Optimal Trend Inflation

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Add firm heterogeneity to otherwise standard sticky price economy

Key conclusions regarding optimal inflation rate change discontinuously

- optimal steady state inflation different from zero
- inflation optimally responds to productivity disturbances
Lots of microeconomic heterogeneity at firm level
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Firm side microdynamics display **systematic trends**:  
- firm life cycle: start small/unproductive, become productive, exit  
- product life cycle: new products, higher quality, initially higher price
- Lots of microeconomic heterogeneity at firm level

- Firm side microdynamics display **systematic trends**:
  - firm life cycle: start small/unproductive, become productive, exit
  - product life cycle: new products, higher quality, initially higher price

- Taking into account firm-level trends
  $\implies$ discontinuously affects optimal inflation
  & rationalizes positive steady state inflation
Sticky price literature concerned with optimal inflation:
abstracts from firm level heterogeneity, except for price heterogeneity
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  abstracts from firm level heterogeneity, except for price heterogeneity

• Technically motivated: aggregating 2-dim. heterogeneity a challenge

_Strong economic implications_: zero inflation optimal
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*Strong economic implications*: zero inflation optimal

Productivity of price adjusting firms equal to productivity of non-adjusting firms
Introduction

- Sticky price literature concerned with optimal inflation:
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- Technically motivated: aggregating 2-dim. heterogeneity a challenge
  *Strong economic implications*: zero inflation optimal
- Productivity of price adjusting firms equal to productivity of non-adjusting firms
- Adjusting firms’ price = price of non-adjusting firms
  \[\implies\] strong force towards zero inflation


idiosyncratic firm level productivity ⇔ without systematic trend

Do not look at optimal inflation

Their results suggest zero inflation optimal:
ave. prod. of adjusting firm ≈ ave. prod. of non-adjusting firm
Add firm life cycle to basic homogeneous firm setup:

- Firm entry & exit
- Measure $\delta$ of randomly selected firms: very negative productivity shock $\Rightarrow$ exit
- Exiting firms replaced by same measure of newly entering firms
- Alternative interpretations of setup possible: product/quality life cycle
Firm-level productivity trends driven by 3 underlying trends:

- **aggregate trend**: productivity gains experienced by all firms
- **experience trend**: firms become more productive over time
- **cohort trend**: productivity level for new cohort of firms
Introduction

- Production function of firm \( j \in [0, 1] \):

\[
Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left( K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),
\]

where \( s_{jt} \) is firm age

\[
A_t = a_t A_{t-1},
\]
\[
Q_t = q_t Q_{t-1},
\]
\[
G_{jt} = \begin{cases} 
1 & \text{if } s_{jt} = 0, \\
g_t G_{jt-1} & \text{otherwise}.
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• Three productivity trends: $a, q$ and $g$
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- Three productivity trends: $a, q$ and $g$

- Measure $\delta$ of firms: productivity drops to zero & exit

- Special cases w/o firm level trends: $\delta = 0$ or if $q_t \equiv g_t$
Figure: Productivity dynamics in a setting with firm entry and exit
Introduction

- Setup naturally generates positive steady state inflation, if young firms initially less productive than non-exiting incumbents.
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In line with young firms being small:

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\[ \text{relative price of adj. to non-adj. firm larger than one} \]
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  → positive rates of inflation optimal in steady state
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  - $\Rightarrow$ positive rates of inflation optimal in steady state

- **Strength of effect independent of turnover rate** $\delta > 0$
  - Discontinuous jump of optimal inflation: $\delta = 0 \rightarrow \delta > 0$
Aggregate NL model in closed form & determine opt. inflation

\[ \Pi = gq, \]

independent of TFP trend and turnover rate \( \delta > 0 \).

Optimal inflation cannot be inferred from aggregate productivity trends - has to know firm level trends & shocks to these trends.

For \( \delta = 0 \), optimal inflation \( \Pi = 1 \).
Introduction

- Aggregate NL model in closed form & determine opt. inflation

- Optimal gross steady state inflation rate

\[ \Pi^* = \frac{g}{q}, \]

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For \( \delta = 0 \): optimal inflation \( \Pi^* = 1 \).
What is the optimal inflation rate of the US economy?


Estimate historically optimal inflation path for the U.S. economy in model-consistent way.
Historically Optimal U.S. Inflation: Baseline Estimation

![Historically Optimal U.S. Inflation: Baseline Estimation](image_url)

(Up to twice these numbers in robustness checks).

