Risk Sharing among Large and Small Countries

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The Question

- Ongoing debate on the degree of international risk sharing between countries and how to increase it.
    - globally: performance of capital markets in achieving risk sharing among countries (Fratzscher and Imbs 2009) and reform of international institutions.
    - in Europe: what policies should be implemented to enhance risk sharing? (e.g. Fatas 1998, or more recently Farhi and Werning 2017, Beblavy et al 2015).
- Countries differ in size and risk profile:
  What are the macroeconomic implications/effects of arrangements that improve sharing of macroeconomic risk among asymmetric countries?
- To what extent can these implications hamper integration, by raising political economy and distributional issues?
- Asymmetry challenges macro modelling.

This Paper

- We present a stylized analysis of risk sharing arrangements (operating either via capital market integration or institutions), with the goal of deriving broad implications for policy and empirical studies.
- For simplicity:
    - two-period model.
    - risk is only fundamental, output process is exogenous (endowment economies).
    - one good model (PPP holds, RER=1).
    - no frictions, except for possibility of market incompleteness.
- In our setup, barring risk sharing (status quo), countries would have identical per capita income on average.
  ...but can differ in size and stochastic properties of business cycle.

Main Message

- Relative income and aggregate demand changes when countries enter either risk or income sharing arrangements (via market integration or institution).
  - The shadow price of domestic (state-contingent) output may differ across borders: in an equilibrium with perfect risk sharing, the value of output flows at market prices may rise or fall.
  - Perfect risk sharing involves an implicit efficient transfer (redistribution) reflecting the "pecuniary" effects of price adjustment.
- Key: enhancing risk sharing, the **level** of national demand may thus rise or fall relative to the status quo.

Implications (1)

Political Economy

- Asymmetries between regions affect political equilibrium and thus degree of risk sharing provided by regional public transfers (e.g. Persson and Tabellini (1996)).
- Because of asymmetries across countries, schemes pursuing income pooling (i.e. redistribution targeting average income across borders) will generally lead to allocations that differ from complete-market, and may not be Pareto improving.

Implications (2)

Quantitative and Empirical Studies

- In quantitative models, approximation around non-stochastic steady state fails to take into account the equilibrium price of risk.
  - Up to first order, asymmetry among countries does not matter and thus income pooling is equivalent to complete markets.
  - Up to second order, neglecting the effect of asymmetry on the steady state may lead to spurious **"welfare reversals"**.
- Measure of risk sharing focused on equalization of (RER adjusted) consumption growth rates may be confounded by step adjustment in consumption levels at time of regime changes (capital market reforms).
  - Consumption growth rates may not be equalised at the time of of a risk-sharing enhancing reform.

Literature

- Theoretical discussion in quantitative studies see Cole and Obstfeld (1991), Kim and Kim (2003), Chari et al (2002), Tille and Pesenti (2004).
- Gains from financial integration depend on the stochastic properties, size and degree of financial development of countries (Rey et al (2016)).
- Discussion of institutions that could enhance risk sharing in the European Union (EU Commission Reflection Paper (2017)).
- A vast literature on estimating level of risk sharing (Asdrubali et al (1996), Bayoumi and Klein (1997), Furceri and Zdzienicka (2015), Sala-i-Martin and Sachs (1991), Sorensen and Yosha (1998), von Hagen and Hepp (2013)).

Outline

- Model (one factor).
- Different Risk Sharing Arrangements: Complete Markets, Income Pooling and Financial Autarky.
- Edgeworth box and country size.
- Welfare.
- Generalization (two factors).

Analytical Framework

- Two-period endowment model with two countries (H and F) differing in size and output process.
- First example: Asymmetric volatility—a single global shock with different factor loadings for each country.
    - In second example we show that results generalize to country-specific shocks.
- We contrast three different arrangements:
    1. complete markets (CM),
    2. income pooling (IP) (equalisation of home and foreign consumption),
    3. financial autarky (FA).
- Welfare analysis under the second order approximation with log utility, $U(C_i) = log(C_i)$ where $i = H, F$.

