Pegging the Interest Rate on Bank Reserves: A Resolution of New Keynesian Puzzles and Paradoxes

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Motivation and overview

- The Great Recession has led central banks to temporarily peg their policy rates near zero.

- The New Keynesian (NK) literature has puzzling and paradoxical implications under a temporary interest-rate peg:
  - forward-guidance puzzle,
  - fiscal-multiplier puzzle,
  - paradox of flexibility,
  - paradox of toil.

- This paper offers a resolution of these puzzles and paradoxes based on a simple and possibly minimal departure from the basic NK model.

- This departure involves the central bank pegging the interest rate on bank reserves (IOR rate) — as central banks did in reality.
Three limit puzzles and paradox

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● **Stark discontinuity** in the permanent-peg or flexible-price limit.
Forward-guidance puzzle and paradox of flexibility

Source: Cochrane (2017a).
Resolution of the puzzles and paradoxes

- The source of these limit puzzles and paradox lies in the basic NK model’s property of exhibiting **indeterminacy under a permanent interest-rate peg**.

- Indeterminacy arises because the central bank pegs the interest rate on a bond serving **only as a store of value** (Canzoneri and Diba, 2005).

- In our model, the central bank pegs the interest rate on bank reserves, which serve to **reduce the costs of banking**.

- Our model delivers **determinacy under a permanent IOR-rate peg**, even under perfectly flexible prices, and therefore solves the limit puzzles and paradox.

- For a related reason, our model can also **solve the paradox of toil** (which says that positive supply shocks are contractionary under a temporary interest-rate peg).
Literature on the NK puzzles and paradoxes


- **Empirical evidence**: Cohen-Setton, Hausman, and Wieland (2017); Garín, Lester, and Sims (2017); Wieland (2016).

- **Attenuations**: Andrade, Gaballo, Mengus, and Mojon (2017); Angeletos and Lian (2016); Del Negro, Giannoni, and Patterson (2015); Farhi and Werning (2017); Gertler (2017); Kaplan, Moll, and Violante (2016); McKay, Nakamura, and Steinsson (2016, 2017); Wiederholt (2015).

- **Resolutions**: Angeletos and Lian (2016); Bilbiie (2017); Cochrane (2017a, 2017b); Gabaix (2016); García-Schmidt and Woodford (2015); Ravn and Sterk (2017).
1. We **solve all** three limit puzzles and paradox with a **simple** departure from the basic NK model.

2. We solve them even for an **arbitrarily small** departure (i.e. arbitrarily small banking costs).

3. We still solve or attenuate them for a **vanishingly small** departure, and also solve the paradox of toil in that case, thus
   - providing an equilibrium-selection device in the basic NK model,
   - closing the gap between the basic sticky-price and sticky-information models.

4. Our resolution of the puzzles and paradoxes **preserves** two standard implications of the basic NK model in normal times:
   - the Fisher effect,
   - the Taylor principle (under a corridor system).
Outline of the presentation

1. Introduction
2. Benchmark Model
3. Resolution of the Puzzles and Paradoxes
4. Comparison with Discounting Models / Robustness Analysis
5. Conclusion
Households

- The representative household (RH) consists of workers and bankers.

- RH’s intertemporal utility function is

\[ U_t = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ u(c_{t+k}) - v(h_{t+k}) - v^b(h^b_{t+k}) \right] \right\} .\]

- Bankers use their own labor \( h^b_t \) and reserves \( m_t \) to produce loans:

\[ \ell_t = f^b(h^b_t, m_t) , \]

where \( f^b \) can be in particular any CES function.

- We can invert \( f^b \) and rewrite bankers’ labor disutility as \( v^b(h^b_t) = \Gamma(\ell_t, m_t) \).

- The FOCs imply \( \ell^\ell_t > l^b_t \) (because \( \Gamma_\ell > 0 \)) and \( l^b_t > l^m_t \) (because \( \Gamma_m < 0 \)).
Firms are monopolistically competitive and owned by households.

They use workers’ labor to produce output:

\[ y_t = f(h_t). \]

They have to borrow their nominal wage bill \( P_t \ell_t = W_t h_t \) in advance from banks, at the gross nominal interest rate \( I_t^\ell \).

Prices are sticky à la Calvo (1983), with a degree of price stickiness \( \theta \in [0, 1) \).
Government and steady state

- The **monetary authority** has two independent instruments:
  - the (gross) nominal interest rate on reserves $I_t^m \geq 1$,
  - the quantity of nominal reserves $M_t > 0$.

