Amplification of Shocks in a Model with Labor and Goods Market Search

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Abstract

The Diamond-Mortensen-Pissarides model with Nash wage bargaining provides a qualitatively appealing theory of unemployment, but its ability to explain the observed magnitude of fluctuations in unemployment remains debated. I extend this model by adding goods market frictions, study the interactions of goods and labor markets, and examine technology and preference shocks as alternative sources of fluctuations. Goods market frictions affect workers' bargaining position, provide a rationale for a high value of non-market activity and also affect its cyclical properties. These frictions can thus amplify the response of unemployment and vacancies to changes in the *measured* labor productivity caused by either technology or preference shocks. I estimate a weekly model using Bayesian methods, and find that the response of vacancies and unemployment to changes in measured labor productivity is about twice as large as in the model with labor search only. In addition, demand shocks account for three quarters of fluctuations and technology shocks for the remaining one quarter. Finally, goods market frictions allow the model to also reproduce the main facts on inventories - procyclical inventory investment, countercyclical inventories-sales ratio, and sales which are more volatile than production.

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1 Introduction

It is widely accepted that heterogeneities and information imperfections make trade in the labor market a decentralized, time consuming and costly activity for firms and workers. Similar complications arise with trade in the goods market. With heterogeneity in characteristics of goods and services, and with costly acquisition of information, consumers have to spend resources to find the goods and services that match their needs and preferences, and to obtain information about their availability at different locations. But while the literature studying departures from the Walrasian labor market by imposing search frictions is quite large (see Pissarides, 2000 for introduction to the literature, and Rogerson, Shimer, & Wright, 2005 for a survey), similar analysis for the goods market is less common.

The aim of this paper is to study how unemployment dynamics is affected by effects that arise from interactions of frictional labor and goods markets. To that end, I extend the standard Diamond-Mortensen-Pissarides labor search-matching model by introducing a goods market search-matching friction, and use it to address two issues. First, the response of unemployment to changes in labor productivity in the basic labor search model is much smaller than in U.S. data; I show that feedback effects between labor market and goods market can result in amplification of shocks in the extended model. Higher employment increases output which can encourage consumers to increase their search for consumption goods; higher search effort by consumers increases profits of firms and thus affects firms' hiring decisions. Moreover, when wages are determined by Nash bargaining, there is an additional effect through the wage channel. In the extended model, goods market frictions affect worker's bargaining position, provide rationale for high value of non-market activity, but also change cyclical properties of the value of non-market activity. This effect arises since higher availability of goods in expansions makes frictions in the goods market less severe from consumer's perspective, thus increasing the value of additional earnings obtained when the worker accepts the job. This results in a downward pressure on worker's outside option in the Nash bargaining and consequently increases incentives for firms to hire new workers.

Second, I examine the driving forces behind unemployment fluctuations, and in addition to technology (supply side) shocks also consider preference (demand side) shocks, that give rise to movements in measured labor productivity similar to those caused by technology shocks. In the model with goods market frictions, a demand shock that increases the search effort by consumers also increases output and measured labor productivity. These shocks can therefore provide an alternative explanation of fluctuations in unemployment over the business cycle. I show that to an economist who would use only the time series usually considered in labor search literature - labor productivity, output, employment, vacancies and wages - preference and technology shocks are observationally equivalent if utility is additively separable and at the same time goods market matching function has unit elasticity of substitution. This means that based on these time series it is impossible to distinguish the case with actual shocks to technology from the case where the true productivity is constant, and changes in measured average labor productivity, output and employment are the result of changes in preferences and demand.

I first explore the qualitative properties of the model without inventories, analyze conditions under which technology and preference shocks can be distinguished, and under which goods market frictions amplify effects of shocks. After that, I discuss the estimation of the model using likelihood based Bayesian methods; this approach is used since in the presence of goods market frictions measured labor productivity becomes endogenous and does not coincide with actual unobserved productivity. The model is first estimated with one shock at a time, to match only the time series for U.S. average labor productivity. When search effort and output supplied by firms are good substitutes, a modest amount of goods market frictions increases the response of vacancy-unemployment ratio to technology shocks by one third. And with low substitutability between search effort and output supplied by firms, shocks to preferences result in response of vacancy-unemployment ratio which is about two and half times larger than the response to technology shocks in model with labor search only.

Finally, I show that extending the model by incorporating inventories allows to determine the relative importance of technology and demand shocks and helps to avoid the issue of non-identification of parameters in estimation. The two sources of fluctuations can be distinguished because technology and preference shocks have different implications for the response of inventory-sales ratio in the model. In particular, this ratio increases in response to a positive technology shock, but decreases in response to a preference shock. In the full model the response of vacancy-unemployment ratio is twice as large as in the model with labor search only, and the model attributes about one quarter of fluctuations to technology shocks and three quarters to preference/demand shocks. In addition, model can match the main facts on inventories - procyclical inventory investment, countercyclical inventories-sales ratio, and sales which are more volatile that production.

The rest of the paper is organized as follows. After the review of related literature, the model is described in Section 2, next, equilibrium is characterized and its qualitative properties are examined in Section 3. In Section 4 I use Bayesian estimation of a weekly model matching the U.S. labor productivity to parametrize alternative shocks, and then compare the implied business cycle properties of unemployment, vacancies and the labor market tightness. Section 5 concludes. Most of the algebra used to derive the characterization of equilibrium, as well as all the proofs are delegated to the appendices.

1.1 Related Literature

The ability of the Diamond-Mortensen-Pissarides search and matching model of the labor market (Diamond, 1982, Pissarides, 1985, Mortensen & Pissarides, 1994, and also Pissarides, 2000 for textbook exposition) to amplify and propagate the technology shocks and the extent to which model generated business cycles statistics match the U.S. data have been widely discussed. Shimer (2005) argues that the basic model calibrated to U.S. data can not generate enough volatility in unemployment, vacancies and in labor market tightness: while surplus of a match increases in expansions, under Nash bargaining wages absorb most of this increase, leaving firms with little incentives to hire new workers. Several papers thus examined different examined wage rigidity (Shimer, 2005, Hall, 2005) and alternative wage bargaining process (Hall & Milgrom, 2008, Mortensen & Nagypal, 2007) as a ways to improve the performance of the model. The wage rigidity required is that wages of workers in new employment relationships are rigid over the business cycle. Given that the empirical evidence available does not support this claim (see Pissarides, 2009 for a detailed discussion), this solutions is not completely without its own problems.

After investigating the puzzle more closely, Hagedorn and Manovskii (2008) have proposed an alternative way to calibrate the two key parameters - worker's bargaining power and the value of the non-market activity - and were able to obtain fluctuations of the right magnitude. However, as shown in Hornstein, Krusell, and Violante (2005) and Costain and Reiter (2008), with this alternative calibration the response of unemployment to changes in unemployment compensation in the model is implausibly large. Several other papers examined modifications of the basic labor search model to see if they improve its quantitative properties; these include among others labor turnover costs (Mortensen & Nagypal, 2007, Pissarides, 2009, Silva & Toledo, 2013), asymmetric information (Guerrieri, 2008, Moen & Rosén, 2011), endogenous home production (Garin & Lester, 2013), and introduction of on-the-job search (Krause & Lubik, 2010, Menzio & Shi, 2011). In all these papers changes in productivity as a result of technology shocks remain the driving source of business cycle fluctuations.

A promising alternative to technology shocks was proposed by Bai, Ríos-Rull, and Storesletten (2012). These authors show that once goods market frictions are incorporated into a traditional RBC model with frictionless labor market, preference shocks generate movements in Solow residual similar to those caused by technology shocks, and also perform well in matching co-movements of main macroeconomic variables. Their results thus motivate to consider preference shocks as an alternative to the technology shocks in the model with labor search. But since labor market is frictionless in their model, the results of interaction of frictions in labor and goods markets are not investigated in their paper. In addition, firms are not allowed to stored unsold goods as inventories, and thus unsold goods simply perish.

There are a few papers that lately started to analyze the interactions of search frictions in labor and goods markets. Lehmann and Van der Linden (2010) investigate the link between inflation and unemployment in a modified labor search model where products are sold in frictional market with demand given by real money holdings of consumers. Kaplan and Menzio (2013) develop a model where shopping externalities lead to multiplicity of equilibria, and where shocks to agents' expectations about future unemployment create self-fulfilling fluctuations even in absence of any shocks to technology or preferences. Huo and Ríos-Rull (2013) develop a neoclassical growth model with tradable and nontradable sector, frictions in goods and labor markets, and with adjustment costs in both physical investment and hiring of new employees. Goods market search frictions exist at the level of varieties household consumes rather than firms' locations as in Bai et al. (2012), search effort is a complement rather than a substitute for the resources spent, and preferences with no wealth effects guarantee that varieties of nontradable goods are a normal good. The paper analyzes the effects of wealth and financial shocks instead of traditionally considered shocks to total factor productivity, and show how the increased desire to save by consumers can induce a recession rather than a boom. This recession arises due to the adjustment cost and labor market frictions; goods market frictions are important quantitatively and amplify the recession. The focus of my paper is different, I examine technology and the preference shocks as alternative source of business cycle fluctuations in the Diamond-Mortensen-Pissarides model with goods market frictions where firms can hold inventories, and study the channels through which search frictions in the goods market amplify the response of unemployment to changes in measured labor productivity.

The two papers that are probably closest to mine are Petrosky-Nadeau and Wasmer (2011) and Michaillat and Saez (2013). Petrosky-Nadeau and Wasmer (2011) consider the standard technology shocks only, and show that in a model with search in credit, labor and goods markets technology shocks are both significantly amplified and propagated by goods market frictions. Their framework is different from the one this paper. There are no inventories in their model, firms and consumers in the goods market form long term matches and price for which output is sold in these matches is determined by bilateral Nash bargaining. Matching frictions in the good market in their model introduce a delay in the reaction of unemployment to technology shocks through firms' response to the evolution of price and congestion in the goods market, but with linear preferences and a simple wage setting rule their approach misses the effects of goods market frictions on outside option of the worker in the wage bargaining process. Moreover, the fact that the technology shock process is parameterized the same way in their models with and without goods market frictions implies that the properties of the measured labor productivity will be different in these models.

The paper by Michaillat and Saez (2013) analyzes the role of demand and supply shocks in shaping the aggregate demand and employment when labor and goods markets are subject to search frictions. In addition, they study the size of the government purchase multiplier and effects of redistributional transfers and changes in minimum wage on output and employment. The focus of their paper is however on theoretical analysis of the short run, and the model they develop is static, with prices that are fixed. If prices and wages were instead determined by Nash bargaining, demand shocks would have no effect on labor market tightness. In contrast, prices in my model are flexible, amplification effects are not driven by price or wage rigidities, and demand shocks play an important role in explaining fluctuations in vacancies and unemployment.

My paper is also related to the large literature studying the behavior of inventories over the business cycle. Two empirical facts, that the inventory-sales ratio is countercyclical, and that sales are more volatile than production turned out to pose quite a challenge in developing models that would be able to replicate them (see Ramey & West, 1999, Bils & Kahn, 2000 and Khan & Thomas, 2007 for further discussion on this issue). As shown here, a relatively simple model with goods market frictions and with demand and supply side shocks can actually match these facts quite well.

2 Model

This section states the problems of the households and the firms, describes how they interact in the labor market and in the goods market and defines the equilibrium. Before going into details, the structure of the model is as follows.

There is a measure one of households, each with measure one of infinitely lived workers. Workers have to search for jobs in the labor market, and search for consumption goods in the goods market. Household pools resources and provides its members insurance against fluctuations that arise due to the uncertain results of search. Preferences are subject to shocks affecting the marginal utility of consumption and marginal disutilities from work and search. These shocks are perfectly correlated across all workers in the economy.

There is also a continuum of firms with measure one which use labor as the only input to produce goods. Goods are sold in market that is subject to search frictions, firms post prices and consumers direct their search effort to acquire goods at a particular price. I assume that workers cannot quit but there is exogenous job destruction. Firms need to open and maintain vacancies to hire new workers. For labor market I employ standard undirected search mechanism with Nash bargaining.

The aggregate state of the economy is $\mathbf{S} = (z, \zeta, N)$, where N is the measure of employed workers after separations take place and (z, ζ) are the exogenous shocks with z being the current technology, ζ the current preference shock. I assume that shocks (z, ζ) follow first order Markov process.

Time is discrete and the timing of events within the period is as follows: (1) shocks are realized;

exogenous job separations occur; (2) each firm decides simultaneously how many vacancies to open and the price for which to sell goods; (3) employed workers produce, unemployed workers search for a job, search and matching in the goods and labor markets takes place; (4) payments are made (goods purchases, dividends, wages); (5) household pools resources and goods purchased, consumption takes place.

2.1 Labor Market

As in the basic labor search-matching model in Pissarides (2000), only unemployed workers search for jobs, search is not directed, and the number of matches of unemployed workers U and vacancies V is given by an aggregate constant returns to scale matching function $m^L(U,V)$. Let $\theta = \frac{V}{U}$ denote the tightness of the market, $\pi^u(\theta) = m^L(1,\theta)$ the probability for an unemployed worker to be hired, and $\pi^v(\theta) = m^L(1/\theta, 1)$ the measure of workers that one vacancy attracts.

I assume that workers value their actions based on the contribution they bring to the utility of the household; worker's surplus from being employed is thus the change in the household's utility from having one additional member employed. When a worker and a vacancy are matched, and the worker accepts the job, wage w is set in every period as a solution to the asymmetric Nash bargaining problem[1]

$$w(\mathbf{S}) = \underset{\hat{w}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w})^{\mu}\hat{\Omega}_n(\hat{w})^{1-\mu} \tag{2.1}$$

where $\hat{W}_n(\hat{w})$ and $\hat{\Omega}_n(\hat{w})$ are the household's and firm's value of a marginal worker employed and earning arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the job is hit by the separation shock δ.

2.2 Goods Market

Acquisition of consumption goods requires active search effort on the side of the consumer to find the goods to purchase. To model these frictions in the goods market I adopt the competitive search mechanism proposed by Moen (1997) - firms post prices and consumers direct their search effort to acquire goods at a particular price. Goods market is thus divided into submarkets, and firm and household's members can choose in which submarket to trade. The amount of goods sold in any submarket is determined by a matching function $m^G(D, TX)$. Here D is the aggregate search effort of all consumers in the particular submarket, T the measure of firms selling in the particular submarket and X is the quantity of goods sold per firm in the submarket.

[1] The timing of payments is however not crucial. Even if wages are constant throughout the duration of employment, as long as at the time when a match is formed the surplus is split according to the Nash bargaining, firms' decisions about hiring are unaffected. Equilibrium allocation is then same as in the case with period by period Nash bargaining.

Assumption 1. *Goods market matching function $m^G(D, TX)$ is constant return to scale, with elasticity of substitution σ.*

Submarkets are indexed by (p, Q) where p is the price of the consumption good and $Q = \frac{T}{D}$ is the tightness of the submarket. Since m^G has constant returns to scale, the amount of goods acquired per unit of search effort by household's shopper is

$$\psi^d(Q, X) = m^G(1, QX)$$

and the probability that a particular unit of good is sold is

$$\psi^x(Q, X) = m^G\Big(\frac{1}{QX}, 1\Big)$$

Consequently, the amount of output successfully sold by a firm supplying x in submarket (p, Q), where the total amount of goods supplied by all firms is TX is

$$x\psi^x(Q, X) = \frac{x}{X}\frac{\psi^d(Q, X)}{Q}$$

The matching process is thus different from the one in Bai et al. (2012). Here, ceteris paribus, an increase in the total supply of goods in the submarket affects the probability that a particular unit of consumption good is sold, whereas in their paper that probability is unaffected. This modified assumption seems intuitive, and as discussed in Section 3 it allows to identify the relative importance of technology and preference shocks. In addition, efficiency in Bai et al. (2012) requires stronger assumption on information available to consumers - equilibrium in is guaranteed to be efficient only if submarket are indexed by price, tightness and also quantity sold; as shown below, only price and tightness are needed in my model.

2.3 Households

As in Merz (1995), I consider households to be extended families consisting of a measure one of workers. All workers are ex-ante identical and their preferences are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \mathrm{u}(c_t, d_t, e_t, \zeta_t)$$

where c_t is consumption, d_t search effort in goods market, e_t is the employment status and $\zeta_t = (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})$ is the preference shock affecting the marginal utility of consumption, marginal disutility of search for consumption good and the disutility of work.

Households own firms, and in the recursive formulation of the household's problem, the individual state of a household, $\mathbf{s} = (a, n)$, is given by wealth in the form of shares a, and the number of members of the household that have a job after separations take place n. Household decides about goods market search effort of its employed and unemployed workers d^n, d^u, consumption allocation c^n, c^u, and about share holdings for next period a'. Each member also decides in which submarket (p, Q) to search for consumption goods, and directs the search to the submarket that delivers the biggest contribution to the utility of the household. I incorporate this through a constraint in the problem of a firm which posts price and decides about quantity sold. In addition, since in equilibrium only one market is going to be active, in the household's problem price of goods, goods market tightness and quantity sold appear as given functions of state $p(\mathbf{S}), Q(\mathbf{S}), X(\mathbf{S})$.

