Banks, Sovereign Risk and Unconventional Monetary Policies

Stéphane Auray† Aurélien Eyquem‡ Xiaofei Ma§

May 12, 2014

Abstract

We develop a two-country model with an explicitly microfounded interbank market and sovereign default risk. Calibrated to the Euro Area, the model performs satisfactorily in matching key business cycle facts on real, financial and fiscal time series. We then use the model to assess the effects of a large crisis and quantify the potential effects of alternative unconventional policies on the dynamics of GDP, sovereign default risk and public indebtedness. We show that quantitative monetary easing is more efficient in stimulating GDP, while qualitative monetary easing relieves financial tensions and sovereign risk more efficiently. In terms of welfare, in the short run, unconventional monetary policies bring sizable welfare gains for households, while the long term effects are much smaller.

Keywords: Recession, Interbank Market, Sovereign Default, Monetary Policy.

JEL Classification: E44, F34, G15.

---

*We thank Gregory Corcos, Olivier Loisel, and Pedro Teles for insightful feedback. We also appreciate helpful discussions with Jordan Roulleau-Pasdeloup and Antoine Vatan. All errors are our own. The authors gratefully acknowledge financial support of the Chair ACPR/Risk Foundation: “Regulation and Systemic Risk”.

†CREST-Ensai, ULCO (EQUIPPE), and CIRPÉE; e-mail: stephane.auray@ensai.fr.
‡Université de Lyon, Université Lumière Lyon 2, CNRS (GATE UMR 5824); e-mail: aurelien.eyquem@ens-lyon.fr.
§CREST-Ensai and Université de Lyon, Université Lumière Lyon 2, CNRS (GATE UMR 5824); e-mail: xiaofei.ma@ensai.fr.
1 Introduction

In this paper, we analyze the interaction between interbank markets and default risk using a two-country dynamic general equilibrium model, with a focus on the transmission of the recent financial crisis and unconventional monetary policies. Interbank markets are at the crossroad of financial and real spheres, as they match creditor and debtor banks. Their dynamics crucially affects the amount of credit in the economy, with effects on investment and GDP. They are also critical in the conduct of monetary policy, as Central Banks implement open market operations to control the interest rate in the overnight interbank market to affect the yield curve. As such, they play a central role in the transmission of monetary policy decisions, as well as in the transmission of potential financial crises. However, despite their apparent importance, there are few widely accepted theoretical analysis about how the interbank market operates. This gap means that, Central Banks were unsure about how to react, when banks stopped lending to each other.

In addition, over the last fifteen years, European banks have increasingly owned government bonds of member countries. In particular, banks of the Core European countries such as France and Germany have gradually become major holders of the sovereign debt issued by Periphery countries like Greece, Italy, Portugal and Spain. As shown by Guerrieri, Iacoviello and Minetti (2012), who used combined data from the Bank for International Settlements and the Bank of France, the ratio of French banks’ holdings of Periphery’s sovereign debt over their holdings of French government debt was 56% in the last quarter of 2009, up from 19% in the first quarter of 2005. As a consequence, European banks were significantly exposed to the sovereign default risk of Periphery countries. This vicious spiral of twin crisis between banks and sovereigns imposes potential losses for banks who invested massively in the sovereign market, and resulted into banks’ reduced ability and propensity to extend lending to the market.
The rising interdependence between interbank and sovereign bonds markets was at the heart of ECB’s concerns about rising sovereign risk in the Euro Area. It was also partly exploited by ECB’s unconventional monetary policies, to release tensions on both markets at the same time. To capture this interdependence, we develop a two-country model based on Guerrieri et al. (2012), who study the international propagation of sovereign debt default, and add an interbank market à la Dib (2010). We quantify the effects of alternative unconventional monetary policies on financial variables (credit, lending rates), real variables (investment, GDP), fiscal variables (debt-to-GDP ratios, sovereign spreads). We also compute welfare losses/gains from the crisis and from the various policies, distinguishing between different types of agents (banks, households, entrepreneurs). Concerning the sovereign risk channel, we differ from Guerrieri et al. (2012) and follow Corsetti, Kuester, Meier and Mueller (2013), assuming that sovereign default risk is increasingly and positively related to a country’s debt-to-GDP ratio. As in Gertler and Kiyotaki (2010), we restrict attention to a purely real model and focus on unconventional policies only.

For the banking sector, our model incorporates an optimizing banking sector with two types of monopolistically competitive banks in the interbank market: “Savings Banks” and “Borrowing Banks”, supplying different banking services and transactions in the interbank market. Savings Banks are financial intermediaries that are net lenders (creditors) in the interbank market, whereas Borrowing Banks are net borrowers (debtors). Banks have monopoly power when setting real deposit and prime lending rates. Savings Banks collect deposits from workers, set real deposit rates, and optimally choose the composition of their portfolio (composed of government bonds and risky interbank lending). Borrowing Banks borrow from Savings Banks in the interbank market and raise bank capital (equity) from bankers (shareholders) in the financial market to satisfy the capital requirement imposed by the regulation.

1The two different banks are necessary to generate heterogeneity, from which different banks can interact in the interbank market (see Dib (2010).
Further, following Goodhart, Sunirand and Tsomocos (2006) and Dib (2010), we assume endogenous strategic default on interbank borrowing. Finally, we introduce unconventional monetary policies in the model. Borrowing Banks can receive injections of money from the Central Bank to prevent freezes on the interbank market, a policy that we refer to as quantitative easing. We also consider a policy by which the Central Bank allows Borrowing Banks to swap a fraction of risky loans for government bonds, thereby improving their balance sheet. We refer to this policy as qualitative easing. We show that quantitative easing is more efficient in stimulating investment and GDP, while qualitative easing is a more effective option to relieve financial tensions and sovereign default risk. In terms of welfare, in the short run, unconventional monetary policies bring welfare gains for households, while the long term effects are much smaller for all types of agents.

The paper is organized as follows. Section 2 relates the paper to the literature. Section 3 describes the model. Section 4 presents the calibration. Section 5 discusses the results and provides the welfare analysis. Section 6 concludes.

2 Literature Review

There are very few studies on the joint frictions in the credit and sovereign markets. The existing studies about the role of banks in global economies pay little attention on sovereign debt problems. Devereux and Yetman (2010) study a two-country economy in which investors hold domestic and foreign assets but are exposed to leverage constraints. They find that if global financial markets are highly integrated, productivity shocks will be propagated through investors’ financial portfolios, which will generate a strong output comovement. Mendoza and Quadrini (2010) build a two-country model with different degrees of financial development. Their model analyzes the cross-country spillover effects of shocks to bank capital. Both Kollmann, Enders and Muller (2011) and Kalemli-Ozcan, Papaioannou and Perri (2012) consider a two-country environment with a global banking
sector, their models generate synchronized business fluctuations across countries.

Our paper relates to some of the recent literature on interbank markets or sovereign default. For instance, Guerrieri et al. (2012) studies the international propagation of sovereign debt default by considering a two-country model calibrated to Europe. They assume that large contractionary shocks in the Periphery trigger sovereign default. Their model shows sizable spillover effects of sovereign default from Periphery to the Core through the financial channel. Mendoza and Yue (2012) construct a small open economy model of sovereign default and business cycles, in which they take default as a strategic decision of the government, but disregard the potential effects of interbank market freezes.

Concerning the interbank market, Allen, Carletti and Gale (2009) build a theoretical model to analyze Central Bank’s intervention on the interbank market but neglect the effects of sovereign default risk. Dib (2010) proposes a micro-founded DSGE model which incorporates an interbank market with two heterogeneous banks according to their liquidity sufficiency. By assuming monopolistic competition among banks, his model studies effects from conventional and unconventional monetary policies. Gertler and Kiyotaki (2010) develop a comprehensive model of the financial sector, they show that the net benefits from Central Bank’s credit market interventions are increasing in the severity of the crisis. Gertler and Karadi (2013) extend the model developed in Gertler and Karadi (2011) to account for qualitative easing on the bond market. However, the model abstracts from sovereign risk and neglects the endogenous response of the government spending during the crisis, as well as the implied effects on debt and default risk. In terms of systemic banking crisis, Boissay, Collard and Smets (2013) build a DSGE model that features a non-trivial banking sector. Their model explains how banking crises break out in the midst of credit intensive booms and bring about particularly deep and long-lasting recessions.

