Heterogeneity and Wage Inequalities over the Life Cycle

Thierry Magnac *(Toulouse School of Economics)*

Sébastien Roux *(Insee, Ined and Crest)*

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Motivation

- Earnings dynamics along the lifecycle affect long term inequalities
- Recent important developments in the modeling of earnings dynamics à la Mincer (1974) with a lot of heterogeneity (Browning et al., 2012, Polachek et al., 2015, Magnac et al., 2018)
- How do these heterogeneities affect what we know about short-term and long-term inequalities?
  Decomposition between permanent and transitory Components?
- Reinterpretation of empirical results on wage dynamics

What We Do

- Reduced form Wage equations disciplined by a human capital investment set-up with fixed individual-specific parameters: level, slope, curvature (Magnac et al., 2018)
- Decomposition of wage profiles in aggregate effects and individual effects
- Decomposition between permanent individual heterogeneity (fixed individual-specific parameters) and transitory individual and time remaining shocks (General ARMA specification)
- Sequential estimation method, first by random effects, and, second, by fixed effects

  $\Rightarrow$ Estimation of covariance wage structure induced by the 3 different dimensions of heterogeneity and implementation of decompositions

Results

Application: Long-period panel, observation of 7,500 males from their entry in 1977 in the labor market to 2007

Results:

- Variance of long-run value of wage profile: same as the cross-section variance of wages after 5 years, but only 60% after 30 years

- 70% of the variance at the beginning of the life-cycle explained by permanent individual heterogeneity, 90% after 30 years

- Relative explanatory power of observed skills and unobserved permanent individual heterogeneity: half-half after 5 years on the LM, 30-70% after 30 years

- Need of 3 dimensions of heterogeneity to properly describe the variance of log wages in cross section:

Levels at the beginning, slope and curvature at the end of the life-cycle

Literature

- Earnings Dynamics (review by Meghir and Pistaferri, 2010): Fitting the empirical covariance structure of (log) earnings using different specifications as HIP or RIP for instance, decomposition between permanent and transitory components

  (Baker, 1997, Guvenen, 2007, Hryshko, 2012, Hoffman, 2019)

- Distinguishing (and/or decomposing) between different theories of wage growth: Human Capital, job search or learning by doing

  Empirical evidence (Rubinstein and Weiss, 2006), Structural modeling (Bowlus and Liu, 2013, Bagger et al., 2014), HK investments vs learning by doing (Heckman et al.,2003, Belley, 2017, Blandin, 2018)

- Multidimensional human capital

  (Sanders et Taber, 2011) for a review, (Gladden and Taber, 2009, Sorensen and Vejlin, 2014) correlation between initial wages and later wage growth

Roadmap

Data

- *Panel DADS*: All individuals born in October of an even year. Administrative data from social security, less attrition or measurement errors

- We consider full-time jobs in the Private sector from 1977 to 2007. No information in 1981, 1983 and 1990 for exogenous reasons.

- Information on job characteristics (labor earning, days of work, full-time status) and **daily wage**:

  - sum of yearly earnings divided by the number of days worked.

  - Wages deflated with "macro" human capital prices (flat spot conditions, cf. Bowlus and Robinson, 2012)

A single Cohort

- Information on age and occupation at entry: definition of "education" groups, interacting these variables
- 7,446 individuals:

  - entering the labor market in 1977,

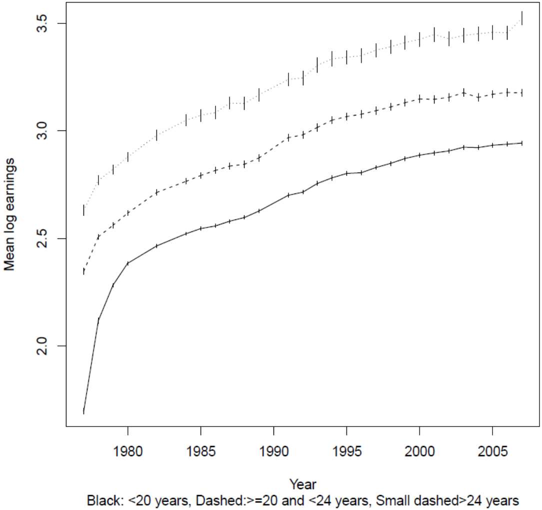
  - aged between 16 and 30 at their entry

