On the Importance of Sales for Aggregate Price Flexibility*

Oleksiy Kryvtsov† and Nicolas Vincent‡

September 2014

Abstract

Macroeconomists have traditionally ignored the behaviour of temporary price markdowns ("sales") by retailers. Although sales are common in the micro price data, they are assumed to be unrelated to macroeconomic phenomena and often filtered out. We challenge this view. First, using the 1996-2012 dataset of the UK CPI monthly price quotes, we document a roughly two-fold increase in the frequency of sales during the Great Recession. Second, we build a New Keynesian macroeconomic model in which temporary sales arise as a pricing mechanism that allows retailers to price-discriminate across consumers with different opportunity costs of time. The model predicts that, during economic slumps, firms facing costs of decreasing regular prices post more sales, and households spend more time looking for sales. The resulting fall in the aggregate price level can be significantly larger than if sales were ignored. When the model is calibrated to match the behavior of sales in the data, it implies that the sales margin leads to a much smaller response of real consumption following a monetary shock.

Keywords: Price dynamics, Sales, Inflation, Monetary neutrality.

---

*The views expressed here are ours, and they do not necessarily reflect the views of the Bank of Canada. We would like to thank Susanto Basu, Marty Eichenbaum, Gee Hee Hong, Kim Huynh, David Jacho-Chávez, Nir Jaimovich, Sergio Rebelo, Joe Vavra and participants at the NBER Price Dynamics 2014 workshop, the Duke Macroeconomics seminar series and the Shanghai Macroeconomic Policies and Business Cycles conference for helpful comments and suggestions. We also thank Ainslie Restieaux and colleagues in the Prices Division at the Office for National Statistics for valuable feedback regarding U.K. CPI data. Claudiu Motoc and Tony Chernis provided superlative research assistance.

†Bank of Canada, 234 Laurier Street Ottawa, Ontario K1A 0G9, Canada. Email: okryvtsov@bankofcanada.ca

‡HEC Montréal, Department of Applied Economics, 3000 Côte-Sainte-Catherine, Montréal, Quebec H3T 2A7, Canada. Email: nicolas.vincent@hec.ca
1 Introduction

Price discounts, or “sales”, are an essential feature of retail price behaviour and an important factor for households’ consumption decisions. A typical sale is associated with a large but temporary price drop that returns close to its pre-sale level. In the last decade or so, access to detailed weekly and monthly price data for a broad variety of retail goods allowed macroeconomists to study implications of retail pricing for aggregate price flexibility. These studies take the stand that although temporary price discounts imply more frequent price changes by retailers,\(^1\) they do not play a role for inflation and business cycle dynamics. The upshot from that literature is that retail price discounts have little to do with macroeconomic phenomena and should be ignored by macroeconomists.\(^2\) In this paper, we challenge this view.

A few recent studies have taken on the task to studying the potential role of sales in macroeconomic models. Kehoe and Midrigan (2008), using modified versions of standard sticky price models, argue that sales are mostly irrelevant for the transmission of monetary shocks, since, due to their temporary nature, sales cannot offset monetary shocks well. Guimaraes and Sheedy (2011) reach similar conclusions using a sticky price model with sales stemming from consumer heterogeneity and incomplete information. In their model, strong strategic substitutability of sales at the micro level implies that their frequency and size barely responds to monetary shocks. Both papers, therefore, predict that the sale margin is not useful for retailers’ price adjustment in response to changes in macroeconomic conditions.

Neither paper, however, tests whether the model predictions are borne out in the data. This is partly due to the fact that the empirical evidence on the cyclical properties of sales is very limited. A recent exception is Coibion, Gorodnichenko and Hong (2014): using a scanner dataset from U.S. grocery stores, they document that sales are mostly acyclical and that most adjustment is done through consumers switching from high- to low-end retailers.

In contrast to these studies, we find that retailers’ adjustment along the sales margin is economically relevant for the response of aggregate price and output to changes in macroe-

---

\(^1\)For example, Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008) find, using U.S. CPI micro data, that on average prices adjust at least once a year, and that excluding sale prices decreases measured price adjustment by around 3 to 5 months. Klenow and Malin (2011) provide an excellent survey of microeconomic evidence on price-setting.

\(^2\)See Eichenbaum, Jaimovich, Rebelo (2011). An important exception is Klenow and Willis (2007) who find that in the U.S. CPI micro data sale-related price changes respond to macro information in the similar way as regular price changes.
conomic conditions. Our contributions are both empirical and theoretical. First, we provide empirical evidence on variations in the sales margin over time. To this end, we use the publicly available micro data underlying the Consumer Price Index (CPI) composed by the U.K. Office for National Statistics (ONS). The data contain monthly prices collected from local retail outlets for a wide range of consumer goods and services over the period from 1996 to 2012. We find that the frequency of sales in the U.K. data is strongly countercyclical: a 1 ppt rise in unemployment rate is associated with a roughly 0.4 ppt increase in the fraction of products on sale. For example, during the Great Recession the fraction of sales almost doubled from 1.8% to 3.8% of observations. This strong correlation between the business cycle and the use of temporary discounts by firms is extremely robust: it does not depend on how sales are identified; it is observed for most products; it is not a product of the exit of low-sale-frequency items; and it survives the use of multiple controls and alternative macroeconomic indicators. In addition, our empirical findings are corroborated by evidence derived from Vavra (2014) based on the U.S. CPI micro data.\(^3\)

Unlike the fraction of sales, the average size and duration of sales in the U.K. are acyclical and much less volatile. Nonetheless, the observed fluctuations in the fraction of sales may well be an important source of aggregate price fluctuations due to two factors. First, sale-related price drops are large on average, between 20% and 25% in the U.K. data. Second, each additional sale generates three times more revenue than a regular-price transaction, and the revenue share of goods on sale may increase during economic slumps as households shift their consumption toward sale prices. Our theoretical contribution is to quantify the aggregate price flexibility due to these two factors.

To this end, in the second part of the paper, we develop a general equilibrium business cycle model with consumer search and price discrimination by monopolistically competitive retailers. Households face i.i.d. time cost of searching for low prices. Those households who draw sufficiently low cost realizations become bargain hunters and find, on average, lower prices. Retailers, in a desire to attract bargain hunters, keep a positive fraction of brands on sale in the store. The return to households from bargain hunting is higher, the larger is

\(^3\)In a recent study, Sudo et al. (2013) find a rise in the frequency of sales in Japan during the 1990s and 2000s at the same time as hours worked were declining and the unemployment rate was rising. This evidence is dominated by strong trends in all three series during the “lost decades”, making it difficult to determine whether the behavior of sales is due to their countercyclical use by retailers or rather structural changes in the Japanese retail industry that are not related to business cycles.
the fraction and size of price discounts. Retailers’ revenue from posting sales increases with households’ willingness to substitute between market work and searching for sales. When the elasticity of substitution is sufficiently high, both the fraction of sales and the fraction of bargain hunters are strongly countercyclical, amplifying procyclical aggregate price dynamics. When matched to key moments of sale-price behavior, the model predicts that in response to, say, an unanticipated monetary contraction, the increases in the fractions of sales and bargain-hunters lead to an aggregate price decrease that is twice as large as the one in the model without time-varying consumer search, or the standard sticky price model with a single price per variety. Accordingly, the size of the real effect in response to monetary shocks is twice as small and can be even smaller depending on the response of the fraction of bargain hunters.

We show that the main mechanism underlying the importance of sale prices for aggregate price flexibility is based on the interaction between the retailers’ price discounting and the households’ search for low prices. At the time of the monetary contraction, some prices fail to decrease due to Taylor-type price adjustment constraints, leading to an increase in retail markup. High profit margins make it desirable for retailers to increase their market share. In our model, they do so through an increase in the fraction of brands on sale. In turn, more aggressive price discounting by retailers increases the return on searching for low prices, leading to a larger number of bargain hunters. The resulting relocation of consumption weight towards lower prices amplifies the fall in both store-average and aggregate price levels. We show that the model’s key prediction – countercyclical shopping intensity – is very much in line with evidence from time-use surveys, household panel scanner datasets, and Google Trends search activity data.

The outline of the paper is as follows. Section 2 presents empirical evidence on the behavior of the frequency and size of sales from the end of the 1990s to 2012 using product-level price quotes from the UK CPI dataset. In Sections 3 to 5 we describe the model with sales and analyze its response to shocks under plausible parameterizations. Section 6 provides evidence on shopping activity over the business cycle as well as the cyclicality of other forms of price discrimination. Section 7 concludes.
2 Empirical evidence on the behavior of sales

2.1 Data

To construct the Consumer Prices Index (CPI), the Office for National Statistics (ONS) surveys the prices for goods and services that are included in the household final monetary consumption expenditure component of the UK National Accounts.\(^4\) The survey includes prices for more than 1100 individual goods or services a month, collected across more than 14,000 retail stores across the UK. The survey excludes housing portion of consumer prices, such as mortgage interest payments, house depreciation, insurance and other house purchase fees.\(^5\) Also expenditures for purposes other than final consumption are excluded, e.g., those for capital and financial transactions, direct taxes, cash gifts.\(^6\)

The goods and services in the CPI are classified into 71 classes, according to the international (European) classification of household expenditure, Classification of Individual Consumption by Purpose (COICOP). A CPI class represents a basic group category, such as “Meat”, “Liquid Fuels” or “New Cars”. Each item in a given class is assigned an item weight, that reflects its relative importance in households’ consumption expenditures.\(^7\) Changes in expenditure weights over time reflect changes in expenditure composition of households’ consumption baskets.

Prices are collected across 13 geographical regions, e.g., London, Wales, East Midlands. There are four levels of sampling for local price collection: locations, outlets within location, items within section and product varieties. For each geographical region locations and outlets based on a probability-proportional-to-size systematic sampling with a size measures based on the number of employees in the retail sector (locations) and the net retail floor space (outlets). The data set contains around 150 locations with an average of more than 90 outlets per location.

Representative items are selected based on a number of factors, including expenditure size

---

\(^4\)Detailed description of the data underlying the CPI, the statistical methodology used, collection and validation of prices, calculation of weights, can be found in the “Consumer Price Indices Technical Manual” (2012). The price quote data are available via ONS website: http://www.ons.gov.uk/ons/datasets-and-tables/index.html.

\(^5\)These prices are used to construct the Retail Prices Index (RPI).

\(^6\)CPI inflation is used by the Government for its inflation target starting in December 2003. Before then, the index was published in the UK as the Harmonised Index of Consumer Prices (HICP).

\(^7\)Weights a calculated based on the Household Final Monetary Consumption Expenditure (HFMCE) and ONS Living Cost and Food Survey (LCF).
and product diversity, variability of price movements, availability for purchase throughout the year (except for certain goods that are seasonal). There are currently over 510 items in the basket. Examples of representative items include: onions, men’s suit, single bed. Finally, for each item-outlet-location, individual products and varieties are chosen by price collectors based on their shelf size and regular stock replenishment.

Most prices are collected monthly, except for some services in household and leisure groups, and seasonal items. For the purpose of this paper, the sample period includes 212 months, from February 1996 till September 2013. The total raw number of observations is over 24.4 million, or about 115,000 per month, though we make some adjustments that are described later in order to make the database amenable to analysis.

In addition to posted prices, the data set also contains information about some good characteristics during price collection. Prices for goods that are on special offer (available to all consumers) or on temporary sale comprise 6.4% of observations in the data set (4.5% weighted). Forced substitutions happen 8.0% (5.5% weighted) of the time, with about a quarter (three quarters) of them corresponding to substitutions for items that are non-comparable (comparable) to previously priced items. An item can be temporarily out-of-stock (2.2% or 1.5% weighted) or permanently missing (0.5% or 0.3% weighted). Finally, a small subset of goods has distinct seasonal patterns and is treated separately: they include some items of clothing, gardening products, holiday products and air fares. For those seasonal goods for which prices are not available, such as clothing, gardening and food, prices are imputed based on prices observed at the end of the previous season or based on prices observed for in-season goods in the same item category; in addition, weights are adjusted in accordance with availability of such goods throughout the year.

We clean the raw database to make it suitable for analysis. First, we create identification numbers that uniquely define item/region/store combinations. Second, if there is more than one observation for the exact same item in a given month, we ignore them. We also delete observations that are not coded as valid by the ONS. Third, we need to deal with product substitutions. To do this, we split the price time series of a given item every time we encounter

---

8Discussions with the ONS indicate that most of these duplicates are due to multiple observations across different locations. Unfortunately, because the ONS did not include the location variable as part of its public database, it is not possible for us to link observations across time in the presence of duplicates, leaving us with no choice but to delete them. Using only items that have never register duplicates over their lifetime has no noticeable impact on our results.
a substition flag. The resulting benchmark dataset contains a total of 20.7 million observations across about 2.3 million unique items. It should be noted that for our empirical analysis, we will mostly focus on items that have at least ten price quotes (17.1 million observations).

