ON THE SIZE OF THE GOVERNMENT SPENDING MULTIPLIER IN THE EURO AREA

Patrick Fève and Jean-Guillaume Sahuc

February 2015
ON THE SIZE OF THE GOVERNMENT SPENDING MULTIPLIER IN THE EURO AREA

Patrick Fève and Jean-Guillaume Sahuc

February 2015
ABSTRACT: This article addresses the existence of a wide range of estimated government spending multipliers in a dynamic stochastic general equilibrium model of the euro area. Our estimation results and counterfactual exercises provide evidence that omitting the interactions of key ingredients at the estimation stage (such as Edgeworth complementarity/substitutability between private consumption and government expenditures, endogenous government spending policy and general habits in consumption) paves the way for potentially large biases. We argue that uncertainty on the quantitative assessments of fiscal programmes could partly originate from these biases.

JEL CLASS.: C32, E32, E62.

KEYWORDS: Government spending multiplier, DSGE models, Estimation bias, Euro area.

RÉSUMÉ: Cet article s’intéresse à l’existence d’un large éventail de valeurs estimées du multiplicateur des dépenses publiques au sein d’un même modèle structurel de la zone euro. Nous montrons à l’aide d’exercices contrefactuels que l’omission, à l’étape de l’estimation, d’interactions entre divers mécanismes essentiels (tels qu’une complémentarité/substituabilité entre consommation privée et dépenses publiques, le degré de stabilisation automatique ou encore une spécification des habitudes de consommation plus générale) donne lieu à des biais potentiellement importants. Nous pensons que l’incertitude sur les évaluations quantitatives des programmes budgétaires pourrait en partie provenir de ce type de biais.


MOTS-CLÉS : Multiplicateur des dépenses publiques, modèles DSGE, biais d’estimation, zone euro.
NON TECHNICAL SUMMARY

Stimulus packages facing the Great Recession and the ongoing consolidation programs in most EU Member States have put fiscal policy at the heart of current economic policy discussions. A particularly debated issue is the evaluation of government spending multipliers, i.e. the increase in output consecutive to an increase in government spending, on which fiscal policy choices are partly based. However, there is not a single figure behind this concept, and a large uncertainty is surrounding its measurement. The value of the multiplier depends on many factors such as the econometric approach, the underlying model, the nature and duration of the fiscal change, or the state of the economy (see among others, Cogan et al., 2010, Ramey, 2011, Auerbach and Gorodnichenko, 2012, Coenen et al., 2012, Feve et al. 2013, or Erceg and Lindé, 2014), leaving the decision maker in trouble.

This paper contributes to this literature by providing an explanation for the wide range of estimated government spending multipliers within a given dynamic stochastic general equilibrium (DSGE) model of the euro area. We argue that omitting the interactions of key ingredients at the estimation stage, through cross-equation restrictions, paves the way for potentially large upward or downward biases. This shows up in our medium-scale model by studying the simultaneous combination of three mechanisms: (i) Edgeworth complementarity/substitutability between private consumption and government spending, (ii) endogeneity of government policy and (iii) general habits in consumption.

The mechanisms underlying the existence of bias in the estimated multiplier are the following. First, Edgeworth complementarity and counter-cyclicality of government expenditures work in opposite directions. Indeed, higher degree of Edgeworth complementarity leads to increase the response of both private consumption and output to a government spending shock, whereas a countercyclical policy acts as an automatic stabilizer. If fiscal policy is wrongly assumed to be exogenous, an econometrician will then under-estimate the true degree of Edgeworth complementarity to match the same correlation pattern of the data (as, for example, the positive relationships between private consumption and public expenditures). This translates into lower estimates of the government spending multiplier. A wrong assessment of the effects of government spending shocks is then obtained when automatic stabilizers are not taking into account. Second, Edgeworth complementarity and habit persistence in consumption work also in opposite directions. A high degree of habit formation tends to reduce (not eliminate) the crowding-out effect of public spending on private consumption. A moderate degree of Edgeworth complementarity is then needed to replicate the properties of the data. If an important dimension of habit formation (in our case, habit stock) is wrongly omitted at the estimation stage, the degree of Edgeworth complementarity will be over-estimated to compensate the lack of habit formation, turning to over-estimate the multiplier. This means that the specification of the utility function matters a lot for when estimating the government spending multiplier.
To address this quantitative issue, we consider a medium-scale DSGE model à la Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. In addition to the three above mentioned features, it includes investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, and nominal price and wage rigidities. Our maximum likelihood estimation with euro area data reveals (i) a high degree of Edgeworth complementarity between private consumption and public expenditures, (ii) a countercyclical endogenous component of government spending, and (iii) a sizeable degree of habit stock. When accompanied by a DSGE-VAR analysis, these outcomes confirm that the data clearly favor a model version including these three new features. We find that the multiplier on impact is around 1.30 (resp. 1.80) when wrongly omitting the endogeneity of government policy (resp. habit formation), to be compared to 1.60 in presence of the three mechanisms. So, the short-run effect of an increase in government spending displays a wide range of estimates, depending on which relevant mechanism is wrongly excluded from the empirical analysis. A similar result holds when it comes to the long-run multiplier which is underestimated (by 25%) or overestimated (by 15%) depending on the restrictive assumption relative to economic policy or preferences. These findings are reinforced when we assume that the zero lower bound on the nominal interest rate binds: The multiplier can be easily divided by two when omitting our key features. It is clear that both downward and upward biases obtained here are not negligible numbers, especially if the model is used to evaluate fiscal programs in the euro area.
1. Introduction

Stimulus packages facing the Great Recession and the ongoing consolidation programs in most EU Member States have put fiscal policy at the heart of current economic policy discussions. A particularly debated issue is the evaluation of government spending multipliers, i.e. the increase in output consecutive to an increase in government spending, on which fiscal policy choices are partly based. However, there is not a single figure behind this concept, and a large uncertainty is surrounding its measurement. The value of the multiplier depends on many factors such as the econometric approach, the underlying model, the nature and duration of the fiscal change, or the state of the economy (see among others, Cogan et al., 2010, Ramey, 2011, Auerbach and Gorodnichenko, 2012, Coenen et al., 2012, Fève et al. 2013, or Erceg and Lindé, 2014), leaving the decision maker in trouble.

This paper contributes to this literature by providing an explanation for the wide range of estimated government spending multipliers within a given dynamic stochastic general equilibrium (DSGE) model of the euro area. We argue that omitting the interactions of key ingredients at the estimation stage, through cross-equation restrictions, paves the way for potentially large upward or downward biases. This shows up in our medium-scale model by studying the simultaneous combination of three mechanisms: (i) Edgeworth complementarity/substitutability between private consumption and government spending, (ii) endogeneity of government policy and (iii) general habits in consumption.\(^1\)

Compared to the models used in Coenen et al. (2013) and Fève et al. (2013), there are four main differences. First, we consider a more general specification of the utility function in order to introduce mechanisms working in opposite directions. Second, we derive an analytical expression for the long-run multiplier in a medium-scale DSGE model. We show that (i) it is an increasing function of the degree of Edgeworth complementarity and (ii) it is independent from general habits or the endogenous component of government spending. Third, we compute the present-value output multiplier when the zero lower bound on the nominal interest rate binds and find that all our results are magnified. Fourth, we study in more details the interactions between our three ingredients at the estimation stage. We conduct several types of counterfactual experiments and show that wrong assumptions about economic policy and preferences translate into a downward bias (when endogenous component of government policy is omitted) or an upward bias (if general habits in consumption is ignored) in the estimated multiplier.

