Financing Efficiency of Securities-Based Crowdfunding*

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Abstract

We analyze early-venture fundraising from dispersed, privately-informed investors. A principal optimally sets the offering price and quantity, and investors communicate their private information by either contributing capital or by abstaining. The principal uses the information conveyed by fundraising amounts to decide whether to invest raised capital in a risky venture. His decision threshold resembles the ubiquitous "all-or-nothing" rules used in Internet-based crowdfunding. The decision threshold hedges investors against bad projects, creating a "loser's blessing" that encourages contributing despite negative private information. The loser's blessing reduces financing efficiency and financing efficiency worsens as the crowd becomes larger.

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1 Introduction

> And for start-ups and small businesses, this bill [JOBS Act] is a potential game changer. Right now, you can only turn to a limited group of investors – including banks and wealthy individuals – to get funding. Laws that are nearly eight decades old make it impossible for others to invest. But a lot has changed in 80 years, and it's time our laws did as well. Because of this bill, start-ups and small business will now have access to a big, new pool of potential investors – namely, the American people. For the first time, ordinary Americans will be able to go online and invest in entrepreneurs that they believe in.
>
> —President Barack Obama, April 5, 2012

The JOBS Act was established to remove financing frictions, enabling start-ups and small businesses to grow. A cornerstone of the legislation is Regulation Crowdfunding, passed on May 16, 2016, which allows US companies to raise capital via the sale of securities to the general public on the Internet.[1] While securities-based crowdfunding is relatively young, its appeal is founded on the success of earlier forms of crowdfunding. In particular, rewards-based crowdfunding (e.g., KickStarter and Indigogo) has harnessed the "wisdom of the crowd" by allowing consumer-investors to communicate their preferences to entrepreneurs. Through pre-sales, only the popular, and likely profitable, products receive sufficient financing and are produced. It is unclear whether or not this efficiency will translate to securities-based campaigns which rely on common value goods (claims to cash flows) rather than private value goods (Boudreau, Jeppesen, Reichstein and Rullani 2018). In other words, can securities-based crowdfunding harness the wisdom of the crowd?

To answer this question, we model an optimal mechanism design problem characterized by key features of crowdfunding. First, dispersed, privately-informed investors are required to finance a new venture. Second, investors communicate by either contributing to the venture or by abstaining.

[1] See Stemler (2013) for details regarding the JOBS Act, specifically Title III (the CROWDFUND Act), and see Massolution (2015) for an overview of worldwide crowdfunding activity.

When investors abstain, they are excluded from any future payoffs. Third, a principal uses the information conveyed by aggregate contributions to either pursue the risky venture or to cancel it and return contributed funds.

Our main finding is that optimally-designed securities-based crowdfunding cannot aggregate the wisdom of the crowd. Including multiple investors creates a rich information environment about a venture's unobservable quality. However, a rich information environment also leads to incentive misalignment between investors and the principal, which the optimal mechanism cannot overcome. First-best information aggregation is often impossible.

In the model, a financially-constrained principal controls a risky project and must raise capital from $N \geq 2$ investors (the agents). In the first period, investors receive private signals (good or bad) and decide whether or not to contribute capital. In the second period, the principal chooses whether to deploy capital to the risky project. The principal's objective is to a design an optimal crowdfunding mechanism to maximize the project's value. To do so, the principal sets the price of the offering and selects the quantity each investor contributes, effectively controlling how many investors must contribute to fund the project.

A key feature of the model is that the principal decides whether to invest in the risky project after receiving investors' contributions. By making the decision after investors have acted, the principal only invests in the risky project when the aggregate information is sufficiently good, and otherwise returns the contributions to investors. The principal's informed decision-making effectively creates an all-or-nothing threshold, which is common in crowdfunding practice. The principal's decision threshold exposes investors more to good projects than bad projects, creating a hedge which we term the loser's blessing. As a result, investors who receive bad signals are willing to contribute in spite of their information.

When investors contribute in spite of their information, aggregate contributions are less informative and financing efficiency suffers. To see this, consider a continuum of investors with $1 million of total capital.[2] Suppose that investors' signals are accurate with 75% probability. If investors only contribute after receiving a good signal, good projects receive \$750,000 and bad projects receive \$250,000. Financing would be perfectly efficient, because the principal would only invest in good risky projects which raise \$750,000. However, this cannot be an equilibrium. An investor could earn a risk-less profit by contributing after observing a bad signal. If her bad signal is incorrect, she obtains ownership in a good project. Conversely, if her signal is correct, her capital is returned because the project is canceled. Due to the discontinuity in payoffs attributed to the loser's blessing, all investors face the same incentive to deviate and a loser's blessing cannot exist in equilibrium. With a continuum of investors, the only equilibria are those in which both good and bad projects are either always financed or never financed, and financing outcomes reflect no information.