Taking prices $p(\mathbf{S})$, $w(\mathbf{S})$, $R(\mathbf{S})$ as given, the household then faces a budget constraint

$$p(\mathbf{S})\big(nc^n + (1-n)c^u\big) + a' = (1 + R(\mathbf{S}))a + w(\mathbf{S})n$$

with shares acting as a numeraire. In addition, search frictions in goods market impose a constraint

$$nc^n + (1-n)c^u = \big(nd^n + (1-n)d^u\big)\psi^d(Q(\mathbf{S}), X(\mathbf{S}))$$

where $\psi^d(Q, X)$ is the amount of goods acquired per unit of search effort in the goods markets, and the search frictions in labor market constraint

$$n' = (1-\delta)n + \pi^u(\theta(\mathbf{S}))(1-n)$$

where $\pi^u(\theta)$ is the probability for an individual to find a job. Since the optimal allocation of consumption and search effort among family members in each period solves

$$\begin{aligned} U(c, d, n, \zeta) = & \max_{c^n, c^u, d^n, d^u} n\mathrm{u}(c^n, d^n, 1, \zeta) + (1-n)\mathrm{u}(c^u, d^u, 0, \zeta) \\ & \text{subject to} \\ & nc^n + (1-n)c^u = c \\ & nd^n + (1-n)d^u = d \end{aligned}$$

where c is the total amount of consumption goods available to household and d is the overall search effort, I can formally set up the household's problem in which it acts as if it had preferences with utility function $U(c, d, n, \zeta)$. To summarize, household's problem written in a recursive form is

$$\begin{aligned} W(\mathbf{s}; \mathbf{S}) = & \max_{c, d, a'} U(c, d, n, \zeta) + \beta \mathbb{E} W(\mathbf{s}'; \mathbf{S}') \qquad (2.2) \\ & \text{subject to} \\ & p(\mathbf{S})c + a' = (1 + R(\mathbf{S}))a + w(\mathbf{S})n \\ & c = d\psi^d(Q(\mathbf{S}), X(\mathbf{S})) \\ & n' = (1-\delta)n + \pi^u(\theta(\mathbf{S}))(1-n) \\ & \mathbf{S}' = G(\mathbf{S}) \end{aligned}$$

2.4 Firms

The individual state of a firm is the number of workers employed n. Each firm chooses in which submarket (p, Q) to sell the goods and at the same time decides how many vacancies v to open. The amount of goods x that the firm can potentially sell is given by

$$x = zf(n - \chi v) - \kappa(v)$$

with $f_l > 0$, $f_{ll} \leq 0$ and $\kappa_v \geq 0$, $\kappa_{vv} \geq 0$ which can be interpreted as a case where some of the workers act as recruiters and thus χv hours of worked are diverted from the production process to hiring, and in addition $\kappa(v)$ costs in terms of goods are incurred for vacancy posting. This specification of the hiring process nests Shimer (2010) as a special case where $f(l) = l$, $\chi = 1$ and $\kappa(v) \equiv 0$, and the benchmark case from Pissarides (2000) if $f(l) = l$, $\chi = 0$ and $\kappa(v) \equiv \kappa$. Each vacancy attracts $\pi^v(\theta)$ new workers. If the firm decides to sell its output x in the (p, Q) submarket, where the aggregate amount of goods being sold is X, then the actual amount of goods sold is given by

$$x\psi^x(Q, X) = \frac{x}{X}\frac{\psi^d(Q, X)}{Q}$$

To highlight the role of inventories, I first assume that goods which are not sold can not be stored, as in Bai et al. (2012) and Petrosky-Nadeau and Wasmer (2011); I introduce the possibility to store unsold goods in Section 4. As discussed in Section 2.3, the firm needs to take into account a constraint which guarantees shoppers in the (p, Q) submarket equilibrium value of search $W_d^*(\mathbf{S})$. Let $M(\mathbf{S})$ be the marginal value of wealth in terms of utility, then

$$W_d(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X)$$

is the value to the household of the marginal search effort in the (p, Q) submarket. Finally, let $m(\mathbf{S}, \mathbf{S}')$ be the stochastic discount factor used to discount future profits. To summarize, the problem that a firm solves is then

$$\Omega(n; \mathbf{S}) = \max_{v,p,Q,x} \left\{p\psi^x(Q, X)x - w(\mathbf{S})n + \mathbb{E}[m(\mathbf{S}, \mathbf{S}')\Omega(n'; \mathbf{S}')]\right\} \tag{2.3}$$

subject to

$$x = zf(n - \chi v) - \kappa(v)$$

$$n' = (1 - \delta)n + \pi^v(\theta(\mathbf{S}))v$$

$$W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X)$$

$$\mathbf{S}' = G(\mathbf{S})$$

2.5 Equilibrium

Definition 1. *Equilibrium is household's value function and decision rules* $(W, g^c, g^d, g^{a'})$ *as functions of* $(\mathbf{s}; \mathbf{S})$*; value function and decision rules* (Ω, g^v, g^p) *as functions of* $(n; \mathbf{S})$*; aggregate allocation* (X, C, D, V)*, tightness* (Q, θ)*, prices* (p, w)*, dividends* R*, law of motion for employment* G^N*, all as functions of* $\mathbf{S}$*; such that*

1. *Value function* W *solves (2.2) and* $(g^c, g^d, g^{s'})$ *are the associated policy functions*
2. *Value function* Ω *solves (2.3) and* (g^v, g^p) *are the associated policy functions*
3. *Household and firm are representative*
4. *Wage* w *solves the Nash bargaining problem (2.1)*
5. *Goods market tightness is* $Q(\mathbf{S}) = \frac{1}{D(\mathbf{S})}$*; labor market tightness* $\theta(\mathbf{S}) = \frac{V(\mathbf{S})}{1-N}$
6. *Law of motion for employment is implied by firm's policy.*

3 Characterization of Equilibrium

In this section I analyze the qualitative properties of the model economy, role of technology and preference shocks, and the interactions of frictions in labor market and goods markets. I start by deriving two functional equations that characterize the dynamics of market tightnesses $Q(\mathbf{S})$ and $\theta(\mathbf{S})$. These are obtained by first deriving the optimality conditions for the household and the firm, and then using them to obtain the solution for the Nash bargaining problem in the labor market, and competitive search problem in the goods market. Details can be found in Appendix A and Appendix B, here I summarize the results. To avoid the notational clutter in what follows I drop the arguments of functions, use g_A to denote derivative of function g with respect to A, and g' to denote value of function g in the next period. Thus for example in equation (3.1) below U_c and U_d stand for $\frac{\partial}{\partial c}U(C(\mathbf{S}), D(\mathbf{S}), N)$ and $\frac{\partial}{\partial d}U(C(\mathbf{S}), D(\mathbf{S}), N)$, ψ^d for $\psi^d(Q(\mathbf{S}), X(\mathbf{S}))$. I also use notation ϵ^A_B for elasticity of A with respect to B.

The goods market with competitive search gives rise to the following intratemporal condition

$$-U_d = (1 - \epsilon^{\psi^d}_Q)\psi^d U_c \tag{3.1}$$

where $\epsilon^{\psi^d}_Q = \frac{\partial \log \psi^d}{\partial \log Q}$.

The labor market behavior is characterized by the following condition, which is the counterpart of the stochastic first order difference equation for market tightness θ in the basic Diamond-Mortensen-Pissarides search matching model

$$\begin{aligned} &\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\psi^x\left(U_c + \frac{U_d}{\psi^d}\right) \\ &= \beta\mathbb{E}\left[\left((1-\mu)z'f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right)(\psi^x)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) + (1-\mu)U_n'\right] \end{aligned} \tag{3.2}$$

Notice that using

$$
\begin{aligned}
&C = \psi^x(Q, X)X \\
&D = 1/Q \\
&X = zf(N - \chi V) - \kappa(V) \\
&V = \theta(1 - N)
\end{aligned}
$$

equations (3.1) and (3.2) can be written in terms of current and next period's (Q, θ, N) and shocks z, ζ only; thus together with the law of motion for labor

$$N' = (1 - \delta)N + \pi^u(\theta)(1 - N) \tag{3.3}$$

they fully characterize the dynamics of (Q, θ, N) in equilibrium.

Note also that the measured average labor productivity in this economy is

$$y = \frac{\psi^x X}{N} \tag{3.4}$$

and that it is affected by technology shock z, the discrepancy between the number of workers N and the number of workers in production $L = N - \chi V$, and the size of the goods market frictions. In the presence of goods market frictions preference shocks ζ_c and ζ_d have an effect on measured average labor productivity y through their effect on both ψ^x and X. This serves as a motivation in Section 4, where I use Bayesian methods to estimate the model and parametrize processes for shocks ζ_c and ζ_d by matching the labor productivity y the model to the labor productivity observed in U.S. data.

3.1 Efficiency

The efficient allocation is defined as an allocation chosen by a social planner facing the same search-matching frictions as the participants in the labor and goods markets in the decentralized economy.

Definition 2. *An allocation is efficient if it solves*

$$
\begin{aligned}
&\mathcal{W}(z, \zeta, N) = \max_{C,D,X,V} \left\{ U(C, D, N, \zeta) + \beta \mathbb{E} \mathcal{W}(z', \zeta', N') \right\} \\
&\textit{subject to} \\
&C = m^G(D, TX) \\
&X = zf(N - \chi V) - \kappa(V) \\
&N' = (1 - \delta)N + m^L(1 - N, V)
\end{aligned}
$$

Given this definition, the following proposition gives the condition under which the equilibrium of the decentralized economy in this paper is efficient.

Proposition 1 (Efficiency)**.** *If $\mu = \epsilon_U^{m^L}$ equilibrium is efficient.*

Thus even with search frictions in the goods market, when this search is competitive and submarkets are indexed by price and market tightness, familiar condition from Hosios (1990) continues to hold, and equilibrium is efficient as long as workers' bargaining power is equal to the elasticity of labor market matching function with respect to unemployment.

3.2 The Role of Goods Market Frictions

The channel through which changes in goods market conditions affect the labor market manifests itself in equation (3.2) by the terms ψ^x and $\frac{U_d}{\psi^d}$ that affect the cost of hiring an extra worker on the left hand side of equation (3.2), and terms $(\psi^x)'$ and $\frac{U_d'}{(\psi^d)'}$ that affect the benefits of hiring this worker on the right hand side of equation (3.2). An increase in the expected probability to successfully sell goods $(\psi^x)'$, or a decrease in the disutility from search for goods $\frac{U_d'}{(\psi^d)'}$ raises future benefits of having an extra worker employed in the similar way as an increase in technology z'. Going back one step, the reason why the new terms appear in the labor market condition (3.2) is that the presence of search frictions in the goods market changes the surplus of the match between a worker and a firm and affects the wage bargaining process. As a result, as shown in Appendix B, with Nash bargaining real wage in equilibrium is

$$\frac{w}{p} = \mu\psi^x(zf_l + \theta(\chi z f_l + \kappa_v)) - (1-\mu)\frac{U_n}{U_c + \frac{U_d}{\psi^d}} \tag{3.5}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption. Compared to a standard model without goods market search, there are however two important differences. First, the value of the marginal product of a worker and the marginal cost of vacancy per worker are multiplied by ψ^x which captures the fact that only a share of goods are actually successfully sold. Second, marginal utility foregone by switching from non-market activity to market activity $-U_n$ is evaluated in terms of consumption goods using $U_c + \frac{U_d}{\psi^d}$ rather than U_c, where $\frac{U_d}{\psi^d}$ captures the disutility from search for consumption good that is needed to be able to spend the extra earned income. Note that this last fact provides rationale for a high value of non-market activity proposed by Hagedorn and Manovskii (2008), as a way to generate fluctuations in labor market tightness in the standard labor search model that would be comparable to those in the data.

Consider now the effects of a positive technology shock z in the economy with goods market frictions. There are several additional channels that affect the wage and the hiring decision of a firm. Since output supplied X increases, the return from search increases for consumers too. Thus, for preferences where the substitution effect dominates the income effect, search effort increases and goods market tightness falls; as a result firms are more likely to sell the goods, which amplifies the impact of initial increase in productivity on return to production. In addition, higher output supplied X and lower goods market tightness Q have opposing effects on disutility from search effort required to purchase the marginal unit of consumption, and thus also on the bargaining position of the worker, wage, and the hiring decision of a firm.

The effects of different shocks can be characterized further if additional assumptions are imposed on preferences and technology. I analyze the behavior of model economy under the assumption of standard separable preferences, and for comparison also under the alternative assumption of preferences for which there is no income effect on search effort.

Assumption 1A. *Utility function of worker is* $\mathrm{u}(c, d, e, \zeta) = \zeta_c u(c) - \zeta_d g(d) - \zeta_n e$ *with relative risk aversion coefficient* $\eta = -\frac{cu''(c)}{u'(c)}$.

Assumption 1B. *Utility function of worker is* $\mathrm{u}(c, d, e, \zeta) = \zeta_c u\big(c - \zeta_d g(d)\big) - \zeta_n e$.

Assumption 2. *Vacancy costs* $\kappa(v)$ *are of the form* $\kappa(v) = z\bar{\kappa}(v)$ *for some* $\bar{\kappa}(v)$ *with* $\frac{d\bar{\kappa}}{dz} = 0$.

Under Assumptions 1A and 2, if in addition the goods market matching function m^G has elasticity of substitution $\sigma = 1$, then preference shocks in this model are in a sense observationally equivalent to technology shocks. That is, a process for technology shock z that generates a particular observed history of average labor productivity y can be replaced by constant technology z and some process for preference shock ζ_d that generates same history y. In addition, observed histories for vacancies, employment, output and wages are also identical. Thus technology shock z and preference shocks ζ_d generate same co-movements of measured labor productivity y and labor market tightness θ.

Proposition 2 (Equivalence of preference and technology shocks)**.**
Under Assumptions 1A and 2

a. iff goods market matching function m^G *has elasticity of substitution* $\sigma = 1$*, then for any history of shocks* (z^t, ζ^t) *and resulting history of average labor productivity, market tightness and employment* $(y^t, \theta^t, Q^t, N^t)$ *which satisfy (3.1)-(3.4), there exist a history* $(\tilde{z}^t, \tilde{\zeta}^t)$ *and* $\tilde{Q}^t$*, with* $\tilde{z}_t = 1$*,* $\tilde{\zeta}_{ct} = \zeta_{ct}$*,* $\tilde{\zeta}_{nt} = \zeta_{nt}$*, such that* $((\tilde{z}^t, \tilde{\zeta}^t), (y^t, \theta^t, \tilde{Q}^t, N^t))$ *also satisfy (3.1)-(3.4).*

b. the histories of real wages $(\frac{w}{p})^t$ *and* $(\frac{\tilde{w}}{\tilde{p}})^t$ *associated with* $(\tilde{z}^t, \tilde{\zeta}^t)$ *and* $(\tilde{z}^t, \tilde{\zeta}^t)$ *are identical also if and only if goods market matching function* m^G *has elasticity of substitution* $\sigma = 1$.

This result has implications for the quantitative analysis: assuming additively separable preferences, Cobb-Douglas goods market matching function and vacancy costs proportional to z implies that the two types of shocks can be distinguished, and their contribution to business cycle fluctuations analyzed only if some data on sales relative to the total supply of output in the market is utilized. To an economist who would use only the time series usually considered in labor search literature - labor productivity, output, employment, vacancies and wages - preference and technology shocks are observationally equivalent, it is impossible to distinguish the case with shocks to technology from the case where the actual technology is constant, and changes in measured average labor productivity, output and employment are the results of changes in preferences and demand.

The next proposition establishes a neutrality result for the case where utility function is logarithmic in consumption.

Proposition 3 (Neutrality of shocks).
Under Assumption 1A if in addition $\eta = 1$ *and* $\sigma = 1$

a. technology shocks z *have no effect on the goods market tightness* Q

b. preference shocks ζ_d *have no effect on the labor market tightness* θ

Under Assumptions 1A and 2, if in addition $\eta = 1$ *and* $\sigma = 1$

c. technology shocks z *have no effect on the labor market tightness* θ

In Proposition 3, labor market tightness θ becomes independent of $\{z, \zeta_d\}$ and depends on $\{\zeta_c, \zeta_n\}$ only; goods market tightness Q becomes completely independent of technology z and the behavior of θ in the labor market, and depends on $\{\zeta_c, \zeta_d\}$ only.

The reason for the neutrality result of the goods market with respect to technology z is that the income and substitution effects for search effort in the goods market cancel: For a given level of employment, improvement in technology z results in higher amount of output X supplied by firms and hence allows agents to consume more even if they decrease their search effort, while the substitution effect motivates greater search effort; and if $\eta = 1$ these two effects exactly offset each other. Then, since the search effort is constant so is the goods market tightness and the probability of selling the good. Note that this holds for any form of vacancy cost $\kappa(v)$, Assumption 2 is not necessary for part a. of the Proposition 3 .

The neutrality of labor market tightness with respect to productivity in the labor search model with additively separable logarithmic utility function and with vacancy costs proportional to z is discussed in Shimer (2010), who emphasizes that it holds for any bargaining power of the worker, and any value of non-market activity (leisure) in his model. As we can see, this result holds even in the model with labor and goods search, for *any* amount of frictions in the goods market. That is, it holds for specification of goods matching function as long as m^G has elasticity of substitution $\sigma = 1$.

Moreover, labor market tightness θ in this model is also neutral with respect to preference shocks ζ_d. Note that this holds for any form of vacancy cost $\kappa(v)$ since Assumption 2 is not necessary for claim b. of the Proposition 3. Thus the neutrality of labor market tightness with respect to preference shock ζ_d is even stronger than the one with respect to technology shocks z, which requires that hiring costs $\chi z f_l + \kappa_v$ are proportional to z.

3.2.1 Comparative Statics

To get more insight about the way changes in preferences and technology work through the model, it is helpful to undertake the comparative statics analysis of the steady state, before proceeding to the quantitative analysis of the business cycle properties of the model. As argued in Mortensen and Nagypal (2007) and Pissarides (2009), since measured labor productivity changes are rather persistent and labor market flows are large, the approximation of dynamics of the DMP model by its steady state elasticities is reasonably accurate.