  - working also in 1978, 1982 and 1984 (to avoid to deal with non participation)

  - 4,670 in 2007

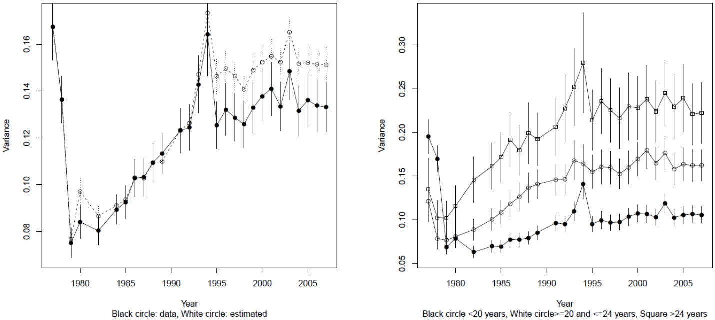
  - 4,873 are in the sample at least 21 years (possibly separate job spells)

Mean log-wages



Note: The small vertical lines represent the 95% confidence intervals.

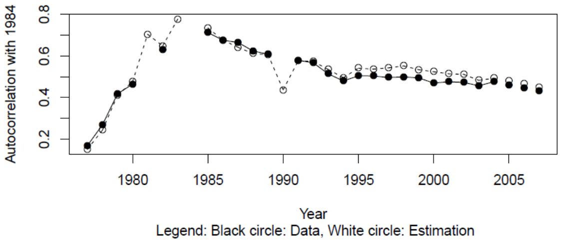
Cross-sectional variance of log-wages

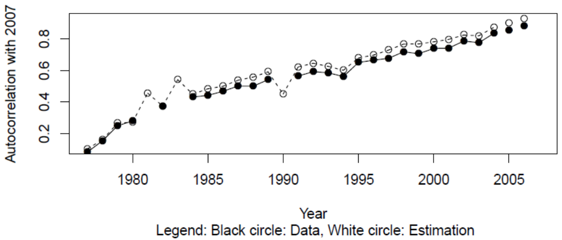


(A) full sample (B) by age of entry

Note: The small vertical lines represent the 95% confidence intervals. Log wage residuals are obtained by regressing log wages on a saturated set of dummies for skill groups and years.

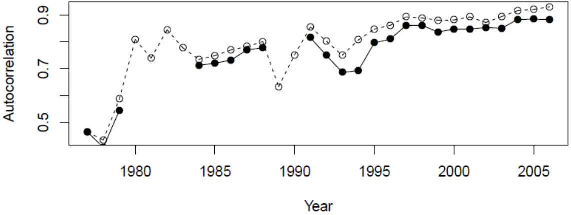
Autocorrelations with 1984 and 2007



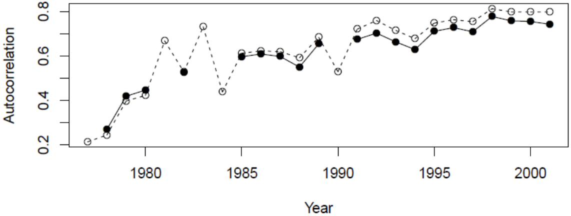


▸ Goodness of Fit

Forward autocorrelations of orders 1 and 6



Legend: Data Black, Estimation dotted



Legend: Data Black, Estimation dotted

Wage equation

Following Magnac et al. (2018), reduced form:

$$\ln w_{it} = \eta_{i1} + \eta_{i2} t + \eta_{i3} \beta^{-t} + v_{it} \tag{1}$$

where $\beta < 1$ (discount rate exogenous and fixed)

- η_{i1}, level: initial endowments in human capital and ability to earn,
- η_{i2}, slope: ability to learn, should be > 0
- η_{i3}, curvature: horizon effect, should be < 0
- v_{it}: log-price of human capital, net of accumulated depreciations (job search, job ladder, dismissals,...)

