Post-Crisis Slow Recovery and Monetary Policy*

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Abstract

In the aftermath of the recent financial crisis and subsequent recession, slow recoveries have been observed and slowdowns in total factor productivity (TFP) growth have been measured in many economies. This paper develops a model that is able to describe a slow recovery resulting from an adverse financial shock in the presence of endogenous TFP growth, and examines how monetary policy should react to the financial shock. The paper shows that in the face of the financial shock, a welfare-maximizing monetary policy rule features a strong response to output. Moreover, compared with this rule, a strict inflation or price-level targeting rule induces a sizable welfare loss as it has no response to output, whereas a nominal GDP growth or level targeting rule performs well because the size of the policy response to inflation plays a minor role for welfare in reacting to the financial shock. To obtain these results, it is crucial to take into account a welfare loss from a permanent decline in the level of consumption caused by a slowdown in TFP growth, since monetary policy has an influence on TFP in the model.

Keywords: Financial shock; Endogenous TFP growth; Slow recovery; Monetary policy; Social welfare measure

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1 Introduction

In the aftermath of the recent financial crisis and subsequent recession, slow recoveries have been observed in many economies. GDP has not recovered to its pre-crisis growth trend in the U.S., while in the Euro area and the U.K., GDP has continued to underperform even its pre-crisis level. As indicated by recent studies, such as Cerra and Saxena (2008) and Reinhart and Rogoff (2009), financial crises tend to be followed by slow recoveries in which GDP scarcely returns to its pre-crisis growth trend, inducing a considerable loss in GDP. Indeed, since its financial crisis in the 1990s, Japan’s GDP has never recovered to its pre-crisis growth trend and has experienced a massive loss. As a main source of this prolonged stagnation in Japan, Hayashi and Prescott (2002) argue a post-crisis slowdown in total factor productivity (TFP) growth. For the recent slow recoveries, TFP growth slowdowns have also been measured, particularly in Europe. To prevent or mitigate post-crisis slow recoveries, how should monetary policy be conducted in terms of social welfare? Should monetary policy make no response to output, as advocated in previous studies, including Schmitt-Grohé and Uribe (2006, 2007)?

The present paper addresses these policy questions. Specifically, we develop a model that is able to describe a slow recovery resulting from an adverse financial shock, and examine how monetary policy should react to the financial shock. The model introduces a financial friction and endogenous TFP growth in an otherwise standard dynamic stochastic general equilibrium (DSGE) model. TFP grows endogenously by increasing the variety of intermediate goods as in Comin and Gertler (2006), who extend the frame-

1 Schmitt-Grohé and Uribe (2006, 2007) demonstrate that welfare-maximizing monetary policy features a muted response to output in a dynamic stochastic general equilibrium model (without financial frictions or endogenous TFP growth).

2 Queraltó (2013) builds a small open-economy real business cycle (RBC) model with the Gertler and Karadi (2011) financial friction and the Comin and Gertler (2006) endogenous TFP growth mechanism to describe post-crisis slow recoveries observed in emerging market economies. Guerron-Quintana and Jinmai (2014) use U.S. time series, including their measured intangible capital, to estimate (mainly shocks in) an RBC model with the Kiyotaki and Moore (2012) financial friction and the Kung and Schmid (2013) endogenous TFP growth mechanism, and show that around the time of Lehman Brothers’ demise, liquidity declined significantly, inducing the U.S. Great Recession.
work of endogenous technological change developed by Romer (1990). The financial 
friction then constrains intermediate-good firms’ borrowing capacity as in Jermann and 
Quadrini (2012). Thus, an adverse shock to the borrowing capacity—which is referred to 
as an adverse “financial shock” following Jermann and Quadrini—induces a slow recovery 
through the mechanism of endogenous TFP growth.

This paper analyzes a class of simple monetary policy rules that adjust the policy 
rate in response to its past rate and the contemporaneous rates of inflation and output 
growth. The paper shows that in the face of the adverse financial shock, a welfare-
maximizing policy rule features a strong response to output. Moreover, compared with 
this rule, a strict inflation or price-level targeting rule induces a sizable welfare loss as it 
has no response to output. By contrast, a nominal GDP growth or level targeting rule 
performs well. This is because the size of the policy response to inflation plays a minor 
role for welfare in reacting to the financial shock. To obtain these results, it is crucial 
to take into account a welfare loss from a permanent decline in the level of consumption 
caused by a slowdown in TFP growth. This is in stark contrast with previous monetary 
policy studies, which employ a model with exogenous TFP growth in policy evaluation. 
In our model, TFP growth is endogenous and monetary policy has an influence on TFP 
and as a consequence, not only the variability of consumption but also the level of 
consumption changes with policy choices and thus constitutes social welfare relevant to 
policy evaluation. Therefore, the level of consumption attained under each policy is a 
crucially important factor in evaluating alternative policies.

The paper also conducts a financial crisis scenario simulation under the monetary 
policy rules mentioned above. In this simulation, a slowdown in TFP growth is much 
less pronounced under the welfare-maximizing monetary policy rule than under the strict 
price-level targeting rule. As a consequence, output recovers to its pre-crisis growth trend 
faster under the welfare-maximizing rule and thus the welfare gain from adopting this 
rule relative to the price-level targeting rule is sizable, as noted above. Under the nominal
GDP level targeting rule, the levels of TFP and output initially overshoot those under the welfare-maximizing rule, but then these levels under the two rules approach each other, implying that the welfare gain from adopting the welfare-maximizing rule relative to the nominal GDP targeting rule is small, as indicated above. However, it is surprising that even in the financial crisis scenario simulation, the initial overshoot induces an initial hike in the interest rate under the nominal GDP targeting rule.

A closely related and complementary study has been done by Reifschneider, Wascher, and Wilcox (2013). These authors conduct optimal-control exercises using a version of the FRB/US model with an ad hoc loss function that reflects the Fed’s dual mandate. They argue plausibly that a significant portion of the recent damage to the supply side of the U.S. economy is endogenous to the weakness in aggregate demand and that such endogeneity provides a strong motivation for a vigorous policy response to a weakening in aggregate demand. This argument has also been demonstrated in our paper, which investigates welfare-maximizing monetary policy using the fully fledged DSGE model augmented with the Jermann and Quadrini (2012) financial friction and shock and the Comin and Gertler (2006) endogenous TFP growth mechanism.

The remainder of the paper proceeds as follows. Section 2 briefly reviews recent post-crisis recoveries. Section 3 presents a DSGE model with a financial friction and endogenous TFP growth. Section 4 confirms that this model is able to describe a slow recovery resulting from an adverse financial shock. Section 5 conducts monetary policy analysis using the model. Section 6 concludes.

2 A Brief Review of Post-Crisis Recoveries

This section briefly reviews the economic developments around recent financial crises to show key features of post-crisis recoveries.3 The crises focused on here are the 2007–08

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3For comprehensive studies on recoveries after financial crises that include not only the crises reviewed in this paper but also others around the globe in other times, see Cerra and Saxena (2008), Reinhart and Rogoff (2009), and Reinhart and Reinhart (2010).
crises in the Euro area, the U.K., and the U.S. and the 1997 crisis in Japan.\textsuperscript{4}

For these financial crises, Fig. 1 plots the developments of four key variables: real GDP per capita, TFP (Solow residual), bank lending, and the CPI inflation rate. In each panel of this figure, a key feature of post-crisis recoveries is detected. First, and most importantly, the post-crisis recoveries were quite slow, as shown in Panel (a). Since the onset of the recent crises, GDP has not recovered to the pre-crisis growth trend in the U.S., while it has continued to underperform even its pre-crisis level in the Euro area and the U.K. Japan’s GDP has never recovered to the pre-crisis growth trend since the 1997 crisis and has experienced a massive loss. This confirms the empirical evidence of Cerra and Saxena (2008), Reinhart and Rogoff (2009), and Reinhart and Reinhart (2010): financial crises tend to be followed by slow recoveries in which economic activity scarcely returns to its pre-crisis growth trend, inducing a considerable loss in GDP.

As a main source of Japan’s prolonged stagnation, Hayashi and Prescott (2002) argue a post-crisis slowdown in TFP growth. For the recent post-crisis slow recoveries, slowdowns in TFP growth have also been measured, particularly in Europe, as can be seen in Panel (b). This is the second key feature of post-crisis recoveries.

The third key feature is that a shrink in financial intermediation was observed during and after the financial crises, as shown in Panel (c). Bank lending in the Euro area, the U.K., and the U.S. all dropped sharply in 2009. Japan’s bank lending was already stagnant due to non-performing-loan problems after the collapse of asset price bubbles in the early 1990s, and dropped further in 1999.

Last, the inflation rate was less stable after the financial crises, as shown in Panel (d). In the Euro area, the U.K., and the U.S., the inflation rate measured by CPI dropped after the 2007–08 crises and then continued to fluctuate.\textsuperscript{5} In Japan, the CPI inflation

\textsuperscript{4}In 1997, Yamaichi Securities—one of the top four securities companies in Japan at that time—failed and Hokkaido Takushoku Bank failed, which was the first failure of a city bank in the nation’s postwar history.

\textsuperscript{5}Although the sharp drop in the CPI inflation rate in the Euro area and the U.S. partly reflected a decline in energy prices, the inflation rate measured by CPI excluding energy decreased as well.
rate was already low after the collapse of asset price bubbles in the early 1990s, and dropped further after the 1997 crisis, falling into deflation.

Based on these features of post-crisis recoveries, the next section develops a model that is able to describe a slow recovery resulting from an adverse financial shock.

3 A DSGE Model for Slow Recoveries

To describe a post-crisis slow recovery like those reported in the preceding section, this paper introduces a financial friction and endogenous TFP growth in an otherwise standard DSGE model. TFP grows endogenously by expanding the variety of intermediate goods as in Comin and Gertler (2006). The financial friction then constrains borrowing capacity of intermediate-good firms as in Jermann and Quadrini (2012). The combination of the financial friction and endogenous TFP growth thus generates a powerful amplification mechanism of a shock to the borrowing capacity, which is called a “financial shock” as in Jermann and Quadrini. This financial shock affects activity of intermediate-good firms and their values, which in turn has a large impact on the economy as a whole by influencing activity not only on the demand side, such as final-good firms and households, but also on the supply side, such as technology adopters and innovators. In particular, the effect on that supply side induces a permanent change in output relative to a balanced growth path through a permanent change in TFP. The possibility of permanent deviations of output and other real variables from a balanced growth path distinguishes our model from those used in the existing literature on monetary policy. This distinguished feature of our model yields a novel implication for policy evaluation based on social welfare.

In the model there are five types of economic agents: firms, technology adopters,

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6 Apart from the financial friction and endogenous TFP growth, our model is quite standard. The model abstracts some typical building blocks of DSGE models, such as consumption habit formation and capital utilization. This is because the model is kept relatively simple to focus on a new mechanism generated by the financial friction and endogenous TFP growth in monetary policy analysis.
innovators, households, and a central bank. Furthermore, firms consist of final-good firms, intermediate-good firms, consumption-good firms, and investment-good firms.\textsuperscript{7} The behavior of each economic agent is described in turn.

\subsection*{3.1 Firms}

Intermediate-good firms play a central role in the model. They engage in various types of activity: production, hiring, capital accumulation, borrowing, dividend payment, and purchase of newly adopted ideas. Final-good firms combine intermediate goods to produce final goods. Consumption-good firms transform final goods into consumption goods and sell them to intermediate-good firms, investment-good firms, technology adopters, innovators, and households. Investment-good firms transform consumption goods into investment goods subject to an adjustment cost and sell them to intermediate-good firms. All the firms are owned by households.

