CoMargin

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Abstract: We present CoMargin, a new methodology to estimate collateral requirements in derivatives central counterparties (CCPs). CoMargin depends on both the tail risk of a given market participant and its interdependence with other participants. Our approach internalizes trading externalities and enhances the stability of CCPs, thus, reducing systemic risk concerns. We assess our methodology using proprietary data from the Canadian Derivatives Clearing Corporation that includes daily observations of the actual trading positions of all of its members from 2003 to 2011. We show that CoMargin outperforms existing margining systems by stabilizing the probability and minimizing the shortfall of simultaneous margin-exceeding losses.

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1. Introduction

In an effort to enhance market stability after the recent financial crisis, G20 member countries mandated the central clearing of standardized derivatives (US Department of Treasury, 2009; European Union, 2012). As a consequence, a growing fraction of the $700 trillion derivatives market, in notional terms (BIS, 2013), is now being channeled through central counterparties (CCPs). The resulting concentration of default risk in CCPs has increased their systemic importance (Acharya et al., 2009; Pirrong, 2011; Duffie, Li, and Lubke, 2010; Duffie, 2013a; Menkveld, 2013), making it necessary for regulators and market participants to assess and potentially re-evaluate the risk management practices of these institutions.[1]

Precautionary measures used to protect CCPs against default include margin (collateral) requirements, minimum capital levels, and contributions to risk mutualization arrangements, such as a default fund. In addition, members are required to segregate their firm (i.e., proprietary) and client accounts (Jordan and Morgan, 1990), and enter into private insurance agreements (Jones and Pérignon, 2013). Of all of these safeguards, however, margin requirements are the most widely used and arguably the most important and effective.[2]

Margining systems commonly used in CCPs, such as the Standard Portfolio Analysis of Risk (SPAN) or the Value-at-Risk (VaR) approach, estimate collateral requirements based on a coverage level of potential losses for an individual contract or portfolio of contracts (Figlewski 1984; Kupiec 1994; Booth et al., 1997; Cotter, 2001; Day and Lewis, 2004; Chicago Mercantile Exchange (CME), 2012). However, by focusing only on individual exposures, these systems ignore the fact that the profits and losses (P&Ls) of clearing members (CMs) are often highly interdependent, and can leave the CCP exposed to simultaneous extreme losses. Therefore, we propose a new margining

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[1] In a derivatives exchange, the clearing house confirms, matches, and settles all trades. Clearing houses operate with a small number of firms, also known as clearing members, who are allowed to submit proprietary and customer trades for clearing. The process of novation allows the clearing house to become the sole counterparty to every trade cleared by its members (Bliss and Steigerwald, 2006; Pirrong, 2011). Thus, the clearing house concentrates credit risk but remains market risk neutral at all times by matching and netting long and short positions.

[2] Acworth (2007) shows that the ratio between aggregate margins and total default protection (i.e., margin + default fund + default insurance + other guarantees) is 88.7% for the Chicago Mercantile Exchange (CME), 95.1% for Eurex, and 96.3% for OCC. The collateral used to satisfy margin requirements in CCPs primarily consists of Treasuries (>90%) and other high-quality liquid assets. Cash accounts for less than 1% of all collected margin.

methodology, called CoMargin, which systematically adjusts collateral requirements as the variability and interdependence of P&Ls increase.

To highlight the importance of P&L interdependence, consider the role of collateral in a CCP. Margin requirements are designed to protect the CCP against losses in clearing member portfolios. However, there are times when these losses exceed the value of pledged collateral, resulting in negative margin account balances. Clearing members facing these "exceedances" may delay payment or even default on their obligations; thus, generating a shortfall in the market as the CCP is obligated to compensate all winning counterparties. Usually, financing the shortfall of a single clearing member over a short period does not impose a hefty financial burden on the CCP; nevertheless, it can still affect market liquidity, particularly during volatile periods. On the other hand, when two or more large clearing members experience simultaneous exceedances due to their P&L interdependence, the resources of the CCP could be eroded to the point of financial distress or even failure. While rare, CCP failures have occurred in the past and proven to be economically significant. Examples include Paris in 1973, Kuala Lumpur in 1983 and Hong Kong in 1987 (Bernanke, 1990; Knott and Mills, 2002; Duffie, 2013b).[3]

P&L interdependence increases (1) with trade crowdedness and (2) underlying asset comovement. Trade crowdedness occurs when clearing members hold similar portfolio positions. Hirshleifer, Subrahmanyam and Titman (1994) argue that this situation arises among large market participants with common informational advantages that lead them to pursue similar directional trades, arbitrage opportunities and hedging strategies.[4] Underlying asset comovement, on the other hand, occurs when asset returns move in the same direction. This is a common phenomenon during economic slowdowns or periods of high volatility.[5]

[3] Default of CMs is of course more frequent. Recent examples in the CME include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon, 2013).
[4] Much of the proprietary trading activity in derivatives exchanges consists of arbitraging futures and OTC or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage, etc.).
[5] The importance of asset comovement has been identified in previous studies. For example, Gemmill (1994) highlights the diversification benefit clearing houses obtain from combining contracts on uncorrelated or weakly correlated assets. Extreme dependence and contagion is discussed in Longin and Solnik (2001), Bae, Karolyi and Stulz (2003), Longstaff (2004), Poon, Rockinger and Tawn (2004), Boyson, Stahel and Stulz (2010), Harris and Stahel (2011), and Christoffersen et al. (2012) among others. Billio, Getmansky, Lo, and Pelizzon (2012) connect extreme financial equity comovements with systemic risk.

To account for P&L interdependence, our CoMargin method estimates the margin requirement of each clearing member conditional on one or several other members being in financial distress. In our context, a clearing member is said to be in financial distress if its losses exceed the VaR of its P&L distribution.[6] By adopting this approach, we obtain individual margin requirements that increase with P&L dependence, stabilize the probability of exceedance events given financial distress, and reduce the risk that the clearing house exhausts its funds due to large or sudden shortfalls.

CoMargin builds on the CoVaR concept introduced by Adrian and Brunnermeier (2011). However, while the objective of CoVaR is to identify systemically important financial institutions, that of CoMargin is to estimate margin requirements that account for P&L interdependence. Thus, CoVaR is defined as the VaR of the banking sector's equity returns conditional on a given institution experiencing a return equal to its VaR. CoMargin, on the other hand, is defined as the VaR of a clearing member's portfolio P&L, conditional on one or several other clearing members experiencing losses that exceed their VaR.

Despite its conditional and multivariate nature, CoMargin is extremely simple to estimate. The process starts by taking the trading positions of all clearing members at the end of the trading day as given. Then, a series of one-day-ahead scenarios based on projected changes in the price and volatility of the underlying assets is used to assess changes in the value of each member's portfolio. For each scenario, we mark-to-model the portfolio of each clearing member and obtain its hypothetical one-day-ahead P&L. Based on these hypothetical P&Ls, we compute margin requirements that target the probability of margin exceedances conditional on the financial distress of other members.

Through mathematical proofs and the use of Monte Carlo simulations, we show that CoMargin has desirable properties that could greatly reduce some of the systemic risk concerns associated with CCPs. First, the CoMargin of a clearing firm increases with the variability and interdependence of

[6] CoMargin echoes recent calls by regulators for focusing on the systemic role of CCPs and joint defaults of clearing members. In November 2013, the Governor of the Bank of England, Mark Carney, stated: "*It is extremely important that CCPs organise themselves to make sure they can provide the necessary resilience plan … [to] cover the failure of one or two major institutions*" (*Risk Magazine*, November 28, 2013).

its P&L but it is insensitive to the variability of other firms' P&Ls. In other words, the CoMargin of a clearing firm does not increase if other firms increase the risk of their trading strategies. Second, our method enhances the stability of the CCP by adjusting the allocation of collateral such that the probability of margin exceedances conditional on the financial distress of other firms remains constant. Third, relative to other methods, CoMargin improves the resilience of the CCP because it minimizes the average shortfall given simultaneous exceedances. Fourth, CoMargin embeds standard margining systems as special cases and, unlike the SPAN system, it can be backtested using formal statistical methods. Formal backtesting is important because it allows managers and regulators to assess the effectiveness of collateral requirements.

Using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC), the CCP of the TMX Montreal Exchange, we empirically assess the performance of CoMargin relative to the VaR and SPAN systems. Our dataset, the first one of its kind to be used in an academic study, includes daily observations of the actual trading positions of all forty-eight CDCC members from 2003 to 2011. Each member holds a proprietary, a client and an omnibus account with the CDCC. At the end of each trading day, we observe the number of long and short positions held in each account in the most actively traded futures contracts in the Canadian market; namely, the three-month Canadian Bankers' Acceptance Futures, the ten-year Government of Canada Bond Futures, and the S&P/TSX 60 Index Standard Futures.

The results show that CoMargin leads to fewer exceedances and lower expected shortfalls than SPAN and VaR margins. CoMargin also leads to exceedances that affect smaller members and that are less clustered through time. The relative performance of our method increases when an exceedance event occurs and, more importantly, when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence suggests that CoMargin could provide more protection to CCPs when they need it most.[7]

[7] These results are robust to equalizing the aggregate margin collections of the VaR and SPAN systems to CoMargin levels. When the P&Ls of clearing members are independent, CoMargin collects the same amount of collateral as existing margining systems. When they are not independent, the additional margin collected under CoMargin depends on the coverage level selected by the CCP. If we were to use a more loose coverage level for CoMargin in our tests, the additional margin would be mechanically reduced. It would even be possible to select the coverage rate such that CoMargin would lead to the *same* aggregate margin as SPAN but, of course, with a different margin allocation across members. However, such exercise would require us to change the coverage level through time.

This paper makes two important contributions to the literature. Our first contribution is methodological. We develop a novel margining system that internalizes market interdependencies and enhances the stability and resiliency of CCPs. Our approach addresses current macro-prudential concerns associated with CCPs, particularly in terms of systemic risk, and offers a general framework to think about collateral allocation in a broad range of financial markets, such as repo, securities lending, and central bank liquidity operations, among others. Our second contribution is empirical. We conduct an extensive analysis of the margining systems widely used in CCPs. To the best of our knowledge, this is the first study that uses actual trading positions of clearing members, which allows us to conduct our assessment without having to assume the trading behavior of market participants.

In addition, our paper complements the nascent literature on derivatives clearing. Duffie and Zhu (2010) show that the optimal number of CCPs needed to minimize counterparty default exposure is one. Biais, Heider and Hoerova (2012) show that central clearing dominates bilateral and decentralized clearing. However, central clearing can only mitigate idiosyncratic risk, which could leave market participants exposed to systematic risk. Menkveld (2013) develops a model for centralized clearing focused on systemic liquidation risk due to the default of market participants.[8] Slive, Witmer and Woodman (2012) show that CCPs choose to clear the most liquid CDS contracts and that CDS liquidity increases slightly after central clearing is introduced. Following Gârleanu and Pedersen (2011), Hedegaard (2013) tests the effect of margins on CME futures prices using data on 597 margin changes on CME futures contracts. He finds no abnormal return after the margin increases. More directly relevant to our study is Jones and Pérignon (2013) who show that the most severe losses of the largest clearing members of the CME tend to occur on the same days. These are exactly the type of events that our CoMargin framework aims to prevent by internalizing P&L dependence.

The remainder of the paper is organized as follows. In Section 2, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In Section 3, we present the theoretical foundations of the CoMargin system. We discuss its implementation in

[8] Other theoretical studies on derivatives clearing include Fontaine, Pérez Saiz and Slive (2012), Koeppl, Monnet and Temzelides (2012), and Acharya and Bisin (2013).

Section 4. In Section 5 we examine its empirical effectiveness and, finally, provide concluding remarks in Section 6.

2. Standard Margining Systems

2.1. Derivatives Market

Consider a derivatives exchange with N clearing members and D derivatives securities (futures, options, credit default swaps, etc.) written on U underlying assets. Let $w_{d,i,t}$ be the number of d contracts in the derivatives portfolio of clearing member i, for $d = 1, \dots, D$ and $i = 1, \dots, N$, at the end of day t, such that:

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix} \tag{1}$$

Initial margins are collected every day from each clearing member to guarantee the performance of their obligations and to guard the clearing house against default over a coverage period of one day. Let $B_{i,t}$ be the initial margin collected by the CCP from clearing member i at the end of day t. This margin is a function of the outstanding trading positions $w_{i,t}$ of member i.

We assume that current exposures are settled every day. Thus, a variation margin amount, $V_{i,t}$, equivalent to the profits or losses in the portfolio of clearing member i on day t, is deposited or withdrawn, respectively, from its margin account at the end of the trading day. Our interest focuses on cases when trading losses exceed margin requirements; that is, when $V_{i,t} \leq -B_{i,t-1}$. In these cases, we say that firm i has experienced an exceedance. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the market that needs to be covered by the CCP. Given the limited funds available to the CCP, simultaneous exceedance events can threaten its stability and survival.

2.2. SPAN Margin

The CME, the world's largest derivatives exchange, introduced the SPAN margining methodology in 1988. It has since become the most widely used margining system in derivatives exchanges around the world. Every day following the market close, clearing houses such as the CDCC, the CME, Eurex, LCH.Clearnet, Nymex and the Options Clearing Corporation (OCC), among others, use the SPAN system to determine the margin requirements of their members.

SPAN is a scenario-based methodology that is used to assess potential changes in the value of the derivatives held by each clearing member. However, SPAN does not take a portfolio-wide approach. Instead, it divides each portfolio into contract families, defined as groups of contracts that share the same underlying asset, and estimates a charge for each family independently. Thus, for a portfolio with $d \leq D$ derivatives written on $u \leq U$ underlying assets, the SPAN system computes initial margin requirements for each of the u contract families.

