

Monetary Policy and Stock Prices in an Open Economy *

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Abstract

This paper studies monetary policy in a two-country “perpetual youth” model where agents can invest their wealth in both stock and bond markets. In our economy the foreign country is financially dominant, in the sense that it hosts the only active equity market where all agents (also residents of the home country) can trade stocks of listed foreign firms. We show that, contrary to what happens in a closed economy, the Central Bank in the economy endowed with an active stock market should grant a dedicated response to movements in stock prices driven only by local productivity shocks. Moreover, we show that the other Central Bank should also optimally respond to stock-price movements in the other economy, as they affect the budget constraint of domestic residents. Optimal monetary policy pursuing price stability requires dedicated response to stock-price dynamics driven by productivity shocks in addition to what implied by the simple existence of the terms of trade channel of policy transmission. Determinacy of rational expectations equilibria and approximation of the optimal policy by simple monetary rules are also investigated.

JEL classification: E43, E44, E52, E58

Key words: Monetary Policy, Stock Prices, Taylor Rule, Open-Economy DSGE Models, Wealth Effects.

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1 Introduction

An interesting recent debate in the field of monetary policy has regarded whether or not the setting of short term interest rates by an optimizing central bank should actively consider movements in stock prices. Such a debate developed in the theoretical framework which generalized the rationale for central banks relying on some kind of Taylor rules in their day to day management of monetary policy via open market operations. However, it has also clearly been inspired by the opportunity to evaluate whether the FED and other central banks have correctly managed monetary policy in the period of fast and growing rise in international equity prices (1995-2000) and in the following phase of rapid and intense downturn which ended in 2002.

Bernanke and Gertler (1999 and 2001), within a Dynamic New Keynesian framework based on the working of a financial accelerator, showed that as long as interest rates react aggressively to expected inflation there is no need to stabilize non-fundamental shocks to stock prices: in fact, an explicit reaction to such shocks, would actually produce perverse and highly implausible dynamic outcomes for both output and inflation. However, Cecchetti and others (2000 and 2002), by exploring in more depth the same theoretical framework, replied that this conclusion cannot be viewed as generally true. On the contrary, it is based on the Taylor Rule adopted and on the objective function that the central bank wishes to optimize. In particular, they show that in order to eliminate the perverse outcomes induced by the central bank reaction to stock-price misalignments, it is sufficient to adopt a Taylor Rule where interest rates respond also to the output gap and not only to the distance of current or expected inflation from the target. Moreover, the same authors argue that by choosing the coefficients in the Taylor Rule so as to minimize a linear combination of inflation and output volatility, the results seem to suggest the opportunity of a moderate, though still positive, reaction to stock prices.

This paper contributes to the debate by extending to an open economy framework a stochastic OLG model with real wealth effects originated by changes in stock prices (Nisticò, 2005). In the closed economy version, the optimal response of a Central Bank pursuing price stability in face of a given swing in stock prices, depends on the shocks underlying the observed dynamics. If the driving forces are supply shocks (to technology or the labor supply) then the real effects of stock prices do not require a dedicated response and the optimal interest rate dynamics is the same as the one emerging from the standard Representative-Agent set up. However, if the driving force is a demand shock, like a shock to the marginal utility of consumption, government expenditure or a fad that affects the equity premium, then positive stock-wealth effects require a dedicated response by the Central Bank. In this case, it is possible to show that interest rates should be adjusted more aggressively than in the Representative-Agent set up.

In an open economy model, an additional channel of transmission of monetary policy impulses is represented by international trade, with changes in the terms of trade affect-

ing output demand and potentially requiring central banks to move interest rates also in function of exogenous shocks to the terms of trade.¹

We introduce explicit consideration of the working of the financial system in a two-country “perpetual youth” model rich enough to include both bond and stock markets. We assume that all consumers have access to invest in the world markets, but that only one active stock market exists. In our model, the stock market is located in the foreign country, which is thereby assumed to be larger and “financially dominant”.²

An additional link, of financial nature, is thus established between the two countries: each agent in the home country, in fact, not only can consume a good produced abroad (making the traditional terms of trade channel effective), but can also access the foreign financial market and allocate his saving in equity shares issued by foreign firms, thus exposing his budget constraint to external financial shocks. By introducing stocks in a perpetual youth model, we can obtain wealth effects linked to movements in stock prices, and by having international capital movements we construct a new channel of transmission of impulses from one country to the other.

We model the Central Bank as pursuing price stability by means of controlling a short-run interest rate, and we characterize its (constrained) optimal monetary policy as the dynamics of an interest rate consistent with the flexible price allocation (the “natural” or *Wicksellian* interest rate, in the sense used by Woodford, 2003). We then compare the implied dynamics of the “natural” interest rate with that emerging from two benchmark cases: the open economy Representative-Agent set up and the closed-economy version of the OLG economy. To keep the model simple, and to focus on what drives the original results with respect to the literature, we limit the stochastic structure of the model and we only deal with pure productivity shocks in the two economies.

We show that, in addition to the standard trade channel that makes a country vulnerable to foreign shocks in the open economy Representative-Agent framework, in our model an additional financial link is represented by the effect of productivity shocks on stock prices. More importantly, we show that, contrary to what happens in the closed economy version of the OLG model, when the national stock market is open to foreign investors and the dynamics of financial wealth has real effects on both national and foreign aggregate consumption, both Central Banks should optimally grant a dedicated response to movements in stock prices driven by productivity shocks; this is always true unless there is perfect symmetry regarding the two countries’ vulnerability to stochastic shocks and their persistence. The reason relies in the fact that each national Central Bank seeks to insulate the domestic inflation rate from exogenous shocks, disregarding the effects on the other economy

¹Under certain restrictions on the structural parameters, however, the domestic and the foreign economies can be shown to be insulated from external shocks, so that the closed-economy analysis still applies.

²This assumption is a simplification and introduces an element of asymmetry in the model, but does not affect our results. We only try to capture the idea that, for many countries, what happens in Wall Street may be more relevant for the domestic economy than what goes on in the smaller local domestic exchange (Mexico is an example, but the point could be applied even to larger open economies, such as Canada).

and calibrating its response only to the share of stock-wealth that is allocated in national portfolios. Our results, therefore, confirm the intuition that optimal national interest rates should react to shocks affecting stock prices (also abroad) as these impact (separately from the terms of trade channel) on both domestic and foreign aggregate demand. Our model thereby suggests that in many countries, if national interest rates had been raised enough to counteract the expansive effect generated by the rapid rise in US stock prices, the 2001-2002 economic slowdown that followed the bursting of the bubble in the world financial system could have been reduced.

From a purely theoretical perspective, our analysis provides a general framework which is able to nest, as special cases for limiting values of the parameters, the previous results obtained in the closed economy version of the OLG model and in the RA open economy model. Such results however, are showed to be valid only in these limiting cases, but not in our more general model.

The remaining of the paper is structured as follows. Section 2 presents a two-country model in which the micro-founded stock-price dynamics has real effects on consumption of both domestic and foreign households. Section 3 analyzes Optimal Monetary Policy in both countries, deriving the conditions for price stability and discussing the issues of implementation and equilibrium determinacy. Section 4 analyzes the macroeconomic implications of adopting, in spite of the optimal policy, different simple instrument rules under alternative policy regimes. In particular, we compare the macroeconomic outcomes of pegging the nominal exchange rate with those derived from using standard Taylor rules where the Central Bank responds to output gap and inflation (either domestic or CPI inflation). We also augment such Taylor rules by allowing both central banks to react to stock-price deviations. Section 5 finally summarizes and concludes.

2 A Structural Two-Country Model with Wealth Effects.

In this section we present a two-country OLG model which draws on and generalizes the closed economy analysis developed in Nisticò (2005). Given the complexities of the links of an open economy model, and in order to explicitly focus on what drives the original contribution of the paper, we simplify the stochastic structure of the model by not allowing for preference shocks or “fads”. We also abstract from considering government consumption and taxes.

Following Obstfeld and Rogoff (1996) and Benigno and Benigno (2003 and 2004), we model the world economy as consisting of a continuum of households and firms in the interval $[0, 1]$, divided in two countries H and F , of dimension n and $(1 - n)$ respectively.

Each household, in each country, supplies labor inputs to firms and demands a bundle of consumption goods consisting of both home and foreign goods.

The productive sector produces a continuum of perishable goods, which are differentiated across countries and with respect to one another. To assign a relevant role to monetary

policy³ we introduce nominal rigidities by assuming that both domestic and foreign firms, each period, face an exogenous probability of optimally changing the price of their good (see Calvo, 1983).

2.1 The Demand-Side.

The demand-side of our economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965). Each period a constant-sized cohort of households enters each country, facing a constant probability γ of dying before the next period begins.⁴ To abstract from population growth the cohort size is set equal to γ .

2.1.1 Intratemporal Allocation.

Each generic household belonging to cohort j in each country derives utility from leisure and the following composite bundle of consumption:

$$C_t(j) = \left[n^{\frac{1}{\theta}} C_{H,t}(j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_{F,t}(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (1)$$

in which $\theta > 0$ is the elasticity of substitution between Home and Foreign goods, and $C_{H,t}(j)$ and $C_{F,t}(j)$ result from Dixit-Stiglitz-aggregation of the consumption goods produced in the two countries:

$$C_{H,t}(j) = \left[\left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n C_t(h, j)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}} \quad C_{F,t}(j) = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 C_t(f, j)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}} \quad (2)$$

in which $\epsilon > 1$ is the elasticity of substitution between the differentiated goods in the intervals $[0, n]$ and $[n, 1]$. We assume such elasticity, reflecting the degree of market power, to be the same across countries.

Total expenditure minimization yields the price-indexes for goods produced in countries H and F and sold in country $i = H, F$

$$P_{H,t}^i = \left[\frac{1}{n} \int_0^n P_t^i(h)^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}} \quad P_{F,t}^i = \left[\frac{1}{1-n} \int_n^1 P_t^i(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}, \quad (3)$$

the consumer-price index (CPI) for country i

$$P_t^i = \left[n(P_{H,t}^i)^{1-\theta} + (1-n)(P_{F,t}^i)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4)$$

³One should notice that, although we do not model money holdings for simplicity, we assign to the Central Bank the role of setting the interest rate according to some optimal “rule”. This is the sense by which we talk about monetary policy.

⁴We interpret the concepts of “living” and “dying” in the sense of being or not being operative in the markets and thereby affecting or not economic activity through decision-making processes. In this perspective, the expected life-time $1/\gamma$ is interpreted as the effective decision horizon. See also Leith and Wren-Lewis (2000), Leith and vonThadden (2004), Nisticò (2005) and Piergallini (2004).

and the total demand for goods produced at home and abroad for generation j

$$C_t(h, j) = \left(\frac{P_t^i(h)}{P_{H,t}^i} \right)^{-\epsilon} \left(\frac{P_{H,t}^i}{P_t^i} \right)^{-\theta} C_t(j) \quad C_t(f, j) = \left(\frac{P_t^i(f)}{P_{F,t}^i} \right)^{-\epsilon} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\theta} C_t(j). \quad (5)$$

Once decided how much to consume overall in period t , each cohort j , depending on the relative prices, determines the allocation of total consumption among the continuum of home and foreign goods available (as compacted in the equations above).

Let's define the Terms of Trade (ToT) as the relative price of foreign goods in terms of home goods ($\mathcal{S}_t \equiv \frac{P_{F,t}^i}{P_{H,t}^i}$). Then, we can express the (log-linear) CPI as⁵

$$p_t^i = n p_{H,t}^i + (1-n) p_{F,t}^i = p_{H,t}^i + (1-n) s_t = p_{F,t}^i - n s_t. \quad (6)$$

We assume that the Law of One Price (LOP) holds, and denote with \mathcal{E}_t the nominal exchange rate defined as the domestic price of foreign currency. Hence, the following relations hold:

$$P_{i,t}^H = \mathcal{E}_t P_{i,t}^F \quad P_t^H = \mathcal{E}_t P_t^F \quad W_t^H = \mathcal{E}_t W_t^F, \quad (7)$$

where W^i denotes the nominal wage denominated in i -currency. The last equation implies that labor is homogeneous and freely mobile across countries.

Aggregation across cohorts yields⁶

$$C_t(h) = \left(\frac{P_t^i(h)}{P_{H,t}^i} \right)^{-\epsilon} \left(\frac{P_{H,t}^i}{P_t^i} \right)^{-\theta} C_t \quad C_t(f) = \left(\frac{P_t^i(f)}{P_{F,t}^i} \right)^{-\epsilon} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\theta} C_t, \quad (8)$$

where C_t denotes world per-capita consumption.