Changes in Technology

Since actual technology z is not directly observable, and only measured average labor productivity y is observed, the relevant elasticity is ϵ_y^θ rather than ϵ_z^θ. The following lemma and proposition thus first restate the equilibrium goods and labor market conditions (3.1) and (3.2) in terms of measured average labor productivity. Then, they establish the relationship between the steady state elasticities of labor market tightness with respect to measured labor productivity in the labor search model $\epsilon_y^{\theta^{LS}}$, and in the goods and labor search model $\epsilon_y^{\theta^{GLS}}$.

Lemma 1. *Under Assumption 2 in the steady state (3.1) and (3.2) can be rewritten as*

$$0 = (1 - \epsilon_Q^{\psi^d})U_C C + U_D D \tag{3.6}$$

$$0 = \Big((1-\mu) f_L - \Big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\Big)(\chi f_L + \bar{\kappa}_V)\Big)\frac{N}{f - \bar{\kappa}} y + (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d} U_C} \tag{3.7}$$

and the real wage (3.5) as

$$\frac{w}{p} = \mu(f_L + \theta(\chi f_L + \bar{\kappa}_V))\frac{N}{f - \bar{\kappa}} y - (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d} U_C} \tag{3.8}$$

Assuming $\mu^{GLS} = \mu^{LS}$ and provided that all denominators are non-zero

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1 + \epsilon_C^{MRS_{CN}}}\left(\frac{1-\sigma}{\sigma}\epsilon_D^{m^G}\epsilon_z^{QX} + \epsilon_D^{MRS_{CN}}\epsilon_z^{Q}\right)\frac{1}{\epsilon_z^{\theta^{GLS}}} \tag{3.9}$$

Proposition 4 (Amplification - shocks to technology)**.** *Suppose that $\mu^{GLS} = \mu^{LS}$. Under Assumptions 1A and 2 $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as m^G has elasticity of substitution $\sigma > 1$. Under Assumptions 1B and 2 $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as u has relative risk aversion coefficient $\eta \leq 1$ and $\sigma \geq 1$; or alternatively $\eta \leq 1$ and $\sigma < 1$ and $\epsilon_D^{g_D}$ is sufficiently small.*

As (3.9) shows, in addition to the channel that works through higher steady state value of non-market activity and thus lower bargaining power, goods market frictions introduce two other channels which can result in amplification of effects that changes in technology have on labor market tightness. These two additional effects work through changes in the value of non-market activity over the business cycle, as a result of goods market search frictions.

First, there is an effect similar to the one that creates a hump shaped profile of consumption in a lifecycle model with preferences that are not separable between consumption and leisure. If preferences are not additively separable and changes in search effort affect marginal rate of substitution between consumption and hours worked, an increase in z causes an additional effect captured in (3.9) by the term $\epsilon_D^{MRS_{CN}} \epsilon_z^Q$. In particular, with GHH preferences from Assumption 1B if $|\epsilon_C^{U_C}| < 1$ consumption and search effort both increase in response to an increase in z. Because of the non-separability in utility, with increased search effort the marginal utility of consumption decreases less compared to the standard labor search model; this in turn means a smaller increase in the value of non-market activity, smaller upward pressure on wage in (3.8), and larger incentives for firms to hire new workers.

Second, there is an effect related to the amount of search effort needed to acquire one unit of consumption good changes over the business cycle. If the elasticity of substitution of the goods market matching function is different from one, changes in D and X affect how severe goods markets frictions are, in the sense that they change the elasticity $\epsilon_Q^{\psi^d}$. An increase in z then results in an additional effect captured in (3.9) by the term $\frac{1-\sigma}{\sigma} \epsilon_D^{m^G} \epsilon_z^{QX}$. To see how this effect works, consider the case where output supplied X and search effort D both increase in response to an increase in z. These two have in general opposing effects on $\epsilon_Q^{\psi^d}$; however, for additively separable preferences from Assumption 1A it holds that $\epsilon_z^{QX} \geq 0$ with equality if $\epsilon_D^{g_D} = 0$ and $\epsilon_C^{U_C} = 0$, and for GHH preferences from Assumption 1B similarly $\epsilon_z^{QX} \geq 0$ with equality if $\epsilon_D^{g_D} = 0$. As a result if $\sigma > 1$ and productivity z increases, goods markets frictions become less severe, that is $\epsilon_Q^{\psi^d}$ increases; this implies a smaller increase in the value of non-market activity, smaller upward pressure on wage in (3.8), and larger incentives for firms to hire new workers.

Changes in Preferences

Consider next the effects of a change in preference parameter ζ_d. Lower disutility from search results in higher search effort, which increases output and measured productivity even if technology z and employment would remain constant. This induced change in measured labor productivity y will however affect firms' incentives to hire new workers, and thus also labor market tightness θ. Similar to the case with changes in technology z discussed above, the relevant elasticity is $\epsilon_y^{\theta^{GLS}}$ rather than $\epsilon_{\zeta_d}^{\theta^{GLS}}$. The relationship between steady state elasticity $\epsilon_y^{\theta^{LS}}$ in response to a change in technology z in the labor search model, and the steady state elasticity $\epsilon_y^{\theta^{GLS}}$ in response to a change in preferences ζ_d in the model with goods and labor search can be summarized as follows.

Proposition 5 (Amplification - preference shocks)**.** *Suppose that* $\mu^{GLS} = \mu^{LS}$*. Under Assumptions 1A and 2* $\epsilon_y^{\theta^{GLS}} \gtreqless \epsilon_y^{\theta^{LS}}$ *when* m^G *has elasticity of substitution* $\sigma \lesseqgtr 1$*.*

Consider a decrease in disutility ζ_d. When search effort by consumers and supply of goods and services by firms are complements in the matching function m^G, higher search effort results in larger incentives for firms to hire more workers in order to increase production, which is now more likely to be sold. Moreover, lower disutility from search per unit of good purchased $\frac{U_D}{\psi^d}$ provides additional incentives to hire more workers, since it creates a downward pressure on wages. The overall effect of the change in measured productivity on labor market tightness is larger than in the model with labor search only. When search effort and supply of goods and services are substitutes in the matching function m^G, incentives for firms to hire more workers in order to increase production are smaller, increase in search effort in much larger, and the result is actually a decrease in ψ^d and an increase in disutility from search per unit of good purchased $\frac{U_D}{\psi^d}$. The overall effect a same change in measured productivity on labor market tightness is consequently smaller than in the model with labor search only.

3.2.2 Worker's Outside Option and Bargaining Power

As already briefly mentioned above, goods market frictions provide some justification for the calibration in Hagedorn and Manovskii (2008), in particular for the choice of a high value of outside option of the worker and a low worker's bargaining power. To see this, consider an extension of the model where in addition to having more leisure, unemployed workers are engaged in home production and also receive unemployment benefit pb financed by a lump sum tax. Consumption c is then a composite good given by $c = g(c_m, c_n)$, where c_m is the amount of market goods and services and $c_n = h(1-n)$ is the amount of home produced goods and services. The wage in this economy is a small modification of (3.5)

$$\frac{w}{p} = \mu\psi^x(zf_L + \theta(\chi zf_L + \kappa_V)) + (1-\mu)\Big(b + \frac{U_C g_{c_n} h_{1-n}}{U_C g_{c_m} + \frac{U_D}{\psi^d}} - \frac{U_N}{U_C + \frac{U_D}{\psi^d}}\Big)$$

and the goods and labor market conditions (3.1) and (3.2) in the steady state become

$$\begin{aligned} 0 &= (1-\epsilon_Q^{\psi^d})\psi^d g_{c_m} U_C + U_D \\ 0 &= \Big((1-\mu)zf_L - \Big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\Big)(\chi zf_L + \kappa_v)\Big)\psi^x - (1-\mu)\Big(b + \frac{g_{c_n} h_{(1-n)}}{\epsilon_Q^{\psi^d} g_{c_m}} - \frac{U_N}{\epsilon_Q^{\psi^d} U_C}\Big) \end{aligned}$$

The steady state condition for labor market implies that the outside option of the worker gets larger, when goods market search frictions become more severe and $\epsilon_Q^{\psi^d}$ becomes smaller. Arguably, given that the productivity of workers in home production and the process through which market and nonmarket goods are combined into the composite consumption good remain unchanged, the bargaining power of the worker μ thus has to be lower, if the same economy is viewed through the lens of the model with goods and labor search, rather than the standard labor search model. This provides yet another channel, in addition to those in Proposition 4, through which goods market frictions amplify effects of changes in productivity: As shown in Hagedorn and Manovskii (2008), increasing the value of non-market activity and at the same time decreasing worker's bargaining power to maintain the same steady state wage leads to wages which are less procyclical, and thus vacancies and unemployment which respond more to changes in productivity.

4 Quantitative Analysis

I consider the case with additively separable preferences

$$\mathrm{u}(c, d, e, \zeta) = \zeta_c \frac{c^{1-\eta}}{1-\eta} - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - \zeta_n e$$

Firms have production technology $zf(l) = zl^{\lambda}$; labor and goods matching functions are $m^L(U, V) = B(\gamma U^{\frac{\nu-1}{\nu}} + (1-\gamma)V^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$ and $m^G(D, TX) = A\big(\alpha D^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(TX)^{\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}$. To calculate steady state elasticities from Section 3 values for the following parameters have to be set: $\beta, \eta, \varphi, \lambda, \mu, \delta, \gamma, \nu, B, \alpha, \sigma, A, \bar{\zeta}_c, \bar{\zeta}_d, \bar{\zeta}_n, \bar{z}$. For the dynamic simulation of the model in addition processes for $z, \zeta_c, \zeta_d, \zeta_n$ also have to be specified. In this section I first describe targets chosen to be matched in the U.S. data to calibrate the above parameters, and then describe the Bayesian estimation procedure used to estimate parameters of the processes for $z, \zeta_c, \zeta_d, \zeta_n$.

4.1 Calibration

Table 1 summarizes the targets and the parameter vales for the benchmark calibration. One period of the model is one fourth of a quarter, so roughly a week, and parameter β is chosen to obtain steady state annual interest rate of 5%. I set $\bar{z}$ to normalize the level of realized output $Y = 1$ and consider the case with constant returns to scale so that $\lambda = 1$. For labor market matching function parameters, I follow Shimer (2010) by setting $\nu = 1$, $\gamma = 0.5$ which implies a symmetric Cobb-Douglas matching function. As in Shimer (2005) I set the value of unemployment benefits b to 0.4 of average labor productivity in the steady state. The case with non-zero unemployment benefits allows a more precise calibration of the outside option of the worker, and implies that even in the case with logarithmic preferences and goods market matching function with unit elasticity of substitution, both technology and preference shocks have an effect on unemployment. Next, as in Hagedorn and Manovskii (2008), I use weekly job finding rate $\pi^u = 0.139$ and separation rate $\delta = 0.0081$; these values imply a steady state unemployment rate $U = 0.055$. Silva and Toledo (2009) and Hagedorn and Manovskii (2008) provide estimates for average costs associated with recruiting, screening and interviewing needed to hire a new worker: the 1982 Employment Opportunity Pilot Project survey, the 1992 Small Business Administration survey, and the findings in Barron, Berger, and Black (1997) suggests that these costs are about 4.5% of new worker's quarterly wages paid. Since a vacancy in the model attracts π^v workers to get one worker $\frac{1}{\pi^v}$ vacancies are needed. To match the above estimated hiring costs, if w is the weekly wage in the model, the total costs of a hire are $\frac{1}{\pi^v} w = 0.045 \times 12 \times w$. Thus for a weekly model I target $\pi^v = \frac{1}{12}\frac{1}{0.045} = 1.8519$. Given the job finding and recruitment rates targeted, since $\frac{\pi^u}{\pi^v} = \theta$ and $\pi^u = B\theta^{1-\gamma}$ the matching efficiency parameter is $B = (\pi^u)^\gamma (\pi^v)^{1-\gamma} = 0.507$.

For the benchmark I set the preference parameters so that $\varphi = 0$ and η targets intertemporal elasticity of substitution equal to 1. I set $\bar{\zeta}_c = 1$ and calibrate $\bar{\zeta}_d$ to normalize the steady state goods market tightness to $Q = 1$.

The calibration of labor market matching function parameters above is based on direct empirical estimates (see Pissarides & Petrongolo, 2001), similar studies are unfortunately not available for the goods market matching function. Bai et al. (2012) and Bai and Ríos-Rull (2013) assume a Cobb-Douglas matching functions with elasticity with respect to demand $\alpha = 0.09$ and $\alpha = 0.25$ respectively, Petrosky-Nadeau and Wasmer (2011) use symmetric Cobb-Douglas matching function with elasticity 0.5. In the benchmark calibration of the goods market matching function I thus set $\sigma = 1$, $\alpha = 0.2$ and calibrate A to obtain steady state fraction of goods purchased $\frac{C}{X} = 0.81$. To get an idea how much the elasticity of substitution in the goods market matching function matters for the quantitative results, I then also consider alternative cases with $\sigma = 0.5$ and $\sigma = 2$.

Finally, to set $\bar{\zeta}_n$ notice that for a given bargaining power μ, value of home production and leisure ζ_n affects wage and through that profits of the firms, hiring, labor tightness θ, and also U. For the model without goods market friction I thus proceed as Shimer (2010), set $\mu = \gamma$ and calibrate $\bar{\zeta}_n$ to match the above mentioned target unemployment rate $U = 0.055$. In the model with goods market search parameters μ and ζ_n can then be set in two alternative ways. In the first μ is kept unchanged and value of home production and leisure ζ_n is recalibrated to maintain the same steady state labor market tightness θ. In the second one ζ_n is kept unchanged and μ is recalibrated. I use the first approach in order to quantify the amplification effect of goods market frictions beyond the effect implied by a lower bargaining power as discussed in Section 3.2.2 above.

Table 1: Calibration

	value	target or source	value
β	0.999	annual interest rate	5%
η	1	coef. of relative risk aversion	1
λ	1.00		
γ	0.50	Shimer (2010)	
μ	0.50	Hosios condition	
δ	0.0081	monthly employment exit prob.	0.100
B	0.482	quarterly recruitment cost	$0.045y$
α	0.20		
A	0.843	capacity utilization rate	0.81
$\bar{z}$	1.313	output	1
ζ_n	0.441	unemployment rate	0.055
ζ_d	0.184	goods market tightness	1.00

4.2 Steady State Elasticities

Table 2 compares the steady state elasticity of vacancy-unemployment ratio with respect to measured average labor productivity ϵ_y^θ for the goods and labor search model in this paper, elasticities from existing labor search models in related papers, and also the empirical counterpart of this elasticity based on the data for U.S economy.

As shown in the top panel of Table 2, the standard deviation of log of the vacancy-unemployment ratio in U.S. for the 1951 to 2003 period is 19.1 times larger than the standard deviation of log average labor productivity. In contrast, as shown in the first line of the second panel the steady state elasticity ϵ_y^θ in Shimer (2005) is only 1.71. Subsequent papers by Hall (2005), Hall and Milgrom (2008) obtain elasticity in their models even larger than the target in the data, by modifying the wage determination mechanism to get less procyclical wages. Hagedorn and Manovskii (2008) maintain Nash bargaining and are able to generate the right amount of fluctuations through different calibration, by setting workers' bargaining power to $\mu = 0.052$ and the value of unemployment to $b = 0.955$. This large value of unemployment however implies a semielasticity of unemployment to changes in unemployment benefits replacement ratio ϵ_b^U which is seven times larger than what is empirically observed in U.S. data (Costain & Reiter, 2008).

Pissarides (2009) and Mortensen and Nagypal (2007) point out that $\frac{\sigma_\theta}{\sigma_y} corr(\theta, y)$ is a more appropriate target then a simple ratio $\frac{\sigma_\theta}{\sigma_y}$, to evaluate any model where productivity shocks are the only source driving of fluctuations. Arguably, other shocks, to preferences, matching efficiency, separation rate, bargaining power or interest rates can to some extent be the reason behind the

Table 2: Comparison of models based on steady state elasticity ϵ_y^θ

US data (1951:2003 period, from Shimer (2005))	
$\frac{\sigma_\theta}{\sigma_y}$	19.10
$\frac{\sigma_\theta}{\sigma_y} corr(\hat{y}, \theta)$	7.56

Labor search models	ϵ_y^θ
Shimer (2005)	1.71
Hall (2005)	81.70
Hall and Milgrom (2008)	42.35
Hagedorn and Manovskii (2008)	23.72
Mortensen and Nagypal (2007)	7.56
Pissarides (2009)	7.25
Silva and Toledo (2013)	4.17

Benchmark labor search model, $\alpha = 0$	ϵ_y^θ
$\eta = 1$, $b = 0.4$	3.69

Goods and labor search model, $\alpha = 0.2$	ϵ_y^θ
$\eta = 1$, $b = 0.4$, $\sigma = 2$	z: 5.03 and ζ_d: 1.10
$\eta = 1$, $b = 0.4$, $\sigma = 0.5$	z: 2.82 and ζ_d: 9.05

fluctuations in vacancy-unemployment ratio observed in data. The choice of $\frac{\sigma_\theta}{\sigma_y} corr(\theta, y)$ as a target is then justified, because this would be the coefficient obtained by running a regression of log of the vacancy-unemployment ratio on log average labor productivity. This yields 7.56 as a target against which Pissarides (2009) and Mortensen and Nagypal (2007) compare the steady state elasticity ϵ_y^θ in their versions of the labor search model which feature labor turnover costs as an additional element. Both papers show that the amount of fixed training costs needed to achieve the target value for the elasticity is quite plausible, in the range of 20% to 40% of the quarterly output of the match. Silva and Toledo (2013) however point out that the crucial detail that matters is the fraction of the labor turnover costs that are sunk at the point when the match is created. In addition, they show that increasing the labor turnover costs has a similar effect on the response of unemployment to changes in unemployment benefits as an increase in the value of unemployment in Hagedorn and Manovskii (2008). Using the available empirical evidence on training costs to discipline the calibration, in addition to restricting the semielasticity ϵ_b^U empirically observed in U.S. data, they find no amplification mechanism generated by fixed labor turnover costs. Their value of elasticity $\epsilon_y^\theta = 4.17$ in the model with labor turnover costs is essentially the same as $\epsilon_y^\theta = 4.18$ in the model without these costs, and is also very close to no labor turnover costs benchmark from Mortensen and Nagypal (2007) where $\epsilon_y^\theta = 3.89$ and Pissarides (2009) where $\epsilon_y^\theta = 3.67$.