\subsection*{3.1.1 Final-good firms}

There is a representative final-good firm. Under perfect competition, this firm produces final goods $x_t$ by combining intermediate goods $x_{j,t}, j \in [0, A_{t-1}]$ to maximize profit $P_t^x x_t - \int_0^{A_{t-1}} P_{j,t}^x x_{j,t}^1 dj$ subject to the CES production function $x_t = (\int_0^{A_{t-1}} x_{j,t}^{1/\theta} dj)^{\theta}$ with $\theta > 1$, given the final-good price $P_t^x$ and intermediate good $j$’s price $P_{j,t}^x$. The first-order condition for profit maximization yields final-good firms’ demand curve for intermediate good $j$

$$x_{j,t} = x_t \left( \frac{P_{j,t}^x}{P_t^x} \right)^{\frac{\theta}{1-\theta}}.$$ \hspace{1cm} (1)

Perfect competition in the final-good market leads to

$$P_t^x = \left[ \int_0^{A_{t-1}} \left( P_{j,t}^x \right)^{\frac{1}{1-\theta}} dj \right]^{1-\theta}.$$ \hspace{1cm} (2)

\textsuperscript{7}Consumption-good firms and investment-good firms are modeled to introduce sticky prices and an investment adjustment cost, respectively.
3.1.2 Intermediate-good firms

There is a continuum of intermediate-good firms. The symmetry among these firms ensures the presence of a representative intermediate-good firm. This firm owns capital \( k_{t-1} \) and a continuum of adopted ideas \( j \in [0, A_{t-1}] \). For each adopted idea \( j \), the intermediate-good firm uses capital \( k_{j,t-1} \) and labor \( n_{j,t} \) to produce intermediate goods \( x_{j,t} \) according to the Cobb-Douglas production function \( x_{j,t} = k_{j,t-1}^\alpha n_{j,t}^{1-\alpha} \) with the capital elasticity of output \( \alpha \in (0, 1) \). The symmetry among adopted ideas implies an identical capital-labor ratio and an identical price for each intermediate good. As a consequence, aggregating the Cobb-Douglas production function over adopted ideas—along with final-good firms’ demand curve (1) (i.e., \( x_{j,t} = x_t A_{t-1}^{\theta} \)) and the final-good price equation (2) (i.e., \( P_t^x = P_j A_{t-1}^{1-\theta} \))—yields

\[
k_{t-1}^\alpha n_t^{1-\alpha} = \int_0^{A_{t-1}} x_{j,t} dj = x_t A_{t-1}^{1-\theta}.
\]  

(3)

where \( k_{t-1} = \int_0^{A_{t-1}} k_{j,t-1} dj \) and \( n_t = \int_0^{A_{t-1}} n_{j,t} dj \).

The intermediate-good firm accumulates capital \( k_t \) and adopted ideas \( A_t \) according to

\[
k_t = (1 - \delta_k) k_{t-1} + \Delta_{k,t},
\]  

(4)

\[
A_t = (1 - \delta_A) A_{t-1} + \Delta_{A,t},
\]  

(5)

where \( \Delta_{k,t} \) is the amount of investment goods purchased from investment-good firms, \( \Delta_{A,t} \) is the number of newly adopted ideas purchased from technology adopters, \( \delta_k \in (0, 1) \) is the depreciation rate of capital, and \( \delta_A \in (0, 1) \) is the obsolescence rate of adopted ideas.

Following Jermann and Quadrini (2012), the intermediate-good firm uses debt and equity. Debt is preferred to equity because of its tax advantage. Given the gross risk-free interest rate \( R_t \), the effective gross interest rate for the firm is \( R_t^e = 1 + (1-\tau)(R_t - 1) \), where \( \tau \) represents the tax benefit. This tax benefit is assumed to be financed by a lump-sum tax on households. The firm starts the period with intertemporal debt \( B_{t-1} \). It is
assumed that the firm must pay for labor $n_t$ and investment $\Delta k_t$ before its production takes place. To finance this payment, the firm raises funds with an intratemporal loan

$$P_t l_t = W_t n_t + Q_t \Delta k_t, \quad (6)$$

where $P_t$ is the price of consumption goods, $W_t$ is the wage, and $Q_t$ is the price of investment goods. The intratemporal loan is repaid with no interest at the end of the period. The capacity of the intratemporal loan $P_t l_t$ and intertemporal debt $B_t$ is constrained by the value of capital held by the firm due to a lack of enforcement. In particular, the firm can default on its debt (both $P_t l_t$ and $B_t$) before the payment for the intratemporal loan is made at the end of the period. In case of default, lenders can foreclose on the capital held by the firm with probability $\xi_t \in (0, 1)$. Then, following Jermann and Quadrini (2012), intratemporal loan $P_t l_t$ is limited by the borrowing constraint

$$P_t l_t \leq \xi_t \left( Q_t k_t - \frac{B_t}{R_t} \right). \quad (7)$$

Here, it is assumed that this borrowing constraint is always binding and that the log-deviation of the foreclosure probability $\xi_t$ from its steady-state value $\xi$ follows the stationary first-order autoregressive process

$$\log \frac{\xi_t}{\xi} = \rho_\xi \log \frac{\xi_{t-1}}{\xi} + \epsilon_{\xi,t}, \quad (8)$$

where $0 \leq \rho_\xi < 1$ and $\epsilon_{\xi,t}$ is white noise and is called a “financial shock.”

Compared with Jermann and Quadrini (2012), two differences in the intratemporal loan equation (6) and the borrowing constraint (7) are worth noting. First, the investment-good price $Q_t$ appears in (7) because the relative price of investment goods can differ from unity due to the presence of an investment adjustment cost explained later. Second, the payment financed by the intratemporal loan accounts for part of the total payment made in the period, while Jermann and Quadrini (2012) assume that total payment must be financed by an intratemporal loan. We choose our formulation for the
intratemporal loan because it can generate much more plausible impulse responses to a monetary policy shock.⁸

After the intratemporal loan arrangement is made, the intermediate-good firm produces and sells products and then pays back the intratemporal loan. Moreover, the firm renews intertemporal debt and pays dividend \( d_t \) to households. Let \( V_t \) denote the price of newly adopted ideas and \( \varphi(d_t) \) be the dividend payment plus associated costs in terms of consumption goods, given by \( \varphi(d_t) = d_t + (\kappa/A_t^*)(d_t - d^* A_t^*)^2 \), where \( \kappa \) is a positive constant, \( A_t^* = A_{t-1}^{(\theta-1)/(1-\alpha)} \), and \( d^* \) is the steady-state value of detrended dividend \( d_t^* = d_t/A_t^* \). The firm’s budget constraint can then be written as

\[
W_t n_t + Q_t \Delta k_t + V_t \Delta A_t + P_t \varphi(d_t) + B_{t-1} = \int_0^{A_{t-1}} P_j^x x_{j,t} dj + \frac{B_t}{R_t^x} = P_t^x x_t + \frac{B_t}{R_t} \quad (9)
\]

where the second equality follows from the zero-profit condition of final-good firms. The presence of the costs of the dividend payment introduces a rigidity that affects the substitution between debt and equity. The presence of \( A_t^* \) in the costs ensures a balanced growth path in the model.

The intermediate-good firm chooses dividend payment \( d_t \), intertemporal debt \( B_t \), labor \( n_t \), capital \( k_t \), and adopted ideas \( A_t \) to maximize the expected discounted value of the present and future dividend payments

\[
E_0 \sum_{t=0}^{\infty} m_{0,t} d_t \quad (10)
\]

subject to (3)–(7) and (9), where \( m_{0,t} \) is the real stochastic discount factor between period 0 and period \( t \). The first-order conditions for this maximization problem are presented in Appendix A. Here, to understand an amplification mechanism embedded

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⁸If the formulation of Jermann and Quadrini (2012) is used instead, the inflation rate rises strongly in response to a contractionary monetary policy shock. This rise is mainly due to an increase in agency costs that appear in an aggregate supply equation. Although this feature might explain the so-called price puzzle, we see the formulation of Jermann and Quadrini (2012) as unsuitable for monetary policy analysis, since the rise in inflation is too large and too sharp to be empirically plausible.
in the model, we present the two key equilibrium conditions

\[ w_t = \frac{(1 - \alpha)s_t (k_{t-1}/n_t)^\alpha}{1/\varphi_t' + \mu_t}, \quad (11) \]

\[ 1 = E_t \left[ \frac{m_{t,t+1} (\theta - 1)s_{t+1}n_{t+1} (k_{t+1}/n_{t+1})^\alpha/A_t + (1 - \delta_A)v_{t+1}/\varphi_t'}{v_t/\varphi_t'} \right], \quad (12) \]

where \( w_t = W_t/P_t \) is the real wage, \( v_t = V_t/P_t \) is the relative price of newly adopted ideas, \( \varphi_t' = \varphi'(d_t) \), \( m_{t,t+1} = m_{0,t+1}/m_{0,t} \), and \( s_t, \mu_t \) are the Lagrange multipliers on the aggregate intermediate-good production function (3) and the borrowing constraint (7).

These two conditions represent the demand curves for labor and newly adopted ideas. If the borrowing constraint tightens, the associated Lagrange multiplier \( \mu_t \) increases and the demand curve for labor shifts inward. The tight borrowing constraint makes it hard for intermediate-good firms to finance the intratemporal loan for hiring labor and thus dampens their demand for labor. This effect on labor through the multiplier \( \mu_t \) in the labor demand curve (11) mitigates the co-movement problem between consumption and labor indicated first by Barro and King (1984), without depending too much on nominal rigidities. A drop in labor decreases the profit from production and the value of newly adopted ideas, \( v_t \). The effect on the value of newly adopted ideas in the demand curve (12) serves as an important amplification mechanism of endogenous TFP. In particular, an increase in \( \mu_t \)—a tightening of the borrowing constraint—causes a decrease in the value of newly adopted ideas through a drop in labor, which in turn makes technology adopters less willing to adopt developed but not yet adopted ideas.\(^9\) Besides, the value of such ideas drops, which makes innovators less willing to develop new ideas.

### 3.1.3 Consumption-good firms

Consumption-good firms consist of wholesalers and retailers. There are a continuum of wholesalers \( i \in [0, 1] \) and a representative retailer. Under perfect competition, this

\(^9\)Another important channel through which a tightening of the borrowing constraint decreases the value of newly adopted ideas in the demand curve (12) is an increase in the marginal value of funds that is represented by an increase in the Lagrange multiplier on the firm’s budget constraint (9), \( \zeta_t = 1/\varphi_t' \).
retailer produces consumption goods $y_t$ by combining wholesale goods $y_{i,t}$ to maximize profit $P_t y_t - \int_0^1 P_{i,t} y_{i,t} di$ subject to the CES production function $y_t = \left( \int_0^1 y_{i,t}^{1/\lambda_p} \right)^{\lambda_p}$ with $\lambda_p > 1$, given the consumption-good price $P_t$ and wholesale good $i$’s price $P_{i,t}$. The first-order condition for profit maximization yields retailers’ demand curve for wholesale good $i$

$$ y_{i,t} = y_t \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_p}{1 - \lambda_p}}. \quad (13) $$

Perfect competition in the consumption-good market leads to $P_t = \left( \int_0^1 P_{i,t}^{1/(1 - \lambda_p)} \right)^{1 - \lambda_p}$.

Under monopolistic competition, each wholesaler $i$ transforms one unit of final goods into one unit of differentiated wholesale good $i$. Hence, the marginal cost of producing each wholesale good equals the final-good price $P^x_t$. In the face of retailers’ demand curve (13) and the marginal cost $P^x_t$, wholesalers set prices of their products on a staggered basis as in Calvo (1983) and Yun (1996). Each period a fraction $\xi_p \in (0, 1)$ of wholesalers sets prices according to the indexation rule $P_{i,t} = \pi P_{i,t-1}$, where $\pi$ is the gross steady-state inflation rate of the consumption-good price, while the remaining fraction $1 - \xi_p$ chooses the price $\tilde{P}_t$ that maximizes the associated profit

$$ E_t \sum_{h=0}^{\infty} \xi^h P_{i,t+h} \left( \pi^h \tilde{P}_t - P^x_{i,t+h} \right) y_{i,t+h} \left( \frac{\pi^h \tilde{P}_t}{P_{t+h}} \right)^{\frac{\lambda_p}{1 - \lambda_p}}, $$

where $M_{i,t+h}$ is the nominal stochastic discount factor between period $t$ and period $t+h$.