We further assume that the production of each good is completely absorbed by private consumption ($Y_t(i') = C_t(i')$, for all $i'=h, f$), and that aggregate demand for both countries is computed by means of the familiar Dixit-Stiglitz aggregators $Y_t^H \equiv \left[\frac{1}{n} \int_0^n Y_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}}$ and $Y_t^F \equiv \left[\frac{1}{1-n} \int_n^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}$. As a consequence, in each country i , we can define the following brand-specific and aggregate demands:

$$Y_t(i') = \left(\frac{P_t^i(i')}{P_{i,t}^i} \right)^{-\epsilon} Y_t^i \quad Y_t^i = \left(\frac{P_{i,t}^i}{P_t^i} \right)^{-\theta} C_t. \quad (9)$$

Since linear approximation implies

$$y_t^H = \theta(1-n)s_t + c_t \quad y_t^F = -\theta n s_t + c_t, \quad (10)$$

aggregating across countries gives the usual income identity:

$$y_t \equiv n y_t^H + (1-n) y_t^F = n\theta(1-n)s_t + n c_t - (1-n)\theta n s_t + (1-n)c_t = c_t. \quad (11)$$

⁵In what follows we denote with lower-case letters the log-deviations of a variable from a symmetric perfect foresight steady state in which $\mathcal{S}_t = 1$. See the Appendix for details.

⁶Since the probability of surviving each period is $(1-\gamma)$ and the size of each newborn cohort was set to γ , the aggregated level across cohorts for each generation-specific variable $X(j)$ are computed as the weighted average $X_t \equiv \sum_{j=-\infty}^t \gamma(1-\gamma)^{t-j} X_t(j)$.

2.1.2 Intertemporal Allocation.

The dynamics of individual and world consumption is determined as the solution of an intertemporal allocation problem. Each household is assumed to have preferences over consumption and leisure, described by a log-utility function. Consumers demand consumption goods and two types of financial assets: state-contingent bonds (with respect to which markets are complete both at national and international level) denominated in foreign currency and equity shares issued by the firms operating in the foreign country. In addition, each consumer in the home country is endowed with an equal amount of non-tradeable shares of the domestic firms.

Consumers born in period j wish to maximize their expected lifetime utility, discounted to account for impatience (β is the intertemporal discount factor) and uncertain lifetime ($(1 - \gamma)$ is the probability of survival across two subsequent periods), by choosing a pattern for real consumption ($C_t(j)$), hours worked ($N_t(j)$) and holdings of financial assets. The latter, at the end of period t , consist of a set of contingent claims whose stochastic nominal payoff in F -currency in period $t+1$ is $B_{F,t+1}(j)$ and a set of equity shares issued by each firm located in the foreign country, $Z_{t+1}(f, j)$. In period t , the relevant discount factor pricing one-period contingent claims is $\mathcal{F}_{t,t+1}$, while the nominal price of equities (denominated in foreign currency) is $Q_{F,t}^*(f)$.

At the beginning of each period, nominal financial wealth (denominated in F currency) of an individual born at time j , $\Omega_{F,t}(j)$, includes the nominal pay-off on both the contingent claims and on the portfolio of equity shares held. Let $D_{F,t}^*(f)$ be the nominal dividend yield paid by firm f . Then, $\Omega_{F,t}(j)$ can be written as:

$$\Omega_{F,t}(j) \equiv \frac{1}{1 - \gamma} \left[B_{F,t}(j) + \int_n^1 \left(Q_{F,t}^*(f) + D_{F,t}^*(f) \right) Z_t(f, j) df \right]. \quad (12)$$

where, following Blanchard (1985), financial wealth carried over from the previous period also pays off the return on the insurance contract that redistributes among survived consumers the financial wealth of the ones who died, thus explaining the presence of $\frac{1}{1-\gamma}$ in equation (12).

The optimization problem faced at time 0 by the representative consumer of cohort j is therefore to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \gamma)^t \left[\log C_t(j) + \log(1 - N_t(j)) \right]$$

subject to a sequence of budget constraints of the form:

$$C_t(j) + \frac{\mathcal{E}_t}{P_t^H} E_t \{ \mathcal{F}_{t,t+1} B_{F,t+1}(j) \} + \frac{\mathcal{E}_t}{P_t^H} \int_n^1 Q_{F,t}^*(f) Z_{t+1}(f, j) df \leq \frac{W_t^H}{P_t^H} N_t(j) + \frac{1}{P_t^H} \int_0^n D_{H,t}^*(h, j) dh + \frac{\mathcal{E}_t}{P_t^H} \Omega_{F,t}(j), \quad (13)$$

for agents living in the Home country (where $D_{H,t}^*(h, j)$ is the cohort j 's share of nominal profits of domestic firm h) and of the form

$$C_t(j) + \frac{1}{P_t^F} E_t \{ \mathcal{F}_{t,t+1} B_{F,t+1}(j) \} + \frac{1}{P_t^F} \int_n^1 Q_{F,t}^*(f) Z_{t+1}(f, j) df \leq \frac{W_t^F}{P_t^F} N_t(j) + \frac{1}{P_t^F} \Omega_{F,t}(j), \quad (14)$$

for F -country households.⁷

First-order conditions for an optimum require the budget constraints (13)–(14) to hold with equality, the following intra-temporal optimality condition with respect to consumption and leisure to be satisfied

$$C_t(j) = \frac{W_t^i}{P_t^i} (1 - N_t(j)), \quad (15)$$

in addition to the inter-temporal conditions with respect to the two financial assets written below:

$$\mathcal{F}_{t,t+1} = \beta \frac{\mathcal{E}_{t+1} P_t^H U_c(C_{t+1}(j))}{\mathcal{E}_t P_{t+1}^H U_c(C_t(j))} = \beta \frac{\mathcal{E}_{t+1} P_t^H C_t(j)}{\mathcal{E}_t P_{t+1}^H C_{t+1}(j)} = \beta \frac{P_t^F C_t(j)}{P_{t+1}^F C_{t+1}(j)} \quad (16)$$

$$Q_t^*(f) = E_t \left\{ \mathcal{F}_{t,t+1} \left[Q_{t+1}^*(f) + D_{t+1}^*(f) \right] \right\}. \quad (17)$$

The nominal price of one equity share is equal to its nominal expected payoff one period ahead, discounted by the stochastic factor $\mathcal{F}_{t,t+1}$. This equilibrium condition defines stock-price dynamics.

From equation (16) one can notice that the equilibrium stochastic discount factor for one-period ahead F -denominated nominal payoffs is the time-discounted growth in the marginal utility of consumption, adjusted for (Home) CPI-inflation and exchange rate depreciation. For k -period ahead nominal payoffs, such stochastic discount factor will therefore be:

$$\mathcal{F}_{t,t+k} = \beta^k \frac{\mathcal{E}_{t+k} P_t^H C_t(j)}{\mathcal{E}_t P_{t+k}^H C_{t+k}(j)} = \beta^k \frac{P_t^F C_t(j)}{P_{t+k}^F C_{t+k}(j)} = \prod_{i=0}^{k-1} \mathcal{F}_{t+i,t+i+1}. \quad (18)$$

The following non-arbitrage conditions determine the nominal gross rates of return ($1 + r_t^i$) on safe one-period bonds paying off one unit of F -currency in period $t+1$ with probability 1 (whose price is therefore $E_t \{ \mathcal{F}_{t,t+1} \}$):

$$(1 + r_t^F) E_t \{ \mathcal{F}_{t,t+1} \} = 1 \quad (19)$$

$$(1 + r_t^H) \mathcal{E}_t E_t \{ \mathcal{F}_{t,t+1} \} = E_t \{ \mathcal{E}_{t+1} \}, \quad (20)$$

From the two equations above, it is easy to derive the Uncovered Interest Parity (UIP) condition:

$$(1 + r_t^H) = E_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} (1 + r_t^F). \quad (21)$$

⁷Note that budget constraints have been deflated with consumer price indexes.

Taking conditional expectations of equation (16), and using the non-arbitrage conditions, one obtains a stochastic Euler equation linking individual consumption growth, exchange rate depreciation and the interest rate:

$$1 = (1 + r_t^H)\beta E_t \left\{ \frac{P_t^H C_t(j) \mathcal{E}_{t+1}}{P_{t+1}^H C_{t+1}(j) \mathcal{E}_t} \right\} = (1 + r_t^F)\beta E_t \left\{ \frac{P_t^F C_t(j)}{P_{t+1}^F C_{t+1}(j)} \right\}. \quad (22)$$

Finally, another requirement for an optimal choice is a “no-Ponzi schemes” condition establishing that the present value of last period’s financial wealth, conditional upon survival, shrinks to zero as time diverges:

$$\lim_{k \rightarrow \infty} E_t \left\{ \mathcal{F}_{t,t+k} (1 - \gamma)^k \Omega_{F,t+k}(j) \right\} = 0. \quad (23)$$

Using equations (17), and definition (12), the equilibrium budget constraint (13) can be rewritten as a stochastic difference equation in the financial wealth $\Omega_{F,t}(j)$:

$$P_t^H C_t(j) + \mathcal{E}_t E_t \{ \mathcal{F}_{t,t+1} (1 - \gamma) \Omega_{F,t+1}(j) \} = W_t^H N_t(j) + \int_0^n D_{H,t}^*(h, j) dh + \mathcal{E}_t \Omega_{F,t}(j). \quad (24)$$

Define as $\mathcal{H}_{F,t}(j)$ the nominal human wealth for cohort j at time t , in F -currency. By using the suitable form of the equilibrium stochastic discount factor (18) and the No-Ponzi condition (23), equation (24) can be solved forward and nominal consumption can be expressed as a linear function of total financial and human wealth:

$$\frac{P_t^H C_t(j)}{\mathcal{E}_t} = \varsigma (\Omega_{F,t}(j) + \mathcal{H}_{F,t}(j)) = P_t^F C_t(j), \quad (25)$$

where $\varsigma \equiv 1 - \beta(1 - \gamma)$ is the propensity to consume out of financial and human wealth, which is common across cohorts and over time.⁸

2.1.3 Aggregation across Cohorts

The solution of the consumer’s problem provides three relevant equilibrium conditions that are specific to each generic cohort j : a static labor supply (15), equations (13)–(14) guaranteeing that the budget constraint holds and the relation linking personal consumption to total personal wealth (equation (25)).

Since these equilibrium conditions are linear in the cohort-specific variables, aggregation yields:

$$C_t = \frac{W_t^i}{P_t^i} (1 - N_t), \quad (26)$$

$$P_t^H C_t + \mathcal{E}_t E_t \{ \mathcal{F}_{t,t+1} \Omega_{F,t+1} \} = W_t^H N_t + \int_0^n D_{H,t}^*(h) dh + \mathcal{E}_t \Omega_{F,t} \quad (27)$$

$$\frac{P_t^H C_t}{\mathcal{E}_t} = \varsigma (\Omega_{F,t} + \mathcal{H}_{F,t}) = P_t^F C_t. \quad (28)$$

⁸The procedure follows Piergallini (2004). Details on the derivation above are available upon request. See also Nisticò (2005) for a closed economy version with a time-varying propensity to consume out of wealth.

where it is to be noticed that the budget constraint is expressed as a stochastic difference equation in aggregate wealth. Below, it will be useful to express equation (26) in log-linear form, as

$$c_t = w_t^i - p_t^i - \varphi n_t,$$

where $\varphi \equiv \frac{N}{1-N}$ is the inverse of the steady-state elasticity of labor supply.

Since the aggregate value of the gross return on the insurance contract equals 1, aggregate financial wealth may be also defined as:

$$\Omega_{F,t} \equiv \left[B_{F,t} + \int_n^1 \left(Q_{F,t}^*(f) + D_{F,t}^*(f) \right) Z_t(f) df \right]. \quad (29)$$

Finally, equation (27) and equation (28) form a system whose solution describes the dynamic path of aggregate consumption

$$\frac{P_t^H C_t}{\mathcal{E}_t} = \sigma E_t \{ \mathcal{F}_{t,t+1} \Omega_{F,t+1} \} + \frac{1}{\beta} E_t \{ \mathcal{F}_{t,t+1} \frac{P_{t+1}^H C_{t+1}}{\mathcal{E}_{t+1}} \}, \quad (30)$$

where the first term (in which $\sigma \equiv \gamma \frac{1-\beta(1-\gamma)}{\beta(1-\gamma)}$) represents the financial wealth effects. Notice that such effects fade out as the probability of exiting the market (γ) goes to zero.⁹

2.1.4 Equilibrium

In equilibrium, the net supply of state-contingent bonds ($B_{F,t} = 0$) is zero. Moreover, the aggregate stock of outstanding equity shares, for each intermediate good-producing firm in country F , must equal 1 ($Z_t(f) = 1$ for all $f \in [n, 1]$).

