

Optimal Trend Inflation

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- Add firm heterogeneity to otherwise standard sticky price economy
- Key conclusions regarding optimal inflation rate change discontinuously
 - optimal steady state inflation different from zero
 - inflation optimally responds to productivity disturbances

- Lots of microeconomic heterogeneity at firm level

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- Firm side microdynamics display **systematic trends**:
 - firm life cycle: start small/unproductive, become productive, exit
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- Firm side microdynamics display **systematic trends**:
 - firm life cycle: start small/unproductive, become productive, exit
 - product life cycle: new products, higher quality, initially higher price
- Taking into account firm-level trends
 - ⇒ discontinuously affects optimal inflation
 - & rationalizes positive steady state inflation

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Strong economic implications: zero inflation optimal
- Productivity of price adjusting firms equal to productivity of non-adjusting firms
- Adjusting firms' price = price of non-adjusting firms
⇒ strong force towards zero inflation

Woodford(2003), Kahn, King & Wolman(2003), Schmitt-Grohé & Uribe(2010)

- Golosov&Lucas (2007), Nakamura&Steinsson (2010)
idiosyncratic firm level productivity \Leftrightarrow without systematic trend
- Do not look at optimal inflation
- Their results suggests zero inflation optimal:
av. prod. of adjusting firm \approx av. prod. of non-adjusting firm

Add firm life cycle to basic homogeneous firm setup:

- Firm entry & exit
- Measure δ of randomly selected firms:
very negative productivity shock \Rightarrow exit
- Exiting firms replaced by same measure of newly entering firms
- Alternative interpretations of setup possible: product/quality life cycle

Firm-level productivity trends driven by 3 underlying trends:

- **aggregate trend:** productivity gains experienced by all firms
- **experience trend:** firms become more productive over time
- **cohort trend:** productivity level for new cohort of firms

- Production function of firm $j \in [0, 1]$:

$$Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where s_{jt} is firm age

$$A_t = a_t A_{t-1},$$

$$Q_t = q_t Q_{t-1},$$

$$G_{jt} = \begin{cases} 1 & \text{if } s_{jt} = 0, \\ g_t G_{jt-1} & \text{otherwise.} \end{cases}$$

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- Three productivity trends: \mathbf{a} , \mathbf{q} and \mathbf{g}
- Measure δ of firms: productivity drops to zero & exit
- Special cases w/o firm level trends: $\delta = 0$ or if $\mathbf{q}_t \equiv \mathbf{g}_t$

Introduction

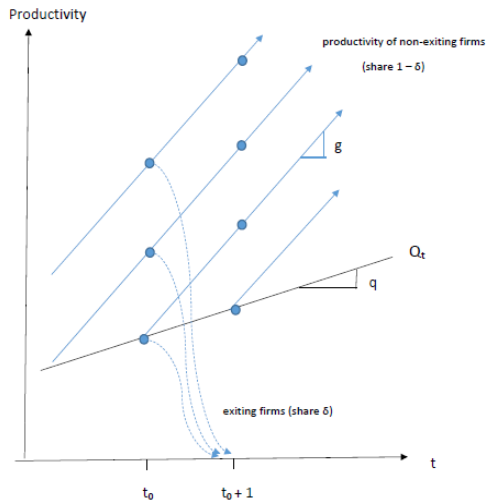


Figure: Productivity dynamics in a setting with firm entry and exit

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- **Strength of effect independent of turnover rate $\delta > 0$**
Discontinuous jump of optimal inflation: $\delta = 0 \rightarrow \delta > 0$

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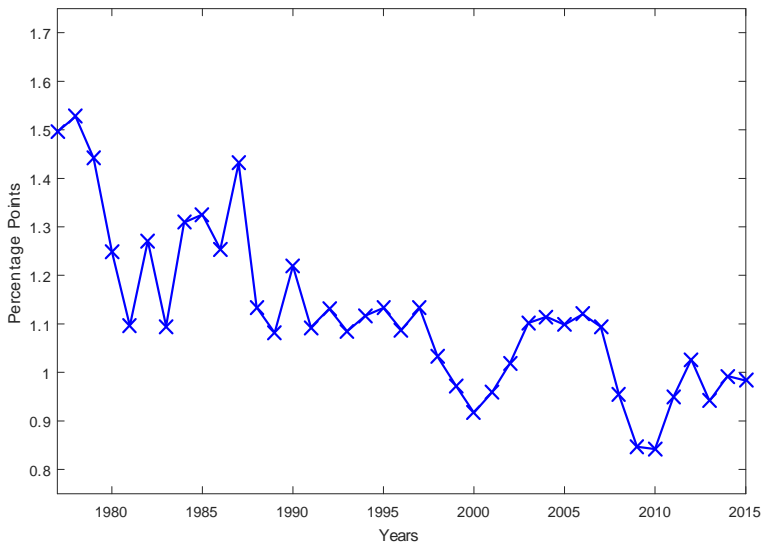
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- Optimal inflation
 - cannot be inferred from aggregate productivity trends
 - has to know firm level trends & shocks to these trends
- For $\delta = 0$: optimal inflation $\Pi^* = 1$.

