US Fiscal Regimes and Optimal Monetary Policy

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(Preliminary and incomplete)

Abstract

Fiscal policy in the US has been documented to have been the leading authority in the ‘60s and the ‘70s (active fiscal policy), while committing to make the necessary fiscal adjustments following Volcker’s appointment (passive fiscal policy). Moreover, while passive, US fiscal policy has at times fluctuated between raising taxes and cutting expenditure keeping the former relatively stable (Clinton expenditure cuts). I analyze those facts through the lens of three-regime New-Keynesian model with a Blanchard-Yaari structure. Fiscal policy is allowed to switch not only between an active and a passive regime, but also between expenditure cuts and tax increases while being passive. Focusing on determinacy regions (à la Leeper (1991)), I show that the model can capture the fall in US debt-to-GDP ratio of the 70s and its subsequent rise following Volcker’s appointment. Performing beliefs counterfactuals, I show that the debt-to-GDP ratio could have been substantially lower had the US fiscal authorities committed to using federal expenditure as the only policy instrument. Finally optimal monetary policy shows that the Fed should increase its reaction to inflation considerably whenever Federal expenditure becomes the fiscal policy instrument.

Keywords: Markov-switching DSGE, Optimal monetary policy, Agents Beliefs

JEL Codes: E2, E5, E6
1 Introduction

Fiscal policy regime switches in DSGE models have attracted much research over the last years ([8], [4], [7], [3]). The main focus in this literature, so far, is on the resulting equilibria following different monetary and fiscal policy regime mixes and on the different effects of shocks depending on the regime of each policy. There is not much evidence, though, as regards whether and how should monetary policy react to time varying fiscal regimes. This paper addresses those two questions.

In the literature on Markov-switching DSGE (MS-DSGE) models analyzing the monetary and fiscal policy mix, the focus is on two regimes. In the first, monetary policy satisfies the Taylor principle and fiscal policy is committed to keep debt on a stable path. In the second, the fiscal authority does not necessarily respond to fluctuations in the debt-to-GDP ratio, while monetary policy does not satisfy the Taylor principle. In the terminology of [7] these two regimes correspond to Active Monetary/Passive Fiscal (AM/PF) and Passive Monetary/Active Fiscal (PM/AF).

The evolution of the debt-to-GDP ratio in the US, along with that of inflation and the real federal funds rate, shows that the US economy has been switching between a PM/AF and an AM/PF since the late ’50s. In particular, the debt-to-GDP ratio declined smoothly from that period until the late ’70s. During the same period US inflation was very persistent and volatile, while real interest rates were fluctuating at a low level. This period corresponds to the PM/AF, as has been widely documented in the literature ([3, 8, 4]). As the economy is hit by inflationary fiscal shocks (either tax cuts or increased in Federal expenditure) the fiscal authority does not commit to a policy in order to stabilize debt. As a result, the monetary authority has to accommodate the increase in inflation (i.e. Taylor principle is violated), leaving the real interest rate to fall, which, in turn, implies higher growth rates. The increase in the latter, along with the fall in the real interest rate determine a fall in the debt-to-GDP ratio. On the other hand, the US debt-to-GDP ratio started rising since the early ’80s following Volcker’s appointment. At the same time, inflation experienced a sharp
drop accompanied with high real interest rates and a deep recession. This period corresponds to an AM/PF regime. In this regime, the central bank is aggressive to inflation fluctuations, while the fiscal authority is committed to make the necessary fiscal adjustments in order to guarantee the stability of the debt-to-GDP ratio.

Typically, MS-DSGE models are used as beliefs-counterfactuals as in [2] in order to explore how would the economy have behaved had the agents been aware of possible regime shifts. A standard result is that the high US inflation volatility during the ’70s could have been avoided if agents were confident about moving to the AM/PF regime. Although such an exercise is very interesting, particularly for the effects of beliefs on the macroeconomy, it shows the weakness of the current MS-DSGE models to capture the actual historical evolution of the economy. Were agents back in the 70s indeed naive, or they were less forward looking? It important to analyze what monetary policy should have been if the MS-DSGE model is indeed able to describe the historical behavior of inflation and output, rather than being used as a counterfactual exercise. The analysis in the MS-DSGE literature so far, rests on the assumption that the private sector during the ’70s has been naive about the shift to disinflation that was about to happen in the ’80s. A question that arises is whether agents we indeed naive, or not? What if they knew about this possibility, but they have been impatient or they cared about the far future less? The weakness of the current MS-DSGE models to describe the actual behavior of the economy stems from two crucial assumptions. First, in the majority of those models, taxes are assumed to be non-distortionary. Second, agents are assumed to be infinitely lived. The model in this paper departs from the second assumption and generates debt-to-GDP and inflation dynamics that are much closer to the actual data than the ones underlined when agents are infinitely lived.

Given that the majority of the models used so far assumes lump-sum, instead of distortionary, taxes, one of the weaknesses, putting the model at odds with empirical evidence, is that fiscal shocks do not have an impact on inflation, while the impact on output is very small, as long as AM/PF regime lasts longer.$^1$ This result holds independently of the pol-

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$^1$This is a standard result in a fixed regime New-Keynesian model where the AM/PF regime lasts forever. [?] refers to the AM/PF regime as the Ricardian regime and to the PM/AF regime as the non-Ricardian. As shown in [3], this holds true in
icy mix (i.e. regime) in place, the reason being that agents are confident that even if the economy lies currently in the PM/AF regime, the necessary fiscal adjustments will eventually be made. An additional reason behind this result is the assumption of infinitely lived agents. This paper assumes a Blanchard-Yaari economy in which agents are no longer infinitely lived. Under this assumption, in a DSGE model where the AM/ PF mix is the only regime, the Ricardian equivalence no longer holds. Further, in the MS-DSGE of this paper, this assumption implies that fiscal shocks will always affect inflation independently of which policy mix (regime) lasts longer or is in place. Consequently, the central bank needs to take into account the inflationary pressures of fiscal shocks, even though agents know that the AM/ PF regime is the most frequent policy mix. Moreover, as shown, it has to take into account the regime of fiscal policy, as well.