Let's now define the indexes of average real dividend payments as well as the average stock market capitalization of F -located firms:

$$D_{H,t} \equiv \frac{1}{nP_t^H} \int_0^n D_{H,t}^*(h) dh \quad D_{F,t} \equiv \frac{1}{(1-n)P_t^F} \int_n^1 D_{F,t}^*(f) df \quad (31)$$

$$Q_{F,t} \equiv \frac{1}{(1-n)P_t^F} \int_n^1 Q_{F,t}^*(f) df. \quad (32)$$

Notice that in equilibrium, given the pricing equation (17), the present discounted nominal value of future financial wealth, $E_t \{ \mathcal{F}_{t,t+1} \Omega_{F,t+1} \}$ is proportional to the current level of the nominal stock-price index:

$$\begin{aligned} E_t \{ \mathcal{F}_{t,t+1} \Omega_{F,t+1} \} &= E_t \left\{ \mathcal{F}_{t,t+1} \left[B_{F,t+1} + \int_n^1 \left(Q_{F,t+1}^*(f) + D_{F,t+1}^*(f) \right) Z_{t+1}(f) df \right] \right\} = \\ &= \int_n^1 E_t \left\{ \mathcal{F}_{t,t+1} \left(Q_{F,t+1}^*(f) + D_{F,t+1}^*(f) \right) df \right\} = \int_n^1 Q_{F,t}^*(f) df = (1-n)P_t^F Q_{F,t}. \end{aligned} \quad (33)$$

⁹The same dynamics for world consumption can be obtained starting from the problem of an agent living in the F -country: $P_t^F C_t = \sigma E_t \{ \mathcal{F}_{t,t+1} \Omega_{F,t+1} \} + \frac{1}{\beta} E_t \{ \mathcal{F}_{t,t+1} P_{t+1}^F C_{t+1} \}$. Details on the derivation available upon request.

Moreover, given the definition of dividends at the firm level ($D_{F,t}^*(f) \equiv P_{F,t}^F(f)Y_t(f) - W_t^F N_t(f)$), one can easily express foreign aggregate real dividends as:

$$\begin{aligned} D_{F,t} &\equiv \frac{1}{(1-n)P_t^F} \int_n^1 D_{F,t}^*(f) df = \frac{1}{(1-n)P_t^F} \int_n^1 \left(P_{F,t}^F(f)Y_t(f) - W_t^F N_t(f) \right) df \\ &= \frac{P_{F,t}^F}{(1-n)P_t^F} \int_n^1 \left(\frac{P_{F,t}^F(f)}{P_t^F} \right)^{1-\epsilon} Y_t^F df - \frac{1}{(1-n)P_t^F} \int_n^1 W_t^F N_t(f) df \\ &= \frac{P_{F,t}^F}{P_t^F} Y_t^F - \frac{W_t^F}{P_t^F} N_t^F. \end{aligned} \quad (34)$$

The demand-side of the world economy is completed by $Y_t = C_t$ and the following aggregate Euler equations

$$C_t - \sigma(1-n)Q_{F,t} = \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} \Pi_{t+1}^H C_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right\} = \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} \Pi_{t+1}^F C_{t+1} \right\} \quad (35)$$

$$Q_{F,t} = E_t \left\{ \mathcal{F}_{t,t+1} \Pi_{t+1}^F \left[Q_{F,t+1} + D_{F,t+1} \right] \right\}. \quad (36)$$

Equation (35) defines the dynamic path of aggregate world consumption. One should notice that an explicit role is played by the dynamics of stock prices. The latter is defined by equation (36), a standard micro-founded asset pricing equation derived from aggregating equation (17) across firms.

Two final remarks before moving to describe the supply side of our economy. First, the benchmark case of infinitely-lived consumers (like the one described in Benigno and Benigno (2004)) is a special case of the one discussed here, corresponding to a zero-probability of death, $\gamma=\sigma=0$. In this case, equation (35) loses the term related to stock prices and collapses to the usual Euler equation for consumption.

$$C_t = \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} \Pi_{t+1}^H C_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right\} = \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} \Pi_{t+1}^F C_{t+1} \right\}. \quad (37)$$

Second, the above Euler equation applies also when $n = 1$. In this case, country H becomes an economy closed to commercial and financial trades (and with no relevant stock market dynamics).

2.2 Supply-Side and Inflation Dynamics.

The world economy produces a continuum of differentiated goods over the interval $[0, 1]$, with a fraction n ($1-n$) produced in country H (F). Each firm, in each country, has access to a stochastic linear technology $Y_t(i') = A_t^i N_t(i')$, with $i' = h, f$ and $i = H, F$, whose country-specific productivity shock is A_t^i . Each firm chooses labor demand in a competitive labor market by minimizing its total real costs subject to the technological constraint. In equilibrium, for each firm in country i , the (log-linear) real marginal cost will hence be

$$mc_{i,t} \equiv \log(\mu MC_{i,t}) = w_t^i - p_{i,t}^i - a_t^i, \quad (38)$$

where $\mu \equiv \frac{\epsilon}{\epsilon-1} = (MC_i)^{-1}$ is the steady state gross markup (i.e., the reciprocal of steady state real marginal costs).

The Dixit-Stiglitz aggregators for national demands make the aggregate production functions equal to $Y_t^i \Xi_t^i = A_t^i N_t^i$, where Ξ_t^i captures relative price dispersion among firms in country i and N_t^i denotes aggregate labor input in country i .

Using equilibrium in the labor market and the definition of the terms of trade, we get the following expressions for the real marginal costs:

$$mc_{H,t} = w_t^H - p_{H,t}^H - a_t^H = c_t + \varphi y_t^H - (1 + \varphi)a_t^H + (1 - n)s_t \quad (39)$$

$$mc_{F,t} = w_t^F - p_{F,t}^F - a_t^F = c_t + \varphi y_t^F - (1 + \varphi)a_t^F - ns_t. \quad (40)$$

We assume that firms set prices according to Calvo's (1983) staggering mechanism. The optimal price-setting strategy for all firms revising their price at time t is then to choose a common optimal price level, $P_t^*(i')$ according to the following log-linear rule:

$$p_t^*(i') \equiv \log(P_t^*(i')/P) = (1 - \alpha^i \tilde{\beta}) E_t \left\{ \sum_{k=0}^{\infty} (\alpha^i \tilde{\beta})^k (mc_{i,t+k} + p_{i,t+k}^i) \right\}, \quad (41)$$

where we set $\tilde{\beta} \equiv \frac{1}{1+r^i}$ (see below).

Given the definition of the producer-price indexes (3) and considering that all firms revising their price at t (a fraction $(1 - \alpha^i)$ of all firms) choose the same price $P_t^*(i')$ while all firms keeping the price constant (a fraction α^i) charge last period's domestic price level, inflation dynamics in country i can be described by a familiar New Keynesian Phillips Curve:

$$\pi_{i,t} \equiv p_{i,t}^i - p_{i,t-1}^i = \tilde{\beta} E_t \pi_{i,t+1} + \lambda^i mc_{i,t}, \quad (42)$$

where $\lambda^i \equiv \frac{(1-\alpha^i)(1-\alpha^i \tilde{\beta})}{\alpha^i}$.

2.3 Steady State and Linearization.

As we show in the Appendix, in a symmetric perfect foresight steady state, the demand-side defines the following set of equilibrium relations:

$$\mathcal{S} = 1 \quad (43)$$

$$Y^H = Y^F = Y = C = \frac{A}{1 + \mu} \quad (44)$$

$$(1 + r^H)^{-1} = (1 + r^F)^{-1} = \tilde{\beta} \equiv \frac{\beta}{1 + \psi} \quad (45)$$

$$\frac{D_F}{(1 + r^F)Q_F} = 1 - \tilde{\beta} \quad (46)$$

$$\frac{Y}{D_F} = \frac{\tilde{\beta}}{1 - \tilde{\beta}} \frac{Y}{Q_F}, \quad (47)$$

where

$$\psi \equiv \gamma(1-n) \frac{1-\beta(1-\gamma)}{(1-\gamma)} \frac{\Omega_F}{P^F Y}. \quad (48)$$

Log-linearization around a perfect foresight steady state yields (see the Appendix for details):¹⁰

$$y_t = c_t \quad (49)$$

$$y_t^H = c_t + \theta(1-n)s_t \quad y_t^F = c_t - \theta n s_t \quad (50)$$

$$E_t\{\Delta e_{t+1}\} = r_t^H - r_t^F = E_t\{\pi_{t+1}^H - \pi_{t+1}^F\} \quad (51)$$

$$c_t = \frac{1}{1+\psi} E_t c_{t+1} + \frac{\psi}{1+\psi} q_{F,t} - \frac{1}{1+\psi} (r_t^i - E_t \pi_{t+1}^i - \tilde{\rho}) \quad (52)$$

$$q_{F,t} = \tilde{\beta} E_t q_{F,t+1} + (1-\tilde{\beta}) E_t d_{F,t+1} - (r_t^F - E_t \pi_{t+1}^F - \tilde{\rho}) \quad (53)$$

$$d_{F,t} = y_t^F + n s_t - \frac{\tilde{\beta} Y / Q_F}{\mu(1-\tilde{\beta})} m c_{F,t} \quad (54)$$

$$\pi_{i,t} = \tilde{\beta} E_t \pi_{i,t+1} + \lambda^i m c_{i,t} \quad (55)$$

$$m c_{H,t} = c_t + \varphi y_t^H - (1+\varphi) a_t^H + (1-n) s_t \quad (56)$$

$$m c_{F,t} = c_t + \varphi y_t^F - (1+\varphi) a_t^F - n s_t. \quad (57)$$

Equations (49) and (50) define the world and country-specific aggregate demands, while equation (51) is derived from the UIP condition and the law of one price.

Equation (52) is the linear approximation of the Euler equation for consumption. Notice that a positive probability of exiting the market ($\gamma, \psi > 0$) affects the degree of smoothing in the inter-temporal path of aggregate consumption. Since agents are uncertain regarding the future, they cannot completely spread over time the effects of current or expected future shocks. Such shocks, therefore, affect current consumption more than in the benchmark case of infinitely living households. Hence, the dynamics of aggregate financial wealth becomes relevant for current aggregate consumption and for the transmission of real and monetary shocks.

Equations (53)–(54) define the pricing equation for equity shares in foreign firms and the dynamics of their dividends. The effect of terms-of-trade dynamics on real dividends are two-fold and conflicting. On one side, an appreciation raises the real value of dividends, for any level of production of F -goods (this is captured by the term $n s_t$). On the other side, an appreciation in the terms of trade makes F -goods less competitive in the world markets and their aggregate demand y_t^F is pushed down; as a consequence, real dividends tend to fall, for any level of world consumption (this substitution effect is captured by the relation $y_t^F = c_t - \theta n s_t$). The more substitutable are H and F -goods, then, the stronger the substitution effect; for $\theta > 1$ the net effect is negative.

¹⁰Variables with no country subscript or superscript denote world levels. Notice that the log-deviation of the interest rate from its steady state is $r_t^i - \tilde{\rho}$, where we set $\tilde{\rho} \equiv \log(1+r^i) = \log(1+r) = -\log \tilde{\beta}$.

Households' finite life horizon (affecting the long-run interest rate) also affects the Phillips Curve expressed in equation (55), as it implies a lower weight on future inflation $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$ and a higher weight on the marginal costs with respect to the standard case. However, as γ goes to zero, our Phillips Curve converges to the latter.

