The Zero Lower Bound and Endogenous Uncertainty*

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ABSTRACT

This paper documents a strong negative correlation between macroeconomic uncertainty and real GDP growth since the Great Recession. Prior to that event the correlation was weak, even when conditioning on recessions. At the same time, many central banks reduced their policy rate to its zero lower bound (ZLB), which we contend contributed to the strong correlation between macroeconomic uncertainty and real GDP growth. To test that theory, we use a model where the ZLB occasionally binds. The model roughly matches the correlation in the data—away from the ZLB the correlation is weak but strongly negative when the ZLB binds.

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1 INTRODUCTION

There is significant interest in understanding the relationship between uncertainty and economic activity. Several papers have found a negative relationship between macroeconomic uncertainty and economic activity in the data. For example, Bloom (2009) shows that unexpected increases in uncertainty, given by stock price volatility, are associated with declines in industrial production in a vector autoregression. Bekaert et al. (2013) and Pinter et al. (2013) find similar relationships.\(^1\)

There is also a large literature that explores how uncertainty affects economic variables in general equilibrium models. Those papers typically focus on how the levels of endogenous variables respond when there is increased uncertainty about exogenous variables, such as technology, government spending, or preference shocks. Some recent examples include Justiniano and Primiceri (2008), Bloom (2009), Fernández-Villaverde et al. (2011a,b), Basu and Bundick (2012), Mumtaz and Zanetti (2013), Bloom et al. (2014), and Christiano et al. (2014).\(^2\) These papers have often found a negative relationship between the level of uncertainty and economic activity.

This paper explores how uncertainty about future economic activity endogenously responds to the state of the economy. We conduct our analysis through the lens of a standard New Keynesian model that imposes a zero lower bound (ZLB) constraint on the short-term nominal interest rate. In this model, as in all rational expectations models, agents make predictions about the future values of economic variables, both exogenous and endogenous. They also make forecasts about the degree of uncertainty surrounding those predictions. Our measure of uncertainty in the model is equivalent to those forecasts, which in a mathematical sense is the expected volatility of the forecast errors regarding future output. We first explore how this measure varies over time in response to changes in the state of the economy in the model. With the theoretical predictions in hand, we then look to see how well the model’s predictions match correlations in the data.

Several important findings emerge from our analysis of the data and model simulations:

1. In the data there is a strong negative correlation between macroeconomic uncertainty and real GDP growth since the Great Recession. Prior to that event the correlation was weak, even when conditioning on recessions.

2. The model predicts an increase in output uncertainty near and at the ZLB. When the nominal interest rate is far from its ZLB, the uncertainty surrounding output is constant and low.

3. The model roughly matches the correlation between uncertainty and real GDP growth in the data—away from the ZLB the correlation is weak but strongly negative when the ZLB binds.

4. Various measures of uncertainty that capture different aspects of the economy for the U.S. and Euro area display similar correlations with real GDP growth.

Our results are important for the growing literature that links uncertainty and economic activity because they show that for particular states of the world there is a causal relationship between economic downturns and uncertainty. The increase in uncertainty that occurs at the ZLB is due to the restriction it places on the central bank. Since the ZLB arises in states where the discount

\(^1\)Others have found a negative relationship between fiscal uncertainty and economic activity [Fernández-Villaverde et al. (2011a), Born and Pfeifer (2013)] and oil price uncertainty and economic activity [Elder and Serletis (2010), Jo (2013), Plante and Traum (2012)]. Johannsen (2013) shows that fiscal uncertainty is more contractionary at the ZLB.

\(^2\)There is also an older literature that examines similar research questions to those posed in the SV literature. See, for example, Leland (1968), Levhari and Srinivasan (1969), and Sandmo (1970).
factor and/or technology are high (i.e., low aggregate demand or high aggregate supply), downward pressure on prices leads to higher real interest rates that further reduce output. Uncertainty increases as the possible realizations of future output are more dispersed and skewed toward losses. Furthermore, as the nominal interest rate approaches its ZLB, the expectational effects of hitting the ZLB also increase output uncertainty. Of course, these results do not rule out that causation can flow in the opposite direction. It merely shows that in at least one case, when the economy is near or at the ZLB, uncertainty is responding to an event that is endogenous to the economy.

Several recent papers document that the ZLB constraint has an important effect on the economy, and that its effect is stronger in the presence of uncertainty. Gust et al. (2013) estimate a nonlinear New Keynesian with a ZLB constraint to quantify how much of the recent decline in output was due to the binding constraint. They find the constraint accounts for about 20 percent of the drop in U.S. GDP from 2008-2009 and, on average, it caused output to be 1 percent lower from 2009-2011 than it would have been without the constraint. Nakov (2008) finds the optimal discretionary monetary policy leads to a more negative output gap at the ZLB when households face uncertainty about the real interest rate than when they have perfect foresight. Nakata (2012) also studies the effects of uncertainty when the ZLB binds by varying the standard deviation of discount factor shocks. He finds higher uncertainty increases the slope of the policy function for output, meaning that positive discount factor shocks lead to a larger reduction in output when the ZLB binds. Basu and Bundick (2012) show that cost and demand uncertainty shocks cause business cycle fluctuations, which become more pronounced when the ZLB binds. Specifically, they find a 1 standard deviation positive demand uncertainty shock causes output to decline by 0.2 percent when the ZLB does not bind and by 0.35 percent when it binds. Moreover, they calculate that demand uncertainty shocks can account for one-fourth of the drop in output in late 2008. These papers suggest output is more sensitive to shocks when the ZLB binds, particularly when there is macroeconomic uncertainty.