Calibration of the benchmark model with labor search in this paper results in elasticity of similar magnitude since $\epsilon_y^{\theta^{LS}} = 3.69$. In comparison, in a model with goods and labor search, $\epsilon_y^{\theta^{GLS}}$ is about 40% larger when the driving force is a productivity shock and goods market matching function has elasticity of substitution $\sigma = 2$, and about 150% larger when the driving force is a preference shock and goods market matching function has elasticity of substitution $\sigma = 0.5$. This amplification is in line with theoretical results in Proposition 4 and Proposition 5.

4.3 Model with a Single Shock

4.3.1 Estimation

To specify the parameters for shock processes $\zeta_c, \zeta_d, \zeta_n, z$, I first consider the model with only one shock at a time; and the process considered is $\log x' = (1-\rho_x)\log\bar{x} + \rho_x \log x + e'_x$ for each shock $x \in \{z, \zeta_c, \zeta_d, \zeta_n\}$. To obtain the autocorrelation coefficients ρ_x and variance of innovations σ_x^2, I estimate a log-linearized weekly model using Bayesian methods, to match quarterly time series for average labor productivity.[2] The labor productivity measure used for estimation is 1951Q1-2010Q4 output per worker in nonfarm business sector. Quarterly labor productivity y_t is calculated as quarterly output Y_t divided by the quarter's employment N_t. Quarterly output is the sum of weekly output, and quarterly employment is given by the average employment in the three months of the quarter. Since for each month employment is measured by the BLS in the second week

$$y_t = \frac{Y_t}{N_t} = \frac{\sum_{i=1}^{12} Y^W_{12t-i+1}}{\frac{1}{3}(N^W_{12t-2} + N^W_{12t-6} + N^W_{12t-10})}$$

Table 3 and Table 4 show the choice of prior distributions, the estimated posterior mode obtained by maximizing the log of the posterior distribution with respect to the parameters, the approximate standard error based on the corresponding Hessian, and also the mean, mode, 10 and 90 percentile of the posterior distribution of the parameters obtained through the Metropolis-Hastings sampling algorithm with four chains and 100000 draws.

The estimated standard deviations for ζ_c and ζ_n shocks in the model without goods market search are very large, much larger than in the model with both search frictions. This is to be expected since the only channel through which they can generate movements in measured labor productivity y is through their effect on θ and productive workforce $L = N - \theta(1-N)$. Thus recruitment needs to vary a lot to match the measured labor productivity, which requires big shocks to preferences.

[2]See An and Schorfheide (2007), Del Negro and Schorfheide (2008) and Lubik (2009) for details regarding Bayesian estimation.

Table 3: Labor search model with one shock. Observables: y

		Prior		Posterior		
		mean	st.dev.	mode	mean	90 % HPD interval
ρ_c	Beta	0.900	0.05	0.9965	0.9963	[0.9945,0.9981]
σ_c	Inverse Gamma	0.010	20.00	0.2552	0.2576	[0.2367,0.2782]
	Log data density 792.27					
ρ_n	Beta	0.900	0.05	0.9964	0.9963	[0.9945,0.9981]
σ_e	Inverse Gamma	0.010	20.00	0.2531	0.2553	[0.2346,0.2749]
	Log data density 792.28					
ρ_z	Beta	0.900	0.05	0.9956	0.9954	[0.9934,0.9976]
σ_z	Inverse Gamma	0.010	20.00	0.0035	0.0035	[0.0033,0.0038]
	Log data density 796.99					

Table 4: Goods and labor search model with one shock. Observables: y

		Prior		Posterior		
		mean	st.dev.	mode	mean	90 % HPD interval
ρ_c	Beta	0.900	0.05	0.9962	0.9960	[0.9941,0.9979]
σ_c	Inverse Gamma	0.010	20.00	0.0198	0.0200	[0.0185,0.0215]
	Log data density 797.27					
ρ_d	Beta	0.900	0.05	0.9957	0.9956	[0.9935,0.9976]
σ_d	Inverse Gamma	0.010	20.00	0.0175	0.0176	[0.0163,0.0189]
	Log data density 796.47					
ρ_n	Beta	0.900	0.05	0.9930	0.9928	[0.9899,0.9959]
σ_n	Inverse Gamma	0.010	20.00	0.1336	0.1349	[0.1237,0.1462]
	Log data density 777.72					
ρ_z	Beta	0.900	0.05	0.9957	0.9955	[0.9935,0.9976]
σ_z	Inverse Gamma	0.010	20.00	0.0044	0.0044	[0.0041,0.0048]
	Log data density 797.71					

4.3.2 Business Cycle Moments

Unitary Elasticity of Substitution

Table 5 and Table 6 show the results of the simulation of models with and without goods market friction, with parameters of shocks set at their posterior means. Comparing panels (A) and (B) in Table 5 we can see that the large shocks to ζ_c required to generate the observed movements in labor productivity cause fluctuation in labor market tightness and recruitment which are 20 times higher than in the data. Moreover, the correlations of all variables with measured labor productivity y have wrong signs - if z is constant, for measured labor productivity y to increase, productive labor $L = N - \theta(1-N)$ has to increase relative to overall labor N, and thus θ has to fall. Shocks to disutility from work ζ_n suffer from the same problem. Thus without goods market frictions technology shocks are the only plausible source of business cycle fluctuations in this model.

Table 5: Summary statistics, U.S. data and labor search model

	(A) U.S. data				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.013	1.00	0.67	0.76	0.013	1.00	-0.89	0.75
θ	0.266	0.34	0.88	0.91	5.060	-0.98	0.89	0.77
V	0.141	0.42	0.89	0.91	3.137	-0.92	0.71	0.60
U	0.131	-0.24	-0.83	0.89	2.282	0.91	-1.00	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.013	1.00	-0.89	0.75	0.014	1.00	1.00	0.78
θ	5.011	-0.98	0.89	0.77	0.051	0.99	0.99	0.78
V	3.101	-0.92	0.71	0.60	0.031	0.93	0.91	0.63
U	2.262	0.91	-1.00	0.83	0.023	-0.94	-0.95	0.83

Table 6: Summary statistics, labor and goods search model, separable preferences, $\sigma = 1$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.98	0.76
θ	0.050	0.99	0.99	0.78	0.465	0.99	0.98	0.78
V	0.031	0.93	0.91	0.63	0.284	0.94	0.87	0.63
U	0.023	-0.94	-0.95	0.83	0.212	-0.92	-0.98	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.018	1.00	-0.99	0.82	0.014	1.00	1.00	0.78
θ	2.492	-0.95	0.89	0.77	0.050	0.99	0.99	0.78
V	1.542	-0.80	0.71	0.60	0.031	0.93	0.91	0.63
U	1.127	0.99	-1.00	0.83	0.023	-0.94	-0.95	0.83

Once goods market search is introduced into the model the situation changes considerably. Panels (A) and (D) of Table 6 document the observational equivalence of technology shocks z and preference shocks ζ_d from Proposition 2: shocks to disutility from search in goods market generate the same fluctuations as technology shocks. Comparing panels (D) for the two economies, with and without goods market search reveals that the volatility of labor market tightness is basically the same in both economies. This is also in line with theoretical analysis in the previous section: Proposition 4 proved that for steady state elasticity ϵ_y^θ there is no amplification in the case with additively separable utility function and goods market matching function with unitary elasticity of substitution. For shocks to marginal utility of consumption ζ_c, the size of the shocks necessary to generate the observed movements in labor productivity falls once the goods market search is introduced. Moreover, correlations of all variables with measured labor productivity y have now correct signs, and fluctuations of labor market tightness and recruitment are closer to those in data.

Non-Unitary Elasticity of Substitution

Table 7 and Table 8 present the moments for the goods and labor search model with elasticity of substitution between D and X in the goods matching function of 0.5 and 2. They confirm the results from Proposition 4, which were already suggested by steady state elasticities in Table 2. In the case with high substitutability and technology shocks, the observed fluctuations in vacancy-unemployment ratio are about 30% larger, compared to the model with labor search only. In the case with low substitutability, preference shocks to disutility from search for goods result in observed fluctuations in vacancy-unemployment ratio that are about 130% larger.

Table 7: Summary statistics, labor and goods search model, separable preferences, $\sigma = 0.5$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.97	0.76
θ	0.121	0.99	0.99	0.78	0.782	0.99	0.96	0.78
V	0.074	0.93	0.90	0.63	0.479	0.94	0.84	0.62
U	0.055	-0.93	-0.96	0.83	0.356	-0.91	-0.99	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.017	1.00	-0.98	0.82	0.014	1.00	1.00	0.78
θ	3.051	-0.96	0.89	0.77	0.039	0.99	0.99	0.78
V	1.888	-0.83	0.71	0.60	0.024	0.92	0.92	0.63
U	1.379	0.99	-1.00	0.83	0.018	-0.94	-0.95	0.83

Table 8: Summary statistics, labor and goods search model, separable preferences, $\sigma = 2$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.99	0.76
θ	0.015	0.99	0.99	0.78	0.272	0.99	0.98	0.78
V	0.009	0.92	0.92	0.63	0.167	0.94	0.89	0.63
U	0.007	-0.94	-0.94	0.83	0.124	-0.92	-0.96	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.018	1.00	-0.99	0.83	0.014	1.00	1.00	0.78
θ	1.869	-0.93	0.89	0.77	0.067	0.99	0.99	0.78
V	1.154	-0.78	0.71	0.60	0.041	0.93	0.91	0.63
U	0.846	1.00	-1.00	0.83	0.030	-0.93	-0.95	0.83

4.3.3 Impulse Response Analysis

The weekly impulse response functions to shocks that generate a one percent increase in measured average labor productivity are shown in Figure 1. As expected given the results so far, the response to technology shock z and preference shock ζ_d are virtually identical for the case with unit elasticity of substitution. Only the behavior of goods market tightness is different. The other two cases imply either a stronger response of unemployment to technology shocks (if the elasticity of substitution between X and D is high) or to preference shocks (if the elasticity of substitution between X and D is low).

To examine further the dynamics of the labor market variables in the model, I next look at the impulse response functions for the model generated quarterly data, and compare them to their empirical counterparts. To that end, I first use quarterly U.S. data on labor productivity, vacancies, unemployment, and employment to estimate a reduced form VAR $\tilde{\mathbf{x}}_t = \sum_{i=1}^{4} \mathbf{A}_i \tilde{\mathbf{x}}_{t-i} + \varepsilon_t$ with $\tilde{\mathbf{x}}_t = (\tilde{y}_t, \tilde{\theta}_t, \tilde{N}_t)'$, where $\tilde{y}_t, \tilde{\theta}_t, \tilde{N}_t$ are the log transformed average labor productivity, vacancy-unemployment ratio and employment, detrended using a third order time polynomial. I then obtain the empirical impulse response functions to a one-standard deviation shock to productivity, using the Cholesky decomposition to orthogonalize shocks with an identification scheme where the shock to productivity is first in the ordering. Afterwards, I run 1000 simulations of the model, each time aggregate the data into quarterly time series and estimate the same VAR on this artificial data. Figure 2 compares the resulting average impulse response functions with empirical counterparts. The top panel shows the case where goods market matching function has unitary elasticity of substitution, and the response of employment and labor market tightness to an increase in the measured productivity is the same in the model without goods market search and with goods market search. This is again in line with results from Proposition 4 and Proposition 5. With non-unitary elasticity of substitution, the model with goods markets frictions performs better than the model with labor search only in terms of amplification, but the problem with the lack of propagation is still present. The response of employment to an increase in measured labor productivity in the model is on impact similar to the response in data, but while in the data employment further increases in the following quarters and the peak occurs after five quarters from the initial shock, in the model this build up is much less pronounced and rather short lived, with peak already in the third quarter.

Figure 1: Impulse response function, weekly model

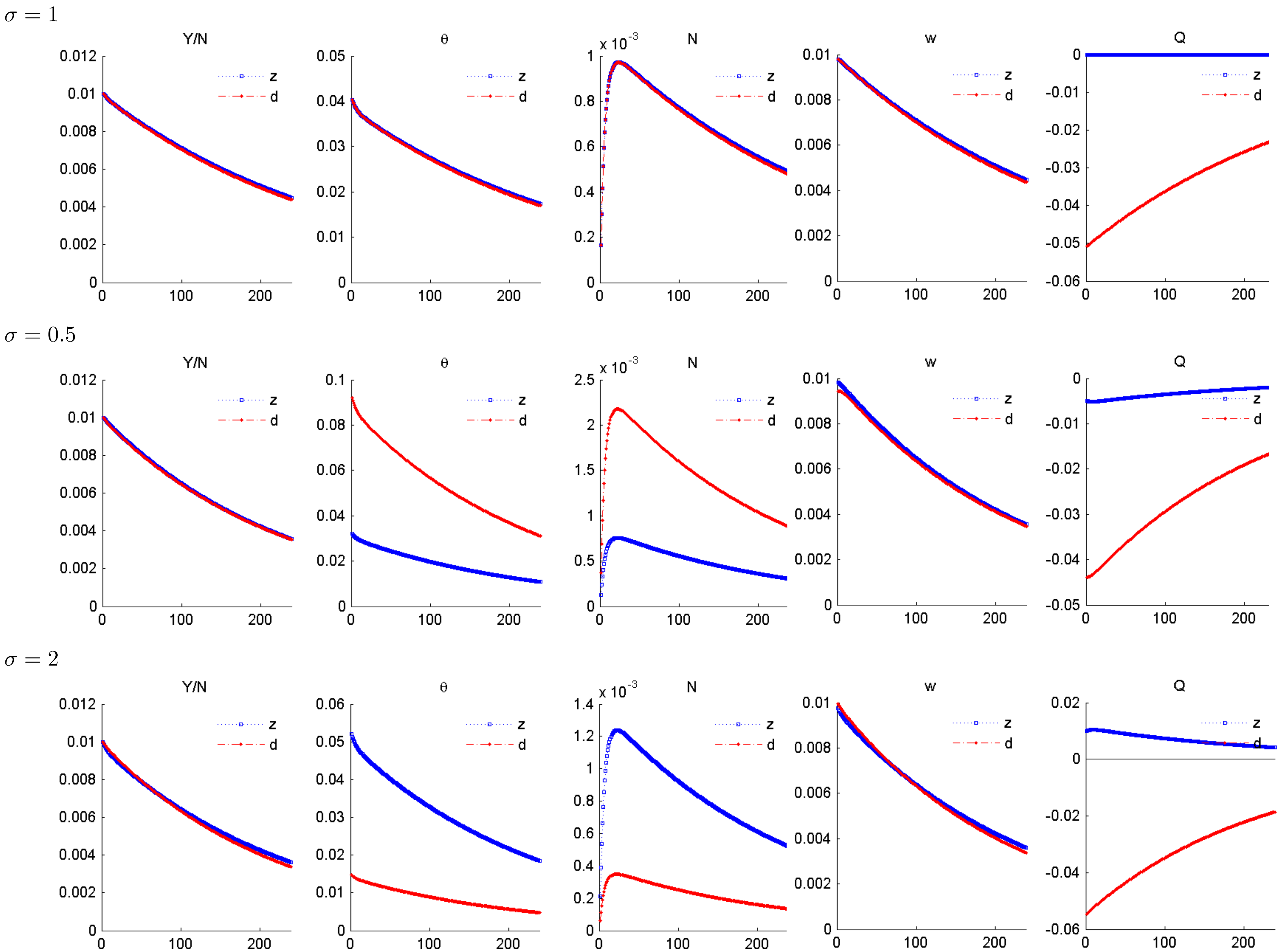
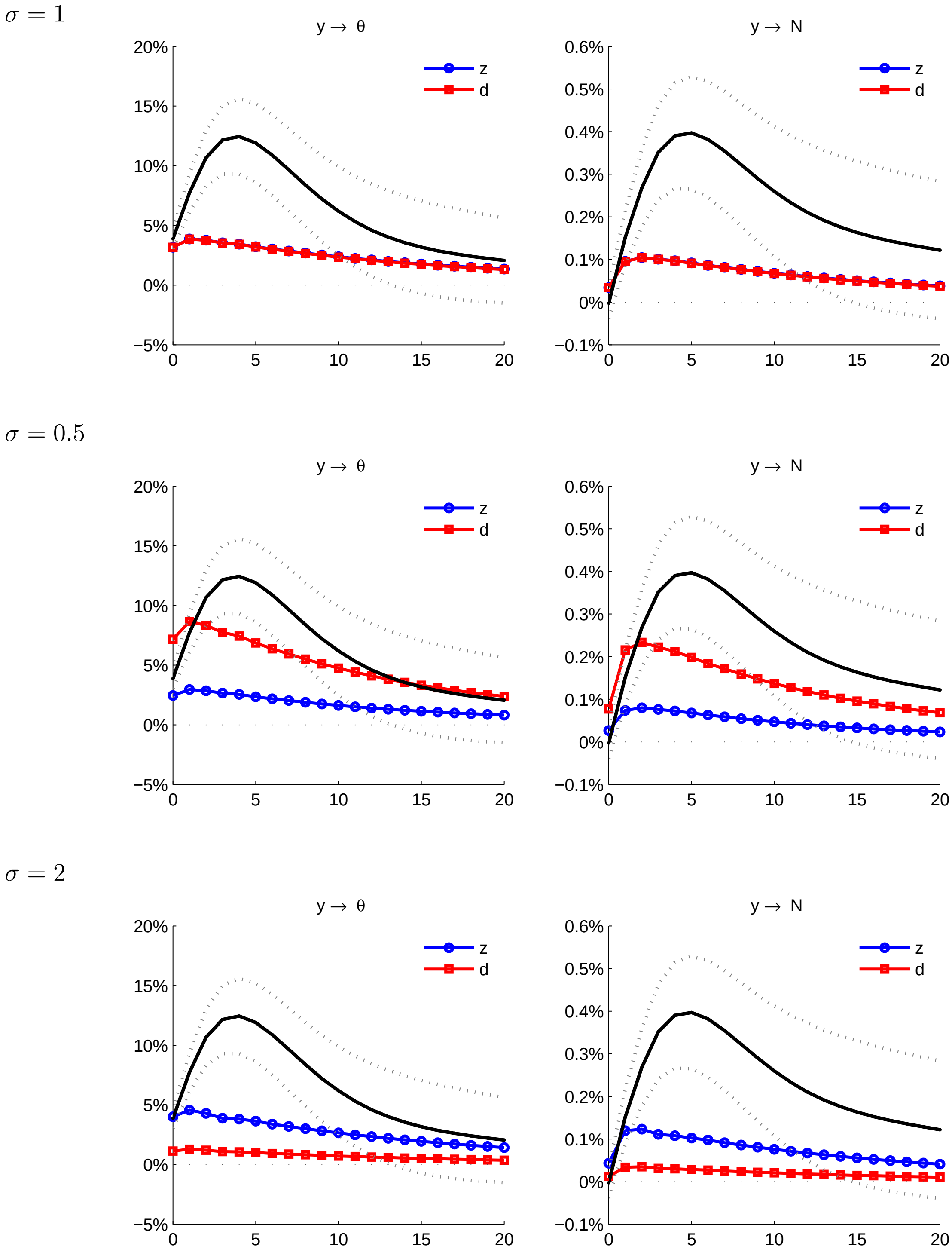


