Imprecise News, Gradual Information Processing and Mini Flash Crashes

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International Workshop on Algorithmic and High Frequency Trading

November 8, 2013 - Banque de France
Motivation

- Improvement in trading technologies enable speculators to trade very fast on news before it gets partially or fully reflected into prices.

- However, in reacting fast to information, speculators take the risk of trading on misleading information as information processing (e.g., checking news accuracy) takes time.

Figure: The “Twitter Crash” of April 2013
Motivation

**Figure**: "Micro Flash Crashes 2006 - 2011", Nanex Research
Research Question

- Empirical studies suggest that high frequency trading has a positive effect on market efficiency (e.g. Brogaard, Hendershott and Riordan (2013), Boehmer, Fong and Wu (2013)).

- Can faster trading on information make financial markets more informationally efficient and yet more prone to mini-flash crashes?

- Is faster trading sufficient to improve efficiency at medium term?
Yes... because information processing is gradual.

- What do we mean by information processing?
  - turning a signal into sound information
  - figuring out if a signal is informative or not

- Processing information takes time
  **Assumption**: faster trading does not accelerate information processing. It remains "gradual".

- But fast trading technologies allows for trading before information processing is terminated.
Findings

- With faster trading, informational efficiency and mini flash crashes are not contradictory.

- Faster trading allows informed traders for trading more times on the same signal (twice vs. once).

- However a low cost of fast trading technologies can reduce the demand for information.

- Overall, faster trading enhances efficiency because the market can better infer signals and their informative nature.

- Faster trading increases the likelihood of price reversals (mini flash crashes) because it involves a risk of trading on noise.

• **Early signal acquisition**: Froot, Scharfstein and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), and Brunnermeier (2005).

• **Market instability**: Golub, Keane and Poon (2012), Gao and Mizrach (2013).

• Carvalho, Klagge and Moench (2009), Engle, Hanse and Lunde (2011).
Baseline Model

- **A 3 periods model.** At $t = 1$ and $t = 2$ the market is open. At $t = 3$, the asset pays off.

- **Asset value.** The asset pay-off is $V$. $V$ is equal to 0 or 1 with equal probabilities.

- **Liquidity Trading.** Some liquidity traders send random orders at $t = 1$ and $t = 2$. At each date, their order flow follows a uniform distribution on the interval $[-Q, Q]$.

- **Market making.** At each date a risk neutral and competitive market maker receives the aggregate order flow of liquidity and informed traders and set a price.
Informed Trading

• **Two types of informed traders** :
  
  → Ex-ante, a mass $\beta$ of risk neutral traders becomes **informed** after paying a cost $C_p$.

  → A fraction $\alpha < \beta$ pays an extra cost $\Delta$ to become **fast** : they acquire a signal $S$ and can trade before and after its processing, resp. at $t = 1$ and $t = 2$,

  → The remaining fraction of traders, $\beta - \alpha$, can only trade after processing $S$, at $t = 2$.

• **Trading constraint** : At each period, the trade size of one informed traders is in the interval $[-1, 1]$.

• **Information acquisition and processing** :
  
  → At $t = 1$, informed traders observe a signal $S$ :

$$S = U \times V + (1 - U) \times \varepsilon,$$

  with $U \in \{0, 1\}$, $Pr[U = 1] = \theta$ and $\varepsilon$ a white noise.

  → At $t = 2$, informed traders observe $S$ and $U$.

  → Information processing is **gradual** : it takes 2 periods.
**Equilibrium Trading Strategies**

\[ \text{Ex-ante} \]

\[ t = 1 \]

\[ t = 2 \]

\( \beta \) traders become informed

\( \alpha < \beta \) informed become fast.

- Fast traders observe \( S \),
  buy if \( S = 1 \) and sell \( S = 0 \).

- Liquidity traders send orders.

- The market maker receives the order flow and set the price \( p_1 \)

- Informed traders observe \( S \) and \( U \).
  If \( U = 1 \), they buy if \( S = 1 \) and sell \( S = 0 \)
  If \( U = 0 \), they trade in the opposite direction of the \( t = 1 \) price change.

- Liquidity traders send orders.