So far, none of the mentioned contributions explores the impact of Central Bank’s
policy on the sovereign default risk through the interbank bank market, which is the main focus of our analysis. Our paper integrates sovereign risk, interbank market, and the Central Bank’s unconventional monetary policy, and studies their joint interaction during a recession.

3 Model

The world economy consists of two countries, Core and Periphery. In each country there are infinitely-lived households, entrepreneurs (capital good producers), final good producers, and bankers. All agents of a given type are homogeneous. In addition, in each country there is a government that purchases final goods financing expenditure with debt and lump-sum taxes. There is one Central Bank in this two-country model, representing the ECB. The final good is produced using labor (non-tradable internationally) and capital. Goods markets are competitive. The banking sector consists of two types of heterogeneous monopolistically competitive banks. We call them “savings” and “borrowing” banks, to indicate that they offer different banking services but interact in the interbank market. The Periphery and the Core have symmetric preferences and technology functional forms, only the calibration will differ. In what follows, we concentrate on the description of the Periphery, and assume that similar relations hold in the Core country. We assume that the size of Periphery Country is 1, while Core Country has a size of \( n \).

Agents’ activities are as follows. In each period, households supply labor to entrepreneurs. Households can save by holding deposits in domestic banks. Entrepreneurs receive loans from banks and invest into physical capital, which they rent to final good producers. Final good producers produce the final good using labor and capital. Saving Banks receive deposits, lend to Borrowing Banks in the interbank market or purchase non-state contingent government bonds issued by both governments. Borrowing Banks make loans to domestic entrepreneurs combining borrowing from the interbank market and bank
capital. Bank capital is held in the form of bonds issued by both governments. Bankers are owners of the two types of banks, from which they receive profits. They consume, save in local government bonds, and accumulate bank capital supplied to Borrowing banks. In our model, the deposit $D_{t-1}$, loan $L_{t-1}$, bank capital $Z_{t-1}$, and physical capital stock $K_{t-1}$ are assumed to be predetermined variables.

### 3.1 Households

We focus on the Periphery country and assume behavioral symmetry in the Core country. Households maximize their expected discounted utility:

$$\max_{\{C_t^H, N_t, D_t\}} E_0 \sum_{t=0}^{\infty} \beta_t^H \left[ \log (C_t^H) - \frac{\chi N_t (N_t)^{1+\eta}}{1 + \eta} \right]$$

subject to

$$C_t^H + D_t + T_t^H + AC_t^H = R^D_{t-1} D_{t-1} + W_t N_t$$

where $C_t^H$ represents households’ consumption, $N_t$ is their labor supply, $D_t$ are households’ holdings of deposits in domestic banks, $AC_t^H = \frac{\phi_D}{2} \left( \frac{D_t}{D} - 1 \right)^2$ denotes quadratic portfolio adjustment costs paid by the household when deviating from the steady-state value of deposit $D$. In addition, $R^D_{t-1}$ is the gross real interest rate on deposit between period $t-1$ and period $t$. $W_t$ is the wage rate, and $T_t^H$ is a lump sum tax imposed on households by the government. First order condition implies that:

$$\lambda_t^H = \frac{1}{C_t^H}$$

$$\lambda_t^H \left[ 1 + \frac{\phi_D}{D} \left( \frac{D_t}{D} - 1 \right) \right] = \beta_H E_t \left[ R^D_{t+1} \lambda_{t+1}^H \right]$$

$$\chi N_t C_t^H (N_t)^{\eta} = W_t$$
3.2 Bankers

Bankers are the owners of the two types of banks, from which they receive profits. They consume, save in domestic government bonds, and accumulate bank capital supplied to Borrowing Banks. Bankers thus enter the period with $Z_{t-1}$ shares of bank capital. Bank capital pays a contingent real return $R^Z_t$, also interpreted as dividend. Bankers maximize their discounted utility function:

$$\max_{\{C^B_t, Z_t, B^B_t\}} \sum_{t=0}^{\infty} \beta^t \log C^B_t$$

subject to

$$C^B_t + B^B_t + Z_t + AC^B_t = R_{t-1} (1 - p_{t-1}) B^B_{t-1} + R^Z_t Z_{t-1} + \Pi^{sb}_t + \Pi^{bb}_t - T^B_t$$

where

$$AC^B_t = \frac{\phi_Z}{2} \left( \frac{Z_t}{Z} - 1 \right)^2 + \frac{\phi_B}{2} \left( \frac{B^B_t}{B^B} - 1 \right)^2$$

and where $C^B_t$ is the consumption of bankers, $B^B_t$ is the amount of local bonds held by bankers paying $R_{t-1}$ with a possibility of default $p_{t-1}$ in period $t - 1$. $\Pi^{sb}_t$ and $\Pi^{bb}_t$ are the profits of savings banks and borrowing banks, respectively, and $T^B_t$ is a lump-sum tax paid to the government. First order condition are

$$\lambda^B_t = \frac{1}{C^B_t}$$

$$1 + \frac{\phi_B}{B^B} \left( \frac{B^B_t}{B^B} - 1 \right) = \beta B E_t \left( \frac{\lambda^B_{t+1}}{\lambda^B_t} R_t (1 - p_t) \right)$$

$$1 + \frac{\phi_Z}{Z} \left( \frac{Z_t}{Z} - 1 \right) = \beta B E_t \left( \frac{\lambda^B_{t+1}}{\lambda^B_t} R^Z_{t+1} \right)$$
3.3 Banks

The banking sector of each country consists of two types of heterogeneous profit-maximizing banks: the Savings Banks (SBs) and Borrowing Banks (BBs).

3.3.1 Savings Banks

Savings Banks (SBs) refer to all financial intermediaries that are net creditors (lenders) in interbank market. There is a continuum of monopolistically competitive, profit-maximizing SBs indexed by \( j \in (0, 1) \). Each SB \( j \) collects fully insured deposits from workers \( D_{j,t} \) and pays a deposit interest rate \( R_{D,j,t}^{D} \), which is optimally set as a markdown over the marginal return of its assets.

The supply of deposits is given by

\[
D_{j,t} = \left( \frac{R_{D,j,t}^{D} - 1}{R_{t}^{D} - 1} \right)^{v_{D}} D_{t-1}
\]

where \( v_{D} > 1 \) is the elasticity of substitution between different types of deposits and \( D_{t} \) is the total amount of deposits. SB \( j \) optimally allocates a fraction \( s_{j,t} \) of deposits to lending in the domestic and foreign interbank markets that return \( R_{t}^{IB,C} \) and \( R_{t}^{IB,P} \), respectively. SB \( j \) also uses the remaining fraction \( 1 - s_{j,t} \) to purchase government bonds, both domestic and foreign. Lending on the interbank market is subject to a default probability \( \delta_{D} \) and induces a quadratic monitoring cost

\[
\Delta_{j,t}^{s} = \frac{\chi_{s}}{2} ((s_{j,t} - \bar{s}) D_{j,t})^{2}
\]

that depends on deviations of \( s_{j,t} \) from a constant target \( \bar{s} \). The remaining fraction of deposits is allocated to bonds and split between domestic bonds and foreign bonds. These assets pay \( R_{t-1}^{P} (1 - p_{t-1}^{P}) \) for bonds issued in the Periphery and \( R_{t-1}^{C} (1 - p_{t-1}^{C}) \) in the Core between period \( t - 1 \) and \( t \), where \( p_{t-1}^{P} \) and \( p_{t-1}^{C} \) are the percentage default rates,
that can also be interpreted as *ex ante* default probabilities in both countries.