Model Description

- Size: H has population n; F has (1-n).
- Endowment is deterministic in period 1. In per capita terms:

$$Y_{H,1} = Y_{F,1}.$$

- Endowment is stochastic in period 2, driven by a common factor ϵ:

$$Y_{H,2} = 1 - \gamma_H \epsilon, Y_{F,2} = 1 - \gamma_F \epsilon.$$

where $E(\epsilon) = 0$, $E(\epsilon^2) = \sigma_\epsilon^2$ and γ_H, γ_F are factor loadings. Note, on average, $\overline{Y}_{i,2} = 1$.

  - Set γ_F =1. Then $|\gamma_H| > 1$ means that Home output is more volatile than Foreign output.
  - The covariance of output depends on sign of γ_H.

- Aggregate output:

$$Y_{W,1} = 1.$$

$$Y_{W,2} = nY_{H,2} + (1-n)Y_{F,2} = 1 - \gamma_W \epsilon.$$

where $\gamma_W = n\gamma_H + (1-n)\gamma_F$.

Complete Markets Allocation

- Consumption **growth rates** in each country are equalised across all states of nature.
- In **levels**, consumption in each country is a fraction of the world endowment (see e.g. Obstfeld and Rogoff (1996)):

$$C_{i,t} = \mu_i Y_{W,t}.$$

where $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}}\right)}{1+\beta}$.

- Up to second order approx, $E\left(\frac{Y_{H,2}}{Y_{W,2}}\right) \approx 1 + (\gamma_W - \gamma_H)\gamma_W \sigma_\epsilon^2$.
- Home and Foreign consumption shares:

$$\mu_H = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_H)}{1+\beta}\sigma_\epsilon^2, \mu_F = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_F)}{1+\beta}\sigma_\epsilon^2.$$

Efficient Consumption Share

Analytical Representation

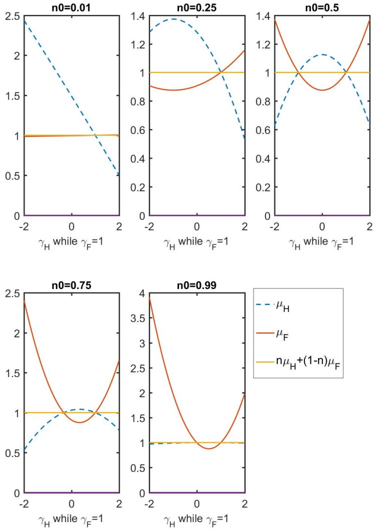
$$\mu_i = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_i)}{1+\beta}\sigma_\epsilon^2$$

where $i = H, F$ and $\gamma_W = n\gamma_H + (1-n)\gamma_F$.

- Up to first order, $\mu_i = 1$ implying that $C_{H,t} = C_{F,t}$, where $t = 1, 2$.
- Up to second order, μ_i can be higher or lower than 1:
  - Home and Foreign consumption share increase in the difference $(\gamma_W - \gamma_i)$.
  - γ_i can be negative which can result in negative γ_W.
  - γ_W is a function of the size of the country: the larger the country the larger its influence on the world output and γ_W.

Efficient Consumption Share

Asymmetric Size and Volatility



Consumption: Levels vs Growth rates

under different risk-sharing arrangements

- **consumption LEVELS are equalized** under IP:

  $C_{H,t} = C_{F,t}$ where $t = 1, 2.$:

  ...as well as (by construction in our example) under FA in period 1:

  $C_{H,1} = 1 = C_{F,1}$
  (recall: $C_{H,2} = 1 - \gamma_H \epsilon \neq C_{F,2}$ yet $\overline{C}_H = \overline{C}_F = 1.$)

- **may not be equalized** under CM, because of the endogenous adjustment of income to the price of risk:

  $C_{H,1} = \mu_H, C_{H,2} = \mu_H Y_{H,2}$ with $\overline{C}_H = \mu_H$.