- The market maker receives the order flow and set the price \( p_2 \).
Price Dynamics

Price dynamics conditional on $S = 0$

$p_0 = \frac{1}{2}$

$p_1 = \frac{1}{2}$

$p_1 = \frac{1-\theta}{2}$

$p_2 = \frac{1-\theta}{2}$

$p_2 = 0$
Trading Profits

- The gross profit of an informed trader who trades before processing, at 
  \( t = 1 \), is

  \[
  \pi_1(\alpha) = \frac{\theta}{2} \times \left( 1 - \frac{\alpha}{Q} \right). 
  \]

- The gross profit of an informed trader who processes first and then trades, 
  at \( t = 2 \), is

  \[
  \pi_2(\alpha, \beta) = \frac{\theta}{2} \times \left[ \left( 1 - \frac{\alpha}{Q} \right) \times \left( 1 - \frac{1}{2 - \theta} \frac{\beta}{Q} \right) + (1 - \theta) \frac{\alpha}{Q} \left( 1 - \frac{\beta}{Q} \right) \right] 
  \]

- An informed trader, who only trades after processing, obtains an expected 
  profit of

  \[
  \pi_2(\alpha, \beta) - C_p 
  \]

  while an informed trader, who trades before and after processing, obtains a 
  total expected profit of

  \[
  \pi_1(\alpha) + \pi_2(\alpha, \beta) - (\Delta + C_p). 
  \]
Equilibrium Conditions

Figure: Equilibrium where $\alpha^* < \beta^*$
**Figure**: Equilibrium where $\alpha^* = \beta^*$
Demand for Information and The Growth of Fast Trading

Proposition

• The demand for information in equilibrium, $\beta^*$, is a U-shape function of the cost of trading fast on information, $\Delta$. It reaches a minimum for $\Delta = \bar\Delta$.

• For $\Delta > \bar\Delta$, some informed traders never trade on information without processing it before, $\beta^* > \alpha^*$, with $\alpha^* = \max \left[ \frac{Q}{2} \left( 1 - 2 \frac{\Delta}{\theta} \right), 0 \right]$.

• For $\Delta \leq \bar\Delta$, no informed traders choose to process information before trading on it, $\beta^* = \alpha^*$.

• There exists a value $C_p^* \in \left[ \frac{\theta}{2}, \frac{1 - \theta}{2 - \theta}, \frac{\theta}{2} \right]$ such that
  $\rightarrow$ if $C_p > C_p^*$ then $\beta^*$ is maximal for $\Delta = 0$
  $\rightarrow$ while if $C_p < C_p^*$ then $\beta^*$ is maximal when $\Delta \geq \frac{\theta}{2}$, and $\alpha^* = 0$. 
Demand for Information and The Growth of Fast Trading

**Figure**: Total demand of information, $\beta^*$, as a function of $\Delta$
Demand for Information and The Growth of Fast Trading

**Figure**: Mass of speculators who trade on information before processing it, $\alpha^*$, as a function of $\Delta$. 
Demand for Information and The Growth of Fast Trading

Figure: $\alpha^*$ and $\beta^*$ as functions of $\Delta$
Informational Efficiency

We measure informational efficiency at date $t$ by the average pricing error at this date, $\mathcal{V}_t = \mathbb{E}[(\tilde{V} - P_t)^2]$

$$\mathcal{V}_1 = \frac{1}{4} - \frac{\theta}{2} \left( \frac{\theta}{2} - \pi_1(\alpha^*) \right)$$

$$\mathcal{V}_2 = \frac{1}{4} - \frac{1}{2} \left( \frac{\theta}{2} - \pi_2(\alpha^*, \beta^*) \right)$$

Proposition

A reduction in the cost of fast trading technologies $\Delta \in [0, \theta/2]$

- always improve informational efficiency at $t = 1$,
- leaves informational efficiency at $t = 2$ when $\Delta > \bar{\Delta}$,
- improves informational efficiency at $t = 2$ if $\Delta < \bar{\Delta}$

→ When $\Delta$ declines, the demand for information may decrease but the market is always more efficient at medium term.