Aggregate Components

- Aggregation of (1) in each education group g (age and skill of entry):

$$\overline{\ln y}_{gt} = \overline{\eta}_{g1} + \overline{\eta}_{g2} t + \overline{\eta}_{g3} \beta^{-t} + \bar{v}_{gt}, \tag{2}$$

  where $\overline{\eta}_{gk} = E(\eta_{ik} \mid i \in g)$ for $k = 1, 2$ or 3,
  $\bar{v}_{gt} = E(v_{it} \mid i \in g)$

- Identification assumption:

$$E(\bar{v}_{gt} | (1, t, \beta^{-t})) = \varphi_{gt},$$

  using flat spot technique (Heckman et al., 1998, Bowlus & Robinson, 2012) from workers who are close to the end of their working life, such as $\overline{\eta}_{g2} t + \overline{\eta}_{g3} \beta^{-t} \simeq 0$.

- Once deflated with φ_{gt}, estimations of $\overline{\eta}_g = (\overline{\eta}_{g1}, \overline{\eta}_{g2}, \overline{\eta}_{g3})'$ with OLS for each g (28 observations per group).

Individual-specific Components

Defining $\eta_{ik}^c = \eta_{ik} - \overline{\eta}_{gk}$, for $k = 1,\ 2,$ or 3 and $v_{it}^c = v_{it} - \bar{v}_{g(i)t}$ and centering the log wage equation (1)

$$u_{it} = \ln y_{it} - \overline{\ln y}_{g(i)t} = \eta_{i1}^c + \eta_{i2}^c t + \eta_{i3}^c \beta^{-t} + v_{it}^c, \qquad (3)$$

v_{it}^c: possible interpretation: frictions in a model of search and mobility

$$E(v_{it}^c \mid \left(1, t, \beta^{-t}\right), \eta_i^c) = 0.$$

- Main identification assumption
- Allowing for initial conditions slightly changes the condition
- Assumption of missingness at random

Two-step estimation procedure

- Some data are missing: T_i observations per individual i, bias of order $1/T_i$ when η_i^c directly estimated.
- First step: Flexible random effect model using the whole sample, parametrized covariance structure of residuals
  Random Effects
- Second step: FGLS individual by individual, FGLS weighted by the inverse of the covariance matrix estimated in the first step.
  Fixed Effects

  Provide estimates of η_i^c (and not only covariance matrix)

In the end, we get individual estimates of η_i

$$\hat{\eta}_i = \widehat{\overline{\eta}}_{g(i)} + \hat{\eta}_i^c.$$

Average Effects Estimation

- Estimates of φ_{gt} from a population of males with potential experience between 25 and 40.
- First loading factor ranging from 2.4 (lowest skill group) to 3.4 (highest skill group)
- Slope factor ranging from 0.0017 to 0.07
- Curvature factor negative as expected
- Covariance structure of loading factors across groups similar to the one observed within groups.

Group averages of individual factor loadings η_g for some groups

Skill group	Age group	Nb Obs	$\overline{\eta}_{g1}$	$\overline{\eta}_{g2}$	$\overline{\eta}_{g3}$
2	17	1268	2.4	0.04	-0.15
			(0.032)	(0.0067)	(0.051)
3	17	1224	2.4	0.039	-0.15
			(0.039)	(0.0056)	(0.04)
1	21	117	2.9	0.052	-0.22
			(0.086)	(0.0085)	(0.068)
2	21	710	2.7	0.047	-0.2
			(0.014)	(0.0024)	(0.018)
3	21	512	2.6	0.041	-0.19
			(0.015)	(0.0026)	(0.019)
1	27	114	3.4	0.047	-0.21
			(0.019)	(0.0045)	(0.034)
2	27	87	3	0.061	-0.32
			(0.02)	(0.0034)	(0.025)
3	27	63	2.7	0.03	-0.079
			(0.036)	(0.005)	(0.039)

Note: Estimation of equation (2). A flat spot deflator is used. Newey West standard errors in parentheses (5 lags).

Covariance structure of individual effects

- v_{it}^c specified as an ARMA(p, q) process. ARMA(3,1) preferred specification.
- Stability of $V(\eta_i^c)$ across different ARMA specifications.