To compute margin requirements for a family of derivatives, the SPAN system simulates one-day-ahead changes in the value of each contract by using sixteen scenarios that vary the price and the volatility of the underlying asset, as well as the time to expiration of the contract (see Table 1). The range of the potential price changes of the underlying asset usually covers 99% of its daily price movements over a historical calibration window. A similar approach is adopted for the volatility. Extreme price changes are used to assess potential changes in deep out of the money options. The scenario analysis yields a risk array for each contract that contains sixteen one-day-ahead potential value changes (i.e., each maturity and each strike price has its own array).[9] The scenario with the worst potential loss for the entire contract family is identified and that loss becomes the first part of the contract family charge.

The second part of the contract family charge consists of a discretionary adjustment that is needed because contracts with different expiration months are assumed to be equivalent in the scenario analysis. In other words, long and short positions written on the same underlying asset but with different expiration months offset each other. Therefore, risk managers are required to add an

[9] The projected price changes of non-linear contracts, such as options, are obtained by using numerical valuation methods or option pricing models.

intra-commodity spread charge to the worst case scenario loss to account for time-spread trading. The resulting value is the contract family charge.

The collateral requirement for an entire portfolio is computed by aggregating the charges across all of its contract families. However, once again, risk managers are required to use discretionary aggregation rules to account for commodity-spread trading (i.e., simultaneous long and short positions in contracts with the same expiration months but written on different but correlated underlying assets). These adjustments are known as *inter-commodity spread charges.*

It is important to note that both intra- and inter-commodity spread charges involve the discretion of risk managers. Thus, these adjustments are rarely consistent across commodities, market conditions or clearing houses. This situation coupled with the fact that the SPAN system targets underlying price and volatility ranges, instead of the probability of portfolio-wide margin-exceeding losses, make the SPAN system inconsistent across time and markets in terms of its P&L coverage.

2.3. VaR Margin

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien, 2002; Berkowitz, Christoffersen and Pelletier, 2012), as well as their regulatory capital requirements (Adrian and Shin, 2013). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing member's one-day-ahead P&L distribution.

Definition 1: The VaR margin of firm i, B_i, corresponds to the α quantile of its P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha \tag{2}$$

The coverage level, determined by α, depends on the CCP's risk aversion and additional available financial resources (e.g. default fund) that could be used if a member defaults.

Like the SPAN system, the VaR margin method is applied on a firm-by-firm basis using a scenario analysis. However, the scenarios are applied to the entire portfolio. More specifically, we consider S scenarios derived from simulated one-day-ahead changes in the value of the price and the volatility of the underlying assets and use them to evaluate each clearing member's entire portfolio. The hypothetical P&L or variation margin of each clearing member is computed by *marking-to-model* its positions in each scenario. Thus, for each clearing member and date t, we obtain a simulated sample of daily profits and losses denoted $\{v_{i,t+1}^{s}\}_{s=1}^{S}$ that can be used to estimate the VaR margin requirement as follows:

$$\hat{B}_{i,t} = -quantile\left(\{v_{i,t+1}^{s}\}_{s=1}^{S}, \alpha\right) \tag{3}$$

Compared to market risk VaR (Jorion, 2007), the estimation of VaR margin is simpler. When estimating market risk VaR, there is only one observation available for each asset on date t. Therefore, the quantile of an asset's return distribution at time t cannot be estimated without making some distributional assumptions.[10] In the context of VaR margin, however, the situation is different because we have S simulated observations of the P&L distribution of each clearing member at time t. This is an ideal situation from an econometric standpoint because the relevant quantile can be directly implied without making any assumptions about the behavior of the P&L distribution over time. Thus, $\hat{B}_{i,t}$, which represents the empirical quantile based on the S simulated observations (equation 3), is a consistent estimate of the P&L VaR when S tends to infinity.

3. CoMargin

3.1. Concept

The VaR and SPAN collateral systems only focus on firm specific risk; that is, the unconditional probability of an individual clearing member experiencing a margin-exceeding loss. By adopting either method, the clearing house guards itself from unique or independent exceedances, but it leaves itself exposed to simultaneous exceedance events. These events, however, tend to be more economically significant because they place a more substantial burden on the resources of the clearing house.

[10] For example, the historical simulation approach broadly used by financial institutions for market risk VaR estimations assumes that asset returns are independent and identically distributed over time. Under these assumptions, the unconditional VaR is stationary and can be estimated from the historical path of asset returns. The estimation of more refined conditional measures also requires some specific assumptions about quantile dynamics. For instance, the CAViaR approach proposed by Engle and Manganelli (2004) assumes an autoregressive process for the quantiles.

Consider the VaR margin of firms i and j. The probability of simultaneous exceedances is given by:

$$\begin{gathered}\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\ = \Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) \times \Pr\left(V_{j,t+1} \leq -B_{j,t}\right)\end{gathered} \tag{4}$$

Equation 4 shows that simultaneous exceedance events tend to happen more frequently not only when firm specific risk increases (i.e., when $\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$ increases), but also when P&L dependence increases (i.e., when $\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right)$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all states of the world. In the second case, firms are more likely to experience these losses at the same time as other firms, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset comovement is high). However, VaR and SPAN margins completely disregard P&L dependence and its potential effect on the stability of the CCP. In the case of the VaR system, risk managers only target unconditional exceedance probabilities by setting a coverage level, $1 - \alpha$, for each clearing member individually. In the case of the SPAN system, risk managers do not have direct control over the unconditional exceedance probabilities, so the clearing house is potentially left even more vulnerable to simultaneous exceedance events.

Now, consider a fully orthogonal market; that is, a market that has firms with orthogonal trading positions and orthogonal underlying asset returns. In this case, firms have orthogonal risk exposures and their exceedance probabilities are independent. Under the VaR system this means:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{5}$$

and

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{6}$$

Equation 6 shows that given a common coverage probability, a fully orthogonal market minimizes the probability of simultaneous exceedance events across clearing members. In addition, a fully orthogonal market provides the best possible level of market stability, regardless of the collateral system being adopted by the clearing house, because once the risk manager selects α, the probabilities of simultaneous events are also fixed (i.e., α^2 for two events, α^3 for three events and so on). Therefore, a fully orthogonal market can be seen as a conceptual construct that provides a common benchmark for all margining systems.

With this in mind and in the spirit of the CoVaR measure of Adrian and Brunnermeier (2011), we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the P&L dependence of clearing members.

The CoMargin of firm i, denoted $B_t^{i|j}$, conditional on the realisation of an event affecting firm j satisfies:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | \mathbf{C}(V_{j,t+1})\right) = \alpha \tag{7}$$

The conditioning event that we consider is the financial distress of firm j, which we define as a loss in its portfolio in excess of its $\alpha\%$ VaR, or equivalently, a loss in excess of its VaR margin; i.e., $\mathbf{C}(V_{j,t+1}) = \{V_{j,t+1} \leq -B_{j,t}\}$.

Definition 2: The CoMargin of firm i conditional on the financial distress of firm j, $B^{i|j}$, corresponds to the α conditional quantile of their joint P&L distribution:

$$\Pr(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}) = \alpha \tag{8}$$

Notice that $B_t^{i|j}$ can be interpreted as the margin level that allows firm i to remain solvent $1-\alpha$ % of the times when firm j is in financial distress, as would be the case in a fully orthogonal market. Through Bayes theorem we know that:

$$\Pr(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}) = \frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]}{\Pr(V_{j,t+1} \leq -B_{j,t})} \tag{9}$$

where the numerator represents the joint probability of i exceeding its CoMargin requirement and j experiencing financial distress. From definitions 1 and 2, we can see that:

$$\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{10}$$

This alternative expression shows that with CoMargin, the probability of a firm exceeding its margin when another one is in distress corresponds to that obtained in a fully orthogonal market (α^2). Put it simply, CoMargin "orthogonalizes" the risk exposures of the CCP; thus, enhancing its stability. Furthermore, comparing equations (6) and (10) shows that when the market is orthogonal, CoMargin and VaR margin requirements are equivalent. Thus, any differences between the collateral requirements of these two systems can be attributed to P&L dependence.

Generalizing the definition of CoMargin to account for more than one conditioning firm is straightforward. With n conditioning firms, where $n < N - 1$, the conditioning event becomes that at least one of the n clearing members is in financial distress:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right]} = \alpha \tag{11}$$

where the probability of observing the conditioning event is:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, \dots, V_{n,t+1}\right)\right] = \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \, or \dots or \, \left(V_{n,t+1} \leq -B_{n,t}\right)\right] \tag{12}$$

3.2. The Properties of CoMargin

We show in this section that CoMargin exhibits several interesting properties that reflect its soundness, stability, ease of implementation and effectiveness. We consider a simple case with two firms that have normally-distributed P&Ls. For simplicity, we consider an unconditional distribution, with respect to past information, and consequently neglect the time index t. Thus, consider $(V_1, V_2)' \sim N(0, \Sigma)$ where:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

In this setting, the CoMargins of both members, denoted ($B^{1|2}$, $B^{2|1}$), are defined by:

$$\Pr\left(V_i \leq B^{i|j} | V_j \leq -B_j\right) = \alpha \tag{13}$$

for $i = 1, 2$, $j \neq i$, and where $B_j = -\sigma_j \Phi^{-1}(\alpha)$ denotes the unconditional VaR of firm j and $\Phi(\cdot)$ the cdf of the standard normal distribution. The conditional distribution of V_i given $V_j < c$, $\forall c \in \mathbb{R}$ is a skewed distribution (Horrace, 2005) and is denoted by $g(\cdot)$. The CoMargin for the firm i is the solution to:

$$\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i, \sigma_j, \rho) du = \alpha \tag{14}$$

$$g\left(u; \sigma_i, \sigma_j, \rho\right) = \frac{1}{\alpha \sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \ \Phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right) \tag{15}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution (Arnold et al., 1993). Using the expression of CoMargin in equation 14, we can illustrate some of its properties:

(1) The CoMargin of firm i increases with the variability of its P&L:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} > 0 \tag{16}$$

See Appendix A1 for the proof. Intuitively, it means that since riskier trading portfolios (as measured by their variability) tend to have larger potential losses, more collateral must be collected to guarantee their performance. Or in simple words, riskier clearing members should post higher margins.

(2) When there is no P&L dependence between firms i and j, CoMargin and VaR margin converge: In this example, P&L dependence is fully characterized by the correlation coefficient ρ; thus,

$$B^{i|j} = B_i \text{ when } \rho = 0 \tag{17}$$

See Appendix A2 for the proof. Notice, however, that this result is not specific to the normally distributed P&Ls. When there is no dependence (linear or otherwise) between the P&L of the two firms, CoMargin and VaR margin are equivalent. In other words, CoMargin nests the VaR margining system.

(3) The CoMargin of firm i increases with the dependence between its P&L and that of other firms: In this example, the only other member is firm j, so:

$$\frac{\partial B^{i|j}}{\partial \rho} > 0 \tag{18}$$

See Appendix A3 for the proof. The intuition behind this property is that a sound margining system should endogenize P&L dependence in order to prevent (or minimize) the occurrence of simultaneous margin-exceeding losses across market participants.

(4) When firms i and j have perfect P&L dependence, their CoMargin converges to an α^2 VaR margin, $B_i(\alpha^2)$:

$$\lim_{\rho \to 1} B^{i|j} = B_i(\alpha^2) \tag{19}$$

See Appendix A4 for the proof. This property shows that the CoMargin is capped by VaR(α^2) and that CoMargin is not explosive when P&L dependence becomes very high.

(5) The CoMargin of firm i does not depend on the variability of the P&L of firm j:

$$\frac{\partial\, B^{i|j}}{\partial \sigma_j} = 0 \tag{20}$$

See Appendix A5 for the proof. This property turns out to be extremely important. Combined with property (3), it shows that a firm's CoMargin increases with its P&L dependence but it does not increase with the risk taking of other market participants.

(6) CoMargin is testable ex-post: One way to assess the effectiveness of a margining system is by backtesting it (Hurlin and Pérignon, 2012). Backtesting aims at identifying misspecified models that lead to either excessive or insufficient coverage for the CCP relative to a target. Therefore, if a margining system cannot be backtested using formal statistical methods, we cannot identify its potential shortcomings and fine tune it to meet its objectives.

VaR margins can be easily backtested because they are defined by the quantile of a P&L distribution. Just like with VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the probability of exceedances conditional on the financial distress of other firms, as defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes:

$$H_0\colon \Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{21}$$

Since the null implies that $E\left[\mathbf{I}\left(V_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha$, then a simple likelihood-ratio (*LR*) test can be used (Christoffersen, 2009). To assess the conditional probability of exceedances, we use the historical paths of the P&Ls for both members i and j; i.e., $\left\{v_{i,t+1}\right\}_{t=1}^{T}$ and $\left\{v_{j,t+1}\right\}_{t=1}^{T}$. The corresponding LR test statistic, denoted $LR_{i|j}$, takes the form:

$$LR_{i|j} = -2\ln[(1-\alpha)^{T-N_{i|j}}\alpha^{N_{i|j}}] + 2\ln\left[\left(1 - \frac{N_{i|j}}{T}\right)^{T-N_{i|j}} \frac{N_{i|j}}{T}^{N_{i|j}}\right] \tag{22}$$

where $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T} \mathbf{I}\left(v_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1} \leq -B_{j,t}\right)$.

3.3. Monte Carlo Simulations

In order to illustrate the performance of the CoMargin system, we now consider the case of four clearing members, where two of them, members 1 and 2, have correlated P&Ls, such that $V \sim N(0, \Sigma)$ where:

$$V = (V_1, V_2, V_3, V_4)' \text{ and } \Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We allow the correlation between the P&Ls of firms 1 and 2, ρ, to increase from 0 to 0.8. As explained above, the rising correlation between the P&Ls of these firms can reflect an increase in the similarity of their trading positions or an increase in the comovement of the underlying assets. Panel A of Figure 1 shows the margin requirements for each clearing member under both the VaR and CoMargin systems for the levels of correlation being considered. To estimate the CoMargin of a given clearing member, we define the conditioning event as at least one of the other three firms being in financial distress.