The rest of the paper is organized as follows. Section 2 describes our measures of uncertainty in the data, computes correlations between those measures and real GDP growth, and explains how the same type of uncertainty is calculated in a theoretical model. Section 3 introduces the theoretical model, the calibration, and the solution method. Section 4 presents our results. Specifically, it illustrates how uncertainty and output are related in our model and demonstrates that the correlations between those variables match equivalent statistics in the data. Section 5 concludes.

2 Empirical and Theoretical Measures of Uncertainty

This section introduces three forward looking measures of uncertainty and shows how they are correlated with real GDP growth. It then describes an analogous measure of uncertainty that naturally arises in stochastic general equilibrium models, which allows us to connect theory to data.

2.1 Data Description Figure 1 displays three alternative measures of economic uncertainty: the Chicago Board Options Exchange (CBOE) S&P 100 Volatility Index (VXO), the dispersion in large manufacturers’ forecasts of business activity from the Business Outlook Survey (BOS), and the dispersion in real GDP forecasts $k$-periods ahead from the Survey of Professional Forecasters (SPF). The shaded regions correspond to recessions, according to the National Bureau of Economic Research. We chose to use these data series because they are forward looking measures of uncertainty and are able to capture changes in people’s expectations over time, as opposed to making predictions about future uncertainty based on statistical relationships (e.g., GARCH model).
Figure 1: Measures of economic uncertainty. Chicago Board Options Exchange (CBOE) Volatility Index (VXO): expected volatility in the S&P 100 over the next 30 days at an annualized rate; Business Outlook Survey (BOS) Forecast Dispersion (FDISP): dispersion in large manufacturers’ forecasts of business activity over the next 6 months; Survey of Professional Forecasters (SPF) real GDP Forecast Dispersion (RGDP-FD): dispersion in real GDP forecasts k-periods ahead. The shaded regions correspond to recessions, according to the National Bureau of Economic Research.

The CBOE VXO measures the expected volatility in the S&P 100 stock market index over the next month at an annualized rate. For example, if the value on the vertical axis is $x$ percent, then people expect there is a 68 percent chance the S&P 100 index will change by $\pm \frac{x}{\sqrt{12}}$ percent over the next month. We aggregate the daily series up to a quarterly frequency so that it is consistent with the frequency of real GDP releases. The Business Outlook Survey (BOS), which is conducted monthly by the Federal Reserve Bank of Philadelphia, asks large manufacturing firms to forecast whether general business activity will increase, decrease, or remain unchanged over the next six months. Following Bachmann et al. (2013), the forecast dispersion (FDISP) in the responses to the survey in period $t$ is given by

$$\text{BOS FDISP}_t = \sqrt{\text{Frac}^+_t \times \text{Frac}^-_t - (\text{Frac}^+_t - \text{Frac}^-_t)^2},$$

where $\text{Frac}^+_t$ (Frac$_t^-$) is the fraction of firms who forecast an increase (decrease) in business activity. We aggregate the monthly BOS FDISP series up to a quarterly frequency and then standardize the values so the vertical axis displays the number of standard deviations from the mean response. The Survey of Professional Forecasters (SPF), which is conducted quarterly by the Federal Reserve Bank of Philadelphia, asks individuals who regularly make forecasts as part of their jobs to predict macroeconomic aggregates for the next four quarters (e.g., inflation, output, interest rates). We
focus on the forecasts of real GDP (RGDP). The inter-quartile forecast dispersion (FD) is given by

$$\text{SPF RGDP FD}_t(k) = 100 \times \left( \log(\hat{y}_{t+k|t-1}^{75}) - \log(\hat{y}_{t+k|t-1}^{25}) \right), \quad k = 0, 1, \ldots, 4,$$

which is the percent difference between the 75th and 25th percentiles of the forecasts made in period $t$ of real GDP in period $t + k$, given historical observations dated $t - 1$ and earlier.\(^3\)

The VXO is the least noisy of our uncertainty measures and is only modestly higher during the 1991 and 2001 recessions. The three spikes in the index correspond to the stock market crash of 1987, the aftermath of the terrorist attacks on 9/11, and the 2008 financial crisis. The dispersion in the BOS and SPF survey data is generally less persistent than the VXO. The SPF data also shows that forecast dispersion increases over longer forecasting horizons. For example, forecasts of real GDP growth next quarter on average differ by 0.43 percent, but the forecasts of real GDP one year ahead differ by 0.86 percent. Both the BOS FDISP and the SPF RGDP FD($k$) temporarily increase during 1991 and 2001 recessions but are persistently higher during the Great Recession.

\[2.2\] **Correlations between Economic Activity and Uncertainty**

Table 1 shows conditional and unconditional correlations between RGDP growth (i.e., the quarter-over-quarter log difference in RGDP) and our three measures of uncertainty. Table 1a is based on the entire data series (1986Q1-2013Q4), whereas table 1b is conditional on data before the Great Recession (1986Q1-2007Q4) and table 1c is conditional on data since the Great Recession (2008Q1-2013Q4). The unconditional correlations between the uncertainty measures and RGDP growth are all negative. Moreover, all of the uncertainty measures positively co-move with each other, but the correlation is modest because they represent uncertainty about different segments of the economy. The correlations conditional on the pre-Great Recession data are much weaker—the positive correlations between the uncertainty measures are smaller and the negative correlations with RGDP growth are closer to zero. In sharp contrast, the correlations conditional on data since the Great Recession are much stronger, as all of the correlations significantly increase.\(^4\)

The results in table 1 suggest that recessions are related to periods of high uncertainty, but, perhaps counterintuitively, we do not find evidence for this relationship in the data. In all of our data series, there is strong evidence of a negative unconditional correlation between uncertainty and economic activity, which is more pronounced when conditioning data since the Great Recession. However, when we extend the time series to include the data since the SPF and BOS surveys began (1968Q4-2014Q1), we find very little difference between the unconditional correlations and the correlations conditional on quarters when the economy is in a recession. Since 1968Q4, the U.S. economy has faced 7 recessions, totaling 34 quarters. The unconditional correlation between RGDP growth and SPF(1) is $-0.19$ whereas the correlation conditional on recessions is only $-0.15$. The equivalent correlations between GDP growth and the BOS are $-0.31$ and $-0.24$. These results suggest that there is a characteristic unique to the Great Recession that is leading to a much stronger negative relationship between our measures of uncertainty and RGDP growth.