- The **fiscal authority** sets exogenously real fiscal expenditures $g_t \geq 0$, and fiscal policy is **Ricardian**.

- We assume that
  - $I_t^m$ is set **exogenously** around some steady-state value $I^m \in [1, \beta^{-1})$,
  - $\mu_t \equiv M_t / M_{t-1}$ is set **exogenously** around the steady-state value $\mu = 1$.

- **Proposition**: *There is a unique steady state, and this steady state has zero inflation.*

- We log-linearize the model around this steady state.
Four equilibrium conditions I

1. Profit maximization by firms leads to the **Phillips curve**

   \[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa_y \hat{y}_t - \kappa_m \hat{m}_t - \kappa_g \tilde{g}_t, \]

   where
   - \( \kappa_y > 0 \) and \( \kappa_g > 0 \) depend (positively) on \( \Gamma_{\ell \ell} \),
   - \( \kappa_m > 0 \) depends (positively) on \( |\Gamma_{\ell m}| \).

2. RH’s indifference between \( b_t \) and \( m_t \) leads to the **interest-rate-spread equation**

   \[ i_t^b - i_t^m = \delta_y \hat{y}_t - \delta_m \hat{m}_t - \delta_g \tilde{g}_t, \]

   marginal opportunity cost of holding reserves
   \hspace{1cm} \text{marginal benefit of holding reserves}

   where
   - \( \delta_y > 0 \) and \( \delta_g > 0 \) depend (positively) on \( |\Gamma_{\ell m}| \),
   - \( \delta_m > 0 \) depends (positively) on \( \Gamma_{mm} \).
The Euler equation gives the standard IS equation

\[ \hat{y}_t = E_t \{ \hat{y}_{t+1} \} - \frac{1}{\sigma} E_t \left\{ i_t^b - \pi_{t+1} \right\} + \tilde{g}_t - E_t \{ \tilde{g}_{t+1} \}. \]

The (first difference of the) reserve-market-clearing condition is

\[ \pi_t = -\Delta \hat{m}_t + \hat{\mu}_t. \]

Using these four equilibrium conditions, we get the dynamic equation in reserves:

\[ E_t \{ LP(L^{-1}) \hat{m}_t \} = i_t^m + E_t \{ Q_{\mu}(L^{-1}) \hat{\mu}_t \} + E_t \{ Q_g(L^{-1}) \tilde{g}_t \}. \]

Lemma: The roots of \( P(X) \) are three real numbers \( \rho, \omega_1, \) and \( \omega_2 \) such that \( 0 < \rho < 1 < \omega_1 < \omega_2. \)
**Proposition:** Setting exogenously $I_t^m$ and $\mu_t$ ensures local-equilibrium determinacy.

The spread equation can be viewed as a **shadow Wicksellian rule** for $i_t^b$.

This rule ensures determinacy **only** because our assumptions on $f^b$ imply that

$$\delta_m \kappa_y - \delta_y \kappa_m > 0.$$

This inequality corresponds to the **Taylor principle** (the nominal interest rate should react by more than one-to-one to the inflation rate in the long run):

$$\Delta i^b = \delta_y \Delta \hat{y} - \delta_m \Delta \hat{m} = \left( \delta_y \frac{\kappa_m}{\kappa_y} - \delta_m \right) \Delta \hat{m} = \frac{\delta_m \kappa_y - \delta_y \kappa_m}{\kappa_y} \pi.$$
Consider the basic NK model, and assume that the central bank

- pegs \( i_t^b = i^b < 0 \) between 1 and \( T \),
- follows the rule \( i_t^b = \phi \pi_t \) from \( T + 1 \) onwards, where \( \phi > 1 \).

Since a permanent \( i_t^b \) peg generates indeterminacy, the dynamic system between 1 and \( T \) has an “excess stable eigenvalue,” so that the economy explodes backward in time starting from the terminal conditions \( \hat{y}_{T+1} = \pi_{T+1} = 0: \)

\[
\lim_{T \to +\infty} (\hat{y}_1, \pi_1) = (+\infty, +\infty).
\]

Now consider our model, and assume that the central bank pegs

- \( i_t^m = i^m < 0 \) between 1 and \( T \),
- \( i_t^m = 0 \) from \( T + 1 \) onwards.