Value added taxes (VAT) are included in the price quotes. The implication is that any change in the VAT rate will automatically lead to a price change. VAT changes, however, were very few over our sample period. The only exceptions are three major changes to the standard VAT rate in the later part of the sample. First, in response to the Great Recession, the Conservative government announced a widespread reduction in the VAT rate from 17.5% to 15% effective on December 1st, 2008. Then, the rate was brought back to 17.5% starting January 1st, 2010. Finally, the standard VAT rate was raised from 17.5% to 20% on January 4, 2011. We control for these events in our analysis.\footnote{Moreover, VAT changes are more akin to regular price changes, while our focus is on temporary sales. For this reasons, they have little incidence on our results.}

2.2 Sales filters

The first challenge when studying temporary sales in micro price data is to identify them. Ideally, we want to discriminate between price drops that are temporary from drops in regular prices. We use three main ways of identifying sales in our dataset.

First, we present results using the “sales flag” from the ONS. The ONS indicates that “sale prices are recorded if they are temporary reductions on goods likely to be available again at normal prices or end of season reductions.” It is important to note that discounts available to only a subset of consumers (e.g. promotions that require coupons or loyalty cards) are ignored, as well as sales on discontinued or defective/refurbished items. Despite the advantage of being directly made available by the statistical agency, there are some issues with the sales flag that require us to make some adjustments. For example, there are a few instances in which the occurrence of a sales flag is accompanied by no recent change in the posted price, or even, in some very rare instances, a price increase. One possible explanation is that the retailer uses some advertising features to gain or retain customers despite not actually changing the price; another is misreporting or coding error. In what follows, we adopt a conservative approach and present results based only on sale flags that correspond to actual sales.\footnote{To be precise, a sales flag is deemed valid only if it coincides with a price quote that is lower than the price that was posted right before the start of the spell of sales flags, which we define as the regular price for the duration of the spell.} That being

\textsuperscript{9}
said, the results using the raw sales indicator are very similar.

Second, we apply a V-shaped-type sales filter, similar to the one used by Nakamura and Steinsson (2008), among others. In this instance, a “sale episode” begins with a price drop and ends as soon as a price increase is registered, as long as this price increase occurs within three months.\(^{11}\) Under this definition, the price increase at the end of the sale need not be as large as the price drop at the beginning of the sale.

We also consider a more restrictive filter whereby a V-shaped sale is initiated by a price drop that is followed within three months by a return to a price equal or higher than the initial price level. For the sales-flag and both V-shaped filters, the unobserved regular price during a sale is assumed to be equal to the last observed regular price.

Finally, we compute a reference price similar in spirit to Eichenbaum, Jaimovich and Rebelo (2011) using a seven-month window. More precisely, for a given month \(t\) we set the reference price equal to the modal price observed between \(t - 3\) and \(t + 3\), as long as there are at least four price observations within that window. To avoid identifying spurious sales that arise from a lag/lead in the adjustment of reference prices, we then apply a procedure similar to Kehoe and Midrigan (2008) to ensure that a change in the reference price coincides with an actual price change. A price observation corresponds to a sale price whenever the posted price is below the reference price.

### 2.3 Preliminary statistics

In Table 1 we present some basic statistics on price dynamics in our dataset. Unless otherwise stated, all moments are weighted using the official CPI category weights. The fraction of price changes is 17.2% over the sample period and the average size of a price change is 11.1% in absolute terms. Price increases are more likely than price decreases (10.7% vs 6.5% of observations respectively). Not surprisingly, we find lower price change frequencies if we focus on regular prices, i.e. price series that were purged of observations for which the posted price differs from the regular price. The probabilities of observing a price change are 14.6%, 12.1% and 7.3% based on the sale flag, V-shaped and reference price filters respectively. Hence, the reference price filter generates significantly stickier price series, largely because it filters out

\(^{11}\)If two price drops occur in a row before the posted price settles to a new regular level, our filter will identify the first price drop as corresponding to a sale. Instead assuming that this episode corresponds to two consecutive drops in the regular price has no impact on our results.
both upward and downward temporary price deviations. All in all, our basic statistics show that prices are stickier in the UK than the US, but more flexible than in Europe.\footnote{See for example Klenow and Kryvtsov (2008) or Nakamura and Steinsson (2008) for the U.S. and Alvarez et al. (2006) for Europe.}

Turning our attention to the main object of study, Table 2 presents a number of basic statistics for sales, using CPI weights for all calculations. The first row shows that the frequency of sales varies significantly depending on the definition used, from 2.75% for the ONS sales flag to 7.66% for the sales based on the reference price filter.\footnote{Though direct comparisons can be difficult, the prevalence of sales seems to be much lower than in the U.S. For instance, Klenow and Kryvtsov (2008) find that about 11% of price observations have a sales flag, while Nakamura and Steinsson (2008) find that sales account for about 7.4% of observations using a V-shaped filter.} Differences are also visible for both the average and median sale sizes: they are much higher for the sales flag (between 20% and 23%) than the three other filters (between 6% and 12%).

To understand why this is the case, we show in Figure 1 the truncated distribution of the size of sales across all observations in our dataset (we drop the alternative V-shaped filter as it shows no significant difference).\footnote{The size of a sale corresponds to the difference between the sale and the regular price, and is therefore not limited to the onset of the sale (initial price drop).} We set the bound of the histogram at 60% since larger sales are rare. Reassuringly, one can immediately notice spikes in the distribution at the familiar discount points: 10% off, 20% off, 25% off, 33% off and 50% off. Second, the three distributions exhibit some differences between -10% and zero: the mass closer to zero is significant for the V-shaped and reference price filters, while there are very few small sales according to the ONS indicator. This seems to indicate that the sales filters commonly used in the literature have a tendency to pick up small price drops that are not advertised as sales by retailers. On the other hand, the three distributions are much more similar for sales larger than 10%, a sensible threshold. For this reason, we focus on sales of at least 10% in our analysis. Under this condition, sale frequencies and sizes become very similar across filters, as can be seen from the bottom portion of Table 2.

### 2.4 The time-series behavior of sales

We now look at the behavior of the frequency of sales during the last 15 years in the UK. For each category and month we compute the proportion of items on sale, and then aggregate them using CPI weights. In the left plot of Figure 2 we show the raw constructed series as
well as the UK unemployment rate. Because of the very strong seasonal patterns of sales, we also report on the right plot the 12-month moving average centered around each month. What is very clear at the onset is that the fraction of items on sale is far from being constant over time: it is around 3.4% at the beginning of the sample in 1997, then declines to a trough of about 1.8% in 2006 before rising back to more than 3.6% by 2011. Also, it is strikingly countercyclical: the fraction of sales moves very closely with the unemployment rate, rising as the economy is slowing down.

Next, we look into the robustness of our main result. First, we show that the countercyclicality of sales is not an artifact of using the sales flag from the ONS. Figure 3 compares the evolution of the fraction of items on sales based on the sales flag as well as the three other filters we described earlier: both the benchmark and more restrictive V-shaped sales filters, as well as the one based on the 7-month reference price. For all series we focus on sales of at least 10%. While there are some differences, the patterns tell a very similar story: there are large swings in the incidence of sales over the sample period, and the prevalence of sales tends to be higher when the economy is weaker.

Second, the volatility and cyclicality of sales are not driven by just a few large categories. The top-left plot of Figure 4 shows the evolution of the fraction of price quotes with a sales flag aggregated using equal weights across categories, as opposed to the weighted statistic reported so far. Another concern could be that some changes in the collection of data or the definition of sales coincided with the onset of the recession and therefore are leading to spuriously identify a high degree of cyclicality. While the fact that the results hold across multiple sales filters should allow us to dismiss this concern, we further note that the cyclicality of sales is not a feature that is common to all categories, and therefore it is unlikely to be generated mechanically. Looking at Figure 4, one can notice a significant rise in the incidence of sales (here defined as the fraction of items with a sales flag) for categories such as “Clothing and footwear”, “Furnishings and other household equipment”, and “Personal care”. Yet, for other categories such as “Books” or “Housing rents and maintenance”, there is no significant rise in sales around 2007-2009.

It should also be noted that the rise in the sale frequency is broad-based: the number of products that experienced a rise in their sale frequency between the periods 2005-2006 and 2009-2010 is almost three times as high as those that saw a drop.
Third, the cyclicality of the fraction of products on sale could either be driven by fluctuations in the incidence of new sales or by changes in the average length of a sale over time; in other words, it may be that retailers put items on sale more often, or that they keep them on sale for longer. We find no evidence of the former in the data: the average duration of a sales spell remained very stable around 1.6 month over our sample period, with no discernible cyclicality nor trend.\footnote{That being said, the fact that we only have monthly frequency data makes it difficult to reliably identify changes in the average length of sales.}

Finally, Figure 5 shows the evolution of the average size of sales over our sample period, under our three main definitions of sales. Under all definitions, there seems to be a rise in the (absolute) size of sales over the sample period. Still, while the size of discounts is not perfectly constant, the fluctuations are significantly more limited in relative terms than what we saw earlier for the frequency of sales. For example, using the V-shaped filter, we find that the average size of sales goes from about 23\% in 1999 to 26\% by the end of 2012. More importantly, there is no noticeable cyclicality for this margin of adjustment, unlike for the sales frequency which basically doubled during the Great Recession.

\section*{2.5 Regression analysis}

Our findings from the previous section show that firms do, at least under some circumstances, alter the prevalence of temporary sales in reaction to macroeconomic fluctuations. While our results appear robust, the graphical analysis used limits our ability to control for various factors that may bias our conclusions. For this reason, we turn our attention to a panel regression approach. The basic specification is standard and given by

\[ s_{it} = \alpha_i + \beta u_t + X_{it}' \Phi + e_{it}, \]  

where \( s_{it} \) is the dependent variable measuring the incidence of sales, \( u_t \) is the unemployment rate, and \( X_{it} \) is a matrix of controls such as month dummies. First, we run the regressions at the category (COICOP) level. In this case, \( s_{it} \) is the fraction of items in category \( i \) whose posted price is at least 10\% below the regular price at time \( t \). All regressions include dummies for the months in which VAT rate changes occurred, as described in Section 2.1. Table 3 summarizes our results. Robustness checks are shown only for the sales flag series for
conciseness, but findings are similar for the other filters.

Our results indicate that a 5-ppt increase in the unemployment rate raises the likelihood of observing a sale by about 1.7 percentage point, which is in line with our earlier graphical findings. Adding a time trend or calendar month dummies does not alter much the economic or statistical significance of this relationship, though it does increase somewhat the $R^2$ of the regression. Even when we include the lagged value of the dependent variable, the coefficient on the unemployment rate remains positive and strongly significant, leading to a slight increase in the long-term effect of unemployment on sales at 0.403 ($= 0.257/(1 - 0.362)$). Running instead a regression where observations are weighted by their CPI weights does not materially affect the findings (column (5)). Finally, the last two columns show our benchmark specification using the two other filters. There again, the unemployment rate is strongly significant statistically, and the economic effects are in the same order of magnitude.

So far, all our analysis has been carried out using the unemployment rate as macroeconomic indicator. While this seems to be a sensible indicator of the financial and economic situation of households, it is hardly the only one. In Table 4 we present results for our benchmark specification but with alternative business cycle indicators on the right-hand side. In keeping with our focus on the consumer side, we use consumer confidence indicators for the UK as compiled by the company GfK on behalf of the European Commission. In these monthly surveys, various questions are asked to a sample of households. We focused, in order, on (1) the aggregate consumer confidence index, (2) the personal financial situation over the next 12 months, (3) the general state of the economy next year, and (4) the intention of making major purchases. To facilitate comparisons, we divided all these variables, as well as the unemployment rate, by their respective time-series standard deviations over the sample. The results in Table 4 show very similar responses across three indicators out of four: a one-standard-deviation change in the explanatory variable leads to a statistically significant change of between 0.4 and 0.5 percentage point in the frequency of sales.

The regressions at a category level suggest that countercyclicality of sales applies at a disaggregate level and does not stem from mere aggregation across categories. To show that...
this point applies at any level of disaggregation, we also conduct panel regressions at individual product level. This will help us to alleviate any concern that our results are driven by some composition bias, for example.\textsuperscript{18}

Under this specification, \( s_{it} \) in equation (1) is an indicator equal to 1 if the item \( i \) is on sale in month \( t \), and 0 otherwise. All \( p \)-values shown are based on robust standard errors clustered at the category level. Results in Table 5 contain little in the way of surprises: the unemployment rate is a statistically significant predictor of the occurrence of a temporary sale; the size of the relationship is in line with the previous graphical and regression analyses; and the results hold for all three filters considered.\textsuperscript{19}

The basic message of this section is clear: there is robust evidence that U.K. firms reacted to macroeconomic conditions in the last 15 years by adjusting the frequency of temporary sales. In the next section, we build a model with sales to investigate the macroeconomic implications of this finding. Before doing that, we corroborate the U.K. facts with some evidence from the U.S. data.

### 2.6 Supporting evidence from the U.S. CPI dataset

As we mentioned in the introduction, empirical studies on the cyclical behavior of sales are very few. In the previous sections, we have done a careful analysis of the publicly available U.K. CPI micro data and documented a very strong countercyclicality of sales. Ideally, we would like to confirm this characteristic from other datasets. This is unfortunately made difficult by the fact that CPI micro data in most countries can be difficult to access. Yet, for the U.S., Vavra (2014) hints at similar conclusions based on the Bureau of Labor Statistics, even though he is not interested in investigating the cyclicality of sales. For example, in Table A-I, he shows that the correlation between the median frequency of price changes and industrial production is -0.55, but only -0.27 once sales and product substitutions are filtered out. This appears to indicate that the frequency of sales (and substitutions) is heavily countercyclical.