Adam & Weber (University of Mannheim & CEPR Deutsche Bundesbank)
Related Literature

- Few papers: inflation ↔ productivity dynamics

- The ones with SS implications find negative inflation rates optimal:
  - Wolman (JMCB, 2011): two sector economy with different sectorial productivity trends and different degree of price stickiness, homogeneous firms in each sector, neg. inflation optimal despite monetary frictions being absent
  - Amano, Murchison & Rennison (JME, 2009): homogeneous firm model with sticky prices and wages & aggregate growth; wages more sticky than prices; to depress wage-markups deflation turns out optimal.
  - Aoki (JME 2001): sticky price & flex price sector, inflation following asymmetric productivity shocks in both sectors, no SS inflation
Zero inflation approx. optimal in models with homogeneous firms
Woodford (2003), Kahn, King & Wolman (2003), Schmitt-Grohé and Uribe (2010)

Zero lower bound cannot justify positive average rates of inflation:
Adam & Billi (2006), Coibion, Gorodnichenko & Wieland (2012)

Brunnermeier and Sannikov (2016): idiosyncratic risk → positive inflation increasingly optimal

Downward nominal wage rigidity may justify positive inflation rates

Positive inflation possibly optimal in models with endogenous entry:
Corsetti & Bergin (2008), Bilbiie, Ghironi & Melitz (2008), Bilbiie, Fujiwara & Ghironi (2014)
Outline of Remaining Talk

1. Sticky price model with $\delta$-shocks (death-shocks)
2. Aggregation & efficient allocation
3. Optimal inflation: main result
4. Optimal inflation for the U.S. economy
Sticky Price Model

- Calvo price stickiness with parameter $\alpha$
  (theoretical results extend to menu cost setting)
- Continuum of sticky price firms, Dixit-Stiglitz aggregate $Y_t$
- Random sample $\delta$ receives $\delta$-shocks
- Firm productivity dynamics as described before
- Competitive labor and capital markets
Sticky Price Model

- Household problem

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left( \frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma} \right)
\]

s.t.

\[
C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} \, dj + \frac{B_{t-1}}{P_t} (1 + i_{t-1}) - T_t
\]

- Existence of balanced growth path:

\[
\beta < (aq)^{\phi\sigma} \quad \text{and} \quad (1 - \delta) (g/q)^{\theta-1} < 1
\]
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Aggregation

- Will spare you the derivation behind the results...
- Highlight key differences relative to homogeneous firm setup
- Abstract from price indexation in the presentation
Aggregate Output, Capital & Labor

- Aggregate output $Y_t$:

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left( K_t^{1 - \frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

with $K_t$, $L_t$ aggregate capital, labor and $F_t \geq 0$ fixed costs

- $\Delta_t$: captures joint distribution of prices & productivities:

$$\Delta_t = \int_0^1 \left( \frac{Q_t}{G_{jt} Q_t - s_{jt}} \right) \left( \frac{P_{jt}}{P_t} \right)^{-\theta} dj \quad (1)$$
Price level: exp.-weighted average of product prices

\[ P_t = \left( \int_0^1 (P_{jt})^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} \]

\[ = \int_0^1 \left( \frac{Y_{jt}}{Y_t} \right) P_{jt} \, dj \]

Price level accounts for prod. substitution (as statistical agencies do)
Price Level

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Price level accounts for prod. substitution (as statistical agencies do)

- Inflation:

\[ \Pi_t = \frac{P_t}{P_{t-1}}. \]
Evolution of the aggregate price under opt. price setting:

\[
P_{t}^{1-\theta} = \left( \delta \right) + \left( 1 - \alpha \right) \left( 1 - \delta \right) \left( p_{t}^{n} \right)^{\theta - 1} - \delta \left( \frac{1}{1 - \delta} \right) P_{t,t}^{*} \left( 1 - \theta \right) + \alpha \left( 1 - \delta \right) P_{t-1}^{1-\theta}
\]

\[
(p_{t}^{n})^{\theta - 1} = \delta + (1 - \delta) \left( p_{t-1}^{n} \frac{g_{t}}{q_{t}} \right)^{\theta - 1}.
\]
Aggregate Price Level Dynamics

\( g_t \equiv q_t \implies \text{no firm level trends and } (p_t^n)^{\theta-1} \to 1 \ \text{and} \)

\[
P_t^{1-\theta} = (\delta + (1 - \alpha)(1 - \delta))(P_{t,t}^*)^{1-\theta} + \alpha(1 - \delta)(P_{t-1})^{1-\theta}
\]

If - in addition - \( \delta = 0 \):

\[
P_t^{1-\theta} = (1 - \alpha)(P_{t,t}^*)^{1-\theta} + \alpha(P_{t-1})^{1-\theta}
\]

Standard price evolution equation in homogeneous firm models.
Attaining efficiency requires
- eliminating firm’s monopoly power by an output subsidy
- choosing $\Delta_t$ in the production function