Since a permanent \( i_t^m \) peg delivers determinacy, the dynamic system between 1 and \( T \) has no excess stable eigenvalue, so that

\[
\lim_{T \to +\infty} (\hat{y}_1, \pi_1) = (\hat{y}_1^p, \pi_1^p).
\]
Consider the same experiment as before, and assume in addition that the government announces at date 1 that $\tilde{g}_T = \tilde{g}^* > 0$ and $\tilde{g}_t = 0$ for $t \neq T$.

In the basic NK model, for the same reason as before, we get

$$\lim_{T \to +\infty} \left( \frac{\partial \hat{y}_1}{\partial \tilde{g}^*}, \frac{\partial \pi_1}{\partial \tilde{g}^*} \right) = (+\infty, +\infty).$$

This result still obtains when $\tilde{g}_T = \zeta^T \tilde{g}^*$ with $0 \ll \zeta < 1$: news about vanishingly distant and vanishingly small fiscal expenditures can have exploding effects today.

In our model, for the same reason as before, we get

$$\lim_{T \to +\infty} \left( \frac{\partial \hat{y}_1}{\partial \tilde{g}^*}, \frac{\partial \pi_1}{\partial \tilde{g}^*} \right) = (0, 0).$$
Paradox of flexibility

- Consider the same experiments as before, but now make $\theta \to 0$ holding $T$ constant.

- In the **basic NK model**, we get

$$\lim_{\theta \to 0} \left( \hat{y}_1, \pi_1, \frac{\partial \hat{y}_1}{\partial \hat{g}^*}, \frac{\partial \pi_1}{\partial \hat{g}^*} \right) = (+\infty, +\infty, +\infty, +\infty).$$

- The reason is that the system’s **excess stable eigenvalue goes to zero** as $\theta \to 0$: under an $i_t^b$ peg, as prices become more and more flexible,
  - the effects of shocks die out more and more quickly forward in time,
  - the economy explodes more and more quickly backward in time.

- In **our model**, for the same reason as before, we get

$$\lim_{\theta \to 0} \left( \hat{y}_1, \pi_1, \frac{\partial \hat{y}_1}{\partial \hat{g}^*}, \frac{\partial \pi_1}{\partial \hat{g}^*} \right) = \left( \hat{y}^f_1, \pi^f_1, \frac{\partial \hat{y}^f_1}{\partial \hat{g}^*}, \frac{\partial \pi^f_1}{\partial \hat{g}^*} \right).$$
Our model solves the limit puzzles and paradox for any given

- (dis)utility and production functions $u, v, v^b, f,$ and $f^b,$
- structural-parameter values $\beta \in (0, 1), \varepsilon > 0,$ and $\theta \in [0, 1),$
- policy-parameter values $l^m \in [1, \beta^{-1})$ and $g \geq 0.$

Now replace $v^b$ by $\gamma v^b$ (and hence $\Gamma$ by $\gamma \Gamma$), where $\gamma > 0$ is a scale parameter.

**Proposition:** As $(l^m, \gamma) \rightarrow (\beta^{-1}, 0)$ with $(\beta^{-1} - l^m) / \gamma$ bounded away from zero and infinity, the steady state and reduced form of our model converge towards the steady state and reduced form of the corresponding basic NK model.

Thus, even an *arbitrarily small* departure from the basic NK model is enough to solve the limit puzzles and paradox.
Consider a **sequence of models** converging towards the basic NK model, each of them solving the limit puzzles and paradox.

Consider the **limit of equilibrium outcomes** along this sequence, for any given
- duration of the IOR-rate peg $T$,
- degree of price stickiness $\theta$.

This limit coincides with a **particular equilibrium** (out of an infinity of equilibria) of the basic NK model under a temporary, followed by a permanent, interest-rate peg.

Thus, we provide an **equilibrium-selection device** in the basic NK model, which is reminiscent of the one developed in the global-games literature (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2000).
Our selected equilibrium

- We show that our **selected equilibrium**
  - exhibits neither the fiscal-multiplier puzzle nor the paradox of flexibility,
  - exhibits an **attenuated** form of the forward-guidance puzzle: inflation and output grow *linearly* with the duration of the peg, not *exponentially*.