The final-good market clearing condition, along with the aggregate intermediate-good production function (3), leads to

$$ \int_0^1 y_{i,t} di = x_t = A^{\theta-1}_{t-1} k^\alpha_{t-1} n^{1-\alpha}_t. \quad (14) $$

Substituting retailers’ demand curve (13) in this equation leads to

$$ y_t = \left( A^{\theta-1}_{t-1} / \Delta_{p,t} \right) k^\alpha_{t-1} n^{1-\alpha}_t, \quad (15) $$

where $\Delta_{p,t} = \int_0^1 \left( P_{i,t} / P_t \right)^{\lambda_p/(1 - \lambda_p)} \, di$ denotes (wholesale-good) price distortion due to the staggered price-setting of wholesalers. This equation shows a standard Cobb-Douglas production function, except that TFP is endogenously determined and given by $(A^{\theta-1}_{t-1} / \Delta_{p,t})$. 

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3.1.4 Investment-good firms

There is a representative investment-good firm, which transforms consumption goods \( i_t \) into investment goods \( \Delta_k,t \) subject to an adjustment cost of the form advocated by Christiano, Eichenbaum, and Evans (2005). This firm’s production function is thus given by

\[
\Delta_k,t = i_t \left[ 1 - S \left( \frac{i_t}{i_{t-1} \gamma_A^{*} - 1} \right)^2 \right],
\]

(16)

where \( S \) is a positive constant and \( \gamma_A^* \) is the gross steady-state rate of balanced growth. The presence of \( \gamma_A^* \) in this equation ensures that the adjustment cost is zero in the steady state. The investment-good firm chooses input \( i_t \) to maximize profit

\[
E_0 \sum_{t=0}^{\infty} m_{0,t} (q_t \Delta_k,t - i_t)
\]

subject to the production function (16), where \( q_t \) is the relative price of investment goods.

3.2 Technology adopters

We turn next to technology adopters. There is a representative technology adopter. This adopter makes an investment \( i_{a,t} \) for technology adoption in terms of consumption goods before a fraction \( \delta_A \) of already adopted ideas \( A_{t-1} \) becomes obsolete. After the obsolescence of the adopted ideas, each developed but not yet adopted idea, which is in the interval between \( A_{t-1}(1 - \delta_A) \) and \( Z_{t-1}(1 - \delta_A) \), is successfully adopted with probability \( \lambda_t \). Thus, the law of motion of adopted ideas is given by

\[
A_t = (1 - \delta_A) [A_{t-1} + \lambda_t (Z_{t-1} - A_{t-1})].
\]

(17)

As in Comin and Gertler (2006), the probability \( \lambda_t \) increases with investment \( i_{a,t} \) such that

\[
\lambda_t = \lambda_0 \left( \frac{A_{t-1}}{A_t} i_{a,t} \right)^\omega,
\]

where \( \lambda_0 \) is a constant and \( 0 < \omega < 1 \). This formulation of the probability \( \lambda_t \) assumes a spillover effect from already adopted ideas \( A_{t-1} \) to individual adoption. The presence of
the scaling factor $A_t^* = A_t^{(\theta-1)/(1-\alpha)}$ keeps the adoption rate constant along a balanced growth path. Overall, the spillover effect is positive as long as $\alpha + \theta < 2$, which holds in our parameterization of the model presented later.

The technology adopter chooses investment $i_{a,t}$ to maximize the value of the idea, $j_t$. The idea, if successfully adopted, is sold to intermediate-good firms at the relative price $v_t$. If it is not successfully adopted, the value of the idea is given by its expected future value $E_t m_{t,t+1} j_{t+1}$. Thus, the present value of the idea is given by

$$j_t = \max_{i_{a,t}} \{-i_{a,t} + (1 - \delta_A) [\lambda_t v_t + (1 - \lambda_t) E_t m_{t,t+1} j_{t+1}]\}. \tag{18}$$

The first-order condition for investment $i_{a,t}$ yields

$$1 = (1 - \delta_A) \lambda_0 \omega \left( \frac{A_{t-1}}{A_t^*} i_{a,t} \right)^{\omega-1} \frac{A_{t-1}}{A_t^*} (v_t - E_t m_{t,t+1} j_{t+1}). \tag{19}$$

The formulation of the probability $\lambda_t$ ensures that investment $i_{a,t}$ increases with the relative price $v_t$. Then, if the value of adopted ideas, $v_t$, declines, investment $i_{a,t}$ decreases as well, which in turn slows the rate of technology adoption. As a consequence, the growth rates of $A_t$ and TFP slow as is clear from the law of motion of adopted ideas (17).

### 3.3 Innovators

There is a representative innovator with the linear R&D technology that transforms one unit of consumption goods into $z_{z,t}$ units of developed ideas. Given the obsolescence rate $\delta_A$, the frontier of developed ideas, $Z_t$, follows the law of motion

$$Z_t = (1 - \delta_A) Z_{t-1} + z_{z,t} i_{d,t}, \tag{20}$$

where $i_{d,t}$ is R&D investment. As in Comin and Gertler (2006), it is assumed that R&D productivity $z_{z,t}$ depends on aggregate variables that are taken as given by innovators. Specifically, it is assumed

$$z_{z,t} = \chi \frac{Z_{t-1}}{(A_t^*)^\rho i_{d,t}^{1-\rho}},$$
where \( \chi \) is a positive constant and \( 0 < \rho < 1 \). Perfect competition among innovators leads to the zero-profit condition

\[
1 = z_{z,t} \left( 1 - \delta_A \right) E_t m_{t,t+1} j_{t+1}.
\]

(21)

In this equation, the left-hand side corresponds to the real cost of one unit of consumption goods, while the right-hand side corresponds to the expected value of ideas developed using one unit of R&D investment. From the formulation of the R&D productivity \( z_{z,t} \), the zero-profit condition (21) implies that R&D investment \( i_{d,t} \) increases with the value of the developed idea, \( j_{t+1} \). This positive relationship between \( i_{d,t} \) and \( j_{t+1} \) serves as another amplification mechanism of endogenous TFP. As shown in the problem of technology adopters, a decline in the value of adopted ideas, \( v_t \), slows the adoption rate \( \lambda_t \) and reduces the value \( j_t \) in (18). A decline in \( j_t \) makes innovators less willing to develop new ideas and slows the growth rate of \( Z_t \) in (20). A slow growth rate of \( Z_t \) in turn reduces the growth rate of \( A_t \), further decreasing TFP growth.

### 3.4 Households

Households are standard as in the literature on DSGE models. There is a continuum of households with measure unity, each of which is endowed with one type of specialized labor \( j \in [0, 1] \). Households have a monopolistic power over wages for specialized labor and the wages are set in a staggered manner as in Erceg, Henderson, and Levin (2000).\(^{10}\) A representative employment agency transforms specialized labor into homogeneous labor and provides the latter labor to intermediate-good firms.

The problem of households consists of three parts: a consumption–saving problem, the employment agency’s problem, and a wage-setting problem. In the consumption–saving problem, each household chooses consumption \( c_t \) and saving \( B_t \) to maximize the

\(^{10}\)This sticky wage is an important factor to describe a slow recovery from the financial shock in the model, as shown later.
utility function

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \psi \frac{n_{jt}^{1+1/\nu}}{1+1/\nu} \right) \]  
(22)

subject to the budget constraint

\[ P_t c_t + \frac{B_t}{R_t} = W_{jt} n_{jt} + B_{t-1} + T_t + \theta_{jt}, \]  
(23)

where \( \beta \in (0,1) \) is the subjective discount factor, \( \nu > 0 \) is the elasticity of labor supply, \( \psi > 0 \) is the coefficient on labor disutility, \( W_{jt}, n_{jt} \) are the wage and the supply of specialized labor \( j \), \( T_t \) is the sum of dividend \( P_t d_t \), firms’ profit, and the lump-sum tax imposed by the government, and \( \theta_{jt} \) is the net cash flow arising from a contingent claim on the opportunity of wage changes. The presence of the contingent claim allows the model to keep a representative-household framework.

The employment agency combines all types of specialized labor \( n_{jt} \) to produce homogeneous labor \( n_t \) using the CES aggregation technology \( n_t = (\int_0^1 n_{jt}^{1/\lambda_w} dj)^{\lambda_w} \) with \( \lambda_w > 1 \). Given the wage of homogeneous labor, \( W_t \), and the wage of each type of specialized labor, \( W_{jt} \), the employment agency chooses the amount of each type of specialized labor, \( n_{jt} \), to maximize profit \( W_t n_t - \int_0^1 W_{jt} n_{jt} dj \) subject to the CES aggregation technology. The first-order condition yields the demand curve for each type of specialized labor

\[ n_{jt} = n_t \left( \frac{W_{jt}}{W_t} \right)^{\frac{1}{1-\lambda_w}}, \]  
(24)

where the corresponding aggregate wage is given by \( W_t = (\int_0^1 W_{jt}^{1/(1-\lambda_w)} dj)^{1-\lambda_w}. \)

The wage of each type of specialized labor is set on a Calvo-style staggered basis. Each period a fraction \( \xi_w \in (0,1) \) of wages is set according to the indexation rule \( W_{jt} = \pi_w W_{jt-1} \), where \( \pi_w = \pi^*_A \) is the gross steady-state wage inflation rate, while the remaining fraction \( 1 - \xi_w \) is set at the wage \( \tilde{W}_t \) that maximizes

\[ E_t \sum_{h=0}^{\infty} (\beta \xi_w)^h \left\{ \Lambda_{t+h} \left( \pi^*_w \tilde{W}_t \right) \left[ n_{t+h} \left( \pi^*_w \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} \right] - \psi \frac{n_{t+h}}{1+1/\nu} \left[ n_{t+h} \left( \pi^*_w \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} \right]^{1+1/\nu} \right\}, \]

where \( \Lambda_t \) is the Lagrange multiplier on the budget constraint (23).
The consumption-good market clearing condition is now given by

\[ y_t = c_t + i_t + i_{a,t} (Z_t - A_t) + i_{d,t} + (\varphi (d_t) - d_t). \]

The output \( y_t \) equals households’ consumption \( c_t \), investment-good firms’ capital investment \( i_t \), technology adopters’ investment \( i_{a,t} (Z_t - A_t) \), innovators’ R&D investment \( i_{d,t} \), and intermediate-good firms’ costs of the dividend payment \( \varphi(d_t) - d_t \).

### 3.5 The central bank

The central bank follows the Taylor (1993)-type rule that adjusts the policy rate in response to its past rate and the contemporaneous rates of consumption-good price inflation and output growth

\[ \log \left( \frac{R_t}{R} \right) = \rho_R \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[ \phi_\pi \log \left( \frac{\pi_t}{\pi} \right) + \phi_{dy} \log \left( \frac{y_t/y_{t-1}}{\gamma^*_A} \right) \right] + \epsilon_{R,t}, \tag{25} \]

where \( R, \pi, \) and \( \gamma^*_A \) are the steady-state rates of policy, inflation, and balanced growth, \( \rho_R \in [0, 1) \) represents the degree of policy rate smoothing, \( \phi_\pi, \phi_{dy} \) are the policy responses to inflation and output growth, and \( \epsilon_{R,t} \) is a monetary policy shock.\(^\text{11}\)

The complete set of equilibrium conditions and the steady state are presented in Appendix B.

### 4 A Slow Recovery from the Financial Shock

This section confirms that the model presented in the preceding section possesses the capability to describe a slow recovery resulting from the financial shock \( \epsilon_{\xi,t} \). To this end, the model is parameterized, linearized around the steady state, and solved for the rational

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\(^{11}\) Any output gap is not included in the monetary policy rules considered in this paper for several reasons. First of all, there are variations in output gaps in terms of definition and measurement. Second and more importantly, in the model, where monetary policy is able to affect TFP, it is not clear which output gap is appropriate in including it in monetary policy rules. For instance, the gap between actual output and potential output that could be obtained in the absence of nominal rigidities seems inappropriate, since inefficiency arises not only from nominal rigidities but also from the endogenous TFP growth mechanism.
expectations equilibrium. Then, impulse responses show how the financial shock induces a slow recovery. Last, two important factors to describe the slow recovery—endogenous TFP growth and sticky wages—are explained.