- Consumption **GROWTH RATES** instead are **equalized under both CM and IP**, not under FA.

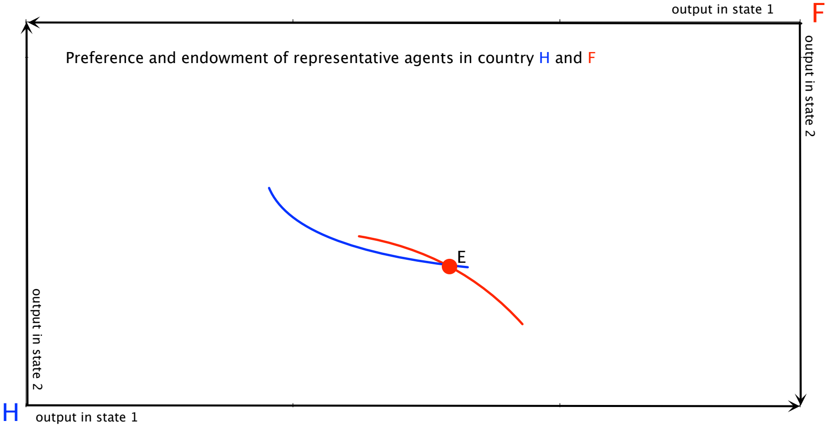
Edgeworth Box Analysis

Unconventional use of the box:

- Individual preferences and endowment of representative agents of two countries H and F.
- But we allow the number of agents in H to be infinitesimal: n goes to 0.
- World price of risk dictated by autarky price in F.
  - In the graph to follow, $\gamma_H > \gamma_F$ and $\gamma_F = 1$. Recall that $Y_{i,2}$ subject to a mean zero shock.

