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Strategic fiscal revaluation or devaluation: why does the labor wedge matter?

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*The views expressed in this paper do not necessarily reflect those of the Banque de France. We would like to thank J. Imbs, T Sopraseuth, L. Patureau and H. Pagès for fruitful discussions on the first version of this paper. This version of the paper also draws on inputs by the participants at Banque de France seminar.

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Abstract

Most European countries suffer from a structural weakness of employment and competitiveness. Can an optimal tax system reinforce European countries in this respect? If so, does this long-term policy act as a devaluation or a revaluation? In this paper, we show that fiscal devaluation can be an optimal policy only if the labor wedge is sufficiently large. Indeed, whereas the terms-of-trade externality calls for a fiscal revaluation, i.e. the use of tariffs by "strategic" countries for extracting a rent from their trade partners, a sufficiently large labor wedge calls for employment subsidies, at the heart of a fiscal devaluation. We show that these subsidies must be financed by VAT instead of tariffs, which are less efficient. Finally, in a multi-country world, we show that, if several countries adopt a similar strategy, the impact of this policy is magnified.

Keywords: Optimal taxation, international trade, labor wedge, general equilibrium model

JEL classification: D51, F42, H21

Résumé

La plupart des pays européens souffrent d’une faiblesse structurelle de l’emploi et de la compétitivité. Les pays européens pourraient-ils se renforcer sur ce plan, s’ils adoptaient un système fiscal optimal ? Si c’était le cas, une telle réforme de long terme agirait-elle comme une dévaluation ou comme une réévaluation ? Dans cet article, nous montrons qu’une dévaluation fiscale peut être optimale uniquement lorsque le coin du travail est suffisamment important. En effet, alors que l’externalité des termes de l’échange appelle à une réévaluation fiscale, c’est-à-dire à ce que les pays "stratégiques" utilisent des tarifs pour extraire une rente de leurs partenaires commerciaux, un coin du travail suffisamment important appelle à verser des subventions à l’emploi qui sont au cœur des dévaluations fiscales. Nous montrons qu’il est plus efficace de financer de telles subventions par la TVA plutôt que par des tarifs. Enfin, dans un monde multi-pays, nous montrons que l’impact de ces réformes est amplifié, lorsque plusieurs pays suivent ce type de stratégie.

Mots-clés : taxation optimale, commerce international, coin du travail, modèle d’équilibre général

Codes JEL : D51, F42, H21
Fiscal devaluation has become a serious policy option in the euro area. The idea behind fiscal devaluation is the following: "a country can use unilateral fiscal policy to generate the same real outcomes as those following a nominal exchange rate devaluation, while keeping the nominal exchange rate fixed". Our paper add to these policy analysis which only focus on the short run, by focusing on welfare gains/losses in the long-run and thus leaving aside short-term nominal rigidities. Our results give new support to this policy: rather than replace the nominal regulation of short-term fluctuations by a new tax system, it can act against the structural weakness of employment observed in many European countries. In the paper, we present our arguments in two steps, in the context of a non-cooperative game where taxes are the strategic instruments chosen by governments.

First, we analyze a pure exchange economy in order to isolate the terms-of-trade externality: we derive therefrom the optimal tax policy in this context, which is a fiscal revaluation. When goods are imperfect substitutes at the international level, the home country planner can compute an allocation by acting as a monopoly vis-à-vis the foreign countries, using the information about the import and export functions. Here, taxes on imports reduce trade between countries, in particular trade in goods produced domestically, and thus lead to a revaluation of domestic goods, which helps increase the real wealth of domestic economic agents. We show that, in a game with more than two players and with heterogeneous tax systems across countries, non-cooperative strategies can lead to greater welfare in strategic countries (i.e. those which can manipulate their tax system) than that obtained in a "free trade" economy.

Second, we extend our analysis to an economy with production where the labor market is imperfectly competitive. We then show that a general result emerges: given that each government has only one fiscal tool for two externalities (terms-of-trade externality and market power on the labor market), it must choose the tax design that reduces the weighted average of these two gaps. These two objectives are antagonists: labor market inefficiencies call for employment subsidies that increase output, whereas the terms-of-trade externality calls for consumption taxes that reduce output. If the employment gap is "small", all taxes on goods are used to subsidize labor, lead to "over-work" and thus to a decrease in welfare. Conversely, if the employment gap is large, consumption tax revenues should be used to "buy" the large rent of the monopolists on the labor market. In this case, the optimal tax system consists in shifting part of the direct tax on labor toward an indirect tax on consumption. Moreover, contrary to the reforms analyzed in the case of the pure exchange economy, we find that such reforms generate positive spillovers for their trade partners. Finally, we show that the VAT is preferable to tariffs when governments decide to tax consumption.
1 Introduction

Fiscal devaluation has become a serious policy option in the euro area. The idea behind fiscal devaluation is the following: "a country can use unilateral fiscal policy to generate the same real outcomes as those following a nominal exchange rate devaluation, while keeping the nominal exchange rate fixed" (Farhi et al. 2012). On the basis of these theoretical results, many European countries have implemented fiscal devaluation: Denmark in 1988, Sweden in 1993, Germany in 2006 and France in 2012. Our paper add to these policy analysis which only focus on the short run: how/why can this policy affect the medium-run allocation? Is this policy welfare-improving? Can this policy increase welfare in a multi-country world where more than one country can strategically manipulate its tax policy?

Rather than replace the nominal regulation of short-term fluctuations by a new tax system (the "New-Keynesian" view) which would simply mimic the impact of another policy instrument (the nominal exchange rate) in a small open economy (the strategies of the other countries being in this case neglected), we show that such a fiscal policy can improve welfare in the long run, even in a multi-country world. Our results therefore give new support to this policy: it can act against the structural weakness of employment observed in many European countries. This could be viewed as a more convincing argument than the one that focuses on output gaps (those between the effective output and the flexible one), which are negligible with respect to the size of long-term inefficiencies. Moreover, we show that this policy can be welfare-improving for "strategic" countries (i.e. those which can manipulate their tax system), even in a non-cooperative game between countries, if there exists "non-strategic" countries. However, we will also see that, in an open economy framework, the terms-of-trade externality might,...

\footnote{As pointed by these authors, this idea was already put forward by Keynes (1931) who showed that a uniform ad valorem tariff on all imports plus a uniform subsidy on all exports has the same effect as a nominal devaluation.}

\footnote{As shown in Lipinska and von Thadden (2009), many European countries have shifted their tax structure in this way over recent years.}

\footnote{Because a devaluation is neutral in a long run perspective, a fiscal devaluation \textit{à la} Farhi et al. 2012 is also neutral in the long run because its objective is only to mimic the changes in the allocation implied by a devaluation.}
on the contrary, call for fiscal revaluation instead of fiscal devaluation. In
our "new" long-term perspective, we determine the conditions under which
fiscal devaluation is an optimal tax policy: we show that fiscal devaluation
is preferable, if the labor market suffers from large inefficiencies.

In the paper, we present our arguments in two steps, in the context of
a non-cooperative game where taxes are the strategic instruments chosen
by governments. We adopt a long-term perspective and we thus leave aside
short-term nominal rigidities.

First, we analyze a pure exchange economy in order to isolate the terms-
of-trade externality: we derive therefrom the optimal tax policy in this con-
text, which is a fiscal revaluation. As recalled by Costinot et al. (2013), this
idea goes back a long way in the international trade literature, notably to
Mill (1844). Indeed, when goods are imperfect substitutes at the interna-
tional level, the home country planner can compute an allocation by acting
as a monopoly vis-à-vis the foreign countries, using the information about
the import and export functions. As shown in Costinot et al. (2013), the
optimal policy allows governments to extract a positive markup if they im-
plement tariffs on imported goods. Here, taxes on imports reduce trade
between countries, in particular trade in goods produced domestically, and
thus lead to a revaluation of domestic goods, which helps increase the real
wealth of domestic economic agents. We show that, in a game with more
than two players and with heterogeneous tax systems across countries4, non-
cooperative strategies can lead to greater welfare in strategic countries5 than
that obtained in a "free trade" economy.

Second, we extend our analysis to an economy with production where the
labor market is imperfectly competitive. We then show that a general result
emerges: given that each government has only one fiscal tool for two exter-
nalities (terms-of-trade externality and market power on the labor market),

\footnote{There exists an asymmetry in tax systems across countries: indeed, tax systems grow
as countries develop. A "fiscal response" is then less easy than a "nominal reaction" based
on the exchange rate, which makes our framework more realistic.}

\footnote{The terms strategic countries and competitive countries are the same as the one put
forward by d’Aspremont et al. (1997) and Gabszewicz and Michel (1992) when they discuss
this type of argument to give foundations to imperfect competitive equilibria (strategic
firms vs competitive firms).}
it must choose the tax design that reduces the weighted average of these two gaps. These two objectives are antagonists: labor market inefficiencies call for employment subsidies that increase output, whereas the terms-of-trade externality calls for consumption taxes that reduce output. If the employment gap is "small" (negligible at the limit), all taxes on goods are used to subsidize labor, lead to "over-work" and thus to a decrease in welfare. Conversely, if the employment gap is large, consumption tax revenues should be used to "buy" the large rent of the monopolists on the labor market. In this case, the optimal tax system consists in shifting part of the direct tax on labor toward an indirect tax on consumption. Moreover, contrary to the reforms analyzed in the case of the pure exchange economy, we find that such reforms generate positive spillovers for their trade partners. Finally, we show that the VAT is preferable to tariffs when governments decide to tax consumption.