\(^1\)These mechanisms have already been considered in the relevant literature: Edgeworth complementarity/substitutability has been used to account for the aggregate interaction between private consumption and public spending (Aschauer, 1985 and Christiano and Eichenbaum, 1992, Fiorito and Kollintzas, 2004); The endogeneity of government spending through a simple feedback rule has been used to account for automatic stabilizers (McGrattan, 1994, Jones, 2002, Leeper et al., 2010); More general specifications of habit formation have been highlighted to accurately reproduce persistence in consumption and replicate the joint behavior of consumption and asset prices (Heaton, 1995, Cantore et al., 2014).
The mechanisms underlying the existence of bias in the estimated multiplier are the following. First, Edgeworth complementarity and counter-cyclicality of government expenditures work in opposite directions. Indeed, higher degree of Edgeworth complementarity leads to increase the response of both private consumption and output to a government spending shock, whereas a countercyclical policy acts as an automatic stabilizer. If fiscal policy is wrongly assumed to be exogenous, an econometrician will then under-estimate the true degree of Edgeworth complementarity to match the same correlation pattern of the data (as, for example, the positive relationships between private consumption and public expenditures). This translates into lower estimates of the government spending multiplier. A wrong assessment of the effects of government spending shocks is then obtained when automatic stabilizers are not taking into account. Second, Edgeworth complementarity and habit persistence in consumption work also in opposite directions. A high degree of habit formation tends to reduce (not eliminate) the crowding-out effect of public spending on private consumption. A moderate degree of Edgeworth complementarity is then needed to replicate the properties of the data. If an important dimension of habit formation (in our case, habit stock) is wrongly omitted at the estimation stage, the degree of Edgeworth complementarity will be over-estimated to compensate the lack of habit formation, turning to over-estimate the multiplier. This means that the specification of the utility function matters a lot for when estimating the government spending multiplier.

To address this quantitative issue, we consider a medium-scale DSGE model à la Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. In addition to the three above mentioned features, it includes investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, and nominal price and wage rigidities. Our maximum likelihood estimation with euro area data reveals (i) a high degree of Edgeworth complementarity between private consumption and public expenditures, (ii) a countercyclical endogenous component of government spending, and (iii) a sizeable degree of habit stock. When accompanied by a DSGE-VAR analysis, these outcomes confirm that the data clearly favor a model version including these three new features. We find that the multiplier on impact is around 1.30 (resp. 1.80) when wrongly omitting the endogeneity of government policy (resp. habit formation), to be compared to 1.60 in presence of the three mechanisms. So, the short-run effect of an increase in government spending displays a wide range of estimates, depending on which relevant mechanism is wrongly excluded from the empirical analysis. A similar result holds when it comes to the long-run multiplier which is underestimated (by 25%) or overestimated (by 15%) depending on the restrictive assumption relative to economic policy or preferences. These findings are reinforced when we assume that the zero lower bound on the nominal interest rate binds: The multiplier can be easily divided by two when omitting

\footnote{Interestingly, Edgeworth complementarity between private consumption and public expenditures seems to be an important channel for euro area fiscal policy (see also Coenen et al., 2013) but not for the US one, as shown by Kormilitsina and Zubairy (2013) and Cantore et al. (2014).}
our key features. It is clear that both downward and upward biases obtained here are not negligible numbers, especially if the model is used to evaluate fiscal programs in the euro area.\(^3\)

The rest of the paper is organized as follows. In the next section, we expound the DSGE model that is subsequently estimated. In Section 2, the quantitative analysis is conducted and a model comparison is done. In Section 3, we inspect the mechanisms at work. The last section contains the concluding remarks.

2. A Medium-Scale Model for the Euro Area

The present section describes our structural model of the euro area economy, which is close to Christiano et al. (2005) and Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. It includes features such as general habits formation, Edgeworth complementarity between private consumption and government spending, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, nominal price and wage rigidities, and countercyclical government expenditures. The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies and the public sector (government and monetary authorities).

2.1. Household Sector.

2.1.1. Employment Agencies. Each household indexed by \(j \in [0, 1]\) is a monopolistic supplier of specialized labor \(N_{j,t}\). At every point in time \(t\), a large number of competitive employment agencies combine households’ labor into a homogenous labor input \(N_t\) sold to intermediate firms, according to \(N_t = \left[ \int_0^1 N_{j,t} \frac{\varepsilon_{w,t}}{\varepsilon_{w,t}} \, dj \right]^{\varepsilon_{w,t}} \). Profit maximization by the perfectly competitive employment agencies implies the labor demand function \(N_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{\varepsilon_{w,t}} \frac{1}{\varepsilon_{w,t}-1} N_t\), where \(W_{j,t}\) is the wage paid by the employment agencies to the household supplying labor variety \(j\), while \(W_t \equiv \left( \int_0^1 W_{j,t} \frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1} \, dj \right)^{\varepsilon_{w,t}-1}\) is the wage paid by intermediate firms for the homogenous labor input sold to them by the agencies. The exogenous variable \(\varepsilon_{w,t}\) measures the substitutability across labor varieties and its steady state is the desired steady-state wage markup over the marginal rate of substitution between consumption and leisure.

\(^3\)We consider Edgeworth complementarity and general habits as useful propagation mechanisms. There are other potential candidates: Non-separable preferences, externalities, deep habits and hand-to-mouth consumers. We do not consider these alternative mechanisms for at least two reasons. First, some of them (externalities) induce severe difficulties at the estimation stage. Second, our estimation results indicate that they do not improve the model’s fit, when our three modeling features are maintained. For instance, the estimated share of hand-to-mouth consumers is too small (less than 7%) to generate enough propagation.
2.1.2. Household’s Preferences. The preferences of the \( j \)th household display time non-separability. Following Heaton (1995), preferences are time additive over a good called "services". In addition, labor supply enters separably in the utility function given by

\[
E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{b,t+s} \left( \log (S_{t+s}) - \frac{N_{j,t+s}^{1+v}}{1+v} + V(G_{t+s}) \right),
\]

where \( E_t \) denotes the mathematical expectation operator conditional upon information available at \( t \), \( \beta \in (0, 1) \) is the subjective discount factor and \( \nu > 0 \) is the inverse of the Frisch labor supply elasticity. \( N_{j,t} \) is labor of type \( j \), \( \varepsilon_{b,t} \) is a discount factor shock, and the services \( S_t \) are defined as

\[
S_t = S_{d,t} - h (1 - \delta_h) S_{h,t},
\]

where \( S_{d,t} \) is the "durability stock" governing the degree of substitutability in consumption decision and \( S_{h,t} \) is the "habit stock" governing the degree of intertemporal complementarity in consumption (Otrok, 2001). The parameter \( h \in [0, 1] \) denotes the degree of habit formation. The two stocks evolve according to

\[
S_{d,t} = \sum_{\tau=0}^{\infty} \delta_d^\tau C_{t-\tau}^* \quad \text{and} \quad S_{h,t} = \sum_{\tau=0}^{\infty} \delta_h^\tau C_{t-\tau}^*,
\]

where \( \delta_d \) and \( \delta_h \) are comprised between 0 and 1 and the consumption bundle \( C_t^* \) is defined as

\[
C_t^* = C_t + \alpha_g G_t.
\]

\( C_t \) and \( G_t \) denote private consumption and public expenditures, respectively. The parameter \( \alpha_g \) accounts for the complementarity/substitutability between private consumption and public expenditures.\(^4\) If \( \alpha_g > 0 \), government spending substitutes for private consumption, with perfect substitution if \( \alpha_g = 1 \), as in Christiano and Eichenbaum (1992). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding-out effect. In the special case \( \alpha_g = 0 \), we recover the standard business cycle model, with government spending operating through a negative income effect on labor supply (see Aiyagari et al., 1992, Baxter and King, 1993). When the parameter \( \alpha_g < 0 \), government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending. Finally, \( V(G_t) \) is a positive concave function, meaning that agents do not necessarily feel worse off when public expenditures increase. Notice that this term has no effect on the equilibrium.