\textsuperscript{18}For example, it could be that items less prone to temporary discounts were more likely to disappear as consumers increased their search intensity during the recession and were being slowly substituted for comparable brands with higher sales frequencies as part of the CPI sampling methodology. While interesting by itself, this exit-and-entry story is different than one by which rms respond to macroeconomic shocks by altering their use of temporary sales.

\textsuperscript{19}In non-reported regressions, we find that sales frequency is roughly similar across regions of the UK, except in London, where it is significantly lower; sales are significantly more prevalent in stores belonging to chains than independent shops; and that sales are most frequent in January, followed by July. We also verified using similar regressions that the size of sales is not responsive to the macroeconomic cycle.
While the original paper does not provide correlations for sales only, the author made available to us the time series for the mean and median frequency of sales and product substitutions he identified using the Nakamura and Steinsson (2008) filter, similar to the one we applied to the U.K. data. Figure 6 depicts these two series (as 12-month moving averages) alongside the U.S. unemployment rate. The similarities with our results for the U.K. are striking: there is a very clear positive correlation between sales/substitutions and unemployment. For example, the mean frequency rose from 6.2% in 2005 to 9% by early 2010, just as unemployment peaked in the U.S. This pattern is not limited to the Great Recession: we can clearly observe spikes in the frequency of sales in the midst of the 1990-91 and 2001 recessions.

Certainly, a definitive answer would require having full access to the BLS micro data and performing an analysis as detailed as the one we did for the U.K. In particular, the plots above included both sales and product substitutions. While our experience with the U.K. data showed that including substitutions or not had no impact on our conclusions, it certainly is true that only a detailed analysis of the BLS micro data would provide a definitive answer. Still, the evidence very much supports the notion that the countercyclicality of sales is not a characteristic specific to the U.K.

3 A general equilibrium model with sales

A model economy is populated by a measure one of infinitely lived ex-ante identical households, indexed by $k$, who are consuming a continuum of differentiated good varieties, indexed by $j$. These varieties are sold by a continuum of retail industries located in isolated “locations” indexed by $i$. There is one retail industry per location, and each industry-location consists of measure one of monopolistically competitive retail firms, each firm selling many highly substitutable brands of a single variety $j$. For example, a beer retailer sells Budweiser, Bud Light, Michelob Light, Coors Light, etc. For simplicity, we will assume brands of the same variety to be perfect substitutes. In all, a given industry-location sells all varieties, and a given variety $j$ is sold in every location. This setup differs from a conventional model with one industry of monopolistically competitive retailers, each producing a single brand of

20 We would like to warmly thank Joseph Vavra for making these aggregated data available to us.
a given variety.

In our setup, a retailer prefers to post different prices for different brands at any point of time. The existence of a non-degenerate price distribution in our model stems from the retailer’s desire to price discriminate in the face of heterogenous household reservation prices at the point of purchase; without such a motive, retailers would prefer a single price for all brands of the same variety. In the model, the dispersion of reservation prices stems from costly search for low prices. We assume that a household faces a random fixed cost of searching in retail store selling variety \( j \), and that this cost is in terms of forgone hours worked. The fixed cost is i.i.d. across time, stores and households, drawn from twice continuously differentiable distribution \( G(z) \), with \( 0 \leq z \leq z_{\text{max}} \). Depending on the realized cost of searching, households will either choose to actively search for a low price for variety \( j \) (and become “bargain hunters” for that variety) or not to search (“workers”).\(^ {21,22} \)

In each period \( t \), the economy experiences one of infinitely many events \( s_t \). Denote by \( s^t = (s_0; \ldots; s_t) \) the history of events up to and including period \( t \). The probability density, as of period 0, of any particular history \( s^t \) is \( \pi(s^t) \), and the initial realization \( s_0 \) is given. The only shocks in this economy are aggregate shocks to the money supply. We assume that the supply of money follows a random-walk process of the form

\[
\log M(s^t) = \log M(s^{t-1}) + \mu(s^t),
\]

where \( \mu(s^t) \) is money growth, a normally distributed i.i.d. random variable with mean 0 and standard deviation \( \sigma_{\mu} \).

### 3.1 Household’s problem

At the beginning of each period, state \( s_t \) is realized, and each household is randomly and uniformly assigned to a “designated” location, \( i^* \). For each variety \( j \), the household then observes the distribution of prices in each store-location \( j(i) \), denoted by \( F_{j(i)}(P, s^t) \), draws

---

\(^{21}\)Consumers therefore are price-discriminated by their search effort (third-degree price discrimination). Another common type of price discrimination is by price elasticity of consumers (second-degree price discrimination), an approach undertaken in Guimaraes and Sheedy (2011).

\(^{22}\)Alternatively, one could think of an environment populated by families, each composed of an infinite number of family members. In this setup, families are randomly allocated across locations and each family member is responsible for shopping for a specific variety. In a given period, a fraction of family members with low cost of searching will become bargain hunters, while the others will be workers. Both environments give rise to exactly the same equilibrium conditions.
the fixed cost of searching $z_j$ and decides whether or not to search, i.e., becomes a bargain hunter or a worker.

Bargain hunters and workers differ in two respects: 1) they face different technology matching them to a store-location $j(i)$, and 2) for each store-location $j(i)$, they face different distribution of prices. Specifically, we assume that workers buy their consumption of variety $j$ at a random location $i$ and at a price randomly drawn from all prices in the store, i.e., they draw one price from distribution $F_{j(i)}(P, s^l)$. Conversely, while bargain hunters must purchase variety $j$ at their designated location $i^*$, active shopping allows them to face on average lower prices. Let $F_{B_{j(i)}}(P, s^l)$ denote the price distribution faced by bargain hunters in store-location $j(i)$. It is assumed that $F_{j(i)}(P, s^l)$ first-order stochastically dominates $F_{B_{j(i)}}(P, s^l)$ for each store-location, i.e., for any price $P$ a worker is more likely to draw a price above $P$ than a bargain hunter. Without any loss of generality, suppose $F_{B_{j(i)}}(P, s^l)$ can be obtained from $F_{j(i)}(P, s^l)$ by applying a functional $F$:

$$F_{B_{j(i)}}(P, s^l) = F\left[F_{j(i)}(P, s^l)\right], \forall j(i), (P, s^l). \tag{2}$$

Condition (2) can be interpreted as characterization of the advantage that bargain hunters have over workers when searching for low prices in a retail store with price distribution $F_{j(i)}(P, s^l)$: they face a lower expected price than the unconditional average price, which is the price the workers face.\textsuperscript{23} We show below that the differences in price distributions faced by bargain hunters and workers, coupled with the randomization of workers across locations, lead to price discrimination by retailers.

In our setup, a household’s utility from consuming a given variety does not depend on the location where it was purchased, and so we can skip the location index hereafter. Since the fixed cost is i.i.d., the household’s search decision can be characterized by a cutoff fixed cost, $z_j^*(s^l)$, for which it is indifferent between searching or not for variety $j$. If the realized fixed cost is low, $z \leq z_j^*(s^l)$, the household will be a bargain hunter and search more intensively, drawing prices from $F_{B_j}(P, s^l)$. Otherwise, it will draw prices from $F_j(P, s^l)$.\textsuperscript{24} Given the

\textsuperscript{23}One can assume explicit searching technology and derive the corresponding $f$ -mapping. For example, we study the case when bargain hunters have two random draws per store, choosing the lowest price from two draws, as opposed to a single draw for workers. A simple version of this technology is studied in Section 4.

\textsuperscript{24}We need to assume that, like workers, bargain hunters also face random prices; otherwise each store would post not more than measure zero of its prices on sale, and all bargain hunters coming to the store would buy only at those prices.
price drawn, $P_{k,j}(s^t)$, household $k$ chooses how much to consume, $c_{k,j}(s^t)$. As we show below, conditional on the same price, consumption demand does not depend on shopper type (bargain hunter or worker). Therefore, the incentive of retailers to lower some prices (i.e. put a fraction of items on sale) stems solely from their willingness to increase their market share by attracting more bargain hunters.

In addition to searching and consuming, households supply working hours in a competitive labour market, and trade money and state-contingent securities. Let $B_k(s^{t+1})$ denote a vector of state-contingent bonds paying household $k$ one dollar in state $s^{t+1}$, and $Q(s^{t+1}|s^t)$ denote the vector of corresponding prices in period $t$ and state $s^t$. We can write each price as

$$Q(s^{t+1}|s^t) = \frac{Q(s^{t+1})}{Q(s^t)},$$

where $Q(s^t)$ is the date 0 price of a bond that pays one dollar if state history $s^t$ is realized.

Household $k$ chooses sequences of consumption varieties $c_{k,j}(s^t)$, total consumption $c_k(s^t)$, leisure $1 - l_k(s^t)$, cutoff search cost $z^*_j(s^t)$, cash holdings, $M_k(s^t)$, and bond holdings $B_k(s^{t+1})$ to maximize lifetime expected utility

$$\sum_t \int \beta^t \pi(s^t) \left[ \frac{c_k(s^t)^{1-\sigma}}{1-\sigma} - \psi(1 - l_k(s^t)) \right] ds^t,$$

where total consumption is derived from consumption of good varieties according to a CES aggregator

$$c_k(s^t) = \left( \int_j \{ E_j[c_{k,j}(s^t)] | z, P, s^t \} \right) \frac{\theta-1}{\theta} dj,$$

where $\theta$ is the elasticity of substitution across varieties, $E_j[\cdot | z, P, s^t]$ denotes the expected value conditional on distributions of cost $z$ and price $P$ for variety $j$ in date $t$ and state $s^t$.

Utility maximization is subject to the budget constraint

$$M_k(s^t) + \int_{s^{t+1}} Q(s^{t+1}|s^t)B_k(s^{t+1}) \leq M_k(s^{t-1}) - \int_j E_j[Pc_{k,j}(s^{t-1}) | z, P, s^{t-1}] dj$$

$$+ W(s^t) \left[ l_k(s^t) - \int_j \left( \int_0^{z^*_j(s^t)} zdG(z) \right) dj \right] + B_k(s^t) + \Pi(s^t) + T(s^t),$$

and a cash-in-advance constraint

$$P_k(s^t)c_k(s^t) \leq M_k(s^t).$$
In the budget constraint, $W(s^t)$ is nominal wage, $\Pi(s^t)$ are dividends paid, $T(s^t)$ are lump-sum government transfers; finally, $\int_0^{z^*_j(s^t)} zdG(z)$ denotes the expected cost of searching for consumer $k$ in store-location $j$, expressed in terms of forgone hours worked. The household combines unspent money holdings from the previous period with labour income, return on bonds, dividends and transfers and splits them into current-period cash and bond holdings to be carried over to the next period.

The first-order conditions for consumption varieties conditional on realization of fixed search cost $z$ and price $P$, yield standard CES demand schedules:

$$c_{k,j}(s^t) = \left( \frac{P}{P_k(s^t)} \right)^{-\theta} c_k(s^t),$$

where for bargain hunters (who drew $z \leq z^*_j(s^t)$) price $P$ is drawn from distribution $F^B_j(P,s^t)$, and for workers ($z > z^*_j(s^t)$) price $P$ is drawn from store distribution $F_j(P,s^t)$.

Hence, conditional on the price drawn, consumption demand is the same regardless of shopper type; the difference between bargain hunters and workers stems from the different price distributions they face, with bargain hunters drawing on average lower prices. The price of total consumption for household $k$, $P_k(s^t)$, is defined as

$$P_k(s^t) = \left( \int_j E_j \left[ P^\alpha_{k,j}(s^t) \mid z, P, s^t \right] dF_j(P,s^t) \right)^{\frac{1}{1-\theta}}.$$

Note that due to i.i.d. search costs, households will end up with identical total consumption $c_k(s^t)$ and price $P_k(s^t)$, so we will drop the household index $k$, denoting them $c(s^t)$ and $P(s^t)$ respectively.