Figure 2: Impulse response function, quarterly data



4.4 Role of Demand Shocks

4.4.1 Model with Inventories

The above results show that the model with goods market search can generate a stronger response of employment to changes in measured labor productivity. The size of the amplification effect however depends the on elasticity of substitution in the goods matching function, and whether the change in the measured labor productivity is caused by a technology or a preference shock. To determine the relative importance of these two types of shocks, I next introduce inventories to the model and use them as additional time series in the estimation. This is motivated by the fact that as shown in Figure 1, the response of the goods market tightness, and thus also the behavior of the fraction of goods sold, is different in response to technology and preference shocks. Intuitively, in the model with inventories a positive shock to technology will result in a build up of inventories relative to sales if the demand does not increase; if on the other hand technology is unchanged and the shock decreases consumers' disutility from search, the result is going to be a drop of inventories relative to sales. Figure 1 also shows that the direction in which goods market tightness moves in response to a technology shock depends on the elasticity of substitution in the goods market matching function. Data on inventories thus provide a source of identification for parameter σ.

I first extend the model by allowing firms to store goods that are not sold, in an attempt to sell them in the next period. Let i' be the amount of goods carried over to the next period

$$i' = (1-\delta_i)(1-\psi^x(Q,X))x$$

where $\delta_i \in (0,1)$ captures the loss of value due to obsoleteness, the fact that some goods will not be demanded at all in the future, and also the storage costs and the inability to store services. The problem of the firm is then a modification of (2.3)

$$\Omega(n,i;\mathbf{S}) = \max_{v,p,Q,x} \left\{ p\psi^x(Q,X(\mathbf{S}))x - w(\mathbf{S})n + \mathbb{E}[m(\mathbf{S},\mathbf{S}')\Omega(n',i';\mathbf{S}')] \right\}$$

subject to

$$x = zf(n-\chi v) - \kappa(v) + i$$

$$n' = (1-\delta)n + \pi^v(\theta(\mathbf{S}))v$$

$$i' = (1-\delta_i)(1-\psi^x(Q,X(\mathbf{S})))x$$

$$W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q,X(\mathbf{S}))$$

$$\mathbf{S}' = G(\mathbf{S})$$

Household's problem remains same as before, and is given by (2.2). Following the same steps as in Appendix A and Appendix B yields a system of equations that characterize the dynamics of goods and labor markets tightnesses Q and θ

$$-U_d = (1-\epsilon_Q^{\psi^d})\psi^d U_c - (1-\delta_i)\beta\mathbb{E}\left[\left(U_c' + \frac{U_d'}{(\psi^d)'}\right)(\Omega_i^r)'\right]$$

$$\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\Omega_i^r\left(U_c + \frac{U_d}{\psi^d}\right)$$

$$= \beta\mathbb{E}\left[\left((1-\mu)z' f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right)(\Omega_i^r)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) + (1-\mu)U_n'\right]$$

where the real marginal value of an inventory good $\Omega_i^r = \frac{\Omega^i}{p}$ evolves according to

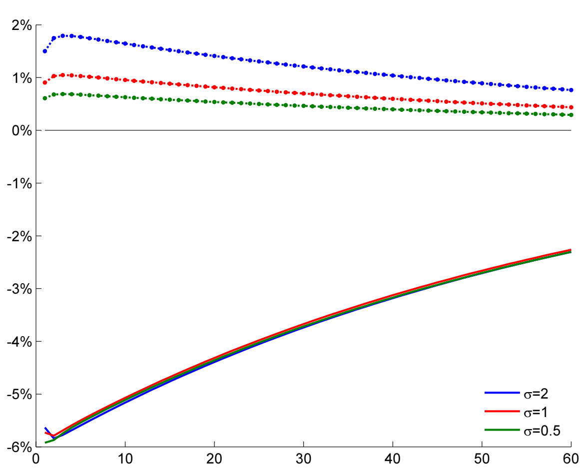
$$\Omega_i^r = \psi^x + (1-\psi^x)(1-\delta_i)\beta\mathbb{E}\left[\frac{U_c' + \frac{U_d'}{(\psi^d)'}}{U_c + \frac{U_d}{\psi^d}}(\Omega_i^r)'\right]$$

4.4.2 Estimation

The two time series used in the estimation are the quarterly average labor productivity $y = \frac{Y}{N}$ and the ratio of inventories to sales $\iota = \frac{I}{C}$. As before, average labor productivity is output per worker in nonfarm business sector constructed by BLS, the ratio of inventories to sales is constructed using data for real nonfarm inventories and real final sales of domestic business from BEA.

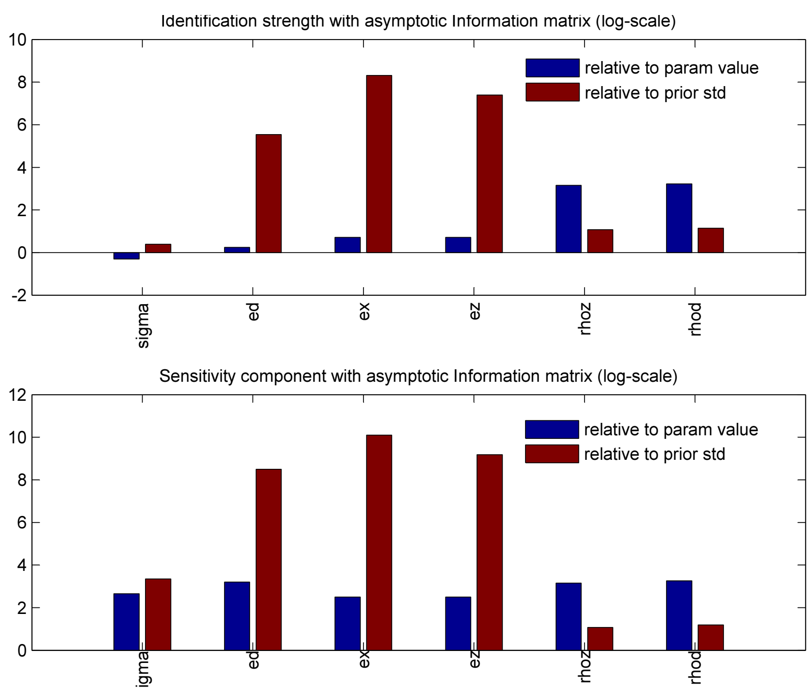
As discussed above, and shown in Figure 3, the behavior of inventories to sales ratio is qualitatively different in response to a technology shock and a preference shock. In addition, the elasticity of substitution in the goods market matching function matters quantitatively for the response to a technology shock. The time series for inventories to sales ratio thus contains information that can be used to determine the contribution of the two types of shocks to the business cycle fluctuations, and to identify the elasticity of substitution in the goods market matching function.

Figure 3: Impulse response function for inventories to sales ratio



I estimate the parameters of the processes for ζ_d and z and the elasticity of substitution in the goods market matching function σ. To verify that all parameters that are estimated are identified, I use the identification tests proposed in Iskrev (2010b). These test is based on the idea that the autocovariogram of the observables with respect to the vector of estimated parameters should have rank equal to the number of the estimated parameters. I find that the Jacobian matrices J_2 and $J(q)$ with $q = 11$ employed in the tests both have full rank, and so parameters are locally identified both in the model and in the data used for estimation. Another issue that can arise in estimation is that some parameters are identified only weakly due to either small sensitivity of moments in the data to that parameter, or due to the high collinearity among some column in Jacobian matrices J_2 and $J(q)$. It is thus useful to inspect identification strength measures from Iskrev (2010a), and the singular value decomposition of the Fisher information matrix as proposed by Andrle (2010). Figure 4 shows the identification strength measures and orders parameters according to the strength of their identification. All estimated parameters affect the behavior of the model, but there is some colinearity present that results in a somewhat weaker identification of the elasticity of substitution σ. This is confirmed by singular value decomposition pattern where the smallest singular value is associated with parameter σ.

Figure 4: Identification strength and sensitivity analysis



Estimation results are shown in Table 9. With exception of σ all parameters are estimated quite tightly, with narrow credible intervals. The confidence interval for the elasticity of substitution is somewhat larger, reflecting the results for the strength of identification. It is however estimated to be significantly below one, so supply and demand are complements in the goods market matching function.

Table 9: Model with two shocks. Observables: y, ι

	Prior			Posterior		
	distribution	mean	st.dev.	mode	mean	90 % HPD interval
ρ_d	Beta	0.800	0.10	0.9926	0.9912	[0.9886,0.9938]
ρ_z	Beta	0.800	0.10	0.9976	0.9976	[0.9975,0.9976]
σ_d	Inverse Gamma	0.050	10.00	0.0080	0.0103	[0.0081,0.0120]
σ_z	Inverse Gamma	0.005	10.00	0.0030	0.0030	[0.0028,0.0032]
$\rho_{z,d}$	Beta	0.000	0.30	-0.8774	-0.7773	[-0.8727,-0.7014]
σ	Gamma	1.000	0.50	0.1018	0.2639	[0.0904,0.4564]
	Log data density 1422.93					

4.4.3 Simulation

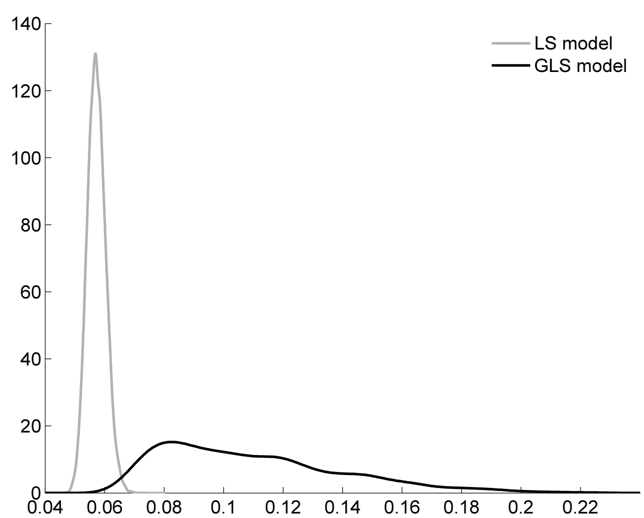
Table 10 shows the main business cycle moments of the model with inventories. Based on the results it is clear that adding goods market frictions improves the ability of the model to replicate the behavior of labor market observed in the U.S. data. Unemployment, vacancies and vacancy-unemployment ratio are about twice as volatile in the extended model as in the model with labor search only in Table 5. To further illustrate this fact, I construct the distribution of the standard deviation of labor market tightness, implied by the posterior distribution of estimated, by simulating the model 100 times for each accepted draw of parameters. The resulting distributions in the model with and without goods market search are plotted Figure 5. The 90% highest posterior density intervals are $[0.052, 0.062]$ and $[0.065, 0.154]$ respectively; the distribution for the model with goods marker search is right skewed with wider support, as a consequences of a wider interval for elasticity of substitution parameter.

The model is also able to match other facts from U.S. data - procyclical inventories and countercyclical inventories-sales ratio. In addition, the variance decomposition reveals that about three quarters of the long run fluctuations in the labor market variables are due to shocks to preferences and the remaining one quarter is due to shocks to technology. Both goods market frictions and demand shocks are therefore important factors in explaining the behavior of unemployment over the business cycle.

Table 10: Summary statistics, labor and goods search model with inventories

	US data			GLS model		
	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$
y	0.013	1.00	0.69	0.014	1.00	0.85
θ	0.262	0.34	0.88	0.102	0.88	0.90
V	0.140	0.42	0.89	0.064	0.80	0.81
U	0.127	-0.25	-0.82	0.047	-0.85	-0.89
I	0.017	0.48	0.48	0.012	0.42	0.37
I/C	0.018	-0.72	-0.45	0.016	-0.66	-0.70

Figure 5: Distribution of standard deviation of labor market tightness in simulations



5 Conclusion

This paper studies the fluctuations of unemployment in a framework that emphasizes the role of consumer demand in determining the output and employment. In particular, it examines the amplification of shocks in a Diamond-Mortensen-Pissarides model after goods market search-matching friction is introduced. When wages are determined by Nash bargaining, goods market frictions affect worker's bargaining position, provide rationale for high value of non-market activity, but also change its cyclical properties. This last effect arises since higher availability of goods in expansions makes frictions in the goods market less severe from consumer's perspective, thus increasing the value of additional earnings obtained when the worker accepts the job. In addition, in the framework analyzed in this paper supply side shocks to technology and demand side shocks to preferences can generate business cycle fluctuations that are observationally equivalent to an economist who would only consider time series for labor productivity, output, employment and wages.

I estimate the model first using data on U.S. average labor productivity only, and show that a modest amount of goods market frictions increases the response of unemployment to technology shocks by one third when search effort and output supplied by firms are good substitutes in the goods market matching function. With low substitutability preference shocks result in response of unemployment which is about two and half times larger than the response to technology shocks in model with labor search only. Afterwards, I add the data on inventories which allows to determine the relative importance of technology and preference shocks. I find that in the full model, with both types of shocks, the response of vacancies and unemployment to changes in measured labor productivity is about twice as large as in the model with labor search only. In addition, demand shocks account for three quarters of fluctuations, and technology shocks for the remaining one quarter. The results of this paper show that both the goods market frictions and the demand shocks play an important role in determining the behavior of unemployment over the business cycle. More work however has to be done to further explore the nature of the demand shocks; preliminary results suggest that news or uncertainty shocks in a framewrok with good market frictions could lead to a similar behavior of the aggregate economy as the one induced by a simple preference shock analyzed in this paper.

References

An, S., & Schorfheide, F. (2007). Bayesian analysis of DSGE models. *Econometric Reviews*, *26*(2-4), 113-172. [22].

Andrle, M. (2010). *A note on identification patterns in DSGE models* (Working Paper Series No. 1235). European Central Bank. [31].

Bai, Y., & Ríos-Rull, J.-V. (2013). *International business cycles redux* (mimeo). [19].

Bai, Y., Ríos-Rull, J.-V., & Storesletten, K. (2012). *Demand shocks that look like productivity shocks* (Working Papers). Federal Reserve Bank of Minneapolis. [3,4,7,9,19].

Barnichon, R. (2010). Building a composite help-wanted index. *Economics Letters*, *109*(3), 175-178. [51].

Barron, J. M., Berger, M. C., & Black, D. A. (1997). *On-the-job training*. W.E. Upjohn Institute for Employment Research. [19].

Bils, M., & Kahn, J. A. (2000). What inventory behavior tells us about business cycles. *American Economic Review*, *90*(3), 458-481. [5].

Costain, J. S., & Reiter, M. (2008). Business cycles, unemployment insurance, and the calibration of matching models. *Journal of Economic Dynamics and Control*, *32*(4), 1120-1155. [3,20].

Del Negro, M., & Schorfheide, F. (2008). Forming priors for dsge models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics*, *55*(7), 1191-1208. [22].

Diamond, P. A. (1982). Wage determination and efficiency in search equilibrium. *Review of Economic Studies*, *49*(2), 217-27. [3].

Garin, J., & Lester, R. (2013). *The cyclicality of unemployment and the relevance of endogenous home production*. [3].

Guerrieri, V. (2008). Heterogeneity, job creation and unemployment volatility. *Scandinavian Journal of Economics*, *109*(4), 667-693. [3].