From the definition of the ToT we also have that domestic inflation rates are related to CPI-inflation rates according to

$$\pi_t^H - \pi_{H,t} = (1-n)\Delta s_t \quad \pi_t^F - \pi_{F,t} = -n\Delta s_t. \quad (58)$$

Using the above relationships together with the law of one price and the UIP condition leads to the stochastic difference equation describing the dynamics of the log-ToT:

$$s_t = E_t s_{t+1} + (r_t^F - E_t \pi_{F,t+1} - \tilde{\rho}) - (r_t^H - E_t \pi_{H,t+1} - \tilde{\rho}). \quad (59)$$

2.4 Equilibrium with flexible prices.

In the limiting case of full price flexibility $\alpha^i \rightarrow 0$. As the price setting rule shows, in this case all firms set prices as a constant markup over nominal marginal costs at all times: $P_t^n(i') = \mu P_{i,t} MC_{i,t}^n = P_{i,t}$, for all $i'=h, f$ and $i=H, F$. As a consequence, real marginal costs match their long-run level at each point in time and in each country, implying $mc_{i,t}^n \equiv \log MC_{i,t}^n - \log MC = 0$.¹¹

From the condition that $mc_{H,t}^n = mc_{F,t}^n$ and the intratemporal allocation described by equation (50) we can rewrite the natural level of the terms of trade, as a decreasing function of the relative foreign productivity:

$$s_t^n = \frac{1+\varphi}{1+\varphi\theta} (a_t^H - a_t^F). \quad (60)$$

It is easy to show that the equation for the natural level of world output becomes:

$$y_t^n = c_t^n = na_t^H + (1-n)a_t^F \equiv a_t, \quad (61)$$

Moreover, it is now possible to link short-run real marginal costs in each country to the world output gap $x_t \equiv y_t - y_t^n$ and the deviations of the ToT from their natural level:

$$mc_{H,t} = (1+\varphi)x_t + (1-n)(1+\varphi\theta)(s_t - s_t^n) \quad (62)$$

$$mc_{F,t} = (1+\varphi)x_t - n(1+\varphi\theta)(s_t - s_t^n). \quad (63)$$

Notice that the positive effect on the marginal costs in country H of an appreciation of the terms of trade stems from two channels: a rise in the product wage and a competitiveness effect shifting aggregate demand from country F to country H (thereby raising real wages through a rise in labor demand).¹²

¹¹The value that each variable takes in the flexible-price equilibrium will be denoted by a superscript n (for "natural").

¹²The effects on real marginal costs in country F are, of course, symmetric.

An equivalent useful expression of real marginal costs involves *domestic* output gaps

$$mc_{H,t} = (1 + \varphi)x_t^H - (1 - n)(\theta - 1)(s_t - s_t^n) \quad (64)$$

$$mc_{F,t} = (1 + \varphi)x_t^F + n(\theta - 1)(s_t - s_t^n), \quad (65)$$

with the relationship between domestic and world output gaps summarized by:

$$x_t = x_t^H - \theta(1 - n)(s_t - s_t^n) = x_t^F + n\theta(s_t - s_t^n).$$

This formulation highlights the role of the terms of trade *for given domestic output*. Holding constant x_t^H an appreciation in the terms of trade still has a positive effect on the product wage, but it also induces a negative substitution effect: through a reduction in the demand and the production of foreign goods, an appreciation in the terms of trade, for given domestic output, reduces average world consumption; since home and foreign goods are substitutable, however, such reduction will call for smoothing in favor of foreign goods, and a lower demand also for domestic goods (the more so the higher θ); on the other hand, lower world consumption implies lower average personal income and thereby higher labor supply. Both these effects will therefore push down real wages to the extent that domestic output (and hence employment) stay unaffected. For $\theta > 1$ the latter effect is dominant and the net effect on real marginal costs is therefore negative.

This formulation, moreover, highlights that openness to international trade generates a cost-push shock that makes real marginal costs no longer proportional to the domestic output gap. Hence, as in Benigno and Benigno (2003) and Benigno (2004), openness introduces a trade-off between domestic inflation and output stabilization: both targets cannot be hit at once at the country level unless the policy makers in the rest of the world behave consistently. However, for $\theta=1$ (Cobb-Douglas aggregate) the substitution and the product-wage effects offset each other; as a result domestic real marginal costs, and hence inflation, are insulated from foreign shocks, and the trade-off is ruled out.

2.5 The Complete Linear Model.

By using the results of the previous sections, the demand-side of the world economy consists of equation (59), the pricing equation of foreign stocks¹³

$$q_{F,t} = \tilde{\beta}E_t q_{F,t+1} + (1 - \tilde{\beta})E_t \{y_{t+1}^F + ns_{t+1}\} \\ - \frac{\tilde{\beta}Y/Q_F}{\mu} E_t mc_{F,t+1} - (r_t^F - E_t \pi_{F,t+1} - \tilde{\rho}) - nE_t \Delta s_{t+1} \quad (66)$$

¹³Equation (66) shows that, in addition to the effects of terms-of-trade *levels* on expected dividends, discussed in Section 2.3, there is also a negative impact of the terms-of-trade expected *appreciation rate*, which imply declining competitiveness and profits in the future and thereby lower current stock prices.

and the two IS-type relations:

$$(1 + \psi)y_t^H = E_t y_{t+1}^H + \psi(q_{F,t} + (1 - n)\theta s_t) - (r_t^H - E_t \pi_{H,t+1} - \tilde{\rho}) - (\theta - 1)(1 - n)E_t \Delta s_{t+1} \quad (67)$$

$$(1 + \psi)y_t^F = E_t y_{t+1}^F + \psi(q_{F,t} - n\theta s_t) - (r_t^F - E_t \pi_{F,t+1} - \tilde{\rho}) + (\theta - 1)nE_t \Delta s_{t+1}. \quad (68)$$

Aggregating across countries ($y_t \equiv ny_t^H + (1 - n)y_t^F$) we obtain the world IS-schedule

$$y_t = \frac{1}{1 + \psi} E_t y_{t+1} + \frac{\psi}{1 + \psi} q_{F,t} - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1} - \tilde{\rho}), \quad (69)$$

Although similar to the analogous expression in Nisticò (2005), an important difference is the quantitative relevance of the stock-wealth effect. This is not only related to the agents' planning horizon ($1/\gamma$), but also to the relative dimension of country F in which the stock market is located, $(1-n)$.¹⁴ As n approaches unity, in fact, the world economy collapses into country H alone, country F and its stock market become negligible and so does the wealth effect on both domestic and the world real activity (as n goes to 1, ψ shrinks to 0, as implied by definition (48)).

The supply-side of the world economy consists of the two New Keynesian Phillips Curves attributing domestic inflationary pressures in each country to domestic non-zero output gaps and foreign unbalances transmitted through the ToT:

$$\pi_{H,t} = \tilde{\beta} E_t \pi_{H,t+1} + \lambda^H (1 + \varphi) x_t^H - (1 - n)(\theta - 1) \lambda^H (s_t - s_t^n) \quad (70)$$

$$\pi_{F,t} = \tilde{\beta} E_t \pi_{F,t+1} + \lambda^F (1 + \varphi) x_t^F + n(\theta - 1) \lambda^F (s_t - s_t^n). \quad (71)$$

The world Aggregate Supply schedule can be obtained via aggregation across countries. Such schedule may assume different formulations depending upon the degree of homogeneity of the two countries with respect to price stickiness. In the general case of heterogeneity, the trade channel transmits local unbalances also to the world economy:

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \lambda(1 + \varphi) x_t - \lambda^R n(1 - n)(1 + \varphi \theta)(s_t - s_t^n), \quad (72)$$

where $\lambda \equiv n\lambda^H + (1 - n)\lambda^F$ and $\lambda^R \equiv \lambda^F - \lambda^H$. If prices at home are stickier than abroad ($\alpha^H > \alpha^F$), then the inflation rate abroad will be more reactive than at home ($\lambda^H < \lambda^F$). Thereby, facing a rise in the terms of trade, marginal costs at home rise as much as they fall abroad, but inflation in country H rises less than it falls in country F , since the latter is more reactive.

In the limiting case of perfect homogeneity ($\alpha^H = \alpha^F = \alpha$ and therefore $\lambda = \lambda^H = \lambda^F$ and $\lambda^R = 0$), the local unbalances cancel each other out and we get the closed economy version of the Phillips curve:

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \lambda(1 + \varphi) x_t. \quad (73)$$

¹⁴Or else, adopting the viewpoint of the Home country, to the degree of domestic "financial" openness.

3 Stock-Price Dynamics and Optimal Monetary Policy for Price Stability.

In this Section, we investigate the monetary policy implications of assuming real effects on output induced by stock-price changes. In particular, we want to address the question of how an Optimal Monetary Policy devoted to achieve price stability should be designed and conducted.

Within the closed-economy versions of DNK models,¹⁵ it is well known that, in absence of a cost push shock that induces a conflict between maintaining zero inflation and zero output gap, the optimal rate of inflation is zero and price stability is therefore the proper target for monetary policy. In a recent contribution, Benigno and Benigno (2003) show that this result can be extended to a Two-Country framework if special conditions are met, namely if the distortions coming from monopolistic competition are the same across countries.

As pointed out earlier, openness to international trade generates a trade-off between domestic inflation and output gap stabilization at the country level: for domestic inflation to be driven to zero the flexible-price equilibrium must prevail not only in the domestic economy but also, and contemporaneously, abroad.

Here, we lay out the dynamic properties of the national and world interest rates consistent with a flexible price allocation at the world level, while in section 3.1 we study the conditions for this particular equilibrium to be determined.

We therefore derive what Woodford (2003) calls the *Wicksellian Natural Rate of Interest*, henceforth rr_t^n , for both economies and the world as a whole. We will also compare our results with the ones obtained in two distinct benchmark cases. First, we compare the natural rates both at national and aggregate level to the limiting case of infinitely-lived households (as for example in Benigno and Benigno (2004)). We show that for country H price stability requires a dedicated policy response to foreign stock-price dynamics, as long as it is driven by pure idiosyncratic productivity shocks or there is asymmetry in these shocks' persistence across countries. Second, we compare the natural rate of interest of the foreign country to the (other) limiting case of the OLG closed economy model (as in Nisticò (2005)). This comparison shows that openness to international trade and free capital movements make stock-price dynamics driven by idiosyncratic supply shocks relevant for monetary policy making also in the country where the stock market is located and equities are traded (as opposed the closed-economy case).

In the case of the Representative Agent setup (henceforth RA, implying $\gamma=\psi=0$), the

¹⁵See Clarida, Galí and Gertler (1999) and Galí (2003)

world and country-specific natural interest rates are given by the following expressions:

$$rr_{RA,t}^n = \rho + E_t \Delta a_{t+1} = \rho + n E_t \Delta a_{t+1}^H + (1-n) E_t \Delta a_{t+1}^F \quad (74)$$

$$rr_{RA,t}^{nH} = \rho + E_t \Delta a_{t+1} + (1-n) \frac{1+\varphi}{1+\varphi\theta} E_t \{\Delta a_{t+1}^H - \Delta a_{t+1}^F\} \quad (75)$$

$$rr_{RA,t}^{nF} = \rho + E_t \Delta a_{t+1} + n \frac{1+\varphi}{1+\varphi\theta} E_t \{\Delta a_{t+1}^F - \Delta a_{t+1}^H\} \quad (76)$$

where $\rho \equiv \log(1+r) = -\log \beta$. Here, the response of the natural interest rate is accommodative with respect to persistent supply shocks (both at the national and the aggregate world level and with respect to both domestic and foreign shocks (for $\theta > 1$)).¹⁶

In the second limiting case, that of a OLG closed economy with stock-wealth effects, swings in stock prices driven by idiosyncratic supply shocks do not require a dedicated response by a Central Bank pursuing price stability. Therefore, with $n=0$:

$$rr_{CE,t}^{nF} = \tilde{\rho} + E_t \Delta a_{t+1}^F. \quad (77)$$

In our more general OLG open-economy model the *Wicksellian natural rate of interest* is obtained as the solution of the system (66)–(69) under the assumption of price flexibility in both countries. Equations (67) to (69) show that simultaneous price stability at the national and the world level implies:

$$rr_t^n = \tilde{\rho} + E_t \Delta y_{t+1}^n + \psi (q_{F,t}^n - y_t^n) \quad (78)$$

$$rr_t^{nH} = \tilde{\rho} + E_t \Delta y_{t+1}^n + \psi (q_{F,t}^n - y_t^n) + (1-n) \frac{1+\varphi}{1+\varphi\theta} E_t \{\Delta a_{t+1}^H - \Delta a_{t+1}^F\} \quad (79)$$

$$rr_t^{nF} = \tilde{\rho} + E_t \Delta y_{t+1}^n + \psi (q_{F,t}^n - y_t^n) + n \frac{1+\varphi}{1+\varphi\theta} E_t \{\Delta a_{t+1}^F - \Delta a_{t+1}^H\} \quad (80)$$

The potential level of (log) average stock market capitalization of country F is then the solution of the stochastic difference equation

$$q_{F,t}^n - y_t^n = \frac{\tilde{\beta}}{1+\psi} E_t \{q_{F,t+1}^n - y_{t+1}^n\} + n(\theta-1) \frac{1-\tilde{\beta}}{1+\psi} \frac{1+\varphi}{1+\varphi\theta} E_t \{a_{t+1}^F - a_{t+1}^H\}. \quad (81)$$

While in the limiting closed-economy case ($n=0$) the solution is simply $q_{F,t}^n = y_t^n = a_t^F$, in general it takes the following form:

$$q_{F,t}^n = y_t^n + \frac{1+\varphi}{1+\varphi\theta} (\kappa^F a_t^F - \kappa^H a_t^H), \quad (82)$$

where $\kappa^i \equiv \frac{n(1-\tilde{\beta})(\theta-1)}{1+\psi-\tilde{\beta}\rho_a^i}$, and ρ_a^i is the degree of persistence of the productivity shock specific to country i .¹⁷ Notice also that in the special case $\rho_a^H = \rho_a^F$ we have $\kappa^H = \kappa^F = \kappa$ and the solution can be written more compactly as $q_{F,t}^n = y_t^n - \kappa s_t^n$.