To cleanly demonstrate the loser's blessing, our base model allows the principal to select only the quantity each investor can contribute. When facing a finite number of investors, the loser's blessing prevents first-best financing efficiency under many non-trivial parameter sets. Even with a small number of investors, the loser's blessing can prevent first-best financing efficiency. As N grows, investors are less likely to be pivotal and the loser's blessing intensifies. As a result, the percentage of parameter sets that support first-best financing efficiency approaches zero. Furthermore, even when first-best equilibria exist, the loser's blessing can give rise to additional, less efficient equilibria.

We extend our base model, allowing the principal to set the price of the offering and introducing investors' information costs. When investors receive signals free of charge, the principal can set the price of the offering such that first-best is always possible. To ensure truth-telling, the principal must lower investors' returns, limiting payoffs from falsely reporting good signals by contributing. In contrast, settings such as IPOs use higher returns to secure truth-telling. When information is costly and N is large, price setting cannot give first-best financing efficiency for ex ante negative valued projects. As N grows, lower returns are required to prevent investors from contributing

[2] While our base model considers a discrete number of investors, we formally solve a model with a continuum of investors in Appendix B.

after observing bad signals. However, returns must remain high enough to compensate investors' information costs when they observe good signals. Both return requirements cannot be satisfied for large N, and thus first-best financing efficiency is impossible for ex ante negative valued projects. Ex ante negative valued projects are of particular interest as crowdfunding candidates because they are less likely to be financed conventionally.

Three key assumptions prevent efficient information aggregation in our model, and distinguish our analysis from several related literatures. First, to capture the realities of Internet-based fundraising, we assume that investors are dispersed and communicate private information by contributing to the project or by abstaining. Contributing conveys good information while abstaining conveys bad information. If an investor abstains, she does not participate in the project's payoff. As such, the net payoff for reporting negative information is fixed and equal to zero. In contrast, IPO models commonly assume richer communication spaces (via allocation discretion or complex price schedules), decoupling investors' signals from their abilities to invest (Benveniste and Spindt 1989, Benveniste and Wilhelm 1990, Biais, Bossaerts and Rochet 2002, Sherman and Titman 2002). While it is reasonable for a limited number of institutional investors to have in-depth communication with underwriters, entrepreneurs cannot feasibly communicate with every potential investor online. Thus, a limited communication space, representative of Internet-based investing, hampers first-best financing efficiency.

Second, we assume that the principal is uninformed ex ante, and the total capital raised is used to optimally undertake the project. In work most closely related to our own, Hakenes and Schlegel (2014) models endogenous information production in the presence of all-or-nothing financing thresholds, showing that too much information production may result. The paper's main analysis assumes that the entrepreneur knows the project's type ex ante, thereby disconnecting the entrepreneur's decision from the financing outcome. The entrepreneur can set a lower all-or-nothing threshold to align incentives for truth-telling, eliminating the loser's blessing. Similarly, in analyzing flexible versus all-or-nothing financing, Chang (2016) disconnects the financing outcome from the entrepreneur's decision. In contrast to these papers, our principal must learn from the crowd and cannot commit to ex post suboptimal decisions.[3] Thus, the loser's blessing's existence relies on both learning and limited commitment.

Third, we assume that investors' are only financially motivated, and do not receive private benefits, for example, from consuming the product. Ellman and Hurkens (2016) and Strausz (2017) show that an optimal crowdfunding mechanism can successfully aggregate the wisdom of the crowd, but these papers assume that capital comes from investor-consumers who have private values for the products they purchase in pre-sales.[4] In contrast, the loser's blessing depends on investors' sharing a common value for the payoff.[5] Thus, our analysis shows that the private-value versus common-value distinction has first-order consequences on financing outcomes.

Our paper is also related to two contemporary papers examining securities-based crowdfunding. Cong and Xiao (2018) studies the effect of all-or-nothing thresholds on the informational efficiency of crowdfunding. In the paper's model, investors are assumed to arrive sequentially, learning from the actions of all prior investors, and the all-or-nothing threshold improves financing efficiency. Our model assumes non-coordinating, strategic investors who move simultaneously, preventing learning. Li (2018) considers the optimal financial contract in securities-based crowdfunding, showing that typical pro rata allocations are suboptimal in aggregating investors' private information. Our model assumes all investors who contribute at the same time are treated equally.