Our paper is related to other contributions in the trade and labor market literature. In the trade literature, the idea of exploiting the terms-of-trade externality is present in the Costinot et al. (2013) paper where it is shown that the optimal policy allows governments to extract a positive markup if they set tariffs on imported goods. Indeed, when goods are imperfect substitutes at the international level, the home country planner can achieve a desired allocation of resources by acting as a monopoly vis-à-vis the foreign countries, using the information about the import and export functions. However, these studies, where only one country can act strategically, may be called into question as foreign countries may react to this type of "aggressive" policy, thus giving rise to a tax competition.7

Indeed, when the labor wedge is sufficiently large, the employment in the economy without optimal tax policy can be lower than its counterpart in an economy without labor market frictions (without labor market frictions, policy makers then reduce optimally the quantities traded to extract a rent in the goods market). Thus, when the labor wedge is sufficiently large, the labor market distortions dominate the terms-of-trade externality.

7The same criticism applies for the fiscal devaluation or nominal devaluation in NK framework à la Farhi et al. 2012.
game. From this result, they deduce that trade agreements remedy this inefficiency and provide welfare gains.\(^8\) However, the argument of Bagwell and Staiger (1999) in favor of the GATT principles is not robust: in a game with more than two players and heterogeneous tax systems across countries, non-cooperative strategies can lead to a greater welfare in strategic countries than that obtained in a "free trade" economy. In the labor macroeconomic literature, a large number of papers underlines the so-called “European employment problem” (see Prescott 2004 and Ljungqvist and Sargent 2008).\(^9\) This "labor wedge" calls for structural policies to raise the total number of hours worked, either by acting on the number of hours worked per worker or on (un)employment: fiscal devaluation, as long as it leads to a reduction in total labor costs, can hence be helpful on this issue. This is shown in Langot et al. (2014). We extend this analysis by taking into account the fiscal competition between "strategic" countries: these countries, acting in a non-cooperative way, can obtain an additional surplus from the "competitive" ones which are the losers in this game.\(^10\)

In the first section of the paper, we present the optimal taxation in a pure exchange economy. In the second section, we extend our results to an economy with production and an imperfectly competitive labor market.

\(^8\)Bagwell and Staiger (2012) argue that trade agreements continue to increase welfare, even if there is imperfect competition on the goods market. This welfare gain still comes from the terms-of-trade externality.

\(^9\)This refers to the substantial decrease in total hours worked in most European countries since the 1970s relative to Anglo-Saxon economies like the USA. Two dimensions must be distinguished: (i) the persistence of a high unemployment rate, which the literature relates to the effect of stringent labor market institutions on the extensive margin of labor, i.e. the number of employees (see e.g. Blanchard and Wolfers 2000), and (ii) a lower amount of hours worked per employee, i.e. the intensive labor margin, which is strongly related to the effect of a too heavy labor tax wedge (see e.g. Rogerson 2006 or Ohanian et al. 2008). On this point, Prescott (2004) mentions that the welfare gains to French households from adopting American taxes (i.e. reducing the effective tax rate on labor by 20 percentage points) “would be equivalent to a 20 percent increase in consumption, with no increase in work effort” (Lucas 2003).

\(^10\)In Costinot et al. (2013), a multi-country analysis is proposed but, only one country has "strategic" tools, and there is no production with imperfect competition in the labor market.
2 International trade, pure exchange economy and optimal taxation

In this section, we present a basic model where welfare gains can only be delivered by trade and tariffs. This allows us to focus on our main argument in favor of fiscal revaluation.\(^{11}\)

The argument is the following. In competitive international trade equilibrium, each country supplies its total endowment of each good to the international trade market (a central market place). Nevertheless, it could be in a country’s interest to restricting its supply of a particular set of initial goods because the resulting equilibrium price leads it to enjoy a higher level of welfare. How to manipulate these quantities? If private agents do not have the possibility of acting strategically\(^{12}\), then governments can manipulate their behavior via a specific tax system that will send new signals to private agents. Given a particular set of taxes, we show that the optimal taxation of a specific country is such that the home country increases its purchases of its specific initial endowment, relative to a world without taxation, and restricts its purchases from foreign countries, which are the victims of the taxation. By doing so, the home country restricts the non-taxed endowment available for the other countries and raises its price. We show that, for certain parameter configurations, the Nash equilibrium between countries is such that strictly positive taxes on imported goods are preferable to the competitive allocation. This means that the negative externality between strategic countries can be counterbalanced by the welfare gains induced by

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\(^{11}\)We leave aside reallocation and other productive gains associated with trade. Using Krugman’s new trade model, Ossa (2011) finds that these gains would also come from a production relocation externality. Finally, in the more recent ‘new new trade’ framework, Melitz and Redding (2012) show that heterogeneity allows trade to increase productivity and this would represent a new source of gains from trade. Arkolakis et al. (2012) defend another view, according to which trade gains depend only on the share of expenditure on domestic goods and on the elasticity of imports with respect to trade costs. However, as the underlying assumptions about demand and trade-cost conditions radically affect the measurement of the aggregate trade elasticity, adding firm heterogeneity might change the trade elasticity and the trade gains (Melitz and Redding 2013).

\(^{12}\)In monopolistic competition, the country can set a price and a quantity for these specific initial endowments. d’Aspremont et al. (1997) show that, even if non-competitive agents may find other non competitive traders competing with them in the same manner, the system of rivalry among non-competitive traders on the various markets may lead to a suboptimal Nash equilibrium where each agent sets a positive markup.
the rent extraction on the competitive countries.

2.1 Main assumptions

- There are \( m = n + 2 \) countries in the world.

- There are 3 goods, and the third is chosen as the numéraire: the price vector is then \( p = (p_1, p_2, 1) \).

- Each country’s endowment of different goods is given by the vector: 
  \[
  y_1 = (1, 0, 0), \quad y_2 = (0, 1, 0), \quad y_i = (0, 0, 1/n) \quad \text{for} \quad i = 3, n + 2, \quad \text{with} \quad n \geq 1.
  \]

- The preferences of the representative household in each country are given by:
  \[
  U_1 = (1 - \alpha_1)^\frac{1}{\sigma} x_{1,1}^{1-\frac{1}{\sigma}} + (\alpha_1 \beta_1)^\frac{1}{\sigma} x_{1,2}^{1-\frac{1}{\sigma}} + \left(\alpha_1 (1 - \beta_1)\right)^\frac{1}{\sigma} x_{1,3}^{1-\frac{1}{\sigma}}
  \]
  \[
  U_2 = (\alpha_2 \beta_2)^\frac{1}{\sigma} x_{2,1}^{1-\frac{1}{\sigma}} + (1 - \alpha_2)^\frac{1}{\sigma} x_{2,2}^{1-\frac{1}{\sigma}} + \left(\alpha_2 (1 - \beta_2)\right)^\frac{1}{\sigma} x_{2,3}^{1-\frac{1}{\sigma}}
  \]
  \[
  U_i = (\alpha_3 \beta_3)^\frac{1}{\sigma} x_{i,1}^{1-\frac{1}{\sigma}} + (\alpha_3 (1 - \beta_3))^\frac{1}{\sigma} x_{i,2}^{1-\frac{1}{\sigma}} + (1 - \alpha_3)^\frac{1}{\sigma} x_{i,3}^{1-\frac{1}{\sigma}} \quad \text{for} \quad i = 3, n + 2
  \]
  where, for all countries \( \sigma > 1, \alpha_i \in (0; 1) \) and \( \beta_i \in (0; 1) \).

- The budget constraints are: 
  \[
  (1 + \tau_i^T_i) \left( x_{i,3} + p_j x_{i,j} \right) + p_i x_{i,i} = y_i \]
  where \( \tau_i^T_i \) is the taxes on imported goods.\(^{13}\) We do not allow the government to discriminate between goods 3 and \( j \) by introducing two different tax rates on imports. This limits our analysis to a second best allocation but facilitates the comparison with the economy with production where only a consumption tax is introduced.

- Governments in countries 1 and 2 can tax imported goods. This tax is denoted \( \tau_i^T_i \), for \( i = 1, 2 \), and is used to finance a lump sum transfer to the representative household. Governments’ budget constraints are:
  \[
  \tau_i^T_i (p_j x_{i,j} + x_{i,3}) = P_i T_i
  \]
  where \( P_i \) is the consumer price index in the country \( i \).

\(^{13}\)We do not introduce export subsidies because only the relative price matters for the optimal consumption sharing rule between goods. Thus, for an arbitrary positive export subsidy, import taxes must be lower.
2.2 Optimal behavior: the net demand functions

2.2.1 Households in countries 1 and 2

For \( i = 1, 2 \) and \( j \neq i \), the program of the representative household is

\[
\begin{align*}
\text{max} & \quad \left\{ (1 - \alpha_j)^{\frac{1}{\sigma}} x_{i,j}^1 \frac{1}{1 - \frac{1}{\sigma}} + (\alpha_i \beta_i)^{\frac{1}{\sigma}} x_{i,j}^2 \frac{1}{1 - \frac{1}{\sigma}} + (\alpha_i (1 - \beta_i))^{\frac{1}{\sigma}} x_{i,j}^3 \frac{1}{1 - \frac{1}{\sigma}} \right\} \\
\text{s.t.} & \quad (1 + \tau_j^i)(x_{i,3} + p_j x_{i,j}) + p_i x_{i,i} = p_i + P_i T_i
\end{align*}
\]

If \( P_i \) denotes the aggregate consumer price index of country 1, given by

\[
P_i = \left[ (1 - \alpha_i)p_i^{1-\sigma} + \alpha_i \beta_i (p_j (1 + \tau_j^i))^{1-\sigma} + \alpha_i (1 - \beta_i) (1 + \tau_j^i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

the solution of the problem, for country \( i = 1 \), is the vector of the demand function

\[
x_1(p) = \frac{p_1 + P_i T_i}{P_i} \left( (1 - \alpha_1) - \frac{P_i}{p_1 + P_i T_i}, \alpha_i \beta_1 \left( \frac{p_2(1 + \tau_j^i)}{P_i} \right)^{-\sigma}, \alpha_i (1 - \beta_1) \left( \frac{1 + \tau_j^i}{P_i} \right)^{-\sigma} \right)
\]

The tax \( \tau_j \) induces two effects: it changes relative prices, and it creates an additional income through the transfer \( T \), which is positive only if \( \tau_j > 0 \).