¹⁶Notice that unitary elasticity of substitution between home and foreign goods ($\theta=1$), coupled with the assumed log-utility function, perfectly insulates the national economies from foreign shocks, and therefore the natural rates respond only to domestic disturbances: $rr_{RA,t}^{ni} = \rho + E_t \Delta a_{t+1}^i$, for $i=H, F$.

¹⁷We assume that the productivity shock in country i follows the stochastic process $a_t^i = \rho_a^i a_{t-1}^i + u_{a,t}^i$.

The potential level of foreign stock prices, therefore, is increasing in local productivity shocks and decreasing in productivity shocks hitting the rest of the world (in our setting, the Home economy).

Plugging the above solution into equations (78) to (80), we obtain the reduced form for the *Wicksellian natural rate of interests*, highlighting the excess interest rate response implied by the presence of stock-wealth effects:

$$rr_t^n = \tilde{\rho} + E_t \Delta y_{t+1}^n + \Psi(\kappa^F a_t^F - \kappa^H a_t^H) = \bar{\rho} + rr_{RA,t}^n + \Psi(\kappa^F a_t^F - \kappa^H a_t^H) \quad (83)$$

$$rr_t^{nH} = \bar{\rho} + rr_{RA,t}^{nH} + \Psi(\kappa^F a_t^F - \kappa^H a_t^H) \quad (84)$$

$$rr_t^{nF} = \bar{\rho} + rr_{RA,t}^{nF} + \Psi(\kappa^F a_t^F - \kappa^H a_t^H), \quad (85)$$

in which $\Psi \equiv \psi \frac{1+\varphi}{1+\varphi\theta}$, and $\bar{\rho} \equiv \tilde{\rho} - \rho = \log(1 + \psi)$ reflects the difference in the long-run interest rate relative to the RA setup. Within the latter framework, this is in fact lower, as a consequence of lower impatience due to a zero-probability of exiting the market.

Equation (84) shows that for country H (which does not have a domestic stock exchange but can freely access the foreign one) the optimal interest rate dynamics implied by a framework in which financial wealth has real effects on output and inflation, displays an additional term which is a direct implication of the presence of stock-wealth effects. As a result, the Central Bank in country H is required to respond to stock-price dynamics if either the driving productivity shock is idiosyncratic to country F , or the driving disturbance is at the world level but the country H features lower productivity persistence. Since the transmission process passes through the real dividends paid by F -located firms, however, the type of response depends on the degree of substitutability across countries. If the substitutability is high enough ($\theta > 1$), in fact, the competitiveness effect is dominant and a stock-price boom (fall) triggers inflationary (deflationary) pressures on domestic prices through this additional financial channel. The “stock-price pass-through” in this case is overheating and partial sterilization of productivity shocks is therefore needed.¹⁸ It is worth noticing that the case of $\theta = 1$, by insulating each economy from foreign shocks, does not imply this additional response.

It is straightforward to see that such a dedicated response shrinks to zero as our OLG framework converges to the RA open-economy setup (in which $\gamma = \psi = 0$) and/or if the relative dimension of the country where the stock market is located becomes negligible (since $\lim_{n \rightarrow 1} \psi = 0$); therefore, the implied differences in the optimal interest rate’s dynamics with respect to the benchmark RA case can entirely be attributed to the fact that changes in foreign stock prices affect the Home economy.

¹⁸Obviously, for ($\theta < 1$), more aggressive accommodation of foreign productivity shocks is needed. Analogously, ($\theta > 1$) requires more aggressive accommodation of *domestic* productivity shocks while ($\theta < 1$) calls for partial sterilization of the latter.

Moreover, notice that equation (85) can also be written as

$$\begin{aligned} rr_t^{nF} &= \tilde{\rho} + E_t \Delta a_{t+1} + n \frac{1 + \varphi}{1 + \varphi \theta} E_t \{ \Delta a_{t+1}^F - \Delta a_{t+1}^H \} + \Psi(\kappa^F a_t^F - \kappa^H a_t^H) \\ &= rr_{CE,t}^{nF} - n \frac{\varphi(\theta - 1)}{1 + \varphi \theta} E_t \{ \Delta a_{t+1}^F - \Delta a_{t+1}^H \} + \Psi(\kappa^F a_t^F - \kappa^H a_t^H). \end{aligned} \quad (86)$$

Compared to the closed-economy version of the OLG model, then, the natural interest rate dynamics for country F displays the usual additional term due to the presence of international trade plus a second additional term, of financial origin, due to the stock-wealth effects. Also in this case, whether the result would be a partial sterilization or a more aggressive accommodation of relative productivity shocks depends on the degree of international substitutability. It is straightforward to see that both terms fade out as $n \rightarrow 0$ and we converge to the closed economy case.¹⁹

In the closed economy version of the OLG model with wealth effects swings in stock prices driven by local supply shocks do not require a deviation in the dynamics of interest rates with respect to the case of no wealth effects. However, openness of country F to international trade and capital movements does require an evaluation of local productivity shocks in relative terms.²⁰ If the productivity shock driving the observed dynamics in stock prices is at the world level and the rest of the world share the same productivity persistence of the domestic economy, then no dedicated response is required (as in the closed-economy case); on the contrary, if the observed stock-market dynamics is driven by a country-specific shock and/or the degrees of persistence are different, then the presence of wealth effects requires an additional dedicated response by the Central Bank of the country hosting the stock exchange *regardless of the country where the shock hits*.

3.1 Implementation: Equilibrium Determinacy.

We now turn to discuss the issue of determinacy, which characterizes all new Keynesian models in which the monetary instrument is assumed to be the nominal interest rate.

In our framework, full stabilization of domestic prices in both countries drives the system to its flexible-price equilibrium (still characterized by the distortions, equal across countries, generated by assuming monopolistically competitive firms), and therefore implies that domestic interest rates follow the optimal dynamics derived in the previous section. However, the optimal dynamic path of interest rates is generally not a sufficient condition for obtaining price stability in this class of models.

Formally, in fact, if we collect the deviations of each variable from its natural level into vector \mathbf{z}_t ,²¹ and assume that each Central Bank can commit itself to the optimal rule

¹⁹The second term is driven to zero by the κ 's, the coefficients measuring the excess effect of local and foreign productivity shifts due to the commercial and financial openness. See equation (82).

²⁰Once again this does not apply to the special case of $\theta = 1$.

²¹Therefore we set $\mathbf{z}_t \equiv [x_t^H \quad x_t^F \quad (q_{F,t} - q_{F,t}^n) \quad \pi_{H,t} \quad \pi_{F,t} \quad (s_t - s_t^n)]'$.

($r_t^i = rr_t^{ni}$), the system dynamics can be written as

$$E_t \mathbf{z}_{t+1} = \mathbf{A} \mathbf{z}_t, \quad (87)$$

where matrix $\mathbf{A} = \mathbf{A}(\vartheta)$ is a function of all structural parameters, collected into vector ϑ . As in all models in this class, though, the assumption that the interest rates are set according to equations (84) and (85) does not imply a unique equilibrium for the system above.²² Hence, if interest rates in both countries were to follow the dynamics derived in the previous section, the flexible price allocation would only be one possible and consistent outcome, but certainly not the only one: $\mathbf{z}_t = 0$ is an indeterminate equilibrium. A revision in expectations would be sufficient to trigger the endogenous dynamics of the system and drive it in anyone of the other infinite alternative equilibria. Persistent endogenous instability would be the result.

In order to rule out self-fulfilling expectations and restore uniqueness of the flexible-price equilibrium the monetary policy instrument rule needs to incorporate a credible threat to deviate from the truly optimal policy in response to deviations of the endogenous variables at stake from the desired equilibrium level.²³

Accordingly, we assume that the Central Banks can credibly commit to follow an instrument rule of the kind

$$r_t^i = rr_t^{ni} + \phi_\pi^i \pi_{i,t} + \phi_y^i x_t^i + \phi_q^i (q_{F,t} - q_{F,t}^n), \quad (88)$$

for $i = H, F$, implying that the interest rates in both countries are set so as to respond to deviations of domestic inflation, domestic output and the stock-price *level* from their natural counterparts.

This assumption affects the matrix formulation (87), and implies now $\mathbf{A} = \mathbf{A}(\vartheta, \phi)$, meaning that matrix \mathbf{A} becomes a function also of the response coefficients collected into ϕ . The latter can therefore be chosen appropriately by the Central Banks in order to guarantee uniqueness of the desired equilibrium.

As emphasized by Benigno and Benigno (2004), moving from a closed-economy to an open-economy framework complicates the analysis of equilibrium determinacy as it requires simultaneous considerations of the policies followed by all countries in the world economy. In our setting, such complications are enhanced by explicitly considering stock-price dynamics.

In order to keep the analysis general enough with respect to the response coefficients in the policy rules of the two countries, we numerically simulate the model within a wide parameter sub-space for the response coefficients. The exercise has been conducted by considering a quarterly calibration of the structural parameters, based on previous studies

²²Blanchard and Kahn (1980) show that the conditions for a unique equilibrium of such a dynamic system, under rational expectations, hinge on the eigenvalues of the coefficient matrix \mathbf{A} : more precisely, the number of the eigenvalues of \mathbf{A} laying inside the unit circle needs to be exactly equal to the number of predetermined variables in the system.

²³See, among others, Benigno and Benigno (2004), Bullard and Mitra (2002) and Galì and Monacelli (2004).

and convention. Specifically, the steady-state net quarterly interest rate r was calibrated at 0.01, implying a long-run real annualized interest rate of 4%,²⁴ and the effective decision horizon was set at 15 years, implying a probability of exiting the markets γ of 0.0167. Accordingly, in order to meet the steady-state restrictions, the intertemporal discount factor β was set at 0.991. The elasticity of substitution among intermediate goods ϵ was set at 21, implying a steady-state net mark-up rate of 5%, the elasticity of substitution between Home and Foreign goods was set equal to 1.5 and $\varphi \equiv \frac{N}{1-N}$ was chosen to be 1/3, consistent with 6 hours worked per day; finally, the dimension of the Home country n was set equal to 0.35, while the probability for firms of having to keep their price fixed for the current quarter was set at 0.75 for both countries, implying that prices are revised on average once a year.²⁵

Figure 1 shows the regions of equilibrium determinacy in the parametric space defined by the combinations of ϕ_π^H and ϕ_π^F for which the conditions on the eigenvalues of matrix \mathbf{A} hold. The plot has been drawn with respect to different possible values of the parameters ϕ_q^i which represent, in each country, the Central Bank's stance towards stock-price dynamics.²⁶

Observation of the two panels in Fig. 1 confirms that the optimal policy (which requires $\phi_\pi^i = \phi_y^i = \phi_q^i = 0$, for all $i=H, F$, and thus corresponds to the axes' origin) cannot be implemented by the two Central Banks. Such policy belongs in fact to the zone in which the equilibrium is indeterminate. The optimal policy can however be implemented through a credible threat to move the interest rate if the actual allocation deviates from the potential one, i.e. if the appropriate conditions on the reaction coefficients are met. Fig. 1 shows that, in case of no reaction to stock prices, the Taylor Principle (a response coefficient of the interest rate to inflation greater than 1) needs to be satisfied contemporaneously by both Central Banks in order for the Equilibrium to be determinate: if either one of the two countries accommodates inflation, in fact, the system generates endogenous instability *no matter how aggressive is the policy in the other country*.

