS_{T_n}} \right] = \frac{\gamma}{1 - \gamma} \sum_{k \geq 0} \frac{ET_n [l_{T_{n+k+1}}]}{(1 + \Phi)^{k+1}} - l_{T_n}
\]

One can write (28) as

\[
\Theta_t = ET_t \left[ \sum_{m \geq 0} \frac{1}{R^m} \frac{I_{T_{n+m}}}{E_{T_{n+m}}} \left[ \frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left( \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}} - 1 + w_{T_{n+m}} \right) \right] \right].
\]

At first-order w.r.t. \( l_t \),

\[
ET_{n+m} \left[ \frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left( \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}} - 1 \right) \right] = ET_{n+m} \left[ \ln \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}} \right]
\]

Thus,

\[
\Theta_t = ET_t \left[ \int_0^{+\infty} \sum_{m \geq 1} \left( \frac{s}{R} \right)^{m-1} e^{-s \lambda} \left[ \int_0^{+\infty} \frac{\gamma}{1 - \gamma} \sum_{k \geq 1} \frac{u^{k-1}}{(k-1)!} \left( \frac{1}{(1 + \Phi)^k} \right) l_{t+s+u} \right] ds \right],
\]

and integrating yields the result.
A.2 Proof of Lemma 5

Suppose $\chi > 1$. Consider two processes $x^1_t$ and $x^2_t$ that satisfy the conditions stated in Lemma 5 with $x^1_0 < x^2_0$. Lemma 2 in Burdzy, Frankel and Pauzner (1998) states that almost surely,

$$x^2_t \geq x^1_t \text{ for all } t \geq 0. \quad (60)$$

This implies in particular that whenever investors switch to being long along a sample path of $(w_t, x^1_t)$, so do they along the sample path of $(w_t, x^2_t)$ that corresponds to the same sample path of $w_t$. This is because it must be that $(w_t, x^2_t)$ is on the right of the frontier $f^2$ whenever $(w_t, x^1_t)$ is on the right of the frontier $f^1$. Thus, the process

$$y_t = x^2_t - x^1_t \quad (61)$$

satisfies

$$0 < y_0 < 1, \quad (62)$$

$$\frac{dy_t}{dt} = \lambda(e_t - y_t), \quad (63)$$

In order to prove the Lemma, we only need to find $\lambda$ such that for all $\lambda \leq \bar{\lambda}$,

$$\Delta = \int_0^{+\infty} \left( \left( \frac{\chi}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda + \rho)v} - \frac{\chi}{\omega - \rho - \lambda} e^{-\omega v} \right) y vdv \geq 0. \quad (64)$$

for all deterministic process $y_t$ that obeys (62) and (63). The result then obtains from taking expectations over all paths of $w_t$.

To prove (64), we introduce the function $\zeta$ that satisfies

$$\left\{ \begin{align*}
\frac{d\zeta(v)}{dv} &= - \left( \frac{\chi}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda + \rho)v} - \frac{\chi}{\omega - \rho - \lambda} e^{-\omega v}, \\
\lim_{v \to +\infty} \zeta &= 0.
\end{align*} \right. \quad (65)$$
Integrating by parts, we have

\[
\Delta = \zeta(0)y_0 + \int_0^{+\infty} \zeta(v) \frac{dv}{dv} dv, \\
= \zeta(0)y_0 + \lambda \int_0^{+\infty} \zeta(v)(\epsilon_v - y_v)dv. 
\]

(65)

(66)

\[
y_v = y_0e^{-\lambda v} + \lambda \int_0^v e^{-\lambda(v-u)}\epsilon_u du, 
\]

(67)

\[
\Delta = y_0 \left( \zeta(0) - \lambda \int_0^{+\infty} \zeta(v)e^{-\lambda v} dv \right) \\
+ \lambda \left[ \int_0^{+\infty} \epsilon_v \left( \zeta(v) - \lambda \int_v^{+\infty} \zeta(u)e^{-\lambda(u-v)} du \right) \right]. 
\]

(68)

(69)

We have

\[
\lim_{\lambda \to 0} \zeta(0) = \frac{\chi - 1}{\rho} > 0, 
\]

(70)

\( \zeta \) is increasing then decreasing beyond a value that stays bounded as \( \lambda \) tends to zero, and \( \int_0^{+\infty} \zeta \) converges. Thus for \( \lambda \) sufficiently small,

\[
\zeta(v) - \lambda \int_v^{+\infty} \zeta(u)e^{-\lambda(u-v)} du 
\]

(71)

is positive for all \( v \geq 0 \), which yields that \( \Delta \) is positive, and concludes the proof.