The table shows that VaR margin remains constant for all firms regardless of their correlation level because this method does not take into account P&L dependence. Consistent with the explanation in the previous section, CoMargin and VaR margin are equal for all firms when $\rho = 0$; that is, when there is no P&L dependence. However, notice that CoMargin is greater than VaR margin when $\rho > 0$; that is, when there is P&L dependence. In addition, CoMargin increases as ρ increases; that is, as P&L dependence increases.

It is important to note that there are two possible reasons why CoMargin could be more effective than VaR margin at reducing conditional margin exceedances. The first reason is that there is an "allocation effect"; that is, CoMargin provides a better allocation of collateral. An alternative reason is that there is a "level effect". This means that CoMargin simply collects more aggregate collateral and, as a consequence, has a higher coverage level. We show that allocations and not aggregate margin collections drive the relative effectiveness of the CoMargin system by reporting a Budget-Neutral VaR (BNVaR) margin. This margining system collects the same amount of aggregate collateral than the CoMargin approach but it does so by distributing additional margin requirements evenly across all clearing members (rather than as a function of their P&L dependence).[11] Thus, this process effectively cancels out the level effect and allows us to assess the merits of each margining system based on their allocations.

The BNVaR margin of firm i at time t, $B_{i,t}^0$, is defined as:

$$B_{i,t}^0 = B_{i,t} + \frac{\sum_{i=1}^{N} B_t^{i|j} - \sum_{i=1}^{N} B_{i,t}}{N} \tag{23}$$

Panels B and C of Figure 1 show the theoretical performance of the VaR, BNVaR and CoMargin systems at the clearing member level. The horizontal line in these charts highlights the values prevalent when $\rho = 0$; that is, when the market is orthogonal. Panel B shows the probability of a

[11] An alternative budget-neutral margin scheme would be to redistribute the additional collateral collected from firms 1 and 2 to firms 3 and 4; that is, to collect the additional collateral from the firms that have uncorrelated P&Ls. A previous version of this paper conducted that experiment and the relative effectiveness of CoMargin is even higher than that reported here. The results are available upon request from the authors.

given clearing member exceeding its margin conditional on at least one other clearing member being in financial distress. When $\rho = 0$, all three margining systems provide the same level of coverage. However, as ρ increases, the VaR and BNVaR margins provide less coverage when at least one clearing member is in financial distress. On the other hand, CoMargin keeps the coverage level constant. Panel C shows the probability of a clearing member exceeding its margin conditional on at least another having an exceedance. In this case, CoMargin keeps the conditional probabilities of the uncorrelated clearing members stable and, unlike VaR and BNVaR, it reduces the conditional probabilities of exceedances among the correlated clearing members; that is, those that are more likely to create systemic risk by experiencing simultaneous exceedances.

Table 2 reports the theoretical performance of the different margining systems at the CCP level. The table reports the unconditional probability of having a minimum number of exceedances, the probability of having additional exceedances given that one has occurred, and the expected shortfalls associated with these events. Panels D, E and F of Figure 1 extend these results to up to four exceedance events. Our findings show once again that when $\rho = 0$, all three margining systems provide the same coverage to the CCP, but as ρ increases, CoMargin provides the best overall coverage.

The unconditional probabilities in Table 2 and Panel D of Figure 1 suggest that BNVaR margin provides the best unconditional coverage as correlation increases. Nevertheless, this result is expected. In our example all four firms are identical except for their correlation level. Since BNVaR collects more aggregate funds than VaR margin and it does so evenly across all clearing members, it is equivalent to a VaR margin with a higher coverage level (i.e., lower α). This higher coverage level embedded in BNVaR reduces the unconditional probability of individual margin exceedances. However, as Panels D and E of Figure 1 show, this does not improve the unconditional and conditional probability of experiencing additional (i.e. simultaneous) exceedance events, particularly as P&L dependence increases. In other words, collecting more VaR margin indiscriminately across clearing members does not optimize the coverage to the CCP.

Lastly, Panel F of Figure 1 shows the shortfall that the CCP is expected to cover given a minimum number of margin exceedances. Notice that both CoMargin and BNVaR margin provide similar results that outperform VaR for $\rho > 0$. CoMargin, however, has a slightly lower shortfall when simultaneous exceedances occur. In addition, recall from Panels D and E that the probability of simultaneous exceedances is lower under the CoMargin system. Therefore, the ex-ante expected shortfall for simultaneous exceedance events under the CoMargin system is less than that under the BNVaR margin system.

Figure 2 repeats the previous exercise but for P&Ls that are jointly Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$. Changing the distributional assumption of the previous exercise from a normal to a Student t multivariate distribution allows us to simulate scenarios where all clearing members have some level of tail dependence in their P&Ls, which is consistent with empirical evidence. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, however, in this case, we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5.[12] Thus, the resulting P&L distributions have progressively fatter tails.

The results of this exercise are consistent with those presented for the Gaussian assumption, but they highlight an important finding: CoMargin is able to capture P&L dependence structures that go beyond correlation. Recall that the P&L dependence structure is fully characterized by correlation only if P&Ls are normally distributed. However, asset prices and P&Ls, particularly those of non-linear portfolios, rarely follow normal distributions. Thus, at least in theory, CoMargin is more robust than other methods for a wide range of P&L distributions.

4. Implementing CoMargin

4.1. Scenario Generation

One common feature of all margining methods is that they are scenario based. As a consequence, generating meaningful scenarios is a crucial stage when setting margin requirements. Unlike the SPAN margining system, VaR margin and CoMargin use a portfolio-wide approach. This allows us to take into account the asset comovement within the portfolio of each clearing member without the need for ad-hoc adjustments (i.e., the inter- and intra-commodity spreads explained in the

[12] We conducted a similar experiment using $\rho = 0$ which leads to the same conclusions. The results are available upon request by contacting the authors.

previous section). In order to assess the potential P&L of the entire portfolio of each clearing member, we need to jointly simulate S vectors of one-day-ahead changes in the underlying asset prices, $\left\{r_{u,t+1}^{s}\right\}_{s=1}^{S}$. While there are different ways to estimate multivariate probability distribution functions, we suggest using a copula to link the marginal probability distribution functions, say $F_1(r_1)$, $F_2(r_2)$,…, $F_U(r_U)$, to form a multivariate probability distribution function, $F(r_1, r_2, \dots, r_U)$, where r_x is the standardized return of underlying asset x and U is the number of assets underlying the derivatives cleared by the CCP.

Using copulas to model the multivariate structure of underlying asset returns is useful in this context. First, marginal distributions do not need to be similar to each other to be linked together with a copula structure. Second, the choice of the copula or multivariate structure is not constrained by the choice of the marginal distributions. Third, copulas can be used with U marginal distributions to cover all of the underlying assets cleared by the CCP (see Oh and Patton, 2012). Finally, the use of copulas allows us to model the tails of the marginal distributions and the tail dependence across underlying assets separately, this feature is particularly important in our case because the likelihood of an extreme underlying asset return might increase either because of fatter tails in the marginal distributions or because of fatter tails in the multivariate distribution function. We use Student t copulas in our modeling because, unlike their Gaussian counterparts, they resemble more closely some of the stylized features of asset returns, such as fat tails in the marginal distributions and multivariate tail dependence.[13] We implement a two-stage semi-parametric approach to estimate a U-dimensional copula for the underlying asset returns. The first stage consists of estimating the empirical marginal distributions of the returns of each underlying asset. The second stage consists of estimating the t-copula parameters, R (correlation matrix) and υ (degrees of freedom), through maximum likelihood. Once the copula parameters are estimated,

[13] A Student t copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully characterized by the variance-covariance matrix of standardized returns and the degrees of freedom, υ. The degrees of freedom define the probability mass assigned to simultaneous extreme returns (both positive and negative); the lower the degrees of freedom, the higher the probability of experiencing simultaneous extreme returns relative to the Gaussian copula. However, as $\upsilon \rightarrow \infty$ the Student t copula converges to its Gaussian counterpart.

we use the implied multivariate structure to simulate potential changes in the price of the underlying assets.[14]

We use a fixed-length estimation window that is rolled daily to simulate new scenarios every day. On each day, once the S potential changes in the price of the underlying assets have been simulated, we mark-to-model all of the derivatives in the portfolio of each clearing member. We then use this simulated sample $\{v_{i,t+1}^s\}_{s=1}^S$ to estimate VaR margin as described in Section 2.3. To estimate CoMargin, we use the simulated samples for all CMs, $\{v_{1,t+1}^s, v_{2,t+1}^s, \dots, v_{n,t+1}^s\}_{s=1}^S$, and follow the procedure described in the following section.

4.2. Estimation

We now show how to estimate CoMargins from simulated P&Ls. In the case of two clearing members, given the simulated sample $\{v_{i,t+1}^s, v_{j,t+1}^s\}_{s=1}^S$, conditional on $B_t^{i|j}$, a simple estimate of the joint probability $\Pr\left[\left(V_{i,t+1} \le -B_t^{i|j}\right) \cap \left(V_{j,t+1} \le -B_{j,t}\right)\right]$, denoted $P_t^{i,j}$, is given by:

$$\hat{P}_t^{i,j} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \le -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1}^s \le -B_{j,t}\right) \tag{24}$$

where $v_{i,t+1}^s$ and $v_{j,t+1}^s$ correspond to the s^{th} simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B_t^{i|j}$. For each time t and for each firm i, we look for the value $B_t^{i|j}$, such that the distance $\hat{P}_t^{i,j} - \alpha^2$ is minimized:

$$\hat{B}_t^{i|j} = \arg\min_{\left\{B_t^{i|j}\right\}} \left(\hat{P}_t^{i,j} - \alpha^2\right)^2 \tag{25}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\{\hat{B}_t^{i|j}\}_{t=1}^T$ for which confidence bounds can be bootstrapped. Following a similar argument, CoMargin can be generalized to n conditioning firms, with $n < N - 1$. In this case, the conditioning event is that at least one of the n clearing members is in financial distress (see Appendix B for details).

[14] These simulations can also be obtained for different correlation matrices. For instance, a pre-defined range can be obtained from 95% confidence intervals used to forecast R through the Dynamic Conditional Correlation method proposed by Engle (2002).

5. Empirical Analysis

5.1. Data and Descriptive Statistics

In this section we compare the empirical performance of the SPAN, VaR and CoMargin systems by using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). The CDCC is the clearing house of the TMX Montreal Exchange. The dataset includes the daily trading positions at market close for each clearing member on the three-month Canadian Bankers' Acceptance Futures (BAX), the ten-year Government of Canada Bond Futures (CGB), and the S&P/TSX 60 Index Standard Futures (SXF) for the forty-eight clearing members active in the CDCC between January 2, 2003 and March 31, 2011. To the best of our knowledge no other academic study has ever used actual clearing member positions.

Table 3 presents a short description of the data. In a derivatives exchange, on any given day, there are many delivery dates available on each underlying asset. Over the sample period there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Thus, the sample includes a total of 113 futures contracts. Table 4 summarizes the specifications of these contracts. The contracts in our sample do not constitute the full set of derivatives cleared by the CDCC. However, they represent a significant portion of its clearing activity. The documentation provided by CDCC states that the BAX, CGB and SXF are among the most actively traded derivatives in Canada. Furthermore, BAX and CGB are the most actively traded cleared interest rate contracts in the country (Campbell and Chung 2003 and TMX Montreal Exchange 2013a,b,c).

Table 5 shows the summary statistics for the contracts in the sample. Panel A shows the aggregate statistics for all 113 contracts and Panels B to D, report the summary statistics by underlying asset. On a typical day, there were approximately 20 active contracts, 12 of them were BAX, four of them were CGB and the remaining four were SXF. On average, contracts remained active for 363 trading days. However, there is a significant dispersion across underlying assets. BAX contracts remained active for 551 days, whereas CGB and SXF contracts remained active for 239 and 237 days, respectively. BAX contracts were also the most actively traded, with an average daily gross open interest of 275,000. The corresponding value for CGB and SXF contracts was less than half of that for BAX at 131,000 and 111,000, respectively.

CDCC members have access to three accounts to submit trades for clearing: a firm, a client and an omnibus account. The firm account is used by clearing members to submit their own trades (i.e., conduct proprietary trading). The client account is used to submit trades on behalf of clearing members' clients. The omnibus account is used for all other clearing activities and is the least active account across all clearing members.

Our analysis includes 21 firm, 23 client and 16 omnibus accounts that were active on at least one day of the sample period. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Due to disclosure restrictions, we are unable to report the owners of each active account by type. However, Table 6 provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period. Notice that this list includes more clearing members than those currently affiliated with the CDCC because some members entered and exited the market during this period.

Since our objective in this section is to present an assessment of our methodology by using actual trading positions, instead of by assuming them, we report results only for the firm or proprietary-trading accounts. We consider this appropriate because these accounts best represent the actual trading decisions of the clearing members. Nevertheless, our results are consistent across all three accounts.

The daily settlement prices for the underlying assets and the futures contracts in the sample were obtained from Bloomberg and are plotted in Figure 3. Panel A shows the time series of underlying asset prices, Panel B shows the underlying asset returns and Panel C shows the settlement futures prices for all delivery dates. Lines in different colours represent different delivery dates. It is evident from Panel B that the volatility of the underlying assets increased dramatically after the onset of the financial crisis in mid-2007. In addition, Panel C shows an increase in the spread of futures prices during the same period, particularly for BAX contracts.

Figure 4 plots the daily stacked P&L values implied from the positions in active firm accounts. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the P&L of the n_t^a clearing members that had an active account on that day. Notice how the volatility of the P&Ls increased dramatically at the beginning of the financial crisis. This is consistent with the trends described for the underlying and futures prices in Figure 3. Therefore, we consider two sub-periods in our analysis. The first one is the pre-crisis period, from January 2, 2003 to July 31, 2007, and the second one is the crisis period, from August 1, 2007 to March 31, 2011.