To derive the optimality conditions for the search decision, denote by $\alpha_j(s^t)$ the probability of drawing a search cost smaller than the threshold level $z^*_j(s^t)$, i.e., $\alpha_j(s^t) = G\left(z^*_j(s^t)\right)$. In other words, $\alpha_j(s^t)$ represents the probability that a household will be a bargain hunter in store $j$. In the problem above we can rewrite the consumption aggregator (3) and the budget constraint (4) as

$$c(s^t) = \left\{ \int_j \left( \alpha_j(s^t) \left( \int c_{k,j}(s^t) dF^B_j(P,s^t) \right)^{\frac{1-\frac{1}{\theta}}{\theta-1}} + \left(1 - \alpha_j(s^t)\right) \left( \int c_{k,j}(s^t) dF_j(P,s^t) \right)^{\frac{1-\frac{1}{\theta}}{\theta-1}} \right) dj \right\}^{\frac{\theta}{\theta-1}}.$$
\[ M_k(s^t) + \int_{s^{t+1}} Q(s^{t+1}|s^t)B_k(s^{t+1}) \leq M_k(s^{t-1}) \]
\[- \int_j \left[ \alpha_j(s^{t-1}) \left( \int P_{kj}(s^{t-1})dF_j^B(P,s^{t-1}) \right) - (1 - \alpha_j(s^{t-1})) \left( \int P_{kj}(s^{t-1})dF_j(P,s^{t-1}) \right) \right] dj \]
\[+W(s^t) \left[ l_k(s^t) - \int_j \Xi(\alpha_j(s^t))dj \right] + B_k(s^t) + \Pi(s^t) + T(s^t), \]

where \( \Xi(\alpha_j(s^t)) = \int_0^{G^{-1}(\alpha_j(s^t))} zdG(z) \) is the expected search cost for a household facing probability \( \alpha_j(s^t) \) of becoming a bargain hunter in store \( j \). Hence, the first-order condition for the cutoff fixed cost, \( z^*_j(s^t) \), can implicitly be written instead as the first-order condition for the probability of searching in store \( j \), \( \alpha_j(s^t) \):

\[ W(s^t)\Xi'(\alpha_j(s^t)) = \left( \frac{\theta}{\theta - 1} D_j^B(s^t) - 1 \right) \left( \int P_{kj}(s^t)dF_j^B(P,s^t) \right) \]
\[- \left( \frac{\theta}{\theta - 1} D_j^W(s^t) - 1 \right) \left( \int P_{kj}(s^t)dF_j(P,s^t) \right), \quad (6) \]

where \( D_j^B(s^t) \) and \( D_j^W(s^t) \) are factors stemming from price dispersion faced by bargain hunters (B) and workers (W) respectively in store \( j \):

\[ D_j^q(s^t) = \left[ \int \left( \frac{P}{P^q_j(s^t)} \right)^{-\theta} dF_j^q(P,s^t) \right]^{-1/\theta}, \quad q = B, W, \]

where \( P^W_j(P,s^t) = F_j(P,s^t) \), and \( P^q_j(s^t) \) is the quantity-weighted mean price faced by type-\( q \) household:

\[ P^q_j(s^t) = \left[ \int P \frac{P^{-\theta}}{P^{-\theta}dF_j^q(P,s^t)} dF_j^q(P,s^t) \right]^{-1/\theta}. \]

Since \( \theta > 1 \), \( D_j^q(s^t) > 1 \), and if the mass of low prices is not too high, bargain hunters face higher consumption risk for a given variety, i.e., \( D_j^B(s^t) > D_j^W(s^t) \), which altogether imply that the right-hand side of (6) is positive for all varieties.

If we denote expected consumption by \( C_j^q(s^t) = \int c_{kj}(s^t)dF_j^q(P,s^t) \), then expected demand can be written as

\[ C_j^q(s^t) = \left( \frac{P_j^q(s^t)D_j^q(s^t)}{P(s^t)} \right)^{-\theta} c(s^t). \]

It has the form of a standard CES demand, in which the expected price \( P_j^q(s^t) \) is marked up
by the dispersion factor $D_q^j(s^t)$: for a given average price in a store, expected consumption is lower than implied by a standard CES demand, because of the uncertainty of consumption of brands across varieties.

Condition (6) equates the marginal expected fixed cost of searching in store $j$ with the marginal utility flow from bargain hunting in that store. Since the latter is positive for any $j$, and since the distribution of the fixed search cost is continuous over $0 \leq z \leq z_{\max}$, the measure of households that draw a sufficiently low fixed cost for a particular store is always a positive-valued function of the state history, i.e., $\alpha_j(s^t) > 0$. This time-varying mix of shopper types is the key difference from Guimaraes and Sheedy (2011), who assume a constant fraction of bargain hunters. In our model, the fraction of bargain hunters is determined endogenously by the value of searching for lower prices through equation (6). We demonstrate below that the importance of sale prices for aggregate price flexibility crucially depends on the interaction of households’ search for lower prices and retailers’ decisions for setting those prices.

The remaining optimality conditions are standard: the first-order condition for hours worked implies

$$\psi P(s^t)c(s^t)^\sigma = W(s^t)$$

while the first-order conditions for state-contingent bonds yield

$$Q(s^{t+1}|s^t) = \beta \pi(s^{t+1}|s^t) \frac{c(s^{t+1})^{-\sigma}}{c(s^t)^{-\sigma}} \frac{P(s^t)}{P(s^{t+1})},$$

where $\pi(s^{t+1}|s^t) = \frac{\pi(s^{t+1})}{\pi(s^t)}$ is the probability of state $s^{t+1}$ conditional on state $s^t$. Similarly, period 0 prices satisfy

$$Q(s^t) = \beta^t \pi(s^t) \frac{c(s^t)^{-\sigma}}{P(s^t)}.$$

### 3.2 Retailer’s problem

Retailers of type $j$ are endowed with a linear production technology that converts hours worked $n_j(s^t)$ into output of brands of variety $j$, $y_j(s^t)$:

$$y_j(s^t) = n_j(s^t).$$

Let $\alpha^{B}_{j(i)}(s^t)$ denote the number of bargain hunters buying consumption from retailer $j$ in location $i$, and $\alpha^{W}_{j(i)}(s^t)$ denote the corresponding number of workers. Due to the random...
allocation of workers across store-locations selling the same variety, a retailer of type \( j \) takes the number of workers as given, and can only increase its market share by retaining bargain hunters in its location. We therefore can focus on a representative retailer \( j \), omitting location index \( i \). This allows us to denote the number of bargain hunters by \( \alpha^B_j(s^t) \) and the number of workers by \( \alpha^W_j(s^t) \), where a bar indicates that this margin is not impacted by the retailer’s pricing decision. Since the search cost is i.i.d. across households, and households are uniformly assigned across locations, the number of households who choose to become a bargain hunter for variety \( j \) in their designated location is equal to the number of bargain hunters in that store-location, i.e.,

\[
\alpha^B_j(s^t) = \alpha_j(s^t) .
\]

(7)

The problem of a retail store selling variety \( j \) is to choose the distribution of prices \( F_j(P,s^t) \) to maximize its profits in period \( t \):

\[
\alpha^B_j(s^t) \int (P - W(s^t)) \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t) \, dF^B_j(P,s^t) \\
+ \alpha^W_j(s^t) \int (P - W(s^t)) \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t) \, dF_j(P,s^t) - \kappa W(s^t) F_j(P,s^t) ,
\]

(8)

subject to the constraints for the number of bargain hunters in store \( j \), \( 6 \) and \( 7 \), and the constraint on the price distribution faced by bargain hunters \( 2 \). The last term in \( 8 \) captures the total fixed cost of posting prices below a given price \( \overline{P} \), expressed in units of labour. Finally, we assume that retailers face costs of adjusting their nominal prices, which we specify explicitly below.

In Section 4 we demonstrate, using a solution of the retailer’s problem for the case of a two-point price distribution, how the differences in price distributions faced by bargain hunters and workers as well as the randomization of workers across locations lead to price discrimination by retailers. First, lower prices create an incentive for households who draw low search costs to become bargain hunters in their designated locations. By itself, this is not a sufficient condition for price discrimination. For example, if each household shops in the designated location regardless of the search cost, the measure of households shopping in a given store-location is fixed so that for every extra bargain hunter the store loses a worker. In this case, due to the concavity of the profit function, smaller price discount yields higher profits, and hence, it is optimal for each retailer to post the same price for all its brands. This
is where the second assumption – randomization of workers across locations – comes into play. It implies that a retailer $j$ in a particular location has no control over the number of workers shopping in its store. In this case, retailers do not internalize the effect their pricing decisions have on the number of workers in their store-location and compete for market share by attracting bargain hunters through the posting of low prices.

### 3.3 Equilibrium

The market-clearing conditions for labor implies that the hours supplied by households in the labour market is equal to total hours demanded by the retail firms:

$$\int_0^1 l_k(s^t) \, dk = \int_0^1 \int_0^1 [n_j (s^t) + \kappa F_j (P, s^t)] \, dj \, di .$$

The market-clearing condition for each good variety states that total consumption of variety $j$ is equal to the total amount of that variety produced:

$$\int_0^1 [\alpha_{k,j} (s^t) c^B_{k,j} (s^t) + (1 - \alpha_{k,j} (s^t)) c^W_{k,j} (s^t)] \, dk = \int_0^1 \left[ \alpha^B_j (s^t) \int \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t) \, dF^B_j (P, s^t) + \alpha^W_j (s^t) \int \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t) \, dF^W_j (P, s^t) \right] \, di .$$

The equilibrium search flow condition states that the total number of households who choose to become workers for variety $j$ is equal to the total number of workers buying variety $j$ across all locations:

$$\int_0^1 (1 - \alpha_{k,j} (s^t)) \, dk = \int_0^1 \alpha^W_j (s^t) \, di .$$

A *symmetric equilibrium* for this economy is a collection of allocations for households: $c (s^t), l (s^t), c_j (s^t), M (s^t), B (s^t), \alpha_j (s^t)$; prices and allocations for firms: $y_j (s^t), n_j (s^t), F_j (P, s^t), F^B_j (P, s^t)$; and aggregate prices $P (s^t), W (s^t), Q (s^t)$, that satisfy household and firm maximization and the market-clearing conditions.

### 4 Model mechanics

In this section, we expose the mechanics of our model under the assumptions of both flexible and sticky prices. For simplicity, throughout the rest of the paper, we will consider equilibria with discrete two-price distributions. That is, stores post price $P^L_j (s^t)$ for a fraction $\gamma_j (s^t)$
of their brands and price $P_j^H(s^t)$ for the remaining fraction of brands, $1 - \gamma_j(s^t)$. Workers buy a brand that they randomly draw in this store. Bargain hunters, on the other hand, face on average lower prices: we assume that bargain hunters are able to increase the probability of their lowest price being $P_j^L(s^t)$ by a factor $f > 1$. This implies that bargain hunters find the low price with probability $f \gamma_j(s^t)$ and high price with probability $1 - f \gamma_j(s^t)$.\(^{25}\)

### 4.1 Model with flexible prices

We first start by studying an environment with fully flexible prices: firms face no constraints to change either $P^L$ or $P^H$.

Denote by $\Pi(P, s^t) = (P - W(s^t))\left(\frac{P}{P(s^t)}\right)^{-\theta} c(s^t)$ the profit function at price $P$; and by $R(P, s^t) = P\left(\frac{P}{P(s^t)}\right)^{-\theta} c(s^t)$ the revenue function at $P$. Then the retailer’s expected profit and revenue from selling to workers in period $t$ and state $s^t$ are:

$$\Pi_j^W(s^t) = \gamma_j(s^t) \Pi(P_j^L(s^t), s^t) + (1 - \gamma_j(s^t)) \Pi(P_j^H(s^t), s^t),$$

$$R_j^W(s^t) = \gamma_j(s^t) R(P_j^L(s^t), s^t) + (1 - \gamma_j(s^t)) R(P_j^H(s^t), s^t).$$

Similarly, the retailer’s expected profit and revenue from selling brands to bargain hunters, $\Pi_j^B(s^t)$ and $R_j^B(s^t)$ are the same as above, where $\gamma_j(s^t)$ is replaced by $f \gamma_j(s^t)$.

The retailer’s problem is to choose prices $P_j^L(s^t), P_j^H(s^t)$ and the fraction of brands on sale, $\gamma_j(s^t)$, to maximize

$$\sum_t \int Q(s^t) \left[\alpha_j(s^t) \Pi_j^B(s^t) + \alpha_j^W(s^t) \Pi_j^W(s^t) - \kappa W(s^t) \gamma_j(s^t)\right] ds^t,$$

subject to the two-price version of the constraint (6) on the number of bargain hunters:

$$W(s^t) \Xi'(\alpha_j(s^t)) = \left(\frac{\theta}{\theta - 1} D_j^B(s^t) - 1\right) R_j^B(s^t) - \left(\frac{\theta}{\theta - 1} D_j^W(s^t) - 1\right) R_j^W(s^t),$$

where the dispersion factor $D_j^W(s^t)$ is a function of the size of discount $P_j^L(s^t)/P_j^H(s^t)$ and

\(^{25}\)For example, if bargain hunters have two price draws (with replacement) and buy consumption at the lowest of the two prices, probability of finding $P_j^L(s^t)$ in this case is $2\gamma_j(s^t)$, i.e., $f = 2$.  

22
the fraction of low prices $\gamma_j (s^t)$:

$$D^W_j (s^t) = \left( \frac{\gamma_j (s^t) \left( \frac{P^L_j (s^t)}{P^H_j (s^t)} \right)^{- \theta} + 1 - \gamma_j (s^t)}{\gamma_j (s^t) \left( \frac{P^L_j (s^t)}{P^H_j (s^t)} \right)^{1 - \theta} + 1 - \gamma_j (s^t)} \right)^{1/\theta},$$

and $D^B_j (s^t)$ is given by the same expression, with $\gamma_j (s^t)$ replaced by $f \gamma_j (s^t)$.

**Optimality of price discrimination**

To gain intuition for why it is optimal for retailers to post different prices, we demonstrate the effect of the search constraint (9) on the retailers’ pricing behaviour. First, suppose that retailers cannot affect search decision by households via this constraint, treating the number of bargain hunters $\alpha_j (s^t)$ as given. It immediately follows from concavity of the retailer’s profit function that it is optimal to charge monopoly price for all its brands: $P^L_j (s^t) = P^H_j (s^t) = P^\ast_j (s^t) \equiv \frac{\theta}{\theta - 1} W (s^t)$, and collect monopoly profits $\Pi^\ast_j (s^t) = \Pi (P^\ast_j (s^t), s^t)$.