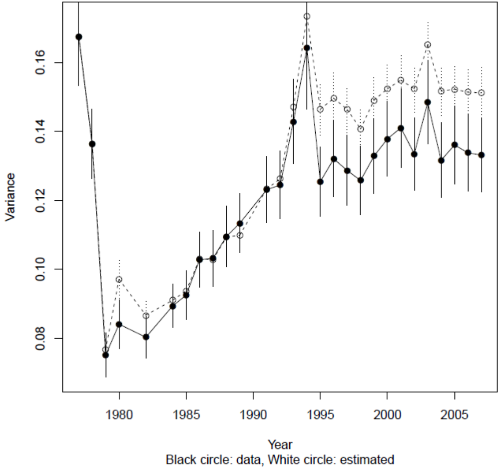
  ▸ Estimated standard errors and correlations of individual effects η_i^c
- Correlations:
  - $\rho_{\eta_1,\eta_2} > 0$, around 0.5, confirmed by literature (Lagakos et al. 2018, Engboom 2018, Guvenen, 2017)
  - $\rho_{\eta_1,\eta_3} < 0$, around -0.6: retraction force at the end of the life-cycle stronger for high-skilled workers
  - $\rho_{\eta_2,\eta_3} << 0$, around -0.95: Stronger retraction force for high-wage growth workers, consistent with structural model (Magnac et al., 2018)
- Structure confirmed with fixed effects estimates, need to have more than 21 observations per individual.

  ▸ Bias corrected covariance matrix of individual effects: fixed and random effect estimation

Initial Conditions and transitory shocks

- Initial Conditions
  - v_{it}^c correlated with η_i^c
  - Negatively with η_{i1}^c and η_{i3}^c, positively with η_{i2}^c
  - Strong transitory conditions that affect the wage process at the beginning of the career.
  - Initial conditions far from the stationnary path.
- Transitory shocks
  - Yearly estimation of the standard deviation of earnings
  - Decreasing over time: diminishing role of individual-specific frictions (Bagger et al.,2014, Bowlus et Liu, 2012)

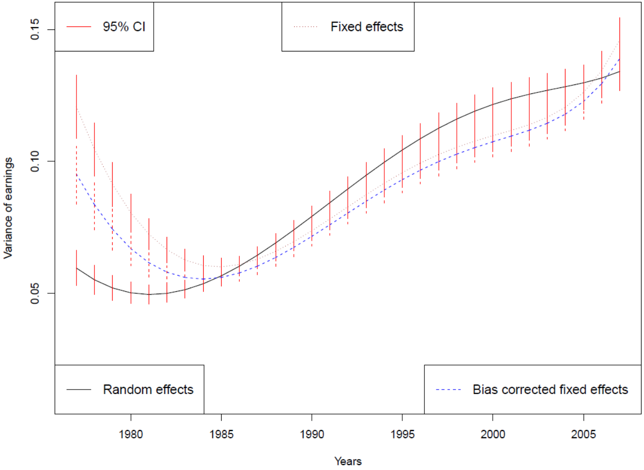
Goodness of fit: Cross-sectional variance of log-wages



(A) full sample

Autocorrelations with 1984 and 2007 | Forward Autocorrelations of orders 1 and 6

Mincer Dip: Variance of the permanent components



Note: The permanent component is $M(\beta)\eta$ defined in equation (6). The sample is restricted to long history profiles (more than 21 periods). "Random effects" are using estimates derived from random effect estimation. "Fixed effects" are using estimates derived from raw fixed effect estimation and "Bias corrected f.e." are the bias corrected version of them.

Correlation between wage and subsequent wage growth induced by permanent components

Table: Covariances of wage level and subsequent growth

Years	All	Low skilled	Medium skilled	High skilled
1977	-0.00594	-0.00573	-0.00665	-0.00688
	(0.000639)	(0.000564)	(0.000885)	(0.00214)
1982	-0.000972	-0.00137	-0.00102	1.04e-05
	(0.000388)	(0.000303)	(0.00052)	(0.00149)
1987	0.00178	0.00113	0.00219	0.00336
	(0.000271)	(0.000198)	(0.000402)	(0.00119)
2002	0.000812	0.000838	0.00141	0.00108
	(0.000269)	(0.000229)	(0.00042)	(0.00108)
2007	0.00524	0.00459	0.00583	0.0109
	(0.000666)	(0.000593)	(0.00114)	(0.00304)
Observations	4873	2942	1433	498

Notes: The coefficient is the covariance between $\eta_{i1} + \eta_{i2} t + \eta_{i3} \beta^{-t}$ and $\eta_{i2} + \beta^{-t}(1/\beta - 1)\eta_{i3}$. Standard errors (heteroskedastic-consistent sampling and parameter uncertainty, 1000 MC simulations) between brackets. The working sample (21+) has 4873 observations