The departure from the assumption of infinitely lived agents implies debt-to-GDP ratio and inflation dynamics that are much closer to the actual data. In fact, when agents are infinitely lived the debt-to-GDP ratio rises substantially relative to actual data, while inflation volatility falls. This is because agents are aware about either the shift to the AM/ PF or that the economy will revert back to that regime even if it departs from it for a number of periods. This means that know that the central bank will shift to fighting inflation aggressively in the future and the fiscal authority will commit again to the necessary fiscal adjustments following fiscal imbalances. This makes inflation less volatile. But, at the same time, agents know that inflation will fluctuate at a considerably lower levels implying a higher debt-to-GDP ratio. On the other hand, when agents are not infinitely lived debt-to-GDP ratio is considerably lower and closer to the actual data, while inflation volatility rises. This is because agents discount future less. Even if they are aware of the shift to the AM/ PF regime, this is not enough to drive inflation volatility down. Hence, even if the monetary and the fiscal authorities are likely to commit to lower inflation and to the fiscal adjustments, respectively, in the near future, agents are less patient and this leads to higher inflation rates and, thereby, a lower debt-to-GDP ratio. Consequently, fiscal shocks have real effects into the economy, implying

\footnote{MS-DSGE model if and only if agents are not aware of regime changes.}
inflationary pressures following tax cuts or increases in federal expenditures.

The optimal policy problem of the monetary authority under discretion is designed, conditional on the fiscal policy regime. For this reason, [16] algorithm is extended to account for regime shifts. Using the values for the structural parameters obtained from the estimation of a similar model for the US by [3], a simple optimal interest rate feedback rule for the Fed is also computed.

The first result from the optimal monetary policy problem is that the Fed should have reacted more aggressively to inflation fluctuations than it did historically, regardless of the fiscal regime. In particular, when fiscal policy is passive, the optimal interest rate rule implies a coefficient on inflation that is substantially higher than the estimated coefficient in [3]. Moreover, even though the optimal interest rate rule suggests deviation from the Taylor principle when fiscal policy is active, the optimal inflation coefficient is again higher than the estimated.

In a counterfactual exercise, the model is simulated under the optimal and the estimated interest rate rule. Under the optimal interest rate rule, the Fed would be able to control inflation better than it did. A substantially higher coefficient on inflation in the Volcker era would have allowed for better anchoring of inflation expectations independently of the fiscal regime. However, due to the presence of a cost-push, a trade off between inflation and output gap stabilization is introduced. This implies that under the optimal interest rate rule, the US output gap is more volatile than it was, historically. Judging, though, the performance of the optimal rule relative to the actual policy of the Fed through a welfare criterion as in [15], it is shown that under the former the US would have been better off in terms of steady state consumption.

The paper is organized as follows. In section 2, a closed economy model with Blanchard-Yaari features for the US is constructed. The model is calibrated and simulated in section 3, while optimal policy design is shown in section 4. Section 5 concludes.
2 The model

The model is a standard New-Keynesian with habits in consumption and endogenous persistence in inflation. In order to study the interaction between fiscal regimes and monetary policy, I develop a Blanchard-Yaari type model of overlapping generations in the spirit of [9]. This specification allows for departures from Ricardian equivalence which implies that debt finance increases in transfers (or decreases in taxes) affects the economy.

Every period new households are born with a fraction $1 - \delta$ of the total population and die with probability $1 - \delta$. Under this structure a debt-finance increase in transfers (or decrease in taxes) will cause a rise in spending since part of the debt will be paid back by future generations. In this respect, the increase in transfers will result in inflationary pressures giving reason for the central bank to intervene in order to keep inflation on target.

Households derive utility from the consumption of goods and supply labor to firms. Each household is the owner of a firm producing a differentiated good. Households receive a wage from labor and profits from firm ownership. Firms operate in a monopolistically competitive market with price stickiness as in [6]. The government imposes lump-sum taxes to households in order to finance its expenditures. The latter is also financed through one period nominal government bond issuance.

2.1 Households

The size of generation $i$ at time $t$ is $(1 - \delta) \delta^{t-i}$ and total population is of measure 1. Households in each generation $i$ choose $\{C_i^t, H_i^t, B_i^t\}$ to maximize

$$U_t = E_t \sum_{s=t}^{\infty} (\beta \delta)^{s-t} \left[ \log(C_s^t - hC_{s-1}^t) - \frac{(H_s^i)^{1+\gamma}}{1 + \gamma} \right]$$

(1)

where $C_i^t, H_i^t$ and $B_i^t$ are consumption, hours worked and government bonds of households of generation $i. \sigma$ is the degree of relative risk aversion and $h$ the degree of habits in consumption. Per capita consumption $C_t$ is a composite consumption index described as
\[ C_t = \left[ \int_0^1 c_t(j)^{\frac{\theta + 1}{\theta}} dj \right]^{\frac{\theta}{\theta - 1}}, \]  

where \( \theta \) captures the intratemporal elasticity of substitution between home and foreign goods. The household in each generation \( i \) chooses \( c_t^i(j) \) to minimize its total expenditure, which implies a demand function for each good \( j \) described by

\[ c_t^i(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\theta} C_t^i, \]  

where \( P_t \) is price index defined as

\[ P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \]  

Capital markets are complete. The household purchases state unstacking bonds \( B_t \) at price \( Q_t \). The budget constraint of the household is summarized as

\[ P_tC_t + Q_tB_t = \frac{1}{\delta} B_{t-1} + W_tH_t^i + \Pi_t - T_t^i \]  

where \( W_t \) is the nominal wage, \( \Pi_t \) are nominal profits that generation \( i \) receives, \( T_t^i \) are lump-sum taxes imposed by the government to generation \( i \) while \( Q_t = R_t^{-1} \).