---

\(^4\)The specification adopted here follows Christiano and Eichenbaum (1992), McGrattan (1994), Finn (1998), among others. An alternative specification is a CES function between \( C_t \) and \( G_t \) (see McGrattan et al., 1997, Bouakez and Rebei, 2007, Coenen et al., 2013). Note that these two specifications yield exactly the same log-linearized equation for the marginal utility of consumption.
Combining the two stocks implies that services at time $t$ are given by

$$S_t = \frac{1 - (\delta_h + h (1 - \delta_h)) L}{(1 - \delta_d L) (1 - \delta_h L)} C_t^*.$$ 

Households are subject to idiosyncratic shocks about whether they are able to re-optimize their wage. Hence, the above described problem makes the choices of wealth accumulation contingent upon a particular history of wage rate decisions, thus leading to households heterogeneity. Combining the assumption of separability between services and labor supply in the utility function with the assumption of a complete set of contingent claims market, all the households will make the same choices regarding consumption and will only differ by their wage rate and supply of labor. This is directly reflected in our notations.

Household $j$’s period budget constraint is given by

$$P_t (C_t + I_t) + T_t + B_t \leq R_{t-1} B_{t-1} + A_{jt,t} + D_t + W_{jt,t} N_{jt,t} + \left(R^h_t u_t - P_t \theta(u_t)\right) \hat{K}_{t-1},$$

where $I_t$ is investment, $T_t$ denotes nominal lump-sum taxes (transfers if negative), $B_t$ is the one-period riskless bond, $R_t$ is the nominal interest rate on bonds, $A_{jt,t}$ is the net cash flow from household’s $j$ portfolio of state contingent securities, $D_t$ is the equity payout received from the ownership of firms, and $R^h_t$ is the rental rate of capital. The capital utilization rate $u_t$ transforms physical capital $\bar{K}_t$ into the service flow of effective capital $K_t$ according to $K_t = u_t \hat{K}_{t-1}$, and the effective capital is rented to intermediate firms at the nominal rental rate $R^h_t$. The costs of capital utilization per unit of capital is given by the convex function $\theta(u_t)$. We assume that $u = 1$, $\theta'(1) = 0$, and we define $\eta_u \equiv \left[\theta''(1) / \theta'(1)\right] / \left[1 + \theta''(1) / \theta'(1)\right]$. The physical capital accumulates according to

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \varepsilon_{i,t} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

where $\delta \in [0, 1]$ is the depreciation rate of capital, and $S(.)$ is an adjustment cost function which satisfies $S(\gamma_z) = S'(\gamma_z) = 0$ and $S''(\gamma_z) = \eta_k > 0$, $\gamma_z$ is the steady-state (gross) growth rate of technology, and $\varepsilon_{i,t}$ is an investment shock.

Households set nominal wages according to a staggering mechanism. In each period, a fraction $\theta_w$ of households cannot choose its wage optimally, but adjusts it to keep up with the increase in the general wage level in the previous period according to the indexation rule $W_{jt,t} = \gamma_z \pi^{1 - \gamma_w} \pi_{t-1}^{\gamma_w} W_{jt,t-1}$, where $\pi_t \equiv P_t / P_{t-1}$ represents the gross inflation rate, $\pi$ is steady-state (or trend) inflation and the coefficient $\gamma_w \in [0, 1]$ is the degree of indexation to past wages. The remaining fraction of households chooses instead an optimal wage, subject to the labor demand function $N_{jt,t}$.

---

5This specification is flexible enough to encompass many popular cases such as time-separable utility ($h = \delta_d = 0$), one lag habit formation ($\delta_h = \delta_d = 0$), and pure durability ($h = 0$).

6Later, we estimate $\eta_u$ rather than the elasticity $\theta''(1) / \theta'(1)$ to avoid convergence problems.
2.2. Business Sector.

2.2.1. Final Good Producers. At every point in time \( t \), a perfectly competitive sector produces a final good \( Y_t \) by combining a continuum of intermediate goods \( Y_t (\zeta), \zeta \in [0, 1] \), according to the technology 
\[
Y_t = \left[ \int_0^1 Y_{\zeta,t} \frac{1}{\varepsilon_{\zeta,t}} d\zeta \right]^{\varepsilon_{p,t}}.
\]
Final good producing firms take their output price, \( P_t \), and their input prices, \( P_{\zeta,t} \), as given and beyond their control. Profit maximization implies
\[
Y_{\zeta,t} = \left( \frac{P_{\zeta,t}}{P_t} \right)^{\varepsilon_{p,t}} Y_t \text{,}
\]
from which we deduce the relationship between the final good and the prices of the intermediate goods
\[
P_{\zeta,t} \equiv \left[ \int_0^1 P_{\zeta,t} \frac{1}{\varepsilon_{\zeta,t}} d\zeta \right]^{\varepsilon_{p,t} - 1}.
\]
The exogenous variable \( \varepsilon_{p,t} \) measures the substitutability across differentiated intermediate goods and its steady state is then the desired steady-state price markup over the marginal cost of intermediate firms.

2.2.2. Intermediate-Goods Firms. Intermediate good \( \zeta \) is produced by a monopolist firm using the following production function 
\[
Y_{\zeta,t} = K_{\zeta,t} \alpha [Z_t N_{\zeta,t}]^{1-\alpha} - Z_t F,
\]
where \( \alpha \in (0, 1) \) denotes the capital share, \( K_{\zeta,t} \) and \( N_{\zeta,t} \) denote the amounts of capital and effective labor used by firm \( \zeta \), \( F > 0 \) is a fixed cost of production that ensures that profits are zero in steady state, and \( Z_t \) is an exogenous labor-augmenting productivity factor whose growth-rate, denoted by \( \varepsilon_{z,t} \equiv Z_t / Z_{t-1} \). In addition, we assume that intermediate firms rent capital and labor in perfectly competitive factor markets.

Intermediate firms set prices according to a staggering mechanism. In each period, a fraction \( \theta_p \) of firms cannot choose its price optimally, but adjusts it to keep up with the increase in the general price level in the previous period according to the indexation rule
\[
P_{\zeta,t} = \pi^{1-\gamma_p} \pi_{t-1}^\gamma P_{\zeta,t-1},
\]
where the coefficient \( \gamma_p \in [0, 1] \) indicates the degree of indexation to past prices. The remaining fraction of firms chooses its price \( P_{\zeta,t}^* \) optimally, by maximizing the present discounted value of future profits
\[
E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ \Pi_{t,s}^p P_{\zeta,t+s}^* Y_{\zeta,t+s} - [W_{t+s} N_{\zeta,t+s} + R_{t+s} K_{\zeta,t+s}] \right\}
\]
where
\[
\Pi_{t,s}^p = \begin{cases} 
\prod_{v=1}^{s} \pi^{1-\gamma_p} \pi_{t+v-1}^\gamma & s > 0 \\
1 & s = 0,
\end{cases}
\]
subject to the demand from final goods firms and the production function. \( \Lambda_{t+s} \) is the marginal utility of consumption for the representative household that owns the firm.