Hagedorn, M., & Manovskii, I. (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review*, *98*(4), 1692-1706. [3,12,17,18,19,20,21].

Hall, R. E. (2005). Employment fluctuations with equilibrium wage stickiness. *American Economic Review*, *95*(1), 50-65. [3,20].

Hall, R. E., & Milgrom, P. R. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, *98*(4), 1653-74. [3,20].

Hornstein, A., Krusell, P., & Violante, G. L. (2005). Unemployment and vacancy fluctuations in the matching model: Inspecting the mechanism. *Economic Quarterly*, 19-50. [3].

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, *57*(2), 279-98. [12].

Huo, Z., & Ríos-Rull, J.-V. (2013). *Paradox of thrift recessions* (mimeo). University of Minnesota. [4].

Iskrev, N. (2010a). *Evaluating the strength of identification in DSGE models. An a priori approach* (mimeo). [31].

Iskrev, N. (2010b). Local identification in DSGE models. *Journal of Monetary Economics*, *57*(2), 189-202. [31].

Kaplan, G., & Menzio, G. (2013). *Shopping externalities and self-fulfilling unemployment fluctuations* (Working Paper No. 18777). National Bureau of Economic Research. [4].

Khan, A., & Thomas, J. K. (2007). Inventories and the business cycle: An equilibrium analysis of (s, s) policies. *American Economic Review*, *97*(4), 1165-1188. [5].

Krause, M. U., & Lubik, T. A. (2010). *On-the-job search and the cyclical dynamics of the labor market* (Working Paper No. 10-12). Federal Reserve Bank of Richmond. [3].

Lehmann, E., & Van der Linden, B. (2010). Search frictions on product and labor markets: Money in the matching function. *Macroeconomic Dynamics*, *14*(01), 56-92. [4].

Lubik, T. A. (2009). Estimating a search and matching model of the aggregate labor market. *Economic Quarterly*, 101-120. [22].

Menzio, G., & Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, *119*(3), 468 - 510. [3].

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, *36*(2), 269-300. [7].

Michaillat, P., & Saez, E. (2013). *A model of aggregate demand and unemployment* (NBER Working Papers No. 18826). National Bureau of Economic Research, Inc. [4].

Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy*, *105*(2), 385-411. [6].

Moen, E. R., & Rosén, A. (2011). Incentives in competitive search equilibrium. *Review of Economic Studies*, *78*(2), 733-761. [3].

Mortensen, D., & Nagypal, E. (2007). More on unemployment and vacancy fluctuations. *Review of Economic Dynamics*, *10*(3), 327-347. [3,15,20,21,22].

Mortensen, D., & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, *61*(3), 397-415. [3].

Petrosky-Nadeau, N., & Wasmer, E. (2011). *Macroeconomic dynamics in a model of goods, labor and credit market frictions* (IZA Discussion Papers No. 5763). Institute for the Study of Labor (IZA). [4,9,19].

Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment vacancies, and real wages. *American Economic Review*, *75*(4), 676-90. [3].

Pissarides, C. A. (2000). *Equilibrium unemployment theory, 2nd edition.* The MIT Press. [1,3,6,9].

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, *77*(5), 1339-1369. [3,15,20,21,22].

Pissarides, C. A., & Petrongolo, B. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, *39*(2), 390-431. [19].

Ramey, V. A., & West, K. D. (1999). Inventories. In J. B. Taylor & M. Woodford (Eds.), *Handbook of macroeconomics* (Vol. 1, p. 863-923). Elsevier. [5].

Rogerson, R., Shimer, R., & Wright, R. (2005). Search-theoretic models of the labor market: A survey. *Journal of Economic Literature*, *43*(4), 959-988. [1].

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, *95*(1), 25-49. [3,18,20].

Shimer, R. (2010). *Labor markets and business cycles.* Princeton University Press. [9,14,18,19].

Silva, J. I., & Toledo, M. (2009). Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics*, *13*(S1), 76-96. [19].

Silva, J. I., & Toledo, M. (2013). The unemployment volatility puzzle: The role of matching costs revisited. *Economic Inquiry*, *51*(1), 836-843. [3,21].

Stole, L. A., & Zwiebel, J. (1996). Intra-firm bargaining under non-binding contracts. *Review of Economic Studies*, *63*(3), 375-410. [41].

Appendix A

Household's optimality conditions

Consider household's problem (2.2). Denoting by λ_1, λ_2 the Lagrange multipliers on first two constraints, the first order conditions and the envelope theorem conditions are

$$
\begin{aligned}
c: &\qquad 0 = U_c - \lambda_1 p - \lambda_2 \\
d: &\qquad 0 = U_d + \lambda_2 \psi^d \\
a': &\qquad 0 = -\lambda_1 + \beta \mathbb{E} W'_a \\
a: &\qquad W_a = \lambda_1 (1+R) \\
n: &\qquad W_n = U_n + \lambda_1 w + (1 - \delta - \pi^u) \beta \mathbb{E} W'_n
\end{aligned}
$$

From the first order conditions for c and d we get for the value of the marginal unit of income

$$\lambda_1 = \frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big)$$

and the envelope theorem yields the following expression for the marginal value of a worker employed under a contract at equilibrium wage $w(\mathbf{S})$

$$W_n = U_n + \Big(U_c + \frac{U_d}{\psi^d}\Big)\frac{w}{p} + (1 - \delta - \pi^u)\beta \mathbb{E} W'_n \tag{A.1}$$

From the first order condition for a and envelope theorem for a'

$$\lambda_1 = \beta \mathbb{E}[\lambda'_1 (1 + R')]$$

Plugging in for λ_1 yields the following Euler equation equalizing the cost of increasing saving in the form of share holdings by a marginal unit and the return from this marginal savings

$$\frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big) = \beta \mathbb{E}\Big[(1 + R')\frac{1}{p'}\Big(U'_c + \frac{U'_d}{(\psi^d)'}\Big)\Big] \tag{A.2}$$

The left hand side corresponds to the utility cost of extra unit of savings: the household could have instead purchased $\frac{1}{p}$ units of good which require utility cost $\frac{U_d}{\psi^d}$ per unit of good because of the goods market search friction, and enjoyed U_c extra utility per unit of good. The right hand side corresponds to the utility benefit of extra unit of savings: the $1 + R$ monetary flow in the next period can be used to purchase extra consumption in the next period. It will be convenient to denote by $M(\mathbf{S})$ the expected discounted utility from marginal unit of share holdings

$$M = \beta \mathbb{E}\Big[(1 + R')\frac{1}{p'}\Big(U'_c + \frac{U'_d}{(\psi^d)'}\Big)\Big] \tag{A.3}$$

The above intertemporal optimality conditions thus states that $\lambda_1 = M$.

Firm's optimality conditions

Since the household is representative adding the full set of Arrow securities would not affect the allocation, and we can use standard complete markets pricing approach to value the firm. Thus define the stochastic discount factor as

$$m(\mathbf{S},\mathbf{S}') = \beta \frac{p(\mathbf{S})}{p(\mathbf{S}')} \frac{U_c(\mathbf{S}') + \frac{U_d(\mathbf{S}')}{\psi^d(Q(\mathbf{S}'))}}{U_c(\mathbf{S}) + \frac{U_d(\mathbf{S})}{\psi^d(Q(\mathbf{S}))}} \tag{A.4}$$

with slight notational abuse $U_c(\mathbf{S}) = \frac{\partial}{\partial c}U(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}))$, $U_d(\mathbf{S}) = \frac{\partial}{\partial d}U(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}))$.

Consider now firm's problem (2.3). After eliminating p, x, n' using the constraints the first order conditions and the envelope theorem condition are

$$Q: \qquad 0 = \left[\left(\frac{\psi^d_Q}{Q} - \frac{\psi^d}{Q^2}\right)\left(\frac{U_d - W^*_d}{\psi^d M} + \frac{U_c}{M}\right) - \frac{\psi^d}{Q}\frac{U_d - W^*_d}{(\psi^d)^2 M}\psi^d_Q\right]\frac{x}{X}$$

$$v: \qquad 0 = -\frac{1}{X}\frac{\psi^d}{Q}p(\chi z f_l + \kappa_v) + \pi^v \mathbb{E}[m\Omega'_n]$$

$$n: \qquad \Omega_n = -w + \frac{1}{X}\frac{\psi^d}{Q}pzf_l + (1-\delta)\mathbb{E}[m\Omega'_n]$$

Using the first order condition for Q one can obtain that the equilibrium price in active market satisfies

$$p = \epsilon^{\psi^d}_Q \frac{U_c}{M} \tag{A.5}$$

where $\epsilon^{\psi^d}_Q = \frac{\partial \log \psi^d}{\partial \log Q}$. Without goods market search friction price of the good p in the market would be equal to the marginal rate of substitution between consumption and savings; with search friction the price is lower, since this helps the firm to attract more shoppers and increases the probability of selling the goods.

Next, applying the envelope theorem and using the first order condition for v together with (A.5) we can obtain the value of a marginal worker to a firm

$$\Omega_n = \left(zf_l + \frac{1-\delta}{\pi^v}(\chi z f_l + \kappa_v)\right)\frac{1}{X}\frac{\psi^d}{Q}p - w \tag{A.6}$$

Finally, combining (A.6) and the first order condition for v yields the job creation condition

$$\frac{1}{X}\frac{\psi^d}{Q}p(\chi z f_l + \kappa_v) = \pi^v \mathbb{E}\left[m\left(\left(z'f'_l + \frac{1-\delta}{(\pi^v)'}(\chi z' f'_l + \kappa'_v)\right)\frac{1}{X'}\frac{(\psi^d)'}{Q'}p' - w'\right)\right] \tag{A.7}$$

Appendix B

Goods market

From (A.5) we have $M = \epsilon_Q^{\psi^d} \frac{U_c}{p}$ and so noting that the right hand side of equation (A.2) was defined in equation (A.3) to be M, we get

$$-U_d = (1 - \epsilon_Q^{\psi^d})\psi^d U_c \tag{B.1}$$

Labor market

Under Nash bargaining wage w is determined as a solution to the following problem

$$w(\mathbf{S}) = \operatorname*{argmax}_{\hat{w}} \hat{W}_n(\hat{w})^{\mu} \hat{\Omega}_n(\hat{w})^{1-\mu}$$

where $\hat{W}_n(\hat{w})$ and $\hat{\Omega}_n(\hat{w})$ are values, to household and firm, of a marginal worker employed under a contract with arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the job is hit by the separation shock δ. I start by deriving these values.

The value function of a household with n members employed and earning equilibrium wage w, and ν members employed and earning arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the they are hit by the separation shock δ is

$$\tilde{W}(\hat{w}, \nu; \mathbf{S}) = \max_{c,d,a'} U(c, d, n + \nu, \zeta) + \beta \mathbb{E} W(s', n'; \mathbf{S}')$$

subject to

$$p(\mathbf{S})c + a' = (1 + R(\mathbf{S}))a + nw(\mathbf{S}) + \nu\hat{w}$$

$$c = d\psi^d(Q(\mathbf{S}), X(\mathbf{S}))$$

$$n' = (1 - \delta)(n + \nu) + \pi^u(\theta(\mathbf{S}))(1 - n - \nu)$$

$$\mathbf{S}' = G(\mathbf{S})$$

The value of a marginal worker earning wage $\hat{w}$ for this households can then be obtained as

$$\hat{W}_n(\hat{w}) = \tilde{W}_\nu(\hat{w}, 0; \mathbf{S}) = \Big(U_c + \frac{U_d}{\psi^d}\Big)\frac{\hat{w} - w}{p} + W_n \tag{B.2}$$

where the last part makes use of equation (A.1).

The value of a firm that employs n worker for equilibrium wage w, and ν workers for arbitrary wage $\hat{w}$ in the current period, and equilibrium wage w thereafter is

$$\tilde{\Omega}(\hat{w},\nu;\mathbf{S}) = \max_{v,p,Q,x}\left\{\frac{x}{X(\mathbf{S})}\frac{\psi^d(Q,X(\mathbf{S}))}{Q}p - w(\mathbf{S})n - \hat{w}\nu + \mathbb{E}[m(\mathbf{S},\mathbf{S}')\Omega(n';\mathbf{S}')]\right\}$$

subject to

$$x = zf(n+\nu-\chi v) - \kappa(v)$$

$$n' = (1-\delta)(n+\nu) + \pi^v(\theta(\mathbf{S}))v$$

$$W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + \psi^d(Q,X(\mathbf{S}))\big(U_c(\mathbf{S}) - pM(\mathbf{S})\big)$$

$$\mathbf{S}' = G(\mathbf{S})$$

Application of envelope theorem yields

$$\tilde{\Omega}_\nu = -\hat{w} + \frac{1}{X}\frac{\psi^d}{Q}pzf_l + (1-\delta)\mathbb{E}[m\Omega'_n]$$

Notice that the first order conditions for Q, v are same as those in Appendix A, and thus we obtain for the value of a marginal worker that the firm employs and pays arbitrary wage $\hat{w}$ in the current period, and equilibrium wage w thereafter

$$\hat{\Omega}_n(\hat{w}) = \tilde{\Omega}_\nu(\hat{w},0) = \left(zf_l + \frac{1-\delta}{\pi^v}(\chi zf_l + \kappa_v)\right)\frac{1}{X}\frac{\psi^d}{Q}p - \hat{w} = w - \hat{w} + \Omega_n \tag{B.3}$$

The Nash bargaining problem is thus[3]

$$w(\mathbf{S}) = \operatorname*{argmax}_{\hat{w}} \hat{W}_n(\hat{w})^\mu \hat{\Omega}_n(\hat{w})^{1-\mu}$$

with $\hat{W}_n(\hat{w}), \hat{\Omega}_n(\hat{w})$ given by (B.2) and (B.3). The first order condition yields a sharing rule

$$W_n = \frac{\mu}{1-\mu}\left(U_c + \frac{U_d}{\psi^d}\right)\frac{1}{p}\Omega_n \tag{B.4}$$

or $\frac{W_n}{p\lambda_1} = \mu\mathcal{S}$ where $\lambda_1 = \frac{1}{p}\big(U_c + \frac{U_d}{\psi^d}\big)$ is the marginal value of wealth for the household and $\mathcal{S} = \frac{\Omega_n}{p} + \frac{W_n}{p\lambda_1}$ is the total surplus of the match.

To derive the wage equation first plug W_n from the sharing rule (B.4) into (A.1), use stochastic discount factor (A.4), and the optimality conditions (A.7) and (A.6) which after a little bit of algebra yields the stochastic wage equation

$$\frac{w}{p} = \mu\frac{1}{X}\frac{\psi^d}{Q}(zf_l + \theta(\chi zf_l + \kappa_v)) - (1-\mu)\frac{U_n}{U_c + \frac{U_d}{\psi^d}} \tag{B.5}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption.

[3]For simplicity, this specification of the wage bargaining problem disregards the impact of losing a marginal worker on the bargaining position of firm with the remaining workers, see Stole and Zwiebel (1996).

Finally, to obtain a stochastic difference equation that characterizes the labor market plug for w' from (B.5) into job creation condition (A.7), and use stochastic discount factor (A.4), to get

$$\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\psi^x\left(U_c + \frac{U_d}{\psi^d}\right)$$
$$= \beta\mathbb{E}\left[\left[(1-\mu)z'f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right](\psi^x)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) + (1-\mu)U_n'\right] \tag{B.6}$$

Appendix C

Proof of Proposition 1

Proof. Consider the social planner's problem

$$\mathcal{W}(z,\zeta,N) = \max_{C,D,X,V}\left\{U(C,D,N,\zeta) + \beta\mathbb{E}\mathcal{W}(z',\zeta',N')\right\}$$

subject to

$$C = m^G(D,X)$$
$$X = zf(N-\chi V) - \kappa(V)$$
$$N' = (1-\delta)N + m^L(1-N,V)$$

First, by combining the first order conditions for C and D we can obtain the intratemporal optimality condition equalizing the cost and the benefit of the marginal search effort

$$-U_D = m_D^G U_C \tag{C.1}$$

Next, using the first order condition for V and C we get for the marginal value of an employed worker

$$\mathcal{W}_N = U_N + \left(zf_L + \frac{1-\delta-m_U^L}{m_V^L}(\chi z f_L + \kappa_V)\right)m_X^G U_C$$

and by shifting this one period forward and plugging back into the first order condition for V we get the following intertemporal optimality condition

$$m_X^G(\chi z f_L + \kappa_V)U_C$$
$$= m_V^L\beta\mathbb{E}\left[\left[z'f_L' + \frac{1-\delta-(m_U^L)'}{(m_V^L)'}(\chi z' f_L' + \kappa_V')\right](m_X^G)'U_C' + U_N'\right] \tag{C.2}$$

To summarize, efficient allocation is characterized by (C.1), (C.2) and constraints

$$C = m^G(D,X)$$
$$X = zf(N-\chi V) - \kappa(V)$$
$$N' = (1-\delta)N + m^L(1-N,V)$$

The equilibrium allocation on the other hand satisfies these three constraints and (B.1), (B.6). It is easy to verify that since $\epsilon_Q^{\psi^d} = \frac{\partial \log \psi^d}{\partial \log Q}$, $\epsilon_\theta^{\pi^v} = \frac{d \log \pi^v}{d \log \theta}$ it holds that

$$m_D^G = (1 - \epsilon_Q^{\psi^d})\psi^d \qquad m_X^G = \frac{1}{X}\psi_Q^d \qquad m_U^L = -\epsilon_\theta^{\pi^v}\pi^u \qquad m_V^L = (1 + \epsilon_\theta^{\pi^v})\pi^v$$

so that (C.1) can be rewritten as

$$-U_D = (1 - \epsilon_Q^{\psi^d})\psi^d U_C$$

and thus (C.2) becomes

$$\begin{aligned}
&\frac{1}{\pi^v}(\chi z f_L + \kappa_V)\psi^x\left(U_C + \frac{U_D}{\psi^d}\right) \\
&= \beta\mathbb{E}\left[\left[(1+\epsilon_\theta^{\pi^v})z'f_L' + \frac{1+\epsilon_\theta^{\pi^v}}{1+(\epsilon_\theta^{\pi^v})'}\left(\frac{1-\delta}{(\pi^v)'} + (\epsilon_\theta^{\pi^v})'\theta'\right)(\chi z' f_L' + \kappa_V')\right](\psi^x)'\left(U_C' + \frac{U_D'}{(\psi^d)'}\right) + (1+\epsilon_\theta^{\pi^v})U_N'\right]
\end{aligned}$$

Clearly, if $\mu = \frac{\partial \log m^L}{\partial \log U} = -\epsilon_\theta^{\pi^v}$ these are exactly the same conditions as (B.1) and (B.6), and so the conditions for efficient allocation coincide with conditions for equilibrium allocation. □

The following lemma is used throughout in subsequent proofs.