Initial wage levels and returns to experience

Table: Time varying correlation of initial levels and returns

Year	All	Low	Med	High
1977	-0.498	-0.674	-0.561	-0.379
	(0.0521)	(0.0425)	(0.0682)	(0.13)
1982	-0.505	-0.683	-0.564	-0.38
	(0.0529)	(0.043)	(0.0705)	(0.134)
1997	-0.186	-0.265	-0.201	0.000268
	(0.0385)	(0.042)	(0.0664)	(0.0796)
2002	0.156	0.207	0.186	0.212
	(0.0346)	(0.0392)	(0.0458)	(0.0765)
2007	0.286	0.389	0.338	0.276
	(0.0387)	(0.0387)	(0.0444)	(0.0888)
Long-run value	-0.373	-0.507	-0.411	-0.247
	(0.0407)	(0.0335)	(0.0596)	(0.0997)

Note: The correlation is $\rho = Corr(\eta_{i1} + \eta_{i3}, \eta_{i2} - \log(\beta)\beta^{-t}\eta_{i3})$ Only observations with more than 21 periods. 4873 observations. First column reports results for centered individual effects while the other columns include aggregate effects. The last three columns for low, medium and high skills.

Impacts of unobserved heterogeneity on mean log wages

Impact of $\{\sigma_j\}_{j=1,.,3}$ on:	Level $\eta_1 \to \eta_1 + \sigma_1$	Slope $\eta_2 \to \eta_2 + \sigma_2$	Curvature $\eta_3 \to \eta_3 + \sigma_3$
Log-wage 1977	0.0112	0.0116	0.0199
	(0.000203)	(0.000336)	(0.00062)
Log-wage 1982	0.0303	0.0263	0.0448
	(0.000549)	(0.000765)	(0.0014)
Log-wage 1992	0.0326	0.0747	0.0778
	(0.00059)	(0.00217)	(0.00242)
Log-wage 2002	0.0326	0.124	0.13
	(0.00059)	(0.00362)	(0.00405)
Log-wage 2007	0.0326	0.149	0.168
	(0.00059)	(0.00435)	(0.00523)
Long-run value	0.0287	0.0573	0.0672
	(0.00052)	(0.00167)	(0.00209)

Note: Average impact on log wages of an increase of a tenth of the standard deviation of: 2nd column, unobserved heterogeneity in the initial human capital; third column, wage growth or returns; fourth column, curvature or horizon.

Variance Decomposition

Fixed Effects	Obs. het. %	Unobs. het. %	Transitory %	Total var.
Log-wage 1977	65	1.45	33.5	0.483
Log-wage 1982	33.5	35.7	30.9	0.138
Log-wage 1992	31.2	47.3	21.5	0.17
Log-wage 2002	28	58	14	0.193
Log-wage 2007	24.5	67.2	8.25	0.207
Long-run value	52.2	42.3	5.51	0.113

Note: The variance (5th column) of each variable (1st column) is decomposed into its component shares which are reported in percentages in column 2 (observed heterogeneity), column 3 (unobserved heterogeneity) and column 4 (transitory component). The share of variance of log 1982 wage (0.138) explained by observed heterogeneity is 33.5

- Similar results with random effects
- Unobs. het. includes also their correlations with initial conditions

Heterogeneity Components

Respective importance of the dimensions of heterogeneity

- Idea: decomposition between observed and unobserved heterogeneity in the three dimensions
- Five experiments:
  1. Observable benchmark: $\eta_i = \overline{\eta}_{g(i)}$
  2. Heterogeneity in level: $\eta_{i1} = \overline{\eta}_{g(i)1} + \hat{\eta}^c_{i1}$,$\eta_{i2} = \overline{\eta}_{g(i)2} + \omega_{21}\hat{\eta}^c_{i1}$, $\eta_{i3} = \overline{\eta}_{g(i)3} + \omega_{13}\hat{\eta}^c_{i1}$
  3. Heterogeneity in slope: η_{i2}, estimated value, η_{i1} and η_{i3}, predicted value knowing $\hat{\eta}^c_{i2}$
  4. Heterogeneity in curvature: η_{i3}, estimated value, η_{i1} and η_{i2}, predicted value knowing $\hat{\eta}^c_{i3}$
  5. Heterogeneity in slope and curvature: η_{i2} and η_{i3}, estimated values, η_{i1}, predicted value knowing $\hat{\eta}^c_{i2}$ and $\hat{\eta}^c_{i3}$