In contrast, when the retailer can attract bargain hunters, it effectively faces constraint (9). In this case, posting a single price for all of its brands is no longer optimal. Specifically, consider a small perturbation by a retailer that, instead of posting a single monopoly price $P^\ast_j (s^t)$ for all its brands, posts an arbitrarily lower price $P^L_j (s^t) < P^\ast_j (s^t)$ for a small fraction of brands $\gamma_j (s^t) = d \gamma$, attracting a small number of bargain hunters: $\alpha_j (s^t) = d \alpha$, and taking the number of workers as given, $\overline{\alpha}_j^W (s^t) \approx 1$. The variation of the retail firm $j$’s profit relative to monopolistic profit $\delta \Pi^\ast_j (s^t) \equiv \Pi_j (s^t) - \Pi^\ast_j (s^t)$ is, up to first order, given by

$$\delta \Pi^\ast_j (s^t) \approx \Pi_j (s^t) d \alpha + d \gamma \left[ \Pi (P^L_j (s^t), s^t) - \Pi (P^\ast_j (s^t), s^t) - \kappa W (s^t) \right].$$

The first term on the right-hand side is the gain in profits due to sales to the extra fraction $d \alpha$ of bargain hunters; this term would have been fully offset by the loss in profits due to smaller number of workers, if that number had directly depended on the retailer pricing decision. The second term is the loss in profits due to sales of the fraction $d \gamma$ of brands at lower price $P^L_j (s^t)$ and the total fixed cost of posting sale prices. From the search constraint (9), we can express $d \alpha$, up to first order, as

$$d \alpha \approx d \gamma \frac{1}{\theta - 1} \frac{f - 1}{W (s^t) \Xi (0)} \left( R \left( P^L_j (s^t), s^t \right) - R \left( P^\ast_j (s^t), s^t \right) \right).$$

(10)
The increase in the fraction of bargain hunters, \( d\alpha \), is proportional to the increase in the number of discounted prices, \( d\gamma \). This is true because while a higher fraction of discounted items raises the probability that both bargain hunters and workers find an item on sale, that probability rises faster for bargain hunters: a 1 percentage point increase in the fraction of sales leads to a \( f > 1 \) percentage points rise in the likelihood that bargain hunters find \( P_L \), but only 1 percentage point for workers. Expression (10) also shows that bargain hunting increases if wages fall, or if the size of price discount increases.

Plugging (10) into the expression for profit variation, we get

\[
\delta \Pi_j^* (s^t) \approx d\gamma \cdot \Pi_j^* (s^t) \left[ \frac{\Pi_j^* (s^t) - \Pi (P_j^L (s^t), s^t) + \kappa W (s^t)}{\Pi_j^* (s^t)} + \frac{f - 1}{\theta - 1} \frac{R(P_j^L (s^t), s^t) - R(P_j^* (s^t), s^t)}{W(s^t)\Xi''(0)} \right].
\]

If the expression in brackets is positive, i.e., if

\[
\frac{f - 1}{\theta - 1} \frac{R(P_j^L (s^t), s^t) - R(P_j^* (s^t), s^t)}{W(s^t)\Xi''(0)} > \frac{\Pi (P_j^L (s^t), s^t) - \Pi (P_j^* (s^t), s^t) + \kappa W (s^t)}{\Pi_j^* (s^t)},
\]

then the benefits stemming from the increase in the market share of the retailer from selling to bargain hunters outweigh the cost of smaller profit margin from selling at lower prices on average, and so the retailer can improve upon single-price monopolistic profit by posting two prices.

**Fraction of sale prices**

What is the desirable number of low prices for a retailer? The first-order condition for the number of brands on sale \( \gamma_j (s^t) \) in the retailer’s problem is

\[
(\alpha_j^W (s^t) + f\alpha_j (s^t)) \left[ \Pi (P_j^H (s^t), s^t) - \Pi (P_j^L (s^t), s^t) \right] + \kappa W (s^t) = \frac{\partial \alpha_j (s^t)}{\partial \gamma_j (s^t)} \Pi_j^B (s^t).
\]

(11)

The left-hand side is the marginal loss from increasing the fraction of sales. It consists of two terms: the first term is the marginal profit loss from selling more goods at a lower price, equal to the product of the marginal increase in the measure of transactions at low price, \( \alpha_j^W (s^t) + f\alpha_j (s^t) \), and the loss in profits per each brand on sale, \( \Pi (P_j^H (s^t), s^t) - \Pi (P_j^L (s^t), s^t) \). The second term is the marginal increase in the total fixed cost of posting sales, \( \kappa W (s^t) \). The right-hand side is the marginal profit from increasing the fraction of sales:
it is the product of the marginal increase in the number of bargain hunters, \( \frac{\partial \alpha_j(s^t)}{\partial \gamma_j(s^t)} \), and the average profits per bargain-hunter, \( \Pi_j^B(s^t) \). By differentiating (9) with respect to \( \gamma_j(s^t) \), we can show that \( \frac{\partial \alpha_j(s^t)}{\partial \gamma_j(s^t)} \) is decreasing in \( \gamma_j(s^t) \), because the utility loss due to price dispersion, given by \( D_j^B(s^t) \) and \( D_j^W(s^t) \), is growing faster for bargain hunters than for workers; or if the marginal cost of searching, \( \Xi''(\alpha_j(s^t)) \), is increasing in the number of bargain hunters. In turn, the retailer’s expected profit per bargain-hunter, \( \Pi_j^B(s^t) \), is also decreasing with \( \gamma_j(s^t) \), since more brands are sold at a lower price. Figure 7 plots the marginal gain and loss from increasing the fraction of brands on sale as functions of the number of brands on sale.

**Size of price discount**

How does the firm pin down the levels of its high and low prices? The corresponding first-order conditions are

\[
N_j^i(s^t) (MR(P_j^i(s^t), s^t) - MC(P_j^i(s^t), s^t)) + \frac{\partial \alpha_j(s^t)}{\partial \ln P_j^i(s^t)} \Pi_j^B(s^t) = 0, \tag{12}
\]

where \( i = H, L \), and \( MR(\cdot, s^t), MC(\cdot, s^t) \) are the marginal revenue and marginal cost functions of a single-price firm respectively:

\[
MR(P, s^t) = -(\theta - 1) \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t),
\]

\[
MC(P, s^t) = -\theta \frac{W(s^t)}{P} \left( \frac{P}{P(s^t)} \right)^{-\theta} c(s^t),
\]

\( N_j^H(s^t) \) and \( N_j^L(s^t) \) are the number of transactions at high and low prices; \( \frac{\partial \alpha_j(s^t)}{\partial \ln P_j^H(s^t)} \) and \( \frac{\partial \alpha_j(s^t)}{\partial \ln P_j^L(s^t)} \) are the marginal changes in the number of bargain hunters due to the increase in the high and low log prices respectively.

Conditions (12) imply that if the retailer could not affect the number of bargain hunters in the store, i.e., if \( \frac{\partial \alpha_j(s^t)}{\partial \ln P_j^H(s^t)} = \frac{\partial \alpha_j(s^t)}{\partial \ln P_j^L(s^t)} = 0 \), the optimal low and high prices would both equate the single-firm marginal revenue and marginal cost, and both would be equal to the optimal monopolistic price \( P_j^*(s^t) \), since, by definition, \( MR(P_j^*(j)) = MC(P_j^*(j)) \). In contrast, if the retailer can indeed attract bargain hunters, search condition (9) implies that \( \frac{\partial \alpha_j(j)}{\partial \ln P_j^H(j)} > 0 \) and \( \frac{\partial \alpha_j(j)}{\partial \ln P_j^L(j)} < 0 \), i.e., a higher high price and a lower low price make bargain-hunting more attractive. Then from (12) it follows that \( MR(P_j^L(s^t), s^t) > MC(P_j^L(s^t), s^t) \) and \( MR(P_j^H(s^t), s^t) < MC(P_j^H(s^t), s^t) \), i.e., retailer’s optimal low (high) price is below
(above) monopolistic price, \( P^L_t (j) < P^*_t < P^H_t (j) \), see Figure 8. This price setting mechanism implies that more elastic search implies larger absolute values of derivatives \( \frac{\partial \alpha_t(j)}{\partial \ln P^L_t(j)} \), \( \frac{\partial \alpha_t(j)}{\partial \ln P^H_t(j)} \), and therefore, larger price discount (lower \( \frac{P_t^L(j)}{P_t^H(j)} \)).

Finally, note that as \( \gamma_j (s^t) \to 0 \), \( P^H_j (s^t) \to P^*_j (s^t) \) and \( P^L_j (s^t) \to \frac{1}{1 + \frac{\Xi''(0)}{\Xi''(0)}} P^*_j (s^t) < P^*_j (s^t) \), i.e., price dispersion does not disappear as the fraction of sale prices goes to zero. This means that there is no equilibrium with a single price. The reason why firms find it optimal to deviate from having a single price for all brands is that for any positive measure of brands on sale, the return for the household of being a bargain-hunter is strictly positive, given by the right-hand side of (9). If the marginal expected fixed cost of searching is not growing too quickly with the probability of searching, e.g., if \( \Xi''(0) \) is low enough, then there always be enough bargain hunters to justify posting low prices by retailers.

### 4.2 Model with sticky prices

Next, we turn to our benchmark version with sticky prices: we assume that retailers face Taylor-type price adjustment constraints for high (regular) prices. Low (sale) prices are assumed to be flexible, although this has virtually no effect on our results, as we show below. Hence, retailer \( j \) sets a new high price in period \( t \), keeps that price fixed for \( N \) periods, i.e. it faces the following pricing constraint:

\[
P^H_j (s^t) = P^H_j (s^{t+1}) = \ldots = P^H_j (s^{t+N-1}) .
\]

(13)

Following the standard Taylor (1980) pricing, such price contracts are evenly staggered, so that in every period a measure \( 1/N \) of retailers resets their prices.

Retailer \( j \)'s problem then is to choose prices \( P^L_j (s^\tau) \), \( P^H_j (s^\tau) \) and the number of prices on sale \( \gamma_j (s^\tau) \) to solve

\[
\max \sum_{\tau=t}^{t+N-1} \int Q(s^\tau) \left[ \alpha_j (s^\tau) \Pi^B_j (s^\tau) + \alpha_j (s^\tau) \Pi^W_j (s^\tau) - \kappa W (s^\tau) \gamma_j (s^\tau) \right] ds^\tau ,
\]

subject to the constraint on bargain hunters (9) and pricing constraints (13).
The first-order conditions yield equations for new (reset) high prices:

\[ P_t^H(j) = \frac{\theta}{\theta - 1} \left( \sum_{\tau=t}^{t+N-1} \int Q(s^\tau)C(s^\tau)P(s^\tau)^{\theta}N_j^H(s^\tau)W(s^\tau)ds^\tau \right) \]

\[ \sum_{\tau=t}^{t+N-1} \int Q(s^\tau)C(s^\tau)P(s^\tau)^{\theta}N_j^H(s^\tau) \left( 1 - \frac{\partial \alpha_j(s^\tau)}{\partial \ln P_j^H(s^\tau)} (\theta-1)N_j^H(s^\tau)R_j^H(s^\tau) \right) ds^\tau. \]

Appendix A contains the full system of equilibrium conditions in the model with sticky prices. The model is solved by applying Blanchard-Khan (1980) method to log-linearized system of stochastic equilibrium equations around deterministic steady state.

5 Results

5.1 Parameterization

Table 6 provides the parameter values that we use in our quantitative simulations. The period is a month, so the discount factor \( \beta = 0.96^{1/12} \). We set \( \sigma = 1 \), a standard value in business cycle literature, and \( \psi = 1 \), which together imply that nominal wages follow the money supply, \( W_t = M_t \).

Following Kryvtsov and Midrigan (2013), we assume that the growth rate of the money supply is serially uncorrelated and choose the standard deviation of money growth, \( \sigma_{\mu} \), to be equal to 0.23% so that the model matches the standard deviation of the exogenous component (identified using the Christiano, Eichenbaum and Evans, 1999, measure of monetary policy shocks) of the U.S. monthly growth rate of the money supply.

We set the elasticity of substitution across good varieties equal to \( \theta = 5 \), implying a 25% markup, which is between the high disaggregate markup estimates consistent with the industrial organization literature and the low macro aggregate markups estimates (e.g. see Basu and Fernald, 1997). Aggregate fluctuations in our model are not sensitive to the level of \( \theta \).

We assume that regular prices change once every 12 months \( (N = 12) \), consistent with our data and existing studies. Although this degree of price stickiness is somewhat higher than that reported in Nakamura and Steinsson (2008) and Klenow and Krysvtsov (2008), it is consistent with the findings Eichenbaum et al. (2011) and Kehoe and Midrigan (2010). We assume that sale prices are fully flexible. As we show below, this assumption has little effect on the overall price flexibility, as firms find it optimal to keep the size of the price discount
close to the steady state. The latter result is consistent with our findings that the size of the average price discount is very acyclical. It is also consistent with Klenow and Kryvtsov (2008) who find that sale prices are roughly as sticky as regular prices.