Changing the composition of bonds portfolio incurs the payment of adjustment cost

$$ Ad^B_{j,t} = \frac{\phi \delta^B}{2} \left( \frac{\delta^B_{j,t}}{\delta^B} - 1 \right)^2 $$

(13)

Meanwhile, changing the composition of the portfolio of interbank lending incurs a similar adjustment cost:

$$ Ad^{IB}_{j,t} = \frac{\phi \delta^{IB}}{2} \left( \frac{\delta^{IB}_{j,t}}{\delta^{IB}} - 1 \right)^2 $$

(14)

Formally, the $j^{th}$ saving bank’s optimization problem is

$$ \max_{\{s_{j,t}, R_{IB,D,P,t}, \delta^{IB}_{j,t}, \delta^{D,P}_{j,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda^B \left\{ s_{j,t} R_{IB,SB,t} + (1 - s_{j,t}) R^B_t - R_{IB,D} - \Delta^s_{j,t} \right\} $$

(15)

where

$$ R^B_t = \delta^B_t \left( 1 - p^P_t \right) + \left( 1 - \delta^B_t \right) R^C_t \left( 1 - p^C_t \right) - Ad^B_{j,t} $$

(16)

and

$$ R^{IB,SB}_{t} = \delta^{IB}_{t} R^{IB,P}_{t} \left( 1 - \delta^{D,P}_{t} \right) + \left( 1 - \delta^{IB}_{t} \right) R^{IB,C}_{t} \left( 1 - \delta^{D,C}_{t} \right) - Ad^{IB}_{j,t} $$

(17)

subject to the constraints above.

In the definition of $R^B_t$, $\delta^B_t$ is the fraction of deposits invested in bonds issued in the Periphery, and $R^P_t$ and $R^C_t$ are the gross returns on bonds in the Periphery and Core countries, respectively.

Similarly, in the definition of $R^{IB,SB}_{t}$, $\delta^{IB}_{t}$ is the fraction of deposits invested in the Periphery interbank market, $R^{IB,P}_{t}$ and $R^{IB,C}_{t}$ are the gross returns of interbank lending in the Periphery and Core countries, respectively; $\delta^{D,P}_{t}$ and $\delta^{D,C}_{t}$ are the default probabilities on interbank lending in the Periphery and Core countries, respectively.
In the symmetric situation, where $s_{j,t} = s_t$ and $R^D_{j,t} = R^D_t$ for all $t > 0$, the first-order conditions of this optimization problem with respect to $s_t, R^D_t, \delta^B_{j,t}$, and $\delta^{IB}_{j,t}$ are:

$$s_t = \bar{s} + \frac{R^{IB,SB}_t - R^B_t}{\chi_{s}D_{t-1}}$$

(18)

$$\frac{1 + v_D}{v_D} (R^D_t - 1) = \left[ s_t R^{IB,SB}_t + (1 - s_t) R^B_t - 1 \right] - \chi_{s} (s_t - \bar{s})^2 D_{t-1}$$

(19)

$$\frac{\delta^B_t}{\delta_B} = 1 + \frac{\delta^B_t}{\phi_{SB}} \left[ R^P_t (1 - p^P_t) - R^C_t (1 - p^C_t) \right]$$

(20)

$$\frac{\delta^{IB}_t}{\delta^{IB}_{t}} = 1 + \frac{\delta^{IB}_t}{\phi_{SB}} \left[ R^{IB,P}_{t} (1 - \delta^P_{t}) - R^{IB,C}_{t} (1 - \delta^C_{t}) \right]$$

(21)

### 3.3.2 Borrowing Banks

Borrowing Banks (BBs) refer to all net debtor (borrower) banks in the interbank market. There is a continuum of monopolistically competitive BBs indexed by $j \in (0, 1)$. BBs borrow from SBs in the interbank market and raise bank capital from bankers to satisfy the capital requirement. We assume that the stock of bank capital $Z_{j,t}$ is held by the BBs as government bonds in a portfolio with fraction $\delta^{BB}_t$ invested in government bonds issued by the Periphery $^2$.

In addition, BBs can receive money from the Central Bank, which can be interpreted as quantitative easing $^3$. To produce loans $L_{j,t}$ supplied to entrepreneurs, each borrowing bank $j$ combines funds received from SBs in the interbank market, $D^{IB}_{j,t}$, plus any exceptional injection of money by the Central Bank, $M_{j,t}$ (quantitative monetary easing). Also, if needed, BBs may swap a fraction $X_{j,t}$ of their risky assets (loans to entrepreneurs) for government bonds from the Central Bank (qualitative monetary easing) $^4$. Through these two channels, the Central Bank can provide liquidity to BBs in times of financial stress.

$^2$ $Z_{j,t} = Z_{t-1}$ in symmetric case.

$^3$ Nominal money divided by the price index.

$^4$ Quantitative monetary easing, which is associated with newly created money, expands banks’ balance sheets; qualitative monetary easing (swapping banks’ assets for government bonds) changes only banks’ assets compositions.
We assume that BBs use the following Leontief technology to produce loans:

\[ L_{j,t} = \min \{ D^{IB}_{j,t} + M_{j,t}; \kappa_{j,t}(Z_{j,t} + X_{j,t}) \} \Gamma_t \]  \hspace{1cm} (22)

where \( \kappa_{j,t} \leq \kappa \) is bank \( j \)'s leverage ratio and \( \kappa \) is the maximum leverage ratio imposed by regulators. \( X_{j,t} \) is the amount of new assets swapped by the Central Bank. \( \Gamma_t \) is a shock to the financial intermediation process affecting credit supply. Leontief technology implies perfect complementarity between interbank borrowing and bank capital, and imposes the capital requirement, which attenuates the real effects of different shocks. When lending to entrepreneurs, BB \( j \) faces the following demand function for loans:

\[ L_{j,t} = \left( \frac{R_{j,t}^L - 1}{R_{L,t}^L - 1} \right)^{-\upsilon_L} L_{t-1} \]  \hspace{1cm} (23)

where \( \upsilon_L > 1 \) is the elasticity of substitution between different types of loans. The BB \( j \) optimally sets the prime lending rate, \( R_{j,t}^L \), as a markup over the marginal cost of producing loans.

The \( j^{th} \) BB’s leverage ratio is given by \( \kappa_{j,t} = \frac{L_{j,t}}{\Gamma_t(Z_{j,t} + X_{j,t})} \), which is subject to the maximum leverage ratio imposed by the regulators, \( \kappa \). Whenever BBs have a lower-than-target leverage ratio, they receive convex gains in the form of

\[ \Delta_{j,t}^{\kappa} = \frac{\chi_{\kappa}}{2} \left( \frac{\kappa - \kappa_{j,t}}{\kappa} \right)^2 \Lambda_{j,t} (Z_{j,t} + X_{j,t}) \]  \hspace{1cm} (24)

Moreover, following Goodhart et al. (2006), we allow BBs to optimally default on a fraction of their interbank borrowing, \( \delta_{j,t}^D > 0 \). The default on interbank lending can be strategic or mandatory (when banks cannot afford to repay their debt). Nonetheless, it is costly for banks to default on the interbank borrowing. In this case, banks must pay
convex penalties in the next period. The \( j \)th bank’s penalty is given by

\[
\Delta_{j,t}^D = \frac{\chi_{\delta D}}{2} \left( \delta_{j,t-1}^D D_{j,t-1}^{IB} \right)^2 R_{t-1}^{IB}
\]  

(25)

where \( \chi_{\delta D} \) is a positive parameter determining the steady-state value of \( \Delta_{j,t}^D \). Furthermore, we assume that borrowing banks bear the same adjustment cost as SBs when they adjust the holding of domestic and foreign government bonds.

The \( j \)th borrowing bank’s optimization problem is thus

\[
\max_{\{R_{j,t}^L, \delta_{j,t}^D\}} \sum_{t=0}^{\infty} \beta_t^B \lambda_t^B \left\{ R_{j,t}^L L_{j,t} - (1 - \delta_{j,t}^D) R_{t}^{IB} D_{j,t}^{IB} - (R_t^Z - R_t^B) Z_{j,t} \right. \\
\left. - \Delta_{j,t}^D + \Delta_{j,t}^\kappa - R_{t}^B M_{j,t} - (R_{j,t} - R_{t}^B) X_{j,t} \right\}
\]

(26)

subject to equations listed above. In the symmetric equilibrium, where \( R_{j,t}^L = R_L, \kappa_{j,t} = \kappa_t, \delta_{j,t}^D = \delta_t^D \), for all \( t > 0 \). The first-order conditions of this optimization problem are:

\[
\frac{\nu_L - 1}{\nu_L} (R_t^L - 1) = \Omega_t - 1
\]

(27)

\[
\delta_t^D = E_t \frac{\lambda_t^B}{\beta_t^B \lambda_{t+1}^B \chi_{\delta B} D_t^{IB}} (\delta_t^D)^2 R_t^{IB} D_t^{IB} + \frac{\chi_{\delta B} \bar{\kappa} - \kappa_t}{\bar{\kappa}^2}
\]