The first order conditions at an interior solution are written as

\[ H_t^i = C_t^{i-\frac{1}{\delta}} \frac{1}{w_t} \]  

\[ 1 = \beta E_t \left[ \frac{R_tP_t}{P_{t+1}} \left( \frac{X_t^i}{X_{t+1}^i} \right) \right] \]  

2.2 Aggregation

Given the overlapping generations structure of the model a variable \( \varsigma_t^i \) has the following
aggregate representation
\[ \zeta_t = \sum_{i=-\infty}^{t} (1 - \delta) \delta^{t-i} \zeta_i \]

Therefore, the aggregate representation of the Euler equation receives the following form
\[ \beta E_t \frac{R_t P_t}{P_{t+1}} X_t = E_t \left[ \frac{(1 - \delta) B_t}{\delta P_{t+1}} \right] + E_t X_{t+1} \] (8)

The aggregate budget constraint is specified as
\[ P_t C_t + Q_t B_t = B_{t-1} + W_t H_t + \Pi_t - T_t + TR_t \] (9)

2.3 Firms

Each firm is the only producer of its good and sets its price in a staggered way as in [6] with a linear production technology
\[ Y_t(j) = A_t L_t(j) \] (10)

where \( A_t \) is a country specific productivity shock at date \( t \) which is assumed to follow a log stationary AR(1) process. Given the Calvo price setting mechanism the price level can be summarized as
\[ P_t = \left[ \omega P_{t-1}^{1-\theta} + (1 - \omega) \bar{p}_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}} \] (11)

At each date, each firm changes its price with a probability \( 1 - \omega \), regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is \( \omega \). There are two kinds of firms, a fraction \( 1 - \zeta \) of backward looking and a fraction \( \zeta \) of forward looking as in [1]. When they reset their price, backward looking firms do not solve any maximization problem, but follow a rule-of-thumb specified as.
\[ p_t^B(j) = P_{t-1} + \pi_{t-1} \] (12)
On the other hand forward looking firms set their price by maximizing the expected discounted value of their profits

\[
max E_t \sum_{s=0}^{\infty} \omega^s Q_{t,s+h} \left\{ p_t^F(j)y_{t+h}(j) - (1 - \tau)W_{t+h}L_{t+s} \right\}
\]

(13)

Therefore the newly set price at date \( t \) is a weighted average

\[
\tilde{p}_t(j) = (1 - \zeta)p_t^B(j) + \zeta p_t^F(j)
\]

(14)

2.4 Fiscal authority

The fiscal authority imposes lump-sum taxes and issues government debt in order to finance expenditures and pay back its debt. The flow budget constraint of the federal government is given by:

\[
B_t = B_{t-1}(1 + r_{t-1}) - T_t + E_t
\]

where \( B_t \) is government debt, \( T_t \) is lump-sum taxes and \( E_t \) is federal expenditures assumed to follow a stationary AR(1) process. Expressing the variables as an output ratio the flow budget constraint receives the following form:

\[
b_t = \left( b_{t-1}(1 + r_{t-1}) \right) / \left( \Pi_t Y_t/Y_{t-1} \right) - \tau_t + \epsilon_t
\]

where all variables are expressed as a fraction of output (GDP), while \( \Pi_t \) is CPI inflation.

2.5 Monetary and Fiscal rules

In this section I describe how Markov switching is introduced into the model. A Markov-switching interest rate rule is specified as

\[
R_t = R_{t-1}^\rho \left( \frac{\pi_t}{\pi} \right)^{\phi_x} x_t^{\phi_x} \left( 1 - \rho \right) e^{\xi_{t,t}}
\]

(15)
while a Markov-switching fiscal rule is specified as

\[
T_t = T_{t-1}^{\rho_t(s_t^*)} \left[ \left( \frac{B_t}{Y_t} \right)^{\gamma_b(s_t)} \left( \frac{G_t}{Y_t} \right)^{\gamma_g(s_t)} \right]^{1-\rho_t(s_t^*)} x_{t-1}^{\gamma_x(s_t)} e^{\varepsilon_{t,t}}
\]

(16)

where \(s_t\) captures the realized policy regime taking values 1 or 2. Specifically, an active monetary policy regime is characterized by \(s_t = 1\) and \(\phi_{\pi}^{s,1} > 1\), while a dovish regime by \(s_t = 2\) and \(0 < \phi_{\pi}^{s,2} < 1\). A passive fiscal regime is characterized \(s_t = 1\) and \(\gamma_{b}(s_t) > 0\), while an active by \(s_t = 1\) and \(\gamma_{b}(s_t) = 0\). Monetary and fiscal policy regime follow a Markov process with transition probabilities \(p_{ji} = P[s_t = i | s_{t-1} = j]\), where \(i, j = 1, 2\). For simplicity I assume that the two policies change regimes simultaneously. Agents know these probabilities. \(\xi_{st}^*\) is a scale parameter, \(\tilde{\pi}\) is the inflation target and \(x_t\) is the output gap, defined as the deviation of output from its natural rate, which corresponds to the efficient flexible price equilibrium. This specification implies that the policy maker and the private sector do not observe the current regime. Therefore, private sector expectations about future inflation, for example, are specified as \(E[\pi_{t+1} | \Omega_t^{-s}]\), where \(\Omega_t^{-s} = \{s_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_t^*, \varepsilon_{t-1}^*\}\) captures its information set. Having assumed a two regime Markov process for monetary policy, the transition probability matrix \(P\) receives the form

\[
P = \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\]

where \(p_{11}\) measures the probability of staying at date \(t\) in regime 1 and \(p_{12}\) the probability of moving to regime 2 at date \(t\) while being in regime 1 at date \(t - 1\). \(p_{22}\) measures the probability of staying in regime 2 at date \(t\) and \(p_{21}\) the probability of moving to regime 1 at date \(t\) while being in regime 2 at date \(t - 1\).