Moreover, Fig. 1 provides a new interesting insight about the links between monetary policy and stock prices. Reacting to deviations of the stock-price level from the one prevailing in the Flexible-Price equilibrium reduces the determinacy zone if this response is activated by the Central Bank of country F , where the stock market is located, while it has no effects if performed in country H . For a given stance towards inflation, therefore, reacting too strongly to non-zero stock-price gaps in the country hosting the stock exchange might be destabilizing because it makes the system subject to potential endogenous fluctuations. This result is common to the CE case.

²⁴Since we concentrate on a symmetric steady state the values reported in the text are meant to refer to both countries as well as to the world economy.

²⁵All the results are robust to changes in the Calvo parameter, in particular to cases of heterogeneity with respect to the producer-price duration.

²⁶Since the effects of different responses to the output gap produce no significant differences with respect to the standard results (namely to mildly relax the Taylor Principle), we focus here on the case in which $\phi_y^H = \phi_y^F = 0$. No loss of generality in the qualitative results follows from this simplification.

Rules Responding to Stock–Price Gaps

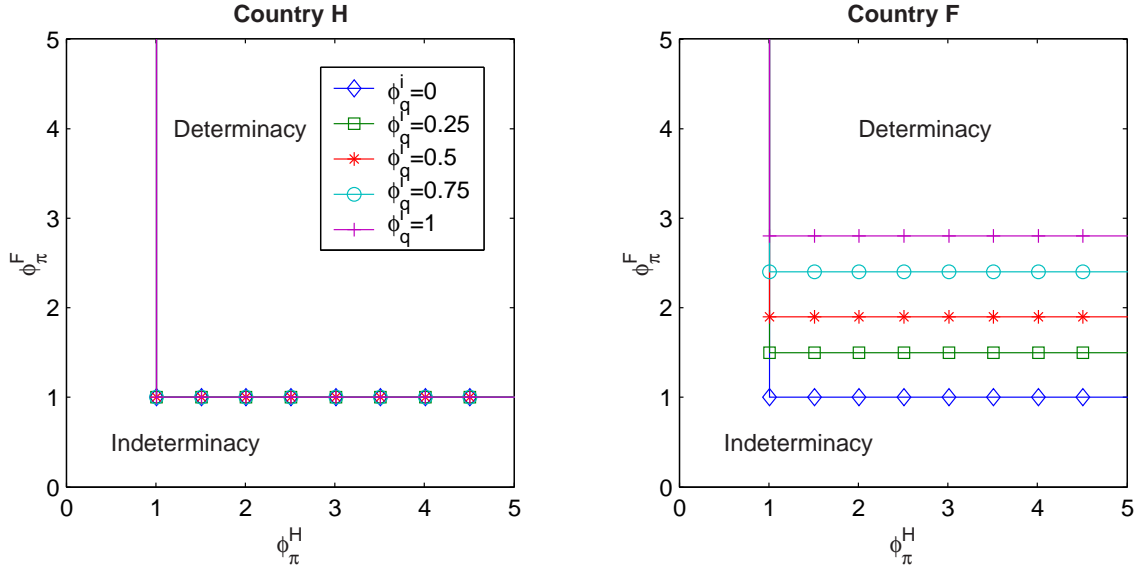


Figure 1: Determinacy Regions for different values of ϕ_q^H (left panel) and ϕ_q^F (right panel).

On the other hand, while no reaction at all yields indeterminacy and potential macro-economic instability, granting a sufficiently aggressive reaction to inflation ($\phi_\pi^H, \phi_\pi^F > 1$) ensures determinacy of the equilibrium even in the absence of any explicit concern about output and stock-price gaps ($\phi_y^H = \phi_y^F = \phi_q^H = \phi_q^F = 0$).

Hence, also in our framework, the simple commitment to the Taylor Principle qualifies as a sufficient condition for equilibrium determinacy, coherently with previous results obtained in either an open economy setting with RA or in the closed economy version of the OLG model. However, in our setting, not only inflation and output but also stock prices end up matching their natural levels at all times.

Common to the closed-economy benchmark is also the result, not showed, that shifting the central bank's concern towards stock-price *growth* eliminates the risks of endogenous instability also in the country hosting the stock exchange: no matter how aggressive the monetary policy against stock prices, the simple commitment to the Taylor principle is sufficient to restore equilibrium determinacy.²⁷

²⁷See Nisticò (2005b)

Table 1: Estimated stochastic properties of the productivity shocks.

Shock	ρ_a^i	σ_a^i	$corr(u_a^H, u_a^F)$	R^2
a^H	0.808 (7.870)	0.0049	0.0593	0.644
a^F	0.683 (11.332)	0.0055		0.466

4 Simple Policy Rules.

In this section, we establish which simple rule best proxies the optimal policy by assessing the macroeconomic performances of alternative monetary policy regimes. This assessment relies on the dynamic response of the economy to different exogenous shocks and on the implied volatility of the main variables.

Let's consider the numerical calibration of the structural parameters discussed above. The persistency, volatility and correlation of the productivity shocks were estimated using quarterly HP-filtered data on labor productivity in Canada and the U.S. for the period spanning from 1987:1 to 2005:1. The estimates obtained are reported in Table 1 (*t*-statistics in brackets).

We first compare the dynamic response of the Home economy to local and foreign productivity shocks under three alternative policy regimes adopted by the local Central Bank. Then we pick the regime that performs better at replicating the response under the optimal policy and assess the dynamic implications of adding an explicit concern about stock prices, in the two forms of a reaction to a stock-price gap and to stock-price growth. In both cases, therefore, the benchmark scenario is the optimal policy implied by a strict Domestic Inflation Targeting derived in the previous section.

The first policy regime considered is a Taylor rule in which the nominal interest rate deviates from its long-run level in response to the *domestic* inflation rate and the output gap.²⁸ We borrow the notation from Galì and Monacelli (2004) and call it DITR:

$$r_t^H = \tilde{\rho} + \phi_\pi^H \pi_{H,t} + \phi_y^H x_t^H; \quad (89)$$

while the second regime consists of a modified Taylor Rule based on the CPI-inflation rate (CITR):

$$r_t^H = \tilde{\rho} + \phi_\pi^H \pi_t^H + \phi_y^H x_t^H. \quad (90)$$

In both specifications above the response coefficients to inflation and the output gap were set to 2.5 and 0.5 respectively.

²⁸See Taylor (1993) and Clarida, Galì and Gertler (1999, 2000)

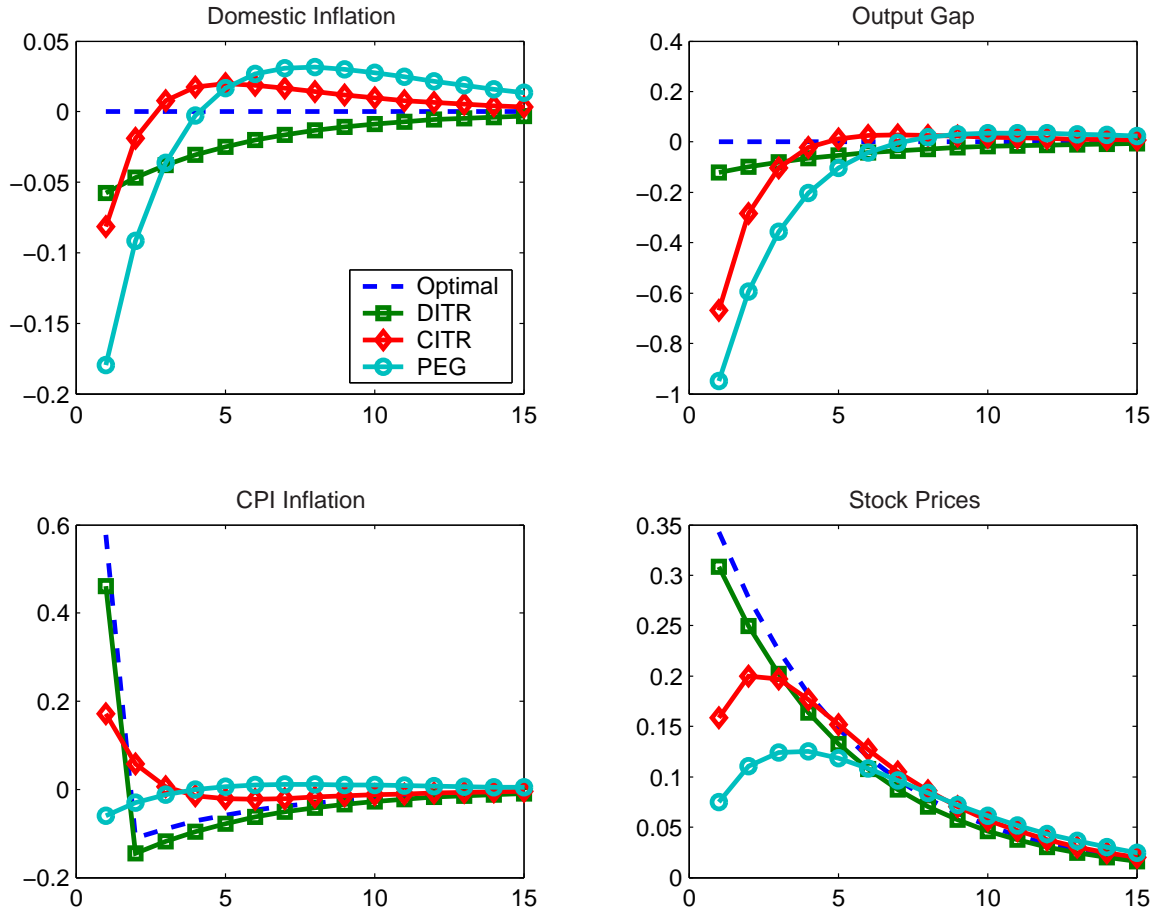


Figure 2: Dynamic responses in the H-country to a local productivity shock: no reaction to stock-price dynamics.

Finally, the third regime is a simple nominal exchange rate peg (PEG), implying

$$e_t = 0 \tag{91}$$

at all times.

We start by assuming that the Central Bank in the other country adopts a strict Domestic Inflation Targeting and hence follows the optimal rule of equation (85).

Figures 2 and 3 show the dynamic response of several variables to a local productivity shock, under the four regimes considered for the Home central bank. Under strict Domestic Inflation Targeting, given the absence of trade-offs between inflation and output stabilization, the reaction of domestic inflation and the output gap are obviously nil at all times (dashed lines), while the nominal and real interest rates fall to accommodate the expansion in potential output. Given the smaller reaction of the foreign interest rate (proportional to

the relative size of country H , which is small), the UIP drives an on-impact depreciation in the nominal exchange rate and the terms of trade, followed by a transition period in which both the exchange rate and the terms of trade appreciate and revert to their long-run levels. The dynamics of the terms of trade under the Optimal Policy has the usual effect of having the CPI inflation rate jump on impact, as a consequence of the initial depreciation in the ToT, while the following progressive appreciation drives the CPI inflation on a dynamics path converging to zero from below.

In our framework, the dynamics of the terms of trade has an additional impact on stock prices, through two conflicting effects: a price effect of ToT level (relatively higher current domestic prices raise dividends for a given demand of F -goods) and a substitution effect of the ToT appreciation rate (a ToT expected appreciation makes F -goods more competitive and raises the global demand for given prices). The second effect is dominant under most parameterizations and the final result for the stock-price level is therefore an initial jump upwards followed by a mean-reverting transition, a dynamics closely mirroring that of the ToT (proportionally to the relative size of country H that here we calibrated at $n = 0.35$).

The graphs suggest that the simple rule whose implications on the global dynamics are closest to those implied by the Optimal Policy is the DITR.²⁹ The CITR (consistently with its aim of holding back the initial response in the CPI inflation) generates a smoother impact and a hump-shaped response of the exchange rate, the terms of trade and the stock-price level, by means of an initial increase in the interest rate that entails the cost of a lower output gap in the first three periods, relative to the DITR. Pegging the nominal exchange rate, on the other hand, by fixing interest and exchange rates, prevents the domestic output from mirroring the expansion in its potential level. The result is a fall in domestic inflation, a consequent increase in the real interest rate and a limited response in the terms of trade which amplify the negative response of both domestic inflation and the output gap.

In order to compare the macroeconomic implications of adding an explicit concern about stock-price dynamics, Figures 4 and 5 compare the response of the same variables under the Optimal Policy and the DITR with those implied by a Taylor Rule that augments the DITR by allowing the interest rate to respond also to stock-price gaps (DI-Gap)

$$r_t^H = \tilde{\rho} + \phi_\pi^H \pi_{H,t} + \phi_y^H x_t^H + \phi_q^H (q_{F,t} - q_{F,t}^n) \quad (92)$$

or to the stock-price growth rate (DI-Gro)

$$r_t^H = \tilde{\rho} + \phi_\pi^H \pi_{H,t} + \phi_y^H x_t^H + \phi_q^H (q_{F,t} - q_{F,t-1}), \quad (93)$$

In both specifications, the new response coefficient was set at 0.75.