Table 7 presents the summary statistics for the firm accounts in the sample. Panel A reports the values for the full sample period and Panels B and C present the values for the pre-crisis and crisis periods, respectively. On a typical day, there were approximately 12 clearing members with active firm accounts. This number remained relatively stable during the pre-crisis and crisis periods. The average account was active for 56% of the days in the sample (1,145 out of 2,066 days). The corresponding proportion is 75% (858 out of 1,148 days) for the pre-crisis period and 56% (516 out of 918 days) for the crisis period. The relatively lower activity during the crisis period was partially influenced by the fact that some clearing members exited the market. The P&L numbers reported in the table focus exclusively on active accounts. The typical active account reported an implied daily loss of $60,000 on the futures contracts listed in the sample. During the pre-crisis period, these accounts reported daily losses of $164,000. However, during the crisis period, the average account reported a daily profit of $65,000. These profits were mostly derived from long positions in BAX contracts. Over the entire sample period, the typical account made an implied loss of $38,000. The corresponding numbers are a loss of $119,000 and a profit of $39,000 for the pre-crisis and crisis periods, respectively.[15]

5.2. Empirical Performance

Using the daily trading positions in each firm account, we compute the initial margin that should be collected from each clearing member under the SPAN, VaR and CoMargin systems. The underlying

[15] It is important to notice that the P&L values reported in this paper are those implied by the positions held by the clearing members in their firm accounts on the contracts included in the sample. The actual accounts of these clearing members, however, included positions in other contracts cleared by the CDCC that are not included in our sample. In addition, our P&L values do not include trading revenues from other sources, such as non-cleared OTC transactions.

price range for the SPAN approach is set at 99% and $\alpha = 2\%$ for the VaR and CoMargin systems. We use a rolling estimation window of 500 trading days in all cases. As mentioned in Section 2.2, the SPAN system is estimated using the sixteen scenarios in Table 1. In this study, as we are only dealing with futures contracts, we ignore scenarios based on volatility changes or extreme scenarios, which are targeted to deep out of the money options.[16] For the VaR and CoMargin systems we consider $S = 100{,}000$ scenarios that are obtained using the methodology described in Section 4.1. Consistent with our earlier discussion and theoretical illustration, we set the financial distress threshold for CoMargin at the VaR margin level of the conditioning firms. The conditioning firms are the two clearing members with the highest one-day-ahead Expected Shortfall, $ES_{i,t+1} = E(V_{i,t+1}|V_{i,t+1} \leq -B_{i,t})$.[17]

For consistency across time periods, we ignore the ad-hoc inter- and intra-commodity spreads used in the SPAN system and impose a minimum margin of $10,000 on all active accounts under all systems. This amount allows us to avoid cases when clearing members are not required to post any collateral because they have matched long and short positions. These cases are likely to result in small exceedances as P&Ls in different contracts do not always offset each other. Thus, imposing a minimum margin amount prevents an upward bias in the number of SPAN exceedances. This amount, however, does not influence the rest of our results as it represents a constant that accounts for less than 0.2% of the average individual daily margin required under all systems.[18]

We find that accounting for P&L dependence increases margin significantly. Table 8 reports the summary statistics for the daily margin collected over the full sample period under different margining methods (Panels A and B). The table also reports budget-neutral margins for the SPAN (BNSPAN) and VaR (BNVaR) systems using the same approach as that described in equation 23. The average aggregate daily margin collected across all clearing members is $112, $101 and $161

[16] The shocks in extreme scenarios 15 and 16 are defined by the risk managers as a percentage increase over the price range (e.g., 20% over the price range). Using an ad hoc extreme change would prevent us from keeping the same 99% confidence interval for all underlying assets. Adding an ad-hoc extreme change only collects more money (i.e., increases the SPAN coverage) but does not change the results.

[17] Alternatively, the CCP may decide to always condition on the same firms that could be selected based on their size, systemic importance, or any other arbitrary reason.

[18] We computed our results under different minimum collateral amounts ranging from $0 to $100,000. The results are consistent in all cases. With a minimum collateral of $0, however, the SPAN system yields a high number of small exceedances.

million for the SPAN, VaR and CoMargin systems, respectively. The fact that CoMargin collects an extra 44% compared to SPAN margin can appear costly at first sight. However, the CCP has the freedom to choose a higher coverage rate to limit the increase in margins.[19] Using budget-neutral margins, we show below that the better performance of CoMargin relative to other systems is not simply the result of collecting higher margins, but rather the outcome of a better collateral allocation process across clearing members.

Panels C and D report the summary statistics for the daily margin collected over the pre-crisis and crisis periods, respectively. As it would be expected given the increased volatility during the financial crisis, both aggregate and individual collateral levels are higher during the crisis period. However, the ranking of margin collections is consistent throughout the full sample period and the two sub-periods being considered. VaR margin consistently collects the least and CoMargin consistently collects the most collateral. Similarly, VaR margin consistently shows the least dispersion and CoMargin consistently shows the most dispersion of collected margin as measured by the standard deviation. This situation arises because CoMargin takes into account the variation of more factors than the other two margining methods (i.e., the factors driving P&L dependence).

Panel A of Figure 5 shows the daily stacked initial margin requirements under the SPAN, VaR and CoMargin systems. The stacking process is the same as that used in the previous section for the P&L values of Figure 4. Notice that all three approaches produce margin requirements that are highly correlated. The average cross-sectional correlation for the full sample period is 0.99 between SPAN and VaR, 0.90 between SPAN and CoMargin, and 0.90 between VaR and CoMargin. The high correlation and low dispersion between the SPAN and VaR systems coupled with the average collection values shown in Table 8, indicate that at the individual clearing member level, SPAN margins behave much like VaR margins but at a higher coverage probability. However, notice that CoMargin is the least correlated of the three systems and shows the widest dispersion. This dispersion is more pronounced during the crisis-period, when P&L dependence is higher. As mentioned in the previous section, this can be explained by the fact that CoMargin converges to

[19] The CCP can also pick different coverage rates for the CoMargin ($B_t^{i|j}$) and the VaR ($B_{j,t}$) in equation (8).

VaR margin as P&L dependence decreases and diverges as P&L dependence increases (see Section 3.2).

Panel B of Figure 5 plots the daily P&L of each active clearing member against its initial margin requirement for the entire time series. The 45 and -45 degree lines are indicated in red. Observations falling below the -45 degree line denote margin-exceeding losses. Notice that of the three margining systems, CoMargin shows the least number of margin exceedances. In addition, unlike the other systems, CoMargin tends to concentrate exceedances in low initial margin, low P&L points. These points represent clearing members with the smallest or least active portfolios; that is, those that are the least likely to pose a systemic threat to the CCP.

Panel C of Figure 5 shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin ($V_{i,t}/B_{i,t-1}$). Once again, the stacking process is the same as that used in Figure 4. Observations with a relative variation margin below -1, the level depicted with a red line, represent margin exceedances. Consistent with the study of the CME margins by Jones and Pérignon (2013), SPAN margins exceedances tend to cluster in time. In contrast, the CoMargin system exhibits the lowest number of simultaneous margin exceedances.

Panels A and B of Table 9 summarize the performance of different margining systems over the full sample period. Panels C and D show the corresponding values for the pre-crisis and crisis periods, respectively. The left panel of the tables measures unconditional performance in terms of the probability of experiencing at least one exceedance, the average number of exceedances, and the expected shortfall if at least one exceedance occurs. The right panel of the tables reports the same measures but conditional on at least one member exceeding its margin.

Consistent with the simulation results presented in Section 3, our empirical results show that the CoMargin system outperforms the SPAN and VaR systems in all dimensions, whether these are estimated unconditionally or conditionally. At the CCP level, notice how CoMargin consistently has the lowest probability of exceedances and the lowest average number of exceedances across the three systems. A lowest number of simultaneous exceedances also allows CoMargin to have the lowest expected shortfall.

In addition, the relative performance of CoMargin increases when we condition on at least one exceedance event. This finding shows that CoMargin does a better job at protecting the CCP from simultaneous exceedances, even after a clearing member has surpassed its margin. Furthermore, Panels C and D indicate that the relative performance of CoMargin also improves during the crisis period, when P&L dependence is more persistent. This indicates that CoMargin tends to provide more protection to the CCP when it is needed most; that is, when simultaneous exceedance events are more likely to occur.

The clearing member-level results in Panels B to D show the probability that a typical clearing member surpasses its margin on any given day, and the expected shortfall associated with this event. Once again, the conditional probabilities and expected shortfalls are lower for CoMargin than for SPAN and VaR margins. This implies that under the SPAN and VaR systems, the typical clearing member is more likely to experience a margin-exceeding loss when another clearing member has exceeded its margin, than under the CoMargin system. In addition, notice how both the conditional and unconditional probabilities increase for the SPAN and VaR systems during the crisis period relative to the pre-crisis period. These results imply that when P&L dependence increases, SPAN and VaR exceedances are more likely than under CoMargin.

Table 9 also shows the performance of the budget-neutral SPAN and VaR systems (BNSPAN and BNVaR, respectively). As explained in Section 3, these artificial constructs allow us to test whether CoMargin performs better that its counterparts due to its allocation of collateral or due to the fact that it collects more funds. At a first glance, the results show that at the CCP and clearing member level, budget-neutral margins tend to perform better than CoMargin in terms of unconditional exceedance probabilities. However, as explained in the previous section, this is consistent with our simulation results. The budget-neutral allocation of collateral increases the coverage level of the SPAN and VaR systems across all clearing members and reduces their unconditional exceedance probabilities.[20]

[20] Results for budget-neutral margins must also be taken with a grain of salt. If there is large dispersion in the size of the clearing members, as is the case in our sample, the budget-neutral scheme can impose unrealistically large additional margins on small clearing members when the largest clearing members have correlated P&Ls.

In all cases, the results show that CoMargin yields the lowest expected shortfalls. This finding is once again consistent with our theoretical results. Relative to the CoMargin system, budget-neutral methods effectively transfer margin requirements from firms with high P&L dependence to those with low P&L dependence; thus, leaving the CCP exposed to simultaneous exceedance events. These simultaneous events account for the higher (conditional and unconditional) expected shortfalls under the budget-neutral systems. Since expected shortfalls ultimately determine the impact on the funds available to the CCP, our findings show that CoMargin allocations enhance the resilience of clearing houses by minimizing the likelihood and economic impact of adverse events.

Figure 6 extends our empirical findings by conditioning on up to three margin exceedance events. The results confirm that CoMargin tends to perform better than the SPAN and VaR systems even after accounting for the additional amount of collateral required (i.e., after using budget-neutral measures).

6. Conclusion

In this paper, we present a new methodology, called CoMargin, to estimate margin requirements in derivatives CCPs. Our approach is innovative because it explicitly takes into account both the individual risk and the interdependence of the P&Ls of market participants. As a result, CoMargin produces collateral allocations that enhance the stability and resilience of CCPs, thus reducing their systemic risk.

We show theoretically and empirically that CoMargin outperforms the widely popular SPAN and VaR margining approaches. CoMargin performs particularly well relative to these alternatives when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence suggests that CoMargin provides more protection to the CCP when it needs it most.

During certain periods, CoMargin significantly increases aggregate collateral levels. However, using budget-neutral methods, we show that the relative performance of CoMargin is driven by its allocation process and not the additional funds collected. Thus, increasing collateral requirements under other systems does not necessarily protect the CCP against simultaneous margin exceedance events.

At a more general level, the paper illustrates the importance of accounting for simultaneous extreme events, or interdependencies, when managing credit risk. Our approach can be seen as a stepping stone that can be generalized and used in different situations, such as estimating collateral requirements for repo and other OTC transactions, assessing capital requirements for banks and insurance companies, or monitoring the accumulation of credit risk across market participants for regulatory purposes.

Finally, notice that we present one of many potential ways to implement our framework. Nevertheless, we welcome and encourage future research to extend and refine our approach. For example, the scenario generating process could be adapted to account for endogenous risk. This feature would consider shocks that not only reflect the past evolution of the underlying variables but also the reaction of clearing firms to specific shocks. As a consequence, the scenarios could become firm-specific and the interdependencies among firms dynamic.

Appendix A: Proofs for the CoMargin Properties (Section 3.2)

[Proof A1]: Let $H(B^{i|j},\sigma_i)$ be a function such that:

$$H(B^{i|j},\sigma_i)=\int_{-\infty}^{-B^{i|j}} g(u,\sigma_i)du-\alpha=0 \tag{A1}$$

Note that we simplified the notation of the pdf $g(u;\sigma_i)$ compared to equation 14. Then, the CoMargin can be defined as an implicit function $B^{i|j}=h(\sigma_i)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \sigma_i}=-\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)} \tag{A2}$$

The derivative $H_B(B^{i|j},\sigma_i)$ can be expressed as follows:

$$H_B\left(B^{i|j},\sigma_i\right)=-g\left(-B^{i|j};\sigma_i\right)<0 \tag{A3}$$

and is negative since $g(u;\sigma_i)$ is a pdf. Thus, the sign of $\frac{\partial B^{i|j}}{\partial \sigma_i}$ is given by the sign of $H_{\sigma_i}(B^{i|j},\sigma_i)$:

$$H_{\sigma_i}\left(B^{i|j},\sigma_i\right)=\frac{\partial}{\partial \sigma_i}\left(\int_{-\infty}^{-B^{i|j}} g(u;\sigma_i)\,du-\alpha\right)=\int_{-\infty}^{-B^{i|j}}\frac{\partial g(u;\sigma_i)}{\partial \sigma_i}du \tag{A4}$$

For simplicity, let us consider the case where $\rho=0$:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i}=\frac{\partial}{\partial \sigma_i}\left(\frac{1}{\sigma_i}\phi\left(\frac{u}{\sigma_i}\right)\right)=-\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)-\frac{u}{\sigma_i^3}\phi'\left(\frac{u}{\sigma_i}\right) \tag{A5}$$

Since $\phi'(x)=-x\,\phi(x)$, we have:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i}=-\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)\left(1-\left(\frac{u}{\sigma_i}\right)^2\right) \tag{A6}$$

For any value of u such that $u<-\sigma_i$, we have $\partial g(u;\rho)/\partial\sigma_i>0$. This condition is satisfied when $u\in\left]-\infty,-B^{i|j}\right]$ since $-B^{i|j}=\sigma_i\Phi^{-1}(\alpha)=-\sigma_i\Phi^{-1}(1-\alpha)$ and $\Phi^{-1}(1-\alpha)>1$ for most of the considered coverage rates (e.g. 1%, 5%). Consequently, the integral defined in equation (A4) is also positive and $H_{\sigma_i}\left(B^{i|j},\sigma_i\right)>0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial \sigma_i}=-\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)}>0 \tag{A7}$$

A similar result can be obtained when we relax the assumption.