→ Gradual trading help dealers to better disentangle these two sources of uncertainty: the signal and its informativeness.
Price Reversals

When the signal $S$ proved to be noise, the model generates price reversal.

- The magnitude of a price reversal is
  \[ M_{\text{Reversal}} = \frac{\theta}{2}. \]

- The likelihood of a price reversal, between $t = 1$ and $t = 3$, is
  \[ p_{\text{Reversal}} = (1 - \theta) \frac{\alpha^*}{Q}, \]

- The likelihood of a quick price reversal, between $t = 1$ and $t = 2$, is
  \[ p_{\text{Quick Reversal}} = (1 - \theta) \frac{\alpha^* \beta^*}{Q^2}. \]

Proposition

- Holding $\theta$ fixed, $p_{\text{Reversal}}$ and $p_{\text{Quick Reversal}}$ increase when $\Delta$ decreases.
- $p_{\text{Reversal}}$ and $p_{\text{Quick Reversal}}$ are inverse U-shape functions of the precision of the signal received by event traders.
Quick Reversals and Mini Flash Crashes

• Empirically, it is natural to define a **mini flash crash** as
  → a **quick reversal**
  → large relative to some measure of normal return volatility.

• We say that a mini-flash crash happens if:
  → there is a quick price reversal between $t = 1$ and $t = 2$,
  → the size of the reversal, equal to $\frac{\theta}{2}$, is larger than $\frac{R}{2}$ where $0 \ll R \leq 1$ is a positive constant less than one.

• We introduce some variation in return volatility by allowing $\theta$ to be stochastic with distribution $f(\theta)$

$$p_{\text{crash}} = \int_{R}^{1} p_{\text{Quick Reversal}}(\theta) f(\theta) d\theta$$
Quick Reversals and Mini Flash Crashes

Example: \( \theta = X^\lambda \) with \( X \) uniformly distributed on \([0, 1]\)

\[
Pr[\theta > R] = 1 - R^{\frac{1}{\lambda}}, \quad E[\theta] = \bar{\theta} = \frac{1}{\lambda + 1}
\]

**Figure:** Likelihood of a flash crash as a function of the mean signal precision \( \bar{\theta} \)
(i) for different values of \( R \): \( R = 10\% \) (plain line), \( R = 30\% \) (dashed line), and \( R = 70\% \) (dotted line),
(ii) for different values of \( \Delta \): \( \Delta = 0.1 \) (plain line), \( \Delta = 0.05 \) (dashed line), and \( \Delta = 0.01 \) (dotted line)
Benchmark Case: Informed Dealer

Price dynamics conditional on $S = 0$

- $p_0 = \frac{1}{2}$
- $p_1 = \frac{1-\theta}{2}$
- $p_2 = 0$
- $p_2 = \frac{1}{2}$
- $1 - \theta$
- $\theta$

Introduction
Model
Equilibrium
Model Implications
Conclusion
Trade Patterns

Proposition

In equilibrium, the covariance between the trades of a trader who trades before and after processing information, at \( t = 1 \) and \( t = 2 \) is:

\[
\text{Cov}(x_1, x_2) = \theta - (1 - \theta) \frac{\alpha^*}{Q},
\]

**Figure**: \( \text{Cov}(x_1, x_2) \) as a function of \( \theta \)
Trade Patterns

Proposition

In equilibrium, the covariance between the first period return \((p_1 - p_0)\) and the trade of an informed trader at \(t = 2\) is:

\[
\text{Cov}(p_1, x_2) = \theta (2\theta - 1) \frac{\alpha^*}{Q}.
\]

**Figure**: \(\text{Cov}(p_1, x_2)\) as a function of \(\theta\)
• When information processing is gradual and news are imprecise, a lower cost for fast trading technologies ameliorates efficiency but generates mini flash crashes.

• A lower cost for fast trading technologies has also a non-monotonic effect on the demand for information.

• The precision level of news has a positive effect on efficiency and a non-monotonic effect on mini flash crashes.

• Depending on the precision level of the news, the trades of fast traders across periods can be positively or negatively correlated on average.

• Similarly, the correlation between the trades of fast traders and past price returns can be positive or negative.