Parameters that are specific to our price-discrimination model are the search efficiency parameter \( f \), the fixed cost of posting price discounts \( \kappa \), and parameters of the distribution of the fixed cost of searching \( G(z) \). The value of \( f \) is set at 2 to match the average discount of \( P_L/P_H = 0.78 \) in the data; according to the model, \( f = 2 \) means that bargain hunting allows households to double the probability of finding the low price. The fixed cost of posting price discounts \( \kappa \) is chosen to match the fraction of sale prices \( \gamma = 0.05 \). This number is in the upper range of values in the UK data presented in this paper, and in other European CPI data studies\(^{26}\); it is somewhat lower than between 7 and 11\% found for the U.S. by Nakamura and Steinsson (2008) and Klenow and Kryvtsov (2008).

Parameters of the distribution of the fixed searching cost \( G(z) \) are chosen to match the average share of revenue at sale prices. For the benchmark model, we choose \( G(z) \) to be uniform: it assigns equal probabilities of any fixed cost draw between 0 and \( z_{\text{max}} \). We choose \( z_{\text{max}} \) so that the model matches the ratio of the share of revenue due to sales to the number of sales. Glandon (2011) reports that for groceries in the US, sales account for 17\% of price quotes yet 40\% of revenue, so that the ratio of revenue share to the fraction of sales is 2.7. Matching this moment also pins down the fraction of bargain hunters in the model at \( \alpha = 0.10 \).

To highlight the main mechanism in the benchmark model, we consider an alternative parametrization of the fixed cost distribution that implies that search for low prices is very inelastic. Such distribution has a step-like shape, splitting the unit probability weight between only two realizations of the fixed cost: zero (with weight \( \alpha \)) and \( z_{\text{max}} \) (with weight \( 1 - \alpha \)).\(^{27}\) Households who face the step-like cost distribution will choose to search with probability \( \alpha \) regardless of the state of the economy. Hence, the key difference between the models with uniform and step-like fixed cost of searching is that in the former model the fraction of bargain hunters will move elastically over the business cycle, while it is virtually fixed in the latter model. Figure 9 plots both fixed cost cdfs. Appendix B provides specifications for both distributions.

\(^{26}\)The fraction of sales is between 0.03 and 0.05 in Austrian CPI data (Baumgartner et al., 2005); 0.03 in Norwegian CPI (Wulfsberg, 2009), 0.02 in French CPI (Baudry et al., 2004; Berardi et al., 2013).

\(^{27}\)In general case, we utilize a flexible two-parameter fixed cost cdf specification, borrowed from Dotsey, King and Wolman (1999) who use it in their state-dependent pricing model.
5.2 Responses to monetary shocks

Our baseline experiment is a negative 1% innovation to the money growth. Figure 10 provides the responses of output, prices, the average fractions of price discounts and the average fractions of bargain hunters in the models with elastic and inelastic fraction of bargain hunters. For output and price responses, the figure also provides the responses in the standard Taylor model with a single retail price.

The figure demonstrates a stark difference in the output and price responses between the economies with elastic and inelastic fractions of bargain hunters. The aggregate price level (top right panel) in the model with fixed fraction of bargain hunters is mimicking that for the standard Taylor model, falling steadily to -1% within one year, as all sticky-price cohorts of retailers get a chance to lower their prices after the shock. Accordingly, the fall in output (top left panel) is immediate, down to around -0.9% at the time of the shock, dissipating to zero after one year, just as in the standard Taylor model.

In contrast, the real effect on output is about twice smaller in the model with a moving fraction of bargain hunters, falling by about half percent on impact and going back to its pre-shock level within a year or so. Such a response in output is due to a faster fall in the aggregate price level, which, unlike in the model with inelastic bargain hunters, immediately drops by 0.5% at the time of the shock.

To show that these results do not depend on the persistence of the monetary shock, we repeat the IRF simulations assuming that the money growth process is serially correlated with persistence parameter equal to 0.536, as in Guimaraes and Sheedy (2011). Figure 11 shows that indeed all the insights from Figure 10 carry over to the case with the persistent shock: the endogenous reaction of bargain hunters leads to a large impact of the sales margin on the response of aggregate prices and output to shocks.

5.3 Understanding impulse responses

The marked differences in the degree of the aggregate price flexibility between the two models are due to the mechanism by which households’ search for lower prices amplifies the incentives for price-discounting by retailers. To understand this mechanism, let us go through the factors that determine the size of the decline in the aggregate price level immediately after the shock. First, we log-linearize the expression for the aggregate price level (5) around the steady state
assuming that both $\gamma$ and $\alpha$ are on average close to zero, and $P^H$ is close to $P$. Denoting the average price discount by $\delta = P^L/P^H$, we obtain that the log deviation of the aggregate price level from its steady state value is, approximately,

$$\hat{P}_t \approx -\frac{P}{\theta - 1} \left( \delta^{1-\theta} - 1 \right) \tilde{\gamma}_t,$$  \hspace{1cm} (14)

where a hat (tilde) denotes the log-linearized (difference-) deviation from the steady state, and steady state values have no subscripts. Expression (14) indicates that the response of the average fraction of sales $\gamma_t$ affects the aggregate price response by shifting the weight from high to low prices. The impact of this shift depends on both the size of the response of the fraction of sales itself and on the size of the price discount. In the elastic (inelastic) case, the discount is equal to $\delta = 0.78 (0.89)$, implying a response of the fraction of sales of $\tilde{\gamma}_t = 1.1 (0.5)$ percentage points (bottom left panel on Figure 10). Plugging these values into (14) gives price level responses of -0.5% (-0.1%) at the time of the shock, as shown on the Figure. Hence, both the average discount and relative responsiveness of the fraction of sales contribute to the five-fold larger response of the price level in the elastic case relative to inelastic case. Here is how the differences in both the size and the number of discounts stem from the difference in the elasticity of search behaviour by households.

First, use pricing equations (12) to approximate the average size of price discount:

$$\delta \approx \left( 1 + \frac{f - 1}{\theta (\theta - 1) \Xi''(\alpha)} \right)^{-1}.$$

As we already established in Section 4.1, larger search elasticity (lower $\Xi''(\alpha)$) and larger search efficiency $f$ imply larger discount. Keeping $f = 2$, and using the steady state value for $\Xi''(\alpha)$ in elastic (inelastic) case, 0.3 (0.6), we obtain, approximately, that the average discount is 0.85 (0.92), close to accurate stead-state value of 0.78 (0.89).

Second, to understand the response in the fraction of sales, $\tilde{\gamma}_t$, note that if that fraction were fixed, sticky regular prices would imply high average markup at the store, increasing $\Pi^B_j(s^t)$ - the expected profit per bargain hunter. The first-order condition for the fraction of sales (11) states that the benefit flow for the retailer (right-hand side) depends on how much extra bargain hunters an additional brand on sale generates, i.e., on the derivative $\frac{\partial \alpha_j(s^t)}{\partial \gamma_j(s^t)}$. In elastic case, the steady state value of this derivative is almost six time larger than
in inelastic case, 1.75 vs 0.29. Since monetary shock does not directly affect the profit loss from posting sales (left-hand side of equation (11)), retailers must bring down their marginal benefit by increasing the number of sales (since both $\frac{\partial \alpha_j(s^t)}{\partial \gamma_j(s^t)}$ and $\Pi_j^B(s^t)$ decrease with $\gamma_j(s^t)$). Altogether, the response of $\tilde{\gamma}_t$ is twice as large in elastic case than in inelastic case.

Finally, the response in the fraction of bargain hunters is approximately

$$\tilde{\alpha}_t \approx \tilde{\gamma}_t \frac{1}{\theta - 1} \frac{f - 1}{W^u(\alpha)} \left( \delta^{1-\theta} - 1 \right),$$

which yields 1.6 percentage points for elastic case and 0.2 percentage points for inelastic case (lower right panel on Figure 10).

Where does the flexibility of the aggregate price response come from? There are three contributing factors: in addition to the increase in the fraction of sale prices, there is also a shift of consumption weight towards sale prices, which in turn is due to the increase in the number of shoppers buying at low prices (the extensive margin) and the increase in average consumption per shopper (the intensive margin). Figure 12 decomposes the response of the true price index from our benchmark experiment, equal to -0.56% on impact, into counterfactual responses with each of the three factors sequentially turned off. The price index that ignores the variations in consumption weights (due to both the intensive and extensive margins), “$P_{CPI}$”, is the model counterpart to a standard fixed-weight CPI index: it decreases by 0.37% after the shock. Therefore, the unmeasured variation in the aggregate price accounts for around one-third of the total variance of the aggregate price level. The index “$P(\text{fixed} \alpha)$” adds the variation in consumption weights due only due to the intensive margin; its response implies that the extensive (intensive) margin accounts for about 1/3 (2/3) of the response due to the cyclical shift in consumption weights. Finally, ignoring fluctuations in the number of brands on sale (given by “$P_{CPI}(\text{regular})$”) brings the impulse response to a mere -0.07% on impact, close to the response in the standard one-price Taylor model or the model with inelastic search.

Our findings therefore broaden the insights from Chevalier and Kashyap (2012) who emphasize the importance of accounting for the shift in consumption weights towards sale prices for the degree of aggregate price flexibility. First, we quantify that the shift in weights in response to a shock can add about a half of variance implied by the constant-weight price index. Second – and this is our main contribution – we show that aggregate price flexibility
stemming from fluctuations in the fraction of sale prices can be even higher than the one implied by the shift in consumption weights.

5.4 What do sales tell us about business cycles?

Our impulse response results demonstrate the importance of accounting for the interaction between retailers’ price-discounting behavior and households’ search for low prices. Countercyclical search for low prices is necessary in making sales important for aggregate price flexibility. This prediction of our model has important implications regarding the nature of the real effects of monetary policy shocks. Namely, in our model, after a monetary contraction markup must rise, or otherwise retailers would not benefit from attracting low-margin shoppers (bargain hunters). We demonstrate this by examining factors driving the increase of the fraction of bargain hunters after the shock.

According to equation (9), the fraction of bargain hunters would increase after the shock if a) the fraction of sales increases, b) the price discount increases, or c) wages decrease (lowering the opportunity cost of searching). We repeat impulse response simulations for variations of the baseline economy in which we shut down each of these three factors. First, we assume that low prices $P_L$ are sticky and can be adjusted every 12 months together with high prices. In this variation of the model, the average price discount is virtually unchanged after the shock. In the second variation, we assume that the fraction of sales is fixed at its steady state level, $\gamma(s^t) = \gamma$. In the third variation, we assume that nominal wages are sticky, given by equation

$$ W(s^t) = W(s^{t-1})^{11/12} M(s^t)^{1/12}, $$

so that it takes around a year for nominal wages to converge to their pre-shock level. In this variation, we will assume that retailers’ marginal cost is still flexible and equal to the money supply, like in the baseline model. Finally, in the fourth variation of the model, we assume that household wages are flexible, and it is the nominal marginal cost that is sticky obeying the law of motion (15).\footnote{It is straightforward to extend our benchmark model to include capital, sticky nominal wages and nominal cost rigidities to explicitly account for reduced forms for sticky wages and marginal cost. Such an extension would not change the point we make here.}

Table 7 provides concurrent responses to the same shock (1% negative impulse to money growth) in the baseline model (column A) and its four variations (columns B through E).
First, as we noted in Section 5, sticky sale prices (column B) make virtually no difference for our results, since retailers find it optimal to vary the fraction of sales, and not the discount, to attract bargain hunters. This arises because the retailer’s value is nearly linear in the fraction of sales: when that fraction is small on average, both retailer’s marginal gains and losses are very inelastic with respect to changes in the fraction, see Figure 7. When retailers cannot adjust the fraction of sales (column C), they can attract bargain hunters only by increasing the price discount. After the shock, the discount increases by 1.2 ppt (-0.06 in the baseline). Since the discount size is a less effective adjustment margin for retailers, the implication is a smaller change in the fraction of bargain hunters than in the baseline model, 1.17 vs 1.62 ppt, respectively. As a result, the shift of consumption weights towards low prices is more limited, resulting in a stickier response of aggregate price and larger response in consumption, -0.68 % (vs -0.44% in the baseline). Even in this case, monetary non-neutrality is smaller by one-third relative to the case with inelastic consumer search. These simulations underline the importance of studying the entire arsenal of tools that retailers use to attract customers: if retailers cannot adjust the fraction of sales in response to a monetary shock, they will adjust along other margins (size of discounts, coupons, etc.).

Our model predicts that the quick fall of household wages after the shock matters relatively little for incentivizing households to look harder for price discounts (column D in Table 7). When nominal wages are sticky, they fall by only about 0.1% after the shock, as opposed to 1% in the baseline model. Since the marginal time cost of searching, in dollars, is equal to \( W(s^t) \alpha(s^t) \), a 0.9% smaller fall in wages implies a smaller rise in the fraction of bargain hunters by only 0.1 ppt (= 0.9\alpha). In addition, retailers post sales less aggressively, amplifying the effect on the fraction of bargain hunters by 0.26 ppt, (i.e., the increase in the fraction is 1.36 ppt vs 1.62 ppt in the baseline model) – still a relatively small number.