One major difference between the current recession and past recessions is the monetary policy response. Unlike past recessions, the Fed began to reduce the federal funds rate to its ZLB starting

\(^3\)For more information about the history of the SPF and the other variables in the survey see Croushore (1993).

\(^4\)Sill (2012) computes an alternative measure of the SPF real GDP forecast dispersion based on the standard deviation of the average probability distribution assigned by the forecasters. We did not include this variable because a consistent sample can only be constructed starting in 1992, but we found that the resulting correlations with the log differences of real GDP are similar to those based on the inter-quartile forecast dispersion (SPF RGDP FD).
\[ \Delta \log(RGDP) \quad \text{VXO} \quad \text{BOS} \quad \text{SPF}(1) \quad \text{SPF}(2) \quad \text{SPF}(3) \quad \text{SPF}(4) \]

\[ \begin{array}{ccccccc}
\Delta \log(RGDP) & - & - & - & - & - & - \\
\text{VXO} & -0.34 & - & - & - & - & - \\
\text{BOS} & -0.27 & 0.21 & - & - & - & - \\
\text{SPF}(1) & -0.22 & 0.50 & 0.18 & - & - & - \\
\text{SPF}(2) & -0.25 & 0.52 & 0.19 & 0.85 & - & - \\
\text{SPF}(3) & -0.21 & 0.59 & 0.13 & 0.74 & 0.90 & - \\
\text{SPF}(4) & -0.13 & 0.47 & 0.12 & 0.61 & 0.82 & 0.89 & - \\
\end{array} \]

(a) Unconditional correlations based on the entire sample (1986Q1-2013Q4).

\[ \begin{array}{ccccccc}
\Delta \log(RGDP) & - & - & - & - & - & - \\
\text{VXO} & 0.00 & - & - & - & - & - \\
\text{BOS} & -0.18 & 0.10 & - & - & - & - \\
\text{SPF}(1) & -0.07 & 0.45 & 0.11 & - & - & - \\
\text{SPF}(2) & -0.10 & 0.48 & 0.16 & 0.83 & - & - \\
\text{SPF}(3) & -0.04 & 0.57 & 0.10 & 0.70 & 0.88 & - \\
\text{SPF}(4) & 0.02 & 0.40 & 0.11 & 0.54 & 0.79 & 0.87 & - \\
\end{array} \]

(b) Correlations conditional on data before the Great Recession (1986Q1-2007Q4).

\[ \begin{array}{ccccccc}
\Delta \log(RGDP) & - & - & - & - & - & - \\
\text{VXO} & -0.77 & - & - & - & - & - \\
\text{BOS} & -0.69 & 0.58 & - & - & - & - \\
\text{SPF}(1) & -0.56 & 0.66 & 0.43 & - & - & - \\
\text{SPF}(2) & -0.52 & 0.61 & 0.34 & 0.93 & - & - \\
\text{SPF}(3) & -0.47 & 0.62 & 0.30 & 0.88 & 0.96 & - \\
\text{SPF}(4) & -0.36 & 0.63 & 0.22 & 0.86 & 0.92 & 0.95 & - \\
\end{array} \]

(c) Correlations conditional on data after the Great Recession (2008Q1-2013Q4).

Table 1: Correlations between real GDP growth ($\Delta \log(RGDP)$) and measures of uncertainty.

in 2008Q1 and has maintained that rate since 2008Q4. We contend that the ZLB is one possible reason for the strong negative correlation between RGDP growth and the higher recent uncertainty.

The correlations between real GDP growth and uncertainty are not unique to U.S. data. The European Central Bank (ECB) has conducted its own survey of professional forecasters (ECB SPF) since 1999Q1.\(^5\) It asks participants to forecast the growth rate of Euro Area real GDP over various time horizons. For example, the survey conducted in 1999Q1 requests forecasts for 1999Q3, given

\(^5\)The Bank of Japan has kept its policy rate near zero since 1995. There is also a monthly survey of Japanese professional forecasters conducted by the Economic Planning Association (known as the ESP) which asks for forecasts of real GDP, but the monthly survey began in mid-2004 and does not provide a large enough sample to analyze. For further information about the ESP and a statistical analysis of the forecasters’ performance see Komine et al. (2009).
the last GDP release is from 1998Q3. Following the U.S. SPF, we calculate the forecast dispersion as ECB SPF $FD_t = |\tilde{y}_{t}^{75} - \tilde{y}_{t}^{25}|$, where $\tilde{y}_t$ is the $x$th percentile of the forecasted growth rates 2-quarters ahead of when the survey is conducted. Over the sample period 1999Q1 to 2013Q4, the unconditional correlation between Euro Area log differences in real GDP and the forecast dispersion is $-0.52$, which is more negative than the values reported for the U.S. but that change in magnitude is likely due to the short sample. The correlation conditional on pre-Great Recession data (1999Q1-2007Q4) is $-0.21$, while the correlation conditional post-Great Recession data (2008Q1-2013Q4) is $-0.46$. While we recognize the sample size is smaller than the U.S. SPF, the qualitative nature of the conditional correlations for the Euro Area echo those for the U.S. Similar to the Federal Funds Rate, the Euribor and ECB refinancing rates moved toward their ZLB starting in 2009Q1, which may explain the stronger correlations since the Great Recession.