4.1 Parameterization of the model

We begin by parameterizing the model. The model parameters are divided into three sets. The first set contains parameters that are standard in DSGE models. The second set pertains to the technology adoption and R&D. The values of the parameters in this set are chosen mainly from Comin and Gertler (2006). The third set pertains to the financial friction. For this set, this paper employs the parameter values calibrated or estimated by Jermann and Quadrini (2012). Table 1 lists the parameterization of the quarterly model.

Regarding the parameters that are standard in DSGE models, the present paper sets the subjective discount factor at $\beta = 0.9975$, the elasticity of labor supply at $\nu = 1$, the labor disutility coefficient $\psi$ so that steady-state labor can be normalized to unity, the capital elasticity of output at $\alpha = 0.36$, the depreciation rate of capital at $\delta = 0.025$, the degrees of price and wage stickiness at $\xi_p = \xi_w = 0.75$, the gross price and wage markups at $\lambda_p = \lambda_w = 1.05$, the steady-state gross inflation rate at $\pi = 1.005$ (i.e., an annualized rate of 2 percent), the degree of policy rate smoothing at $\rho_R = 0.6$, and the policy responses to inflation and output growth at $\phi_\pi = 1.5, \phi_{dy} = 0.2$. These parameter values are more or less within the values calibrated or estimated in previous studies with DSGE models.

Next, we turn to the parameters that pertain to the technology adoption and R&D. As in Comin and Gertler (2006), the gross price markup of intermediate goods is set at $\theta = 1.55$, so that the resulting overall gross price markup $\lambda_p \theta$ is approximately 1.6. The coefficient on the adoption probability, $\lambda_0$, is set so that the annualized steady-state adoption probability of 10 percent can be attained. The obsolescence rate of ideas is
set at $\delta_A = 0.025$ (i.e., an annualized rate of 10 percent) following Bilbiie, Ghironi, and Melitz (2012). The elasticity of the adoption probability is set at $\omega = 0.95$. The elasticity of newly adopted ideas is set at $\rho = 0.8$. The coefficient on R&D productivity, $\chi$, is set so that an annualized steady-state balanced growth rate of 2 percent can be attained.$^{12}$

Last, the values of the model parameters that pertain to the financial friction are explained. Following Jermann and Quadrini (2012), the present paper sets the coefficient on the investment adjustment cost at $S = 0.04$, the steady-state probability of foreclosure at $\xi = 0.1634$, the coefficient on the dividend payment cost at $\kappa = 0.146$, the tax benefit at $\tau = 0.35$, and the financial shock persistence at $\rho_\xi = 0.97$.\(^{13}\)

### 4.2 Impulse responses to the financial shock

Using the parameterization presented above, this subsection analyzes impulse responses to the financial shock $\epsilon_{\xi,t}$.\(^{14}\)

Fig. 2 plots impulse responses of output, labor, consumption, total investment (i.e., the sum of capital investment, technology adoption investment, and R&D investment), the (annualized) inflation rate, the (annualized) interest rate, intratemporal loans, and TFP to an adverse financial shock. The solid line, called the “benchmark,” represents the case of the model presented above. In this figure, the magnitude of the shock is set at $\epsilon_{\xi,1} = -0.01$.\(^{15}\) When this shock hits the economy in period 1, the foreclosure probability $\xi_t$ drops by 1 percentage point. This drop tightens the borrowing constraint

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$^{12}$For details and discussions on the values of parameters pertaining to the technology adoption and R&D, see Comin and Gertler (2006), Comin, Gertler, and Santacreu (2009), and Queraltó (2013).

$^{13}$The value of the coefficient on the investment adjustment cost of $S = 0.04$ is smaller than those calibrated or estimated in previous studies with DSGE models, including Christiano, Eichenbaum, and Evans (2005). This is because the financial friction plays the role of an investment adjustment cost in the model.

$^{14}$By analyzing impulse responses to the monetary policy shock $\epsilon_{R,t}$, we confirm that the model possesses standard properties for monetary policy analysis. That is, in response to a contractionary monetary policy shock, the interest rate rises and then output, labor, consumption, and investment all decline. Inflation decreases as well. Overall, these impulse responses are consistent with those in canonical DSGE models.

$^{15}$In the U.S. estimate by Jermann and Quadrini (2012) during the period 1984:Q1–2010:Q2, the standard deviation of the financial shock in their model is about 1 percent.
(7) and makes it difficult for intermediate-good firms to raise funds for their economic activity, thereby reducing real activity of the overall economy. Indeed, as shown in the figure, output, labor, consumption, total investment, and intratemporal loans, as well as inflation, all decline substantially, inducing a recession. In reaction to the declines in inflation and output growth induced by the adverse financial shock, the monetary policy rule (25) lowers the interest rate. More importantly, TFP falls permanently through the endogenous mechanism embedded in the model. The decline in output causes a decrease in the profit of intermediate-good firms and thereby leads to a decline in the value of newly adopted ideas $v_t$ according to intermediate-good firms’ demand curve for the ideas (12). This decline in $v_t$ causes technology adopters to be less willing to adopt developed but not yet adopted ideas and as a consequence, adoption investment $i_{a,t}$ falls in accordance with equation (19). In addition, this fall in $i_{a,t}$ reduces the probability of technology adoption, $\lambda_t$, and thus the value of unadopted ideas, $j_t$, in accordance with equation (18). Furthermore, this reduction in $j_t$ lowers the incentives of innovators to develop new ideas, resulting in a decrease in R&D investment $i_{d,t}$ from equation (21). Due to this endogenous mechanism, TFP drops permanently in response to the adverse financial shock. Consequently, output, consumption, and total investment do not return to the steady-state balanced growth path. In the figure, output drops below the steady-state balanced growth path by about 0.7 percentage point and then recovers less than half of the drop, remaining below the path by 0.4 percentage point even after 40 quarters (10 years). From these observations, we confirm that the model possesses the capability to describe a slow recovery resulting from the financial shock.

4.3 Important factors to describe a slow recovery from the financial shock

Before proceeding to monetary policy analysis, this subsection investigates which factor is important to describe a slow recovery from the financial shock in the model. In Fig. 2,
the dashed line, labeled the “exogenous growth,” represents the case of the model with exogenous TFP growth to understand the role of the endogenous TFP growth mechanism embedded in the model. The dotted line, called the “flexible wage,” represents the model with flexible wages (but endogenous TFP growth) to see the role of sticky wages.

The exogenous TFP growth model removes the mechanism of endogenous TFP growth from the benchmark model by assuming that TFP grows exogenously at the same steady-state rate, as shown in Panel (h) of the figure. There is neither R&D nor technology adoption in the model. In short, the model is a standard DSGE model with the financial friction and shock as in Jermann and Quadrini (2012). In response to the financial shock, output drops below the balanced growth path by about 0.5 percentage point, about two-thirds of the drop in the benchmark model. It then returns to the path, in sharp contrast with the relatively permanent decline in output in the benchmark model. Therefore, the mechanism of endogenous TFP growth is a crucially important factor to describe the slow recovery from the financial shock.

Another important factor to describe the slow recovery is sticky wages. In response to the financial shock, output drops below the steady-state balanced growth path by about 0.6 percentage point and then approaches the path faster than in the benchmark model. Although output does not permanently return to the steady-state balanced growth path due to the presence of the endogenous TFP growth mechanism, the magnitude of the permanent decline in output is much smaller than in the benchmark model. In 40 quarters (10 years), output approaches the balanced growth path much more closely than in the benchmark model. As indicated by Justiniano, Primiceri, and Tambalotti (2010), sticky wages make a wage markup countercyclical, so that they mitigate the co-movement problem between consumption and labor indicated first by Barro and King (1984). In the benchmark model, sticky wages amplify the effect of the financial shock as consumption and labor nearly co-move in response to the financial shock. By contrast, in the model with flexible wages, the adverse financial shock induces an increase in consumption,
featuring the co-movement problem. This increase in consumption dampens the effect on output through the mechanism of endogenous TFP growth and subdues declines in output and TFP caused by the financial shock. Thus, sticky wages are another crucially important factor to describe the slow recovery from the financial shock.

5 Monetary Policy Analysis

This section examines how monetary policy should react to the financial shock. To this end, we begin by deriving a welfare measure from the utility functions of households. With this welfare measure, a welfare-maximizing monetary policy rule is computed and characterized. Last, under this rule and others, a financial crisis scenario simulation is carried out.

5.1 Welfare measure

The welfare measure is the unconditional expectation of the average utility function over households, given by

$$SW = (1 - \beta)E \left[ \int_0^1 \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \psi \frac{n_{jt}^{1+1/\nu}}{1+1/\nu} \right) dj \right],$$

(26)

where $E$ is the unconditional expectation operator and the scaling factor $(1 - \beta)$ is multiplied for normalization. Because TFP grows endogenously over time, a deterministic growth trend is subtracted from this welfare measure $SW$ to make the resulting welfare measure stationary. Letting $SW^*$ denote such a stationary welfare measure, Appendix C shows that this welfare measure can be approximated around the steady state, up to the second order, as

$$SW^* \approx - \left[ \frac{Var(c_t^*)}{(c^*)^2} + \frac{\beta}{1 - \beta} \frac{Var(\gamma_{A,t}^*)}{(\gamma_A^*)^2} + \frac{\psi}{\nu} Var(n_t) + \frac{\psi \lambda_w \left(1 + \frac{\lambda_w(1+1/\nu)}{\lambda_w-1}\right)}{\lambda_w - 1} Var(\Delta_{w,t}) \right]$$

$$+ \frac{\varepsilon_c}{c^*} + \frac{\beta \varepsilon_{\gamma_A}}{1 - \beta \gamma_A^*} - \psi \varepsilon_n + \frac{\lambda_w}{\lambda_w - 1} \varepsilon_{\Delta_w},$$

(27)
where $Var$ denotes the unconditional variance operator, $c^*_t (= c_t/A^*_t)$ is detrended consumption, $c^*$ is its steady-state value, $\gamma^*_{A,t} (= A^*_t/A^*_{t-1})$ is the gross balanced growth rate, $\Delta w_t$ represents wage dispersion, and $\varepsilon_x = E(x_t) - x$ is the “bias” between the unconditional mean and the steady-state value of variable $x_t$. Note that in the second-order approximation the bias can exist, that is, the unconditional mean is not necessarily consistent with the steady-state value.\footnote{In the first-order approximation, the unconditional mean and the steady-state value are identical, so that there is no bias (i.e., $\varepsilon_x = 0$) for any variable $x_t$.} The approximation (27) shows that the stationary welfare measure $SW^*$ is negatively related to the unconditional variances of detrended consumption, the balanced growth rate, labor, and wage dispersion and the bias in labor and is positively related to the bias in the other three. A distinguished feature of the welfare measure (27) lies in the presence of the terms related to the balanced growth rate $\gamma^*_{A,t}$. In standard DSGE models where TFP is exogenous, the unconditional variance and the bias of the balanced growth rate are also exogenous and independent of policy. In our model, however, TFP is endogenous and dependent on policy, so that the $\gamma^*_{A,t}$-related terms constitute social welfare relevant to policy evaluation.