The loser's blessing is related to the idea that price feedback can hamper the incorporation of information into prices.[6] Goldstein and Guembel (2008), Bond, Goldstein and Prescott (2010),

[3]Hakenes and Schlegel (2014) conjecture that without knowledge of the project's type, the entrepreneur would have to accept a degree of false reporting, i.e. a loser's blessing.

[4]Schwienbacher (2015) and Chemla and Tinn (2018) do not analyze optimal mechanisms, but do show situations in which reward-based crowdfunding successfully aggregates the wisdom of the crowd.

[5]Grüner and Siemroth (Forthcoming) analyze a general equilibrium model of securities-based crowdfunding in which investors receive common-valued financial returns from their crowdfunding contributions. The loser's blessing does not arise because there is no principal-agent information asymmetry, and the investors receive private benefits from the products that are produced.

[6]While not directly relying on prices, the analysis of informational cascades in Bikhchandani, Hirshleifer and Welch (1992) features individuals who may ignore their own information. In their setting, this is due to learning in a sequential-action game, while in our simultaneous-action setting, discrete payoff jumps lead investors' actions to diverge from their beliefs regarding project quality.

Goldstein, Ozdenoren and Yuan (2013) and Edmans, Goldstein and Wei (2015) provide models in which price feedback either alters investors' trading strategies or obfuscates the signal in prices. In each of these models, active trading markets allow investors to somewhat coordinate and at least partially communicate their information to real decision-makers. The mechanism for our results differs due to two key features of crowdfunding. First, there is no market in which investors can trade and incorporate their information into prices. In crowdfunding, prices are fixed and fundraising quantities convey information. Second, primary market financing creates a discrete payoff structure; investors can only profit when they invest.[7] Thus, the feedback and associated inefficiencies we identify are due to discrete investor payoffs generated by the principal's optimal threshold rule.

Our analysis challenges the notion that securities-based crowdfunding will harness the wisdom of the crowd. When investors are dispersed and strictly profit motivated, the loser's blessing reduces the information content of raised capital. Entrepreneurs undertake bad projects, forgo profitable ones, and aggregate welfare suffers. Our analysis is positive in the sense that we show all-or-nothing thresholds are optimal, which is consistent with standing regulations. Our analysis is also normative in showing that realizing securities-based crowdfunding's potential depends on mitigating the loser's blessing. In particular, platforms may be designed to attract consumer-investors who reap private benefits by contributing, for example, by catering to local businesses or novel products. Alternatively, the loser's blessing may be mitigated by setting high prices for crowdfunding offerings to lower expected returns and dissuade investors from free-riding.

[7] The asymmetric payoff structure separates our analysis from the voting literature, in which collective actions take place when there are enough votes in favor of a proposal. While investors can be pivotal in both settings, only those who contribute to a crowdfunding campaign are committed to the action (investment). For analyses of information aggregation in voting settings, see e.g., Feddersen and Pesendorfer (1997), Feddersen and Pesendorfer (1998) and Bond and Eraslan (2010).

2 Base Model

We model a game in which a risk-neutral, financially-constrained principal controls a risky project and must raise capital from $N \geq 2$ ex ante homogeneous, privately-informed, risk-neutral investors (the agents). The game takes place over two periods: at $t = 1$ investors receive private signals and decide whether or not to contribute capital, and at $t = 2$ the principal chooses how to deploy contributed capital. The principal's objective is to design an optimal mechanism that maximizes the project's expected value. In the base model, all surplus accrues to investors and it is equivalent to interpret the principal's objective as maximizing investor surplus.[8]

The principal decides whether to deploy raised capital, K, to either the risky project (r) or to a safe asset (s). We denote the principal's choice of how to deploy the capital as $d \in \{r, s\}$. The risky project's quality, F, is either good (G) or bad (B) with equal probability. The project has a minimum scale of $\underline{K} \geq 0$ and a maximum scale of $\overline{K} \geq \underline{K}$. Good projects generate a gross rate of return $\Delta > 1$ (net return $\delta > 0$). Bad projects generate a gross rate of return 0. The risky project's net payoff is,

$$V(K, F|r) = \begin{cases} \Delta \mathbb{1}_G K - K & K \leq \overline{K} \\ \Delta \mathbb{1}_G \overline{K} - \overline{K} & K > \overline{K}, \end{cases} \tag{1}$$

in which $\mathbb{1}_G$ equals 1 for good projects and 0 for bad projects. We normalize the gross return of the safe asset to 1. The safe asset's net payoff is,

$$V(K, F|s) = 0 \ \forall K. \tag{2}$$

Note that the principal must deploy capital to the safe asset if $K < \underline{K}$.