Lemma 2. *Suppose that $f(x, y)$ is homogeneous of degree 1 and has elasticity of substitution σ_{xy}. Then $\sigma_{xy} = \frac{f_x f_y}{f_{xy} f}$.*

Proof. Since f has elasticity of substitution σ_{xy}

$$\sigma_{xy} = \frac{\frac{1}{x f_x} + \frac{1}{y f_y}}{-\frac{f_{xx}}{(f_x)^2} + \frac{2 f_{xy}}{f_x f_y} - \frac{f_{yy}}{(f_y)^2}}$$

thus

$$f_y y + f_x x = \sigma_{xy}\left[-xy\left[f_{xx}\frac{f_y}{f_x} + f_{yy}\frac{f_x}{f_y}\right] + 2 f_{xy} xy\right]$$

Using the fact that f is HOD 1 we have

$$\begin{aligned}
f &= -\sigma_{xy}\left[xy\left[f_{xx}\frac{f_y}{f_x} + f_{yy}\frac{f_x}{f_y}\right] + f_{xx}x^2 + f_{yy}y^2\right] = -\sigma_{xy}\left[f_{xx}\frac{x}{f_x}(f_x x + f_y y) + f_{yy}\frac{y}{f_y}(f_y y + f_x x)\right] \\
&= -\sigma_{xy}\left[\frac{f_{xx}x}{f_x} + \frac{f_{yy}y}{f_y}\right] f
\end{aligned}$$

Now use the fact that f_x and f_y are HOD 0 and simplify further to get

$$1 = -\sigma_{xy}\left[\frac{-f_{xy}y}{f_x} + \frac{-f_{xy}x}{f_y}\right] = \sigma_{xy}\frac{f_{xy} f}{f_x f_y}$$

□

Proof of Proposition 2

Proof.

a. First, using $y = \frac{\psi^d}{QN}$ and $\psi^d_Q = \epsilon^{\psi^d}_Q \frac{\psi^d}{Q}$ we have $\psi^d_Q = \epsilon^{\psi^d}_Q Ny$, and since under Assumption 2 $\kappa = z\bar{\kappa}$ and $X = z(f - \bar{\kappa})$, the goods and labor market equilibrium conditions (3.1) and (3.2) become

$$-U_d D = (1 - \epsilon^{\psi^d}_Q) U_c C \tag{C.3}$$

$$\begin{aligned} &\frac{1}{\pi^v}\frac{\chi f_l + \bar{\kappa}_v}{f - \bar{\kappa}}\epsilon^{\psi^d}_Q N y U_c \\ &\quad = \beta \mathbb{E}\left[\Big((1-\mu)f_l' + \Big(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\Big)(\chi f_l' + \kappa_v')\Big)\frac{1}{f' - \bar{\kappa}'}(\epsilon^{\psi^d}_Q)' N' y' U_c' + (1-\mu)U_n'\right] \end{aligned} \tag{C.4}$$

Consider now any arbitrary history of shocks (z^t, ζ^t) and resulting history of measured average labor productivity, market tightness and employment $(y^t, \theta^t, Q^t, N^t)$ which satisfy equilibrium conditions (C.3), (C.4) and the law of motion for labor (3.3). Now let $\tilde{z}_t = 1$; because m^G is strictly increasing in both its arguments we can find a unique $\tilde{Q}_t$ such that

$$\frac{\psi^d(\tilde{Q}_t, \frac{\tilde{z}_t}{z_t}X_t)}{\tilde{Q}_t} = \frac{\psi^d(Q_t, X_t)}{Q_t} \tag{C.5}$$

and set $\tilde{D}_t = 1/\tilde{Q}_t$ and $\tilde{X}_t = \tilde{z}_t f\big(N_t - \chi\theta_t(1 - N_t)\big) - \tilde{z}_t\bar{\kappa}\big(\theta_t(1 - N_t)\big)$. This guarantees that

$$C_t = \psi^d(\tilde{Q}_t \tilde{X}_t)/\tilde{Q}_t$$
$$y_t = \frac{\psi^d(\tilde{Q}_t, \tilde{X}_t)/\tilde{Q}_t}{N_t}$$

hold. Then let $\tilde{\zeta}_{dt}$ be such that (C.3) holds with $(Q_t, D_t, X_t, \zeta_{dt})$ replaced by $(\tilde{Q}_t, \tilde{D}_t, \tilde{X}_t, \zeta_{dt})$, that is

$$-U_d\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big)\tilde{D}_t = (1 - \epsilon^{\psi^d}_Q(\tilde{Q}_t, \tilde{X}_t))U_c\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big)C_t$$

The last equilibrium condition that is left to be verified is (C.4). Since by Assumption 1A preferences are additively separable the optimal allocation for household satisfies $c_e = c_u = c$ and $d_e = d_u = d$ and so

$$U(c, d, n, \zeta) = \zeta_c u(c) - \zeta_d g(d) - \zeta_n n$$

which implies that

$$U_c\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big) = U_c\big(C_t, D_t, N_t, (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})\big) \tag{C.6}$$

$$U_n\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big) = U_n\big(C_t, D_t, N_t, (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})\big) \tag{C.7}$$

and thus (C.4) will hold for $((\tilde{z}^t, \tilde{\zeta}^t), (y^t, \theta^t, \tilde{Q}^t, N^t))$ as long as

$$\epsilon^{\psi^d}_Q(\tilde{Q}_t, \tilde{X}_t) = \epsilon^{\psi^d}_Q(Q_t, X_t) \tag{C.8}$$

It now only remains to be shown that this condition can only be satisfied if $\epsilon_Q^{\psi^d}(Q,X) \equiv const$. Since by construction

$$\epsilon_Q^{\psi^d} = m_Q^G(1,QX)\frac{Q}{m^G(1,QX)} = m_{(TX)}^G(D,TX)\frac{TX}{m^G(D,TX)}$$

and $\tilde{X}_t = \frac{\tilde{z}_t}{z_t}X_t$, condition (C.8) is equivalent to

$$m_{(TX)}^G\Big(\tilde{D}_t, T\frac{\tilde{z}_t}{z_t}X_t\Big)\frac{T\frac{\tilde{z}_t}{z_t}X_t}{m^G\Big(\tilde{D}_t, T\frac{\tilde{z}_t}{z_t}X_t\Big)} = m_{(TX)}^G(D_t,TX_t)\frac{TX_t}{m^G(D_t,TX_t)}$$

and since m^G is homogeneous of degree 1 and $m_{(TX)}^G$ homogeneous of degree 0, this implies $\tilde{D}_t = \frac{\tilde{z}_t}{z_t}D_t$. But then (C.5) would imply

$$\frac{\tilde{z}_t}{z_t}m^G(D_t,TX_t) = m^G(\tilde{D}_t,T\tilde{X}_t) = \frac{\psi^d(\tilde{Q}_t,\frac{\tilde{z}_t}{z_t}X_t)}{\tilde{Q}_t} = \frac{\psi^d(Q_t,X_t)}{Q_t} = m^G(D_t,TX_t)$$

or $\tilde{z}_t = z_t$ which is a contradiction. Thus (C.8) can only be satisfied if matching function m^G is actually such that $\epsilon_Q^{\psi^d}(Q,X) \equiv const$.

b. Since the wage is given by (B.5), using (B.1), (C.5) and (C.6)-(C.7) we obtain

$$\frac{w}{p} = \mu\frac{1}{\tilde{X}}\frac{\tilde{\psi}^d}{\tilde{Q}}(\tilde{z}f_l + \theta(\chi\tilde{z}f_l + \tilde{z}\bar{\kappa}_v)) - (1-\mu)\frac{\tilde{U}_n}{\epsilon_Q^{\psi^d}\tilde{U}_c}$$

Thus the observed histories of real wages under history of shocks (z^t,ζ^t) and under the alternative history $(\tilde{z}^t,\tilde{\zeta}^t)$ are identical only if $\epsilon_Q^{\psi^d}(\tilde{Q}_t,\tilde{X}_t) = \epsilon_Q^{\psi^d}(Q_t,X_t)$ for all t, which as shown in part a. only holds if $\epsilon_Q^{\psi^d}(Q,X)$ is actually constant.

□

Proof of Proposition 3

Proof. Under Assumption 1A with $u(c) = \log c$, since preferences are additively separable, optimal allocation for household satisfies $c_e = c_u = c$, $d_e = d_u = d$ and so

$$U(C,D,N,\zeta) = \zeta_c \log C - \zeta_d g(D) - \zeta_n N$$

Then

a. Since $m_D^G = (1-\epsilon_Q^{\psi^d})\psi^d$ the goods market equation (3.1) can be written as

$$0 = m_D^G(D,TX)U_C\big(m^G(D,TX),D,N,\zeta\big) + U_D\big(m^G(D,TX),D,N,\zeta\big)$$

where $X = zf(N-\chi V) - \kappa(V)$. Then, under Assumption 1A with $u(c) = \log c$, this simplifies to

$$\zeta_d g_D D = \zeta_c \epsilon_D^{m^G} \tag{C.9}$$

Because

$$\frac{\partial \epsilon_D^{m^G}}{\partial (TX)} = \frac{m_{D,(TX)}^G D}{m^G}\Big(1 - \frac{m_D^G m_{TX}^G}{m_{D,(TX)}^G m^G}\Big) = -\frac{1}{TX}\frac{1}{\sigma}\epsilon_{(TX)}^{m^G}\epsilon_D^{m^G}(1-\sigma)$$

if the matching function m^G has elasticity of substitution $\sigma = 1$ then (C.9) does not depend on X, thus the search effort D and consequently also goods market tightness Q in equilibrium does not react to changes in productivity z.

b. Under Assumption 1A with $u(c) = \log c$ labor market equation (3.2) becomes

$$\frac{1}{X}\epsilon_Q^{\psi^d}\frac{K}{\pi^v}\zeta_c = \beta\mathbb{E}\Big[\Big[(1-\mu)z' f_l' + \Big(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\Big)K'\Big]\frac{1}{X'}(\epsilon_Q^{\psi^d})'\zeta_c' + (1-\mu)U_n'\Big] \tag{C.10}$$

where $K = \chi z f_L(N - \chi\theta(1-N)) + \kappa_V(\theta(1-N))$ and $X = zf(N - \chi\theta(1-N)) - \kappa(\theta(1-N))$. Moreover, ζ_d does not enter the equation above explicitly, so any changes in ζ_d can only affect labor market indirectly through changes in Q and $\epsilon_Q^{\psi^d}$. Since

$$\epsilon_Q^{\psi^d} = \frac{\partial m^G(1,QX)}{\partial Q}\frac{Q}{m^G(1,QX)} = \frac{m_{(TX)}^G(1,QX)XQ}{m^G(1,QX)} = \frac{m_{(TX)}^G(D,TX)TX}{m^G(D,TX)} = \epsilon_{(TX)}^{m^G}$$

and

$$\frac{\partial \epsilon_{(TX)}^{m^G}}{\partial D} = \frac{m_{D,(TX)}^G TX}{m^G}\left(1 - \frac{m_D^G m_{(TX)}^G}{m_{D,(TX)}^G m^G}\right) = -\frac{1}{D}\frac{1}{\sigma}\epsilon_{(TX)}^{m^G}\epsilon_D^{m^G}(1-\sigma)$$

if the matching function m^G has elasticity of substitution $\sigma = 1$ then Q does not enter equation (C.10), and thus the labor market tightness θ is unaffected by shocks to the disutility from search ζ_d.

c. If Assumption 2 also holds in addition to Assumption 1A, then $K = z\bar{K}$ and $X = z\bar{X}$ where $\bar{K} = \chi f_L(N - \chi\theta(1-N)) + \bar{\kappa}_V(\theta(1-N))$ and $\bar{X} = f(N - \chi\theta(1-N)) - \bar{\kappa}(\theta(1-N))$; thus the labor market equation (C.10) simplifies even further and becomes

$$\frac{1}{\bar{X}}\epsilon_Q^{\psi^d}\frac{\bar{K}}{\pi^v}\zeta_c = \beta\mathbb{E}\Big[\Big[(1-\mu)f_l' + \Big(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\Big)\bar{K}'\Big]\frac{1}{\bar{X}'}(\epsilon_Q^{\psi^d})'\zeta_c' + (1-\mu)U_n'\Big]$$

Since z does not enter this equation explicitly, if the matching function m^G has elasticity of substitution $\sigma = 1$ so that using results from parts a. and b. neither Q nor $\epsilon_Q^{\psi^d}$ react to changes in productivity z, then labor market tightness θ is also unaffected by shocks that change the productivity z. □

Proof of Lemma 1

Proof. First, using $y = \frac{\psi^d}{QN}$ and $\psi^d_Q = \epsilon_Q^{\psi^d} \frac{\psi^d}{Q}$ we have $\psi^d_Q = \epsilon_Q^{\psi^d} N y$, and since under Assumption 2 $\kappa = z\bar{\kappa}$ and $X = z(f - \bar{\kappa})$, it is straightforward to show that the goods and labor market equilibrium conditions (3.1) and (3.2) become

$$
\begin{aligned}
0 &= (1 - \epsilon_Q^{\psi^d}) U_C C + U_D D \\
0 &= \Big((1-\mu) f_L - \Big(\mu\theta + \frac{1-\beta(1-\delta)}{\pi^v}\Big)(\chi f_L + \bar{\kappa}_V)\Big) \frac{N}{f - \bar{\kappa}} y + (1-\mu) \frac{U_N}{\epsilon_Q^{\psi^d} U_C}
\end{aligned}
$$

and similarly real wage (3.5) can be rewritten as

$$
\frac{w}{p} = \mu(f_L + \theta(\chi f_L + \bar{\kappa}_V)) \frac{N}{f - \bar{\kappa}} y - (1-\mu) \frac{U_N}{\epsilon_Q^{\psi^d} U_C}
$$

Note that $m_D^G = (1 - \epsilon_Q^{\psi^d})\psi^d$ and $\frac{\psi^d_Q}{X} = m^G_{(TX)}$, thus in the steady (3.1) and (3.2) can be also rewritten as

$$
\begin{aligned}
0 &= m_D^G U_C + U_D \\
0 &= \Lambda z m^G_{(TX)} U_C + (1-\mu) U_N
\end{aligned}
$$

where $\Lambda = (1-\mu) f_L - \big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\big)(\chi f_L + \bar{\kappa}_V)$.