Let $SW^*_b$ and $SW^*_a$ denote the values of the welfare measure $SW^*$ attained under the benchmark monetary policy rule (i.e., the rule (25) with the benchmark parameterization presented in Table 1) and under an alternative monetary policy rule and let $\Delta SW = SW^*_a - SW^*_b$. Then, this difference also equals the corresponding difference in terms of the welfare measure (26), that is, $\Delta SW = SW_a - SW_b$, where $SW_b$ and $SW_a$ denote the values of the welfare measure (26) under the benchmark rule and under the alternative rule, because the subtracted deterministic growth trend is identical between $SW_b$ and $SW_a$. Therefore, the welfare difference $\Delta SW$, if it is positive, represents the welfare gain from adopting the alternative rule relative to the benchmark rule. Moreover, $\eta = 1 - (1 - 2\Delta SW)^{1/2}$ represents the welfare gain in terms of permanent increase in consumption,
because by definition this welfare gain measure \( \eta \) must satisfy

\[
SW_a = (1 - \beta) E \left[ \int_0^1 \sum_{t=0}^{\infty} \beta^t \left( \log((1 + \eta) c_{b,t}) - \psi \frac{n_{b,t}^{1+1/\nu}}{1 + 1/\nu} \right) d\gamma \right],
\]

where \( \{c_{b,t}, \{n_{b,t}\}\} \) is the pair of equilibrium consumption and labor under the benchmark monetary policy rule, and then it follows

\[
SW_b + \Delta SW = SW_a = SW_b + \log(1 + \eta) \approx SW_b + \left( \eta - \frac{1}{2} \eta^2 \right),
\]

where the last approximation uses the second-order approximation to \( \log(1 + \eta) \).

Using the welfare measure (27) and the welfare gain measure \( \eta \), the next subsections analyze a welfare-maximizing monetary policy rule in reaction to the financial shock.

### 5.2 Features of a welfare-maximizing monetary policy rule in reaction to the financial shock

This paper considers a class of simple monetary policy rules that adjust the current policy rate in response to its past rate and the contemporaneous rates of inflation and output growth. Specifically, two forms of such rules are analyzed. One form is, of course, the rule (25). The other is the so-called “first-difference rule,” where the change in the policy rate responds to its past change and the current rates of inflation and output growth\(^\text{17}\)

\[
\log \left( \frac{R_t}{R_{t-1}} \right) = \rho_R \log \left( \frac{R_{t-1}}{R_{t-2}} \right) + (1 - \rho_R) \left[ \phi_\pi \log \left( \frac{\pi_t}{\pi} \right) + \phi_{dy} \log \left( \frac{y_t}{y_t} \right) \right].
\]

Moreover, in the rule of the form (25), the specification of \( \phi_{dy} = 0 \) is called “strict inflation targeting,” while the specification of \( \phi_\pi = \phi_{dy} \) is called “nominal GDP growth targeting.” In the rule of the form (28), the specification of \( \phi_{dy} = 0 \) is called “strict price-level targeting” and the specification of \( \phi_\pi = \phi_{dy} \) is called “nominal GDP level targeting,” because these specifications are implied respectively by such targeting rules.\(^\text{18}\) For each

\(^{17}\)For first-difference rules, see, e.g., Orphanides (2003).

\(^{18}\)For recent discussions on nominal GDP level targeting, see, for example, Woodford (2012) and English, Lopez-Salido, and Tetlow (2013). One point emphasized here is that our specifications of the
specification considered, three requirements on its coefficients are imposed, following Schmitt-Grohé and Uribe (2007). First of all, the coefficients guarantee local determinacy of the rational expectations equilibrium. Second, they satisfy $1 \leq \phi_\pi \leq 10$, $0 \leq \phi_{dy} \leq 10$, and $0 \leq \rho_R \leq 0.99$. Last, they meet the condition on the volatility of the policy rate, $2(Var(R_t))^{1/2} < R$.

In deriving a welfare-maximizing monetary policy rule, this paper focuses on the financial shock only. That is, such a rule is derived under the condition that only the financial shock occurs in the economy. This exclusive focus allows us to characterize a welfare-maximizing monetary policy rule from the perspective of the financial shock, which not only constitutes one of the most important driving forces in U.S. business cycles, as argued by Jermann and Quadrini (2012), but also causes a slow recovery in our model as shown above. Therefore, the financial shock is worth analyzing independently from other shocks. In computing the welfare-maximizing rule, the standard deviation of the financial shock is set at 1 percent, which is close to the shock’s standard deviation Jermann and Quadrini (2012) estimate using U.S. data during the period 1984:Q1–2010:Q2.

For each specification of the monetary policy rules considered, Table 2 shows a welfare-maximizing combination of its coefficients in reaction to the financial shock. In this table, three findings are detected. First, a welfare-maximizing monetary policy rule features a strong response to output. Within the specifications considered, the welfare-maximizing rule is the rule (25) with $\phi_\pi = 1$, $\phi_{dy} = 10$, and $\rho_R = 0.99$.\footnote{This welfare-maximizing rule attains higher welfare—0.03 percentage point permanent increase in consumption—than the welfare-maximizing rule of the form (28), where $\phi_\pi = 2.1$, $\phi_{dy} = 1.9$, and $\rho_R = 0.38$. Because in the former rule the policy response to output growth of $\phi_{dy} = 10$ hits the upper bound on the response considered and the one to inflation of $\phi_\pi = 1$ hits the lower bound, the welfare difference between the two rules could increase if the ranges of the coefficients in the rules widen.} Hence, the welfare-maximizing rule contains a much stronger response to output relative to in-

\footnote{This welfare-maximizing rule attains higher welfare—0.03 percentage point permanent increase in consumption—than the welfare-maximizing rule of the form (28), where $\phi_\pi = 2.1$, $\phi_{dy} = 1.9$, and $\rho_R = 0.38$. Because in the former rule the policy response to output growth of $\phi_{dy} = 10$ hits the upper bound on the response considered and the one to inflation of $\phi_\pi = 1$ hits the lower bound, the welfare difference between the two rules could increase if the ranges of the coefficients in the rules widen.}
The finding that the welfare-maximizing rule calls for a strong response to output contrasts starkly with previous monetary policy studies including Schmitt-Grohé and Uribe (2006, 2007), who argue that welfare-maximizing monetary policy features a muted response to output. This difference between their result and ours lies in the shocks considered in deriving welfare-maximizing policy. Our paper focuses only on the financial shock, whereas Schmitt-Grohé and Uribe (2006, 2007) and others consider mainly a TFP shock and a government spending shock. Indeed, if only TFP shocks are considered in our model, a welfare-maximizing monetary policy rule features a muted response to output as in the previous studies. The table also shows that the welfare gain from adopting the welfare-maximizing rule relative to the benchmark rule (i.e., the rule (25) with $\phi_\pi = 1.5$, $\phi_{dy} = 0.2$, and $\rho_R = 0.6$) is huge. The welfare gain from the welfare-maximizing rule is a permanent increase in consumption of 14.71 percentage points. This gain is more than 30 times larger than the one of 0.48 percentage points in the model with exogenous TFP growth. Moreover, the table demonstrates that the huge welfare gain arises mostly from an improvement in the bias of the balanced growth rate $\gamma^*_A, t$ in the welfare measure (27). This improvement explains more than 90 percent of the total welfare gain. An adverse financial shock generates a slowdown in TFP growth and hence balanced growth, and thereby causes a permanent decline in consumption. This decline induces a welfare loss, which is captured by the bias in the balanced growth rate. In the model, monetary policy has an influence on TFP. Thus, the strong policy response

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20 As indicated below, the size of the policy response to inflation plays a minor role for welfare in reacting to the financial shock, and therefore we emphasize a strong policy response to output as one of the most important features of the welfare-maximizing rule. The strong policy response to output is partially due to the financial shock, since in the model with exogenous TFP growth (shown in the second to last row of Table 2) the welfare-maximizing rule of the form (25) contains a policy response to output growth of $\phi_{dy} = 10$ as well, although it also contains a policy response to inflation of $\phi_\pi = 6.7$, which is much stronger than the one of $\phi_\pi = 1$ in the benchmark model.

21 In this case (of only TFP shocks), the welfare-maximizing monetary policy rule of the form (25) contains $\phi_\pi = 3.1$, $\phi_{dy} = 0$, and $\rho_R = 0$. Moreover, if the financial friction and endogenous TFP growth are removed from the benchmark model and the policy response to real wage growth, $\phi_{dw}$, is introduced in the rule (25) as in Schmitt-Grohé and Uribe (2007), the welfare-maximizing rule has $\phi_\pi = 3.8$, $\phi_{dy} = 0$, $\phi_{dw} = 3.0$, and $\rho_R = 0.99$, so that it shows no response to output in line with their result.
to output under the welfare-maximizing rule subdues the slowdown in TFP growth and thereby ameliorates social welfare through an improvement in the bias.

Second, the strict inflation or price-level targeting rule induces a sizable welfare loss relative to the welfare-maximizing rule. Even the optimal strict inflation targeting rule—where $\rho_R = 0$ and $\phi_\pi = 10$, the latter of which hits the upper bound on the response considered—yields lower welfare by 0.69 percentage point permanent decline in consumption than the welfare-maximizing rule, and the optimal strict price-level targeting rule—where $\rho_R = 0$ and $\phi_\pi = 10$, the latter of which hits the upper bound—generates lower welfare by 0.47 percentage point permanent decline in consumption. This is because these rules have no policy response to output and thus cannot directly mitigate a slowdown in TFP growth caused by the financial shock.

Last, the nominal GDP growth or level targeting rule performs well, even compared with the welfare-maximizing rule. Indeed, the welfare gain from adopting the welfare-maximizing rule relative to the optimal nominal GDP growth targeting rule—where $\rho_R = 0.92$ and $\phi_\pi = \phi_{dy} = 10$, the last two of which hit the upper bounds on the responses considered—is a permanent increase in consumption of only 0.04 percentage point, while the one relative to the optimal nominal GDP level targeting rule—where $\rho_R = 0.21$ and $\phi_\pi = \phi_{dy} = 1.2$—is a permanent increase in consumption of only 0.02 percentage point. Because the actual policy responses to inflation in the welfare-maximizing rule, the nominal GDP growth targeting rule, and the nominal GDP level targeting rule, $\phi_\pi (1 - \rho_R)$, are 0.01, 0.84, and 0.91, the welfare-maximizing rule contains a much weaker response to inflation than the two nominal GDP targeting rules. This implies that the size of the policy response to inflation plays a minor role for welfare in reacting to the financial shock.

22 Because the last two rows of Table 2 show that the welfare-maximizing rule of the form (25) features an aggressive policy response to inflation of $\phi_\pi = 6.7$ in the model with exogenous TFP growth and of $\phi_\pi = 4.6$ in the model with flexible wages, the combination of endogenous TFP growth and sticky wages could cause the welfare-maximizing rule to contain a weak response to inflation in the benchmark model.
5.3 Financial crisis scenario simulations

Under the welfare-maximizing monetary policy rule and other rules analyzed above, this subsection conducts simulations in an illustrative financial crisis scenario.

In the scenario, the economy is hit by an adverse financial shock of \( \epsilon_{\xi,t} = -0.04 \) for three periods \((t = 1, 2, 3)\) and this is anticipated by all economic agents in the model when the first shock emerges in period 1. A financial shock of this size occurred in the U.S. during the Great Recession, as can be seen in the estimated series of the financial shock by Jermann and Quadrini (2012). The anticipated financial shocks subsequent to the emergence of the first shock seem reasonable, since once a financial crisis happens, the resulting financial turbulence tends to continue unfolding.

Fig. 3 plots the developments of output, total investment, intratemporal loans, (year-on-year) inflation, the (annualized) interest rate, and TFP under the benchmark monetary policy rule (the solid line), the welfare-maximizing rule (the dashed line), the optimal nominal GDP level targeting rule (the dotted line), and the optimal strict price-level targeting rule (the dot-dashed line) in the financial crisis scenario. In this figure, three findings are detected. First, in response to the severe financial shocks, a slowdown in TFP growth is much less pronounced under the welfare-maximizing rule than under the benchmark rule. As a consequence, output and total investment approach the pre-crisis balanced growth path under the welfare-maximizing rule, while they do not under the benchmark rule, implying that the welfare gain from adopting the former rule relative to the latter is huge, as shown above. The inflation rate then rises under the welfare-maximizing rule, whereas it drops under the benchmark rule, reflecting a much weaker policy response to inflation under the former rule (i.e., \( \phi_n(1 - \rho_R) = 0.01 \) under the welfare-maximizing rule, whereas \( \phi_n(1 - \rho_R) = 0.60 \) under the benchmark rule).