(28)

where \( \Omega_t \) is the marginal production cost of loans:

\[
\Omega_t = \Gamma_t^{-1} \left[ (1 - \delta_t^D) R_t^{IB} + E_t \beta_B \chi_{\delta B} \frac{\lambda_{t+1}^B}{\lambda_t^B} (\delta_t^D)^2 R_t^{IB} D_t^{IB} + \frac{\chi_{\delta B} \bar{\kappa} - \kappa_t}{\bar{\kappa}^2} \right]
\]

(29)

3.4 Entrepreneurs

In each country, there is a continuum of identical infinitely-lived entrepreneurs who borrow from local BBs, consume, and accumulate physical capital that depreciates at rate \( \delta \).
Entrepreneurs maximize their lifetime utility:

$$\max_{\{C_t^E, K_t, L_t\}} \sum_{t=0}^{\infty} \beta_t^E \log C_t^E$$

subject to the budget constraint:

$$C_t^E + K_t + R_t^L L_{t-1} + AC_t^E = (R_{K,t} - \delta) K_{t-1} + L_t - T_t^E$$

where

$$AC_t^E = \frac{\phi_t^E}{2} \left( \frac{I_t}{I} - 1 \right)^2 + \frac{\phi_{L_t}^E}{2} \left( \frac{L_t}{L_{t-1}} - 1 \right)^2$$

denotes quadratic portfolio adjustment costs paid by the entrepreneurs to change the investment, and the holdings of loans. The investment $I_t$ is defined as:

$$I_t = K_t - (1 - \delta) K_{t-1}$$

In addition, entrepreneurs are subject to a constraint that limits their leverage to a constant fraction $m$ of their capital holdings:

$$L_t = \rho_E L_{t-1} + (1 - \rho_E) m K_t$$

where the parameter $\rho_E$ captures the elasticity of the loan limit to the current capital choice of the entrepreneur. Therefore, the FOCs of the program are:

$$\lambda_t = \frac{1}{C_t^E}$$

$$\lambda_t \left[ 1 + \frac{\phi_t^E}{\delta K} \left( \frac{I_t}{\delta K} - 1 \right) - \frac{\beta_t^E \phi_t^E (1 - \delta)}{\delta K} \left( \frac{I_{t+1}}{\delta K} - 1 \right) \right] - m (1 - \rho_E) \tilde{\lambda}_t = \beta_t^E E_t \left[ \lambda_{t+1} (R_{K,t+1} - \delta) \right]$$

$$\lambda_t \left[ \frac{\phi_{L_t}^E}{L_{t-1}} \left( \frac{L_t}{L_{t-1}} - 1 \right) - 1 \right] + \tilde{\lambda}_t = E_t \beta_{t+1} \lambda_{t+1} \left[ \frac{\phi_{L_{t+1}}}{L_t^2} \left( \frac{L_{t+1}}{L_t} - 1 \right) - R_{L,t+1} \right] + \beta_{t+1} \rho_E$$
where $\lambda_t$ and $\tilde{\lambda}_t$ are the Lagrange multipliers for constraint (31) and (34).

### 3.5 Final Good Producers

The representative final good producer operates a Cobb-Douglas production function given by

$$Y_t = A_t (K_{t-1})^\alpha (N_t)^{1-\alpha}$$

where $Y_t$ is the output of domestic final good, $K_{t-1}$ is the capital stock rented from entrepreneurs at the rate, $R_{K,t}$, $N_t$ is the labor input and $A_t$ is a stochastic process for productivity. Producers make zero profits because of perfect competition and constant returns to scale. With competitive factor market and profit maximizing producers, we have:

$$W_t = (1 - \alpha) \frac{Y_t}{N_t}$$

$$R_{K,t} - 1 = \alpha \frac{Y_t}{K_{t-1}}$$

### 3.6 Governments

Government spending is financed through taxes on households, entrepreneurs, bankers and through debt. The government budget constraint in the Periphery country is

$$B_t - R_{t-1} (1 - p_{t-1}) B_{t-1} = G_t - (T^H_t + T^B_t + T^E_t)$$

(38)

where $G_t$ is exogenous government spending, $B_t$ is the amount of government bonds issued in the Periphery. Public expenditure is financed at time $t$ by raising taxes, issuing additional debt or partially defaulting on debt issued in the past.
We assume that government spending has a counter-cyclical component seeking to stabilize the business cycle. In addition, we impose a cut on government spending when the probability of default rises:

\[
G_t = \rho B G_{t-1} + (1 - \rho B) (G - \sigma (Y_t - Y) - \gamma (B_{t-1} - B)) + \varepsilon_{gt} \tag{39}
\]

### 3.7 Sovereign Default Risk

There are mainly two strands of literatures about sovereign default. On one hand, Eaton and Gersovitz (1981), Arellano (2008) and others have modeled default as a strategic decision of the government. On the other hand, Bi (2012) considers default as the consequence of the government’s inability to raise funds used to honor its debt obligations. Under both approaches, the probability of sovereign default is closely and non-linearly related to the level of public debt to GDP. We adopt the approach from Corsetti et al. (2013), in which the \textit{ex ante} probability of default, \( p_t \), at a certain level of sovereign indebtedness, \( b_t = B_t / (4Y_t) \), will be given by the cumulative distribution function of the beta distribution:

\[
p_t = F_{\text{beta}} (b_t / b_{\text{max}}, \alpha_{b\theta}, \beta_{b\theta}) \tag{40}
\]

where \( b_{\text{max}} \) denotes the upper end of the support for the debt-to-GDP ratio. We focus on the \textit{ex ante} probability of sovereign default, because the \textit{ex ante} default risk is crucial for the pricing of government debt.

### 3.8 Equilibrium

We assume that the Core country is characterized by similar equilibrium conditions. An equilibrium is defined as a sequence of prices and quantities such that all agents solve their optimization problems, and that prices clear the markets:

1. Production factor markets
2. Loan markets

3. Bank capital markets

\[ Z_{t-1}^P = \frac{L_{t-1}^P}{\Gamma_t^P \kappa_t^P} - X_t^P \]
\[ Z_{t-1}^C = \frac{L_{t-1}^C}{\Gamma_t^C \kappa_t^C} - X_t^C \]

4. The goods market

\[ Y_t^P + nY_t^C = \left( C_t^{H,P} + C_t^{B,P} + C_t^{E,P} \right) + n \left( C_t^{H,C} + C_t^{B,C} + C_t^{E,C} \right) \]
\[ (K_t^P - (1 - \delta) K_{t-1}^P) + n \left( K_t^C - (1 - \delta) K_{t-1}^C \right) \]
\[ + G_t^P + G_t^C + \text{adjustment/monitoring costs} \]
\[ - \text{gains from excess bank capital} \]

5. The interbank market

\[ \delta_t^{IB,P} s_t^P D_{t-1}^P + n\delta_t^{IB,C} s_t^C D_{t-1}^C = D_{t-1}^{IB,P} = \frac{L_{t-1}^P}{\Gamma_t^P} - M_t^P \]
\[ n \left( 1 - \delta_t^{IB,C} \right) s_t^C D_{t-1}^C + (1 - \delta_t^{IB,P}) s_t^P D_{t-1}^P = \ nD_{t-1}^{IB,C} = n \left( \frac{L_{t-1}^C}{\Gamma_t^C} - M_t^C \right) \]

6. Sovereign debt markets

\[ B_t^P = B_t^{B,P} + \delta_t^{B,P} \left[ (1 - s_t^P) D_{t-1}^P + Z_{t-1}^P + X_t^P \right] \]
\[ + \ n\delta_t^{B,C} \left[ (1 - s_t^C) D_{t-1}^C + Z_{t-1}^C + X_t^C \right] \]
\[ B_t^C = B_t^{B,P} + (1 - \delta_t^{B,P}) \left[ (1 - s_t^P) D_{t-1}^P + Z_{t-1}^P + X_t^P \right] \]
\[ + \ n(1 - \delta_t^{B,C}) \left[ (1 - s_t^C) D_{t-1}^C + Z_{t-1}^C + X_t^C \right] \]
3.9 Shock Processes

The economy is subject to productivity, government expenditure, interbank tension, conventional and unconventional monetary policy shocks.