2.3. Public Sector. The stationary component of government spending is given by
\[
\frac{G_t}{Z_t} = g \tilde{G}_{t} \varepsilon_{g,t},
\]
where \( g \) denotes the deterministic steady-state value of \( G_t/Z_t \). \( \epsilon_{g,t} \) is a government spending shock. The endogenous component of the policy \( \tilde{G}_t \) is assumed to follow the simple rule

\[
\tilde{G}_t = \left( \frac{Y_t}{\gamma_z Y_{t-1}} \right)^{\phi_g}.
\]

The parameter \( \phi_g \) is the policy rule parameter linking the stationary component of government policy to demeaned output growth. If \( \phi_g > 0 \), the policy rule contains a procyclical component that triggers an increase in government expenditures whenever output growth is above its average value. In contrast, if \( \phi_g < 0 \), the policy rule features a countercyclical component, and thus reflects automatic stabilizers (see e.g. Jones, 2002, and Fève et al., 2013). In both cases however, assessing the degree of pro- or counter-cyclicality of the overall level of government spending requires taking the stochastic trend in productivity into account. For example, assuming that \( \phi_g = 0 \), the growth rate of government expenditures would still be positively correlated with total factor productivity growth.

The monetary authorities follow a generalized Taylor rule by gradually adjusting the nominal interest rate in response to inflation and output growth:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\phi_r} \left( \frac{\pi_t}{\pi} \right)^{\phi_{\pi}} \left( \frac{Y_t}{\gamma_z Y_{t-1}} \right)^{\phi_y} (1-\phi_r) \epsilon_{r,t},
\]

where \( \epsilon_{r,t} \) is a monetary policy shock.

2.4. Market Clearing and Stochastic Processes. Market clearing condition on final goods market is given by

\[
Y_t = C_t + I_t + G_t + \theta (u_t) \bar{K}_{t-1}.
\]

\[
\Delta_p Y_t = (u_t \bar{K}_{t-1})^a [Z_t N_t]^{1-a} - Z_t F
\]

where \( \Delta_p = \int_0^1 \left( \frac{P_{t+s}}{P_t} \right)^{-\frac{p}{p-1}} \, d\zeta \) is a measure of the price dispersion.

Regarding the properties of the stochastic variables, productivity and monetary policy shocks evolve according to \( \log (\epsilon_{x,t}) = \zeta_{x,t} \), with \( x \in \{z,r\} \). The remaining exogenous variables follow an AR(1) process \( \log (\epsilon_{x,t}) = \rho_x \log (\epsilon_{x,t-1}) + \zeta_{x,t} \), with \( x \in \{b,i,g\} \), except the substitutabilities across labour varieties and across differentiated intermediate goods which are assumed to follow ARMA(1,1) processes, \( \log (\epsilon_{x,t}) = (1 - \rho_x) \log (\epsilon_x) + \rho_x \log (\epsilon_{x,t-1}) + \zeta_{x,t} - \phi_x \zeta_{x,t-1} \), with \( x \in \{w,p\} \), in order to capture the moving average, high frequency component of both wages and inflation. In all cases, \( \zeta_{x,t} \sim i.i.d. N (0, \sigma_x^2) \).

3. Quantitative Analysis

In this section, our formal econometric procedure is expounded. We then present the estimation results of an unconstrained (hereafter referred as benchmark) and a constrained (hereafter referred as Smets-Wouters) version of the model. The Smets-Wouters model corresponds to the case when our three
ingredients are absent \((\varphi_g = \alpha_g = \delta_d = \delta_h = 0)\). Finally, we discuss the implications for the estimated government spending multipliers.

3.1. **Data and Econometric Approach.** The quarterly euro area data run from 1985Q1 to 2007Q4 and are extracted from the AWM database compiled by Fagan et al. (2005), except hours worked and the working age population. Inflation is measured by the first difference of the logarithm of GDP deflator (YED), the short-term nominal interest rate is a three month rate (STN), and real wage growth is the first difference of the logarithm of nominal wage (WRN) divided by GDP deflator. Private consumption growth is constructed by multiplying real private consumption (PCR) times the private consumption deflator (PCD), divided by GDP deflator and transformed into first difference of the logarithm; Private investment growth is defined as the aggregate euro area total economy gross investment minus general government investment, scaled by GDP deflator and transformed into first difference of the logarithm; government spending growth is defined as the sum of nominal general government final consumption expenditure (GCN) and nominal government investment (GIN), scaled by GDP deflator and transformed into first difference of the logarithm. Real variables are divided by the working age population, extracted from the OECD Economic Outlook. Ohanian and Raffo (2012) have build a new dataset of quarterly hours worked for 14 OECD countries. We have then made a weighted (by country size) average of their series of hours worked for France, Germany and Italy to obtain a series of total hours for the euro area. Interestingly, the series thus obtained is very close to that provided by the ECB on the common sample, i.e. 1999-2007. The growth of total hours worked is the first difference of the logarithm of total hours worked.

We follow the Bayesian approach to estimate various versions of the model (see An and Schorfheide, 2007, for an overview). Let \(\theta\) denote the vector of structural parameters to be estimated and \(X_T \equiv \{X_t\}_{t=1}^T\) the sample of observable data, where

\[
X_t = 100 \times [\Delta \log C_t, \Delta \log I_t, \Delta \log G_t, \Delta \log (W_t/P_t), \Delta \log N_t, \pi_t, R_t].
\]

We use the Kalman filter to calculate the likelihood \(L(\theta, X_T)\), and then combine the likelihood function with a prior distribution of the parameters to be estimated, \(\Gamma(\theta)\), to obtain the posterior distribution, \(L(\theta, X_T)\Gamma(\theta)\). Given the specification of the model, the posterior distribution cannot be recovered analytically but may be computed numerically, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters.

We use growth rates for the non-stationary variables in our data set (private consumption, private investment, government spending and the real wage) and express gross inflation, gross interest rates and the first difference of the logarithm of hours worked in percentage deviations from their sample mean. We write the measurement equation of the Kalman filter to match the seven observable series with their model counterparts. Before taking the model to the data, we induce stationarity by getting
ride of the stochastic trend component $Z_t$ and we log-linearized the resulting system in the neighborhood of the deterministic steady state. Thus, the state-space form of the model is characterized by the state equation $\Xi_t = A(\theta)\Xi_{t-1} + B(\theta)\xi_t$, $\xi_t \sim i.i.d. N(0, \Sigma_{\xi})$, where $\Xi_t$ is a vector of endogenous variables, $\xi_t$ is a vector of innovations to the seven structural shocks, and $A(\theta)$ and $B(\theta)$ are complicated functions of the model’s parameters. The measurement equation is given by $X_t = C(\theta) + D\Xi_t$, where $D$ is a selection matrix and $C(\theta)$ is a vector that is function of the structural parameters.

The benchmark model contains twenty two structural parameters, excluding the parameters relative to the exogenous shocks. We calibrate seven of them: the discount factor $\beta$ is set to 0.99, the inverse of the Frisch labor supply elasticity $\nu = 2$, the capital depreciation rate $\delta$ is equal to 0.025, the parameter $\alpha$ in the Cobb-Douglas production function is set to 0.30 to match the average capital share in net (of fixed costs) output (McAdam and Willman, 2013), the steady-state price and wage markups $\varepsilon_p$ and $\varepsilon_w$ are set to 1.20 and 1.35 respectively (Everaert and Schule, 2008), and the steady-state share of government spending in output is set to 0.20 (the average value over the sample period). The remaining fifteen parameters are estimated.