A.3 Complement to the proof of Proposition 3

We prove here that \( f_1 \) is Lipschitz with a constant that is smaller than that of \( f_0 \), that we denote \( K_0 \). Suppose by contradiction that two points \((w_t, x_t)\) and \((w'_t, x'_t)\) on \( f_1 \) satisfy

\[
x'_t > x_t, \\
x'_t - x_t < \frac{1}{K_0}. 
\]

(72)

(73)

We compare the paths \( x'_{t+u} \) and \( x_{t+u} \) corresponding to pairs of paths of \( w'_{t+u} \) and \( w_{t+u} \) that satisfy for all \( u \geq 0 \)

\[
w_{t+u} - w'_{t+u} = w_t - w'_t. 
\]

(74)

It must be that for such pairs of paths:

\[
x'_{t+u} - x_{t+u} \leq (x'_t - x_t)e^{-\lambda u}. 
\]

(75)
Otherwise it would have to be the case that \((w',x')\) can be on the right of \(f_0\) when \((w,x)\) is not. Let \(T\) denote the first time at which this occurs. It must be that

\[
K_0 e^{-\lambda T} (x'_t - x_t) \geq w_{t+T} - w'_{t+T} = w_t - w'_t,
\]

(76)
a contradiction with (73). Thus along such paths of \(w'_{t+u} - w_{t+u}, x'_{t+u} - x_{t+u}\) shrinks at least as fast as when investors switch to being short all the time. Together with (73), this implies that the expected return on the carry trade cannot be the same in \((w_t,x_t)\) and \((w'_t,x'_t)\), a contradiction.

**A.4 Proof of Proposition 6**

The first point is a particular case of Theorem 2 in Burdzy, Frankel, and Pauzner (1998). To prove the second point, notice that as \(\sigma \to 0\), starting from a point on the frontier,

\[
E_t [x_{t+v}] \simeq (1 - x_t) (1 - (1 - x_t) e^{-\lambda v}) + x_t^2 e^{-\lambda v}
\]

(77)
because the system bifurcates upwards with probability \(1 - x_t\) and downwards with probability \(x_t\) in the limit. Plugging this in (40) and writing that the expected return is zero yields a slope of the frontier equal to

\[
-(\overline{l} - l)(\chi - 1)
\]
as \(\lambda \to 0\). This means that the absolute value of the slope of the frontier varies as \(\chi\) w.r.t. \(\gamma, \Phi\) for \(\sigma, \lambda\) sufficiently small. This proves the proposition.

**A.5 Proof of Lemma 7**

We have

\[
I_{T_{n+1}} - R(1 - w_{T_n}) = R \left( \left( \frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi} - 1 + w_{T_n} \right),
\]

(78)

\[
\simeq R \left( w_{T_n} - E_{T_n} \left[ \sum_{k \geq 0} \frac{l_{T_{n+k}}}{(1 + \Phi)^k} \right] \right),
\]

(79)

\[
= R \left( w_{T_n} - l_{T_n} - \int_0^{+\infty} \sum_{k \geq 1} \frac{s^{k-1} e^{-s}}{(k-1)!(1 + \Phi)^k} E_{T_n} [l_{T_{n+s}}] \, ds \right),
\]

(80)

\[
= R \left( w_{T_n} - l_{T_n} - \frac{1}{1 + \Phi} \int_0^{+\infty} e^{-\omega s} E_{T_n} [l_{T_{n+s}}] \, ds \right).
\]

(81)

This proves the lemma.
References


Fleming, J. Marcus (1962): “Domestic financial policies under fixed and floating exchange rates” *IMF Staff Papers* 9, 369–379


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