Government spending shock is given by equation (39). The left structural shocks follow AR(1) processes:

$$\log(\Lambda_t) = (1 - \rho_\Lambda) \log(\Lambda) + \rho_\Lambda \log(\Lambda_{t-1}) + \varepsilon_{\Lambda_t}$$

where \(\Lambda_t = \{A_t, \Gamma_t, M_t, X_t\}\); \(\Lambda \geq 0\) is the steady-state value of \(\Lambda_t\); \(\rho_\Lambda \in (0, 1)\); and \(\varepsilon_{\Lambda_t} \sim N(0, \sigma_\Lambda)\).

4 Calibration

We calibrate the model to the Euro Area. The time unit is a quarter. The Periphery comprises Greece, Italy, Portugal and Spain while the Core is made of remaining members of the monetary union. We choose the Households’ discount factor, \(\beta_H\), to be symmetric and equal to 0.9975, implying an annual real interest rate on deposits of 1%. The inverse of the Frisch elasticity of labor supply is \(\eta = 2\). The discount factors of bankers and entrepreneurs are set equal to 0.997 and 0.99, respectively.

On the production side, the depreciation rate \(\delta\) is 0.03 and the capital share in production \(\alpha\) is 0.33. We set investment adjustment costs \(\phi^F_E = 2.5\) to match the relative volatility (standard deviation) of investment as much as possible. Entrepreneurs also pay a convex cost for adjusting loans; we set \(\phi^E_L = 0.05\) as in Guerrieri et al. (2012). We set the parameter governing entrepreneurs’ working capital constraint \(\rho_E = 0.75\). \(m\) is chosen so that its value matches the ratio of outstanding loans over capital stocks.

Moving to the government, the debt ceiling parameter \(\bar{b}\) is 3.4 for the Periphery bloc and 2.4 for the Core bloc. These choices imply a debt-to-GDP ratio equal to 0.85 in the
Periphery and 0.6 in the Core, in line with data from the IMF Economic Outlook for 2010. Steady-state tax rates are calibrated to represent 45\% of the consumption of each type of agent and remain constant over simulations. In terms of sovereign default risk, we follow Corsetti et al. (2013), and choose parameters $\alpha_{bg} = 3.70$, $\beta_{bg} = 0.54$, $b_{\text{max}} = 2.56$. In the public spending process, we assume a $\rho_B = 0.81$ persistence, the coefficient on default risk is $\gamma = 0.025$ in the Core and $\gamma = 0.05$ in the Periphery, and the coefficient on output is $\sigma = 0.25$ in the Core and $\sigma = 0$ in the Periphery. Those different coefficient are set to match differences in cyclical patterns of public spending between the Core and Periphery, as well as sovereign spread volatilities.

Households pay a cost for adjusting deposits, we set $\phi_D = 0.05$. In the banker’s problem, the parameter measuring the convexity of the adjusting cost function, $\phi_B$, denote how costly it is for the bankers to adjust domestic and foreign bonds: we set $\phi_B = 0.01$.

To apportion the steady-state holdings of government debt to households, Periphery and Core banks, we adopt the calibration in Guerrieri et al. (2012), i.e. 41.7\% the share of public debt of the Periphery is held by the Core bloc, 18.8\% is held by Periphery households, and 39.5\% is allocated to Periphery banks. For the Core government debt, 11\% of their collective sovereign debt is held by households in Core countries, and 70\% of the total is assigned to Core banks.

In the interbank sector, the parameter $\chi_s$ determines the ratio of bank lending to total assets held by the savings banks $s_t$. It is set so that the steady-state value of $s_t$ is equal to 0.64 for the Periphery and 0.86 for the Core, which corresponds to the historical ratio observed in the data\(^5\). The parameter $\chi_\kappa$ is set so that we could have a reasonable value for $\nu_L$, i.e. around 4.

Based on the Basel II minimum required bank capital ratio of 8\%, we assume that the

\(^5\)This is significantly smaller than in the US (0.82), which might have helped Europe to offset the impact from the 2008 subprime crisis.
maximum imposed bank leverage, $\bar{\kappa}$, is 5.36 for Periphery and 4.03 for Core\textsuperscript{6}. Similarly, we calibrate $\chi_{\delta D}$, the parameter determining the total cost of banks’ defaults on interbank borrowing, so that the probability of default in the interbank market equals to 0.2% in annual term.

The parameters $\nu_D$ and $\nu_L$, which measure the degrees of monopoly power of savings and Borrowing banks, are set to match the historical averages of real deposit and loan rates, $R^D$ and $R^L$.

Table 1 summarizes the parameterization and shows the implied steady-state value of key variables in the model.

\textsuperscript{6}Before Basel II, the maximum ratio of bank asset/capital is the reciprocal of required capital ratio, which equals to 12.5. However, loans are only a part of bank assets. The leverage ratio in our model equals to loans/bank capital.
Table 1: Parameter and steady-state values

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>Periphery</th>
<th>Core</th>
<th>Steady-state values</th>
<th>Periphery</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_H$</td>
<td>0.9975</td>
<td>0.9975</td>
<td>$A$</td>
<td>1.00</td>
<td>1.21</td>
</tr>
<tr>
<td>$\beta_B$</td>
<td>0.997</td>
<td>0.997</td>
<td>$Y$</td>
<td>3.9860</td>
<td>6.1067</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>0.99</td>
<td>0.99</td>
<td>$W$</td>
<td>1.9249</td>
<td>2.5581</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.00</td>
<td>2.00</td>
<td>$N$</td>
<td>0.3481</td>
<td>0.2619</td>
</tr>
<tr>
<td>$m$</td>
<td>0.4580</td>
<td>0.4550</td>
<td>$\bar{s}$</td>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>0.03</td>
<td>$s$</td>
<td>0.64</td>
<td>0.86</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>0.33</td>
<td>$\bar{\kappa}$</td>
<td>5.3619</td>
<td>4.0387</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>0.75</td>
<td>0.75</td>
<td>$\kappa$</td>
<td>4.9330</td>
<td>3.7156</td>
</tr>
<tr>
<td>$\nu_D$</td>
<td>3.35</td>
<td>2.88</td>
<td>$\Gamma$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\nu_L$</td>
<td>3.88</td>
<td>3.86</td>
<td>$R^D$</td>
<td>1.0025</td>
<td>1.0025</td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>0.00019</td>
<td>0.00015</td>
<td>$R^L$</td>
<td>1.007</td>
<td>1.007</td>
</tr>
<tr>
<td>$\chi_{\kappa}$</td>
<td>0.08</td>
<td>0.06</td>
<td>$R^{IB}$</td>
<td>1.004</td>
<td>1.004</td>
</tr>
<tr>
<td>$\chi_D$</td>
<td>128.93</td>
<td>84.73</td>
<td>$R^{P/C}$</td>
<td>1.0093</td>
<td>1.0047</td>
</tr>
<tr>
<td>$\phi_D$</td>
<td>0.05</td>
<td>0.05</td>
<td>$R^Z$</td>
<td>1.0030</td>
<td>1.0030</td>
</tr>
<tr>
<td>$\phi_Z$</td>
<td>70</td>
<td>70</td>
<td>$R^K$</td>
<td>1.0387</td>
<td>1.0387</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>0.01</td>
<td>0.01</td>
<td>$\delta^D$</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\phi_I^E$</td>
<td>2.50</td>
<td>2.50</td>
<td>$D/Y$</td>
<td>4.55</td>
<td>4.80</td>
</tr>
<tr>
<td>$\phi^E_L$</td>
<td>0.05</td>
<td>0.05</td>
<td>$T^H/Y$</td>
<td>0.2115</td>
<td>0.2117</td>
</tr>
<tr>
<td>$\phi_D^B$</td>
<td>0.01</td>
<td>0.01</td>
<td>$C^H/Y$</td>
<td>0.4699</td>
<td>0.4704</td>
</tr>
<tr>
<td>$\phi_M^B$</td>
<td>0.01</td>
<td>0.01</td>
<td>$Z/Y$</td>
<td>0.7913</td>
<td>1.0435</td>
</tr>
<tr>
<td>$\rho^B$</td>
<td>0.81</td>
<td>0.81</td>
<td>$B^B/Y$</td>
<td>1.9168</td>
<td>0.4452</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.05</td>
<td>0.025</td>
<td>$T^B/Y$</td>
<td>0.0079</td>
<td>0.0069</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.00</td>
<td>0.25</td>
<td>$C^B/Y$</td>
<td>0.0175</td>
<td>0.0154</td>
</tr>
<tr>
<td>$\alpha_{BS}$</td>
<td>3.70</td>
<td>3.70</td>
<td>$\Pi^{SB}/Y$</td>
<td>0.0036</td>
<td>0.0043</td>
</tr>
<tr>
<td>$\beta_{BV}$</td>
<td>0.54</td>
<td>0.54</td>
<td>$\Pi^{BB}/Y$</td>
<td>0.0137</td>
<td>0.0136</td>
</tr>
<tr>
<td>$b_{max}$</td>
<td>2.56</td>
<td>2.56</td>
<td>$K^E/Y$</td>
<td>8.5223</td>
<td>8.5203</td>
</tr>
<tr>
<td>$size$</td>
<td>1</td>
<td>2.58</td>
<td>$T^E/Y$</td>
<td>0.0146</td>
<td>0.0147</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L/Y$</td>
<td>3.9035</td>
<td>3.8772</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$C^E/Y$</td>
<td>0.0324</td>
<td>0.0326</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$D^{IB}/Y$</td>
<td>3.9035</td>
<td>3.8772</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$G/(sizeY)$</td>
<td>0.2237</td>
<td>0.2260</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$B/(sizeY)$</td>
<td>3.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