Observation of the figures above highlights that adding a reaction to stock-price dynamics produces some beneficial effects on the performance of the DITR in terms of domestic

²⁹This result is extremely robust to all specifications relative to the policy regime adopted abroad: whatever is the conduct of the Central Bank in country F , within country H the DITR always outperforms all other regimes.

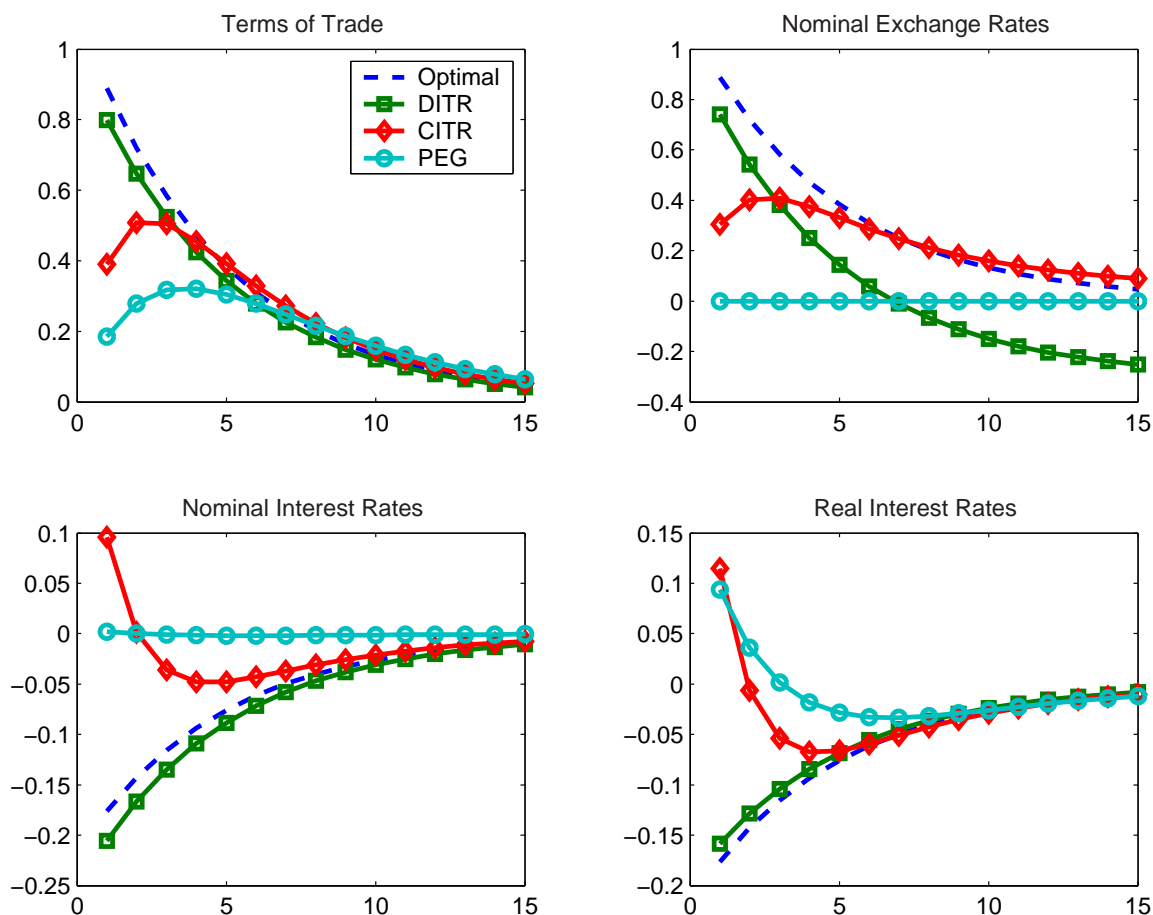


Figure 3: Dynamic responses in the H-country to a local productivity shock: no reaction to stock-price dynamics.

inflation and the output gap without bearing a cost in terms of the dynamic response of other variables in the model, whose response under the Optimal Policy was already well replicated by the DITR. When the productivity shock originates at home, reacting to stock-price growth yields the greatest gains (relative to the DI-Gap). These result in a dynamic path of domestic inflation half as distant from the optimal one as the one prevailing under DITR. The effect on the output gap is less striking, and for both variables the impact effect is less appealing than under a DITR: the stock-price level jumps on impact and generates a large positive growth rate in the first period, which drives a reduced accommodative response of the interest rates.

When analyzing the dynamic response to a shock originated in the Foreign country, however, the conclusions are reversed. While the DITR keeps on being the outperforming regime in the Home country, adding a reaction to stock-price growth yields a more volatile

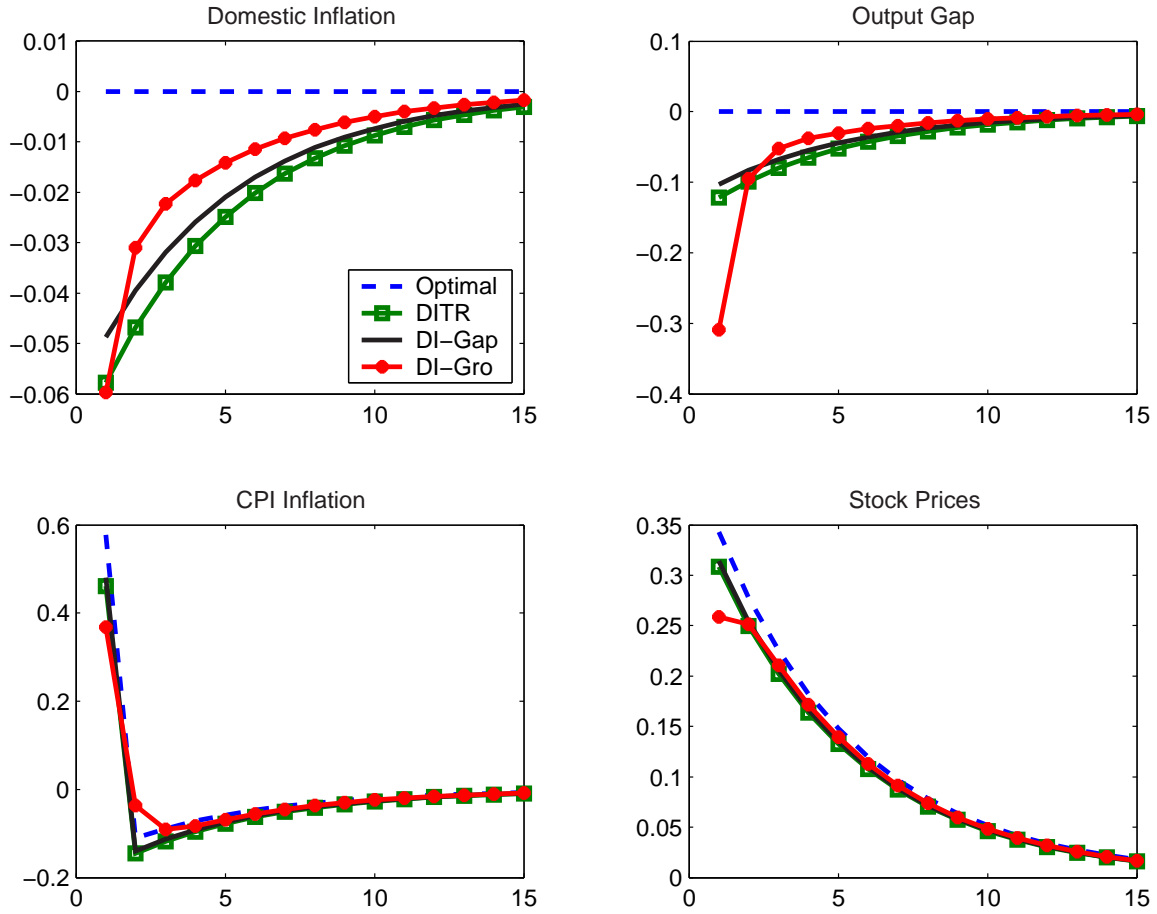


Figure 4: Dynamic responses in the H-country to a local productivity shock when the CB responds to stock-price dynamics.

response in both domestic inflation and output, while reacting to the stock-price gap replicates the dynamic performance of the DITR and therefore stems as best-performing (see Figure 6 in Appendix A.)

Overall, as reported in Table 2, moving from a DITR to a DI-Gap results in stability gains for the output gap, domestic inflation and the interest rates, while reacting to stock-price growth seems to be more stabilizing only with respect to stock prices. A Central Bank in the Home country whose loss function is the familiar weighted average of the volatility of output gap, inflation and interest rates, then, would find in DI-Gap the simple rule to follow.³⁰

³⁰We have repeated the analysis under the assumption that the Central Bank in the Foreign country adopts either a DITR or a DI-Gro regime and obtained similar qualitative results.

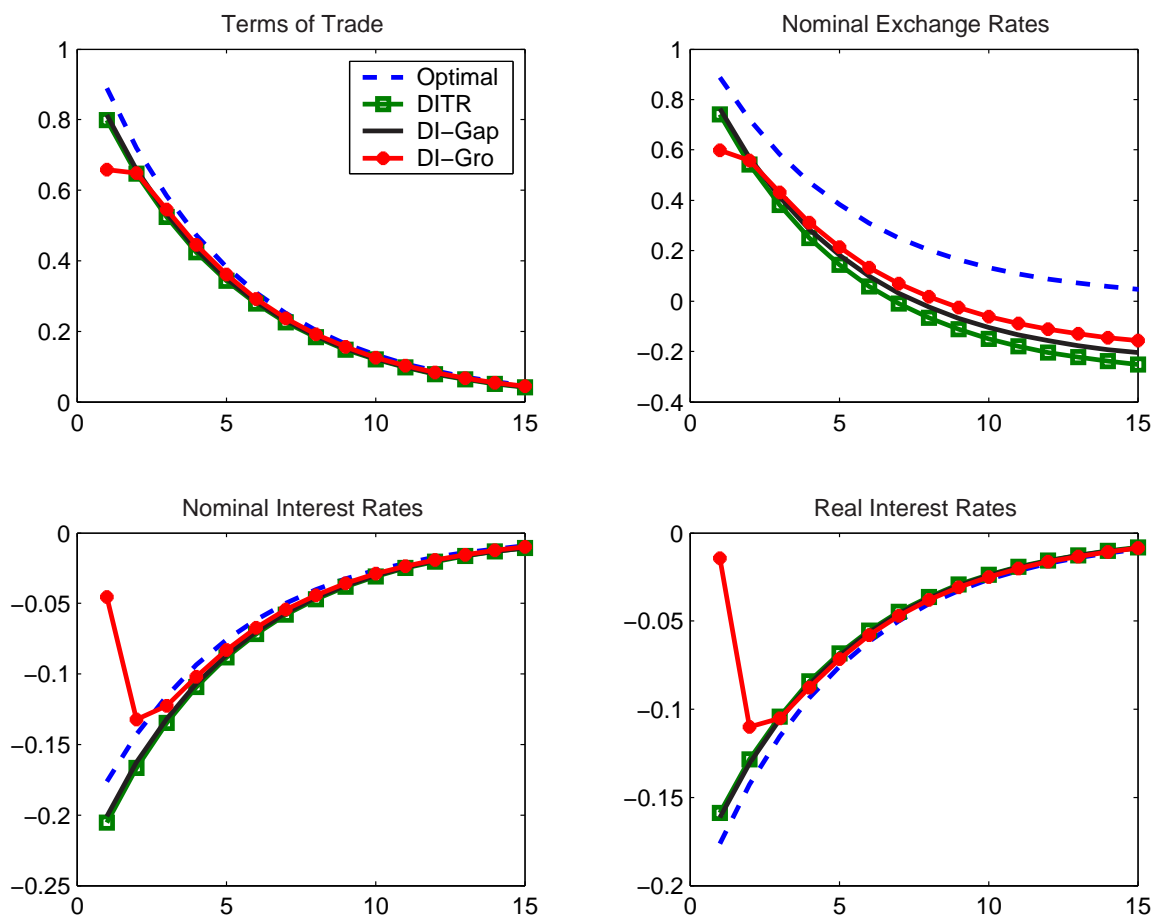


Figure 5: Dynamic responses in the H-country to a local productivity shock when the CB responds to stock-price dynamics.