Given that \( y_1 = (1; 0; 0) \), the vector of the net demand functions is:

\[
x_1^*(p) = \frac{p_1 + P_i T_i}{P_i} \left( (1 - \alpha_1) - \frac{P_i}{p_1 + P_i T_i}, \alpha_i \beta_1 \left( \frac{p_2(1 + \tau_j^i)}{P_i} \right)^{-\sigma}, \alpha_i (1 - \beta_1) \left( \frac{1 + \tau_j^i}{P_i} \right)^{-\sigma} \right)
\]

For country \( i = 2 \), given that the endowment is \( y_2 = (0; 1; 0) \), we obtain

\[
x_2^*(p) = \frac{p_2 + P_2 T_2}{P_2} \left( \alpha_2 \beta_2 \left( \frac{p_1(1 + \tau_j^i)}{P_2} \right)^{-\sigma}, (1 - \alpha_2) \left( \frac{p_2}{P_2} \right)^{-\sigma} - \frac{P_2}{p_2 + P_2 T_2}, \alpha_2 (1 - \beta_2) \left( \frac{1 + \tau_j^i}{P_2} \right)^{-\sigma} \right)
\]

where the aggregate price index in country 2 is given by

\[
P_2 = \left[ \alpha_2 \beta_2 (p_1 (1 + \tau_j^i))^{1-\sigma} + (1 - \alpha_2)p_2^{1-\sigma} + \alpha_2 (1 - \beta_2) (1 + \tau_j^i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

2.2.2 Households in countries \( i \geq 3 \)

As regards these countries, the program of the representative household is:

\[
\begin{align*}
\text{max} & \quad \left\{ (\alpha_3 \beta_3)^{\frac{1}{\sigma}} x_{i,1}^1 \frac{1}{1 - \frac{1}{\sigma}} + (\alpha_3 (1 - \beta_3))^{\frac{1}{\sigma}} x_{i,2}^2 \frac{1}{1 - \frac{1}{\sigma}} + (1 - \alpha_3)^{\frac{1}{\sigma}} x_{i,3}^3 \frac{1}{1 - \frac{1}{\sigma}} \right\} \\
\text{s.t.} & \quad p_1 x_{i,1} + p_2 x_{i,2} + x_{i,3} = 1/n
\end{align*}
\]
Using the definition of the consumer price index, which is
\[ P_3 = \left[ \alpha_3 \beta_3 p_1^{1-\sigma} + \alpha_3 (1 - \beta_3) p_2^{1-\sigma} + (1 - \alpha_3) \right]^{\frac{1}{1-\sigma}} \]
we can deduce from the previous program the vector of optimal demands:
\[ x_i(p) = \left( \frac{1}{n} \right) \left( \alpha_3 \beta_3 \left( \frac{p_1}{P_3} \right)^{-\sigma}; \alpha_3 (1 - \beta_3) \left( \frac{p_2}{P_3} \right)^{-\sigma}; (1 - \alpha_3) \left( \frac{1}{P_3} \right)^{-\sigma} \right) \]

As in the Dixit-Stiglitz model, the imperfect substitutability between goods can lead to a positive markup in a framework where firms set their prices. Thus, with this demand function, countries 1 and 2 can manipulate their taxes in order to mimic the behavior of a monopolist and thus extract a rent from private agents of countries \( i \geq 3 \). The net demand functions are:
\[ x_i^*(p) = \left( \alpha_3 \beta_3 \left( \frac{p_1}{P_3} \right)^{-\sigma} \left( \frac{1}{n} \right); \alpha_3 (1 - \beta_3) \left( \frac{p_2}{P_3} \right)^{-\sigma} \left( \frac{1}{n} \right); -p_1 x_{i,1}(p) - p_2 x_{i,2}(p) \right) \]

In these countries, there are no taxes.

### 2.3 General equilibrium of a pure exchange world

If private agents across countries do not display a strategic behavior, the net demand functions coincide with the competitive net demand function. Thus, the Walras law determines the equilibrium price vector as the solution of \( \sum_i x_i^*(p) = 0 \) where \( x_i^*(p) = x_i(p) - y_i, \forall i \), are the net demand functions. We solve the asymmetrical equilibrium because, in each strategic country, it is necessary to account for the direct and indirect effects of its own tariffs and those of its competitor.

Before solving for \( p \), it is necessary to integrate governments’ budget constraints in the net demand function of each country. Using the demand functions, the governmental budget constraints in countries \( i = 1, 2 \) lead to:
\[
\frac{\tau_j (1 + \tau_j)^{-\sigma} \left[ \alpha_i \beta_i \left( \frac{p_j}{T_i} \right)^{1-\sigma} + \alpha_i (1 - \beta_i) \left( \frac{1}{T_i} \right)^{1-\sigma} \right]}{1 - \tau_j (1 + \tau_j)^{-\sigma} \left[ \alpha_i \beta_i \left( \frac{p_j}{T_i} \right)^{1-\sigma} + \alpha_i (1 - \beta_i) \left( \frac{1}{T_i} \right)^{1-\sigma} \right]} p_j \frac{T_i}{P_i} = T_i
\]
where \( j = 1, 2 \) and \( j \neq i \). If we define \( \Psi_i(p_j, P_i) = \alpha_i \beta_i \left( \frac{p_j}{P_i} \right)^{1-\sigma} + \alpha_i (1 - \sigma)}
β_i) \left( \frac{1}{P_i} \right)^{1-\sigma}, \text{ the demand functions of country } i \text{ become:}

\begin{align*}
x_{i,i} & = (1 - \alpha_i) \left( \frac{p_i}{P_i} \right)^{-\sigma} \frac{1}{1 - \tau_i^j(1 + \tau_i^j)^{-\sigma}\Psi_i(p_j, P_i) P_i} \\
x_{i,j} & = \alpha_i \beta_i \left( \frac{(1 + \tau_i^j)p_j}{P_i} \right)^{-\sigma} \frac{1}{1 - \tau_i^j(1 + \tau_i^j)^{-\sigma}\Psi_i(p_j, P_i) P_i} \\
x_{i,3} & = \alpha_i(1 - \beta_i) \left( \frac{1 + \tau_i^j}{P_i} \right)^{-\sigma} \frac{1}{1 - \tau_i^j(1 + \tau_i^j)^{-\sigma}\Psi_i(p_j, P_i) P_i}
\end{align*}

Because the countries 1 & 2 have symmetrical preferences, the demand functions of the country \( j \) are identical.\(^\text{14}\) These net demands show that taxes have different effects, for a given value of the price vector \( p \):

- As regards the share of each good in the consumption basket (the first two terms of the demand functions), import taxes raise the relative prices of foreign goods and thus favor domestic goods.

- As regards their redistributive impact (the third term of the demand function), import taxes increase transfers and thus the consumer wealth, but this effect is attenuated by the size of the tax revenues that decline with the tax.

- As regards the purchasing power of the endowment of the consumer (the last term of the demand functions), import taxes, by raising the price of the consumer basket, reduce the agents’ wealth.

Given the definition of the consumer price indices \( P_1 \) and \( P_2 \), the equi-

\(^{14}\text{In country 2, the government’s budget constraint leads to}

\begin{align*}
\tau_i^2 \left[ \rho_2 \beta_2 \left( \frac{p(1 + \tau_i^j)}{P_i} \right)^{1-\sigma} \left( \frac{p + T_2}{p(1 + \tau_i^j)} \right) + \alpha_2(1 - \beta_2) \left( \frac{1 + \tau_i^j}{P_i} \right)^{1-\sigma} \left( \frac{p + T_2}{1 + \tau_i^j} \right) \right] & = P_2 T_2
\end{align*}
librium price of good 1 is then the solution of the following equation:

\[
\sum_{i=1}^{n+2} x_{i,1}(p) = 1
\]

\[
(1 - \alpha_1) \left( \frac{p_1}{P_1} \right)^{-\sigma} \left( \frac{1}{1 - \tau_1 \Gamma(1 + \tau_1)} - \sigma \Psi_1(p_2, P_1) \right) \frac{p_1}{P_1} + \alpha_2 \beta_2 \left( \frac{(1 + \tau_2^2)p_1}{P_2} \right)^{-\sigma} \left( \frac{1}{1 - \tau_2 \Gamma(1 + \tau_2)} - \sigma \Psi_2(p_1, P_2) \right) \frac{p_2}{P_2} + \sum_{i=3}^{n+2} \alpha_3 \beta_3 \left( \frac{p_i}{P_i} \right)^{-\sigma} \frac{1/n}{P_i} = 1
\]

In the same way, we get the equilibrium price of good 2: \( \sum_{i=1}^{n+2} x_{i,2}(p) = 1 \).

The last market, where good 3 is traded, is at equilibrium via the Walras law.