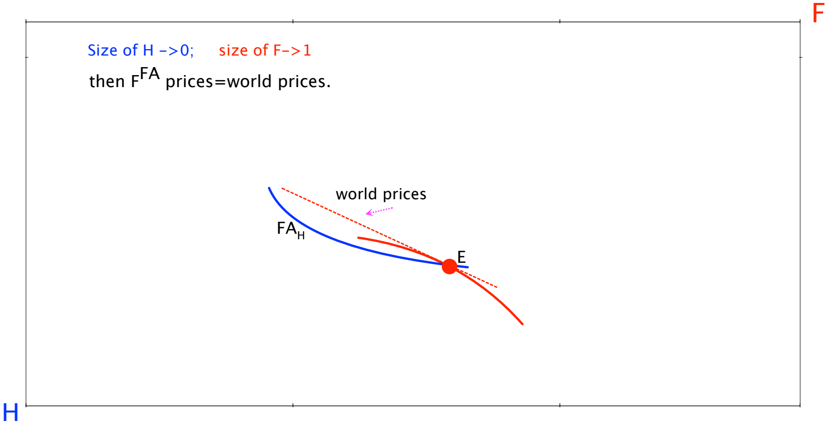
Edgeworth Box

Financial Autarky



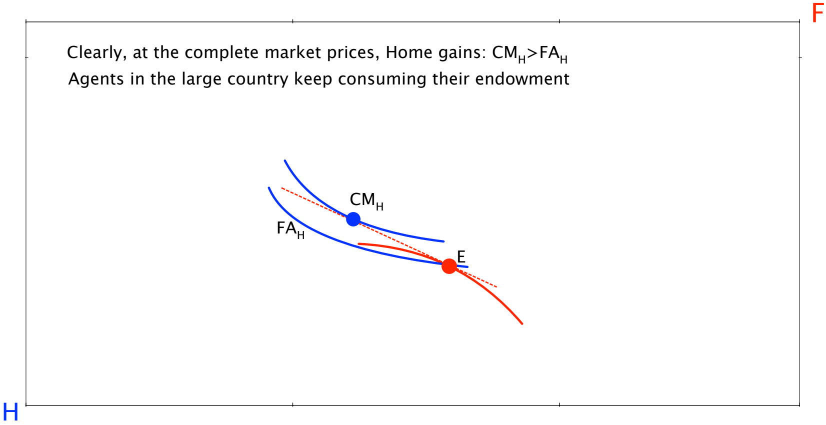
Edgeworth Box

Budget Constraint and Autarky World Prices



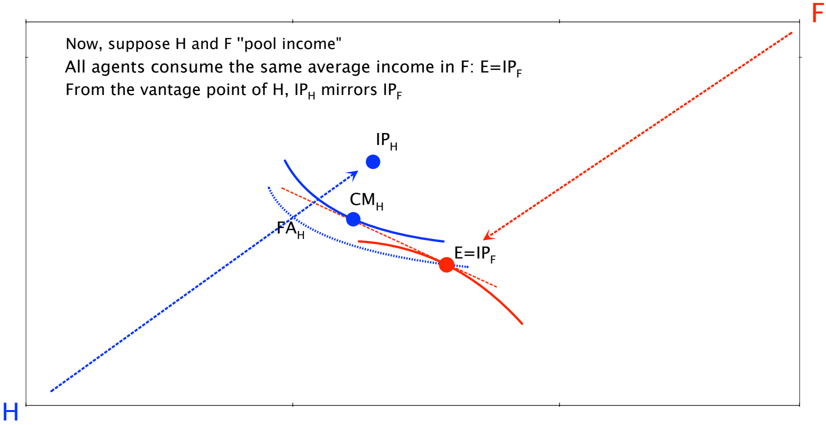
Edgeworth Box

Complete Markets



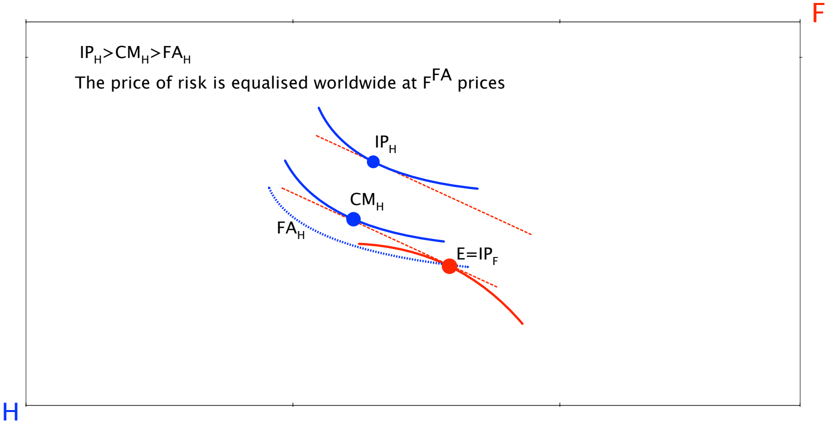
Edgeworth Box

Income Pooling



Comparing Risk Sharing Arrangements

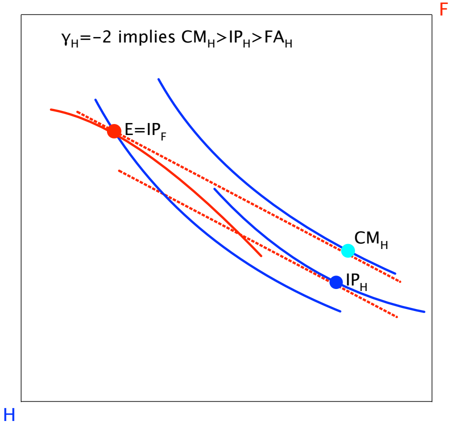
IP is a good deal for Home: case of high volatility of output with $\gamma_H = 2$