Counterfactual variances of log wages permanent components

Years	$\eta_1 = \bar{\eta}_{1g}$ $\eta_2 = \bar{\eta}_{2g}$ $\eta_3 = \bar{\eta}_{3g}$	Het. in Levels	Growth	Curv.	Growth and Curv.	All Perm. Comp.
1977	0.306	0.312	0.307	0.308	0.311	0.313
	(0.0169)	(0.0169)	(0.017)	(0.017)	(0.017)	(0.017)
1982	0.0449	0.0924	0.0489	0.0547	0.0679	0.094
	(0.0181)	(0.0181)	(0.0181)	(0.0181)	(0.0181)	(0.0182)
1992	0.0523	0.107	0.0849	0.0818	0.085	0.133
	(0.0221)	(0.0221)	(0.0221)	(0.0221)	(0.0221)	(0.0222)
2002	0.0533	0.108	0.144	0.136	0.144	0.165
	(0.0313)	(0.0313)	(0.0313)	(0.0313)	(0.0313)	(0.0313)
2007	0.05	0.105	0.18	0.188	0.188	0.189
	(0.0402)	(0.0402)	(0.0402)	(0.0402)	(0.0402)	(0.0402)
Long-run	0.0579	0.1	0.0771	0.0799	0.0801	0.106
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.0201)

Note: Only observations with more than 21 periods. 4873 observations. The counterfactuals are described in the text and measure the influence of each component of heterogeneity, in levels, growth and curvature.

Counterfactual variances of log wages permanent components - High Skills

Years	$\eta_1 = \bar{\eta}_{1g}$ $\eta_2 = \bar{\eta}_{2g}$ $\eta_3 = \bar{\eta}_{3g}$	Het. in Levels	Growth	Curv.	Growth and Curv.	All Perm. Comp.
1977	0.049	0.0636	0.0501	0.0522	0.0582	0.0642
	(0.0677)	(0.0677)	(0.0677)	(0.0677)	(0.0678)	(0.0681)
1982	0.0643	0.171	0.0698	0.0802	0.11	0.172
	(0.074)	(0.074)	(0.074)	(0.074)	(0.0741)	(0.0743)
1992	0.0871	0.211	0.132	0.135	0.135	0.248
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
2002	0.092	0.216	0.216	0.226	0.226	0.287
	(0.112)	(0.112)	(0.112)	(0.112)	(0.112)	(0.112)
2007	0.0837	0.207	0.262	0.307	0.322	0.333
	(0.126)	(0.126)	(0.126)	(0.126)	(0.126)	(0.127)
Long-run	0.0725	0.168	0.0988	0.108	0.114	0.177
	(0.0814)	(0.0815)	(0.0814)	(0.0814)	(0.0816)	(0.0818)

Note: High Skills. Only observations with more than 21 periods. 498 observations. The counterfactuals are described in the text and measure the influence of each component of heterogeneity, in levels, growth and curvature.

Conclusion

- Description of wage profiles with three individual factor loadings: level, slope and curvature.
- Estimation method combining random and fixed effect models.
- We ensure that the main features of our wage model are consistent with human capital theories of wages.
- Main conclusion: importance of considering at least three dimensions of heterogeneity when describing wage profile variances, in particular heterogeneous horizon effects.
- Limits:
  - Issue of missing data: go beyond the selection of balanced panels or missing at random assumption.
  - Proposing other ways to model transitory components
  - ...

Random Effects

- We use:

$$\begin{aligned} E(u_i \mid \eta_i^c) &= M(\beta)\eta_i^c, \\ V(u_i \mid \eta_i^c) &= V(v_i^c \mid \eta_i^c) \equiv \Omega(\eta_i^c), \end{aligned}$$

where $M(\beta)$ is a $T \times 3$ matrix of stacked $(1, t, \beta^{-t})$

- The covariance structure of u_i can be deduced:

$$V(u_i) = V(E(u_i \mid \eta_i^c)) + E(V(u_i \mid \eta_i^c)) = M(\beta)V(\eta_i^c)M(\beta)' + E(\Omega(\eta_i^c))$$