2.4 The optimal taxation in a non-cooperative world

In order to determine the optimal taxation in each country, we obtain the solution of the Ramsey problem where governments maximize private agents’ welfare subject to the following constraints: (i) the optimal demand function of households (participation condition), (ii) the price vector (market equilibria) and (iii) the government’s budget constraint. If consumer price indices are denoted by \( P_i = \mathcal{P}(p_1, p_2, \tau_i) \) in country \( i \) and the market equilibrium of good \( j \) by \( 0 = \mathcal{E}_j(p_1, p_2, P_1, P_2, \tau_j^1) \), the Ramsey problem, after integrating the optimal demand function in the planner’s objective, is

\[
\max_{\tau_j} \left\{ \frac{\sigma}{\sigma - 1} \left( \frac{p_1}{P_1} - \frac{1}{1 - \tau_j \Gamma(1 + \tau_j)} - \sigma \Psi_1(p_2, P_1) \right) \right\}^{\frac{\sigma - 1}{\sigma}}
\]

s.c. \( P_1 = \mathcal{P}_1(p_1, p_2, \tau_1^1) \)

\( P_2 = \mathcal{P}_2(p_1, p_2, \tau_2^1) \)

\( 0 = \mathcal{E}_1(p_1, p_2, P_1, P_2, \tau_1^1) \)

\( 0 = \mathcal{E}_2(p_1, p_2, P_1, P_2, \tau_2^1) \)

This problem has no analytical solution. We thus propose illustrating its properties numerically.
2.5 Numerical illustrations

In this section, we propose illustrating our theoretical results using numerical examples. In order to provide an assessment in consumption units of the welfare gains/losses induced by the policies, we compute the following indicator

\[
U_1(\tau^1) = \tau^2 = 0) = (1 - \alpha_1)^{\frac{1}{1 - \beta}} \frac{[x_{1,1}(1 + \Delta_1)]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + (\alpha_1 \beta_1)^{\frac{1}{2}} \frac{x_{1,2}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + (\alpha_1(1 - \beta_1))^\frac{1}{2} \frac{x_{1,3}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}
\]

\[
U_1(\tau_1^*, \tau_2^*) = (1 - \alpha_1)^{\frac{1}{1 - \beta}} \frac{[x_{1,1}^*]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + (\alpha_1 \beta_1)^{\frac{1}{2}} \frac{[x_{1,2}^*]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + (\alpha_1(1 - \beta_1))^\frac{1}{2} \frac{[x_{1,3}^*]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}
\]

\[
\Rightarrow \Delta_1 = \left[ \frac{\sigma - 1}{\sigma} U_1(\tau^1 = \tau^2 = 0) - U_1(\tau_1^*, \tau_2^*) \right]^{\frac{\sigma - 1}{\sigma - 1}} - 1
\]

Thus, \( \Delta_i \) is the reduction, in percentage, of the domestic goods consumption in an economy without taxes, which leads to the same utility level obtained in an economy with taxes \( (U_i) \). When \( \Delta_i \) is negative, this indicates that welfare is larger in an economy with the optimal taxation, implying a subsidy when there is no tax. The sign of the welfare gap, i.e. the difference between the welfare of a non-competitive country with taxes and that of a non-competitive country without taxes, should depend on the preference parameters of non-competitive countries \( \alpha \) and \( \beta \). We thus scan their range \( \alpha \in [0, 0.5] \) and \( \beta \in [0, 1] \), for a given elasticity of substitution between goods \( (\sigma) \) and for a given degree of openness of the competitive countries \( (\alpha_3) \). We choose \( \sigma = 4 \) and \( \alpha_3 = 0.4 \). The values for the parameter \( \sigma \) is consistent with an elasticity of substitution between 1.5 and 5 reported in the literature: an elasticity of substitution equal to 1.5 is consistent with estimates from macroeconomic data (see for example Backus et al. 1994), whereas Imbs and Mejean (2009) find a value of around 5 by taking into account sectoral heterogeneity. Given the steady-state of the price index \( P_3 \), choosing a value of 0.4 for \( \alpha_3 \) implies a share of foreign goods in real revenue of around 20\%, which is consistent with the import penetration (the ratio of imports over total expenditures) observed for China on average over the last 20 years.

For given values for \( \alpha_3 \) and \( \sigma \), the left panel of Figure 1 shows that there exists a set of couples \( \{\alpha, \beta\} \) that leads to \( \Delta < 0 \): the non-cooperative tax
equilibrium can be welfare-improving for strategic countries. However, it is not always the case. The combination of a high value for $\alpha$ and a low value for $\beta$ gives a large weight to the rise in the purchasing power of domestic households induced by the general equilibrium price effect.

Figure 1: Welfare gains/losses - The impact of $\alpha$, $\beta$

When parameter $\beta$ rises, the volume of trade between countries 1 and 2 becomes more substantial and that with competitive countries decreases. Thus, when $\beta$ increases, the negative impact of the non-cooperative tax game between countries 1 and 2 becomes dominant. Without a home bias ($\alpha$ close to 0.5) and with international trade taking place predominantly within the area where countries are strategic players, it seems that protectionism is not an optimal policy.

Nevertheless, if - as it is observed in the largest European countries, like Germany and France - the share of imported goods represents approximately 25% (which corresponds to $\alpha$) and between 40% and 50% (approximately
39% for Germany and 47% for France) of these imports comes from countries outside the euro area ($\beta = 1/2$ is a maximum), then the gains from protectionism are slightly positive\(^{15}\). The right panel of Figure 1 gives the optimal tax rate. This optimal tax decreases with the share of trade conducted with the other strategic country, while a high degree of openness leads the Ramsey planner to set high taxes.

Figure 2: Welfare gains/losses - The impact of $\sigma$, $\alpha_3$

\[ \alpha = 0.25, \beta = 0.25. \]

Is this prediction sensitive to private agents’ preferences in the competitive countries? Figure 2 provides an answer to this question by illustrating the impact of $\alpha_3$ on welfare gains/losses, for given values\(^{16}\) of

\(^{15}\)It should be noted that if we consider foreign trade with European countries, the estimated value for $\beta$ would be close to $2/3$ for France and 0.57 for Germany. We restrict the set of strategic countries to the euro area because these countries share the same currency, and can thus only have fiscal strategies.

\(^{16}\)We choose these values because they are close to the observed ones and they represent the thresholds for the efficiency of protectionism in our framework.
\{\alpha, \beta\} = \{0.25; 0.4\} and for a set of values for \sigma. Parameter \alpha_3 corresponds to the degree of openness of competitive countries. The larger this parameter, the higher the demand addressed by these countries to strategic countries. Thus, unsurprisingly, the right panel of Figure 2 shows that the tax increase with \alpha_3. The left panel of Figure 2 shows that the welfare gains of strategic countries rises when the competitive countries are more dependent (\alpha_3 large).

How are these results affected by the elasticity of substitution between goods (\sigma)? When \sigma increases, the elasticity of substitution rises, and the monopoly power of strategic countries is thus reduced. The left panel of Figure 2 shows that this leads the welfare gains of the strategic countries to decrease. Indeed, a large value of \sigma allows competitive countries to mitigate the impact of protectionist taxation chosen by strategic countries.

3 International trade, economies with production and optimal taxation

This section extends our basic framework to economies with production. This enables us to discuss the interaction between the terms-of-trade externality analyzed in Section 2, and under-employment due to imperfect competition on the labor market. Indeed, European countries are characterized by a sub-optimal employment rate due to imperfect competition in the labor market: the level of production is lower than that of a purely competitive free exchange economy. Thus, labor market inefficiencies call for employment subsidies that increase output, whereas the terms-of-trade externality calls for tariffs that reduce output. There is a conflict between these two gaps.\(^{17}\)

In a second best allocation, where the planner has only one instrument, a consumption tax used to finance an employment subsidy\(^{18}\), we show that (i)

---

\(^{17}\)On the one hand, the planner wants domestic goods to be rarer and therefore to decrease production. On the other, he wants to reduce inefficiencies in the labor market and thus to increase supply.

\(^{18}\)In the tariffs case, we assume, as in the previous section, that there is only one tax rate for all imported goods and that the employment subsidy is determined by the government surplus generated by the revenues from these tariffs. In the VAT case, we also assume that there is one tax rate and that fiscal revenues are used to finance an employment subsidy. Thus, in both cases, there is only one free tool at the government’s disposal.
if the consumption tax is a tariff on imported goods, then taxes must be used to exploit the terms-of-trade externality, whereas (ii) if the consumption tax is a VAT, then taxes must be used to reduce labor market inefficiencies. Indeed, the terms-of-trade externality calls for the use of tariffs, which have the advantage of distorting relative prices, whereas the "tax base" argument is in favor of a reduction of the tax wedge in order to reduce the labor wedge.

The set of assumptions is exactly the same as in the previous section except that an economy with production and an imperfectly competitive labor market is now considered:

- \( a_i \) denotes each country’s technological ability to produce the 3 goods: 
  
  for the sake of simplicity, it is assumed that \( a_1 = (A_1, 0, 0) \), \( a_2 = (0, A_2, 0) \) and \( a_i = (0, 0, A) \). Thus, the production functions are \( y_1 = A_1 h_1^\eta \), \( y_2 = A_2 h_2^\eta \) and \( y_i = A h_i \), with \( A = 1/n \) for \( i \geq 3 \).