2.6 The model in log-linear form

The model is log-linearized around the zero inflation unique steady state.\(^2\) The set of

\(^2\)In a separate appendix I provide the conditions that are necessary and sufficient to guarantee that the steady state is unique and independent of regime switches.
log-linearized equilibrium conditions are summarized in the following table:

<table>
<thead>
<tr>
<th>Log-linearized equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phillips Curve</strong></td>
</tr>
<tr>
<td>( \pi_t = \gamma^f E_t \pi_{t+1} + \gamma^b \pi_{t-1} + \kappa \hat{x}_t )</td>
</tr>
<tr>
<td><strong>Demand curve</strong></td>
</tr>
<tr>
<td>( \dot{Y}<em>t = E_t \hat{Y}</em>{t+1} - (i_t - E_t \pi_{t+1}) + \frac{1-\delta}{\delta(1-g_y)} b_t + \frac{1}{1-g_y} E_t (\hat{g}_{t+1} - \hat{g}_t) )</td>
</tr>
<tr>
<td><strong>Government Budget Constraint</strong></td>
</tr>
<tr>
<td>( b_t = \frac{1}{\beta} b_{t-1} + \frac{1}{\beta} b(i_{t-1} - \pi_t - \hat{Y}<em>t + \hat{Y}</em>{t-1} - a_t) - \hat{\tau}_t + \hat{g}_t )</td>
</tr>
<tr>
<td><strong>Monetary policy rule</strong></td>
</tr>
<tr>
<td>( i_t = \rho i_{t-1} + (1-\rho) (\phi_x(s_t) \hat{x}<em>t + \phi_x(s_t) \pi_t) + \sigma_i \varepsilon</em>{i,t} )</td>
</tr>
<tr>
<td><strong>Fiscal rule</strong></td>
</tr>
<tr>
<td>( \tau_t = \rho_q (s_t^F) \tau_{t-1} + (1 - \rho_q) [\gamma_b(s_t) b_{t-1} + \gamma_g(s_t) g_t] + \gamma_x(s_t) x_{t-1} + \sigma_{\tau} \varepsilon_{\tau,t} )</td>
</tr>
<tr>
<td><strong>Resource constraint</strong></td>
</tr>
<tr>
<td>( \dot{Y}_t = \hat{C}_t + \frac{1}{1-g_y} \hat{g}_t )</td>
</tr>
</tbody>
</table>

### 3 Calibration

Given the Markov-Switching structure of the model, standard solution techniques cannot be applied in order to find a solution. In the recent literature on Markov-switching DSGE models, various alternative techniques for solving such models have been suggested ([7], [11], [17], [10]). The technique I use is that of [10]. The virtue of that technique is that it is able to find all possible minimal state variable (MSV) solutions. Moreover, the algorithm is able to find whether the MSV solution is stationary (mean square stable).\(^3\) The model can be written in the following form

\[
A(s_t) X_t = B(s_t) X_{t-1} + \Psi(s_t) \varepsilon_t + \Pi(s_t) \eta_t \tag{17}
\]

where \( \varepsilon_t \) is a \( 4 \times 1 \) vector of i.i.d. stationary exogenous shocks and \( \eta_t \) is an \( 2 \times 1 \) vector.\(^3\)

\(^3\)For an extensive argument regarding the merits of the solution technique used in this paper over the alternative ones see Farmer et al. (2011) and the references therein.
of endogenous random variables. According to that technique the MSV equilibrium of the model takes the form

$$X_t = g_{1,s_t}X_{t-1} + g_{2,s_t}\varepsilon_t$$  \hspace{1cm} (18)$$

In order for the above minimal state variable solution to be stationary it must be that the eigenvalues of

$$(P \otimes I_{24})\text{diag} [\Gamma_1 \otimes \Gamma_1, \Gamma_2 \otimes \Gamma_2]$$

where $\Gamma_j = A(j)V_j$ for $j = 1, 2$. And where $V_j$ is a $11 \times 6$ matrix resulting from the Schur decomposition of $A(j)^{-1}B(j)$. In the present model the largest eigenvalue was found to be equal to 0.81, implying, thus, that the MSV solution is stationary. The impulse responses and the moments of the variables of interest are then derived from that stationary solution.

Before the design of the optimal monetary policy, the model is calibrated using the estimated monetary and fiscal feedback rules from . The values of the model parameters are summarized at table 1 below.

### 3.1 Impulse Responses

The impulse responses, illustrated in figures 1, 2,3 and 4 report the impulse responses of inflation, output, debt-to-GDP ratio and tax revenues as a fraction of GDP. The responses reported are those from the standard New Keynesian model (blue lines) where agents are infinitely lived ($\delta = 1$) and those from the benchmark model in which agents are not infinitely lived ($\delta = 0.97$). The reason of this comparison is to show the different debt dynamics resulting from relaxing the assumption of infinitely lived agents.

As mentioned in the introduction MS-DSGE models with infinitely lived agents are able to model regime shifts, but they are used rather as counterfactual models. That is, as models in order to explore what the path of the economy would have been had agents taken into account the possibility of a future regime shift. As a result, those models as they stand they are unable to capture the historical behavior of key macroeconomic variables. Why do we need to make the assumption that agents did not take into account the possibility of
a regime change? In this section, it is shown that by relaxing the assumption of infinitely lived agents one can, first, account for the fact that agents are not naive as regards future policy shifts and, second, that the MS-DSGE benchmark model can capture the behavior of the US economy quite satisfactorily. The key message is that even though agents are aware of possible policy shifts, this does not necessarily has as important effects as it would had they been assumed to be infinitely lived. What is crucial, it is the duration of the regime. As long as, the average duration of their active years lasts on average less than the average duration of a specific regime, then the effects of a possible future regime shift are much less pronounced than if they were infinitely lived. In the latter case, they are certain that they will experience the regime shift and its effects. Therefore, the effects of a regime shift are showing up even before the regime change. However, if they are not infinitely live, then they are not certain that they will experience the regime shift and its effects. Consequently, they do not necessarily change their intertemporal decisions, which implies completely different dynamics for inflation, output and debt-to-GDP ratio.