5 Simulations

In this section we run two types of exercises. First, we compute the business cycle moments induced by the model and compare them to the key business cycle moments in the
Euro Area. Second, we run non-linear simulations of the model to assess the impact of quantitative and qualitative easing policies followed by the ECB during the crisis.\textsuperscript{7} The idea is to compare the relative efficiency of alternative interventions both on the interbank and the sovereign debt markets. We also introduce a scenario when the market treats Core government bond as “safe” asset, \textit{i.e.} a safe haven. The last part of the section proceeds to a welfare analysis of each policy.

\section*{5.1 Business Cycle Moments}

We first report the business cycle properties of the economy when driven by three types of shocks: productivity shocks, financial shocks and public spending shocks. Our calibration for those shocks follows Dib (2010) and assumes $\rho_A = \rho_T = 0.8$, $\sigma_A = 0.01$ and $\sigma_T = 0.003$. The public spending rule features a 0.81 persistence ($\rho_B = 0.81$) and shock volatilities are adjusted to match the volatility of public spending relative to the volatility of GDP, \textit{i.e.} $\sigma_G = 0.0087$ in the Core and $\sigma_G = 1.01$ in the Periphery. The artificial time series generated by the linearized solution of the model are then HP-filtered with $\lambda = 1600$ and we report the standard deviations, autocorrelations and correlations with output. Those results are compared to business cycle moments computed from the data. The results are reported in Table 2.

Business cycle moments concerning GDP and its main components (consumption, investment and public spending) are correctly reproduced. GDP is more volatile in the Periphery than in the Core. The relative volatility of consumption is less than one in the Core and close to one in the Periphery. The relative volatility of investment is higher in the Core than in the Periphery, although its level is a bit larger than in the data.\textsuperscript{8} The relative volatility of public spending is perfectly matched with data but the latter was

\textsuperscript{7}So far, the most important instrument from ECB is the reverse transaction (applicable on the basis of repurchase agreements or collateralized loans). Its influence on the economy should be a mix of the quantitative and qualitative monetary easing.

\textsuperscript{8}A calibration abstracting from financial shocks allows to match those moments almost exactly.
Table 2: Business cycle moments

<table>
<thead>
<tr>
<th>Variables (x)</th>
<th>Core</th>
<th>Periphery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ</td>
<td>ρ</td>
</tr>
<tr>
<td>Output² (y)</td>
<td>1.27</td>
<td>0.79</td>
</tr>
<tr>
<td>Consumption²</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>Public spending²</td>
<td>0.82</td>
<td>0.68</td>
</tr>
<tr>
<td>Investment²</td>
<td>3.00</td>
<td>0.75</td>
</tr>
<tr>
<td>IB rate² (r²)</td>
<td>0.23</td>
<td>0.89</td>
</tr>
<tr>
<td>Loan rate²</td>
<td>0.10</td>
<td>0.89</td>
</tr>
<tr>
<td>Deposit rate²</td>
<td>0.09</td>
<td>0.91</td>
</tr>
<tr>
<td>Sov. long rates² (r)</td>
<td>0.04</td>
<td>0.68</td>
</tr>
<tr>
<td>Sov. Spreads² (spr)</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>IB. Spreads²</td>
<td>0.02</td>
<td>0.52</td>
</tr>
<tr>
<td>Debt to GDP²</td>
<td>3.75</td>
<td>0.78</td>
</tr>
<tr>
<td>Loans to GDP²</td>
<td>2.66</td>
<td>0.86</td>
</tr>
<tr>
<td>Deposits to GDP²</td>
<td>4.38</td>
<td>0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlations (ρ_{x,x})</th>
<th>r²</th>
<th>spr</th>
<th>r³</th>
<th>spr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt to GDP²</td>
<td>−</td>
<td>−0.50</td>
<td>0.10</td>
<td>−</td>
</tr>
<tr>
<td>Public spending²</td>
<td>−</td>
<td>0.02</td>
<td>−0.08</td>
<td>−</td>
</tr>
<tr>
<td>Loans to GDP²</td>
<td>0.07</td>
<td>−</td>
<td>−</td>
<td>0.34</td>
</tr>
<tr>
<td>Deposits to GDP²</td>
<td>0.36</td>
<td>−</td>
<td>−</td>
<td>−0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables (x)</th>
<th>Core</th>
<th>Periphery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ</td>
<td>ρ</td>
</tr>
<tr>
<td>Output (y)</td>
<td>1.41</td>
<td>0.63</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.89</td>
<td>0.35</td>
</tr>
<tr>
<td>Public spending</td>
<td>0.81</td>
<td>0.69</td>
</tr>
<tr>
<td>Investment</td>
<td>6.34</td>
<td>−0.02</td>
</tr>
<tr>
<td>IB rate (r²²)</td>
<td>0.33</td>
<td>−0.29</td>
</tr>
<tr>
<td>Loan rate</td>
<td>0.26</td>
<td>0.23</td>
</tr>
<tr>
<td>Deposit rate</td>
<td>0.11</td>
<td>0.61</td>
</tr>
<tr>
<td>Sov. long rates (r)</td>
<td>0.08</td>
<td>0.63</td>
</tr>
<tr>
<td>Sov. Spreads (spr)</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>IB. Spreads</td>
<td>0.05</td>
<td>0.73</td>
</tr>
<tr>
<td>Debt to GDP</td>
<td>1.49</td>
<td>0.65</td>
</tr>
<tr>
<td>Loans to GDP</td>
<td>1.50</td>
<td>0.66</td>
</tr>
<tr>
<td>Deposits to GDP</td>
<td>1.40</td>
<td>0.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlations (ρ_{x,x})</th>
<th>r²²</th>
<th>spr</th>
<th>r³²</th>
<th>spr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt to GDP</td>
<td>−</td>
<td>0.37</td>
<td>0.26</td>
<td>−</td>
</tr>
<tr>
<td>Public spending</td>
<td>−</td>
<td>0.29</td>
<td>0.23</td>
<td>−</td>
</tr>
<tr>
<td>Loans to GDP</td>
<td>0.22</td>
<td>−</td>
<td>−</td>
<td>0.04</td>
</tr>
<tr>
<td>Deposits to GDP</td>
<td>0.27</td>
<td>−</td>
<td>−</td>
<td>0.12</td>
</tr>
</tbody>
</table>

See Appendix for the details of the data.
used as a target to calibrate public spending shocks. In terms of persistence, the model produces a little bit less persistence than observed in the data, especially for investment and consumption. Comovements are reproduced at least qualitatively, although investment is less pro-cyclical than consumption while the opposite pattern characterizes the data. Public spending is weakly counter-cyclical in the Core while weakly pro-cyclical in the Periphery, as observed in the data.