2.3 Measure of Endogenous Uncertainty

Macroeconomic models typically include exogenous variables that follow a specified process. While several papers examine the implications of time-varying means of these variables, there is also a recent and growing segment of the literature that introduces stochastic volatility (SV) into dynamic stochastic general equilibrium models. Our work differs from these papers in that we focus on how uncertainty about future macroeconomic variables varies endogenously in response to the state of the economy.

To help illustrate our measure of endogenous uncertainty, it is useful to first describe how one measures uncertainty in a model with SV. As an example, suppose a model includes an exogenous random variable $x$, such as technology or government spending, that evolves according to $x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_t$, where $\rho_x < 1$ and $\varepsilon$ is white noise. SV is then introduced into the model by assuming the standard deviation of the shock is time-varying and evolves according to an exogenous process specified by the modeler, which relaxes the common assumption of homoscedastic innovations. Given the linear process governing $x$, the expected forecast error, $FE_x$, equals

$$
\mu_{FE,t+1} \equiv E_t[FE_{x,t+1}] = E_t[(x_{t+1} - E_t x_{t+1})] = 0.
$$

Although the forecast error is mean zero, there is uncertainty about its future value. That uncertainty is measured by the volatility of the forecast error, which is given by

$$
\sqrt{E_t[FE_{x,t+1}^2]} = \sqrt{E_t[(x_{t+1} - E_t x_{t+1})^2]} = \sqrt{E_t[(x_{t+1} - \rho_x x_t)^2]} = \sqrt{E_t \sigma_{x,t+1}^2}.
$$

Models that include SV in various shocks are able to match features of the data that models with homoscedastic errors cannot match, but they do not explain why volatility changes over time because the uncertainty is exogenous. However, regardless of whether a model includes SV, there is always a certain amount of uncertainty that is endogenous. We quantify the endogenous uncertainty by following the logic of the SV literature. Suppose $y$ is any endogenous variable in the model, such as output. Since the model is nonlinear, the expected forecast error is no longer zero. Thus, the amount of uncertainty surrounding $y$, $k$ periods in the future, is given by

$$
\sigma_{y,t}(k) \equiv \sqrt{E_t[(FE_{y,t+k} - E_t[FE_{y,t+k}])^2]} = \sqrt{E_t[(y_{t+k} - E_t y_{t+k} - \mu_{FE,t+k})^2]},
$$

which will vary over time like it does in the SV specification, except that it may change endogenously in response to the state of the economy. That is, given the realizations of the shock, the model will endogenously generate recurring periods of both high and low uncertainty.

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6We found similar correlations using the European Commission’s Business and Consumer surveys. The survey of industry asks manufacturers whether they expected their production to increase, decrease, or remain unchanged.
3 Economic Model, Calibration, and Solution Method

We use a conventional New Keynesian model in which the ZLB on the short-term nominal interest rate occasionally binds due to discount factor and/or technology shocks.

3.1 Households A representative household chooses \( \{c_t, n_t, b_t \}_{t=0}^{\infty} \) to maximize expected lifetime utility, given by, 
\[
E_0 \sum_{t=0}^{\infty} \beta_t c_t^{1-\gamma} / (1 - \gamma) - \chi n_t^{1+\eta} / (1 + \eta),
\]
where \( 1/<gamma> \) is the elasticity of intertemporal substitution, \( \beta_t \) is the Frisch elasticity of labor supply, \( c_t \) is consumption of the final good, \( n_t \) is labor hours, \( E_0 \) is an expectation operator conditional on information available in period \( 0, \beta_0 \equiv 1, \) and \( \beta_t = \prod_{j=1}^{t} \beta_j \) for \( t > 0 \). \( \beta_j \) is a time-varying discount factor that evolves according to
\[
\beta_j = \bar{\beta} (\beta_{j-1}/\bar{\beta})^{\rho_\beta} \exp(\varepsilon_j),
\]
where \( \bar{\beta} \) is the steady-state discount factor, \( 0 \leq \rho_\beta < 1, \) and \( \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \). These choices are constrained by \( c_t + b_t = w_t n_t + i_t - b_{t-1} / \pi_t + d_t, \) where \( \pi_t = p_t / p_{t-1} \) is the gross inflation rate, \( w_t \) is the real wage rate, \( b_t \) is a 1-period real bond, \( i_t \) is the gross nominal interest rate, and \( d_t \) are profits from intermediate firms. The optimality conditions to the household’s problem imply
\[
w_t = \chi n_t^{\eta} c_t^\gamma,
\]
\[
1 = i_t E_t[\beta_{t+1}(c_t / c_{t+1})^{\gamma} / \pi_{t+1}].
\]

3.2 Firms The production sector consists of monopolistically competitive intermediate goods firms who produce a continuum of differentiated inputs and a representative final goods firm. Each firm \( f \in [0, 1] \) in the intermediate goods sector produces a differentiated good, \( y_t(f) \), with identical technologies given by \( y_t(f) = z_t n_t(f) \), where \( n_t(f) \) is the level of employment used by firm \( f \). \( z_t \) represents the level of technology, which is common across firms and follows
\[
z_t = \bar{z}(z_{t-1}/\bar{z})^{\rho_z} \exp(\upsilon_t),
\]
where \( \bar{z} \) is steady-state technology, \( 0 \leq \rho_z < 1 \), and \( \upsilon_t \sim N(0, \sigma_\upsilon^2) \). Each intermediate firm chooses its labor supply to minimize its operating costs, \( w_t n_t(f) \), subject to its production function.