- What is the optimal inflation rate of the US economy?
- Use establishment-level data from Business Dynamics Statistics (US Census Bureau): all private U.S. establishments 1977-2015.
- Estimate historically optimal inflation path for the U.S. economy in model-consistent way

Historically Optimal U.S. Inflation: Baseline Estimation



- Few papers: inflation \Leftrightarrow productivity dynamics
- The ones with SS implications find negative inflation rates optimal:
 - Wolman (JMCB, 2011): two sector economy with different sectorial productivity trends and different degree of price stickiness, homogeneous firms in each sector, neg. inflation optimal despite monetary frictions being absent
 - Amano, Murchison & Rennison (JME, 2009): homogeneous firm model with sticky prices and wages & aggregate growth; wages more sticky than prices; to depress wage-markups deflation turns out optimal.
- Aoki (JME 2001): sticky price & flex price sector, inflation following asymmetric productivity shocks in both sectors, no SS inflation

- Zero inflation approx. optimal in models w homogeneous firms
Woodford (2003), Kahn, King & Wolman (2003), Schmitt-Grohé and Uribe (2010)
- Zero lower bound cannot justify positive average rates of inflation:
Adam & Billi (2006), Coibion, Gorodnichenko & Wieland (2012)
- Brunnermeier and Sannikov (2016): idiosyncratic risk \rightarrow positive inflation increasingly optimal
- Downward nominal wage rigidity may justify positive inflation rates
Kim & Ruge-Murcia (2009), Benigno & Ricci (2011), Schmitt-Grohé & Uribe (2013), Carlsson & Westermarck (2016)
- Positive inflation possibly optimal in models with endogenous entry:
Corsetti & Bergin (2008), Bilbiie, Ghironi & Melitz (2008), Bilbiie, Fujiwara & Ghironi (2014)

Outline of Remaining Talk

- 1 **Sticky price model with δ -shocks (death-shocks)**
- 2 Aggregation & efficient allocation
- 3 Optimal inflation: main result
- 4 Optimal inflation for the U.S. economy

Sticky Price Model

- Calvo price stickiness with parameter α
(theoretical results extend to menu cost setting)
- Continuum of sticky price firms, Dixit-Stiglitz aggregate Y_t
- Random sample δ receives δ -shocks
- Firm productivity dynamics as described before
- Competitive labor and capital markets

- Household problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma} \right)$$

s.t.

$$C_t + K_{t+1} + \frac{B_t}{P_t} =$$

$$(r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} dj + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t$$

- Existence of balanced growth path:

$$\beta < (aq)^{\phi\sigma} \quad \text{and} \quad (1 - \delta)(g/q)^{\theta-1} < 1$$

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- Will spare you the derivation behind the results...
- **Highlight key differences** relative to homogeneous firm setup
- Abstract from price indexation in the presentation

- Aggregate output Y_t :

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

with K_t , L_t aggregate capital, labor and $F_t \geq 0$ fixed costs

- Δ_t : captures joint distribution of prices & productivities:

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right) \left(\frac{P_{jt}}{P_t} \right)^{-\theta} dj \quad (1)$$

- Price level: exp.-weighted average of product prices

$$\begin{aligned} P_t &= \left(\int_0^1 (P_{jt})^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \\ &= \int_0^1 \left(\frac{Y_{jt}}{Y_t} \right) P_{jt} dj \end{aligned}$$