- The labor used to produce each good is a CES aggregate: 

  \[
  h_i = \left( \int_0^1 \frac{\nu_i^{-1}}{h_{i,k}} \, dk \right)^{\frac{\nu_i}{\nu_i - 1}} \quad \text{with} \quad \nu_i > 1, \quad \text{the elasticity between two skills}
  \]

  The labor demand for each skill is 

  \[
  h_{i,k} = \left( \frac{w_{i,k}}{W_i} \right)^{-\nu_i} h_i \quad \forall k \quad \text{where} \quad W_i = \left( \int_0^1 w_{i,k}^{-\nu_i} \, dk \right)^{\frac{1}{1-\nu_i}}
  \]

  We allow \( \nu_i \) to have a country-specific value in order to account for the asymmetries in the labor market institutions.

- There is perfect competition in the goods markets. Corporate profits are \( \Pi_i = p_i y_i - \eta p_i A_i h_i^{\eta-1} h_i = (1 - \eta) p_i y_i \), where \( \tau_f \) denotes the payroll tax. The first order condition (FOC) of the firm’s program leads to 

  \[
  \eta p_i A_i h_i^{\eta-1} = (1 + \tau_f^i) W_i, \quad \text{and hence} \quad \eta p_i A_i h_i^\eta = (1 + \tau_f^i) W_i h_i.
  \]

  Given that for \( i > 2 \), we have \( \eta = 1, \, \Pi_i = 0 \) for \( i > 2 \). Moreover, for \( i > 2 \), we assume that labor supply is inelastic and normalized to unity, leading to \( W_i = A = 1/n \).

- Finally, the tax reform consists in introducing a tax on imported goods \( \tau_I \) that allows the government to finance employment subsidies \( \tau_f \). The
government’s budget constraint is thus, for \( i = 1, 2 \) and \( j \neq i \),

\[
\tau_j^t (p_j x_{i,j} + x_{i,0}) + \tau_j^f W_i h_i = 0 \quad \text{if tariffs}
\]

\[
\tau_j^c P_i C_i + \tau_j^f W_i h_i = 0 \quad \text{if VAT}
\]

implying that \( \tau_j < 0 \). This budget constraint implies that the government has only one tax instrument, the other tax rate being set in order to balance government accounts. This set of taxes restricts our analysis to a second best allocation.

- The price indices are:

\[
P_i = \left[ \left( 1 - \alpha_i \right) p_{i}^{1-\sigma} + \alpha_i \beta_i \left( p_j (1 + \tau_j^f) \right)^{1-\sigma} + \alpha_i (1 - \beta_i) \left( 1 + \tau_j^c \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

If tariffs

\[
P_i = \left[ \left( 1 - \alpha_i \right) p_{i}^{1-\sigma} + \alpha_i \beta_i p_j^{1-\sigma} + \alpha_i (1 - \beta_i) \right]^{\frac{1}{1-\sigma}}
\]

If VAT

Given that labor supply is assumed to be inelastic in countries \( i \geq 3 \), the solutions of household behavior are the same as in the previous section.

### 3.1 Household behavior in countries 1 and 2

In each country (1 or 2), households supply a specific skill \( k \) and sets its wage \( w_{i,k} \). Their optimal decisions are the solution to the following program:

\[
\max \left\{ \log(C_i) - \kappa \frac{h_{i,k}^{1+\rho}}{1+\rho} \right\}
\]

s.t. \( (1 + \tau_j^t) (x_{i,3} + p_j x_{i,j}) + p_i x_{i,i} = w_{i,k} h_{i,k} + \Pi_i \) if tariffs

\( (1 + \tau_j^c) (x_{i,3} + p_j x_{i,j} + p_i x_{i,i}) = w_{i,k} h_{i,k} + \Pi_i \) if VAT

\[
C_i = \left( (1 - \alpha_i) \frac{x_{i,k}^{\frac{\sigma-1}{\sigma}}} + (\alpha_i \beta_i) \frac{x_{i,j}^{\frac{\sigma-1}{\sigma}}} + (\alpha_i (1 - \beta_i)) \frac{x_{i,3}^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}}
\]

\[
h_{i,k} = \left( \frac{w_{i,k}}{W_i} \right)^{-\nu} h_i
\]

It should be noted that in the VAT case, the budget constraint is simply \((1 + \tau_j^c) P_i C_i = w_{i,k} h_{i,k} + \Pi_i \). The FOC of the minimization of the consumption
basket cost leads to:

\[
\begin{align*}
x_{i,i} &= (1 - \alpha_i) \left( \frac{p_i}{P_i} \right)^{-\sigma} C_i \\
x_{i,j} &= \alpha_i \beta_i \left( \frac{1 + \tau_i}{1 + \tau_j} \right)^{-\sigma} C_i \\
x_{i,3} &= \alpha_i (1 - \beta_i) \left( \frac{1 + \tau_i}{1 + \tau_3} \right)^{-\sigma} C_i
\end{align*}
\]

A first trivial property may be deduced from these demand systems: VAT, contrary to tariffs, is neutral in terms of its impact on the composition of the goods basket. We derive therefrom that it is not possible for VAT to reach an allocation where the government can "efficiently" use its market power. Conversely, given that tariffs distort the relative price between domestic and foreign goods, this tax can change the composition of the consumer’s goods basket and thus generates a market power for the home country via the terms-of-trade externality.

At the symmetric equilibrium in the labor market of country \(i\), the wage setting rule leads to

\[
\frac{W_i}{\tau_i'} = \frac{\nu_i}{\nu_i - \kappa h'_i} \rho_i C_i
\]

Workers apply a markup \(\frac{\nu_i}{\nu_i - \kappa h'_i}\) over their marginal rate of substitution \(MRS = \kappa h'_i C_i\) when they set their real wage.

### 3.2 Labor market equilibrium and taxes

In this section, we analyze the labor market equilibrium, given the government’s budget constraint. In order to separate the effect of taxes on the good market and that of employment subsidies, we first present a simplified case where consumption taxes are redistributed through a lump-sum transfer.

#### 3.2.1 The cases without payroll tax subsidies

We start by assuming that there are no payroll tax subsidies, i.e. \(\tau_i' = 0\), and the consumption tax revenues are redistributed via a lump-sum transfer, i.e. \(\tau_i (p_j x_{i,j} + x_{i,0}) = P_i T_i\) if tariffs, and \(\tau_i P_i C_i = P_i T_i\) if VAT. In this case, we
have

\[
\frac{W_i}{P_i} = \frac{\text{Tariiffs}}{\nu_i - 1 - \kappa \theta_i^* C_i} \quad \frac{W_i}{P_i} = \frac{\text{VAT}}{\nu_i - 1 - \kappa \theta_i^* W_i h_i + \Pi_i + P_i T_i} \quad \frac{W_i}{P_i} = \frac{\text{VAT}}{\nu_i - 1 - \kappa \theta_i^* W_i h_i + \Pi_i + P_i T_i}
\]

Given the definition of the price index, we have \( P_i C_i = W_i h_i + \Pi_i + P_i T_i \) in the case of tariffs and \( (1 + \tau_i^c) P_i C_i = W_i h_i + \Pi_i + P_i T_i \) in the case of VAT.

In the tariffs case, the government’s budget constraint and the demand functions lead to \( \tau_i^c (1 + \tau_i^c) - \sigma \Psi_i(p_j, P_i) \frac{\nu_i - 1}{\kappa \eta} = P_i T_i \), implying the following expression of the hours worked

\[
\left( \frac{1}{1 - \tau_i^c (1 + \tau_i^c) - \sigma \Psi_i(p_j, P_i) \frac{\nu_i - 1}{\kappa \eta}} \right)^{\frac{1}{1 + \rho}} = h_i
\]

First, if there are no frictions on the labor market (\( \frac{\nu_i - 1}{\kappa \eta} \rightarrow 1 \)), then \( h_i < h_i^* \), where \( h_i^* \) is the employment level in the free trade world, i.e. \( h_i^* = \left( \frac{\kappa \eta}{\nu_i - 1} \right)^{\frac{1}{1 + \rho}} \). Indeed, we have \( \frac{1}{1 - \tau_i^c (1 + \tau_i^c) - \sigma \Psi_i(p_j, P_i) \frac{\nu_i - 1}{\kappa \eta}} > 1 \). This shows that the introduction of tariffs which enable the home country to generate a rent for itself, reduces the incentives to work (the counterpart of the monopoly power created by the taxes in the international goods market). Second, if there are frictions on the labor market (\( \frac{\nu_i - 1}{\kappa \eta} > 1 \)), tariffs move away the labor market equilibrium from that of a free trade economy without labor market frictions (\( h_i^* \)). Indeed tariffs create a markup on the goods market that is added to the markup on the labor market. Thus, if tariffs generate a surplus for the domestic economy on the goods market, they degrade the allocation on the labor market. For policy makers, there is a cost and a benefit to the introduction of tariffs, unlike the case of an endowment economy. Finally, it should be noted that the impact of a change in tariffs on the labor market equilibrium is ambiguous. First, because the direct impact of \( \tau_i \) is indeterminate, and second because \( \tau_i \) changes the equilibrium prices, and thus has an indirect impact through this channel.

Given that the labor demand leads to \( \Pi_i = (1 - \eta) p_i y_i \) and thus \( \frac{W_i}{P_i} = \frac{1 - \eta}{\eta} h_i \), we obtain in the VAT case:

\[
\left( \frac{1 + \tau_i^c}{\nu_i - 1 - \eta} \right)^{\frac{1}{1 + \rho}} = h_i
\]
This shows that the equilibrium price variations have no impact on the hours worked. Moreover, if there are no labor market frictions \((\nu_i \nu_{i-1} \to 1)\), then \(h_i < h^*_i\) because \(1 + \tau^*_i > 1\): the redistribution of VAT leads to a wealth effect. If policy makers focus only on the labor market, this result suggests that they should set \(\tau^*_i = 0\). As in the case of tariffs, the introduction of labor market frictions \((\nu_i \nu_{i-1} \to 1)\), increases the labor wedge if \(\tau^*_i > 0\).