According to these developments of inflation and output growth, the benchmark rule

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23 According to Fig. 2 of Jermann and Quadrini (2012), their estimated financial shock went down by 4 percentage points, from about 1 percent to about −3 percent during the recent U.S. Great Recession.
lowers the interest rate below zero, while the interest rate cut is subdued substantially under the welfare-maximizing rule.

Second, under the nominal GDP level targeting rule, the levels of TFP, output, and total investment initially overshoot those under the welfare-maximizing rule, but then these levels under the two rules approach each other, implying that the welfare gain from adopting the welfare-maximizing rule relative to the nominal GDP targeting rule is small, as indicated above. However, it is surprising that even in the financial crisis scenario, the initial overshoot induces an initial rise in inflation and hence an initial hike in the interest rate under the nominal GDP targeting rule. Under the welfare-maximizing rule its weak policy response to inflation \((\phi_{\pi}(1 - \rho_R) = 0.01)\) leads to an initial cut in the interest rate, albeit an initial increase in inflation.

Last, under the strict price-level targeting rule, inflation is stabilized by the strong policy response to inflation \((\phi_{\pi}(1 - \rho_R) = 10)\) much more than under the welfare-maximizing rule, which contains the weak response to inflation \((\phi_{\pi}(1 - \rho_R) = 0.01)\). Yet the strict price-level targeting rule cannot directly mitigate a slowdown in TFP growth caused by the severe financial shocks, since it has no response to output. Consequently, output and total investment recover to the pre-crisis balanced growth path more slowly than under the welfare-maximizing rule, and thus the welfare loss from the price-level targeting rule relative to the welfare-maximizing rule is sizable, as noted above.\(^{24}\)

\section{Concluding Remarks}

This paper has developed a model that can describe a slow recovery resulting from an adverse financial shock like the slow recoveries observed in many economies after the

\(^{24}\)Appendix D addresses the question of whether a discretionary deviation from the benchmark monetary policy rule by a monetary policy shock can achieve the output growth attained under the welfare-maximizing monetary policy rule. It is shown that in the presence of a zero lower bound on the (nominal) interest rate, the regime shift from the benchmark rule to the welfare-maximizing rule could work much better in the financial crisis scenario than the constrained discretion under the benchmark rule as long as the shift has no credibility problem.
recent financial crisis and subsequent recession. Specifically, the Jermann and Quadrini (2012) financial friction and shock and the Comin and Gertler (2006) endogenous TFP growth mechanism have been introduced in an otherwise standard DSGE model. With this model, the paper has examined how monetary policy should react to the financial shock. It has been shown that in the face of the financial shock, a welfare-maximizing monetary policy rule features a strong response to output. Moreover, compared with this rule, a strict inflation or price-level targeting rule induces a sizable welfare loss, whereas a nominal GDP growth or level targeting rule performs well. To obtain these results, it is crucial to take into account a welfare loss from a permanent decline in consumption caused by a slowdown in TFP growth. In the model, TFP growth is endogenous and monetary policy has an influence on TFP, and as a consequence, not only the variability of consumption but also the level of consumption changes with policy choices and thus constitutes social welfare relevant to policy evaluation.

The paper has also conducted a financial crisis scenario simulation under the monetary policy rules. In this simulation, a slowdown in TFP growth is much less pronounced under the welfare-maximizing rule than under the strict price-level targeting rule and as a consequence, output recovers to its pre-crisis growth trend faster under the welfare-maximizing rule. Under the nominal GDP level targeting rule, the levels of TFP and output initially overshoot those under the welfare-maximizing rule, but then these levels under the two rules approach each other. It is surprising that even in the financial crisis scenario, the initial overshoot induces an initial hike in the interest rate under the nominal GDP targeting rule.

This paper has studied interest rate policy only. After lowering the policy rate virtually to the zero lower bound, central banks in advanced economies have been underpinning economic recovery in the wake of the recent global financial crisis by using unconventional policy tools, such as forward guidance and asset purchases. The analysis of these unconventional policies in the model is left for future work.
Appendix

A The intermediate-good firm’s problem

This appendix solves the problem of the representative intermediate-good firm. This firm chooses dividend payment \( d_t \), intertemporal debt \( b_t (= B_t/P_t) \), labor \( n_t \), capital \( k_t \), and adopted ideas \( A_t \) to maximize the expected value of the present and future dividend payment

\[
E_0 \sum_{t=0}^{\infty} m_{0,t} d_t
\]

subject to (3)–(7) and (9). Substituting the capital accumulation equation (4), the adopted idea equation (5), and the intratemporal loan equation (6) in the borrowing constraint (7) and the budget constraint (9) yields

\[
\xi_t \left( q_t k_t - \frac{b_t}{R_t} \right) \geq w_t n_t + q_t (k_t - (1 - \delta_k)k_{t-1}),
\]

\[
p_t^x x_t + \frac{b_t}{R_t} = w_t n_t + q_t (k_t - (1 - \delta_k)k_{t-1}) + v_t (A_t - (1 - \delta_A)A_{t-1}) + \varphi (d_t) + \frac{b_{t-1}}{\pi_t},
\]

where \( q_t = Q_t/P_t \) is the relative price of investment goods, \( w_t = W_t/P_t \) is the real wage, \( p_t^x = P_t^x/P_t \) is the relative price of final goods, and \( v_t = V_t/P_t \) is the relative price of newly adopted ideas. Assume that the borrowing constraint is always binding. Then, letting \( s_t, \mu_t \), and \( \varsigma_t \) denote the Lagrange multipliers on the aggregate intermediate-good production function (3), the borrowing constraint, and the budget constraint, the first-order conditions with respect to \( d_t, k_t, n_t, b_t \), and \( A_t \) are

\[
s_t = \frac{1}{\varphi_t},
\]

\[
1 = E_t \left[ m_{t,t+1} \frac{\alpha s_{t+1} (n_{t+1}/k_{t+1})^{1-\alpha} + (1 - \delta_k)q_{t+1} (\varsigma_{t+1} + \mu_{t+1})}{q_{t} (s_t + \mu_t (1 - \xi_t))} \right],
\]

\[
w_t = \frac{(1 - \alpha) s_t (k_{t-1}/n_t)^{\alpha}}{s_t + \mu_t},
\]

\[
1 = E_t \left[ m_{t,t+1} \frac{R_t^{\gamma} (s_t)_{t+1} + \frac{R_t^{\gamma} \mu_t \xi_t}{\pi_{t+1} s_t}}{\pi_t s_t R_t} \right],
\]

\[
1 = E_t \left[ m_{t,t+1} \frac{(\theta - 1) s_{t+1} x_{t+1} A_{t-\theta} + (1 - \delta_A) s_{t+1} v_{t+1}}{s_t v_t} \right],
\]

31
where \( m_{t,t+1} = m_{0,t+1}/m_{0,t} \) and \( \phi'_t = \phi'(d_t) = 1 + 2\kappa(d_t/A^*_t - d^*) \). Substituting (A1) and the aggregate intermediate-good production function in the first-order conditions (A2)–(A5) leads to

\[
1 = E_t \left[ \frac{\alpha s_{t+1} (n_{t+1}/k_t)^{1-\alpha} + (1 - \delta_k) q_{t+1} (1/\phi'_t + \mu_{t+1})}{q_t [1/\phi'_t + \mu_t (1 - \xi_t)]} \right],
\]

\[
w_t = \frac{(1 - \alpha) s_t (k_{t-1}/n_t)^\alpha}{1/\phi'_t + \mu_t},
\]

\[
1 = E_t \left[ m_{t,t+1} \frac{R_t^* \phi'_t}{\pi_{t+1} \phi'_{t+1}} \right] + \mu_t \xi_t \phi'_t \frac{R_t^*}{R_t},
\]

\[
1 = E_t \left[ m_{t,t+1} \frac{(\theta - 1) s_{t+1} n_{t+1} (k_t/n_{t+1})^\alpha / A_t + (1 - \delta_A) v_{t+1}/\phi'_t}{v_t/\phi'_t} \right].
\]

### B Equilibrium conditions and the steady state of the model

This appendix presents equilibrium conditions and the steady state of the model.

#### B.1 Equilibrium conditions

The equilibrium conditions are presented in terms of detrended variables. With 24 variables, \( y_t^* = y_t/A_t^* \), \( c_t^* = c_t/A_t^* \), \( i_t^* = i_t/A_t^* \), \( i_{o,t}^* = i_{o,t}A_t-1/A_t^* \), \( i_{d,t}^* = i_{d,t}/A_t^* \), \( d_t^* = d_t/A_t^* \), \( k_t^* = k_t/A_t^* \), \( b_t^* = b_t/A_t^* \), \( s_t^* = s_t/A_t^{\theta-1} \), \( v_t^* = v_tA_t-1/A_t^* \), \( j_t^* = j_tA_t-1/A_t^* \), \( a_t = A_t/Z_t \), \( w_t^* = w_t/A_t^* \), \( \pi_t \), \( \Delta_{p,t} \), \( F_{p,t} \), \( K_{p,t} \), \( F_{w,t} \), \( K_{w,t} \), \( n_t \), \( R_t \), \( q_t \), \( \gamma_{A,t} \), and \( \mu_t \), where \( A_t^* = A_{t-1}^{\theta-1} \), the system of the equilibrium conditions consists of the following 24 equations.

**Intermediate-good firms:**

\[
\xi_t \left( q_t k_t^* - \frac{b_t^*}{R_t} \right) = w_t^* n_t + q_t \left( k_t^* - (1 - \delta_k) \frac{k_{t-1}^*}{\gamma_{A,t}^*} \right), \quad (B1)
\]

\[
0 = \theta s_t^* \phi'_t \left( \frac{k_{t-1}^*}{\gamma_{A,t}^*} \right)^\alpha n_t^{1-\alpha} + \frac{b_t^*}{R_t} - w_t^* n_t - q_t \left( k_t^* - (1 - \delta_k) \frac{k_{t-1}^*}{\gamma_{A,t}^*} \right) - v_t^* (\gamma_{A,t} - (1 - \delta_A)) - \left[ d_t^* + \kappa (d_t^* - d^*)^2 \right] - \frac{b_{t-1}^*}{\gamma_{A,t}^* n_t} \frac{d_t^*}{n_t}, \quad (B2)
\]

\[
k_t^* = (1 - \delta_k) \frac{k_{t-1}^*}{\gamma_{A,t}^*} + \left[ 1 - S \left( \frac{i_t^*}{i_{t-1}^*} \right)^2 \right] i_t^*, \quad (B3)
\]
\[ 1 = E_t \left[ \frac{\beta c_t^* s_{t+1}^* (\gamma_{A,t+1} n_{t+1}/k_t^*)^{1-\alpha} + (1 - \delta) q_{t+1} (1/\varphi_{t+1} + \mu_{t+1})}{q_t [1/\varphi'_t + \mu_t (1 - \xi_t)]} \right], \quad (B4) \]
\[ w_t^* = \frac{s_t^*}{1/\varphi'_t + \mu_t} (1 - \alpha) \left( \frac{k_{t-1}^*}{\gamma_A^* n_t} \right)^\alpha, \quad (B5) \]
\[ 1 = E_t \left[ \frac{\beta c_t^* \varphi'_t R_t^*}{\gamma_{A,t+1}^* c_{t+1}^* \varphi'_{t+1} \pi_{t+1}} + \mu_t \xi_t \varphi'_t R_t^* \right], \quad (B6) \]
\[ 1 = E_t \left[ \frac{\beta c_t^* (\theta - 1) s_{t+1}^* n_{t+1} \left[ k_t^* / (\gamma_{A,t+1}^* n_{t+1}) \right]^{\alpha} + (1 - \delta_A) v_{t+1}^* / \varphi'_t}{v_t^*/\varphi'_t} \right]. \quad (B7) \]