Comparing Risk Sharing Arrangements

But income pooling may also be a bad deal for Home

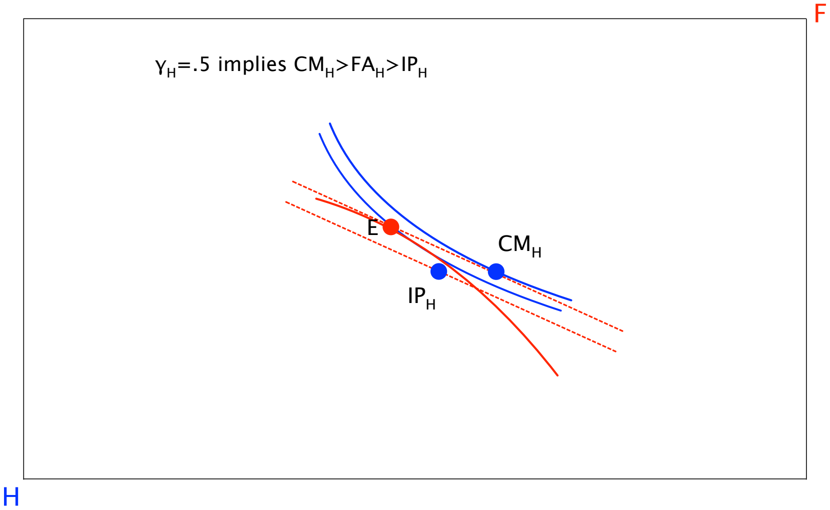
output variance high but covariance negative $\gamma_H = -2$, $(\gamma_F = 1)$



Comparing Risk Sharing Arrangements

Home may even be better off in FA

volatility of output not too high, e.g. $\gamma_H = 0.5$, $(\gamma_F = 1)$



Welfare analysis in the general case

Second Order Approximation

We now reconsider welfare ranking using:

$$E(log(C_H)) = E(log(\overline{C}_H) + \frac{1}{\overline{C}_H}E(C_H - \overline{C}_H) - \frac{1}{\overline{C}_H^2}E(C_H - \overline{C}_H)^2$$

Home loss under different risk sharing arrangements:

- FA: $L_H^{FA} = \frac{\beta}{2}\gamma_H^2\sigma_\epsilon^2$.
- CM: $L_H^{CM} = -\beta(\gamma_W - \gamma_H)\gamma_W\sigma_\epsilon^2 + \frac{\beta}{2}\gamma_W^2\sigma_\epsilon^2$.
- IP: $L_H^{IP} = \frac{\beta}{2}\gamma_W^2\sigma_\epsilon^2$.

Ranking Welfare

as a Function of Size and Volatility

Table 1: **Ranking of CM, FA, IP for Home and Foreign country depending on home size n and factor loading γ_H (setting $\gamma_F = 1$):**

n	γ_H				
0.01	< -199	$(-199, -99)$	$(-99, -0.98)$	$(-0.98, 1)$	> 1
0.25	< -7	$(-7, -3)$	$(-3, -0.6)$	$(-0.6, 1)$	> 1
0.5	< -3	$(-3, -1)$	$(-1. - 0.33)$	$(-0.33, 1)$	> 1
0.75	< -1.67	$(-1.67, -0.33)$	$(-0.33, -0.14)$	$(-0.14, 1)$	> 1
0.99	< -1.02	$(-1.02, -0.01)$	$(-0.01, -0.005)$	$(-0.005, 1)$	> 1
home	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$	$CM \succ FA \succ IP$	$IP \succ CM \succ FA$
foreign	$CM \succ FA \succ IP$	$CM \succ IP \succ FA$	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$

- For $-99 < \gamma_H < -.01$ (given $\gamma_F = 1$), implying negative covariance of output, the Home country prefers:
  - complete markets if its size is **small** (n=0.01).
  - income pooling if its size is **large** (n=.99).

Generalization: two factor model

Model Description

- A two-period endowment model of two countries: home of size n and foreign of size (1-n).
- Deterministic endowment in period 1:

$$Y_{H,1} = Y_{F,1} = 1.$$

- Stochastic endowment in period 2:

$$Y_{H,2} = 1 - \epsilon_H, Y_{F,2} = 1 - \epsilon_F.$$

$$Y_{W,2} = nY_{H,2} + (1-n)Y_{F,2} = 1 - n\epsilon_H - (1-n)\epsilon_F.$$

- $E(\epsilon_H) = E(\epsilon_F) = 0$, $E(\epsilon_H^2) = \sigma^2_{\epsilon_H}$, $E(\epsilon_F^2) = \sigma^2_{\epsilon_F}$, $E(\epsilon_H \epsilon_F) = \sigma_{\epsilon_H \epsilon_F}$.
- $U(C_i) = log(C_i)$ where $i = H, F$.