**Table 1. Prior Densities and Posterior Estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior Smets-Wouters Model</th>
<th>Posterior Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habit in consumption, $h$</td>
<td>$B[0.70,0.10]$</td>
<td>0.852</td>
<td>0.813 [0.745,0.886]</td>
</tr>
<tr>
<td>Habit stock, $\delta_h$</td>
<td>$B[0.50,0.20]$</td>
<td>–</td>
<td>0.429 [0.181,0.671]</td>
</tr>
<tr>
<td>Durability stock, $\delta_d$</td>
<td>$B[0.50,0.20]$</td>
<td>–</td>
<td>0.256 [0.062,0.444]</td>
</tr>
<tr>
<td>Capital utilisation cost, $\eta_u$</td>
<td>$B[0.50,0.10]$</td>
<td>0.586</td>
<td>0.600 [0.454,0.750]</td>
</tr>
<tr>
<td>Investment adj. cost, $\eta_k$</td>
<td>$G[4.00,1.00]$</td>
<td>4.379</td>
<td>4.302 [2.805,5.765]</td>
</tr>
<tr>
<td>Edgeworth compl., $\alpha_g$</td>
<td>$U[-3.30,1.30]$</td>
<td>–</td>
<td>-0.647 [-0.986,-0.316]</td>
</tr>
<tr>
<td>Policy endogeneity, $\varphi_g$</td>
<td>$U[-2.45,2.45]$</td>
<td>–</td>
<td>-0.782 [-1.032,-0.536]</td>
</tr>
<tr>
<td>Growth rate of technology, $\log(\gamma_z)$</td>
<td>$G[0.45,0.10]$</td>
<td>0.306</td>
<td>0.329 [0.217,0.439]</td>
</tr>
<tr>
<td>Calvo price, $\theta_p$</td>
<td>$B[0.66,0.10]$</td>
<td>0.823</td>
<td>0.815 [0.746,0.885]</td>
</tr>
<tr>
<td>Calvo wage, $\theta_w$</td>
<td>$B[0.66,0.10]$</td>
<td>0.808</td>
<td>0.765 [0.680,0.855]</td>
</tr>
<tr>
<td>Price indexation, $\gamma_p$</td>
<td>$B[0.50,0.15]$</td>
<td>0.195</td>
<td>0.197 [0.068,0.324]</td>
</tr>
<tr>
<td>Wage indexation, $\gamma_w$</td>
<td>$B[0.50,0.15]$</td>
<td>0.458</td>
<td>0.389 [0.185,0.592]</td>
</tr>
<tr>
<td>Monetary policy-smoothing, $\varphi_r$</td>
<td>$B[0.75,0.15]$</td>
<td>0.842</td>
<td>0.849 [0.815,0.883]</td>
</tr>
<tr>
<td>Monetary policy-inflation, $\varphi_{\pi}$</td>
<td>$G[1.70,0.30]$</td>
<td>1.509</td>
<td>1.562 [1.266,1.845]</td>
</tr>
<tr>
<td>Monetary policy-output growth, $\varphi_y$</td>
<td>$G[0.125,0.05]$</td>
<td>0.158</td>
<td>0.163 [0.067,0.255]</td>
</tr>
</tbody>
</table>
Table 1. Prior Densities and Posterior Estimates (cont’d)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Smets-Wouters Model</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage markup shock persistence, ρ₇</td>
<td>B[0.60,0.15]</td>
<td>0.867</td>
<td>[0.788,0.945]</td>
</tr>
<tr>
<td>Intertemporal shock persistence, ρ₈</td>
<td>B[0.60,0.15]</td>
<td>0.298</td>
<td>[0.132,0.449]</td>
</tr>
<tr>
<td>Investment shock persistence, ρ₉</td>
<td>B[0.60,0.15]</td>
<td>0.872</td>
<td>[0.785,0.961]</td>
</tr>
<tr>
<td>Price markup shock persistence, ρ₁₀</td>
<td>B[0.60,0.15]</td>
<td>0.855</td>
<td>[0.722,0.965]</td>
</tr>
<tr>
<td>Government shock persistence, ρ₁₁</td>
<td>B[0.60,0.15]</td>
<td>0.948</td>
<td>[0.914,0.982]</td>
</tr>
<tr>
<td>Wage markup shock (MA part), ϱ₇</td>
<td>B[0.60,0.15]</td>
<td>0.670</td>
<td>[0.518,0.821]</td>
</tr>
<tr>
<td>Price markup shock (MA part), ϱ₁₀</td>
<td>B[0.60,0.15]</td>
<td>0.567</td>
<td>[0.388,0.756]</td>
</tr>
<tr>
<td>Wage markup shock volatility, σ₇</td>
<td>IG[0.25,2.00]</td>
<td>0.144</td>
<td>[0.113,0.172]</td>
</tr>
<tr>
<td>Intertemporal shock volatility, σ₈</td>
<td>IG[2.00,2.00]</td>
<td>2.819</td>
<td>[1.916,3.685]</td>
</tr>
<tr>
<td>Investment shock volatility, σ₉</td>
<td>IG[0.25,2.00]</td>
<td>0.273</td>
<td>[0.212,0.333]</td>
</tr>
<tr>
<td>Price markup shock volatility, σ₁₀</td>
<td>IG[0.25,2.00]</td>
<td>0.123</td>
<td>[0.096,0.150]</td>
</tr>
<tr>
<td>Productivity shock volatility, σ₁₁</td>
<td>IG[0.25,2.00]</td>
<td>0.813</td>
<td>[0.713,0.910]</td>
</tr>
<tr>
<td>Government shock volatility, σ₁₂</td>
<td>IG[0.25,2.00]</td>
<td>0.942</td>
<td>[0.825,1.059]</td>
</tr>
<tr>
<td>Monetary policy shock volatility, σ₁₃</td>
<td>IG[0.25,2.00]</td>
<td>0.126</td>
<td>[0.113,0.172]</td>
</tr>
</tbody>
</table>

Log-marginal likelihood                        -462.546        -438.369
Posterior odds ratio                           0.000           1.000
Bayes factor                                   $3.161 \times 10^{10}$ –
Kass-Raftery statistics                         48.354 –

Note: This table reports the prior distribution, the mean and the 90 percent confidence interval (within square brackets) of the estimated posterior distribution of the structural parameters. The Bayes factor between model i and j is computed as $\exp(L(X_T|M_i) - L(X_T|M_j))$ where $L$ stands for log-marginal likelihood. The Kass-Raftery statistics is computed as twice the logarithm of the Bayes factor. $B[.,.]$, $G[.,.]$, $U[.,.]$ and $IG[.,.]$ stand for Beta distribution, Gamma distribution, Uniform distribution and Inverse-Gamma distribution, respectively.

The prior distribution is summarized in Table 1. Our choices are in line with the literature, especially with Smets and Wouters (2007), Sahuc and Smets (2008) and Justiniano et al. (2010). Importantly, we specify for $\varphi_g$ and $\alpha_g$ uniform priors centered on 0 and $-1$, and with a standard deviation of 1.00 and 1.33, respectively, to reflect our agnostic view concerning these key parameters. Finally, the rate of depreciation $\delta_d$ and the weight parameter on habit stock $\delta_h$ are assumed to follow a Beta distribution centered on 0.50 with a standard error of 0.20.

3.2. Estimation Results. The estimation results are reported in Table 1, together with the posterior mean and the 90% confidence interval. Several results are worth commenting on.
First, the two model versions display very similar estimated values of the common structural parameters. Neither the parameters related to real rigidities nor those related to nominal rigidities are affected by the presence of Edgeworth complementarity, endogenous government policy and general time non-separable preferences. Note also that the parameters related to monetary policy are almost insensitive.