Turning to financial variables, the model matches well the volatility of interbank interest rates, but the predicted persistence are too low and the correlation with GDP are negative which is at odds with data. In terms of variance decomposition, the model is mainly driven by productivity shocks and the latter induce a negative correlation between interest rates and GDP, while standard RBC models predict a positive one. In the model, the collateral constraint applied to entrepreneurs disconnects the dynamics of capital accumulation from its marginal productivity, and ties the latter to the quantity of loans. After a positive productivity shock, output rises, as well as deposits, so the liquidity in interbank markets increases as well, allowing saving banks to offer lower deposit rates. This increase in interbank liquidity lowers the marginal production cost of loans, and then the lending rate, which raises demand for loans, and thus the capital stock. This mechanism also explains why the model produces a positive correlation between the loans (or deposits) to GDP ratio and the interbank interest rate. Loans to GDP and deposits to GDP ratios are slightly more volatile than output but not as much as in the data; the correlation with GDP is strong and negative while weak and mixed in the data. The negative correlation of interest rates with GDP is mainly responsible for this counterfactual cyclical behavior.

Finally, moments related to public finance are correctly matched. The volatilities of sovereign rates and sovereign spreads are close to data, although slightly more volatile
especially for sovereign spreads.\footnote{In the model, long term rates are computed as the discounted sum of short rates with a discount factor of 0.85 to match the maturity of sovereign rates from the data (10 years or equivalently 40 quarters). The sovereign spread is defined as the difference between Periphery and Core long term sovereign rates. The interbank spread is defined as the difference between the Core sovereign long term rate and the “long term” interbank rate.} Volatility of debt to GDP ratio is smaller than in data but the model produces a more volatile ratio for the Periphery, which is in line with the data. In terms of the correlation with GDP, sovereign rates and GDP are negatively correlated in the model, which is at odds with data. Sovereign spreads are strongly and negatively correlated with GDP while this correlation is weaker in the data. The correlation between sovereign debt and GDP is negative as in the data. Finally, the correlation between loans to GDP and interbank interest rate is positive, corresponding to what we observed in the data. Other correlations, especially the correlation between sovereign rates and debt to GDP ratio (or between sovereign rates and public spending) could arguably be better matched.

Overall, however, the model does a satisfactory job in matching so many key business cycle facts on real, financial and fiscal time series. We consider it as reliable enough to perform additional simulations to quantify the effects of the Great recession and unconventional monetary policies on key aggregates and welfare.

5.2 The Great Recession and Central Bank Interventions

We start our analysis with the benchmark scenario were a large recession is simulated feeding the model with negative TFP shocks. Although it is clear that the great recession was not caused by negative TFP shocks, the latter induce changes in macroeconomic aggregates that match particularly well the observed patterns. In addition, according to Meza and Quintin (2007), TFP falls remarkably during financial crises, while changes in capital stock and labor hoarding are only secondary roles.

We start our simulation from the second quarter in 2008. Shocks are adjusted to
reproduce the percentage deviations of aggregate output in the Euro Area starting in 2008Q2. The model is solved using a Newton-type, and the adjustment of key variables is depicted in Figure 1.

**Figure 1**: Benchmark Simulation. Negative TFP shocks without Central Bank Intervention.

In Figure 1, aggregate output falls by around 4.5% below its 2008 level, which is exactly what is observed in the data. The fall in GDP raises mechanically debt-to-GDP ratios above their steady-state levels by more than 4pp in the Periphery and 3pp in the Core. The automatic component of public spending aimed at stabilizing GDP further feeds the rise in debt to GDP ratios. As a consequence default risk rises in both countries, but more markedly for the Periphery, explaining the rise in sovereign spreads.

We then consider three possible interventions from the Central Bank:
1. **Quantitative Monetary Easing.** According to our definition, quantitative monetary easing is an option of the Central Bank to provide liquidity to banks with difficulties to borrow from the interbank market. The Central Bank tries to release the interbank tensions by injecting cash directly to those who are short of liquidity. Normally, central banks use this instrument to determine the overnight interest rate. In reality, banks can use these cash either to replenish their tier 1 capital, or to provide loans. Considering both possibilities would complicate our analysis. Therefore, we assume that banks are forced by the contract to make loans. In other words, central bank inject money to banks and let them lend to entrepreneurs.

2. **Qualitative Monetary Easing.** When banks face the deterioration of loan quality, the Central Bank can help them improve their balance sheet strength by allowing these banks to swap a fraction of risky loans for government bonds. In this way, banks’ balance sheets are improved. This policy is quite different because the transmission mechanism does not go through loans directly but through bonds markets. However this policy will also affect loans through a variety of channels. First, qualitative monetary easing will release the tensions on sovereign markets, lowering the return on bank capital and therefore the marginal production cost of loans. Second, because the loan production function has a Leontief technology, producing loans requires a good capitalization of bank and good conditions on the interbank market. As a consequence, an intervention of the very same size could in principle have similar effects on GDP and other key macroeconomic variables.

3. **Qualitative Monetary Easing with the Existence of a Safe Haven.** As a final possibility, we consider an intervention of the Central Bank by which banks are offered the possibility to change the composition of their portfolio in favor of safe assets, *i.e.* bonds from the Core government. Indeed, when sovereign default risk in countries from the Periphery soars, investors tend to choose bonds issued in the Core
countries as a safe haven. Since the fall of 2012, the ECB has conducted outright open market operations, i.e. unlimited sovereign bond purchases in secondary sovereign bond markets. In September 2012 the ECB announced the technical features it had decided upon for such operations, named Outright Monetary Transactions (OMT). This operation allowed banks to replace their risky Periphery bonds by Core bonds. In our model, this scenario is modeled with a temporary shock on $\delta_t^{B,P}$.

Figure 2 and 3 report the marginal effects of the three potential monetary policies on both countries.

**Figure 2:** Periphery Country: Marginal Effect of Eurozone–wide 10pp GDP Quantitative Easing, Qualitative Easing, and Qualitative Easing with Safe Haven

A quantitative monetary easing amounting to 10% of Eurozone GDP yields a 0.5 percentage points gain in Periphery and Core output after 5 periods. Loans rise about 0.7
pp above their level without intervention both in the Periphery and the Core. Leverage ratios rise as well but moderately. The impact on debt-to-GDP ratio is also remarkable, as the latter falls by 1.8 pp in the Periphery and by 1.3 pp after 5 periods in the Core. As a result, sovereign default risk is relieved, more importantly for the Periphery than for the Core: the probability of default is cut by 0.05 pp in the Periphery while only by 0.01 pp in the Core. The marginal effects of a quantitative monetary easing are also quite persistent over time, even when the size of intervention is massively reduced. Indeed, with a 0.95 persistence, the size of shock is only $0.95^{25} = 0.2774$ after 25 periods while the marginal effect on output remains above half of its initial rise.

**Figure 3**: Core Country: Marginal Effect of Eurozone–wide 10pp GDP Quantitative Easing, Qualitative Easing, and Qualitative Easing with Safe Haven

The effects of a qualitative monetary easing are very similar to those of a quantitative easing except for the effect on leverage ratio. The former falls persistently under its steady-
state level while it rises under a quantitative monetary easing. This is clearly not a surprise as the initial goal of the qualitative easing is to help banks maintain the quality of their capital, while quantitative easing provides them with required liquidity. Nevertheless, since liquidity and capital are both important in the production of loans, its marginal effects on loans are positive as well. From a quantitative perspective, in comparison to the quantitative easing, the resulting increase in loans is both less important and less persistent (0.3 pp in the Periphery against 0.6 pp after 20 periods). As a result, the effect of qualitative easing on capital stock and thus GDP is less important. On sovereign markets however, qualitative easing does a better job than quantitative easing in limiting the increase in debt-to-GDP ratios, reducing globally the risk of sovereign default and, to a lesser extent, the spread between Periphery and Core sovereign debt. Transmission of the intervention goes through a fall of sovereign interest rates. As banks arbitrate between sovereign bonds and interbank lending, interbank interest rates fall simultaneously with government bond yields. Lending rates decline due to reduced cost on producing loans, which stimulates credit demand from the market.

With the existence of a safe haven, the effects of the intervention are qualitatively identical to the quantitative monetary easing except for the sovereign spread. Investment in the safe haven lowers Core government bond yield while raises the Peripheral interest rate. As a result, sovereign spread rises above its steady-state level. With the presence of a safe haven, unconventional policies implemented by the Central Bank tend to be less efficient in reducing Periphery country’s sovereign risk.