In the three model variations considered so far (columns B, C, D), markups rise by roughly half of a percentage point after the monetary contraction, making it a good time for retailers to increase their market share. In the last simulation, we limit the rise in the markup on impact by assuming sticky marginal cost (column E in Table 7). In this case, retailers barely change their prices after the shock. The fraction of bargain hunters increases only by 0.1 ppt due to the one-percent fall in household wages, implying only a small effect on the aggregate price and, therefore, a large consumption response at -0.94%, very close to the inelastic case.
Hence, in our model, countercyclical markups represent the central impetus that leads retailers to make a more intensive use of sales during slumps.\footnote{A growing consensus in macroeconomic literature is that nominal cost rigidities, rather than countercyclical markups account for most of the monetary non-neutrality, see Christiano et al. (2005). In contrast, our theory, combined with countercyclical consumer search in the data, implies that countercyclical markups are indeed important for the real effects of monetary shocks. Kryvtsov and Midrigan (2013) arrive to a similar conclusion, based on the study of the observed behaviour of inventories.}

In sum, our findings imply that changes in households’ shopping activity are associated, to a large extent, with retailers’ pricing over the cycle, and less so with changes in the opportunity cost of time. Coibion, Gorodnichenko, Hong (2014) and Nevo and Wong (2014) emphasize the role of the decline in the opportunity cost of time in explaining the increased shopping activity during slumps. Our paper, therefore, shows that the rise in return to shopping due to more frequent sales in recessions can also be a powerful driving mechanism behind fluctuations in households’ shopping time.

6 Discussion

In this section, we present some empirical evidence on price search behavior over the business cycle as well as discuss how a broader view of price discrimination may allow us to reconcile our findings with those of Coibion, Gorodnichenko and Hong (2012).

6.1 Price search behavior during the Great Recession

In the previous section, we presented a model in which price discrimination gave rise to sales and altered the cyclical properties of aggregate variables. As we made it clear, the key mechanism behind this result is that households search for bargains more intensively in recessions. In what follows, we discuss both exisiting and novel evidence in support of this central prediction of our model.

Time-use surveys have been a useful source of evidence on search and shopping activity over the business cycle or across households. For example, Aguiar et al. (2013) find a significant increase in time spent shopping for a typical household during the 2008-2009 recession based on the American Time Use Survey. They document that about 7% of foregone market hours are allocated to increased shopping time. This is in line with evidence from Aguiar and Hurst (2007), who document important fluctuations in the shopping margin over the lifecycle.
and Kruger and Mueller (2010), who find that unemployed individuals devote 15% to 30% more time to shopping than the employed. Kaplan and Menzio (2014) use the Kilts-Nielsen Consumer Panel Dataset to show that “households with more non-employed members pay less for the same basket of goods than households whose members are all employed: the average price index for non-employed households is between 1% and 4.5% lower than the price index for employed households.” Using a similar dataset, Nevo and Wong (2014) estimate that household consumption declined by 60% less than market expenditures due to reallocation of time from market work to home production and shopping. All this evidence supports the assumption that on average, consumers search for lower prices more intensively during economic downturns.

Next, we turn to a new source of information on search behavior: Google Trends. A few years ago, Google made it possible to extract time-series indexes of how intensively internet users in a given location search online for specific search phrases relative to the total number of searches done through their engine. While these indexes are automatically normalized (Google does not make the raw data available), this is not problematic for our objective of determining whether search for low prices was significantly higher during the 2008-2009 recession.

The first challenge in this type of analysis is to pick relevant Google search terms. For example, using the term “sale” by itself is problematic: it is too general and will include searches that are not relevant for our purposes (items that are “for sale”) in addition to those that are (products that are “on sale”). For this reason, we settled on two search terms that we considered closely related to bargain hunting behavior: “clearance” and “on sale”. That being said, our findings are similar across a wide range of other search terms, such as “outlet”, “deal of the day”, “shoes on sale”, “half off deals”, “promotional code”, etc.

In Figure 13, we plot the unemployment rate alongside Google search activity between January 2004 and December 2013, which is normalized to be equal to 100 in December 2006, for both the U.S. and the U.K. The evidence is very clear: for both countries, during the Great Recession there is a strong rise in online search using terms related to bargain hunting. For example, search activity in the U.K. for the term “clearance” tripled between early 2007 and mid-2009. In the U.S., the indexes for both terms have been trending downward alongside
unemployment since 2010.\footnote{For conciseness, we present only graphical evidence for two search terms in two regions. We also performed a more detailed regression-based analysis using seven search terms and US-state-level data and reached very similar conclusions. Results are available from the authors upon request.}

In summary, both the evidence from time-use surveys as well as from online search activity appear to support our central modelling feature: search activity for low prices rises during recessions.

### 6.2 Other forms of price discrimination over the cycle: the case of coupons

In Section 2, we documented a very robust link between aggregate economic downturns and a more intensive use of sales by U.K. retailers. In the U.S., preliminary evidence from the CPI micro data also seems to support the countercyclicality of sales. Yet, a recent study by Coibion, Gorodnichenko and Hong (2012) finds that sales are mostly acyclical. They reach this conclusion using the IRI Marketing Dataset, a large scanner database that covers about 30 product categories across grocery stores in 50 distinct U.S. markets, with up to 150 stores per market (see Bronnenberg et al., 2008, for a description of the database). Based on the same dataset, we confirm their findings about temporary sales: while some markets/categories exhibit an increase in the incidence of sales during the Great Recession, these movements are in general not particularly significant, either statistically or economically.

We do, however, find another interesting feature: the use of coupons by retailers (generally through weekly flyers), an information available in the dataset, almost doubled during the 2008-2009 recession. This can be seen in the left-hand plot of Figure 14 where we show the 12-month moving average of the frequency of retailer coupons between 2004 and 2011 alongside the unemployment rate. Notice, however, that the frequencies are really low and at first sight unlikely to be of much economic significance.\footnote{The use of coupons varies widely across products, being particularly prevalent for categories such as carbonated beverages and cold cereal.} This should not be surprising, since most coupons are distributed by manufacturers and would therefore not be accounted for in the IRI database. Nonetheless, our finding is corroborated by evidence from industry sources that points to substantial increases in, both, distribution and redemption of coupons during the last recession (see for example Inmar Inc, 2013, or NCH, 2013). It also coincides with a dramatic rise in Google searches for the term “coupons” over the same time period (right-hand plot of Figure 14).
Our findings for coupon usage are corroborated by Nevo and Wong (2014), who use the Nielsen Homescan dataset to document an increase in shopping intensity in the U.S. during the Great Recession. In particular, Nevo and Wong find that shopping activities increased by 1.5 to 2 percentage point on average, and that U.S. counties that experienced larger rises in unemployment also experienced on average higher increases across various shopping activities: sales, coupons, generic products, or large-size items.

In sum, our evidence suggests that price discrimination can take many forms, and is likely to vary across products and markets. For example, coupons are a much less prevalent price-promotion tool in the U.K. While sales have been the focus of this paper due to their prominence in the discussion surrounding price dynamics, it may be worthwhile for macroeconomists to look at whether other pricing tools, such as coupons, can have a significant role to play in the transmission of shocks.

7 Conclusions

Temporary price discounts ("sales") are an important feature of pricing behaviour of retailers. The recent literature argues that sales are mostly irrelevant for macroeconomists under the assumption that they are not significantly affected by business cycles and represent high frequency phenomena that have little impact on the predictions of macro models. We revisit this debate and argue that the dynamics of the observed sales behaviour can indeed be important for aggregate price flexibility. First, using the UK CPI micro data covering the period from 1996 to 2012, we demonstrate that sales (that normally would be filtered out by macroeconomists) are correlated with the business cycle: in particular, the frequency of sales is strongly countercyclical and doubled during the last recession. We also show that the impact of sales on the effective aggregate price level can be very large once we take into account their disproportional weight in total spending.

Second, we study the propagation of monetary shocks in an environment where sales respond to macroeconomic conditions in line with the empirical evidence. We build a general equilibrium business cycle model with consumer search and price discrimination by retailers. Our model, calibrated to price-discount behaviour in the data, predicts that sale prices represent a significant source of price flexibility. In response to an unanticipated monetary contraction, the increase in consumer search activity and more aggressive discounts by re-
tailers lead to a much faster decrease in the aggregate price level and substantially smaller response of real output.

Our takeaway: focusing on posted regular or reference prices may lead macroeconomists to miss important aspects of pricing over the business cycle. More generally, we believe future research should be aimed at investigating the cyclical properties and aggregate implications of the numerous price discrimination strategies used by firms. Furthermore, evidence on retailers' price-discounting behaviour, combined with evidence on households' search for low prices can provide useful insights for the debate on the driving forces of business cycles.
References


Figures

Figure 1: Distribution of the size of sales for the three main filters
Figure 2: Frequency of sales flags (raw and 12-month moving average) and unemployment rate

Figure 3: The evolution of the frequency of sales (alternative filters)
Figure 4: Frequency of sales (sales flag), selected categories

Figure 5: The evolution of the size of sales
Figure 6: Frequency of sales and substitutions in U.S. CPI data and unemployment rate
Figure 7: Profit loss and gain from 1 ppt increase in the fraction of low prices

Figure 8: Decision for high and low price levels
Figure 9: (Inverse) Distributions of the fixed cost of searching

Figure 10: Responses to a negative 1% impulse to money growth
Figure 11: Responses to a negative 1% impulse to persistent money growth

Figure 12: Decomposition of aggregate price response to a negative 1% impulse to money growth
Figure 13: Google search activity for bargain hunting-related terms and unemployment rate (12-month centered moving averages)

Figure 14: Frequency of retailer coupons in the IRI Marketing scanner dataset and Google search activity for term “coupons”
## Tables

### Table 1: Summary statistics for posted and regular prices

|                  | $P(\Delta p_{it} \neq 0)$ | $P(\Delta p_{it} > 0)$ | $P(\Delta p_{it} < 0)$ | $|\Delta p_{it}|_{\Delta p_{it} \neq 0}$ |
|------------------|-----------------------------|-------------------------|-------------------------|------------------------------------------|
| **All prices**   | 0.172                       | 0.107                   | 0.065                   | 0.111                                    |
| **Regular prices** |                             |                         |                         |                                          |
| Sales flag       | 0.146                       | 0.095                   | 0.050                   | 0.080                                    |
| V-shaped         | 0.121                       | 0.084                   | 0.037                   | 0.087                                    |
| Reference price  | 0.073                       | 0.053                   | 0.020                   | 0.095                                    |

### Table 2: Summary statistics for temporary sales

<table>
<thead>
<tr>
<th></th>
<th>Sales flag</th>
<th>V-shaped</th>
<th>V-shaped (alt.)</th>
<th>Reference price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All sales</strong></td>
<td>Frequency</td>
<td>2.63%</td>
<td>4.95%</td>
<td>3.87%</td>
</tr>
<tr>
<td></td>
<td>Mean size</td>
<td>-22.4%</td>
<td>-11.9%</td>
<td>-11.2%</td>
</tr>
<tr>
<td></td>
<td>Median size</td>
<td>-20.0%</td>
<td>-7.0%</td>
<td>-6.1%</td>
</tr>
<tr>
<td><strong>Sales of at least 10%</strong></td>
<td>Frequency</td>
<td>2.14%</td>
<td>2.04%</td>
<td>1.58%</td>
</tr>
<tr>
<td></td>
<td>Mean size</td>
<td>-26.1%</td>
<td>-23.9%</td>
<td>-23.1%</td>
</tr>
<tr>
<td></td>
<td>Median size</td>
<td>-22.2%</td>
<td>-20.1%</td>
<td>-20.0%</td>
</tr>
</tbody>
</table>
### Table 3: Panel regression results at the category level

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales flag</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_t$</td>
<td>0.356***</td>
<td>0.377***</td>
<td>0.339***</td>
<td>0.257***</td>
<td>0.320***</td>
<td>0.466***</td>
<td>0.368***</td>
</tr>
<tr>
<td>$s_{i,t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.362***</td>
<td></td>
</tr>
<tr>
<td>Month dum.</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Time trend</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>CPI weights</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.022</td>
<td>0.049</td>
<td>0.052</td>
<td>0.175</td>
<td>0.041</td>
<td>0.041</td>
<td>0.034</td>
</tr>
<tr>
<td>F-stat</td>
<td>8.44</td>
<td>5.35</td>
<td>5.13</td>
<td>43.28</td>
<td>3.84</td>
<td>8.79</td>
<td>11.10</td>
</tr>
</tbody>
</table>

### Table 4: Panel regression results - alternative macroeconomic indicators

<table>
<thead>
<tr>
<th></th>
<th>Sales flag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>0.0050***</td>
</tr>
<tr>
<td>Cons. confidence</td>
<td>-0.0041***</td>
</tr>
<tr>
<td>Fin. situation next year</td>
<td>-0.0047***</td>
</tr>
<tr>
<td>Economy next year</td>
<td>-0.0018*</td>
</tr>
<tr>
<td>Major purchases</td>
<td>-0.0041***</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.051</td>
</tr>
<tr>
<td>F-stat</td>
<td>5.36</td>
</tr>
</tbody>
</table>

### Table 5: Panel regression results at the product level

<table>
<thead>
<tr>
<th></th>
<th>V-shaped</th>
<th>Ref. price</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td>(9)</td>
<td>(6)</td>
</tr>
<tr>
<td>$u_t$</td>
<td>0.373***</td>
<td>0.309***</td>
</tr>
<tr>
<td>$s_{i,t-1}$</td>
<td>-</td>
<td>0.202***</td>
</tr>
<tr>
<td>Month dummies</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Time trend</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>F-stat</td>
<td>4.40</td>
<td>58.01</td>
</tr>
</tbody>
</table>
Table 6: Parameterization (benchmark model)