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- Inflation:

$$\Pi_t = P_t / P_{t-1}.$$

Aggregate Price Level Dynamics

Evolution of the aggregate price under opt. price setting:

$$P_t^{1-\theta} = \underbrace{(\delta)}_{\substack{\text{new} \\ \text{firms}}} + \underbrace{(1-\alpha)(1-\delta)}_{\text{old adj.firms}} \underbrace{\frac{(p_t^n)^{\theta-1} - \delta}{1-\delta}}_{\substack{\text{rel. price} \\ \text{factor}}} \underbrace{P_{t,t}^*}_{\substack{\text{opt} \\ \text{price} \\ \text{new} \\ \text{firm}}}^{1-\theta} + \underbrace{\alpha(1-\delta)}_{\substack{\text{old firms,} \\ \text{w/o adj.}}} P_{t-1}^{1-\theta}$$

$$(p_t^n)^{\theta-1} = \delta + (1-\delta) \left(p_{t-1}^n \frac{g_t}{q_t} \right)^{\theta-1} .$$

Aggregate Price Level Dynamics

$g_t \equiv q_t \implies$ no firm level trends and $(p_t^n)^{\theta-1} \rightarrow 1$ and

$$P_t^{1-\theta} = (\delta + (1-\alpha)(1-\delta))(P_{t,t}^*)^{1-\theta} + \alpha(1-\delta)(P_{t-1})^{1-\theta}$$

If - in addition - $\delta = 0$:

$$P_t^{1-\theta} = (1-\alpha)(P_{t,t}^*)^{1-\theta} + \alpha(P_{t-1})^{1-\theta}$$

Standard price evolution equation in homogeneous firm models.

Conditions Insuring Efficiency

- Attaining efficiency requires
 - eliminating firm's monopoly power by an output subsidy
 - choosing Δ_t in the production function

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

equal to

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- $\Delta_t = \Delta_t^e$ decentralized by prices satisfying

$$P_{jt} \propto \frac{1}{G_{jt} Q_{t-s_{jt}}}$$

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Efficiency under Sticky Prices

- **Proposition:** Suppose there is an appropriate output subsidy and initial prices in $t = -1$ reflect firms' relative productivities. The eq. allocation under sticky prices is efficient if

$$\Pi_t^* = \left(\frac{1}{\delta (rp_t)^{\theta-1} + (1-\delta)} \right)^{\frac{1}{\theta-1}}$$

where rp_t is the **relative productivity between new and old firms:**

$$rp_t \equiv \frac{A_t Q_t}{a_t g_t \frac{A_{t-1} Q_{t-1}}{\Delta_{t-1}^e}}$$

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- Result is exact, achieves full efficiency, and is independent of initial productivity distribution.
- With homogeneous firms ($rp_t \equiv 1$ or $\delta = 0$):

$$\Pi_t^* \equiv 1.$$

Familiar result: price stability optimal at all times.

- Consider setting **with firm level trends** ($rp_t \neq 1$ and $\delta > 0$)

Efficiency under Sticky Prices

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- Steady state inflation ($g_t \equiv g$, $q_t \equiv q$) is

$$\lim \Pi_t^* = \frac{g}{q}$$

SS inflation *positive* when $g > q$

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- Consider setting **with firm level trends** ($rp_t \neq 1$ and $\delta > 0$)
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$$\lim \Pi_t^* = \frac{g}{q}$$

SS inflation *positive* when $g > q$

- SS independent of turnover rate $\delta > 0$:
 - fewer unproductive firms enter \rightarrow lower inflation
 - existing firms accumulate experience for longer \rightarrow higher inflation

- Derived optimal nonlinear inflation dynamics, but linearization still instructive:

$$\pi_t^* = (1 - \delta)\pi_{t-1}^* + \delta \left(\frac{g_t}{q_t} - 1 \right) + O(2)$$

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- Positive experience shock (g_t): persistent rise in opt. inflation
- Positive cohort shock (q_t): persistent drop in opt inflation

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The Optimal U.S. Inflation Rate

- Quantify **historically optimal inflation path** Π_t^* for U.S. economy
- Allow for arbitrary historical stochastic disturbances and potentially sub-optimal historical inflation rates Π_t

The Optimal U.S. Inflation Rate

- Theoretical result showing how to recover Π_t^* given

- values for (α, δ, θ)
- the observed historical inflation rate Π_t
- the ratio \bar{L}_t^c / \bar{L}_t , where