**Monetary policy shock.** The first line in each figure reports the responses to a contractionary monetary policy shock. In the AM/PF regime inflation and output fall in both models. The debt-to-GDP ration rises persistently, while tax revenues fall. The rise in the debt-to-GDP ratio is caused by the rise in the real interest rate and the slowdown in economic activity. The latter also causes the short fall in tax revenues. As the effects of the shock tend to die out, inflation starts to revert back to its steady state and the real interest rate starts to fall pushing tax revenues upwards and initiating the debt repayment. The responses in the benchmark model do not differ after all shocks but after tax cuts and increases in government expenditure. Under the PM/AF regime inflation rises in the benchmark model while it falls in the standard MS-DSGE model. This is because in the benchmark case agents are not infinitely lived and they know that the monetary authority will allow inflation to rise in order for the debt process to be stabilized. When agents are assumed to be infinitely lived instead, inflation falls because they expect the economy to shift in the near future to the AM/PF regime, pushing thus inflation downwards following a monetary policy shock. In
fact, as the expectation of a switch to the AM/PF regime in the future goes to zero, inflation rises instead of falling. Hence, in the benchmark case, the fact that agents average effective years are on average less than the average duration of the PM/AF regime makes the effects of the future switch to the AM/PF regime weaker. As regards the debt-to-GDP ratio, its response to a monetary policy shock is significantly dampened in the PM/AF regime. When agents are not infinitely lived they do not expect to be alive once the economy switches to the AM/PF regime. Therefore, they do not expect inflation to be less volatile any time soon in the near future. Also they expect low real interest rates which push the debt-to-GDP further down making it less vulnerable to all kinds of shocks in the PM/AF regime. Tax revenues increase persistently in the quarters after the shock. This is driven by the fact that output stays above its steady state for more than 16 quarters, implying thus a persistent increase in tax revenues. The rise in inflation following the monetary policy shock and the weak response to inflation, keeps the real interest rate negative for a long period leading to the persistent rise in economic activity and, hence, in tax revenues.

**Government expenditure and Tax revenues.** In the standard MS-DSGE model inflation and output are completely unaffected following fiscal shocks. This is because agents know that the fiscal authority is committed to perform the necessary fiscal adjustments in order for debt to be repaid. This is also because agents are infinitely lived and the AM/PF regime lasts longer than the PM/AF regime. On the other hand in the benchmark model the real economy is affected following fiscal shocks. As also explained above, this is because agents discount the future a lot (or are less patient than in the standard MS-DSGE model), making thus the real economy sensitive to fiscal shocks independently of which regime is the one that dominates. As regards debt-to-GDP ratio its responses are again dampened in the benchmark case relative to the standard MS-DSGE model. Tax revenues response, though, is amplified following fiscal shocks. This is due to the fact that fiscal shocks affect the real economy in the benchmark model. The rise in output due to tax cuts or increased federal expenditure is further strengthened by the rise in the debt-to-GDP ratio (see equation (8)) a

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4Here, I follow the terminology of [3] where dominant is the regime with the higher duration on average.
channel that is shut-off when agents are infinitely lived. Since agents are not infinitely lived they do not expect the fiscal authority to start reacting to debt fluctuations any time soon. Hence, this boosts economic activity even more, leading to higher tax receipts.

**Productivity shock.** As in an standard NK model inflation falls after a positive productivity shock, while output rises. Again, the responses of all the variables do not differ significantly in the AM/PF regime. At the PM/AF regime, thought, the differences in variables responses are non-negligible. Output response is dampened, while inflation rises persistently in the periods following the shock. This is the reason behind the amplified response of the debt-to-GDP ratio in this regime. It falls for 3 quarters after the shock and then reverts back to the steady state much faster than in the standard MS-DSGE model. When in the PM/AF regime, agents do not expect the central bank to start being more aggressive to inflation fluctuations any time soon in the near future. This leads to the persistent rise in inflation and hence to the significantly dampened response of the debt-to-GDP ratio relative to the standard MS-DSGE model. As in the previous shocks, the response of tax revenues is again amplified. They rise persistently in the periods after the shock. As with fiscal shocks this because it takes output a larger number of periods to revert back to the steady state in the benchmark model than in the standard MS-DSGE model.

### 3.2 Model simulation

In this section the model is simulated for 180 periods. The PM/AF regime is assumed as the initial regime. In the simulation exercise a random regime change is allowed which takes place in period 40, where the economy switches to the AM/PF regime. Figure 5 depicts the paths of inflation, output gap and debt-to-GDP ratio from both the benchmark and the standard MS-DSGE model.

In the figure it is clear that inflation in the benchmark model (red dashed line) is considerably more volatile. The assumption of infinitely lived agents instead makes inflation less volatile (blue line), since agents are certain about the switch to the AM/PF regime. Consequently, the effects of a possible switch to the latter regime on agents inflation expectations
become apparent even before the regime change date. On the other hand, when agents are
not infinitely lived and even though certain about the shift to the AM/PF regime, they know
that they are less likely to live until the actual regime change takes place. This implies that
the effects of the expectation of the switch to the AM/PF become weaker while the economy
is still at the PM/AF regime causing a higher inflation volatility.

The debt-to-GDP ratio fluctuates at considerably lower level in the benchmark model. In
particular, when agents are infinitely lived the debt-to-GDP ratio is higher in both regimes.
In the PM/AF regime the ratio is falling in the benchmark model until the regime change
date (period 40). In the standard MS-DSGE model instead, the ratio rises for 20 periods,
then falls for the next 20 until the regime change date and then jumps abruptly to 95%.
Overall, the debt-to-GDP ratio is considerably higher when agents are infinitely lived for
140 periods where it falls below that from the benchmark model. This is driven by the fact
that infinitely lived agents are certain about probability of the switch to the AM/PF regime
along with the fact that this regime is the most frequent one. Hence, they expect inflation
to be fluctuating less in the near future, which implies that the central bank will not be
accommodating higher inflation so that debt to be stabilized.