To conclude the labor market analysis, a general result emerges: if policy makers’ objective is an employment level lower than \(h^\text{mc}_i\) where \(h^\text{mc}_i = \left(\frac{\nu_i}{\nu_{i-1}} \frac{\kappa_1}{\eta}\right) - 1\), then taxes on goods market (tariffs or VAT), redistributed through lump-sum transfers, may be used. Conversely, if policy makers’ objective is an employment level that satisfies \(h_i > h^\text{mc}_i\), this policy is not appropriate. Thus, in those economies where workers have a strong market power, as is the case in Europe, it is unlikely that the objective will be to reduce employment: taxes on the goods market must be accompanied by measures in favor of employment.

### 3.2.2 The case with payroll tax subsidies

A simple employment policy is to use consumption tax revenues to subsidize the labor cost: we then introduce a payroll tax subsidy. Given that labor demand leads to \(\eta p_i A_i h^{\eta - 1}_i = p_i MPH_i = (1 + \tau^*_i) W_i\) where \(MPH\) is the marginal product of a hour worked, the labor market equilibria are

\[
\begin{align*}
\frac{p_i}{P_i} MPH_i &= (1 + \tau^*_i) \frac{\nu_i}{\nu_{i-1}} MRS_i & \text{Tariffs} \\
\frac{p_i}{P_i} MPH_i &= (1 + \tau^*_i)(1 + \tau^*_f) \frac{\nu_i}{\nu_{i-1}} MRS_i & \text{VAT}
\end{align*}
\]

The equilibrium amount of hours worked is given by:\(^{19}\)

\[
h_i = \left(\frac{\nu_i}{\nu_{i-1}} \frac{\kappa_1}{\eta} [1 + (1 - \eta) \tau^*_f] \right)^{-\frac{1}{1+\tau^*_f}}
\]

If tariffs or VAT

---

\(^{19}\)Labor demand also leads to \(\Pi_i = (1 - \eta) p_i y_i\). We deduce \(\frac{\eta_i}{(1 + \tau^*_f)W_i} = \frac{1 - \eta}{\eta} h_i\). Given the definition of the price index, we have \(P_i C_i = W_i h_i + \Pi_i\) in the tariffs case and \((1 + \tau_i) P_i C_i = W_i h_i + \Pi_i\) in the VAT case.
The two tax rates \( \{\tau_I, \tau_f\} \) are linked by the government’s budget constraint, which can be rewritten as follows:\(^{20}\)

\[
1 + (1 - \eta)\tau_f = \mathcal{B}^I(\tau_f) \\
\text{where } \mathcal{B}^I(\tau_f) = \frac{\eta}{\eta + (1-\eta)(1+\tau_f)} - \Psi(p_j, P_i)
\]

\[
1 + (1 - \eta)\tau_f = \mathcal{B}^c(\tau_c) \\
\text{where } \mathcal{B}^c(\tau_c) = \frac{\eta}{\eta + (1-\eta)(1+\tau_c)} - \tau_c
\]

with \( \mathcal{B}^c(\tau_c) \in (0, 1) \), because \( \tau_f < 0 \).\(^{21}\) This first result shows that using consumption tax revenues (from tariffs or VAT) to finance employment subsidies offsets labor market frictions: we have \( \nu_i \nu_i - 1 > 1 \) but \( 1 + (1 - \eta)\tau_f < 1 \), \( \forall \tau_f < 0 \). Thus, if labor market frictions are sufficiently large, consumption tax and employment subsidies may be viewed as complementary policies for improving welfare. Another way of interpreting this result is the following: even if labor market frictions are "small", there exists a "small" consumption tax (tariffs or VAT) that enables the policymaker to improve the labor market allocation.

The government’s budget constraint leads to: \( \mathcal{B}^c(\tau_c)' < 0 \) whereas \( \mathcal{B}^I(\tau_f)' \gtrless 0 \). Integrating these results in the labor market equilibrium, we deduce the number of hours worked as a function of \( \tau_z \), for \( z = I, c \):  

\[
\mathcal{H}_z(\tau_z) = \left( \frac{\nu_i - \kappa \mathcal{B}^z(\tau_z))}{\nu_i - 1} \right)^{-\frac{1}{1+\eta}}
\]

with \( h_{mc} < \mathcal{H}_z(\tau_z) \leq h^*_c \) for large values of \( \frac{\mu_i}{\nu_i - 1} \). Without labor market frictions \( (\frac{\mu_i}{\nu_i - 1} \rightarrow 1) \), a labor wedge equal to zero \( (\mathcal{H}_z_{\nu_i \rightarrow \infty}(\tau_z) = h^*_c) \) calls for \( \tau_f = 0 \) and thus \( \tau_f = \tau_c = 0 \). This shows that rent creation on the goods market necessarily has a welfare cost due to over-work when there is no labor market frictions. Conversely, if the structural labor wedge is sufficiently large \( (\frac{\mu_i}{\nu_i - 1} >> 1) \), there could exist a complementarity between consumption taxes and employment subsidies. Is there a difference between tariffs and VAT at this level of the analysis?

Intuitively, these two policies act directly on the relative price though their effects on the marginal cost via the employment subsidy, but have different effects on the consumption choices: tariffs act directly on the sharing

\(^{20}\)This constraint shows that the government has only one free tax tool.

\(^{21}\)The equilibrium on the labor market is defined if and only if, \( 1 + (1 - \eta)\tau_f > 0 \Leftrightarrow \frac{1}{\nu_i - 1} > -\tau_f \).
rule between domestic and foreign goods (the composition and the level of the goods basket), while VAT only acts on the level of the aggregate consumption basket.

If we focus only on this employment margin, then tariffs associated with employment subsidies can reduce the labor wedge. However, the impact of this policy depends on the substitution between goods, because tariffs are targeted at foreign goods. Thus, if the other strategic country can raise its price using symmetrical tariffs, some of the effect of the domestic tariffs is crowded out by the strategic response of the other non-cooperative country. This is why the sign of the derivative of \( B^I(\tau^I_i) \) is undetermined. We can therefore conclude that when the weight of the response of the other strategic country is low in the optimal choice of domestic consumers, then \( B^I(\tau^I_i)' < 0 \). The policymaker is then incited to increase tariffs to reduce the labor wedge. Conversely, if the response of the other strategic country has a large weight in the consumption choices of the domestic agent, one can have \( B^I(\tau^I_i)' > 0 \). In this case, the policymaker is hampered in his rent-seeking on the goods market by the increase in the labor wedge that this policy induces.

If we now analyze the impact of VAT, accompanied by employment subsidies on the labor market, the conclusions are different because this impact does not depend on the response of relative prices. Consumption taxes can thus always improve the allocation in the labor market if they are used to subsidize employment. This result is just the product of a "tax base" effect in favor of consumption (it disappears in our model if \( \eta = 1 \), implying that \( dh_i/d\tau^I_f = 0 \)).

3.3 General Equilibrium

The general equilibrium combines all the previously discussed effects: (i) for a given net income, the impact of taxes on the equilibrium quantities traded in goods markets (these effects are the same as in an endowment economy), (ii) the effect of a change in taxes on income, and (iii) the income changes linked to labor market adjustments.

Given the labor market equilibrium \( \mathcal{H}^z(\tau^z_i) \), we deduce \( y_i \equiv \mathcal{Y}(\tau^z_i) = A_i h^I_i \equiv A_i(\mathcal{H}^z(\tau^z_i))^\eta \), as a function in \( \tau^z_i \), for \( z = I, c \). At equilibrium, given
For a given level of income and the increase in the goods basket and disappears in the demand functions.

VAT case, this rise in household income is equal to the rise in the price of goods. Moreover, these demand functions show that the two basic effects of the tax changes the relative prices in favor of domestic goods, and (ii) it decreases the purchasing power of consumers via the increase in $P_i$ in the tariffs case, or in the "after tax" price $(1 + \tau_i^c)P_i$ in the VAT case.

For a given level of $p_iy_i$ and thus of $W_i$ and $h_i$, these expressions show that household incomes $I_i$ increase with consumption taxes (tariffs or VAT), as in an endowment economy or in an economy with production and transfers: the redistribution of the consumption tax (tariffs or VAT) via an employment subsidy raises the firm’s redistributed profits. It should be noted that in the VAT case, this rise in household income is equal to the rise in the price of the goods basket and disappears in the demand functions.

The demand functions are:

\[
\begin{align*}
I_i &= \frac{1}{1-\tau_i^j(1+\tau_i^c)}\Psi(p_j,P_i) \Rightarrow I_i = p_iy_i(1 + \tau_i^c) \\
I_i &= \frac{1}{1-\tau_i^j(1+\tau_i^c)}\Psi(p_j,P_i)
\end{align*}
\]

For a given level of $p_iy_i$ and thus of $W_i$ and $h_i$, these expressions show that household incomes $I_i$ increase with consumption taxes (tariffs or VAT), as in an endowment economy or in an economy with production and transfers: the redistribution of the consumption tax (tariffs or VAT) via an employment subsidy raises the firm’s redistributed profits. It should be noted that in the VAT case, this rise in household income is equal to the rise in the price of the goods basket and disappears in the demand functions.

Moreover, these demand functions show that the two basic effects of the tax on imported goods remain: (i) the tax changes the relative prices in favor of domestic goods, and (ii) it decreases the purchasing power of consumers via the increase in $P_i$ in the tariffs case, or in the "after tax" price $(1 + \tau_i^c)P_i$ in the VAT case.