[Proof A2]: If $=0$, the last term in equation 15 becomes $\Phi\left(-B_j/\sigma_j\right)=\Phi(\Phi^{-1}(\alpha))=\alpha$ since $B_i=-\sigma_i\Phi^{-1}(\alpha)$. Consequently, the CoMargin of firm i is the solution of the following integral:

$$\int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi(\frac{u}{\sigma_i})\, du = \alpha \tag{A8}$$

By properties of the normal distribution, we have $B^{i|j} = -\sigma_i \Phi^{-1}(\alpha) = B_i$.

[Proof A3]: Let $F(B^{i|j}, \rho)$ be a function such that:

$$F(B^{i|j}, \rho) = \int_{-\infty}^{-B^{i|j}} g(u; \rho)\, \mathrm{du} - \alpha = 0 \tag{A9}$$

Note that we simplified the notation of the pdf $g(u; \rho)$ compared to equation 14. Then, the CoMargin can be defined as an implicit function $B^{i|j} = f(\rho)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j}, \rho)}{F_B(B^{i|j}, \rho)} \tag{A10}$$

where $F_\rho(.)$ and $F_B(.)$ denote respectively the first derivative of the F function with respect to ρ and B. For any function $H(x)$ defined as:

$$H(x) = \int_{-\infty}^{-b(x)} h(t)\, dt \tag{A11}$$

we have $H'(x) = h(b(x)) \times \partial b(x)/\partial x$. Consequently, the derivative $F_B(B^{i|j}, \rho)$ can be expressed as follows:

$$F_B\left(B^{i|j}, \rho\right) = -g\left(-B^{i|j}; \rho\right) < 0 \tag{A12}$$

and is negative since $g(u; \rho)$ is a pdf. Thus, the sign of $\partial B^{i|j}/\partial \rho$ is given by the sign of $F_\rho(B^{i|j}, \rho)$:

$$F_\rho\left(B^{i|j}, \rho\right) = \frac{\partial}{\partial \rho}\left(\int_{-\infty}^{-B^{i|j}} g(u; \rho)\, du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u; \rho)}{\partial \rho} du \tag{A13}$$

Given the expression of the pdf $g(u; \rho)$ we have:

$$\begin{aligned} \frac{\partial g(u; \rho)}{\partial \rho} &= \overbrace{-\frac{1}{\alpha \sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right)}^{A} \\ &\quad \times \left(\frac{-u/\sigma_i \sqrt{1-\rho^2} - (B_j/\sigma_j + \rho u/\sigma_i)\rho(1-\rho^2)^{-1/2}}{1-\rho^2}\right) \\ &= A \times \left(\frac{1}{1-\rho^2}\right)^{3/2} \times \left(\frac{u}{\sigma_i} + \frac{\rho B_j}{\sigma_j}\right) \end{aligned} \tag{A14}$$

This function is positive for any value of u such that $u \leq \rho B_i = -\rho \sigma_i \Phi^{-1}(\alpha)$ with $-\rho \sigma_i \Phi^{-1}(\alpha) > 0$. Since $B^{i|j} \geq 0$ by definition, this condition is satisfied for the interval $\left]-\infty, -B^{i|j}\right]$ and $F_\rho\left(B^{i|j}, \rho\right) > 0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j},\rho)}{F_B(B^{i|j},\rho)} > 0 \quad \text{(A15)}$$

[Proof A4]: For $\rho = 1$, the pdf $g\left(u;\sigma_i,\sigma_j,\rho\right)$ in equation 14 is degenerated. However, when ρ tends to one, we have:

$$\lim_{\rho\to 1} \Phi\left(\frac{-\mathrm{B_i}/\sigma_j - \rho u}{\sqrt{1-\rho^2}}\right) = 1 \quad \text{(A16)}$$

as long as $u < \frac{-\mathrm{B_i}}{\sigma_j} = \Phi^{-1}(\alpha)$. If we assume that the standardized CoMargin for i is larger than the standardized VaR margin for i, i.e., $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$, then we have:

$$\lim_{\rho\to 1} g(u) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \quad \text{(A17)}$$

And consequently the CoMargin corresponds to the VaR margin defined for a coverage rate α^2 since:

$$\lim_{\rho\to 1} \int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi\left(\frac{x}{\sigma_i}\right) dx = \alpha^2 \quad \text{(A18)}$$

$$\lim_{\rho\to 1} B^{i|j} = -\,\sigma_i \Phi^{-1}(\alpha^2) \quad \text{(A19)}$$

We can check that condition $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$ is satisfied since $\Phi^{-1}(\alpha^2) \leq \Phi^{-1}(\alpha)$.

[Proof A5]: Since $B_j = -\sigma_j \Phi^{-1}(\alpha)$, the pdf $g(.)$ in equation 15 can be rewritten as:

$$g\left(u;\sigma_i,\sigma_j,\rho\right) = \frac{1}{\alpha\sigma_i} \times \phi(\frac{u}{\sigma_i}) \times \Phi\left[\frac{\Phi^{-1}(\alpha) - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right] \quad \text{(A20)}$$

As $g(.)$ does not depend on σ_j, $\partial B^{i|j}/\partial\sigma_j = 0$.

Appendix B: CoMargin with *n* Conditioning Firms

With n conditioning firms, $n < N-1$, the conditioning event of the CoMargin is that at least one of the n clearing members is in financial distress. Thus, the definition of CoMargin becomes:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right]} = \alpha \tag{B1}$$

where the probability to observe the conditioning event is:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] = \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \, or \, \ldots \, or \, \left(V_{n,t+1} \leq -B_{n,t}\right)\right] \tag{B2}$$

Using Poincaré's formula for the probability of the union of events, we can see that:

$$\begin{aligned}
\Pr\left[\mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right] = & \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\
& \underbrace{- \sum_{1 \leq j_1 < j_2 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right)\right]}_{\boldsymbol{2\ events}} \\
& \underbrace{+ \sum_{1 \leq j_1 < j_2 < j_3 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right) \cap \left(V_{j_3,t+1} \leq -B_{j_3,t}\right)\right]}_{\boldsymbol{3\ events}} \\
& \ldots \underbrace{+(-1)^{\boldsymbol{n-1}} \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \cap \ldots \cap \left(V_{n,t+1} \leq -B_{n,t}\right)\right]}_{\boldsymbol{n\ events}}
\end{aligned} \tag{B3}$$

Thus, the probability of the conditioning event can be rewritten as follows:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right] = n\alpha - P_t^n \tag{B4}$$

where P_t^n denotes the sum of the probabilities of all common events (for two events, three events, etc.). An estimator of this value, $\hat{P}_t^n$, can be obtained from the simulated path $\left\{V_{1,t+1}^s, .., V_{n,t+1}^s\right\}_{s=1}^S$. When the financial distress events of the conditioning firms are mutually exclusive, however, the probability of the conditioning events simplifies to $n\alpha$. Therefore, an estimator of the CoMargin of firm i conditional on n clearing members, $B_t^{i|n}$, is the solution of the program:

$$\hat{B}_t^{i|n} = \arg\min_{\left\{B_t^{i|n}\right\}} \left(\frac{\hat{P}_t^{i,n}}{n\alpha - \hat{P}_t^n} - \alpha\right)^2 \tag{B5}$$

where $\hat{P}_t^{i,n}$ denotes the estimator of $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, .., V_{n,t+1}\right)\right]$.

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Table 1: Scenarios used in the SPAN system

Scenario	Underlying Asset Price Change	Volatility Change	Time to Expiration
1	0	+ volatility range	$-1/252$
2	0	- volatility range	$-1/252$
3	$+1/3$ x price range	+ volatility range	$-1/252$
4	$+1/3$ x price range	- volatility range	$-1/252$
5	$-1/3$ x price range	+ volatility range	$-1/252$
6	$-1/3$ x price range	- volatility range	$-1/252$
7	$+2/3$ x price range	+ volatility range	$-1/252$
8	$+2/3$ x price range	- volatility range	$-1/252$
9	$-2/3$ x price range	+ volatility range	$-1/252$
10	$-2/3$ x price range	- volatility range	$-1/252$
11	$+3/3$ x price range	+ volatility range	$-1/252$
12	$+3/3$ x price range	- volatility range	$-1/252$
13	$-3/3$ x price range	+ volatility range	$-1/252$
14	$-3/3$ x price range	- volatility range	$-1/252$
15	Positive extreme change	0	$-1/252$
16	Negative extreme change	0	$-1/252$

Note: The table shows the sixteen scenarios used to determine the contract family charge in the SPAN system. Price and volatility ranges usually cover 99% of the data points over a rolling historical estimation window. Positive and negative extreme changes are designed to assess the effect of deep out of the money options.

Table 2: Theoretical performance of VaR and CoMargin systems

	Jointly Normally Distributed P&Ls				Jointly Student t Distributed P&Ls			
	Unconditional		Conditional on One Exceedance		Unconditional		Conditional on One Exceedance	
	Prob. of at least one Exceedance	Expected Shortfall	Prob. of Additional Exceedances	Expected Shortfall	Prob. of at least one Exceedance	Expected Shortfall	Prob. of at least one Exceedance	Expected Shortfall
	ρ = 0				**ν = 30**			
VaR	0.185	0.084	0.076	0.451	0.177	0.094	0.123	0.531
CoMargin	0.185	0.084	0.076	0.451	0.115	0.056	0.074	0.485
BNVaR	0.185	0.084	0.076	0.451	0.108	0.052	0.088	0.485
	ρ = 0.4				**ν = 10**			
VaR	0.179	0.084	0.109	0.466	0.171	0.119	0.151	0.695
CoMargin	0.138	0.060	0.069	0.433	0.081	0.053	0.090	0.650
BNVaR	0.129	0.055	0.083	0.430	0.077	0.051	0.104	0.658
	ρ = 0.8				**ν = 5**			
VaR	0.165	0.084	0.193	0.505	0.164	0.175	0.191	1.068
CoMargin	0.110	0.048	0.062	0.432	0.051	0.060	0.129	1.171
BNVaR	0.077	0.033	0.144	0.428	0.049	0.059	0.141	1.193

Note: This table presents the theoretical performance of the VaR (equation 2), CoMargin (equation 10), and Budget-neutral VaR (BNVaR, equation 23) systems, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and reports the results for different levels of the correlation parameter,ρ, that range from 0 to 0.8. The right panel shows the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_\upsilon(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

Table 3: Description of the data used in the empirical analysis

Item	Number	Comments
Clearing members	48	There is entry and exit in the sample, so the number of clearing members varies over time.
Trading Days	2,066	The sample period is from January 2, 2003 to March 31, 2011.
Underlying Assets	3	The three underlying assets are: 1. Yield on the three-month Canadian bankers' acceptance. 2. Yield on the ten-year Government of Canada Bond Futures 3. Level of the S&P/TSX 60 Index
Three-Month Canadian Bankers' Acceptance Futures Contracts (BAX)	45	Delivery dates range from January 2003 to December 2013.
Ten-Year Government of Canada Bond Futures Contracts (CGB)	34	Delivery dates range from March 2003 to June 2011.
S&P/TSX 60 Index Standard Futures Contracts (SXF)	34	Delivery dates range from March 2003 to June 2011.
Total futures contracts	113	These represent all the futures contracts (i.e., all delivery dates) written on the three underlying assets during the sample period.
Active firm accounts	21	We report results only for this type of account.
Active client accounts	23	
Active omnibus accounts	16	

Note: The table presents an overview of the dataset used in the empirical analysis, which was obtained from the Canadian Derivatives Clearing Corporation. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset.

Table 4: Specifications of the contracts included in the empirical analysis

	S&P/TSX 60 Index Standard Futures (SXF)	**Three-Month Canadian Bankers' Acceptance Futures (BAX)**	**Ten-Year Government of Canada Bond Futures (CGB)**
Underlying Interest	The S&P/TSX 60 Index C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Expiration Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index : 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Settlement	Cash settlement	Cash settlement	Physical delivery of eligible Government of Canada Bonds.
Trading Hours (EST)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Table 5: Summary statistics of the contracts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
			Panel A: All Contracts		
Active contracts per day	19.81	20.00	0.9279	13.00	21.00
Trading days per contract	362.25	253.00	221.72	6.00	756.00
			Panel B: BAX Contracts		
Active contracts per day	11.99	12.00	0.14	8.00	13.00
Trading days per contract	550.58	699.00	249.25	6.00	756.00
Open interest long	137.81	131.32	49.21	48.35	310.97
Open interest short	-137.81	-131.32	49.21	-310.97	-48.35
Open interest gross	275.61	262.65	98.42	96.71	621.94
			Panel C: CGB Contracts		
Active contracts per day	3.93	4.00	0.49	1.00	5.00
Trading days per contract	238.91	253.00	42.73	55.00	255.00
Open interest long	65.26	60.81	27.36	16.15	176.97
Open interest short	-65.26	-60.81	27.36	-176.97	-16.15
Open interest gross	130.52	121.62	54.72	32.30	353.94
			Panel D: SXF Contracts		
Active contracts per day	3.89	4.00	0.43	1.00	4.00
Trading days per contract	236.32	250.00	42.58	52.00	255.00
Open interest long	55.28	55.09	14.01	24.40	98.49
Open interest short	-55.28	-55.09	14.01	-98.49	-24.40
Open interest gross	110.56	110.17	28.02	48.79	196.99

Note: The table shows the summary statistics of the 113 futures contracts included in the empirical analysis. These contracts are divided according to their underlying assets as follows: Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF). Over the sample period (January 2, 2003 to March 31, 2011), there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Open interest values are reported in thousands.