- We relate this attenuation of the forward-guidance puzzle to **price-level stationarity** ($p_\infty = p_0$) under a temporary IOR-rate peg ($i_t^m = i_t^b = i^*$ for $1 \leq t \leq T$):

  $$\hat{y}_1 = \hat{y}_\infty - \frac{T i^*}{\sigma} + \frac{p_\infty - p_1}{\sigma} = - \frac{T i^*}{\sigma} - \frac{\pi_1}{\sigma}.$$

- We also show that our selected equilibrium does not exhibit the **paradox of toil**, and relate this feature to **inflation inertia** in our model.
Consider the **basic NK model**, and assume that the central bank
- pegs \( i_t^b = 0 \) between 1 and \( T \),
- follows the rule \( i_t^b = \phi \pi_t \) from \( T + 1 \) onwards, where \( \phi > 1 \).

Consider a cost-push shock \( \hat{\phi}^* > 0 \) between 1 and \( T \):

\[
\hat{y}_t = \mathbb{E}_t\{\hat{y}_{t+1}\} + \sigma^{-1}\mathbb{E}_t\{\pi_{t+1}\}, \quad \text{(IS)}
\]
\[
\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \hat{y}_t + \kappa \phi \hat{\phi}^*. \quad \text{(PC)}
\]

We get sequentially:
- \( \pi_{T+1} = \hat{y}_{T+1} = 0 \) (from the rule at dates \( t \geq T + 1 \)),
- \( \pi_T > 0 \) and \( \hat{y}_T = 0 \) (from IS and PC at date \( T \)),
- \( \pi_{T-1} > 0 \) and \( \hat{y}_{T-1} > 0 \) (from IS and PC at date \( T - 1 \))...

In our selected equilibrium, we have \( \pi_{T+1} < 0 \) because of the **inertia introduced by the state variable** (the stock of reserves), and hence \( \hat{y}_{T+1} < 0 \), and hence \( \hat{y}_T < 0 \) (from IS at date \( T \)); and we show that \( \hat{y}_t < 0 \) for \( 1 \leq t \leq T \).
Comparison with other equilibria

- Our selected equilibrium differs from the **standard equilibrium**, which exhibits all four puzzles and paradoxes.

- It also differs from Cochrane’s (2017a) **backward-stable and no-inflation-jump equilibria**:
  - our equilibrium exhibits (a weak form of) the forward-guidance puzzle,
  - our equilibrium makes inflation negative at the start of a liquidity trap.

- It belongs to the set of **local-to-frictionless equilibria** (Cochrane, 2017a), as it does not exhibit the paradox of flexibility.

- It behaves like the equilibrium of Mankiw and Reis’s (2002) **sticky-information model** in terms of exhibiting or not the puzzles and paradoxes (Carlstrom, Fuerst, and Paustian, 2015; Kiley, 2016).

- So it brings the canonical sticky-price model **at par with** its sticky-information cousin in terms of their ability to solve or attenuate the four puzzles and paradoxes.
Standard, backward-stable, and no-inflation-jump equilibria

\[ \text{it} - rt = -2\% \text{ between } t = 0 \text{ and } t = 5, \text{ shown by vertical dashed lines, and } \text{it} = rt \text{ otherwise.} \]

The thick lines show the backward-stable equilibrium, the no-jump equilibrium, and the standard equilibrium discussed below. Thinner lines show a range of additional possible equilibria.

Source: Cochrane (2017a).
Discounting models

- We consider the class of “discounting models” with a reduced form of type

\[
\hat{y}_t = \xi_1 \mathbb{E}_t \{\hat{y}_{t+1}\} - \frac{\xi_2}{\sigma} \mathbb{E}_t \left\{ i_t^b - \pi_{t+1} \right\}, \\
\pi_t = \beta \xi_3 (\theta) \mathbb{E}_t \{\pi_{t+1}\} + \kappa (\theta) [\hat{y}_t - \xi_4 (\theta) \mathbb{E}_t \{\hat{y}_{t+1}\}],
\]

where \( \beta \in (0, 1) \), \( \sigma > 0 \), \( \xi_1 > 0 \), \( \xi_2 > 0 \), and, for all \( \theta \in (0, 1) \), \( \xi_3 (\theta) \geq 0 \), \( \xi_4 (\theta) \in [0, 1) \), and \( \kappa (\theta) > 0 \), with \( \lim_{\theta \to 0} \xi_3 (\theta) < +\infty \) and \( \lim_{\theta \to 0} \kappa (\theta) = +\infty \).