We also get an additional effect: the impact of subsidies on labor costs, which increases the level of output $\mathcal{Y}(\tau_i^j)$, for $z = I, c$. This effect on $\mathcal{Y}(\tau_i^j)$ depends on the function $\mathcal{B}(\tau_i^j)$, for $z = I, c$, as discussed in the section devoted on the labor market equilibrium analysis.

Finally, we have the general equilibrium impact linked to price adjustments. For $i > 2$, the equilibrium price is the solution of $\sum_{i=1}^{n+2} x_{i,j}(p) = 1 \Leftrightarrow E_i^c(p_1, p_2, P_1, P_2, \tau_i^c, \tau_j^c) = 0$, for $z = I, c$, whereas for $i, j = 1, 2$ and $j \neq i$,
we have:\footnote{Given this price vector \((p_1, p_2)\), the last market, where the good 3 is traded, is at the equilibrium via the Walras law.}

If tariffs
\[
\mathcal{Y}(\tau^I) = (1 - \alpha_i) \left( \frac{p_i}{P_i} \right)^{-\sigma} \frac{1}{1 - \tau^I_i(1 + \tau^I_j)^{-\sigma} \Psi(p_j, P_j)} \frac{p_i \mathcal{Y}(\tau^I_j)}{P_i} \\
+ (\alpha_j \beta_j) \left( \frac{p_i(1 + \tau^I_j)}{P_j} \right)^{-\sigma} \frac{1}{1 - \tau^I_i(1 + \tau^I_j)^{-\sigma} \Psi(p_i, P_i)} \frac{p_j \mathcal{Y}(\tau^I_j)}{P_j} \\
+ (\alpha_3 \beta_3) \left( \frac{p_i}{P_3} \right)^{-\sigma} \frac{1}{P_3}
\]

\[\iff 0 = \mathcal{E}^I_i(p_1, p_2, P_1, P_2, \tau^I_1, \tau^I_2)\]

If VAT
\[
\mathcal{Y}(\tau^I_i) = (1 - \alpha_i) \left( \frac{p_i}{P_i} \right)^{-\sigma} \frac{p_i \mathcal{Y}(\tau^I_j)}{P_i} \\
+ (\alpha_j \beta_j) \left( \frac{p_i}{P_j} \right)^{-\sigma} \frac{p_j \mathcal{Y}(\tau^I_j)}{P_j} + (\alpha_3 \beta_3) \left( \frac{p_i}{P_3} \right)^{-\sigma} \frac{1}{P_3}
\]

\[\iff 0 = \mathcal{E}^I_i(p_1, p_2, P_1, P_2, \tau^I_1, \tau^I_2)\]

In the case of tariffs, there are two possibilities depending on the weight of the opposing strategic country: either its weight is small and therefore \(B^I(\tau^I_i)' < 0\), or the weight is sufficiently large to ensure \(B^I(\tau^I_i)' > 0\). If the weight of the opposing strategic country is sufficiently small, then tariffs, accompanied by employment subsidies, lead to an increase in supply. This additional production is redistributed as income (wages and dividends), and hence increases demand. The shift in demand is smaller than that of supply because domestic goods are only a share of the consumer basket. This limits the decline in the equilibrium price but increases the quantity traded in this market. As we initially assumed the weight of the other strategic country to be low, the rise in its demand can be considered negligible in this qualitative analysis of market adjustments: the potential positive spillovers between countries are thus negligible. Even if they are not null, the adjustments of the foreign country go in the same direction as those of the home country.

Conversely, when the weight of the opposing strategic country is sufficiently large, tariffs, accompanied by employment subsidies, reduce supply.
This fall in production is accompanied by a reduction in incomes (wages and dividends) and hence by a decrease in demand. As previously, the shift in demand is smaller than that of supply because domestic goods are only a share of the consumer basket. This limits the decline in the equilibrium price but accentuates the decreases in the quantity traded in this market. Given that in this case the weight of the other strategic country is large, the decrease in its demand (the other strategic country acts symmetrically) is not negligible and we conclude that its reaction accentuates the contraction in this market: there are significant negative spillovers between countries.

It appears therefore that the positive impact of tariffs with respect to an endowment economy is amplified when openness is small, while their negative impact is accentuated when openness is large.

In the VAT case, we always have $B^c(\tau^c_i)' < 0$: the policy leads to a permanent positive supply shock. Contrary to the tariffs case, a large openness creates unambiguously positive spillovers. Indeed, as the other strategic country acts symmetrically, there are two positive permanent shocks in the goods market produced by the home country: the tax policy of the home country shifts its supply and demand curves to the right, whereas the tax policy of the foreign country accentuates the shift in the demand curve. Indeed, even if goods are substitutes, income increases in both countries raise the demands of domestic and foreign consumers for all goods. Therefore, even if VAT does not enable policy-makers to fully exploit the terms-of-trade externality, its use for financing payroll tax subsidies generates correlated supply shocks and, hence positive spillovers in both non-cooperative countries.

### 3.4 The Optimal Ramsey Taxation

The previous intuitive discussion can be clarified by the solution to the optimal taxation problem, which determines the optimal trade-off between two externalities (the one related to terms-of-trade and the one related to labor market imperfect competition) for a planner with only one instrument. In order to determine the optimal taxation in each country $i = 1, 2$, we solve the Ramsey problem, after integrating the optimal demand function in the
planner’s objective:

$$\max_{\tau_i^z} \left\{ \eta \log \left( \mathcal{H}^z(\tau_i^z) \right) - \log \left( \mathcal{F}^z(\tau_i^z) \right) + \log \left( \frac{p_i}{P_i} \right) - \kappa \frac{\left( \mathcal{H}^z(\tau_i^z) \right)^{1+\rho}}{1+\rho} \right\}$$

s.c. $P_i = \mathcal{P}^z_i(p_i, p_j, \tau_i^z)$ for $i, j = 1, 2$, and $j \neq i$

$$0 = \mathcal{E}^z_i(p_1, p_2, P_1, P_2, \tau_i^z, \tau_j^z) \text{ for } i = 1, 2$$

$$\mathcal{H}^z(\tau_i^z) = \left( \frac{\nu_i - \kappa}{\nu_i - 1} \right)^{-\frac{1}{1+\rho}}$$

$$\mathcal{B}^z(\tau_i^z) = \begin{cases} \eta \left( \frac{\eta + \alpha(1-\eta)\tau_i^z (1+\tau_i^z)}{\eta (1+\tau_i^z)} \right)^{-\sigma} \Psi(p_j, P_i) & \text{if } z = I \\ \eta \left( \frac{\eta + \alpha(1-\eta)\tau_i^z (1+\tau_i^z)}{\eta (1+\tau_i^z)} \right)^{-\sigma} \Psi(p_j, P_i) & \text{if } z = c \end{cases}$$

$$\mathcal{F}^z(\tau_i^z) = \begin{cases} 1 - \tau_i^z (1 + \tau_i^z)^{-\sigma} \Psi(p_j, P_i) & \text{if } z = I \\ 1 & \text{if } z = c \end{cases}$$

The employment subsidy $\tau_i^f$ is then deduced from the government’s budget constraint.

### 3.5 Numerical illustrations

We consider double asymmetries between "strategic" and "competitive" countries: "strategic" countries can use taxation as a policy instrument, but they have the competitive disadvantage of having structural distortions on their labor markets. This leads them to use tax revenues as labor subsidies.

In this section, we illustrate the differences between tariffs and VAT, used to finance payroll tax subsidies. The main results are the following. First, in the case of tariffs, the non-competitiveness of labor markets does not change the main results of Section 2: tariffs may distort relative prices in favor of the strategic countries, even if part of these gains is lost via the Nash equilibrium or turned into losses when the weight of the trade between strategic countries is sufficiently large. Second, if policy makers use VAT, they always improve the welfare of private agents living in strategic countries: fiscal reforms in strategic countries generate positive spillovers for their trade partners. Even if this policy is less efficient than the introduction of tariffs, it prevents the economy from recording some welfare losses, and the positive spillovers partially offset this low efficiency.

For the numerical illustrations, we use the same parameters for preferences as in the section on pure exchange economies. For the labor market
of strategic countries, we set a markup of 25%, i.e. a substitution elasticity \( \nu = 5 \). This markup value represents half of the labor wedge estimation of Gali et al. (2007). This is consistent with the view that the other components of the labor wedge observed in the data come from other distortions, such as the tax wedge. For the elasticity of output with respect to labor \( \eta \), we choose the average share of wages, which is around 0.6 in Europe. We set the weight of leisure \( \kappa \) such that \( h(\tau = 0) = 0.33 \). Finally, in order to compare the results of the economy with production with those of a pure exchange economy, the scale parameter of the production function \( A_i \) is calibrated in order to have an equilibrium output equal to the one in the “laissez faire” economy: the scale of the economy with production is the same as that of a pure exchange economy where the endowment of goods is normalized to unity.

3.5.1 When tariff revenues are used to subsidize employment

As in the case of the endowment economy, we first analyze the impact of the openness of the strategic countries (parameters \( \alpha \) and \( \beta \)), and then the substitutability between goods (\( \sigma \)), the openness of the competitive economy (\( \alpha_3 \)) and finally the degree of labor market rigidity (\( \nu \)).