Table 6: Clearing members included in the empirical analysis

Number	Name	Number	Name
1	Newedge Canada Inc.	25	Morgan Stanley Canada LTD.
2	RBC Dominion Securities Inc.	26	CFG Financial Group Inc.
3	Union Securities LTD.	27	MF Global Canada Co.
4	T.D. Securities Inc.	28	Haywood Securities Inc.
5	BMO Nesbitt Burns LTD.	29	Goldman Sachs Canada Inc.
6	Macquarie Private Wealth Inc.	30	Timber Hill Canada Co.
7	UBS Securities Canada Inc.	31	Credit Suisse Securities
8	Desjardins Securities Inc.	32	CIBC World Markets Inc.
9	Macquarie Capital Markets Inc.	33	NBCN Clearing Services Inc.
10	Name not reported	34	HSBC Securities (Canada) Inc.
11	Merrill Lynch Canada Inc.	35	Mackie Research Capital Corporation
12	Odlum Brown LTD.	36	Benson-Quinn GMS Inc.
13	Penson Financial Services Inc.	37	Scotia Capital Inc.
14	Dundee securities corporation	38	E*trade Canada Securities Corporation
15	Daex Commodities Inc.	39	Raymond Kames LTD.
16	Canaccord Capital Corporation	40	Lévesque Beaubien Geoffrion Inc.
17	Friedberg Mercantile Group LTD.	41	TD Waterhouse Canada Inc.
18	W.D. Latimer Co. LTD.	42	Citigroup Global Markets Canada Inc.
19	Canadian Imperial Bank of Commerce (CIBC)	43	National Bank of Canada
20	Jones, Gable & Co. LTD.	44	J.P. Morgan Securities Canada Inc.
21	Name not reported	45	Merrill Lynch Canada Inc.
22	Timber Hill Canada Company	46	Name not reported
23	Laurentian Bank Securities Inc.	47	Fidelity Clearing Canada ULC
24	Deutsche Bank Securities LTD.	48	Maple Securities Canada LTD.

Note: The table provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period (January 2, 2003 to March 31, 2011). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Notice that this list includes more clearing members than those currently affiliated with the Canadian Derivatives Clearing Corporation (CDCC) because some of them entered and exited the market during the sample period.

Table 7: Summary statistics of the firm accounts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
		Panel A: Full Sample period			
Active accounts per day	11.64	12.00	1.09	8.00	15.00
Active days for an account	1145.19	1420.00	911.72	3.00	2066.00
Daily P&L across clearing members	-60.92	-97.80	2659.44	-15014.20	17502.52
P&L over time	-37.83	0.43	160.57	-455.52	237.50
		Panel B: Pre-Crisis period			
Active accounts per day	11.96	12.00	0.95	9.00	15.00
Active days for an account	858.00	1148.00	431.12	3.00	1148.00
Daily P&L across clearing members	-163.50	-156.15	2027.37	-6813.50	10381.36
P&L over time	-119.12	-0.78	225.46	-671.62	39.80
		Panel C: Crisis period			
Active accounts per day	11.24	11.00	1.13	8.00	15.00
Active days for an account	516.05	684.00	418.29	3.00	918.00
Daily P&L across clearing members	65.18	-57.43	3280.36	-15014.20	17502.52
P&L over time	39.83	-0.60	135.28	-110.76	484.42

Note: The table presents the summary statistics of the 23 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011 and there are $N = 48$ clearing members in the sample. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011. P&L values are reported in thousands of dollars and are estimated only for active firm accounts.

Table 8: Daily margin requirements under the SPAN, VaR and CoMargin systems

	Mean	Median	St.Dev.	Min	Max
			Panel A: Aggregate Market (CCP level)		
SPAN	112.04	105.71	38.49	49.83	328.77
VaR	101.40	95.80	36.03	42.90	301.97
CoMargin	161.31	156.20	64.93	56.89	475.27
BNSPAN	161.31	156.20	64.93	56.88	475.27
BNVaR	161.31	156.20	64.93	56.88	475.27
			Panel B: Per Clearing Member		
SPAN	9.76	9.16	3.54	3.83	29.86
VaR	8.83	8.26	3.28	3.30	27.45
CoMargin	14.14	13.41	6.08	4.38	43.01
BNSPAN	14.14	13.41	6.08	4.38	43.01
BNVaR	14.14	13.41	6.08	4.38	43.01
			Panel C: Per Clearing Member (Pre-Crisis)		
SPAN	8.49	8.32	2.38	3.83	14.98
VaR	7.72	7.47	2.15	3.30	13.79
CoMargin	11.94	11.62	4.45	4.38	25.98
BNSPAN	11.94	11.62	4.45	4.38	25.98
BNVaR	11.94	11.62	4.45	4.38	25.98
			Panel D: Per Clearing Member (Crisis)		
SPAN	11.36	10.85	4.07	4.88	29.86
VaR	10.21	9.51	3.87	4.46	27.45
CoMargin	16.89	16.21	6.69	5.64	43.01
BNSPAN	16.89	16.21	6.69	5.64	43.01
BNVaR	16.89	16.21	6.69	5.64	43.01

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation 2) and CoMargin (equation 10) systems for the 23 active firm accounts during the full sample period, from January 2, 2003 to March 31, 2011 (Panels A and B), during the pre-crisis period, from January 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from August 1, 2007 to March 31, 2011 (Panel D). The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 23. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 9: Performance of the SPAN, VaR and CoMargin systems

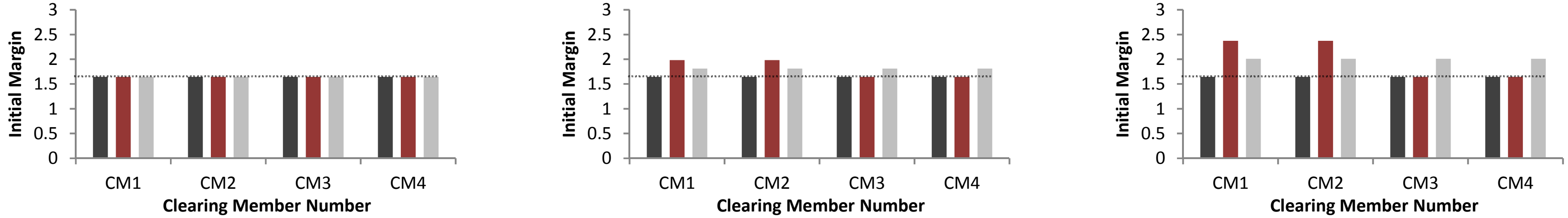
	Unconditional			Conditional on at least one exceedance		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
			Aggregate Market (CCP level)			
SPAN	0.09	0.15	0.35	0.36	1.63	3.78
VaR	0.14	0.25	0.44	0.42	1.80	3.20
CoMargin	0.07	0.10	0.13	0.28	1.44	1.85
BNSPAN	0.02	0.02	0.16	0.38	1.47	10.15
BNVaR	0.02	0.03	0.16	0.36	1.47	9.45
			Panel B: Per Clearing Member			
SPAN	0.01	-	0.03	0.14	-	0.34
VaR	0.02	-	0.04	0.15	-	0.29
CoMargin	0.01	-	0.01	0.12	-	0.16
BNSPAN	0.00	-	0.01	0.12	-	0.90
BNVaR	0.00	-	0.01	0.12	-	0.84
			Panel C: Per Clearing Member (Pre-Crisis)			
SPAN	0.01	-	0.01	0.11	-	0.08
VaR	0.02	-	0.01	0.13	-	0.09
CoMargin	0.01	-	0.00	0.10	-	0.05
BNSPAN	0.00	-	0.00	0.10	-	0.17
BNVaR	0.00	-	0.00	0.10	-	0.18
			Panel D: Per Clearing Member (Crisis)			
SPAN	0.02	-	0.06	0.17	-	0.63
VaR	0.03	-	0.08	0.18	-	0.48
CoMargin	0.01	-	0.02	0.14	-	0.30
BNSPAN	0.00	-	0.03	0.14	-	1.40
BNVaR	0.00	-	0.03	0.14	-	1.21

Note: The table compares the empirical performance of the SPAN, VaR (equation 2) and CoMargin (equation 10) systems computed for the 23 active firm accounts over the sample period, from January 2, 2003 to March 31, 2011 (Panels A and B), during the pre-crisis period, from January 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from August 1, 2007 to March 31, 2011 (Panel D). The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 23. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

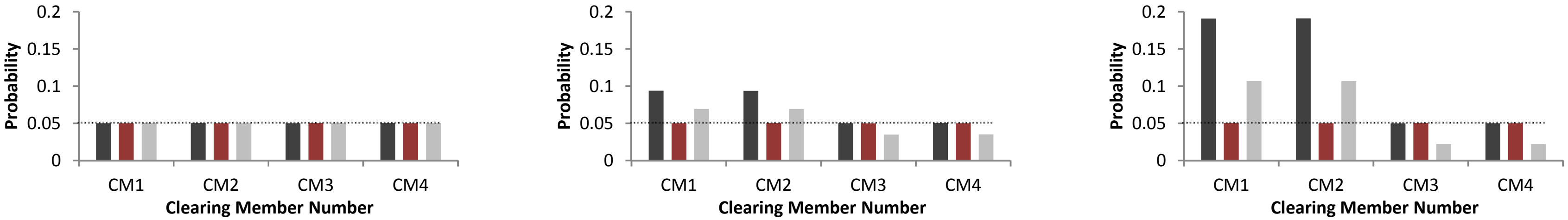
Figure 1: Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls

ρ = 0	ρ = 0.4	ρ = 0.8

Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given clearing member exceeding its margin conditional on at least another clearing member being in financial distress



Panel C: Probability of a clearing member exceeding its margin conditional on at least another clearing member having a margin exceedance

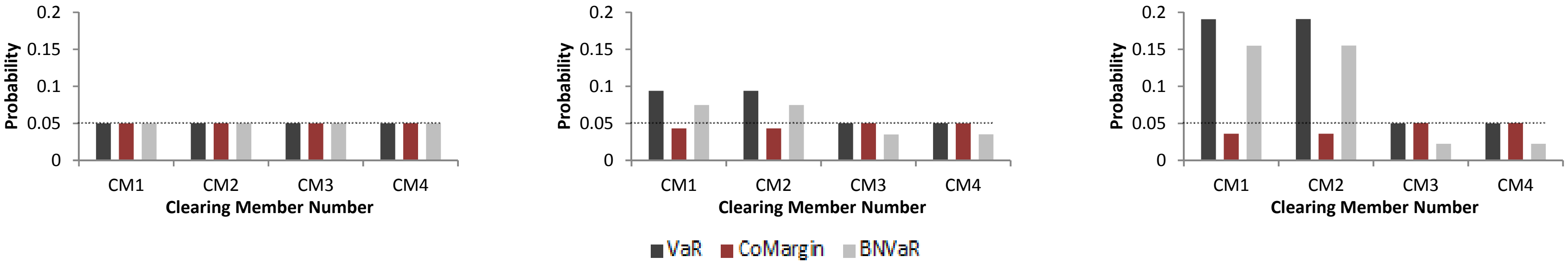
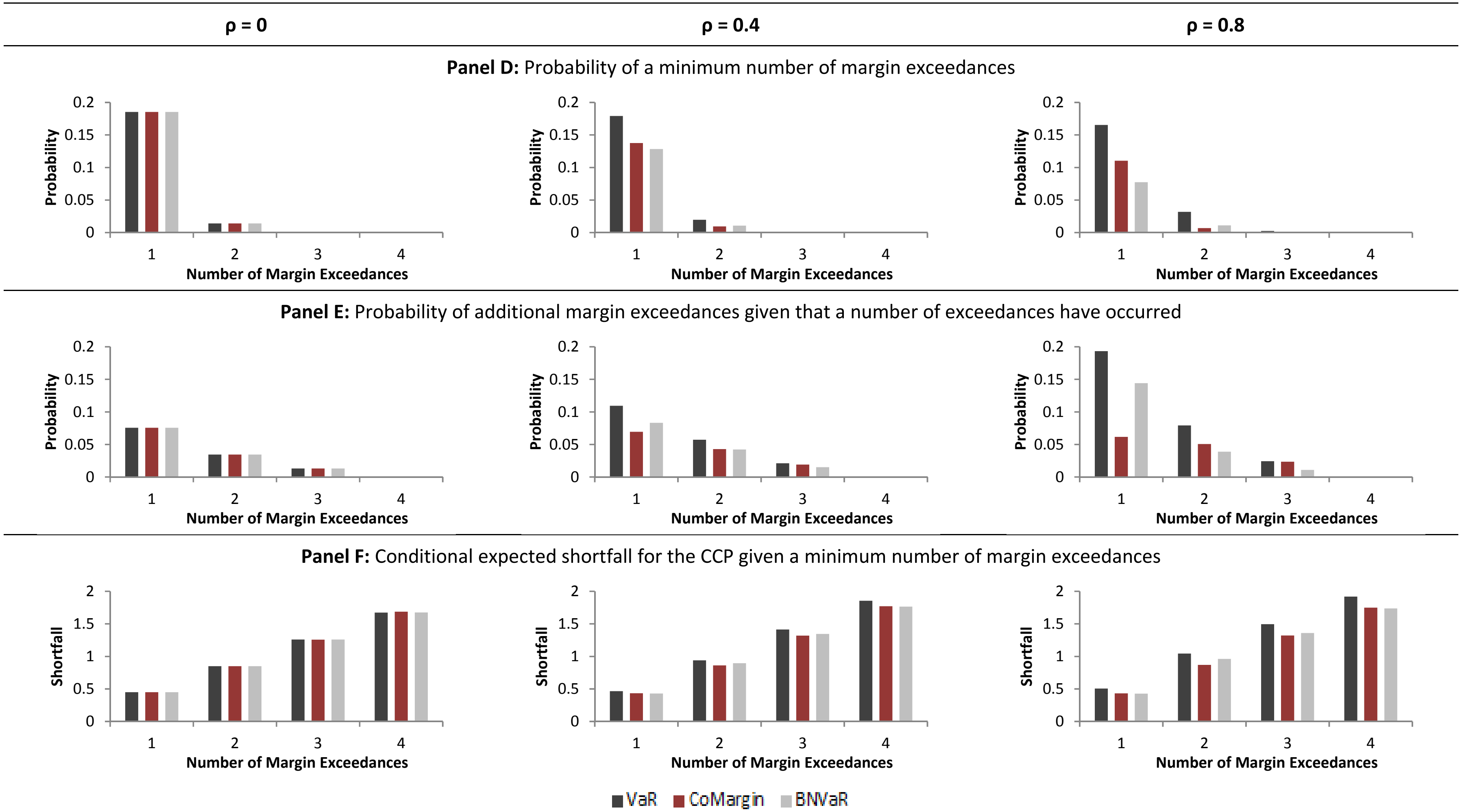