- This class of models nests, as special cases,
  - the basic NK model,
  - Gabaix’s (2016) benchmark model,
  - Angeletos and Lian’s (2016) model,
  - Bilbiie’s (2017) two models,
  - McKay, Nakamura, and Steinsson’s (2017) model,
  - Ravn and Sterk’s (2017) model with risk-neutral equity investors.

- Most of these models have been shown to be able to solve or attenuate the forward-guidance puzzle.
We highlight four properties of discounting models (thus generalizing Cochrane, 2016), and we show how our model is different:

1. these models do not solve the **paradox of flexibility**;

2. they require a **sufficiently large departure** from the basic NK model to solve the forward-guidance puzzle;

3. they cannot solve the forward-guidance puzzle without generating a **negative** long-term relationship between $\pi_t$ and $i^b_t$;

by contrast, our model generates the **Fisher effect**, i.e. a **one-to-one** long-term relationship between $\pi_t$ and $i^b_t$;
these models cannot solve the forward-guidance puzzle without having non-standard and far-reaching implications for equilibrium determinacy in “normal times;”

by contrast, in our model, under a corridor system, the spread equation becomes
\[ \hat{m}_t = \left( \frac{\delta_y}{\delta_m} \right) \hat{y}_t - \left( \frac{\delta_g}{\delta_m} \right) \tilde{g}_t, \]
and the Phillips curve can be rewritten as
\[ \pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \left( \frac{\delta_m \kappa_y - \delta_y \kappa_m}{\delta_m} \right) \hat{y}_t - \left( \frac{\delta_m \kappa_g - \delta_g \kappa_m}{\delta_m} \right) \tilde{g}_t; \]

therefore, the reduced form of our model is then isomorphic to the basic NK model’s reduced form for any given rule for \( i^b_t \);

as a consequence, our model then inherits all the standard implications of the basic NK model for equilibrium determinacy in normal times.
In our benchmark model, the stock of nominal reserves is **exogenous**.

We endogenize it by considering the rule \( M_t = P_t R(P_t, y_t) \), with \( R_P < 0 \) and \( R_y \leq 0 \).

The steady state is still unique, and we derive a simple sufficient **condition for determinacy** under a permanent IOR-rate peg.

**This condition is met**

- arguably for all relevant calibrations and all values of \( \theta \) (paradox of flexibility),
- necessarily for \((I^m, \gamma)\) sufficiently close to \((\beta^{-1}, 0)\) (basic-NK-model limit).

The shadow rule for \( i^b_t \) is still **Wicksellian**:

\[
i^b_t = i^m_t + \delta y \hat{y}_t - \delta m \hat{m}_t - \delta g \bar{g}_t = i^m_t + \delta y \hat{y}_t - \delta m \left( -\nu P \hat{P}_t - \nu y \hat{y}_t \right) - \delta g \bar{g}_t.
\]

**spread equation**   **nominal-reserves rule**
Robustness check #2: Household cash

- In our benchmark model, the central bank controls bank reserves; but in reality, it controls the monetary base (bank reserves and cash).

- We introduce household cash, through a cash-in-advance constraint, into our benchmark model.

- Again, the steady state is still unique, and we derive a simple sufficient condition for determinacy under a permanent IOR-rate peg.

- Again, this condition is met
  - arguably for all relevant calibrations and all values of $\theta$ (paradox of flexibility),
  - necessarily for $(I^m, \gamma)$ sufficiently close to $(\beta^{-1}, 0)$ (basic-NK-model limit).

- Again, the shadow rule for $i_t^b$ is still Wicksellian:

$$i_t^b = i_t^m + \delta_y \hat{y}_t - \delta_m \hat{m}_t - \delta_g \hat{g}_t = i_t^m + \delta_y \hat{y}_t - \delta_m \left[ \frac{1}{1 - \alpha_c} \left( \frac{\hat{M}_t}{P_t} \right) - \frac{\alpha_c}{1 - \alpha_c} \hat{c}_t \right] - \delta_g \hat{g}_t.$$
Our model solves, even for an arbitrarily small departure from the basic NK model, the forward-guidance puzzle, the fiscal-multiplier puzzle, the paradox of flexibility.

It still solves or attenuates them for a vanishingly small departure, and also solves the paradox of toil in that case, thus providing an equilibrium-selection device in the basic NK model, closing the gap between the basic sticky-price and sticky-information models.

It preserves two standard implications of the basic NK model in normal times: the Fisher effect, the Taylor principle (under a corridor system).

Our resolution is essentially robust to the endogenization of nominal reserves, the introduction of household cash.