Second, the parameter estimates are in line with previous results (Smets and Wouters, 2003, Sahuc and Smets, 2008, Coenen et al., 2013). For example, the adjustment cost parameter is $\eta_k \approx 4.30$ and the parameter related to capital utilization is $\eta_u \approx 0.60$. In addition, the wage indexation parameter is $\gamma_w \approx 0.40$ in the two model versions, higher than the price indexation parameter $\gamma_p \approx 0.20$. This reflects a now standard result that the euro area data do not require too high a degree of price indexation. The probability that firms are not allowed to re-optimize their price is $\theta_p \approx 0.82$, implying an average duration of price contracts of about 15 months. The probability of no wage change is $\theta_w \approx 0.80$, implying an average duration of wage contracts of about 20 months. All these numbers are consistent with the results reported in the survey done by Druant et al. (2012). The monetary policy parameters $(\phi_{\pi}, \phi_y) \approx (1.51, 0.16)$ and $\phi_r \approx 0.85$ indicate that the monetary authorities act very gradually with a large weight on inflation.

Third, the three retained mechanisms are essential as they heavily increase the fit of the model, as shown by the improvement of the log-marginal likelihood in Table 1. The posterior odds ratios offer a complementary way of seeing this. Starting from a prior distribution on model versions with equal probability (1/2), we obtain that the benchmark model represents the whole probability mass. Similarly, a Bayes factor beyond 100 and a Kass and Raftery’s (1995) statistics beyond 10 provide decisive and very strong evidence in favour of the benchmark model.

The estimated value for $\alpha_g$ is negative suggesting strong Edgeworth complementarity between private consumption and public expenditures. This result is in the line with those obtained in Coenen et al. (2013) for the euro area and Feve et al. (2013) for the United States. Moreover, the endogenous component of government expenditures is negatively related to output growth. 7 Finally, the parameters related to habit stock and durability are positive and significant. This implies that the habit parameter $h$ is higher in the Smets-Wouters model since such a version omits habit stock in preferences.

Another procedure to compare models is the DSGE-VAR methodology. The DSGE-VAR approach has been suggested as a tool for studying misspecification of a DSGE model and allowing the cross-equation restrictions of the DSGE model to be relaxed in a flexible manner (Del Negro and Schorfheide, 2004). The basic idea is to (i) use a VAR model as an approximating model for the DSGE model and (ii) construct a mapping from the DSGE model to the VAR parameters, leading to a set of cross-restrictions for the VAR model. Deviations from these restrictions may be interpreted as evidence for DSGE model misspecification.

7We also investigate various forms of the endogenous component of government spending and obtain that the specification adopted here is preferred by the data.
In a Bayesian framework, one can specify a prior distribution for deviations from the DSGE model restrictions, whose tightness is scaled by a single hyperparameter \( \lambda \). The values \( \lambda = \infty \) and \( \lambda = 0 \) correspond to the two polar cases in which the cross-coefficient restrictions are strictly enforced and completely ignored (unrestricted VAR). The log-marginal likelihood function of \( \lambda \in (0, \infty] \) provides an overall assessment of the DSGE model restrictions that may be viewed as more informative than a comparison of the two polar cases. An estimate of \( \lambda \) is then used as a way to evaluate DSGE models: If \( \lambda \) is small the prior on the VAR coefficients is diffuse, if it is large, most of the prior mass on the VAR coefficients concentrates near the DSGE model restrictions. Figure 1 shows how the shape of the posterior distribution of the \( \lambda \) changes across our two alternative models. Relative to the benchmark model, the marginal likelihood associated with the Smets-Wouters model is lower for any value \( \lambda \). Most importantly, the posterior mass shifts to the left, toward a lower value of \( \lambda \) (looser prior): The estimated value of the hyperparameter is 0.70 in the benchmark model and 0.49 in the Smets-Wouters model. We interpret these findings as strong evidence that the benchmark specification improves the fit of the DSGE model.

Finally, we compare the two model versions by computing the forecast error variance decomposition of consumption growth, government spending growth and total hours growth. This decomposition is reported in Figure 2, where the forecast horizon is set to one. In the benchmark model, the
government spending shock explains 6% (resp. 20%) of the variance of consumption growth (resp. total hours growth), whereas it is almost zero (resp. 10%) in the Smets-Wouters model. At the same time, the share of the variance of government expenditures is the same in the two model versions (around 56%), meaning that our key additional ingredients allow to reinforce the transmission mechanism of government policy.

Figure 2. Variance Decomposition of a Selection of Variables

3.3. Implications for Government Spending Multipliers. We can now investigate the quantitative implications of these two model versions for the government spending multipliers. Two types of multipliers are considered.

First, we are interested in the short-run output multiplier, defined as the present value of additional output over a $q$-period horizon that is generated by a change in the present value of government spending (see Mountford and Uhlig, 2009),

$$E_t \sum_{i=0}^{q} \hat{\beta}^i \Delta Y_{t+i}$$

where $\hat{\beta} \equiv \beta / \gamma_z$ is the inverse of the steady-state real interest rate. This notion embodies the full dynamics associated with exogenous fiscal actions and properly discounts future macroeconomic effects. It is obtained from the parameter estimates of Table 1. At $q = 0$, the present-value multiplier equals the impact multiplier. An important issue when it comes to the evaluation and to the comparison of
short-run multipliers between several structural models is the degree of persistence of the government policy shock, i.e. \( \rho_g \) in our notation (see Aiyagari et al., 1992, and Campbell, 1995). In our case, this is not problematic because the autoregressive parameter is almost identical across model versions. In addition, as we have shown that the common structural parameters are very similar, we can suspect that the short-run multipliers are mainly driven by our three mechanisms.

Second, we consider the long-run output multiplier, defined as the increase in steady-state output consecutive to an increase in steady-state government spending expenditures. Interestingly, this multiplier can be easily derived, even in a medium-scale DSGE model, since the steady state is independent from both real and nominal rigidities. From this definition and the structure of the benchmark model, the following proposition states key properties of the long-run multipliers.

**Proposition 1.** Under the benchmark model:

1. The long-run output multiplier is

\[
\frac{\Delta y}{\Delta g} = \frac{(1 + s_f) (1 - \alpha_g)}{1 + s_f - s_i + v (s_c + \alpha_g s_g)},
\]

where \( y = Y / Z, g = G / Z, s_c, s_i \) and \( s_g \) denotes the consumption to output ratio, investment to output ratio and government expenditures to output ratio, respectively. The parameter \( s_f \) is defined by \( F / y \), where the fixed cost \( F \) is assumed to be constant.

2. The long-run multiplier is a decreasing function of the Edgeworth complementarity parameter \( \alpha_g \).

**Proof.** See the appendix.

This multiplier is obtained from the elasticities of non-stochastic steady-state employment and output with respect to public spending. The steady-state real interest rate is independent from all frictions (both real and nominal), the government endogenous policy and the exogenous process of public spending. It follows that the output to total hours ratio is constant and thus the real wage is unaffected. Importantly, the long-run multiplier directly depends on two structural parameters related to preferences \( (\alpha_g \) and \( v \)) as well as on the great ratios of the economy (so implicitly on \( \beta, \delta, \alpha \)). Given the fact that \( \beta, \delta, \alpha \) and \( v \) are calibrated prior to estimation, the long-run multiplier (i) depends only on the estimated value of the Edgeworth complementarity parameter \( \alpha_g \), and (ii) unambiguously decreases with \( \alpha_g \). From this property and the estimation results of Table 1, we can directly expect a higher long-run multiplier in our benchmark model than in the Smets-Wouters model. We now quantify this discrepancy relative to the benchmark model.

Let us first consider the short-run dynamics. Figure 3 displays the short-run output multiplier of the two model versions together with its 90% confidence interval. The impact multiplier associated with the benchmark model is around 1.60 and then steadily goes back to a value close to 0.80. Notice that the short-run multiplier remains greater that one during almost five years. This result is in contrast with the Smets-Wouters model for which the multiplier is around one on impact but then decreases towards
0.40 (see Cogan et al., 2010, for a similar number). In addition, the two confidence intervals do not overlap on the whole horizon.