<table>
<thead>
<tr>
<th>A. Calibrated Parameters</th>
<th>B. Targets (steady state)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>θ</strong> elast. subst. goods</td>
<td><strong>Markup</strong></td>
</tr>
<tr>
<td><strong>f</strong> search efficiency</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>κ</strong> fixed cost of posting discounts *</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>z_{max}</strong> max fixed search cost *</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Price discount</td>
</tr>
<tr>
<td></td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Fraction of discounts</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Δ Revenue share of sales</td>
</tr>
<tr>
<td></td>
<td>from Δγ=1ppt</td>
</tr>
</tbody>
</table>

* in units of time

C. Assigned Parameters

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>β</strong> discount factor</td>
<td>0.96^{1/12}</td>
<td></td>
</tr>
<tr>
<td><strong>σ</strong> risk aversion</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>I/N</strong> frequency of price changes</td>
<td>1/12</td>
<td></td>
</tr>
<tr>
<td><strong>ρ_μ</strong> serr corr of money shock</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>σ_μ</strong> stdev of money shock impulse</td>
<td>0.23</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Responses to a negative 1% impulse to money growth (on impact)

<table>
<thead>
<tr>
<th></th>
<th>A. Baseline</th>
<th>B. Sticky $P^L$</th>
<th>C. Fixed $γ$</th>
<th>D. Sticky wages (household)</th>
<th>E. Sticky marginal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg frac of sales, ppt</td>
<td>1.08</td>
<td>1.05</td>
<td>0.00</td>
<td>0.93</td>
<td>0.06</td>
</tr>
<tr>
<td>Avg discount, ppt</td>
<td>-0.06</td>
<td>-0.01</td>
<td>1.20</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Wage (households), %</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.08</td>
<td>-1.00</td>
</tr>
<tr>
<td>Marginal cost, %</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.08</td>
</tr>
<tr>
<td>Consumption, %</td>
<td>-0.44</td>
<td>-0.45</td>
<td>-0.68</td>
<td>-0.50</td>
<td>-0.94</td>
</tr>
<tr>
<td>Frac of bargain-hunters, ppt</td>
<td>1.62</td>
<td>1.63</td>
<td>1.17</td>
<td>1.36</td>
<td>0.09</td>
</tr>
<tr>
<td>Avg markup, ppt</td>
<td>0.44</td>
<td>0.45</td>
<td>0.68</td>
<td>0.50</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Note: Responses are given in the month of the impulse
Appendix A. Equilibrium conditions for sticky price model

Denote the following auxiliary variables:

\[
\begin{align*}
\Pi^{L}_{t,j} &= (P^{L}_{t,j} - W_{t}) \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{-\theta} c_{t}, \\
\Pi^{H}_{t,j} &= (P^{H}_{t,j} - W_{t}) \left( \frac{P^{H}_{t,j}}{P_{t}} \right)^{-\theta} c_{t}, \\
\Pi^{L}_{1,j} &= \gamma_{t,j} \Pi^{L}_{t,j} + (1 - \gamma_{t,j}) \Pi^{L}_{t,j}, \\
\Pi^{H}_{1,j} &= f_{t,j} \Pi^{L}_{t,j} + (1 - f_{t,j}) \Pi^{L}_{t,j}, \\
\Pi^{W}_{t,j} &= \gamma_{t,j} \Pi^{L}_{1,j} + (1 - \gamma_{t,j}) \Pi^{L}_{1,j}, \\
\Pi^{B}_{t,j} &= f_{t,j} \Pi^{L}_{1,j} + (1 - f_{t,j}) \Pi^{L}_{1,j}, \\
R^{L}_{t,j} &= P^{L}_{t,j} \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{-\theta} c_{t}, \\
R^{H}_{t,j} &= P^{H}_{t,j} \left( \frac{P^{H}_{t,j}}{P_{t}} \right)^{-\theta} c_{t}, \\
R^{W}_{t,j} &= \gamma_{t,j} R^{L}_{t,j} + (1 - \gamma_{t,j}) R^{L}_{t,j}, \\
R^{B}_{t,j} &= f_{t,j} R^{L}_{t,j} + (1 - f_{t,j}) R^{L}_{t,j}, \\
D^{W}_{t,j} &= \frac{\left( \gamma_{t,j} \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{-\theta} + 1 - \gamma_{t,j} \right)^{1-1/\theta}}{\gamma_{t,j} \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{1-\theta} + 1 - \gamma_{t,j}}, \\
D^{B}_{t,j} &= \frac{\left( f_{t,j} \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{-\theta} + 1 - f_{t,j} \right)^{1-1/\theta}}{f_{t,j} \left( \frac{P^{L}_{t,j}}{P_{t}} \right)^{1-\theta} + 1 - f_{t,j}},
\end{align*}
\]

and \(D^{B}_{t,j}\) and \(D^{W}_{t,j}\) are derivatives of \(D^{B}_{t,j}\) and \(D^{W}_{t,j}\) with respect to \(\frac{P^{L}_{t,j}}{P_{t}}\); and \(D^{B}_{2t,j}\) and \(D^{W}_{2t,j}\) are derivatives of \(D^{B}_{t,j}\) and \(D^{W}_{t,j}\) with respect to \(\gamma_{t,j}\);

Equilibrium consists of \(5N + 5\) equations for \(5N + 5\) variables \(P^{L}_{t,j}, P^{H}_{t,j}, \gamma_{t,j}, \alpha^{W}_{t,j}, \alpha_{t,j}, P_{t}, c_{t}, W_{t}, R_{t}, M_{t}\) including:

- equation for \(P^{H}_{t,0}\)

\[
P^{H}_{t,0} = \frac{\theta}{\theta - 1} \frac{E_{t} \sum_{t=0}^{t+N-1} \beta^{t} c_{t}^{1-\sigma} P^{H}_{t} - N^{H}_{t,t-1} W_{t}}{E_{t} \sum_{t=0}^{t+N-1} \beta^{t} c_{t}^{1-\sigma} P^{H}_{t} - N^{H}_{t,t-1} (1 - \Delta^{H}_{t,t-1})},
\]

- \(N - 1\) equations for \(P^{H}_{t,1}, \ldots, P^{H}_{t,N-1}\)

\[
P^{H}_{t,0} = P^{H}_{t+1,1} = \ldots = P^{H}_{t+N-1,N-1},
\]
- N equations for $P_{t,0}^L, \ldots, P_{t,N-1}^H$

$$P_{t,j}^L = \frac{\theta}{\theta - 1} + \frac{1}{\Delta_{t,j}^L} W_t,$$

where

$$N_{t,j}^H = (1 - \gamma_{t,j}) \alpha_{t,j}^W + \alpha_{t,j} (1 - f_{t,j}),$$

$$N_{t,j}^L = \gamma_{t,j} \left( \alpha_{t,j}^W + \alpha_{t,j} f_{t,j} \right),$$

$$\Delta_{t,j}^H = \frac{\partial \alpha_{t,j}}{\partial \ln P_{t,j}^H} \left( \frac{\Pi_{t,j}^B}{(\theta - 1) N_{t,j}^H R_{t,j}^H} \right),$$

$$\Delta_{t,j}^L = - \frac{\partial \alpha_{t,j}}{\partial \ln P_{t,j}^L} \left( \frac{\Pi_{t,j}^B}{(\theta - 1) N_{t,j}^L R_{t,j}^L} \right),$$

$$\frac{\partial \alpha_{t,j}}{\partial \ln P_{t,j}^H} = \frac{D_{t,j}^B R_{t,j}^B}{W_t \Xi''(\alpha_{t,j})} \frac{1}{\theta - 1} \left[ - \left( \frac{D_{t,j}^B P_{t,j}^L}{D_{t,j}^B} \right) - (\theta - 1) \left( \frac{1 - f_{t,j}(\gamma_{t,j})}{R_{t,j}^H} \right) \left( \frac{D_{t,j}^B P_{t,j}^H}{D_{t,j}^B} \right) \right],$$

$$\frac{\partial \alpha_{t,j}}{\partial \ln P_{t,j}^L} = \frac{D_{t,j}^W R_{t,j}^W}{W_t \Xi''(\alpha_{t,j})} \frac{1}{\theta - 1} \left[ - \left( \frac{D_{t,j}^W P_{t,j}^L}{D_{t,j}^W} \right) - (\theta - 1) \left( \frac{1 - f_{t,j}(\gamma_{t,j})}{R_{t,j}^W} \right) \left( \frac{D_{t,j}^W P_{t,j}^H}{D_{t,j}^W} \right) \right],$$

- N equations for the fraction of bargain hunters $\alpha_{t,j}$

$$W_t \Xi''(\alpha_{t,j}) = \left( \frac{\theta}{\theta - 1} D_{t,j}^B - 1 \right) R_{t,j}^B - \left( \frac{\theta}{\theta - 1} D_{t,j}^W - 1 \right) R_{t,j}^W,$$

- N equations for the fraction of low prices $\gamma_{t,j}$

$$\left( \alpha_{t,j}^W + f_{t,j} \right) \left[ \Pi_{t,j}^H - \Pi_{t,j}^L \right] + \kappa W_t = \frac{\partial \alpha_{t,j}}{\partial \gamma_{t,j}} \Pi_{t,j}^B,$$
where
\[
\frac{\partial \alpha_{t,j}}{\partial \gamma_{t,j}} = \frac{1}{W_t \Xi''(\alpha_{t,j})} \frac{f}{\theta - 1} [D_{2t,j}^B R_{t,j}^B + D_{t,j}^B (R_{t,j}^L - R_{t,j}^H)]
\]
\[
- \frac{1}{W_t \Xi''(\alpha_{t,j})} \frac{1}{\theta - 1} [D_{2t,j}^W R_{t,j}^W + D_{t,j}^W (R_{t,j}^L - R_{t,j}^H)] ,
\]
- \(N\) equations for the fraction of workers of variety \(j\)
\[
\bar{\alpha}_{t,j}^W = 1 - \alpha_{t,j} ,
\]
- equation for the price of aggregate consumption \(P_t\)
\[
P_t = \left[ \frac{1}{N} \sum_{j=0}^{N-1} \alpha_{t,j} \left[f \gamma_{t,j} \left( P_{t,j}^L \right)^{1-\theta} + (1 - f \gamma_{t,j}) \left( P_{t,j}^H \right)^{1-\theta} \right] + \frac{1}{N} \sum_{j=0}^{N-1} \bar{\alpha}_{t,j}^W \left[ \gamma_{t,j} \left( P_{t,j}^L \right)^{1-\theta} + (1 - \gamma_{t,j}) \left( P_{t,j}^H \right)^{1-\theta} \right] \right]^{\frac{1}{1-\theta}} ,
\]
- the cash-in-advance constraint
\[
M_t = P_t c_t ,
\]
- a labour-market condition
\[
\frac{W_t}{P_t} = \psi c_t^\sigma ,
\]
- equation for the risk-free rate
\[
R_t^{-1} = \beta E_t \left( \frac{c_{t+1}^{-\sigma} P_t}{c_t^{-\sigma} P_{t+1}} \right) ,
\]
- and an exogenous AR(1) process (in logs) for money growth \(\mu_t = \frac{M_t}{M_{t-1}}\):
\[
\ln \mu_{t+1} = (1 - \rho_\mu) \ln \mu + \rho_\mu \ln \mu_t + \varepsilon_{\mu t} ,
\]
where i.i.d. innovations \(\varepsilon_{\mu t}\) drawn from \(N(0, \sigma^2_\mu)\).

**Appendix B. Fixed search cost distributions**

We utilize a flexible two-parameter fixed cost cdf specification, borrowed from Dotsey, King and Wolman (1999) who use it in their state-dependent pricing model:
\[ \Xi'(\alpha) = z_{\text{max}} \frac{\arctan\left(\frac{\alpha - c_1}{c_2}\right) - \arctan\left(\frac{-c_1}{c_2}\right)}{\arctan\left(\frac{1-c_1}{c_2}\right) - \arctan\left(\frac{-c_1}{c_2}\right)}, \]

where \( \Xi'(0) = 0 \) and \( \Xi'(1) = z_{\text{max}} \). This implies

\[ \Xi''(\alpha) = \frac{1}{c_2 c_3} \frac{1}{1 + \left(\frac{\alpha - c_1}{c_2}\right)^2}, \]

\[ [\Xi'(\alpha)]^{-1} = G(z) = c_1 + c_2 \tan(c_3 \alpha - c_4), \]

\[ \Xi'''(\alpha) = -\frac{1}{c_2^2 c_3} \frac{2\left(\frac{\alpha - c_1}{c_2}\right)}{\left[1 + \left(\frac{\alpha - c_1}{c_2}\right)^2\right]^2}, \]

where

\[ c_3 = \frac{1}{z_{\text{max}}} \left(\arctan\left(\frac{1-c_1}{c_2}\right) - \arctan\left(\frac{-c_1}{c_2}\right)\right), \]

\[ c_4 = -\arctan\left(\frac{-c_1}{c_2}\right). \]

In the model with inelastic search, we used the following parameter values: \( c_1 = 0.119, c_2 = 6 \cdot 10^{-4} \) and \( z_{\text{max}} = 1 \).