Figure 1 Continued: Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls

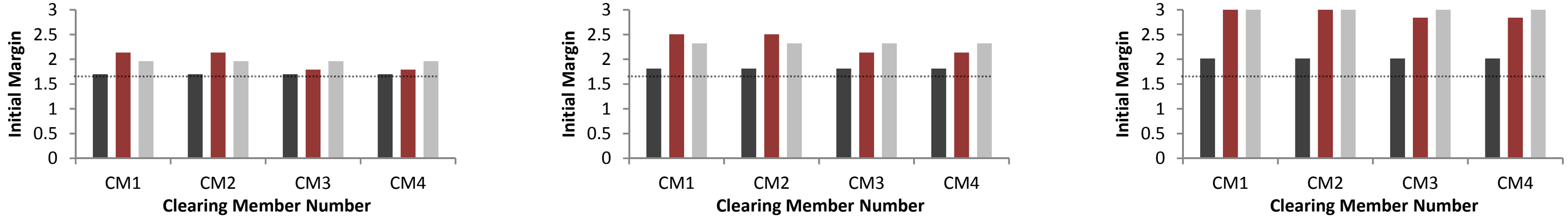


Note: This figure presents the theoretical performance of VaR margin (equation 2), CoMargin (equation 10), and BNVaR margin (equation 23). We consider four firms with jointly normally distributed P&Ls, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. We report our results for different levels of the correlation parameter, ρ, that range from 0 to 0.8.

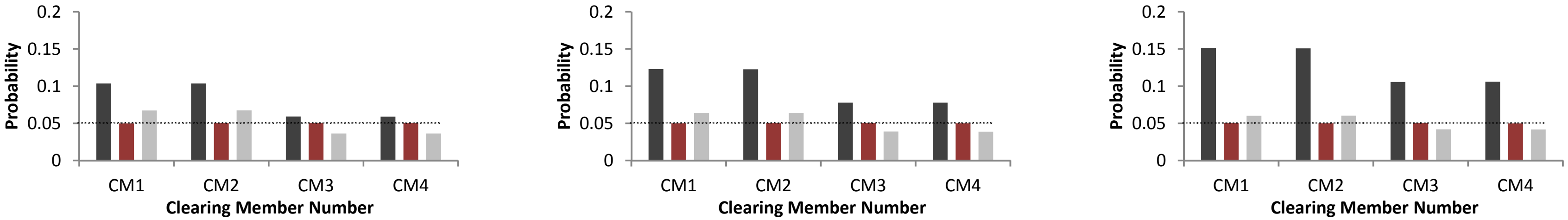
Figure 2: Theoretical performance of VaR and CoMargin systems assuming jointly Student t distributed P&Ls

v = 30	v = 10	v = 5

Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given clearing member exceeding its margin conditional on at least another clearing member being in financial distress



Panel C: Probability of a clearing member exceeding its margin conditional on at least another clearing member having a margin exceedance

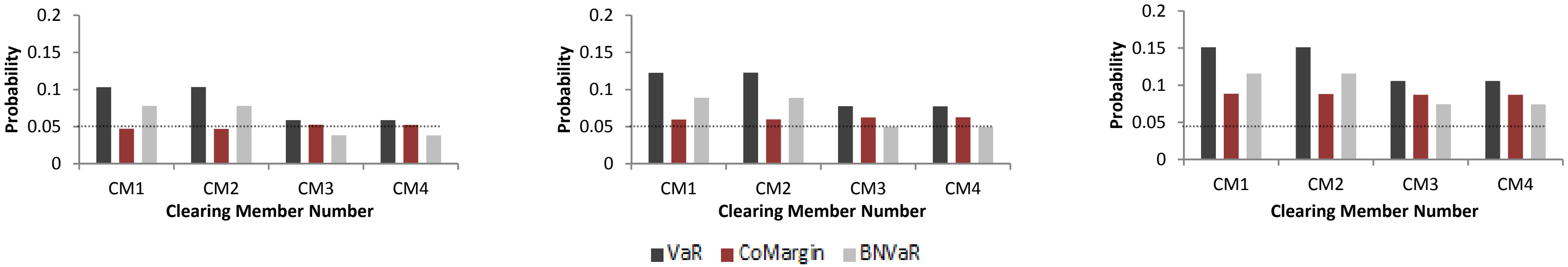
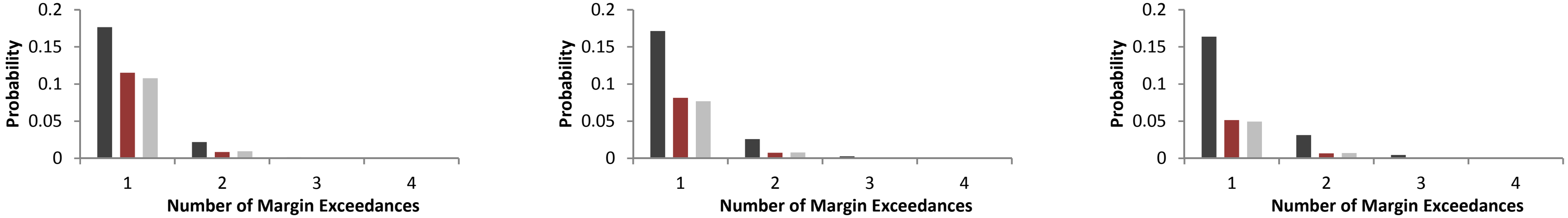


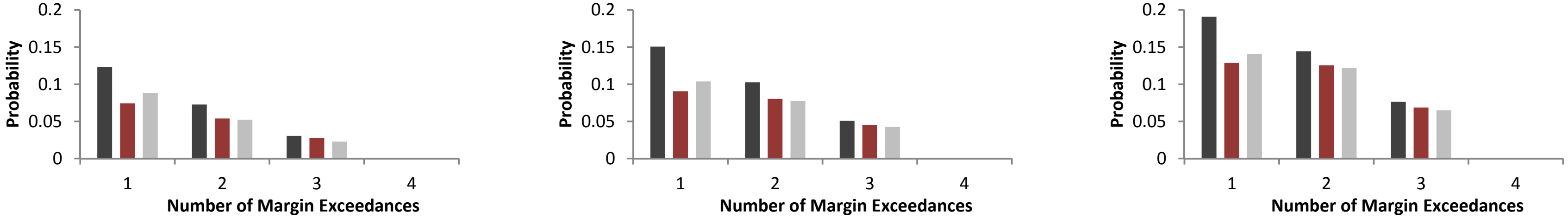
Figure 2 Continued: Theoretical performance of VaR and CoMargin systems assuming jointly Student t distributed P&Ls

v = 30	v = 10	v = 5

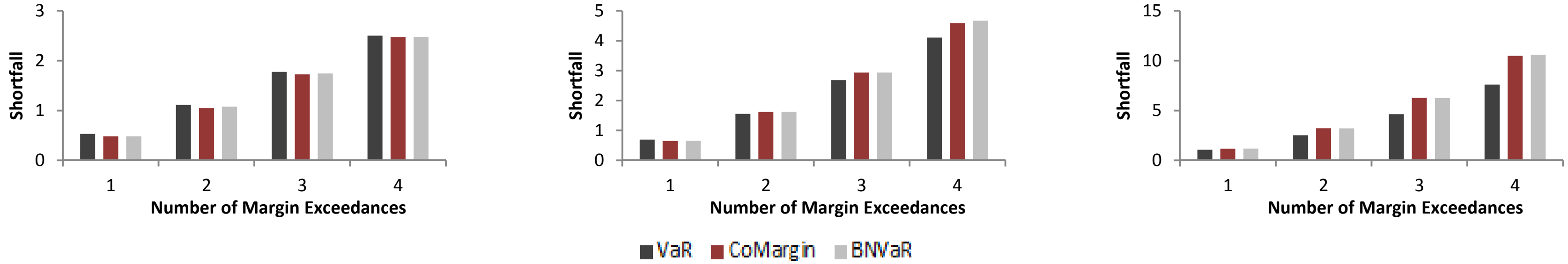
Panel D: Probability of a minimum number of margin exceedances



Panel E: Probability of additional margin exceedances given that a number of exceedances have occurred



Panel F: Conditional expected shortfall for the CCP given a minimum number of margin exceedances

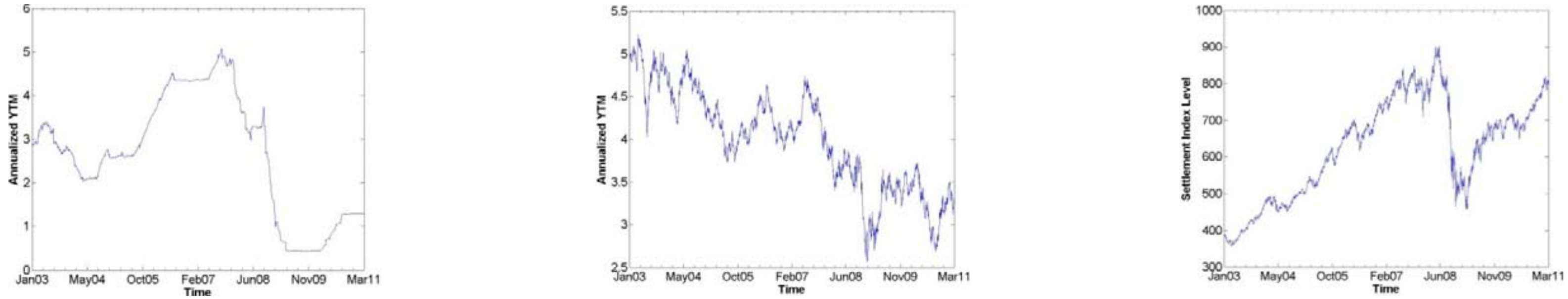


Note: This figure presents the theoretical performance of VaR margin (equation 2), CoMargin (equation 10), and BNVaR margin (equation 23). We consider four firms with joint Student t distributed P&Ls with degrees of freedom υ, such that $V \sim t_{\upsilon}(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$, $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\rho = 0.4$. We report our results for different levels of the degrees of freedom parameter than range from 30 to 5.

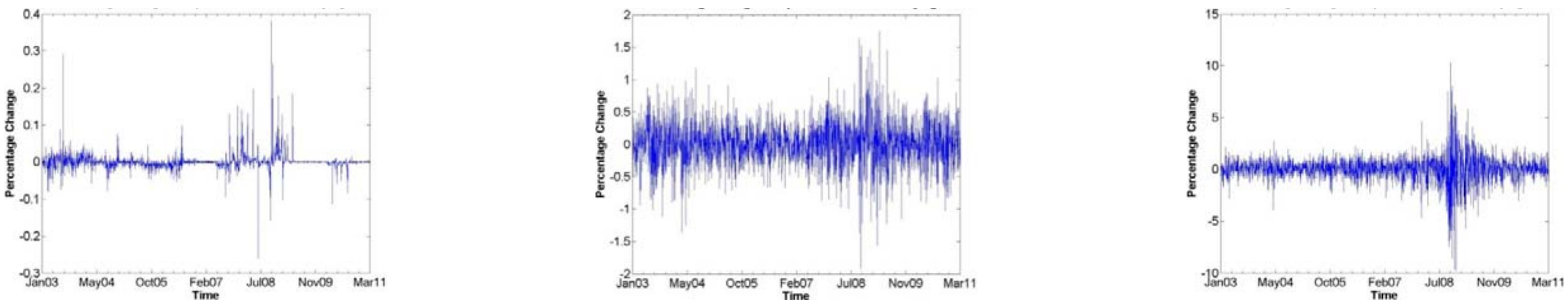
Figure 3: Underlying assets and futures contracts used in the empirical analysis

Three-month Canadian Bankers' Acceptance (BAX)	Ten-year Government of Canada Bond (CGB)	S&P/TSX 60 Index Standard (SXF)