Consumption-good firms:

\[ 1 = (1 - \xi_p) \left( \lambda_p \frac{K_{p,t}}{F_{p,t}} \right)^{\frac{1}{1-\lambda_p}} + \xi_p \left( \frac{\pi}{\pi_t} \right)^{\frac{1}{1-\lambda_p}}, \quad (B8) \]
\[ F_{p,t} = \frac{y_t^*}{c_t^*} + \beta \xi_p E_t \left( \frac{\pi}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1}, \quad (B9) \]
\[ K_{p,t} = \frac{y_t^*}{c_t^*} \theta s_t^* \varphi'_t + \beta \xi_p E_t \left( \frac{\pi}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1-\lambda_p}} K_{p,t+1}, \quad (B10) \]
\[ y_t^* \Delta_{p,t} = (k_t^*/\gamma_{A,t}^*)^\alpha n_t^{1-\alpha}, \quad (B11) \]
\[ \Delta_{p,t} = (1 - \xi_p) \left( \lambda_p \frac{K_{p,t}}{F_{p,t}} \right)^{\frac{1}{1-\lambda_p}} + \xi_p \left( \frac{\pi}{\pi_t} \right)^{\frac{\lambda_p}{1-\lambda_p}} \Delta_{p,t-1}, \quad (B12) \]
\[ y_t^* = c_t^* + i_t^* + i_{a,t}^* \left( \frac{1}{a_t} - 1 \right) + i_{d,t}^* + \kappa (d_t^* - d^*)^2. \quad (B13) \]

Investment-good firms:

\[ 1 = q_t \left[ 1 - S \left( \frac{i_t^*}{i_{t-1}^*} \frac{\gamma_{A,t}^*}{\gamma_A^*} - 1 \right)^2 - 2S \left( \frac{i_t^*}{i_{t-1}^*} \frac{\gamma_{A,t}^*}{\gamma_A^*} - 1 \right) \frac{i_t^*}{i_{t-1}^*} \frac{\gamma_{A,t}^*}{\gamma_A^*} \right] \]
\[ + E_t \beta c_t^* q_{t+1} 2S \left( \frac{i_{t+1}^*}{i_t^*} \frac{\gamma_{A,t+1}^*}{\gamma_A^*} - 1 \right) \left( \frac{i_{t+1}^*}{i_t^*} \right)^2 \frac{\gamma_{A,t+1}^*}{\gamma_A^*}, \quad (B14) \]

Technology adopters:

\[ j_t^* = -i_{a,t}^* + (1 - \delta) \left[ \lambda_0 (i_{a,t}^*)^\omega v_t^* + E_t \frac{\beta c_t^*}{\gamma_{A,t+1}^* c_{t+1}^*} (1 - \lambda_t) j_{t+1}^* \right], \quad (B15) \]
\[ 1 = (1 - \delta) \lambda_0 \omega (i_{a,t}^*)^\omega \frac{1}{k} \left( v_t^* - E_t \frac{\beta c_t^*}{\gamma_{A,t+1}^* c_{t+1}^*} j_{t+1}^* \right), \quad (B16) \]
\[ \gamma_{A,t+1} = (1 - \delta_A) \left[ \lambda_0 (i_{a,t}^*)^\omega (1/a_t - 1) + 1 \right]. \quad (B17) \]
Innovators:

$$\frac{a_t}{a_{t+1}} \gamma_{A,t+1} = \chi (i_{d,t}^*)^\rho + 1 - \delta_A,$$

$$1 = \frac{1}{a_t} \chi \left( \frac{1}{i_{d,t}^*} \right)^{1-\rho} (1 - \delta_A) E_t \frac{\beta}{c_t} \frac{c_t^*}{c_{t+1}} j_{t+1}^*.$$

(B18)

(B19)

Households:

$$1 = E_t \frac{\beta}{c_t} \frac{c_t^*}{c_{t+1}^*} \frac{R_t}{\pi_{t+1}},$$

(B20)

$$1 = (1 - \xi_w) \left( \frac{1}{w_t^*} \frac{\lambda_{w}}{F_{w,t}} \right)^{1-\lambda_{w}(1+1/v)} + \xi_w \left( \frac{w_{t-1}^* \gamma_{A,t}^* \pi}{w_t^* \gamma_{A,t+1}^* \pi_{t+1}} \right)^{1-\lambda_{w}},$$

(B21)

$$F_{w,t} = \frac{n_t}{c_t^*} + \beta \xi_w E_t \left[ \left( \frac{w_t^*}{w_{t+1}^*} \right)^{1-\lambda_{w}} \left( \frac{\gamma_{A,t}^*}{\gamma_{A,t+1}^* \pi_{t+1}} \right)^{1-\lambda_{w}} F_{w,t+1} \right],$$

(B22)

$$K_{w,t} = \lambda_w \psi n_{t+1}^* + \beta \xi_w E_t \left[ \left( \frac{w_t^*}{w_{t+1}^*} \right)^{\lambda_{w}(1+1/v)} \left( \frac{\gamma_{A,t}^*}{\gamma_{A,t+1}^* \pi_{t+1}} \right)^{\lambda_{w}(1+1/v)} K_{w,t+1} \right].$$

(B23)

Central Bank:

$$\log \left( \frac{R_t}{R_t} \right) = \rho_R \log \left( \frac{R_{t-1}}{R_t} \right) + (1 - \rho_R) \left[ \phi \log \left( \frac{\pi_t}{\pi} \right) + \phi_{dy} \log \left( \frac{y_{t-1}^*}{y_t^*} \frac{\gamma_{A,t}}{\gamma_{A,t+1}^*} \right) \right] + \epsilon_{R,t}.$$ (B24)

B.2 The steady state

The strategy for computing the steady state is to set a target value for labor $n$, the technology adoption rate $\lambda$, and the growth rate of $A_t$, $\gamma_A$, and pin down the parameter values of $\psi$, $\chi$, and $\lambda_0$ instead. Labor is normalized to unity, $n = 1$. In the steady state, the costs of dividend payment are zero, so that $\varphi' = 1$. The parameter value of $d^*$ will be determined later.

Equation (B14) implies $q = 1$. From equation (B20), the interest rate is given by

$$R = \frac{\pi \gamma_A^*}{\beta},$$

where $\gamma_A^* = \frac{\phi_{-1}}{\phi A}$. Equation (B17) determines

$$a = \left[ \frac{1}{\lambda} \left( \frac{\gamma_A}{1 - \delta_A} - 1 \right) + 1 \right]^{-1}.$$
From equations (B8)–(B10), the marginal cost is given by

\[ s^* = \frac{1}{\theta \lambda p}. \]

From equation (B6), the Lagrange multiplier on the borrowing constraint is given by

\[ \mu = \left( \frac{R - R^*}{R^*} \right) \frac{1}{\xi}, \]

where \( R^* = 1 + (1 - \tau) (R - 1) \). Because \( n = 1 \), equations (B3)–(B5), (B7), and (B11) determine capital, physical capital investment, the wage, the value of the newly adopted idea, and output respectively as

\[ k^* = \left[ \frac{\gamma_A}{\beta (1 + (1 - \xi) \mu) - (1 - \delta_k) (1 + \mu)} \right]^{1/\gamma_A^*} \gamma_A^*; \quad i^* = \left( 1 - \frac{1 - \delta_k}{\gamma_A^*} \right) k^*, \]

\[ w^* = \frac{s^*}{1 + \mu} (1 - \alpha) \left( \frac{k^*}{\gamma_A^*} \right)^\alpha; \quad v^* = \frac{(\theta - 1) s^* (k^*/\gamma_A^*)^\alpha}{\gamma_A^*/\beta - 1 + \delta_A}; \quad y^* = (k^*/\gamma_A^*)^\alpha. \]

Equation (B1) determines the intertemporal borrowing as

\[ b^* = R \left( k^* - \frac{w^* + i^*}{\xi} \right). \]

Equation (B2) pins down the parameter value of \( d^* \) as

\[ \bar{d} = \theta s^* \left( \frac{k^*}{\gamma_A^*} \right)^\alpha + b^* \left( \frac{w^* + i^* + v^* (\gamma_A - (1 - \delta_A)) + \frac{b^*}{\pi \gamma_A^*}}{R^*} \right). \]

Solving equations (B15) and (B16) for \( j^* \) and \( i_a^* \) yields

\[ i_a^* = \left[ \frac{\gamma_A/\beta - (1 - \lambda) (1 - \delta_A) - \left( 1 - \delta_A \right) (1 + \omega \lambda)}{\gamma_A/\beta - (1 - \delta_A) (1 - \lambda + \omega \lambda)} \right] (1 - \delta_A) \omega \lambda v^*, \]

\[ j^* = \frac{-i_a^* + (1 - \delta_A) \lambda v^*}{1 - (\beta/\gamma_A) (1 - \lambda) (1 - \delta_A)}. \]

The parameter of the technology adoption rate is pinned down as \( \lambda_0 = \lambda / (i_d^*)^\omega \). Solving equations (B18) and (B19) for \( i_d^* \) and \( \chi \) yields

\[ i_d^* = \frac{\gamma_A - (1 - \delta_A) \beta}{a} (1 - \delta_A) \frac{\beta}{\gamma_A} j^*; \quad \chi = \frac{\gamma_A - (1 - \delta_A)}{(i_d^*)^p}. \]

Equation (B13) determines consumption as

\[ c^* = y^* - \left[ i^* + \frac{1}{a} (i_a^* - 1) + i_d^* \right]. \]
Equations (B21)–(B23) imply
\[
\psi = \frac{w^*}{\lambda_w c^*}, \quad F_w = \frac{1}{1 - \beta \xi_w c^*}, \quad K_w = \frac{\lambda_w \psi}{1 - \beta \xi_w}.
\]

Finally, equations (B9) and (B10) are solved as
\[
F_p = \frac{1}{1 - \beta \xi_p c^*}, \quad K_p = \frac{1}{1 - \beta \xi_p c^*} \theta s^*.
\]

C  The second-order approximation to the welfare of households

This appendix derives a second-order approximation around the steady state to the unconditional expectation of the average utility function over households, given by (26). Substituting the demand curve for each type of specialized labor, (24), in equation (26) yields

\[
SW = (1 - \beta) \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \frac{\psi}{1 + 1/\nu} n_t \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{\lambda_w (1+1/\nu)} \frac{1}{1 - \lambda_w} dj \right) \right] + \log \left( A_t \right)
\]

Let \( \Delta_{w,t} \) denote wage dispersion given by

\[
\Delta_{w,t} = \left[ \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{\lambda_w (1+1/\nu)} d\frac{1}{1 - \lambda_w} \right]^{(1 - \lambda_w) \nu \lambda_w (1+1/\nu)}.
\]

Then, \( \Delta_{w,t} \) can be expressed recursively as

\[
\Delta_{w,t} = \left[ (1 - \xi_w) \bar{w}_t \rho \frac{\lambda_w (1+1/\nu)}{1 - \lambda_w} + \xi_w \left( \Delta_{w,t-1} \frac{w_{t-1}^*}{w_t^*} \frac{\gamma_{A}^* \pi_{t-1}^*}{\gamma_{A,t}^* \pi_t^*} \right) \right]^{\left( \frac{1}{1 - \lambda_w} \right)^{1 - \lambda_w} \nu \lambda_w (1+1/\nu)}, \quad (C1)
\]

where the newly set relative wage \( \bar{w}_t = \bar{W}_t/W_t \) is given by

\[
\bar{w}_t = \left( \frac{1}{F_{w,t}} \right)^{\frac{1}{1 - \lambda_w}} \lambda_w (1+1/\nu) \]

Using \( \Delta_{w,t} \) and \( c_t^* = c_t/A_t^* \), the welfare measure \( SW \) can be rewritten as

\[
SW = (1 - \beta) \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t^*) + \log(A_t^*) - \frac{\psi}{1 + 1/\nu} n_t \int_0^1 \Delta_{w,t} \frac{1}{1 - \lambda_w} \right]. \quad (C2)
\]
Because \( \log(A^*_t) \) follows the random-walk process \( \log(A^*_t) = \log(A^*_{t-1}) + \log(\gamma^*_{A,t}) \), subtracting \( (1 - \beta) \sum_{t=0}^{\infty} \beta^t \log(\bar{A}_t) \), where \( \bar{A}_t \) is the deterministic trend governed by \( \bar{A}_t = \gamma^*_{A} \bar{A}_{t-1} \), from both sides of (C2) makes the resulting welfare measure \( SW^* \) stationary, given by

\[
SW^* = SW - (1 - \beta) \sum_{t=0}^{\infty} \beta^t \log(\bar{A}_t)
\]

\[
= (1 - \beta) \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \log(c^*_t) + \log(\bar{A}_t) - \frac{\psi}{1 + 1/\nu} \eta t^{1+1/\nu} \Delta w_{t-1} \right) \right], \tag{C3}
\]

where \( \bar{A}_t = A^*_t / \bar{A}^*_t \).