Using a Dixit and Stiglitz (1977) aggregator, the representative final goods firm purchases \( y_t(f) \) units from each intermediate goods firm to produce the final good, \( y_t \equiv [\int_0^1 y_t(f)(\theta - 1)/\theta df]^{\theta/(\theta - 1)}, \) where \( \theta > 1 \) measures the elasticity of substitution between the intermediate goods. The final goods firm maximizes its profits, which determines its demand for intermediate good \( f \), \( y_t(f) = (p_t / p_d)^{-\theta} y_t, \) where \( p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)} \) is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price, \( adj_t(f) \), which emphasizes the negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997), \( adj_t(f) = \varphi[p_t(f) / (\bar{\pi} p_{t-1}(f)) - 1]^2 y_t / 2, \) real profits of firm \( f \) are \( d_t(f) = [(p_t / p_t) y_t(f) - (w_t n_t(f) + adj_t(f))] \), where \( \varphi \geq 0 \) controls the size of the adjustment cost and \( \bar{\pi} \) is the steady-state gross inflation rate. Each intermediate goods firm chooses its price, \( p_t(f) \), to maximize the expected discounted present value of real profits \( E_t \sum_{k=t}^{\infty} \lambda_{t,k} d_k(f), \) where \( \lambda_{t,t} \equiv 1, \lambda_{t,t+1} = \beta_{t+1}(c_t / c_{t+1})^{\gamma} \) is the stochastic pricing kernel between periods \( t \) and \( t + 1 \) and \( \lambda_{t,k} \equiv \prod_{j=t+1}^{k} \lambda_{j-1,j}. \) In a symmetric equilibrium, all intermediate goods firms make the same decisions and the optimality condition reduces to
\[
\varphi \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = (1 - \theta) + \theta(w_t / z_t) + \varphi E_t \left[ \lambda_{t,t+1} \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right] \frac{\pi_{t+1} y_{t+1}}{\bar{\pi} y_t}. \]
In the absence of price adjustment costs (i.e., $\varphi = 0$), the real marginal cost ($w_t/z_t$) equals $(\theta - 1)/\theta$, which is the inverse of a firm’s markup of price over marginal cost.

### 3.3 Monetary Policy and Equilibrium

Each period, the central bank sets the gross nominal interest rate according to

$$i_t = \max \{1, \bar{r}(\pi_t/\pi^*)^{\phi_y}(y_t/\bar{g})^{\phi_y} \},$$

where $\pi^*$ is the inflation rate target and $\phi_y$ and $\phi_y$ are the policy responses to inflation and output.\(^7\)

The resource constraint is given by $c_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]y_t \equiv y_t^{adj}$, where $y_t^{adj}$ includes the value added by intermediate firms, which is their output minus quadratic price adjustment costs. A competitive equilibrium consists of sequences of quantities $\{c_t, n_t, b_t, y_t^{adj}\}_{t=0}^{\infty}$, prices $\{w_t, i_t, \pi_t\}_{t=0}^{\infty}$, and exogenous variables $\{\beta_t, z_t\}_{t=0}^{\infty}$ that satisfy the household’s and firm’s optimality conditions \((2),(3),(5)\), the production function, $y_t = z_t n_t$, the monetary policy rule \((6)\), the stochastic processes \((1),(4)\), the bond market clearing condition, $b_t = 0$, and the resource constraint.

### 3.4 Calibration and Solution Method

We calibrate the model at a quarterly frequency using values common in the monetary policy literature. The risk-free real interest rate is set to 2 percent annually, which implies a steady-state quarterly discount factor, $\hat{\beta}$, equal to 0.995. The Frisch elasticity of labor supply, $1/\eta$, is set to 3 and the leisure preference parameter, $\chi$, is set so that steady-state labor equals 1/3 of the available time. Our baseline coefficient of relative risk aversion, $\gamma$, is set to 1, but we also consider other values. The price elasticity of demand between intermediate goods, $\theta$, is calibrated to 6, which corresponds to an average markup of price over the real wage rate equal to 20 percent. The costly price adjustment parameter, $\varphi$, is set to 59.11, which is similar to a Calvo (1983) price-setting specification in which prices change on average once every four quarters. In the policy sector, the steady-state gross inflation rate, $\pi$, is calibrated to 1.006 so that the annual inflation rate target is 2.4 percent. We set the monetary response to changes in inflation, $\phi_x$, equal to 1.5 and the response to changes in output, $\phi_y$, equal to 0.1. Mendes (2011) and Richter and Throckmorton (2014) shows that a rational expectations solution requires that the persistence and standard deviation of the stochastic processes are not too large. We set $\rho_{\beta} = 0.8$ and $\sigma_{\beta} = 0.0025$, which are the same values used in Fernández-Villaverde et al. (2012). Steady-state technology, $\bar{z}$, is normalized to 1, and we set $\rho_{z} = 0.9$ and $\sigma_{z} = 0.0025$.

We solve the model using the policy function iteration algorithm described in Richter et al. (2013), which is based on the theoretical work on monotone operators in Coleman (1991). This solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use piecewise linear interpolation to approximate future variables that show up in expectations, since this approach more accurately captures the kink in the policy functions than continuous functions, and Gauss-Hermite quadrature to numerically integrate. Those techniques capture the expectational effects of going to and returning to the ZLB. For a more formal description of the numerical algorithm see the appendix in Richter et al. (2013).

### 4 Theoretical and Empirical Results

We first build intuition on why the ZLB leads to a strong negative correlation between output and uncertainty by assuming technology is fixed, so that the ZLB events are endogenous due to

\(^7\)Clarida et al. (1999) and others establish that interest rate smoothing is empirically relevant. However, we exclude that component since it significantly reduces the number of ZLB events in a simulation. See Dong (2012) for details.
positive discount factor shocks. We also explain how households’ degree of risk aversion affects this relationship. Then we turn to the model with both a discount factor and technology process and show that both processes are necessary to match correlations in the data.