**Figure 3. Short-Run Output Multiplier**

![Short-Run Output Multiplier](image)

*Note:* The solid and dashed lines correspond to the mean of the short-run multiplier in the benchmark model and in the Smets-Wouters model, respectively. The light and dark grey areas correspond to the 90 percent confidence interval in the benchmark and Smets-Wouters models, respectively.

**Figure 4. Dynamic Responses**

![Dynamic Responses](image)

*Note:* The solid and dashed lines correspond to the mean of the short-run multiplier in the benchmark model and in the Smets-Wouters model, respectively. The light and dark grey areas correspond to the 90 percent confidence interval in the benchmark and Smets-Wouters models, respectively.
Figure 4 reports the responses of other aggregate variables. This figure shows huge differences between the two model versions. First, the benchmark model displays a positive and persistent response of private consumption after an increase in public spending. In contrast, the response of consumption is persistently negative in the Smets-Wouters model. Second, the decrease in investment is more pronounced in the benchmark model than in the Smets-Wouters one. The reason is that Edgeworth complementarity leads to reduce private saving, despite the increase in labor supply. Third, the effects on inflation and the nominal interest rate of the government policy shock are more pronounced. Such a demand shock triggers higher inflation pressures in the benchmark model than in the Smets-Wouters model, due to a larger increase in the real marginal cost. For the nominal interest rate this is a direct consequence of the Taylor rule. The dynamic responses obtained from the Smets-Wouters model appear outside the confidence interval in the short run.

We now investigate the effect of a permanent change in government spending. Figure 5 reports the empirical distribution of the long-run multiplier to output obtained from the benchmark model, together with the average long-run multiplier for the two model versions. This figure clearly shows that the two model versions yield very different estimates of the long-run multiplier. Indeed, the average multiplier is around 1.04 in the benchmark model whereas it is two times smaller (0.56) when we shut down together the three mechanisms that we put forward. Such a difference is clearly not neutral if the model is used to assess recovery plans or consolidation programmes in the euro area.

![Figure 5. Empirical Distribution of the Long-Run Output Multiplier](image)

*Note:* The vertical dashed lines correspond to the mean of the long-run output multiplier in the benchmark model and in the Smets-Wouters model, respectively.
In the spirit of Christiano et al. (2011), we also check the value of the government spending multiplier when the zero lower bound on the nominal interest rate binds. We consider a discount rate shock that is sufficiently large so that the zero lower bound on the nominal interest rate binds during six periods, and we assume that government expenditures increase in the first period. We can then compute the differences between the percent deviation of output and government expenditures from steady state that results from both the original shock and the increase in government purchases and those that result only from the original shock. The present-value output multiplier is then defined as a discounted ratio of these differences in output and in government spending. Figure 6 shows that (i) the output multiplier becomes very large when the zero bound binds in both model versions and (ii) the peak value of the multiplier in the benchmark version is more than twice (3.30) that obtained in the Smets-Wouters model (1.50). Note that these values clearly depend on the duration during which the nominal interest rate is constant: A longer period would imply a stronger effect on the multiplier. Where does this boost effect come from? The implied higher discount rate leads to a substantial decline in investment, hours worked, output and consumption. This fall in output is associated with a fall in marginal costs and substantial deflation. Since the nominal interest rate is zero, the real interest rate rises sharply and increases desired saving. This larger fall in output is undone by an increase in government expenditures. In presence of complementarity between private consumption and public spending, the effect on output of a rise in government spending is reinforced by the positive impact on private consumption.

Figure 6. Short-Run Output Multiplier When the Zero Bound is Binding
4. Inspecting the Mechanisms

Having shown that the benchmark model generates both short-run and long-run multipliers that exceed unity, we now inspect the key mechanisms at work after a shock to government spending. To do so, we conduct two types of counterfactual experiments. First, we shut down one or several transmission mechanisms by altering a parameter (or a combination thereof) and we re-estimate the model using Bayesian techniques (Counterfactual #1 in Table 2). Second, we perturb the same set of parameters while keeping all the other at the estimated values obtained in our benchmark case (Counterfactual #2 in Table 2). If the implied government spending multiplier is strongly affected in both types of counterfactual experiments, the isolated mechanism plays an essential role in the transmission of fiscal shocks. This simply means that other forces (or parameters) are not adjusted, meaning that altering the parameter reveals the mechanism at work. Conversely, if the other parameters adjust to fit the data, there exist potentially other forces that propagate the government spending shock. In this case, we may obtain very different multipliers in the two counterfactual experiments.

We concentrate our analysis on our three modeling features: (i) Edgeworth complementarity, (ii) endogenous component of government spending, and (iii) general habits in consumption. All our experiments are reported in Tables 2 and 3. Table 2 reports the present-value output multiplier on impact \((q = 0)\), one year after impact \((q = 5)\) and two years after impact \((q = 9)\) and the long-run multiplier (i.e. after a permanent change in the level of government spending). Remember that this multiplier depends only on the estimated value of Edgeworth complementarity, \(\alpha_g\), since the other parameters in the formula remain constant (see Proposition 1). Table 3 reports the structural parameters estimates under Counterfactual #1.

<table>
<thead>
<tr>
<th>Table 2. Estimated Effects of Government Spending Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
</tr>
<tr>
<td>1stQ</td>
</tr>
<tr>
<td>1.618</td>
</tr>
<tr>
<td>Smets-Wouters</td>
</tr>
<tr>
<td>1stQ</td>
</tr>
<tr>
<td>1.065</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counterfactual #1</th>
<th>Counterfactual #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1stQ</td>
<td>5thQ</td>
</tr>
<tr>
<td>No Edgeworth Complementarity</td>
<td>1.091</td>
</tr>
<tr>
<td>No feedback Rule</td>
<td>1.279</td>
</tr>
<tr>
<td>No Habit Stock</td>
<td>1.654</td>
</tr>
<tr>
<td>No Local Durability</td>
<td>1.611</td>
</tr>
<tr>
<td>One Lag Habit</td>
<td>1.815</td>
</tr>
</tbody>
</table>
4.1. Edgeworth Complementarity. Let us first consider the role played by Edgeworth complementarity. We set $\alpha_g = 0$. The third line of Table 2 reports the quantitative results. We see that both the short and long-run multipliers decrease heavily. This result applies for the two counterfactual experiments. So, no other parameters can adjust to compensate the role played by Edgeworth complementarity. This is confirmed by comparing the two first columns of Table 3 (benchmark and no Edgeworth complementarity). Indeed, the other model’s parameters remain almost identical under the benchmark specification and the constrained version. The large decrease in the government spending thus results in the crowding out effect on consumption in the absence of Edgeworth complementarity. This mechanism thus appears essential for a proper transmission mechanism of government spending shocks.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_g$</td>
<td>-0.650</td>
<td>0.000</td>
<td>-0.275</td>
<td>-0.696</td>
<td>-0.658</td>
<td>-0.776</td>
</tr>
<tr>
<td>$\varphi_g$</td>
<td>-0.745</td>
<td>-0.606</td>
<td>0.000</td>
<td>-0.759</td>
<td>-0.746</td>
<td>-0.807</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.812</td>
<td>0.842</td>
<td>0.829</td>
<td>0.886</td>
<td>0.768</td>
<td>0.799</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>0.613</td>
<td>0.580</td>
<td>0.566</td>
<td>0.618</td>
<td>0.614</td>
<td>0.625</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>3.962</td>
<td>4.156</td>
<td>3.874</td>
<td>3.955</td>
<td>3.985</td>
<td>4.000</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.829</td>
<td>0.832</td>
<td>0.817</td>
<td>0.833</td>
<td>0.834</td>
<td>0.835</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>0.784</td>
<td>0.776</td>
<td>0.760</td>
<td>0.794</td>
<td>0.788</td>
<td>0.826</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.166</td>
<td>0.159</td>
<td>0.161</td>
<td>0.162</td>
<td>0.165</td>
<td>0.159</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>0.366</td>
<td>0.389</td>
<td>0.383</td>
<td>0.376</td>
<td>0.369</td>
<td>0.419</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.853</td>
<td>0.855</td>
<td>0.850</td>
<td>0.850</td>
<td>0.854</td>
<td>0.846</td>
</tr>
<tr>
<td>$\phi_z$</td>
<td>1.558</td>
<td>1.553</td>
<td>1.588</td>
<td>1.544</td>
<td>1.564</td>
<td>1.507</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.144</td>
<td>0.145</td>
<td>0.149</td>
<td>0.146</td>
<td>0.145</td>
<td>0.143</td>
</tr>
</tbody>
</table>

Note: (1) No Edgeworth complementarity, (2) No feedback rule, (3) No habit stock, (4) No local durability, and (5) One lag habit.