The simulated series for the debt-to-GDP ratio are plotted along with the actual data
series. The data series correspond to the period spanning from November 1969 to May
2012. The debt-to-GDP ratio implied by the benchmark model captures the dynamics of
the actual series more satisfactorily than the debt-to-GDP ratio implied by the standard
MS-DSGE model. Therefore, the assumption of agents that are not infinitely lived brings
the model closer to the actual data series. The novelty of this result rests on the fact that the
benchmark MS-DSGE model does not work as a counterfactual model. Even though agents
are aware of a possible shift to another regime, the fact that they are infinitely lived implies
that they care less about the far future. This turns out to be a necessary element to make
the MS-DSGE model able to match with the actual data better.
3.3 Counterfactual exercise

In this section, two counterfactual exercises are performed. In the first the benchmark model is simulated assuming that the AM/PF regime stays forever and the evolution of inflation and debt-to-GDP ratio dynamics are compared to those from the baseline calibration. In the second, the same exercise is performed but now assuming that the PM/AF regime stays forever.

3.3.1 Fixed AM/PF regime

In this section the model is simulated assuming that the AM/PF regime is fully credible. That is, assuming that this policy mix will stay forever. What is interesting to explore is how important is this assumption about agents beliefs and, hence, for the evolution of inflation and the debt-to-GDP ratio.

The left panel of figure 6 illustrates the path of inflation under the fixed AM/PF regime (blue line) along with the path of inflation from the baseline calibration. Clearly, inflation would have been considerably less volatile had the two authorities adopted the AM/PF mix forever. Not surprisingly, the fact that the central bank always reacts aggressively to inflation fluctuations, while the fiscal authority commits to the necessary fiscal adjustments keep inflation lower compared to the baseline calibration. The consequence, though, of low inflation is a higher debt-to-GDP ratio. The central bank does not need to allow for higher inflation to stabilize debt, implying positive real interest rate, which, in turn, keeps debt-to-GDP ratio at a higher level than in the benchmark case.

3.3.2 Fixed PM/AF regime

In this section the model is simulated assuming that the PM/AF regime is fully credible. In this case, the debt-to-GDP ratio would have been considerably lower than that from the baseline calibration, as shown in the right panel of figure 6. This is due to the fact that the monetary authority is accommodating the higher inflation, keeping real interest rates low. The latter pushes growth upwards leading to higher tax revenues, implying further downward
pressures to the debt-to-GDP ratio. The effect is further enhanced by the low real interest rate. Looking at the evolution of inflation, it is clearly more volatile than if the economy was switch between the two regimes. Again, this is the trade off that the economy has to incur in order to keep a low debt-to-GDP ratio.

4 Optimal monetary policy when fiscal policy switches regimes

The monetary authority seeks to minimize the welfare loss derived from a second order approximation to the utility function of the representative household in the spirit of [13]. The welfare criterion, thus, is specified as follows:

$$L_t = \pi_t^2 + \chi_x x_t^2 + \chi_\pi \Delta(\pi_t - \pi_{t-1})^2 + \chi_\Delta \Delta(y_t - y_{t-1})^2 + \lambda i_t^2$$

where the weights $\chi_\pi$, $\chi_x$ and $\chi_\Delta$ are functions of the structural parameters of the model, while the term $\lambda i_t^2$ is an ad hoc interest rate stabilization objective.\(^\text{5}\) Note, that given the cashless economy assumed, the second order welfare loss function does not assign a weight on interest rate fluctuations. However, the latter are introduced in an ad hoc manner in the second order welfare criterion. This is so because the Central bank is also interested in minimizing the variability in the federal funds rate.

The focus in this paper is on optimal discretionary policy. Under discretion the central bank takes the future path of variables as given. However, the present model introduces persistence in output and inflation. This implies that the central bank actions today affect the path of the variables tomorrow, even though, a discretionary policy is followed.\(^\text{6}\) Consequently, the optimal policy problem of the central bank can be solved using dynamic programming. The approach followed is that of [5], while the algorithm to solve the optimal policy problem is that of [16] extended to account for regime switches.

**Formulation.** The policy maker chooses the control $i_t$ (i.e. the interest rate rule) which minimizes the expected value of the intertemporal loss function, stated in the previous section

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\(^{5}\) A detailed derivation of the welfare criterion is provided in a separate appendix and is available upon request.

\(^{6}\) For a more detailed analysis of this issue under discretion see [18]
and summarized as

$$\sum_{t=0}^{\infty} \beta^t L(h_t, i_t)$$

subject to $h_0, s_0$ given, and the model describing the economy

$$h_{t+1} = A(s_{t+1})h_t + B(s_{t+1})i_t + C\varepsilon_{t+1} \quad t \geq 0$$

where $L(h_t, i_t)$ is the period loss function, $\beta$ is the discount factor, $h_t$ is a $11 \times 1$ vector of state variables, $i_t$ is the control variable (i.e. the interest rate) and $\varepsilon_t$ is a $4 \times 1$ vector of white noise shocks with variance covariance matrix $\Sigma_\varepsilon$ and $C$ is a $11 \times 4$. The loss function (13) can be conveniently expressed as follows

$$L(h_t, i_t) = h_t'Rh_t + i_tQi_t$$

where $R$ is a $11 \times 11$ positive definite matrix and $Q$ is a scalar. The matrices $A$ and $B$, as already mentioned, are stochastic and take on different values depending on the regime $s_t, t = 1, 2$.