4.1 Model with Constant Technology We begin our analysis by assuming technology is constant (i.e., $z_t = \bar{z}$ for all $t$). In this case, the model contains only one state variable, $\hat{\beta}_{-1}$, which is exogenous and ranges from $\pm 1.85$ percent of its steady-state value. In all our results, a hat denotes percent deviation from steady state (i.e., for some generic variable $x$ in levels, $\hat{x}_t \equiv 100 \times (x_t - \bar{x})/\bar{x}$) and a tilde denotes a percent of steady state (i.e., $\hat{\sigma}_{x,t}(k) = 100 \times \sigma_{x,t}(k)/\bar{x}$).

The top panel of figure 2 plots the decision rules for adjusted output, $\hat{y}_{t+1}^{adj}$, (left panel) and the standard deviation of the forecast error for adjusted output, $\hat{\sigma}_{y_{t+1}^{adj}}$, (right panel) as a function of $\hat{\beta}_{-1}$ for three values of risk aversion: $\gamma = 1$ (solid line), $\gamma = 0.5$ (dashed line), and $\gamma = 2$ (dash-dotted line). The shaded regions indicate where the ZLB binds, which depends on the value of $\gamma$. When $\gamma = 1 (\gamma = 0.5, \gamma = 2)$, the ZLB binds in states where $\hat{\beta}_{-1} > 0.95 (\hat{\beta}_{-1} > 0.89, \hat{\beta}_{-1} > 0.99)$. Our results are based on the one-period ahead ($k = 1$) forecast error, unless specified otherwise.

The other panels of figure 2 plot the probability density function of future adjusted output as a percent deviation from its mean, $100 \times (\hat{y}_{t+1}^{adj}/E_t[\hat{y}_{t+1}^{adj}]) - 1$. The middle panel fixes $\gamma = 1$ and plots the density function for three values of the discount factor state: $\hat{\beta}_{-1} = 0$ (solid line), $\hat{\beta}_{-1} = -1.49$ (dashed line), and $\hat{\beta}_{-1} = 1.49$ (dash-dotted line). The bottom panel fixes $\hat{\beta}_{-1} = 1.49$ so the ZLB binds and plots the density function across the alternative degrees of risk aversion. The density functions are informative because they relate the slope of the policy function for $\hat{y}_{t+1}^{adj}$ to $\hat{\sigma}_{y_{t+1}^{adj}}$. That is, they illustrate how forecast error volatility changes across the state space and across different parameters. We begin by discussing how uncertainty changes across the discount factor states.

The discount factor is a proxy for aggregate demand because it determines households’ degree of patience. When the discount factor is low (high), households are impatient (patient), and less (more) willing to postpone consumption to future periods. Firms respond to the higher (lower) demand by increasing (decreasing) their prices and output. Hence, the policy function for adjusted output is downward sloping (top left panel). In discount factor states where the ZLB does not bind, the slope of the policy function for output is essentially constant, which means the distribution of future adjusted output values is symmetric and virtually identical for states outside the ZLB. The distribution of adjusted output in these states is also narrower than in states where the ZLB binds (middle panel). Thus, the standard deviation of the forecast error for adjusted output ($\hat{\sigma}_{y_{t+1}^{adj}}$) is relatively small and nearly constant across states where the ZLB does not bind (top right panel).

In states where the ZLB binds, demand is low, which causes firms to reduce their prices. With the nominal interest rate stuck at its ZLB and inflation below its steady state, the real interest rate rises sharply, which leads to even lower output than if the ZLB did not bind. A steeper policy function for adjusted output in states where the ZLB binds widens the distribution of possible adjusted output values next period and skews it toward output losses. For example, when $\hat{\beta}_{-1} = 0$, a $\pm 1$ standard deviation discount factor shock (i.e., $\pm 0.25$ percent) causes adjusted output to move from its steady state by $\pm 0.3$ percent. When $\hat{\beta}_{-1} = 1$, the same change in the discount factor can cause output to decrease by $-0.8$ percentage points or increase by $0.4$ percentage points. Thus, the standard deviation of the forecast error for adjusted output sharply increases and is skewed toward output losses in states where the ZLB binds. These results show that when the nominal interest rate is far from its ZLB, the amount of uncertainty that arises endogenously in the model is low.
Figure 2: Policy function for adjusted output (top left panel) and the standard deviation of the forecast error for adjusted output (top right panel). The horizontal axes in these panels display the discount factor state, which is in percent deviations from steady state. The vertical axis in the top left panel shows adjusted output in percent deviations from its steady state. The vertical axis in the top right panel shows the standard deviation of the forecast error for adjusted output as a percent of steady-state adjusted output. The shaded regions in these panels indicate where the ZLB binds. The middle and bottom panels show the density function of future adjusted output. The horizontal axes displays adjusted output as a percentage change from its mean. The vertical axes shows the probability density value.
and independent of the state of the economy. As the nominal interest rate declines, the probability of lower output rises, which increases the amount of uncertainty even before the ZLB binds.

To build more intuition about the relationship between the slope of the policy function and our measure of uncertainty, we now turn to discussing how uncertainty changes across the alternative degrees of risk aversion. When households are more risk averse (i.e., a higher $\gamma$), they are less willing to intertemporally substitute consumption goods. Thus, they are less sensitive to changes in the real interest rate, which means the policy function for adjusted output is relatively flatter across the entire state space, including states where the ZLB binds. For example, if $\beta_{-1} = 1$, then a $\pm 1$ standard deviation discount factor shock causes adjusted output to range from $-0.5$ to $-1.1$ percent below its steady state when $\gamma = 2$ and from $-1.4$ to $-4.1$ percent when $\gamma = 0.5$. A much flatter policy function for adjusted output leads to a narrower and more symmetric distribution for future adjusted output (bottom panel), which explains why households face less uncertainty at and away from the ZLB when they are more risk adverse. Regardless of the value of $\gamma$, however, the amount of uncertainty is unaffected by discount factor shocks when the nominal interest rate is far from its ZLB and increases sharply when it approaches and eventually hits its ZLB.