4.2. Endogenous Government Spending. In this exercise, we assume that government spending is exogenous ($\varphi_g = 0$). In Counterfactual #1, the model’s parameters remain unaffected, with the noticeable exception of $\alpha_g$. When $\varphi_g = 0$, the estimated value of $\alpha_g$ decreases (it is divided by more than 2) and both the short-run and long-run multipliers decreases, i.e. by around 0.30 points which is not a negligible figure. This results comes from the interplay of Edgeworth complementarity and countercyclicality of government spending. Indeed, these two mechanisms work in opposite directions in terms of generating a correlation pattern between output or consumption and government expenditures. Edgeworth complementarity tends to increase this correlation, because it induce that agents
are willing to consume more. A countercyclical component in the government policy rule reduces this correlation, by construction. So, in order to replicate a given correlation pattern between output and government spending, a higher degree of Edgeworth complementarity is needed to compensate a highly countercyclical government policy. This mechanically translates to a higher government spending multiplier. When the government policy is assumed to be exogenous, there is no need for high Edgeworth complementarity, thus yielding a smaller multiplier. Consequently, omitting the endogeneity of government spending may mask sizeable crowding-in effects on private consumption. In Counterfactual #2, setting $\phi_g = 0$ has very little effect. Only the impact multiplier increases by 0.03 point. This is because, we shut down the automatic stabilizer effect of government spending. Notice that the long-run multiplier is the same since $\alpha_g$ is maintained to its benchmark value. The results show that omitting the endogenous policy rule at the estimation stage would lead an analyst to under-estimate the government spending multiplier at all horizons.

4.3. Habit Formation on Consumption. We consider three cases. In the first one, we eliminate the habit stock specification ($\delta_h = 0$). For the second case, we consider a pure habit model ($\delta_d = 0$). In the third case, we specify a one lag habit, as usual in the DSGE literature ($\delta_h = \delta_d = 0$). Overall, our results show that omitting habit formation can have potentially strong effects on the estimated government spending multiplier through the estimation of $\alpha_g$ (Counterfactual# 1 in Table 2). For example, if we compare the results obtained in the one lag habit case with those of the benchmark case, we obtain a sizeable difference in the estimated multipliers. Both the short-run and the long-run multipliers are now overestimated, because the estimated value of $\alpha_g$ increases (in absolute value). The reason for this result is the following. Suppose that the econometrician will seek to estimate the Edgeworth complementarity, but she wrongly omits habit stock in consumption. Habit formation creates intertemporal complementarity in consumption decision and thus tends to limit the crowding out effect of public spending on private consumption. Because habit stock is (wrongly) absent in the estimated model, there is a need for a higher degree of Edgeworth complementarity to match the data, and thus this implies a higher multiplier. This is illustrated by the two columns "benchmark" and (5) in Table 3. For example, a typical version of the Smets and Wouters model considers only one lag in habit formation. This yields to estimate $\alpha_g = -0.776$ (to be compared to $\alpha_g = -0.650$) and then an impact multiplier of 1.81, 0.20 point over the benchmark one. Notice that the effects of local durability remain small.

4.4. Summing Up. The previous analysis shed light on two opposite forces affecting the estimation of the government spending multiplier. On the one hand, omitting Edgeworth complementarity (together with endogenous government spending policy) leads to under-estimate the multiplier. On the other hand, ignoring habit stock in utility function tends to over-estimate the multiplier. The estimation of the benchmark model and the counterfactual exercises clearly show that (i) the first mechanism dominates and (ii) the multiplier is mainly driven by the degree of complementarity between private consumption and public spending.
5. Concluding Remarks

This paper has assessed the main mechanisms at work when estimating government spending multipliers in a DSGE model of the euro area. First, we have shown that the degree of Edgeworth complementarity between private and public consumption is essential to fit the data and thus lead to larger multiplier. Moreover, we have explored the consequences of misspecifying the government spending rule and habit formation on the estimated government spending multiplier. We have notably shown that omitting these last two features may exert a severe downward or upward biases on the estimated multipliers and thus can seriously affect the quantitative assessment of fiscal stimulus.

In our framework, we deliberately abstracted from relevant details in order to highlight, as transparently as possible, the empirical link between the three mechanisms we pushed forward. However, the recent literature insists on other modeling or policy issues that might potentially affect our results. We mention two of them. First, we assumed lump-sum taxes to finance government deficit but a more realistic case could consider distortionary taxes instead. In this latter case, we expect an even greater difference between the two model versions because taxes (labor income or VAT) would need to go up much more in the Smets-Wouters model for a given spending hike. Second, our paper focused on the size of the government spending multiplier in the euro area as a whole, abstracting from any form of heterogeneity (especially fiscal) among its members. Our framework could be extended to a model of a monetary union to account for cross-country spillovers.


**APPENDIX: PROOF OF PROPOSITION 1**

(1) At the deterministic steady-state (see the online appendix for more details), the log-linearized production function is \( \hat{\gamma} = \alpha \hat{k} + (1 - \alpha) \hat{n} \), where \( \hat{\gamma} = (y / (y + F)) \hat{y} \). From the Euler equation on consumption, we get that \( (y + F) / k \) and \( i / k \) are constant and independent of \( g \), implying that \( \hat{\gamma} = \hat{k} = \hat{i} \). Plugging this in the production function yields \( \hat{\gamma} = \hat{n} \). Otherwise, from the real wage equation, it comes that \( \hat{\omega} = (1 + F / y) (1 - \alpha_g) (g / y) \hat{\gamma} \), or equivalently \( \hat{\omega} = (1 + F / y) (1 - \alpha_g) (g / y) \hat{\gamma} \).

Knowing that \( \Delta y = y \times \hat{\gamma} \) and \( \Delta g = g \times \hat{\gamma} \), we deduce the long-run multiplier formula.

(2) Differentiating the multiplier with respect to \( \alpha_g \) implies

\[
\frac{\partial (\Delta y / \Delta g)}{\partial \alpha_g} = -\frac{(1 - s_f) (1 + s_f - s_i + v(s_c + s_g))}{[1 + s_f - s_i + v(s_c + \alpha_g s_g)]^2} < 0. \]

\( \blacksquare \)


522. F. Ferroni and B. Klaus, “Euro Area business cycles in turbulent times: convergence or decoupling?,” November 2014


Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : www.banque-france.fr

For a complete list of Working Papers published by the Banque de France, please visit the website: www.banque-france.fr

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE
49- 1404 Labolog
75049 Paris Cedex 01
tél : 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81
e-mail : 1404-ut@banque-france.fr