- Specification of $\Omega(\eta_i^c)$:
  - ARMA process (up to 3,3) whose initial conditions can depend on η_i^c.
  - Time heteroskedasticity of innovations
- Pseudo-likelihood estimation (cf. Alvarez and Arellano, 2004) of $V(\eta_i^c)$ (6), ARMA process (6 max), initial conditions (18 max) and time heteroskedasticity (28)

▸ Two-step estimation procedure

Fixed Effects

- Accounting for initial conditions and covariance structure of residuals, we prove:
$$\hat{\eta}_i^c = \hat{B} u_i,$$
where $\hat{B}$ is a function of random effect parameters.
Without initial conditions, and no missingness:
$B = \left(M(\beta)' \Omega^{-1} M(\beta)\right)^{-1} M(\beta)' \Omega^{-1}$
- Expression of the bias on the variance: Consider the unfeasible estimate, $\tilde{\eta}_i^c = B u_i = \eta_i^c + B v_i^c$, we get:
$$V(\tilde{\eta}_i^c) = EV(\tilde{\eta}_i^c \mid \eta_i^c) + VE(\tilde{\eta}_i^c \mid \eta_i^c) = B \Omega B' + V(\eta_i^c)$$
First term of order $1/T_i$.
- Feasible estimate: $\hat{\eta}_i^c = \hat{B} u_i = \tilde{\eta}_i^c + \left(\hat{B} - B\right) v_i^c,$
Second term of order $1/\sqrt{N}$.
- In the paper, adaptation to the case with initial conditions.

▸ Two-step estimation procedure

Estimated standard errors and correlations of individual effects η_i^c

	1-1	...	2-3	3-1	3-2	3-3
σ_{η_1}	.302		.304	.306	.300	.298
	(.001)		(.003)	(.003)	(.003)	(.004)
σ_{η_2}	.038		.036	.038	.037	.037
	(.005)		(.001)	(.001)	(.001)	(.001)
σ_{η_3}	.255		.248	.258	.247	.242
	(.005)		(.005)	(.005)	(.006)	(.007)
ρ_{η_1,η_2}	.473		.610	.505	.485	.365
	(.016)		(.013)	(.017)	(.020)	(.030)
ρ_{η_1,η_3}	- .604		- .729	- .636	- .620	- .509
	(.003)		(.012)	(.016)	(.019)	(.029)
ρ_{η_2,η_3}	- .946		- .941	- .946	- .943	- .944
	(.023)		(.003)	(.002)	(.003)	(.004)

Note: The first line corresponds to the ARMA specification (AR-MA) used for the random effect estimation. Standard errors in parentheses.

▸ Covariance structure of individual effects

Bias corrected covariance matrix of individual effects: fixed and random effect estimation

Sample	$Var(\eta_1)$	$Cov(\eta_1,\eta_2)$	$Cov(\eta_1,\eta_3)$	$Var(\eta_2)$	$Cov(\eta_2,\eta_3)$	$Var(\eta_3)$
21	0.18	0.012	-0.13	0.0034	-0.026	0.22
	(0.029)	(0.0032)	(0.03)	(0.00061)	(0.0049)	(0.043)
24	0.14	0.014	-0.12	0.0041	-0.03	0.23
	(0.017)	(0.0027)	(0.022)	(0.00061)	(0.0047)	(0.037)
27	0.077	0.0046	-0.04	0.0017	-0.011	0.079
	(0.0046)	(0.00064)	(0.0047)	(0.00015)	(0.001)	(0.0074)
28	0.067	0.0036	-0.031	0.0015	-0.0097	0.067
	(0.0049)	(0.00067)	(0.0047)	(0.00016)	(0.0011)	(0.0074)
21+	0.11	0.0078	-0.07	0.0025	-0.017	0.13
	(0.0073)	(0.00095)	(0.0092)	(0.00018)	(0.0015)	(0.013)
RE	0.093	0.0059	-0.05	0.0015	-0.0093	0.066
	(0.0036)	(0.00051)	(0.004)	(0.00011)	(0.00079)	(0.0059)

Notes: The first lines are obtained using fixed effect estimates. Sample periods = number of observed periods. Standard errors (heteroskedastic-consistent sampling and parameter uncertainty, 1000 MC simulations) between brackets. The working sample (21+) has 4873 observations

▸ Covariance structure of individual effects