The Bellman equation. The policy maker in a Markov-switching environment needs to find the interest rate rule that is state-contingent. This rule describes the way that the control variable, the interest rate, should be set as a function of both the state variables and the regime occurring at date $t$. Therefore, a Bellman equation is associated with each regime. The regime $j$ dependent Bellman equation is specified, thus, as follows

$$V(h_t, j) = \max_{i_t} \left\{ L(h_t, i_t) + \beta \Sigma_{i=1}^{2} p_{ji} E_t [V(h_{t+1}, i)] \right\}$$

where $V(h_t, j)$ is a function of the state variables $h_t$, the regime prevailing at date $t$ and represents the continuation value of the optimal dynamic programming problem at $t$. The value function for this problem is

$$V(h_t, j) = h_t'P_jh_t + d_j, \quad j = 1, 2$$
where \( P_j \) is a \( 11 \times 11 \) symmetric positive semidefinite matrix, while \( d_i \) is a scalar. The optimal policy is described by

\[
i(h_t, j) = -F_j h_t, \quad j = 1, 2
\]

where \( F_j \) is a \( 11 \times 1 \) matrix, depending on \( P_j \). That is, matrix \( F_j \) specifies the coefficients in the policy rule of the central bank. Those coefficients are regime specific. Maximizing, thus, the Bellman subject to the constraints, the matrix \( F_j \) is specified as

\[
F_j = \left( Q + \beta p_{j1} B'_1 P_i B_1 + \beta p_{j2} B'_2 P_i B_2 \right)^{-1} \beta \left( p_{j1} A'_1 P_i B_1 + p_{j2} A'_2 P_i B_2 \right)
\]

where matrix \( P_i \) has been already determined by a set of interrelated Riccati equations, which specify a system with the following form

\[
P_j = R + \beta p_{j1} A'_1 P_i A_1 + \beta p_{j2} A'_2 P_i A_2 - \ldots
\]

\[
-\beta^2 \left( p_{j1} A'_1 P_i B_1 + p_{j2} A'_2 P_i B_2 \right) \left( Q + \beta p_{j1} B'_1 P_i B_1 + \beta p_{j2} B'_2 P_i B_2 \right)^{-1} \left( p_{j1} B'_1 P_i A_1 + p_{j2} B'_2 P_i A_2 \right)
\]

4.1 An implementable rule

Given the structure of the dynamic programming problem, the optimal interest rate rule described by (37) includes 24 elements. This means that the home central bank must react to all the endogenous variables of the model. Clearly such a rule is not implementable. In this section I look at a simple interest rate rule that can either replicate the optimal allocation or generate losses that are negligible relative to the optimal allocation.

The task of specifying an implementable rule is challenging given the complexity of the model. This is because of three reasons. First, the rule must be a time varying one. Such a rule must either replicate the optimal allocation or generate negligible welfare costs in both regimes. Second, the coefficients in the rule must be such that determinacy is achieved in
each regime conditionally. Third, the rule must be such that the MS-DSGE model has a stationary minimum state variable solution.

I experiment with a number of different simple interest rate rules. The family of rules considered includes the standard Taylor rule (either lagged of forward looking), a strict inflation targeting rule, nominal income rules in the spirit of [14], and Taylor rules expanded by international variables (i.e. real exchange rate and/or relative prices). For each of these rules I do a grid search. Specifically, I limit my attention to policy coefficients in the interval [0, 5]. In each step of the search I compute the associated welfare costs. The only rule that is able to generate nearly zero welfare costs is one in which the home nominal interest rate reacts to inflation and the lagged output gap. It is specified as follows

\[ i_t = \rho i_{t-1} + (1 - \rho) (\phi_p (s_t) \pi_t + \phi_x (s_t) \hat{x}_t) \]  

The values of the coefficients and the resulting welfare costs relative to the optimal discretionary policy are summarized at table 2 below. Welfare costs are defined as the percentage increase in steady state consumption that would make the household indifferent between the allocation under rule (29) and the optimal allocation. Determinacy is achieved in each regime conditionally and the corresponding minimum state variable solution is stationary with a maximum eigenvalue of 0.9682.

### 4.2 Simulation under the optimal simple rule

In this section a counterfactual exercise is performed in order to assess the evolution of inflation and output under the optimal rule relative to the the interest rate rule estimated by [3]. Specifically, the model is initially simulated under the baseline calibration as described at table 1. In order to explore how inflation and the output gap would have behaved had the Fed adopted the optimal interest rate rule (26), the model is simulated after imposing the latter.

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7The size of the interval is arbitrary. I do not consider negative values as they are hard to interpret.

8A detailed description of the calculation of the welfare costs is provided in the appendix.
In the simulation exercise the PM/AF regime is assumed as the initial state and the model is simulated for 180. A random regime change date is allowed, which takes place after 40 periods (quarters). From period 40 onwards the economy switches to the AM/PF regime. The simulated paths for inflation and output gap under each interest rate rule are illustrated in figure 7.

Inflation and output fluctuate less under the optimal rule (26) relative to the interest rate rule in the baseline calibration. Note, though, that due to the cost push shock, the central bank trades off lower inflation volatility for higher output volatility in the AM/PF regime. The opposite holds for output and inflation volatility trade off in the PM/AF regime.

Finally, the conditional volatilities of inflation, output under the optimal interest rate rule relative to their counterparts from the baseline calibration are computed and summarized at table 3 below. Inflation is 0.8958 times (or 10%) less volatile under the optimal rule, while the output is 0.6942 times (or 33%) less volatile. On the other hand, output is slightly less volatile (approximately 2%) in the PM/AF regime, while inflation is 30% less volatile.

5 Conclusions

A Blanchard-Yaari closed economy model for the US is developed in order to determine the optimal policy of the Fed when fiscal policy switches regimes over time. Using the values for the interest rate and the fiscal feedback rule coefficients as estimated by [3], I show that inflation and output gap are more volatile when active fiscal policy is accompanied by a passive monetary policy.