5 Summary and Conclusions.

This paper presents an open-economy version of a Dynamic New Keynesian OLG economy where agents are assumed to be able to diversify their financial portfolios by simultaneously holding bonds and equities. In particular, an active stock market is located in the Foreign country, which we view as larger and “financially dominant”. This market is open to Home country consumers, thereby inducing a financial channel for international transmission of disturbances.

We characterize the optimal monetary policy sequences of interest rates that guarantee flexible-price allocations in the two countries as well as in the world economy. We show that (contrary to what happens for a closed economy) such sequences do require both Central Banks to actively respond to stock-price dynamics driven by relative local productivity

Table 2: Implied Volatility in the Home Country

	Optimal	DITR	DI-Gap	DI-Gro	CITR	PEG
Output Gap	0.000	0.124	0.104	0.296	0.492	0.843
Domestic Inflation	0.000	0.049	0.041	0.049	0.081	0.140
CPI Inflation	0.444	0.428	0.429	0.533	0.193	0.047
Interest Rates	0.150	0.181	0.175	0.237	0.286	0.237
Terms of Trade	0.971	0.926	0.932	0.974	0.773	0.482
Stock Prices	0.583	0.565	0.568	0.545	0.603	0.686

Note: Standard Deviations in %; Country F follows the Optimal Rule

shocks, which affect consumers' budget constraint through real wealth effects. We compare our results with two benchmark cases, the open-economy representative-agent model and the closed-economy version of our OLG economy. These can be viewed as particular cases of our more general setting, for given values of our structural parameters. However, we show that the result that monetary policy should not grant a dedicated response to stock-price dynamics induced by pure productivity shocks (obtained in the two benchmark cases considered) does not hold in our more general framework.

In our model, optimal monetary policy is defined in terms of price stability only, as the task of eventually correcting distortions due to the presence of monopolistically competitive firms is not considered and we abstract from cost push shocks that would induce a conflict between inflation and output stabilization. Although the Wicksellian interest rate sequences that we obtain are not sufficient to guarantee uniqueness of the flexible price allocation, we show that if the Central Banks can credibly commit to an instrument rule in which the Taylor principle is satisfied, the optimal policy can be easily implemented. We also show, however, that an aggressive reaction to deviations in stock-price *levels* by the Central Bank in the Foreign country might induce endogenous and persistent macroeconomic instability (as in the closed economy benchmark), while this destabilizing implications do not hamper policy makers in the Home country.

Analysis of the macroeconomic implications of simple (and therefore operational) monetary policy rules, finally, shows that in the Home country the performance of the Optimal Policy in terms of dynamic response and second moments of the target variables are best replicated by a *domestic inflation-based* Taylor Rule, and that some beneficial effects are yielded *at no costs* from augmenting the DITR with a reaction to deviations of the stock-price index from its potential level. The result that in the Home country reacting to stock-price gaps does not yield indeterminacy *per se*, therefore, turns out to be crucial in differentiating the optimal conduct for the two countries.

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A Appendix

The Perfect Foresight Steady State. We need to show that in our model, the symmetric Perfect Foresight Steady State drives the equilibrium Terms of Trade to 1.

Recall that, in each period and in each country, the aggregate demand of domestic output is linked to world output through the following expressions

$$Y_t^H = \left(\frac{P_t^H}{P_{H,t}^H} \right)^\theta Y_t \quad Y_t^F = \left(\frac{P_t^F}{P_{F,t}^F} \right)^\theta Y_t. \quad (\text{A.1})$$

From the law of one price and the definitions of ToT ($S_t \equiv P_{F,t}^i/P_{H,t}^i$) and CPI, moreover, the following also holds:

$$Y_t^H = \left(\frac{P_t^H}{P_{H,t}^H} \frac{P_{F,t}^F}{P_t^F} \right)^\theta Y_t^F = \left(\frac{\mathcal{E}_t P_t^F}{\mathcal{E}_t P_{H,t}^F} \frac{P_{F,t}^F}{P_t^F} \right)^\theta Y_t^F = \left(\frac{P_{F,t}^F}{P_{H,t}^F} \right)^\theta Y_t^F = \mathcal{S}_t^\theta Y_t^F \quad (\text{A.2})$$

$$\left(\frac{P_t^H}{P_{H,t}^H} \right)^{1-\theta} = n + (1-n)\mathcal{S}_t^{1-\theta} \quad \left(\frac{P_t^F}{P_{F,t}^F} \right)^{1-\theta} = n\mathcal{S}_t^{\theta-1} + (1-n). \quad (\text{A.3})$$

implying that in a Perfect Foresight Steady State we have:

$$Y^H = \left(\frac{P^H}{P_H^H} \right)^\theta Y \quad Y^F = \left(\frac{P^F}{P_F^F} \right)^\theta Y \quad Y^H = \mathcal{S}^\theta Y^F \quad (\text{A.4})$$

$$\left(\frac{P^H}{P_H^H} \right)^{1-\theta} = n + (1-n)\mathcal{S}^{1-\theta} \quad \left(\frac{P^F}{P_F^F} \right)^{1-\theta} = n\mathcal{S}^{\theta-1} + (1-n). \quad (\text{A.5})$$

It is now possible to show that a steady state featuring symmetry (about the long-run level of the productivity index: $A^H=A^F=A$) yields:

$$C = Y = \frac{W^i}{P^i} (1 - N^i) = \frac{1}{\mu} \frac{P_i}{P^i} (A - Y^i) \quad (\text{A.6})$$

$$\left(\frac{P^i}{P_i} \right)^{1-\theta} Y^i = \frac{1}{\mu} (A - Y^i). \quad (\text{A.7})$$

and also:

$$\frac{A}{\mu} = Y^H \left[\frac{1}{\mu} + \left(\frac{P}{P_H} \right)^{1-\theta} \right] = Y^F \left[\frac{1}{\mu} + \left(\frac{P}{P_F} \right)^{1-\theta} \right] \quad (\text{A.8})$$

$$Y^H \left(\frac{1}{\mu} + n + (1-n)\mathcal{S}^{1-\theta} \right) = \mathcal{S}^{-\theta} Y^H \left(\frac{1}{\mu} + n\mathcal{S}^{\theta-1} + (1-n) \right) \quad (\text{A.9})$$

$$\frac{1}{\mu} + n + (1-n)\mathcal{S}^{1-\theta} = \frac{\mathcal{S}^{-\theta}}{\mu} + n\mathcal{S}^{-1} + (1-n)\mathcal{S}^{-\theta}, \quad (\text{A.10})$$

Claim: Equation (A.10) implies $\mathcal{S} = 1$.

Proof: Equation (A.10) defines the condition that $v(\mathcal{S}) = 0$, where we define

$$v(\mathcal{S}) \equiv \left(\frac{1}{\mu} + n\right)\mathcal{S}^\theta - n\mathcal{S}^{\theta-1} + (1-n)\mathcal{S} - \left(\frac{1}{\mu} + (1-n)\right),$$

and recall that $\theta \geq 1$.

First, notice that $v(0) = -\left(\frac{1}{\mu} + (1-n)\right) < 0$ and that $\lim_{\mathcal{S} \rightarrow \infty} v(\mathcal{S}) = +\infty$, given that the term with power θ dominates.

For $\theta = 1$ the function $v(\mathcal{S})$ becomes linear in the terms of trade and the proof that condition (A.10) holds is straightforward:

$$\begin{aligned} v(\mathcal{S}) &\equiv \left(\frac{1}{\mu} + n\right)\mathcal{S} - n + (1-n)\mathcal{S} - \left(\frac{1}{\mu} + (1-n)\right) = \frac{1+\mu}{\mu}\mathcal{S} - \frac{1+\mu}{\mu} \\ v(\mathcal{S}) = 0 &\iff \mathcal{S} = 1. \end{aligned}$$

For $\theta > 1$ the function $v(\mathcal{S})$, though non-linear, is strictly increasing in \mathcal{S} , with a unique zero in $\mathcal{S}=1$. In order to show this, take the first derivative

$$v'(\mathcal{S}) = \theta \frac{1+n\mu}{\mu} \mathcal{S}^{\theta-1} - (\theta-1)n\mathcal{S}^{\theta-2} + (1-n)$$

and notice that it is positive for all positive values of the terms of trade. In fact: $v'(0) = (1-n) > 0$, $v'(1) = \theta \frac{1+n\mu}{\mu} - \theta n + 1 = \theta \frac{1}{\mu} + 1 > 0$, $v'(+\infty) = +\infty > 0$ since the first term is dominant and finally $\theta \frac{1+n\mu}{\mu} > (\theta-1)n$.

As a result, the equation $v(\mathcal{S})=0$ has a unique solution, and we already showed that this corresponds to $\mathcal{S}=1$:

$$v(1) = \frac{1}{\mu} + n - n + (1-n) - \frac{1}{\mu} - (1-n) = 0.$$

□

Given the steady state value of the ToT, the long-run levels of the other relevant variables fulfill:

$$C = Y = Y^H = Y^F = \frac{A}{1+\mu}.$$

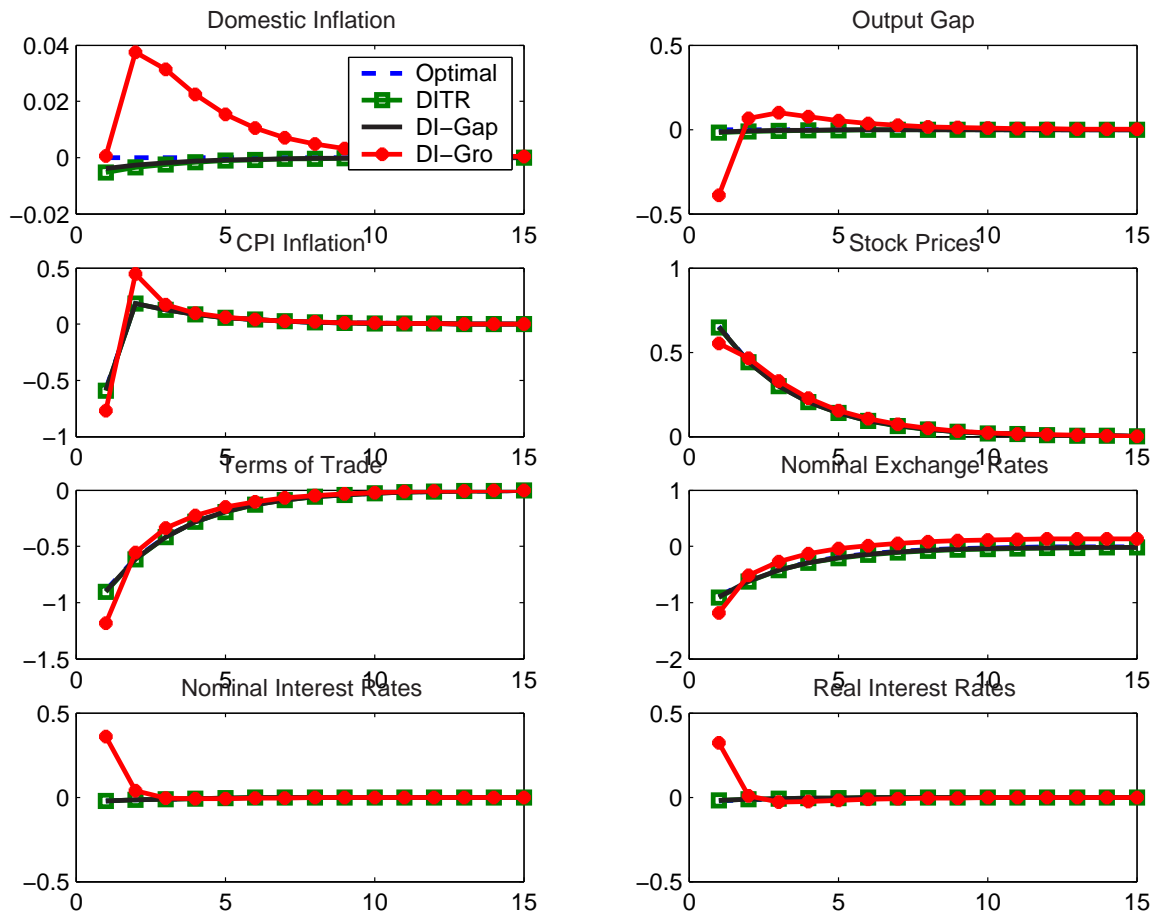


Figure 6: Dynamic responses in the H-country to a Foreign productivity shock when the local CB responds to stock-price dynamics and the Foreign CB follows the optimal Policy.