We now approximate the stationary welfare measure \( SW^* \) around the steady state up to the second order. The term related to detrended consumption \( c^*_t \) in (C3) is approximated around the steady state as

\[
(1 - \beta) \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \log(c^*_t) \right] \approx (1 - \beta) \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[ \log(c^*) + \frac{c^*_t - c^*}{c^*} - \left( \frac{c^*_t - c^*}{c^*} \right)^2 \right]
\]

\[
= \log(c^*) + \frac{Ec^*_t - c^*}{c^*} - \frac{1}{(c^*)^2} \mathbb{E} (c^*_t - c^*)^2
\]

\[
= \log(c^*) + \frac{\varepsilon_c}{c^*} - \frac{1}{(c^*)^2} \mathbb{E} [(c^*_t - Ec^*_t) + (Ec^*_t - c^*)]^2
\]

\[
\approx \log(c^*) + \frac{\varepsilon_c}{c^*} - \frac{\text{Var} (c^*_t)}{(c^*)^2}, \tag{C4}
\]

where \( \varepsilon_c = E(c^*_t) - c^* \) denote the bias associated with detrended consumption. In the second-order approximation, the unconditional mean of \( c^*_t \), \( E(c^*_t) \), is not necessarily consistent with the steady-state value \( c^* \). However, we treat \( \varepsilon_c^2 \approx 0 \), because the approximation is accurate up to the second order.

Next, we approximate the term related to \( \bar{A}_t \) in (C3) around the steady state. Without loss of generality, we can assume \( \bar{A}_0^* = A_0^*/\bar{A}_0^* = 1 \) because \( A_0^* \) is predetermined. Then, \( \bar{A}_t \) can be expressed as

\[
\bar{A}_t = \frac{\gamma^*_{A,t} A_{t-1}^*}{\gamma^*_{A} \bar{A}_{t-1}^*} = \prod_{h=1}^{t} \frac{\gamma^*_{A,h}}{\gamma^*_{A}}.
\]
Then, the term related to $A_t^*$ in (C2) is rewritten as

$$(1 - \beta) E \left[ \sum_{t=0}^{\infty} \beta^t \log \left( A_t^* \right) \right] = (1 - \beta) \sum_{t=1}^{\infty} \beta^t E \left[ \log \left( \prod_{h=1}^{t} \frac{\gamma_{A,h}^*}{\gamma_A^*} \right) \right]$$

$$= (1 - \beta) \sum_{t=1}^{\infty} \beta^t E \left[ \prod_{h=1}^{t} \log \left( \frac{\gamma_{A,h}^*}{\gamma_A^*} \right) \right]$$

$$= E \left[ \log \left( \frac{\gamma_{A,h}^*}{\gamma_A^*} \right) \right] (1 - \beta) \sum_{t=1}^{\infty} \beta^t t$$

$$= \frac{\beta}{1 - \beta} E \left[ \log \left( \frac{\gamma_{A,h}^*}{\gamma_A^*} \right) \right]. \quad \text{(C5)}$$

Thus, the term related to $A_t^*$ in (C2) is approximated around the steady state as

$$(1 - \beta) E \left[ \sum_{t=0}^{\infty} \beta^t \log \left( A_t^* \right) \right] \approx \frac{\beta}{1 - \beta} \left( \frac{\varepsilon_{A}^*}{\gamma_A^*} - \frac{Var \left( \gamma_{A,t}^* \right)}{(\gamma_A^*)^2} \right),$$

where $\varepsilon_{A}^* = E(\gamma_{A,t}^*) - \gamma_A^*$.

The term related to labor in (C2) is approximated around the steady state as

$$(1 - \beta) E \sum_{t=0}^{\infty} \beta^t \frac{\psi}{1 + 1/\nu} n_t^{1 + \frac{1}{\nu}} \Delta_{w,t} \Delta_{w,t}^{1-\lambda_w}$$

$$\approx \frac{\psi}{1 + 1/\nu} \left( \frac{1}{1 + \frac{1}{\nu}} \right)^{n_t - n} \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} \right)^2 \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} - 1 \right)^2 (\Delta_{w,t} - \Delta_w)$$

$$+ \left( \frac{1}{1 + \frac{1}{\nu}} \right)^{n_t - n} \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} \right)^2 \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} - 1 \right) \left( \Delta_{w,t} - \Delta_w \right)$$

$$+ 2 \left( \frac{1}{1 + \frac{1}{\nu}} \right)^{n_t - n} \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} \right)^2 \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} - 1 \right)^2 \left( \Delta_{w,t} - \Delta_w \right)$$

$$\approx \frac{\psi}{1 + 1/\nu} \left[ 1 + \left( \frac{1}{1 + \frac{1}{\nu}} \right) \varepsilon_n + \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} \right)^2 \varepsilon_{\Delta_w} \right.$$

$$+ \left( \frac{1}{1 + \frac{1}{\nu}} \right)^{n_t - n} \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} \right)^2 \left( \frac{\lambda_w (1 + 1/\nu)}{1 - \lambda_w} - 1 \right) \left( \Delta_{w,t} - \Delta_w \right) \varepsilon_{\Delta_w} \] \right), \quad \text{(C6)}$$

where $\varepsilon_n = E(n_t) - n$ and $\varepsilon_{\Delta_w} = E(\Delta_{w,t}) - \Delta_w$ and where $n = \Delta_w = 1$ is used to derive the approximation.

From (C4)–(C6), the second-order approximation to $SW^*$ in (C3) around the steady state is given by (27), where terms independent of monetary policy are subtracted.
D Regime shift versus constrained discretion

This appendix addresses the question of whether a discretionary deviation from the benchmark monetary policy rule by a monetary policy shock can achieve the output growth attained under the welfare-maximizing monetary policy rule. Put differently, the question we ask here is: which is better, the regime shift from the benchmark rule to the welfare-maximizing rule or the constrained discretion under the benchmark rule?

Fig. 4 plots the developments of output, (year-on-year) inflation, the (annualized) interest rate, and TFP under the welfare-maximizing rule (the dashed line) and the benchmark rule with monetary policy shocks that almost achieve the output growth attained under the welfare-maximizing rule (the solid line), along with the benchmark rule (the dotted line). This figure shows that the constrained discretion under the benchmark rule substantially subdues declines in TFP, output, and inflation caused by the severe financial shocks and that its policy performance is comparable to that of the welfare-maximizing rule. However, the figure also demonstrates that such constrained discretion lowers the interest rate below zero and hence its policy performance is unachievable in the presence of a zero lower bound on the (nominal) interest rate. In this sense, the regime shift from the benchmark rule to the welfare-maximizing rule could work much better in the financial crisis scenario than the constrained discretion under the benchmark rule as long as the shift has no credibility problem.
References


19 Reifschneider, Dave, William Wascher, and David Wilcox. (2013) “Aggregate Supply in the United States: Recent Developments and Implications for the Conduct of


Table 1: Parameterization of the quarterly model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.9975</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Coefficient on labor disutility</td>
<td>0.6688</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital elasticity of output</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\xi_p, \xi_w$</td>
<td>Consumption-good price or wage stickiness</td>
<td>0.75</td>
</tr>
<tr>
<td>$\lambda_p, \lambda_w$</td>
<td>Gross consumption-good price or wage markup</td>
<td>1.05</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady-state gross inflation</td>
<td>1.005</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Policy rate smoothing</td>
<td>0.6</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Policy response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_{dy}$</td>
<td>Policy response to output growth</td>
<td>0.2</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Gross intermediate-good price markup</td>
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</tr>
<tr>
<td>$\lambda_0$</td>
<td>Coefficient on adoption probability</td>
<td>0.4825</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Elasticity of adoption probability</td>
<td>0.95</td>
</tr>
<tr>
<td>$\delta_A$</td>
<td>Obsolescence rate of ideas</td>
<td>0.025</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity of newly adopted ideas</td>
<td>0.8</td>
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<tr>
<td>$\chi$</td>
<td>Coefficient on R&amp;D productivity</td>
<td>1.5742</td>
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<tr>
<td>$S$</td>
<td>Coefficient on investment adjustment cost</td>
<td>0.04</td>
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<tr>
<td>$\xi$</td>
<td>Steady-state probability of foreclosure</td>
<td>0.1634</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coefficient on dividend payment cost</td>
<td>0.146</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax benefit</td>
<td>0.35</td>
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<tr>
<td>$\rho_\xi$</td>
<td>Financial shock persistence</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Standard parameters in DSGE models

Technology adoption and R&D

Financial friction and shock
Table 2: Welfare-maximizing combinations of coefficients of monetary policy rules in reaction to the financial shock.

<table>
<thead>
<tr>
<th>Policy rule specification</th>
<th>$\phi_\pi$</th>
<th>$\phi_{dy}$</th>
<th>$\rho_R$</th>
<th>$Var(R_t)$</th>
<th>Welfare gain $\eta$</th>
<th>$\gamma^*_A$-bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule (25)</td>
<td>1</td>
<td>10</td>
<td>0.99</td>
<td>0.075</td>
<td>14.71%</td>
<td>92.9%</td>
</tr>
<tr>
<td>Strict inflation targeting</td>
<td>10</td>
<td>—</td>
<td>0</td>
<td>0.128</td>
<td>14.01%</td>
<td>93.0%</td>
</tr>
<tr>
<td>Nominal GDP growth targeting</td>
<td>10</td>
<td>10</td>
<td>0.92</td>
<td>0.178</td>
<td>14.66%</td>
<td>92.9%</td>
</tr>
<tr>
<td>Rule (28)</td>
<td>2.1</td>
<td>1.9</td>
<td>0.38</td>
<td>0.173</td>
<td>14.68%</td>
<td>92.9%</td>
</tr>
<tr>
<td>Strict price-level targeting</td>
<td>10</td>
<td>—</td>
<td>0</td>
<td>0.102</td>
<td>14.23%</td>
<td>92.9%</td>
</tr>
<tr>
<td>Nominal GDP level targeting</td>
<td>1.2</td>
<td>1.2</td>
<td>0.21</td>
<td>0.165</td>
<td>14.68%</td>
<td>92.9%</td>
</tr>
<tr>
<td>Rule (25) in the exogenous TFP model</td>
<td>6.7</td>
<td>10</td>
<td>0.97</td>
<td>0.119</td>
<td>0.48%</td>
<td>—</td>
</tr>
<tr>
<td>Rule (25) in the flexible wage model</td>
<td>4.6</td>
<td>3.6</td>
<td>0.81</td>
<td>0.202</td>
<td>1.52%</td>
<td>94.5%</td>
</tr>
</tbody>
</table>

Note: For each of the monetary policy rules, the welfare gain $\eta$ denotes the one from adopting this rule relative to the benchmark rule (i.e., the rule (25) with $\phi_\pi = 1.5$, $\phi_{dy} = 0.2$, and $\rho_R = 0.6$) in terms of permanent increase in consumption, and the term “$\gamma^*_A$-bias” shows the fraction of the welfare gain arising from an improvement in the bias of the balanced growth rate $\gamma^*_A$ in the total welfare gain.
Figure 1: Economic developments around recent financial crises. Note: In each panel of the figure, the scale of years at the top is for Japan only, while that at the bottom is for all other economies. The data on TFP come from the Conference Board Total Economy Database.
Figure 2: Impulse responses to an adverse financial shock.
Figure 3: Financial crisis scenario simulations.
Figure 4: Regime shift vs. constrained discretion