Complete Market allocation

in the two-factor model

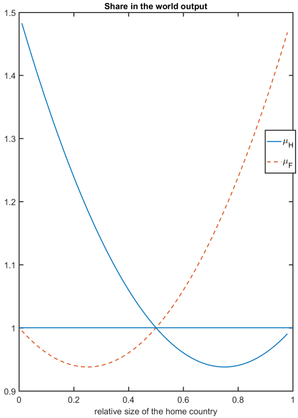
- Consumption in each country is a fraction of the world endowment with $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}}\right)}{1+\beta}$.
- $E\left(\frac{Y_{H,2}}{Y_{W,2}}\right) \approx 1 + n(n-1)\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + 2n(1-n)\sigma_{\epsilon_H\epsilon_F}$.
- Home consumption share:

$$\mu_H = 1 + \frac{\beta(1-n)(n(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_H}) - (1-n)(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_F}))}{1+\beta}.$$

- if $n \to 0$ then $\mu_H = 1 + \frac{\beta(\sigma^2_{\epsilon_F} - \sigma_{\epsilon_H\epsilon_F})}{1+\beta}$.
- In general, $\frac{\partial \mu_H}{\partial \sigma^2_{\epsilon_H}} = -\frac{\beta(1-n)n}{1+\beta}$ and $\frac{\partial \mu_H}{\partial \sigma_{\epsilon_H\epsilon_F}} = \frac{\beta(1-n)(2n-1)}{1+\beta}$.

Optimal Consumption Shares

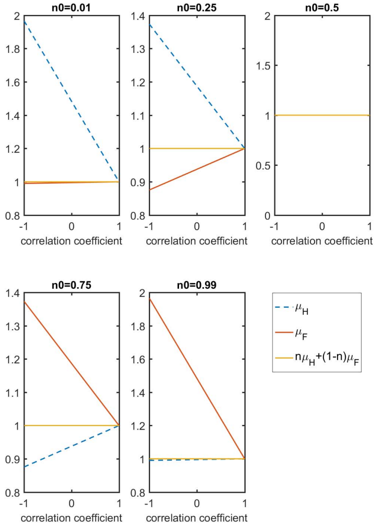
$$\sigma_{\epsilon_H} = \sigma_{\epsilon_F} = 1,\ \sigma_{\epsilon_H \epsilon_F} = 0$$



$$\mu_H = 1 + \frac{\beta \sigma_\epsilon^2 (n-1)(2n-1)}{1+\beta},\ \mu_F = 1 + \frac{\beta \sigma_\epsilon^2 n(2n-1)}{1+\beta}.$$

Optimal Consumption Shares

$$\sigma_{\epsilon_H} = \sigma_{\epsilon_F} = 1,\ \sigma_{\epsilon_H \epsilon_F} \in [-1, 1]$$



What do we learn?

- Countries that are ex ante identical but for size, may end up with a different level of (consumption) demand with full risk sharing.
- Smaller countries gain. Intuitively, the equilibrium price of risk is close to the FA prices of the larger country.
- The gains are larger, the more negative the covariance of output is.