\bar{L}_t average employment per establishment in t

\bar{L}_t^c average employment of continuing establishments in t

- Data for \bar{L}_t^c / \bar{L}_t taken from the Business Dynamic Statistics

The Optimal U.S. Inflation Rate

Proposition: Suppose an efficient output subsidy, initial prices reflect relative productivities, and no fixed costs in production ($F_t \equiv 0$). The optimal inflation rate Π_t^* then satisfies

$$\left(\frac{\Delta_t}{\Delta_t^e}\right)^{-1} \left(\frac{1 - \alpha(1 - \delta)(\Pi_t)^{\theta-1}}{1 - \alpha(1 - \delta)(\Pi_t^*)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}} = \frac{1 - (1 - \delta)\bar{L}_t^c/\bar{L}_t}{1 - (1 - \delta)(\Pi_t^*)^{\theta-1}} \quad \text{for } t \geq 0, \quad (2)$$

where Δ_t/Δ_t^e evolves recursively according to

$$\begin{aligned} \frac{\Delta_t}{\Delta_t^e} &= \left[1 - \alpha(1 - \delta)(\Pi_t^*)^{\theta-1}\right] \left(\frac{1 - \alpha(1 - \delta)(\Pi_t)^{\theta-1}}{1 - \alpha(1 - \delta)(\Pi_t^*)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}} \\ &\quad + \alpha(1 - \delta) \left(\frac{(\Pi_t)^\theta}{\Pi_t^*}\right) \frac{\Delta_{t-1}}{\Delta_{t-1}^e}, \end{aligned}$$

with $\Delta_{-1}/\Delta_{-1}^e = 1$.

- Special case with historically optimal inflation ($\Pi_t = \Pi_t^*$)....

The Optimal U.S. Inflation Rate

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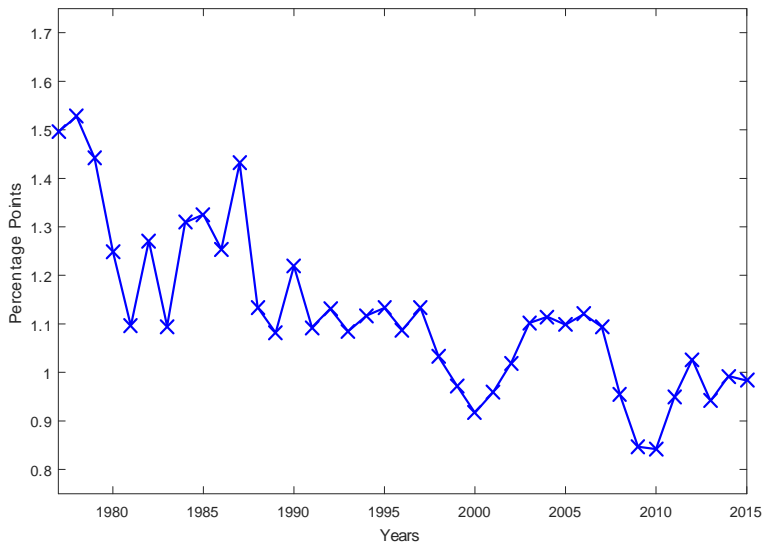
- Relative firm size determines optimal inflation!
- Related firm size: a measure of relative productivities given demand elasticity θ

The Optimal U.S. Inflation Rate

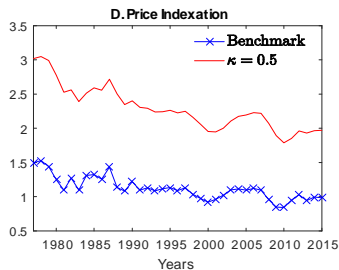
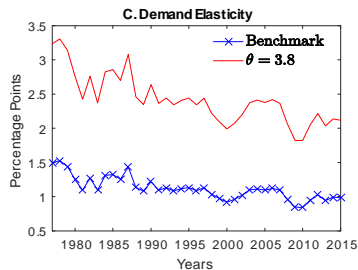
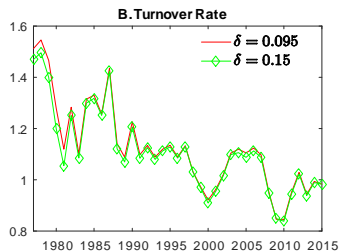
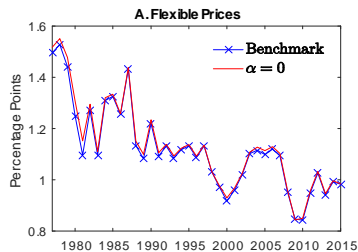
- Baseline parameters (annual model, BDS data annual):