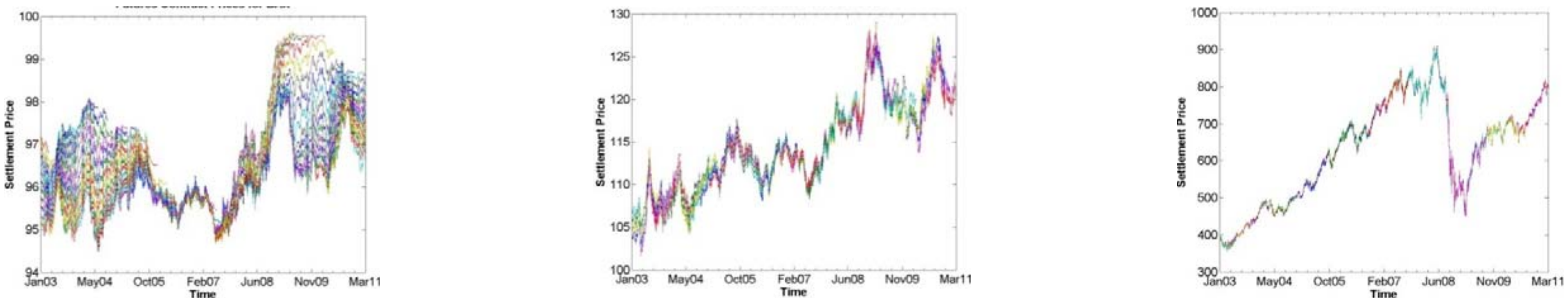
Panel A: Underlying Asset Prices



Panel B: Underlying Asset Returns

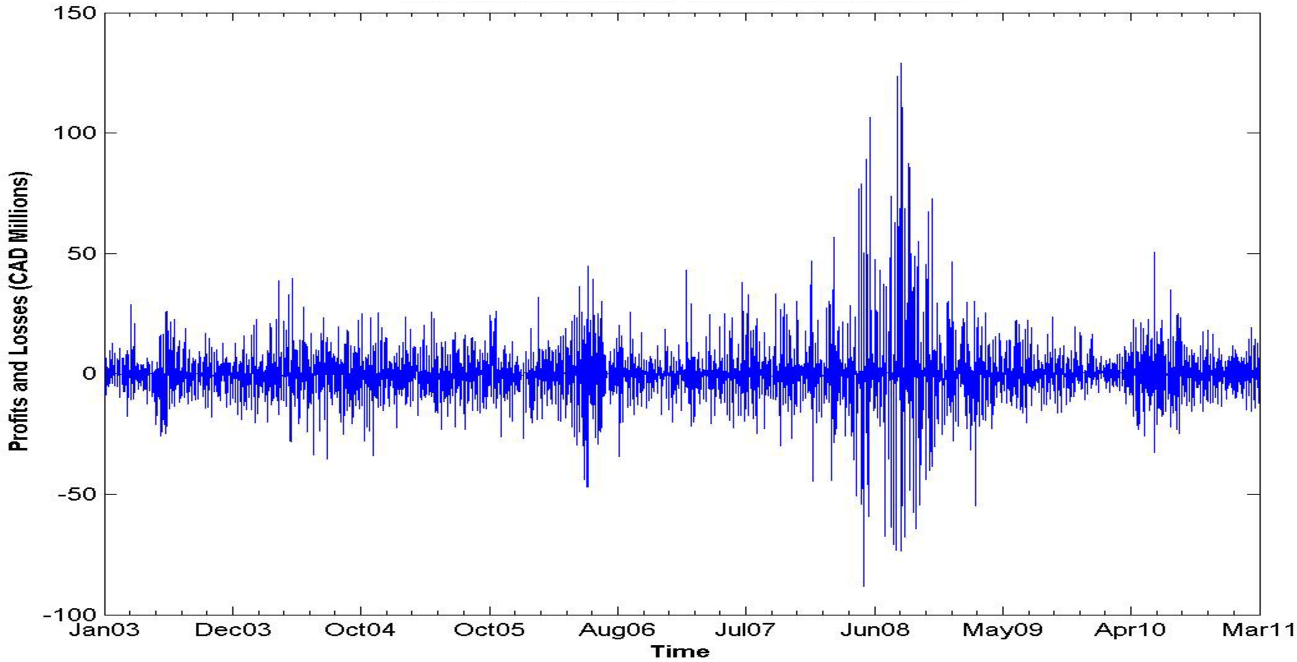


Panel C: Futures Prices (All Maturities)



Note: Panel A presents the daily annualized settlement yield for the Three-month Canadian Bankers’ Acceptance, the annualized yield on the Ten-year Government of Canada Bond and the settlement level of the S&P/TSX 60 Index. Panel B shows the daily returns (i.e., percentage changes) of the variables presented in Panel A. Panel C presents the daily settlement futures prices for the futures contracts written on the Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF), traded in the Montreal Exchange. Lines in different colours represent different delivery dates. The sample period is from January 2, 2003 to March 31, 2011. Source: Bloomberg.

Figure 4: P&L for active firm accounts

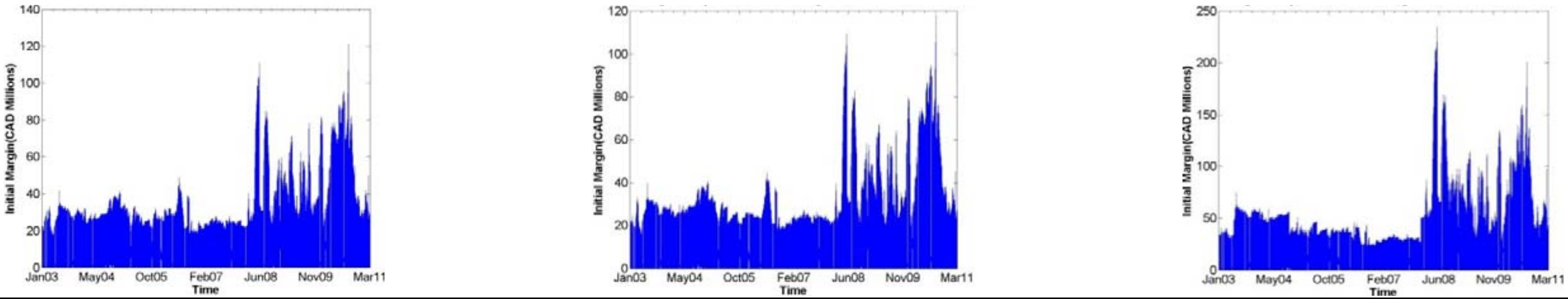


Note: The figure shows the daily stacked P&L implied from the positions of the 23 active firm accounts included in the sample; that is, accounts with an open interest (i.e., long or short position) in at least one underlying asset at the end of the trading day. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the P&L of the n_t^a clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011 and there are $N = 48$ clearing members in the sample.

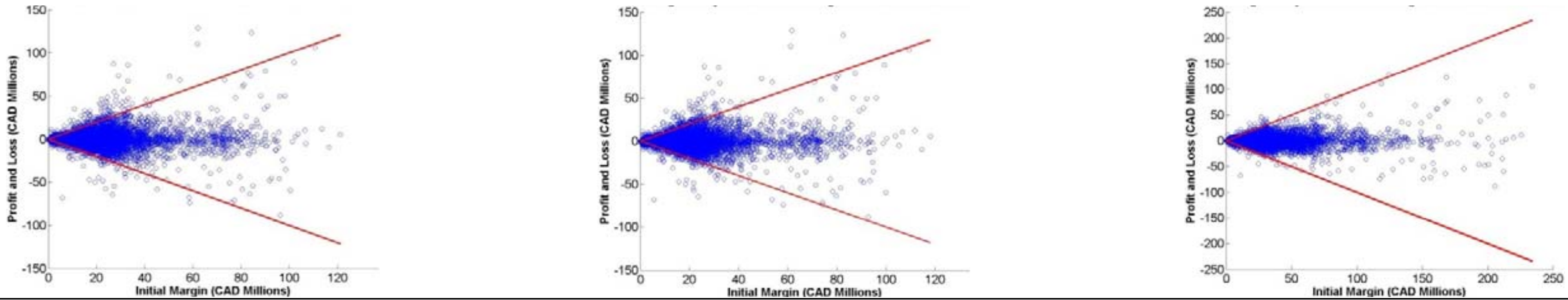
Figure 5: SPAN, VaR and CoMargin collateral requirements over the full sample period

SPAN	VaR	CoMargin

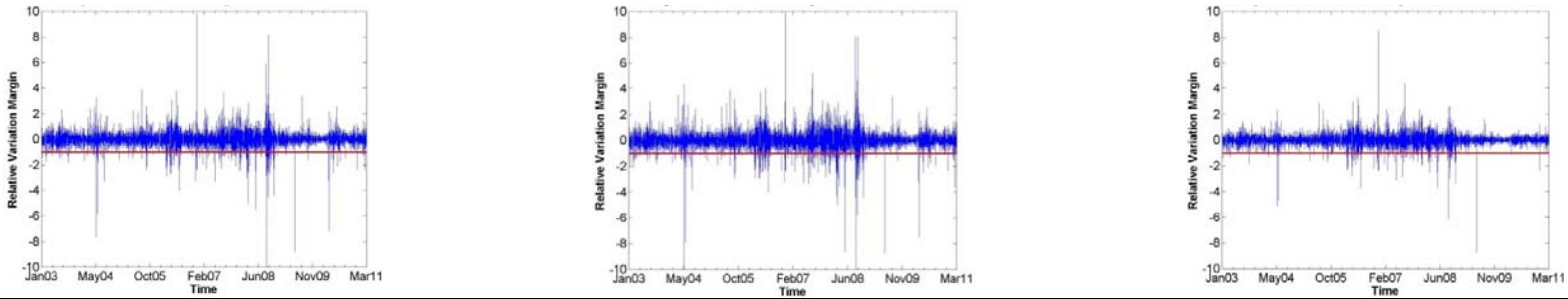
Panel A: Initial Margin



Panel B: Daily P&L and Initial Margins



Panel C: Relative Variation Margin

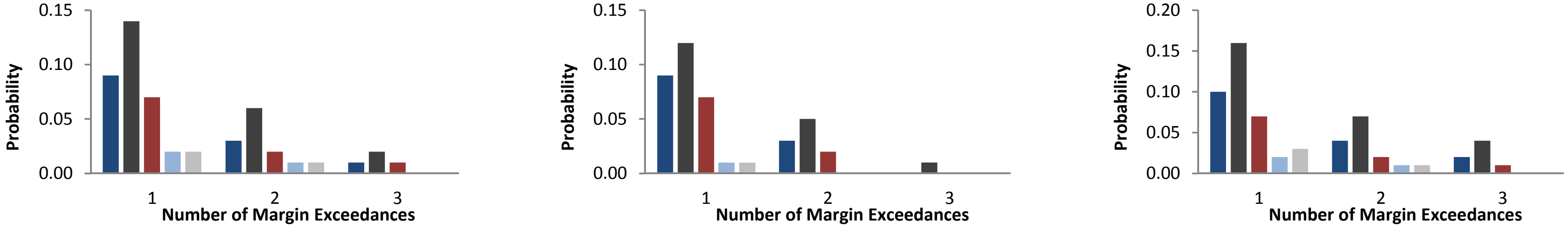


Note: The figure shows the implied margin requirements from the positions in the 23 active firm accounts of the $N = 48$ clearing members in the sample under the SPAN, VaR and CoMargin systems. Panel A shows the daily stacked initial margin requirements. Panel B plots the daily implied P&L against its initial margin requirement. Panel C shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin. The stacking method used in panels A and C is as follows: for each date t, $n_t^a \in N$ observations are plotted, which correspond to the observations of the n_t^a clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011.

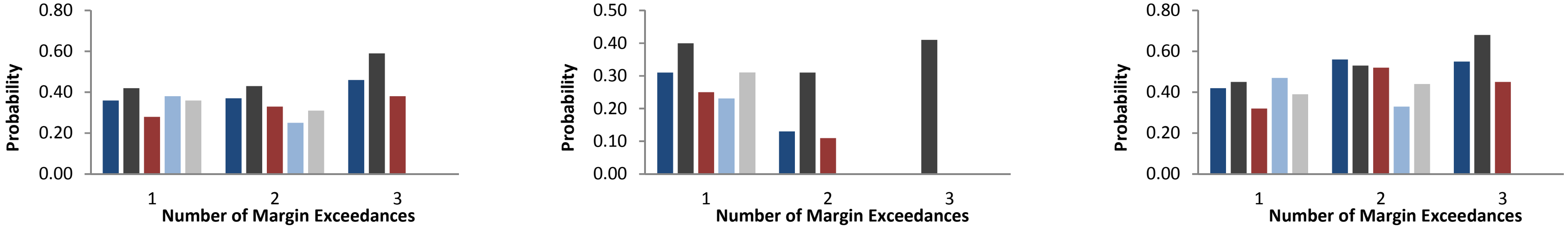
Figure 6: Empirical performance of the SPAN, VaR and CoMargin systems

Full Sample	Pre-Crisis	Crisis

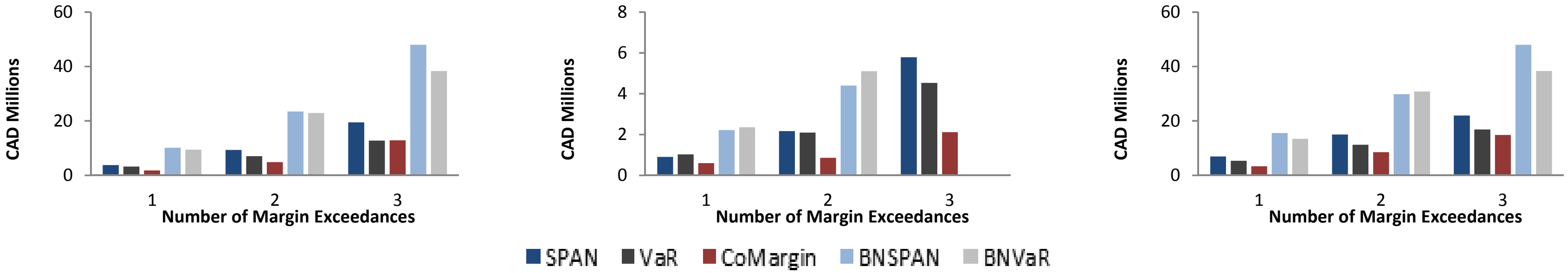
Panel A: Probability of a minimum number of margin exceedances



Panel B: Probability of additional margin exceedances given that a number of exceedances has occurred



Panel C: Conditional expected shortfall for the CCP given a minimum number of margin exceedances



Note: The figure compares the empirical performance of the SPAN, VaR (equation 2) and CoMargin (equation 10) systems computed for the 23 active firm accounts in the sample. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 23. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011.