5.3 Welfare Analysis

From the experiments above we compute the welfare losses from the Great Recession in comparison to a situation where the economy would have remained in the steady-state, and the welfare gains from the three unconventional monetary policies. In each country,
we compute the welfare gains/losses for households, bankers and entrepreneurs. The computation is made either using 30 quarters after the beginning of the recession (for short term analysis), or 200 quarters after the recession (for long term analysis). Table 3 reports the results of the three policies, for each type of agent in both regions of the monetary union.

**Table 3:** Welfare gains, in percentage of steady-state consumption. Negative signs indicate welfare losses.

<table>
<thead>
<tr>
<th>Short run (30 quarters)</th>
<th>Core H</th>
<th>B</th>
<th>E</th>
<th>Av.</th>
<th>Periphery H</th>
<th>B</th>
<th>E</th>
<th>Av.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Recession</td>
<td>-0.8601</td>
<td>6.1982</td>
<td>-10.0287</td>
<td>-3.8826</td>
<td>-0.6778</td>
<td>2.5586</td>
<td>-12.1292</td>
<td>-2.6585</td>
</tr>
<tr>
<td>Quantitative Easing</td>
<td>0.0132</td>
<td>-0.4214</td>
<td>0.1322</td>
<td>0.0245</td>
<td>0.0034</td>
<td>-0.1849</td>
<td>0.0033</td>
<td>-0.0061</td>
</tr>
<tr>
<td>Qualitative Easing</td>
<td>0.0078</td>
<td>-0.0595</td>
<td>-0.0931</td>
<td>-0.0017</td>
<td>0.0012</td>
<td>0.0042</td>
<td>-0.2974</td>
<td>-0.0360</td>
</tr>
<tr>
<td>QE with Safe Haven</td>
<td>0.0134</td>
<td>-0.4389</td>
<td>0.1360</td>
<td>0.0242</td>
<td>0.0024</td>
<td>-0.1689</td>
<td>-0.0425</td>
<td>-0.0127</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long run (200 quarters)</th>
<th>Core H</th>
<th>B</th>
<th>E</th>
<th>Av.</th>
<th>Periphery H</th>
<th>B</th>
<th>E</th>
<th>Av.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Recession</td>
<td>-0.2598</td>
<td>4.1564</td>
<td>-3.5580</td>
<td>-1.0627</td>
<td>-0.1947</td>
<td>1.7393</td>
<td>-3.2641</td>
<td>-0.6652</td>
</tr>
<tr>
<td>Quantitative Easing</td>
<td>0.0064</td>
<td>-0.2391</td>
<td>0.0575</td>
<td>0.0072</td>
<td>0.0042</td>
<td>-0.0963</td>
<td>-0.0237</td>
<td>-0.0018</td>
</tr>
<tr>
<td>Qualitative Easing</td>
<td>0.0016</td>
<td>-0.0353</td>
<td>-0.0274</td>
<td>-0.0043</td>
<td>-0.0010</td>
<td>0.0022</td>
<td>-0.0997</td>
<td>-0.0146</td>
</tr>
<tr>
<td>QE with Safe Haven</td>
<td>0.0064</td>
<td>-0.2498</td>
<td>0.0575</td>
<td>0.0063</td>
<td>0.0035</td>
<td>-0.0846</td>
<td>-0.0319</td>
<td>-0.0034</td>
</tr>
</tbody>
</table>

Notes: H denotes households, B bankers and E entrepreneurs. Av. is the weighted average welfare gain/loss using steady-state consumptions to compute weights.

In the short run, Table 3 shows that the recession generates large welfare losses for households (between 0.68% and 0.86%) and very large losses for entrepreneurs (between 10% and 12%) while welfare gains for bankers. However, given the small importance of bankers consumptions in aggregate consumption (only 3% of total consumption), the average welfare effect is around 3% of steady-state consumption. The welfare loss for entrepreneurs is larger in the Periphery but the welfare loss for households is larger in the Core. The effects of unconventional monetary policies in the short run are small and yield welfare gains for households in both the Core and the Periphery. Quantitative easing and QE with safe haven produce important welfare gains for entrepreneurs in the Core (around 0.13%) but welfare losses for bankers in both countries. The effects of QE with safe haven
is very similar to a standard quantitative monetary easing, except that at the presence of safe haven, households and entrepreneurs gain more in the Core but less in the Periphery. This is due to unbalanced investment in Core government bonds that brings excess of welfare to the Core agents. The qualitative monetary easing has small and mixed effects on bankers, while brings large welfare losses to entrepreneurs in the Periphery country (around 0.3%). The average effects of unconventional monetary policies is globally small (around 0.02% of steady-state consumption) and mostly negative.

Long run effects are much smaller, as gaps from the steady-state are progressively closed over time. Qualitatively however, most results applying to the short run are preserved, except that quantitative easing brings welfare losses to entrepreneurs in the Periphery, and that qualitative easing brings small welfare losses to households in the Periphery. In the long run, quantitative easing and QE with safe haven bring welfare gains to households, and welfare losses to bankers. Quantitatively however, the welfare effects of unconventional monetary policies in the long run remain quite small, close to negligible.

6 Conclusion

Our results imply that Central Bank’s intervention can be beneficial during a recession: both quantitative and qualitative monetary easing help alleviate output decline, relieve financial tensions (including interbank, credit market, and sovereign interest rates), and reduce sovereign default risk. Particularly, quantitative monetary easing is more efficient in stimulating output; and qualitative monetary easing is a good choice to limit sovereign risk. The presence of a safe haven tends to partially neutralize the benefit of QE on Periphery’s sovereign problem, due to the misleading belief in “safe” assets. Finally, alternative potential interventions from the Central Bank can bring some welfare gains for households in short run with potential losses for entrepreneurs and/or bankers. The long run welfare effects are close to negligible.
References


Boissay, Frédéric, Fabrice Collard, and Frank Smets, “Booms and Banking Crises,” 2013.


and ____, “QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool,” International Journal of Central Banking, 2013, 9 (1), 5–53.


A Appendix: Data

• (a) GDP, consumption, public spending, investment: OECD quarterly data in volume (expenditure approach) in PPP dollars from 1970Q01 to 2013Q03. The log of aggregates is HP-filtered with \( \lambda = 1600 \) to get business cycle components. Core denotes average moments for Belgium, France, Germany and the Netherlands. Periphery denotes average moments for Italy, Portugal and Spain. Standard deviation is 100 times the standard deviation for GDP. Other standard deviations are relative to the standard deviation of GDP.

• (b) Short-run interest rate is the average 3-month refund rate in percent per annum for the Euro Area from Eurostat from 1999Q01 to 2013Q03. Figures are the same for the Core and the Periphery by definition. Standard deviation is 100 times the standard deviation for the (HP-filtered component of the) log of one plus the rate, where the rate is transformed to percent per quarter. The correlation with output is calculated against an aggregate measure of GDP for the Euro Area.

• (c) Loans to GDP and Deposits to GDP are build using quarterly OECD data on sectoral financial accounts for financial agents and institutions from 1999Q01 to 2013Q3. Loans and Deposits are nominal so country-specific GDP deflators are used to get real quantities. The resulting time series are then divided by quarterly GDP. The log of ratios is taken and HP-filtered. Core denotes average moments for Belgium and France, and Periphery denotes average moments for Italy and Spain. Standard deviations are 100 times the standard deviation of the ratio.

• (d) Sovereign long term rates are the 10-years yield in percent per annum on sovereign bonds taken from the OECD financial database and debt to GDP ratios are build using quarterly data on Gross Nominal Debt, deflating total debt using GDP deflators and dividing by quarterly GDP in annual levels. The dataset goes from 2000Q01 to
2011Q04. Rates are transformed as the short-term interest rate. Spreads are computed against German rates, and treated as other rates, *i.e.* transformed to quarterly rates and considered as the log of one plus the rate before HP-filtering and computing moments. Core denotes average moments for Belgium, France, Germany, and the Netherlands and Periphery denotes average moments for Portugal and Spain.

- (e) Interest rates are MFI interest rates on outstanding amounts of euro-denominated deposits and loans by euro area residents in percent per annum taken from the ECB database. Loans are up to 1 year to non-financial institutions, and deposits are up to 2 years from households. The dataset goes from 2003Q01 to 2014Q01. Rates are transformed as the short-term interest rate, and are considered as the log of one plus the rate before HP-filtering and computing moments. The correlation with output is calculated against an aggregate measure of GDP for the Euro Area.