Analyzing the optimal policy of the Fed conditional on the fiscal regime, it is shown that the Fed should have reacted more aggressively to inflation fluctuations whenever fiscal policy was estimated to be passive. Moreover, its reaction to inflation should have been more aggressive as well, whenever fiscal policy was estimated to be active, even though the Taylor principle is not satisfied. Under such policy, the Fed could have achieved a better control of inflation, giving up some output gap stabilization. Assessing though the performance of the
optimal policy using an appropriate welfare criterion, it is shown that should the Fed had followed the underlying optimal interest rate rule, the US would be better off.
References


Table 1: Baseline calibration parameter values

<table>
<thead>
<tr>
<th>Structural Parameters</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>elasticity of substitution</td>
</tr>
<tr>
<td>$1/(1-\delta)$</td>
<td>planning horizon</td>
</tr>
<tr>
<td>$\omega$</td>
<td>price stickiness</td>
</tr>
<tr>
<td>$h$</td>
<td>habit parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>forward looking firms</td>
</tr>
</tbody>
</table>

Monetary Policy Rule Coefficients

Active: $\phi_{\pi,1} = 2.3522$  $\phi_{x,1} = 0.1527$  $\rho_1 = 0.8132$
Passive: $\phi_{\pi,2} = 0.6244$  $\phi_{x,2} = 0.3716$  $\rho_2 = 0.8480$

Fiscal Policy Rule Coefficients

Active: $\phi_{b,1} = 0.0000$  $\phi_{G,1} = 0.7045$  $\phi_{x,1} = 0.0869$  $\rho_1 = 0.8921$
Passive: $\phi_{b,1} = 0.0327$  $\phi_{G,1} = 0.7045$  $\phi_{x,1} = 0.0869$  $\rho_1 = 0.7306$

Probabilities

$p_{AM/\pi F} = 0.99$  $p_{PM/\pi F} = 0.985$
### Table 2: Implementable Rule

<table>
<thead>
<tr>
<th></th>
<th>$\phi_\pi$</th>
<th>$\phi_x$</th>
<th>$\rho$</th>
<th>Welfare Costs</th>
</tr>
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<tbody>
<tr>
<td>AMPF</td>
<td>3.53</td>
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<td>0.88</td>
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<tr>
<td>PMAF</td>
<td>0.96</td>
<td>0.35</td>
<td>0.88</td>
<td>$3 \times 10^{-6}$</td>
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### Table 3: Relative Volatilities

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM/PF</td>
<td>0.8958</td>
<td>0.6942</td>
</tr>
<tr>
<td>PM/AF</td>
<td>0.7079</td>
<td>0.9729</td>
</tr>
</tbody>
</table>
Figure 1 - Inflation impulse responses
Figure 2 - Output impulse responses

A M / P F

MP

TAX

DEMAND

PRODUCTIVITY

COST PUSH

EXPENDITURE

Quarters

4 8 12 16 20

Quarters

4 8 12 16 20

P M / A F

0

0

0

0

0

0

0

0
Figure 3 - Debt-to-GDP impulse responses
Figure 4 - Tax Revenues impulse responses
Figure 5 -

Inflation

Output

Debt-to-GDP
Figure 6 - Counterfactual Exercise

In Debt-to-GDP, the counterfactual exercise shows a comparison of different policy scenarios. The blue line represents the Fully Credible AM/PF, the green line represents the Benchmark, and the gray line represents the PM/AF. The graphs display the values over time, with the x-axis representing time periods and the y-axis representing the percentage of GDP.

In Inflation, similar comparisons are made. The blue line represents the Fully Credible PM/AF, the green line represents the Benchmark, and the gray line represents the PM/AF. The graphs display the inflation rates over time, with the x-axis representing time periods and the y-axis representing inflation rates.
Figure 7 - Inflation and Output Gap under the Optimal Rule
Appendix

Derivation of the Welfare cost measure

In order to gauge the importance of reacting optimally conditional on the regime of fiscal policy, I compute the welfare costs associated with a suboptimal interest rate rule as the one used in the baseline calibration, obtained from the estimates of [3]. Welfare costs are computed as in [12]. In particular, I compute the percent increase in steady state consumption that would make the home representative agent indifferent between the time invariant policy and the optimal regime specific home monetary policy.

Given that the economy in the MS-DSGE model can lie in either of the two regimes, I compute the costs of changes in those conditional on each regime. Such a comparison is valid given that the steady state is unique and independent of monetary and fiscal policy regime changes, each regime is determinate conditionally, and the minimum state variable solution (18) is stationary.

Let \( r_s \) be the policy regime where the home central bank follows the regime specific optimal policy prescribed by (29). Let \( ri \) be the policy regime where the home central bank follows the suboptimal interest rate rule as described at table 2. Let \( \varsigma^{s,ri} \) capture the welfare cost of not implementing a regime specific optimal policy, but rather the suboptimal rule and be measured as the percent increase in steady-state consumption that would make the individual indifferent between the allocation obtained under the suboptimal rule \( ri \) and the regime specific optimal policy \( rs \). More specifically, \( \varsigma^{PF,ri} \) denotes the welfare cost when fiscal policy is passive and \( \varsigma^{AC,ri} \) denotes the welfare cost when fiscal policy is active. Formally, \( \varsigma^{PF} \) and \( \varsigma^{AF} \) are defined by

\[
\frac{1}{1 - \beta} \bar{U}((1 + \varsigma^{s,ri}) \bar{C}, \bar{L}) + W^{s,ri} + t.i.p. = \frac{1}{1 - \beta} \bar{U}(\bar{C}, \bar{L}) + W^{rs} + t.i.p
\]

with

\[
W^{s,ri} = -\frac{1}{2} \Theta \bar{C} U_CE_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} X'_t RX_t
\]
\[ W^{s,rs} = -\frac{1}{2} \Theta \bar{C} U_C E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} X_t' R X_t \]

for \( s = PF, AF \). Solving for \( \varsigma^{s,ri} \) I obtain the expression for the welfare costs associated with policy regime \( s \) expressed in percentage terms

\[ \varsigma^s \times 100 = \left[ \left( \frac{(1-\sigma)(W^{rs} - W^{s,ri})}{C^{1-\sigma}(1-\beta)^{-1}} + 1 \right)^{\frac{1}{1-\sigma}} - 1 \right] \times 100 \]

for \( s = PF, AF \).