Consumption Level

Two-factor model

1. By construction, we have an example in which consumption levels in period 1 are equalized under financial autarky.

   $$C_{H,1} = 1, C_{H,2} = 1 - \epsilon_H, \overline{C}_H = 1.$$

2. as well as with income pooling:

   $C_{H,t} = C_{F,t}$ where $t = 1, 2.$:

   $$C_{H,1} = 1, C_{H,2} = (1 - n\epsilon_H - (1-n)\epsilon_F), \overline{C}_H = 1.$$

3. Under complete markets, however, because of the endogenous adjustment of income to the price of risk, Consumption is not equalized in period 1.

   $$C_{H,1} = \mu_H, C_{H,2} = \mu_H(1 - n\epsilon_H - (1-n)\epsilon_F), \overline{C}_H = \mu_H.$$

Welfare

Two-factor model

Home loss under different risk sharing arrangements:

- FA: $L_H^{FA} = \frac{\beta}{2}\sigma^2_{\epsilon_H}$.
- CM: $L_H^{CM} = -\beta(1-n)(n(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_H}) - (1-n)(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_F})) + \frac{\beta}{2}(n^2\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + \ 2n(1-n)\sigma_{\epsilon_H\epsilon_F})$.
- IP: $L_H^{IP} = \frac{\beta}{2}(n^2\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + 2n(1-n)\sigma_{\epsilon_H\epsilon_F})$.

Welfare Ranking

as a Function of Size and Volatility

Table 2: **Ranking of CM, IP, FA for Home and Foreign country depending on home size n and $\sigma^2_{\epsilon_H}$ ($\sigma_{\epsilon_F} = 1$ and $\sigma_{\epsilon_H \epsilon_F} = 0$)**

n	$\sigma^2_{\epsilon_H}$			
0.01	< 0.98	$(0.98, 99)$	$(99, 199)$	> 199
0.25	< 0.6	$(0.6, 3)$	$(3, 7)$	> 7
0.5	< 0.33	$(0.33, 1)$	$(1, 3)$	> 3
0.75	< 0.14	$(0.14, 0.33)$	$(0.33, 1.67)$	> 1.67
0.99	< 0.005	$(0.005, 0.01)$	$(0.01, 1.02)$	> 1.02
home	$CM \succ FA \succ IP$	$CM \succ IP \succ FA$	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$
foreign	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$	$CM \succ FA \succ IP$

- For $0.01 < \sigma^2_{\epsilon_H} < 99$ (given $\sigma_{\epsilon_F} = 1$) Home country prefers:
  - complete markets if its size is **small** (n=0.01).
  - income pooling if its size is **large** (n=.99).

Conclusions

- We reconsider the implications of risk sharing arrangements among countries that differ in size and business cycle.
- As the shadow price of future contingent output differs across borders, reforms that enhance risk-sharing necessarily lead to a change in relative income and (consumption) demand vis-à-vis the status quo.
- Potential implications for the political economy of institutional reform and capital market integration.
  - large countries would prefer arrangement implementing income pooling while small countries would prefer complete markets.
- Macro modeling of risk sharing:
  - Asymmetries among countries underline importance of second order approximation around the stochastic steady state.

Research Directions

- Modeling risk sharing among asymmetric countries.
- Portfolio adjustment as a function of risk sharing arrangements.
- Unemployment and labor income risk insurance.
- Interaction of financial frictions with other distortions.
    - In our examples, pecuniary externalities from moving to complete markets are efficient (since there are no distortions other than incomplete markets).
      For an example in which these externalities may lower welfare (because of nominal rigidities and single monetary policy), see Auray and Equyem (2014).

Appendix

Complete Markets: Utility Maximisation

$$\max U(C_H) = u(C_{H,1}) + \sum_{s=1}^{S} \beta\pi(s)u(C_{H,2}(s))$$

subject to:

$$C_{H,1} + \sum_{s=1}^{S} \frac{p(s)}{1+r} C_{H,2}(s) = Y_{H,1} + \sum_{s=1}^{S} \frac{p(s)}{1+r} Y_{H,2}(s)$$

Home and Foreign Euler equations:

$$C_{H,2}(s) = \left[\pi(s)\beta\frac{(1+r)}{p(s)}\right]^{\frac{1}{\rho}} C_{H,1};\ C_{F,2}(s) = \left[\pi(s)\beta\frac{(1+r)}{p(s)}\right]^{\frac{1}{\rho}} C_{F,1}$$