The results reported in Figure 3 must be compared to those obtained for an endowment economy (Figure 1). First, this comparison shows that the economy with production shares exactly the same properties as the endowment economy: protectionism is more efficient when the share of trade with competitive economies increases, due to the low weight of negative interactions between strategic countries. This also shows that positive spillovers between strategic countries are negligible. However, the most interesting result is the following: for all values of \( \{ \alpha, \beta \} \), the gains for the economy with production are larger than those for the endowment economy. This shows that, beyond the rent extraction related to the terms-of-trade externality, tariffs on imported goods generate revenues used to subsidize employment. This additional gain can be small because the policymaker focuses on his external objective (for which his instrument is the most efficient), the reduction of the negative impact of the labor market externality being a "by-product"
of the trade policy.

These conclusions are robust to a sensitivity analysis with respect to the parameters \( \{\alpha_3, \sigma\} \), as shown in Figure 4 (to be compared with Figure 2). When non-strategic countries show a large degree of openness (\( \alpha_3 \) large), it is easier for strategic countries to extract a surplus from international trade. However, when the elasticity of substitution between goods increases, this surplus extraction is limited and this is also the case for welfare gains. When goods are highly substitutable, consumers can avoid paying tariffs: it is easier for them to change their goods basket. The monopoly power of each strategic country is therefore reduced.

Finally, Figure 5 shows that a lower markup on the labor market leads to lower welfare gains. This confirms the previous results: because tariffs can efficiently exploit the terms-of-trade externality, the employment subsidy is only a by-product of this optimal tax policy. In this second best allocation,
Figure 4: Welfare gains/losses - The impact of $\sigma$, $\alpha_3$

\[ \alpha_3 = 0.4, \sigma = 4, \nu = 5 \& \eta = 0.6. \]

tariffs are first and foremost used to distort relative prices, leading the planner to closely mimic the behavior of the planner in an endowment economy.

### 3.5.2 When VAT revenues are used to subsidize employment

First, Figure 6 shows that VAT always improves the welfare of private agents living in strategic countries. This contrasts with the results obtained in the case of tariffs. It appears that the advantage of VAT stemming from the "tax base" effect enables policy makers to significantly reduce the labor wedge, and hence improve welfare. This positive supply shock hits simultaneously the domestic and foreign strategic countries. Contrary to tariffs (when they have a positive impact), the size of this supply shock is not reduced by the openness of these economies. In the VAT case, openness becomes an advantage. If openness is large ($\alpha$ close to 0.5) and if trade is concentrated among strategic countries ($\beta$ close to 1), the policy reaches its largest positive effect.
Figure 5: Welfare gains/losses - The impact of $\sigma$, $\nu$

\[
\alpha = 0.25, \beta = 0.5, \alpha_3 = 0.4 \& \eta = 0.6.
\]

on welfare. The fiscal reform of each country generates positive spillovers in other economies: the reform increases domestic supply, but also the domestic and foreign demand functions. Because the foreign strategic country also implements a symmetrical policy, the improvement in supply in both countries increases the wealth of both populations, and thus demands for domestic and imported goods in all countries. Thus, fiscal reforms in both strategic countries mutually reinforce their long run gains. Conversely, if trade is concentrated among non-strategic countries ($\beta$ close to 0), the policy reaches its lowest positive effect on welfare. These results give a measure of the positive impact of interactions between strategic countries.

As shown in Figure 6, if the value of $\alpha$ is close to zero, we have a benchmark assessment of the transfer from direct to indirect taxation: this shows the impact of this policy in a closed economy. For $\beta$ close to zero (no interaction with the other strategic country), a large share of trade with non-
strategic countries reduces the positive impact of the policy (Figure 6): given that non-strategic countries do not change their supply, part of the wealth increase of domestic consumers is used to absorb the price increases in the market for good 3. Conversely, when $\beta$ is close to one (interactions only targeted towards the other strategic country), domestic consumers take advantage of the two positive supply shocks.

If we compare these results with those obtained in the case of tariffs (comparison between Figures 6 and 3), it appears that VAT avoids incurring welfare losses. However, VAT is less effective than tariffs, for those parameters that lead to an improvement in welfare. Thus, it seems that the best tax strategy depends on the degree of openness, especially as regards strategic countries. VAT should be opted for when trading with strategic countries is large, while tariffs should be chosen when trading with these countries is small.

$\alpha_3 = 0.4, \sigma = 4, \nu = 5 \& \eta = 0.6$. 
Figure 7: Welfare gains/losses - The impact of $\sigma$, $\alpha_3$

$\alpha_3 = 0.4$, $\sigma = 4$, $\nu = 5$ & $\eta = 0.6$.

Figure 7 shows that the impacts of $\alpha_3$ and $\sigma$ are the opposite of those obtained in the case where tax policy uses tariffs. In the VAT case, the greater the openness of the non-strategic countries ($\alpha_3$ close to one), the larger their weight in the equilibrium of each good market, and thus the lower the impact of demand shifts in this market. The openness of non-strategic countries therefore mitigates the positive impact of VAT. In the VAT case, a high elasticity of consumption is synonymous with a low price sensitivity of goods that are in fixed quantities (the goods produced by non-strategic countries). Indeed, in both strategic countries, private agents’ wealth increases after the policy reform, and thus demands for each good rises. When the elasticity of substitution is large, it is easy for consumers to change their goods basket and thus to lessen the negative impact of a price increase in good 3.

Finally, results in Figure 8 show how the impact of VAT depends on the frictions in the domestic economy, but is not highly sensitive to the interna-
Figure 8: Welfare gains/losses - The impact of $\sigma, \nu$

\[\alpha = 0.25, \beta = 0.5, \alpha_3 = 0.4 & \eta = 0.6.\]

tional trade determinants, summarized here by the elasticity of substitution between goods. As discussed in the analytical part, we thus obtain the strong result that, when the markup on the labor market converges to zero, the optimal policy is a tax equal to zero. Indeed, with a uniform tax on all consumption goods, policy makers cannot discriminate between goods in order to extract an additional surplus from trade with non-strategic countries. The tax policy is therefore used to fight the market power of the workers, with the complementarities discussed previously: the tax policy of the other strategic country raises demand addressed to the domestic economy.

4 Conclusion

This paper shows that fiscal devaluation can be an optimal tax policy in a multi-country world, if the labor wedge is sufficiently large. When labor
market distortions imply a low amount of hours worked, as is the case in the large majority of European countries, a shift from direct to indirect taxation generates a positive supply shock, by reducing the tax wedge on labor. If several countries adopt a similar strategy, the impacts of this policy can be magnified. This long run analysis of fiscal devaluation lends new support to this policy. We also show that, if the labor wedge is small, with production being close to its efficient level, then the optimal policy becomes fiscal revaluation, where the efficient instruments are tariffs. In this case, our multi-country framework enables us to show that non-cooperative strategies reduce the efficiency of this type of "aggressive" policy.

Very simple numerical exercises provide an illustration of these theoretical results and show that the deep parameters for which fiscal devaluation is welfare-improving at general equilibrium are highly likely. Thus, we deduce that the actual tax reforms, in Denmark (1987), Germany (2007) and France (2012), which consist in shifting the tax burden from direct taxation of labor (employers’ social contributions) to indirect taxation of consumption or specific imported goods, can be welfare-improving.

The main shortcut of our analysis is to focus on a static problem. Even in the euro area, the position of the trade balance is not the same across countries. Thus, the debt dynamics of foreign trade can change the solution of this game between countries. Future research will be devoted to this analysis.
A Computing the welfare loss

\[ W^{nt} = \log(C_i) - \kappa \frac{h_{i,k}^{1+\rho}}{1+\rho} \]

\[ C_i = \left( (1 - \alpha_i) x_{i,i}^{-\frac{1}{\sigma}} + (\alpha_i \beta_i) x_{i,j}^{-\frac{1}{\sigma}} + (\alpha_i (1 - \beta_i)) x_{i,j}^{-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \]

\[ \tilde{x}_{i,i} = (1 + \Delta) x_{i,i} \]

\[ W^{nt} = \frac{\sigma}{\sigma-1} \log \left( C_i^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_i)^{\frac{1}{\sigma}} (\tilde{x}_{i,i}^{\frac{\sigma-1}{\sigma}} - x_{i,i}^{\frac{\sigma-1}{\sigma}}) \right) - \kappa \frac{h_{i,k}^{1+\rho}}{1+\rho} \]

\[ = W + \frac{\sigma}{\sigma-1} \log \left( 1 + (1 - \alpha_i)^{\frac{1}{\sigma}} C_i^{-\frac{\sigma-1}{\sigma}} \left( \tilde{x}_{i,i}^{\frac{\sigma-1}{\sigma}} - x_{i,i}^{\frac{\sigma-1}{\sigma}} \right) \right) \]

\[ = W + \frac{\sigma}{\sigma-1} \log \left( 1 + (1 - \alpha_i)^{\frac{1}{\sigma}} \left( \frac{x_{i,i}}{C_i} \right)^{\frac{\sigma-1}{\sigma}} \right) \left( (1 + \Delta)^{\frac{\sigma-1}{\sigma}} - 1 \right) \]

\[ (1 + \Delta)^{\frac{\sigma-1}{\sigma}} - 1 = \frac{\exp \left( \frac{\sigma-1}{\sigma} (W^{nt} - W) \right) - 1}{(1 - \alpha_i)^{\frac{1}{\sigma}} \left( \frac{x_{i,i}}{C_i} \right)^{\frac{\sigma-1}{\sigma}}} \]

\[ \Delta = \left( \frac{\exp \left( \frac{\sigma-1}{\sigma} (W^{nt} - W) \right) - 1}{(1 - \alpha_i)^{\frac{1}{\sigma}} \left( \frac{x_{i,i}}{C_i} \right)^{\frac{\sigma-1}{\sigma}}} + 1 \right)^{\frac{\sigma}{\sigma-1}} - 1 \]
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