If we denote $\mathbf{Q} = (Q, \theta)'$ and $\mathbf{x} = (z, \zeta)'$ we can thus define function $\mathbf{G}$ in order to write the above two conditions as $\mathbf{0} = \mathbf{G}(\mathbf{Q}, \mathbf{x})$. Applying Implicit Function Theorem, we then obtain $\mathbf{Q} = \mathbf{F}(\mathbf{x})$ and $\mathbf{G}_\mathbf{Q} d\mathbf{Q} = -\mathbf{G}_\mathbf{x} d\mathbf{x}$, afterwards using Cramer's rule we get

$$
\epsilon_z^{\theta^{GLS}} = -\frac{G_Q^1 G_z^2 - G_Q^2 G_z^1}{G_Q^1 G_\theta^2 - G_Q^2 G_\theta^1} \frac{z}{\theta}
$$

and then, since $G_Q^2 dQ + G_\theta^2 d\theta + G_z^2 dz = 0$

$$
\epsilon_z^Q = -\frac{G_z^2 z}{G_Q^2 Q} - \frac{G_\theta^2 \theta}{G_Q^2 Q} \epsilon_z^{\theta^{GLS}}
$$

It is straightforward to verify that

$$
\begin{aligned}
G_z^2 &= -(1-\mu)\Bigg(1 - \frac{\epsilon_D^{m^G}}{\sigma} + (1 - \epsilon_D^{m^G})\epsilon_C^{MRS_{CN}}\Bigg) \frac{U_N}{z} \\
G_\theta^2 &= -(1-\mu)\Bigg(\epsilon_\theta^\Lambda - \frac{\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X + \epsilon_C^{MRS_{CN}} \epsilon_{TX}^{m^G} \epsilon_\theta^X + \epsilon_N^{MRS_{CN}} \epsilon_\theta^N\Bigg) \frac{U_N}{\theta} \\
G_Q^2 &= (1-\mu)\Bigg(\frac{\epsilon_D^{m^G}}{\sigma}\big(1 + \sigma \epsilon_C^{MRS_{CN}}\big) + \epsilon_D^{MRS_{CN}}\Bigg) \frac{U_N}{Q}
\end{aligned}
$$

so that

$$
\epsilon_z^Q = \frac{1 - \frac{\epsilon_D^{m^G}}{\sigma} + (1 - \epsilon_D^{m^G})\epsilon_C^{MRS_{CN}} - \Big(-\epsilon_\theta^\Lambda + \frac{\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X - \epsilon_C^{MRS_{CN}} \epsilon_{TX}^{m^G} \epsilon_\theta^X - \epsilon_N^{MRS_{CN}} \epsilon_\theta^N\Big)\epsilon_z^{\theta^{GLS}}}{\frac{\epsilon_D^{m^G}}{\sigma}\big(1 + \sigma \epsilon_C^{MRS_{CN}}\big) + \epsilon_D^{MRS_{CN}}} \tag{C.11}
$$

Note that in the model with labor search only, the steady state θ satisfies

$$0 = G(\theta, z) = \Lambda z U_C + (1-\mu) U_N$$

and since

$$G_z = -(1-\mu)\big(1 + \epsilon_C^{MRS_{CN}}\big)\frac{U_N}{z}$$
$$G_\theta = -(1-\mu)\Big(\epsilon_\theta^\Lambda + \epsilon_C^{MRS_{CN}}\epsilon_\theta^X + \epsilon_N^{MRS_{CN}}\epsilon_\theta^N\Big)\frac{U_N}{\theta}$$

we get

$$\epsilon_z^{\theta^{LS}} = -\frac{G_z}{G_\theta} = \frac{1 + \epsilon_C^{MRS_{CN}}}{-\epsilon_\theta^\Lambda - \epsilon_C^{MRS_{CN}}\epsilon_\theta^X - \epsilon_N^{MRS_{CN}}\epsilon_\theta^N} \tag{C.12}$$

Because steady state θ and N are calibrated to be the same in the model with labor search and the model with goods and labor search, assuming that preferences and technology are such that the steady state elasticities $\epsilon_C^{U_C}$, $\epsilon_N^{U_C}$, $\epsilon_C^{U_N}$, $\epsilon_N^{U_N}$, ϵ_L^f, $\epsilon_L^{f_L}$ are same in both models, and also $\mu^{LS} = \mu^{GLS}$ we can use (C.12) to substitute for ϵ_θ^Λ into (C.11) to obtain after some rearrangements

$$\frac{1}{\epsilon_z^{\theta^{LS}}} = \left(1 - \frac{1}{\sigma}\frac{1+\sigma\epsilon_C^{MRS_{CN}}}{1+\epsilon_C^{MRS_{CN}}}\epsilon_D^{m^G}(1 + \epsilon_z^Q + \epsilon_\theta^X\epsilon_z^{\theta^{GLS}}) - \frac{\epsilon_D^{MRS_{CN}}}{1+\epsilon_C^{MRS_{CN}}}\epsilon_z^Q\right)\frac{1}{\epsilon_z^{\theta^{GLS}}} \tag{C.13}$$

Since the model with labor search only measured productivity is given by $y = \frac{X}{N}$ thus

$$\epsilon_z^{y^{LS}} = 1 + \big(\epsilon_\theta^X - \epsilon_\theta^N\big)\epsilon_z^{\theta^{LS}} \tag{C.14}$$

Similarly, since in the model with goods and labor search $y = \frac{m^G}{N}$ we have

$$\epsilon_z^{y^{GLS}} = 1 + \big(\epsilon_\theta^X - \epsilon_\theta^N\big)\epsilon_z^{\theta^{GLS}} - \epsilon_D^{m^G}\big(1 + \epsilon_z^Q + \epsilon_\theta^X\epsilon_z^{\theta^{GLS}}\big) \tag{C.15}$$

Because π^u, δ and the steady state θ and N are same in both models, we can combine (C.14) and (C.15) to obtain

$$\frac{\epsilon_z^{y^{GLS}}}{\epsilon_z^{\theta^{GLS}}} = \frac{\epsilon_z^{y^{LS}}}{\epsilon_z^{\theta^{LS}}} + \Big(1 - \epsilon_D^{m^G}(1 + \epsilon_z^Q + \epsilon_\theta^X\epsilon_z^{\theta^{GLS}})\Big)\frac{1}{\epsilon_z^{\theta^{GLS}}} - \frac{1}{\epsilon_z^{\theta^{LS}}}$$

Finally, use (C.13) to substitute for $\frac{1}{\epsilon_z^{\theta^{LS}}}$ and get

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1+\epsilon_C^{MRS_{CN}}}\left(\frac{1-\sigma}{\sigma}\epsilon_D^{m^G}\epsilon_z^{QX} + \epsilon_D^{MRS_{CN}}\epsilon_z^Q\right)\frac{1}{\epsilon_z^{\theta^{GLS}}}$$

□

Proof of Proposition 4

Proof. First, note that $\epsilon_z^{QX} = \epsilon_z^Q + 1 + \epsilon_\theta^X \epsilon_z^\theta$ and that given $\mathbf{G}$ as defined in the proof of Lemma 1

$$\epsilon_z^Q = -\frac{G_z^1 z}{G_Q^1 Q} - \frac{G_\theta^1 \theta}{G_Q^1 Q} \epsilon_z^{\theta^{GLS}}$$

Since

$$G_z^1 = -\left(1 + \sigma \epsilon_C^{MRS_{CD}}\right) \frac{1 - \epsilon_D^{m^G}}{\sigma} \frac{U_D}{z}$$
$$G_\theta^1 = -\left(\left(1 + \sigma \epsilon_C^{MRS_{CD}}\right) \frac{1 - \epsilon_D^{m^G}}{\sigma} \epsilon_\theta^X + \epsilon_N^{MRS_{CD}} \epsilon_\theta^N\right) \frac{U_D}{\theta}$$
$$G_Q^1 = -\left(\frac{1 - \epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G} \epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}\right) \frac{U_D}{Q}$$

we have

$$\epsilon_z^Q = \frac{-\frac{1-\epsilon_D^{m^G}}{\sigma}\left(1 + \sigma \epsilon_C^{MRS_{CD}}\right) - \left(\frac{1-\epsilon_D^{m^G}}{\sigma}\left(1 + \sigma \epsilon_C^{MRS_{CD}}\right)\epsilon_\theta^X + \epsilon_N^{MRS_{CD}} \epsilon_\theta^N\right)\epsilon_z^{\theta^{GLS}}}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G} \epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}}$$

and

$$\epsilon_z^{QX} = -\frac{(\epsilon_C^{MRS_{CD}} + \epsilon_D^{MRS_{CD}})(1 + \epsilon_\theta^X \epsilon_z^{\theta^{GLS}}) + \epsilon_N^{MRS_{CD}} \epsilon_\theta^N \epsilon_z^{\theta^{GLS}}}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G} \epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}}$$

Under Assumptions 1A condition (3.9) then yields

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1 + \epsilon_C^{U_C}} \frac{1 - \sigma}{\sigma} \epsilon_D^{m^G} \frac{(-\epsilon_C^{U_C} + \epsilon_D^{U_D})(1 + \epsilon_\theta^X \epsilon_z^{\theta^{GLS}})}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G} \epsilon_C^{U_C} + \epsilon_D^{U_D}} \frac{1}{\epsilon_z^{\theta^{GLS}}}$$

Therefore if $\sigma > 1$ we have $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$.

Under Assumptions 1B condition (3.9) then yields

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1 + \epsilon_C^{U_C}} \left(\frac{1 - \sigma}{\sigma} \frac{\epsilon_D^{m^G} \epsilon_D^{g_D}(1 + \epsilon_\theta^X \epsilon_z^{\theta^{GLS}})}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G} \epsilon_C^{U_C} + \epsilon_D^{U_D}} + \epsilon_D^{U_C} \epsilon_z^Q\right) \frac{1}{\epsilon_z^{\theta^{GLS}}}$$

Therefore $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as u has relative risk aversion coefficient $-\epsilon_C^{U_C} \leq 1$ and $\sigma \geq 1$; or alternatively $-\epsilon_C^{U_C} \leq 1$ and $\sigma < 1$ and $\epsilon_D^{g_D}$ is sufficiently small. □

Proof of Proposition 5

Proof. Since in the model with goods and labor search $y = \frac{m^G}{N}$ we have

$$\epsilon_{\zeta_d}^y = \left(\epsilon_\theta^X - \epsilon_\theta^N\right)\epsilon_{\zeta_d}^{\theta^{GLS}} - \epsilon_D^{m^G}\left(\epsilon_{\zeta_d}^Q + \epsilon_\theta^X \epsilon_{\zeta_d}^{\theta^{GLS}}\right) \tag{C.16}$$

and thus for $\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{\epsilon_{\zeta_d}^y}{\epsilon_{\zeta_d}^{\theta^{GLS}}}$ we obtain

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = -\epsilon_D^{m^G} \frac{\epsilon_{\zeta_d}^Q}{\epsilon_{\zeta_d}^{\theta^{GLS}}} + (1 - \epsilon_D^{m^G})\epsilon_\theta^X - \epsilon_\theta^N$$

Given function $\mathbf{G}$ as defined in the proof of Lemma 1 we get using Cramer's rule

$$\frac{\epsilon_{\zeta_d}^Q}{\epsilon_{\zeta_d}^{\theta^{GLS}}} = \frac{-\frac{G_{\zeta_d}^1 G_\theta^2 - G_{\zeta_d}^2 G_\theta^1}{G_Q^1 G_\theta^2 - G_Q^2 G_\theta^1}\frac{\zeta_d}{Q}}{-\frac{G_Q^1 G_{\zeta_d}^2 - G_{\zeta_d}^1 G_Q^2}{G_Q^1 G_\theta^2 - G_\theta^1 G_Q^2}\frac{\zeta_d}{\theta}} = -\frac{G_{\zeta_d}^1 G_\theta^2 - G_{\zeta_d}^2 G_\theta^1}{G_{\zeta_d}^1 G_Q^2 - G_Q^1 G_{\zeta_d}^2}\frac{\theta}{Q}$$

and with additively separable preferences from Assumptions 1A since $G_{\zeta_d}^2 = 0$

$$\frac{\epsilon_{\zeta_d}^Q}{\epsilon_{\zeta_d}^{\theta^{GLS}}} = \frac{\epsilon_\theta^\Lambda - \frac{\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X + \epsilon_{(TX)}^{m^G}\epsilon_C^{U_C}\epsilon_\theta^X - \epsilon_N^{U_N}\epsilon_\theta^N}{\frac{\epsilon_D^{m^G}}{\sigma}(1 + \sigma\epsilon_C^{U_C})}$$

Plugging this back yields

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{-\epsilon_\theta^\Lambda - \epsilon_C^{U_C}\epsilon_\theta^X + \epsilon_N^{U_N}\epsilon_\theta^N}{\frac{1}{\sigma} + \epsilon_C^{U_C}} + \epsilon_\theta^X - \epsilon_\theta^N \tag{C.17}$$

In the model with labor search only with a shock to z we have from (C.12) and (C.14) under Assumption 1A that

$$\frac{1}{\epsilon_y^{\theta^{LS}}} = \frac{-\epsilon_\theta^\Lambda - \epsilon_C^{U_C}\epsilon_\theta^X + \epsilon_N^{U_N}\epsilon_\theta^N}{1 + \epsilon_C^{U_C}} + \epsilon_\theta^X - \epsilon_\theta^N \tag{C.18}$$

comparing (C.17) and (C.18) the result follows immediately. □

Appendix D

Data

Seasonally adjusted average output per worker in nonfarm business sector y, and seasonally adjusted output in the nonfarm business sector Y are both constructed by the BLS using National Income and Product Accounts and Current Employment Survey, they are series PRS85006163 and PRS85006043 respectively.

Seasonally adjusted composite help wanted index V is constructed following the approach in Barnichon (2010) and combines help-wanted advertising index and the online help-wanted index constructed by Conference Board. Seasonally adjusted unemployment U is constructed by BLS from Current Population Survey, it's the series LNS13000000. Both V and U are quarterly averages of monthly series.

In simulations, quarterly average labor productivity y_t is calculated as quarterly output Y_t divided by the quarter's employment N_t, with quarterly output given by the sum of weekly output, and quarterly employment given by the average employment in the three months of the quarter. Since for each month employment is measured by the BLS in the second week quarterly productivity is calculated as

$$y_t = \frac{\sum_{i=1}^{12} Y_{12t-i+1}^W}{\frac{1}{3}(N_{12t-2}^W + N_{12t-6}^W + N_{12t-10}^W)}$$

The ratio of inventories to sales ι is constructed using data for real nonfarm inventories and real final sales of domestic business from BEA, from Table 5.8.6 of the National Income and Product Accounts.

Log-linearized model

Let $\hat{x} = \log(x/\bar{x})$ denote the percentage deviation of variable x from its steady state $\bar{x}$. The log-linearized system of equations for $(\theta, Q, \Omega_i^r, I, N, y, \iota)$ is

$$\frac{1}{\bar{\pi}^v}\Big(-\hat{\pi}^v + \hat{z} + \hat{f}_l + \hat{\Omega}_i^r + \bar{\epsilon}_Q^{\psi^d} + \hat{U}_c \Big)$$
$$= \beta\mathbb{E}\Bigg[-\mu\bar{\theta}\hat{\theta}' - \frac{1-\delta}{\bar{\pi}^v}(\hat{\pi}^v)' + \Big(1-\mu-\mu\bar{\theta}+\frac{1-\delta}{\bar{\pi}^v}\Big)\Big(\hat{z}' + \hat{f}_l' + (\hat{\Omega}_i^R)' + (\hat{\epsilon}_Q^{\psi^d})' + \hat{U}_c'\Big)$$
$$- \Big(1-\mu-\mu\bar{\theta}-\frac{1-\beta(1-\delta)}{\beta\bar{\pi}^v}\Big)\frac{1}{b\bar{\epsilon}_Q^{\psi^d}\bar{U}_c - \bar{U}_n}\big(b\bar{\epsilon}_Q^{\psi^d}\bar{U}_c(\hat{\epsilon}_Q^{\psi^d} + \hat{U}_c') - \bar{U}_n\hat{U}_n'\big)\Bigg]$$

$$\hat{U}_d = \hat{\psi}^d - \frac{\bar{\epsilon}_Q^{\psi^d}}{1-\bar{\epsilon}_Q^{\psi^d}}\hat{\epsilon}_Q^{\psi^d}$$
$$+ \frac{1}{\bar{U}_c - (1-\delta_i)\beta\Big(\bar{U}_c + \frac{\bar{U}_d}{\bar{\psi}^d}\Big)\bar{\Omega}_i^r}\Bigg[\bar{U}_c\hat{U}_c - (1-\delta_i)\beta\bar{\Omega}_i^r\mathbb{E}\Big[\bar{U}_c\hat{U}_c' + \frac{\bar{U}_d}{\bar{\psi}^d}(\hat{U}_d' - (\psi^d)') + \Big(\bar{U}_c + \frac{\bar{U}_d}{\bar{\psi}^d}\Big)(\hat{\Omega}_i^r)'\Big]\Bigg]$$

$$\hat{\Omega}_i^r = (1-\beta(1-\delta_i))\hat{\psi}^x + (1-\bar{\psi}^x)\beta(1-\delta_i)\mathbb{E}\big[(\hat{\epsilon}_Q^{\psi^d})' + \hat{U}_c' - \hat{\epsilon}_Q^{\psi^d} - \hat{U}_c + (\Omega_i^r)'\big]$$

$$\hat{I}' = -\frac{\bar{\psi}^x}{1-\bar{\psi}^x}\hat{\psi}^x + \hat{X}$$

$$\hat{N}' = \Big(1 - \frac{\delta}{1-\bar{N}}\Big)\hat{N} + \delta(\hat{\theta} + \hat{\pi}^v)$$

$$\hat{y} = \hat{Y} - \hat{N}$$

$$\hat{\iota} = -\frac{1}{1-\bar{\psi}^x}\hat{\psi}^x$$

where $(\hat{Y}, \hat{C}, \hat{X}, \hat{F})$ are eliminated using

$$\hat{Y} = \hat{C} + (1-\delta_i)\frac{1-\bar{\psi}^x}{\bar{\psi}^x}(\hat{I}' - \hat{I})$$
$$\hat{C} = \hat{\psi}^x + \hat{X}$$
$$\hat{X} = \big(1-(1-\delta_i)(1-\bar{\psi}^x)\big)\hat{F} + (1-\delta_i)(1-\bar{\psi}^x)\hat{I}$$
$$\hat{F} = \hat{z} + \frac{\lambda}{\bar{N} - \bar{\theta}(1-\bar{N})}((1+\bar{\theta})\bar{N}\hat{N} - (1-\bar{N})\bar{\theta}\hat{\theta})$$

and given the choice of the functional forms we also have

$$\hat{U}_c = \hat{\zeta}_c - \eta\hat{C} \qquad \hat{U}_d = \hat{\zeta}_d \qquad \hat{U}_n = \hat{\zeta}_n$$
$$\hat{f}_l = \frac{\lambda-1}{\bar{N} - \bar{\theta}(1-\bar{N})}((1+\bar{\theta})\bar{N}\hat{N} - (1-\bar{N})\bar{\theta}\hat{\theta})$$
$$\hat{\pi}^v = -\gamma\hat{\theta}$$
$$\hat{\psi}^x = -\frac{\alpha(\bar{Q}\bar{X})^\rho}{\alpha(\bar{Q}\bar{X})^\rho + 1 - \alpha}(\hat{Q} + \hat{X})$$
$$\hat{\epsilon}_Q^{\psi^d} = -\frac{\alpha(\bar{Q}\bar{X})^\rho}{\alpha(\bar{Q}\bar{X})^\rho + 1 - \alpha}\rho(\hat{Q} + \hat{X})$$