Parameter		Assigned value
Price stickiness	α	0.0915
δ -shock probability	δ	11.5%
Demand elasticity	θ	7

The Optimal U.S. Inflation Rate



The Optimal U.S. Inflation Rate



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- Lower demand elasticities/price indexation: optimal inflation rates double

The Welfare Costs of Strict Price Stability

- Suppose MP implements $\Pi = 1$ in an economy where $\Pi^* \neq 1$
- Analytical result: strictly positive welfare costs even in the limit $\delta \rightarrow 0$
- Numerical illustration highlighting the source of welfare distortions

Assumptions for the analytical result:

- there is an optimal output subsidy and initial prices reflect initial productivities
- there are no aggregate productivity disturbances and $\delta > 0$
- fixed costs of production are zero ($f = 0$)
- disutility of work is given by

$$V(L) = 1 - \psi L^\nu, \text{ with } \nu > 1, \psi > 0.$$

- $g/q > \alpha(1 - \delta)$, so that a well-defined steady state with strict price stability exists
- consider the limit $\beta(\gamma^e)^{1-\sigma} \rightarrow 1$

The Welfare Costs of Strict Price Stability

Proposition: Consider a policy implementing the optimal inflation rate Π_t^* , which satisfies $\lim_{t \rightarrow \infty} \Pi_t^* = \Pi^* = g/q$. Let $c(\Pi^*)$ and $L(\Pi^*)$ denote the limit outcomes for $t \rightarrow \infty$ for consumption and hours under this policy. Similarly, let $c(1)$ and $L(1)$ denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

$$L(1) = L(\Pi^*)$$

and

$$\frac{c(1)}{c(\Pi^*)} = \left(\frac{1 - \alpha(1 - \delta)(g/q)^{\theta-1}}{1 - \alpha(1 - \delta)} \right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1 - \alpha(1 - \delta)(g/q)^{-1}}{1 - \alpha(1 - \delta)(g/q)^{\theta-1}} \right)^{\phi} \leq 1. \quad (3)$$

For $g \neq q$ the previous inequality is strict and

$$\lim_{\delta \rightarrow 0} c(1)/c(\Pi^*) < 1$$

The Welfare Costs of Strict Price Stability

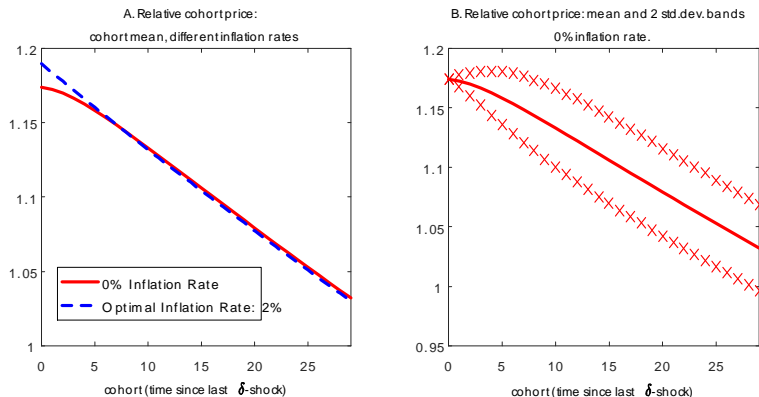


Figure: Relative prices and inflation

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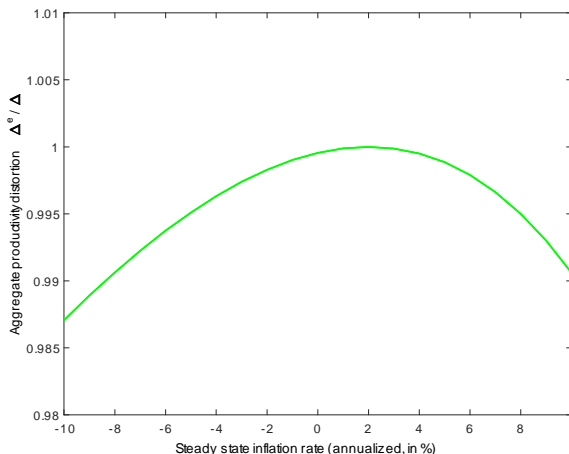


Figure: Aggregate productivity as a function of gross steady state inflation (optimal inflation rate is 1.02)