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left( K_t^{1 - \frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

equal to

$$\Delta_t = \Delta^e_t = \left( \int_0^1 \left( \frac{Q_t}{G_{jt} Q_t - s_{jt}} \right)^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}$$
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$\Delta_t = \Delta^e_t$ decentralized by prices satisfying

$$P_{jt} \propto \frac{1}{G_{jt} Q_{t-s_jt}}$$
Outline of Remaining Talk

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Efficiency under Sticky Prices

**Proposition:** Suppose there is an appropriate output subsidy and initial prices in \( t = -1 \) reflect firms’ relative productivities. The eq. allocation under sticky prices is efficient if

\[
\Pi_t^* = \left( \frac{1}{\delta (r_{pt})^{\theta - 1} + (1 - \delta)} \right)^{\frac{1}{\theta - 1}}
\]

where \( r_{pt} \) is the relative productivity between new and old firms:

\[
r_{pt} \equiv \frac{A_t Q_t}{a_t g_t \frac{A_{t-1} Q_{t-1}}{\Delta_{t-1}^e}}.
\]
**Proposition:** Suppose there is an appropriate output subsidy and initial prices in \( t = -1 \) reflect firms’ relative productivities. The eq. allocation under sticky prices is efficient if

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\Pi_t^* = \left( \frac{1}{\delta (r_p)^{\theta-1} + (1 - \delta)} \right)^{\frac{1}{\theta-1}}
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Result is exact, achieves full efficiency, and is independent of initial productivity distribution.
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where $r_{pt}$ is the relative productivity between new and old firms:

$$r_{pt} \equiv \frac{A_t Q_t}{a_t g_t \frac{A_{t-1} Q_{t-1}}{\Delta e_{t-1}}}.$$

Result is exact, achieves full efficiency, and is independent of initial productivity distribution.

With homogeneous firms ($r_{pt} \equiv 1$ or $\delta = 0$):

$$\Pi_t^* \equiv 1.$$

Familiar result: price stability optimal at all times.
Consider setting *with firm level trends* \((r_{pt} \neq 1 \text{ and } \delta > 0)\)
Consider setting **with firm level trends** \((rp_t \neq 1 \text{ and } \delta > 0)\)

Steady state inflation \((g_t \equiv g, \; q_t \equiv q)\) is

\[
\lim \Pi_t^* = \frac{g}{q}
\]

SS inflation *positive* when \(g > q\)
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\[
\lim \Pi^*_t = \frac{g}{q}
\]

SS inflation \textit{positive} when \(g > q\)

SS independent of turnover rate \(\delta > 0\):
- fewer unproductive firms enter \(\rightarrow\) lower inflation
- existing firms accumulate experience for longer \(\rightarrow\) higher inflation
Derived optimal nonlinear inflation dynamics, but linearization still instructive:

\[ \pi_t^* = (1 - \delta)\pi_{t-1}^* + \delta \left( \frac{g_t}{q_t} - 1 \right) + O(2) \]
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With \( \delta \approx 0.12 \) in annual calibration: small but persistent inflation responses
Efficiency under Sticky Prices

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- With \( \delta \approx 0.12 \) in annual calibration: small but persistent inflation responses

- Positive experience shock \((g_t)\): persistent rise in opt. inflation
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With $\delta \approx 0.12$ in annual calibration: small but persistent inflation responses

Positive experience shock ($g_t$): persistent rise in opt. inflation

Positive cohort shock ($q_t$): persistent drop in opt inflation
1. Sticky price model with $\delta$-shocks
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Quantify historically optimal inflation path $\Pi_t^*$ for U.S. economy

Allow for arbitrary historical stochastic disturbances and potentially sub-optimal historical inflation rates $\Pi_t$
The Optimal U.S. Inflation Rate

- Theoretical result showing how to recover $\Pi_t^*$ given
  - values for $(\alpha, \delta, \theta)$
  - the observed historical inflation rate $\Pi_t$
  - the ratio $\bar{L}_t^c / \bar{L}_t$, where
    - $\bar{L}_t$ average employment per establishment in $t$
    - $\bar{L}_t^c$ average employment of continuing establishments in $t$
- Data for $\bar{L}_t^c / \bar{L}$ taken from the Business Dynamic Statistics
Proposition: Suppose an efficient output subsidy, initial prices reflect relative productivities, and no fixed costs in production ($F_t \equiv 0$). The optimal inflation rate $\Pi_t^*$ then satisfies

\[
\left( \frac{\Delta_t}{\Delta^e_t} \right)^{-1} \left( \frac{1 - \alpha (1 - \delta) (\Pi_t)^{\theta-1}}{1 - \alpha (1 - \delta) (\Pi_t^*)^{\theta-1}} \right) = \frac{1 - (1 - \delta) \frac{\bar{L}_c}{\bar{L}_t}}{1 - (1 - \delta) (\Pi_t^*)^{\theta-1}}
\]