Next, we simulate the model for 500,000 quarters and plot a 100 quarter snapshot of the simulation where there is a long ZLB event. The ZLB binds in 1.2 percent of quarters in the entire simulation. The top panel of figure 3 plots the simulated paths of adjusted output, $y^{adj}$, (left axis,
solid line) and the standard deviation of the forecast error for adjusted output, $\tilde{\sigma}_{y^{adj}}$ (right axis, dashed line), which is yet another way to visualize the correlation between output and endogenous uncertainty in the model. The bottom panel plots the path of the nominal interest rate (solid line) and the notional interest rate—the interest rate that would occur in the absence of a ZLB constraint (dashed line). The shaded region indicates periods when the ZLB binds. At the beginning of the simulation, the ZLB binds for 11 consecutive quarters, which means the notional interest rate is negative. Outside of the ZLB, the nominal interest rate is equal to the notional interest rate.

There are three important takeaways from this simulation. One, the uncertainty surrounding output is time-varying, both inside and outside the ZLB. When the ZLB does not bind, the level of uncertainty is constant, except in periods when the nominal interest rate is near its ZLB. In those situations, the high probability of hitting the ZLB in the next period leads to higher uncertainty that persists for several quarters. The closer the nominal interest is to the ZLB, the higher the uncertainty, which means expectational effects play a key role. When the ZLB binds, $\tilde{\sigma}_{y^{adj}}$ is as much as three times larger than its value outside the ZLB. The magnitude of the uncertainty depends on the notional (or shadow) interest rate. That value indicates how likely it is for the nominal interest rate to rise and exit the ZLB in the near-term. The lower the value of the notional interest rate, the less likely the economy will exit the ZLB and the higher the uncertainty.

Two, there is a weak negative correlation between $\hat{y}^{adj}$ and $\tilde{\sigma}_{y^{adj}}$ when the ZLB does not bind but a strong negative correlation between those variables when it does bind. The strength of those correlations depends on the likelihood of entering and staying at the ZLB. When the nominal interest rate is far from its ZLB, there is no correlation since the level of uncertainty shows little variation. In periods when the nominal interest rate is near or at its ZLB due to negative discount factor shocks, the level of uncertainty is high and adjusted output is low.

Three, it is possible for uncertainty to decline while the ZLB binds. That outcome occurs when the nominal interest rate is far below zero and there is a negative discount factor shock. The reason this feature of the model is important is because our measures of uncertainty in the data have continued to fluctuate since many central banks reduced their policy rates to the ZLB in 2008.

4.2 Model with Variable Technology

Now assume technology varies across time according to the process in (4). The model contains two state variables, $\hat{z}_{-1}$ and $\hat{\beta}_{-1}$, which range from $\pm 2.5$ and $\pm 1.85$ percent of their steady-state values, respectively. Figure 4 shows the policy function for adjusted output (left panel) and the standard deviation of the forecast error for adjusted output (right panel). The shaded regions indicate where the ZLB binds, which represents 26 percent of the state space. A low (high) level of technology increases (reduces) firms’ marginal cost of production. Firms respond by decreasing (increasing) their production and raising (reducing) their prices. The central bank responds by increasing (decreasing) the nominal interest rate. This means that the nominal interest rate will hit its ZLB given a sufficiently high level of technology. In those states, the model dynamics are similar to what occurs in high discount factor states. That is, lower inflation drives up the real interest rate, causing a sharp decline in adjusted output and a large increase in uncertainty, even though technology is well above its steady state.\(^8\)

The amount of uncertainty surrounding output is mostly unaffected by the level of technology when the nominal interest rate is far from its ZLB. In that situation, uncertainty is extremely stable and low, even when technology and the discount factor are both below their steady-state values.

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\(^8\)For a more thorough description of the dynamics in a model with and without capital see Gavin et al. (2014).
Figure 4: Policy function for adjusted output (left panel) and the standard deviation of the forecast error for adjusted output (right panel). The horizontal (vertical) axes in these panels display technology (the discount factor), which is in percent deviations from steady state. The contours in the left panel display adjusted output in percent deviations from its steady state. The contours in the right panel show the standard deviation of the forecast error for adjusted output as a percent of steady-state adjusted output. The shaded regions indicate where the ZLB binds.

Figure 5: Top panel: simulated paths of adjusted output (solid line) and the standard deviation of the forecast error for adjusted output (dashed line). Bottom panel: simulated path of the nominal interest rate (solid line) and notional interest rate (dashed line). The horizontal axes display the time period. In the top panel, the left vertical axis displays adjusted output in percent deviations from its steady state. The right vertical axis shows the standard deviation of the forecast error for adjusted output as a percent of steady-state adjusted output. In the bottom panel, the vertical axis displays the nominal/notional interest rates as a net percent. The shaded region indicates periods when the ZLB binds.
In sharp contrast, when the nominal interest rate nears its ZLB, regardless of whether it is due to unusually high levels of technology or a high discount factor, forecast error volatility rises. When technology and the discount factor simultaneously increase and move away from their respective steady-state values, adjusted output rapidly declines, which drives up uncertainty at the ZLB. Thus, variable technology represents another source of uncertainty, but only when the ZLB binds.