for $t \geq 0$, (2)

where $\Delta_t / \Delta^e_t$ evolves recursively according to

\[
\frac{\Delta_t}{\Delta^e_t} = \left[ 1 - \alpha (1 - \delta) (\Pi_t^*)^{\theta-1} \right] \left( \frac{1 - \alpha (1 - \delta) (\Pi_t)^{\theta-1}}{1 - \alpha (1 - \delta) (\Pi_t^*)^{\theta-1}} \right)^{\frac{\theta}{\theta-1}}
\]

\[+ \alpha (1 - \delta) \left( \frac{\Pi_t^\theta}{\Pi_t^*} \right) \frac{\Delta_{t-1}}{\Delta^e_{t-1}}, \]

with $\Delta_{-1} / \Delta^e_{-1} = 1$.

• Special case with historically optimal inflation ($\Pi_t = \Pi_t^*$)....
Special case with historically optimal inflation ($\Pi_t = \Pi_t^*$):

$$\Pi_t^* = \left( \frac{L_c}{L_t} \right)^{\frac{1}{\theta-1}}$$

Relative firm size determines optimal inflation!

Related firm size: a measure of relative productivities given demand elasticity $\theta$
Baseline parameters (annual model, BDS data annual):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price stickiness</td>
<td>$\alpha$ 0.0915</td>
</tr>
<tr>
<td>$\delta$-shock probability</td>
<td>$\delta$ 11.5%</td>
</tr>
<tr>
<td>Demand elasticity</td>
<td>$\theta$ 7</td>
</tr>
</tbody>
</table>
The Optimal U.S. Inflation Rate

Figure: Optimal U.S. inflation $\pi_t$, benchmark estimation. Adam & Weber (University of Mannheim & CEPR Deutsche Bundesbank).
The Optimal U.S. Inflation Rate

A. Flexible Prices

B. Turnover Rate

C. Demand Elasticity

D. Price Indexation

Figure:

Optimal inflation for the United States, alternative parameter assumptions

Adam & Weber (University of Mannheim & CEPR Deutsche Bundesbank)

Trend Inflation

May 2018
Conclusions

- Aggregate in closed form a sticky price model with firm level productivity trends
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- Aggregate in closed form a sticky price model with firm level productivity trends
- Firm level productivity trends key for optimal inflation rate in sticky price models

\[ \pi = q^{>1} \]

Productivity disturbances have persistent effects on optimal inflation. Optimal US inflation: dropped from approx 1.5% in 1977 to 1.0% in 2015.

Lower demand elasticities/price indexation: optimal inflation rates double.
Conclusions

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Suppose MP implements $\Pi = 1$ in an economy where $\Pi^* \neq 1$

Analytical result: strictly positive welfare costs even in the limit $\delta \to 0$

Numerical illustration highlighting the source of welfare distortions
Assumptions for the analytical result:

- There is an optimal output subsidy and initial prices reflect initial productivities.
- There are no aggregate productivity disturbances and $\delta > 0$.
- Fixed costs of production are zero ($f = 0$).
- Disutility of work is given by $V(L) = 1 - \psi L^\nu$, with $\nu > 1$, $\psi > 0$.
- $g/q > \alpha (1 - \delta)$, so that a well-defined steady state with strict price stability exists.
- Consider the limit $\beta (\gamma^e)^{1-\sigma} \to 1$. 
Proposition: Consider a policy implementing the optimal inflation rate \( \Pi_t^* \), which satisfies \( \lim_{t \to \infty} \Pi_t^* = \Pi^* = g/q \). Let \( c(\Pi^*) \) and \( L(\Pi^*) \) denote the limit outcomes for \( t \to \infty \) for consumption and hours under this policy. Similarly, let \( c(1) \) and \( L(1) \) denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

\[
L(1) = L(\Pi^*)
\]

and

\[
\frac{c(1)}{c(\Pi^*)} = \left( \frac{1 - \alpha (1 - \delta) (g/q) \theta^{-1}}{1 - \alpha (1 - \delta)} \right)^{\frac{\phi \theta}{\delta - 1}} \left( \frac{1 - \alpha (1 - \delta) (g/q)^{-1}}{1 - \alpha (1 - \delta) (g/q)^{\theta^{-1}}} \right)^{\phi} \leq 1.
\]

For \( g \neq q \) the previous inequality is strict and

\[
\lim_{\delta \to 0} \frac{c(1)}{c(\Pi^*)} < 1
\]
The Welfare Costs of Strict Price Stability

Figure: Relative prices and inflation
Figure: Aggregate productivity as a function of gross steady state inflation (optimal inflation rate is 1.02)