**Figure 5** shows a 100 quarter window of a 500,000 quarter simulation, where the ZLB binds in 2.5 percent of quarters. The presence of technology increases the volatility of output and the likelihood of spikes in uncertainty, since variable technology is an additional source of movements in the nominal interest rate. The ZLB binds for 18 quarters at the beginning of the simulation and at three other periods in time for a shorter duration. During each ZLB event, adjusted output declines while the level of uncertainty significantly increases. That uncertainty is highest when the notional interest rate is well below zero. Overall, the model with variable technology preserves the simulation properties of the model with constant technology—uncertainty is time varying at and away from the ZLB and it is negatively correlated with changes in output. However, that correlation is weaker because variable technology adds new states where it is possible for output remain unchanged or even fall when uncertainty increases.

### 4.3 Connections Between Model Predictions and the Data

Table 2 shows conditional and unconditional correlations in the model and compares them to analogous correlations in the data. The correlations in the model between log differences in adjusted output and the \( k \)-period ahead standard deviation of the adjusted output forecast error are based on a 500,000 quarter simulation. The “Unconditional” correlations are based on the entire simulation, while the “Out of ZLB” correlations are conditional on \( r > 1 \) and the “In the ZLB” correlations are conditional on \( r = 1 \). The correlations in the data between log differences of RGDP and the respective measure of volatility are based on different data ranges. The “Unconditional” sample uses data from 1986Q1-2013Q4, while the “Out of ZLB” sample ranges from 1986Q1-2007Q4 and the “In the ZLB” sample ranges from 2008Q1-2013Q4. The 1-period ahead model correlations are compared to those based on the VXO and SPF\((1)\), the 2-period ahead model correlations are compared to the BOS, and the \( k \)-period ahead model correlations are compared to the SPF\((k)\), \( k > 1 \). All of the correlations in this section are based on our baseline risk aversion parameter value, \( \gamma = 1 \).

There are several remarkable similarities between the correlations in the model and the data. One, the correlations from both variants of the model are qualitatively similar to the data from the “Unconditional” sample and the “In the ZLB” sample. In both of these samples, the model correlations are negative, and they are more negative with the “In the ZLB” sample.\(^9,10\) Both of these facts are consistent with the data. Two, with the “Out of the ZLB” sample, the model with constant technology fails to predict the weak correlation between the growth rate of output and uncertainty in the data, except for the BOS. However, by adding the technology process to the model, its predictions better match the weak correlations with the VXO and SPF\((k)\). Three, adding the technology process also qualitatively matches the decreasingly negative correlations between

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\(^9\)We also calculated the correlations from the model with interest rate smoothing in the monetary policy rule. The correlations conditional on the small fraction of quarters where the ZLB binds (0.15 percent) are similarly negative as those without interest rate smoothing. Moreover, the unconditional correlation was close to zero, since the interest rate is rarely at or near zero, which supports our finding that there is a weak correlation outside of the ZLB in the data.

\(^{10}\)The risk aversion parameter, \( \gamma \), also affects the conditional correlations in the model. Lowering \( \gamma \) leads to a weaker correlation away from the ZLB and a stronger correlation at the ZLB. Increasing \( \gamma \) leads to the opposite changes.
Table 2: Comparison between conditional and unconditional correlations in the model and the data. The correlations in the model are between log differences of adjusted output and the SD of the adjusted output FE $k$-periods ahead. The correlations in the data are between log differences of RGDP and alternative measures of uncertainty.


(b) Model: Subset of a 500,000 quarter simulation where the ZLB does not bind ($i > 1$). Data: 1986Q1-2007Q4.

(c) Model: Subset of a 500,000 quarter simulation where the ZLB binds ($i = 1$). Data: 2008Q1-2013Q4.

real GDP growth and the SPF($k$) that occur with the “In the ZLB” sample as $k$ increases. Without the technology process, the model predicts these correlations slightly increase as $k$ increases, but with variable technology they decrease as they do in the data. Four, the correlations predicted by the model are closest in magnitude to those based on the SPF($k$), which is a measure of the real GDP forecast dispersion, rather than stock market volatility (VXO) or uncertainty in a particular sector of the economy (BOS). All of these findings provide evidence that the ZLB is a source of the elevated uncertainty in the data since the Great Recession.
5 Conclusion

This paper documents there is a strong negative relationship between macroeconomic uncertainty and real GDP growth since the Great Recession. Prior to that event, the correlation between those two variables was weak, even when conditioning on quarters when the economy was in a recession. Why does the Great Recession lead to a stronger negative correlation compared to previous recessions? One possible answer is the ZLB on short-term nominal interest rates. During the Great Recession many central banks around the world sharply reduced their policy rates and effectively hit the ZLB for the first time in their history. We contend that those policies contributed to the strongly negative correlation between uncertainty and output growth during the Great Recession.

To test this theory, we use a model where the nominal interest rate occasionally hits its ZLB due to technology and discount factor shocks. We find that when the nominal interest rate is far from its ZLB, the level of uncertainty surrounding output is constant and low. However, when the nominal interest rate approaches, and eventually hits its ZLB, uncertainty rises sharply. This result occurs in the model because the dispersion in future output values widens in states of the economy where the ZLB binds (i.e., output is more sensitive to shocks). The magnitude of the uncertainty depends on the notional interest rate, which determines the probability of exiting the ZLB. These results imply that the correlation between output and uncertainty is state dependent—outside of the ZLB the correlations are weak, but they are strongly negative when the ZLB binds. Thus, our model is able to roughly match correlations between uncertainty and real GDP growth in the data, both qualitatively and quantitatively. While it may not be the case that the ZLB is the only factor driving the higher uncertainty, our results provide strong evidence that it is an important factor.

References