The principal maximizes project value by choosing how much capital each investor can contribute. We assume that investors are financially unconstrained, so the principal may choose any

[8] In Section 3, we allow the principal to set prices and the project's expected value is no longer equivalent to investor surplus.

positive value for investors' contribution amount $\kappa \geq 0$. By choosing κ, the principal effectively chooses how many investors must contribute to meet the minimum cost, $\underline{K}$, and how many must contribute to fully use the project's capacity, $\overline{K}$. As examples, $\kappa = \overline{K}$ allows one investor to fully finance the project, while $\kappa = \underline{K}/N$ requires every investor to contribute to meet the minimum project scale.

To represent a key feature of Internet-based fundraising, we restrict security payments in one way. Investors must contribute capital to receive a security payment. While this restriction is natural in our venture financing setting, it distinguishes our analysis from literatures on voting and secondary markets. In the voting literature, all agents are affected by the voting outcome, whereas in our model, investors who abstain from contributing receive zero net payoff. In secondary markets, agents can often short an asset, incorporating negative information into its price. In our model, investors can only communicate information through the contribution decision, and the only negative signal is to abstain.

We can interpret the principal as one of two natural parties in the crowdfunding industry. First, we can interpret the principal as a fundraising platform that serves as an intermediary between an entrepreneur and investors. The fundraising platform raises investor capital and chooses whether to pass that capital onto the entrepreneur (risky deployment) or to return it to investors (safe deployment). Second, we can interpret the principal as a benevolent, financially-constrained entrepreneur. The entrepreneur solicits investor capital and chooses whether to deploy it to his start-up project (risky deployment) or to cancel the project and return capital to investors (safe deployment).

Each investor i receives a costless, conditionally i.i.d. signal $\hat{F} \in \{\hat{G}, \hat{B}\}$ such that,

$$\Pr(F = G|\hat{G}) = \Pr(F = B|\hat{B}) = \alpha > \frac{1}{2}. \tag{3}$$

Signals are private information. We denote the contribution strategy of investor i as, $\vec{\pi}_i = \{g_i, b_i\}$, in which $g_i \in [0,1]$ and $b_i \in [0,1]$ are investor i's probabilities of contributing based on signals of $\hat{G}$

and $\hat{B}$. We denote all other investors' strategies, not including investor i, as $\vec{\pi}_{-i}$.

The principal and investors are rational, and both prior beliefs and the existence of private signals are common knowledge. Consequently, investors' equilibrium strategies are common knowledge. We restrict our analysis to equilibria in which the investors play symmetric strategies. Furthermore, the principal updates his beliefs according to Bayes' Rule, and we denote the principal's posterior beliefs, conditional on raising K dollars from investors, as,

$$\rho(n, \vec{\pi}) = \Pr(F = G | K, \kappa, \vec{\pi}), \tag{4}$$

in which,

$$n = \frac{K}{\kappa}, \tag{5}$$

denoting the number of contributing investors. Given the principal's posterior beliefs, if $K \geq \underline{K}$, the principal deploys capital to the risky project if and only if,

$$\rho(n, \vec{\pi})\Delta - 1 \geq 0. \tag{6}$$

The explicit form of the principal's posterior beliefs is given by,

$$\rho(n, \vec{\pi}) = \frac{\Pr(n \cap G)}{\Pr(n \cap G) + \Pr(n \cap B)}, \tag{7}$$

in which,

$$\begin{aligned} \Pr(n \cap G) \quad = \quad & \frac{1}{2} \sum_{y=0}^{N} \sum_{x=\max(0, n-(N-y))}^{\min(y,n)} \left(\alpha^y \prod_{j \in X} g_j \prod_{j \in Y, j \notin X} (1 - g_j) \right) \times \\ & \left((1-\alpha)^{N-y} \prod_{j \in Z, j \notin Y} b_j \prod_{j \notin Z, j \notin Y} (1 - b_j) \right) \binom{N}{y} \binom{y}{x} \binom{N-y}{n-x}, \end{aligned} \tag{8}$$

and,

$$\Pr(n \cap B) = \frac{1}{2}\sum_{y=0}^{N}\sum_{x=\max(0,n-(N-y))}^{\min(y,n-1)}\left((1-\alpha)^y \prod_{j\in X,} g_j \prod_{j\in Y, j\notin X}(1-g_j)\right)\times$$
$$\left(\alpha^{N-y}\prod_{j\in Z, j\notin Y} b_j \prod_{j\notin Z, j\notin Y}(1-b_j)\right)\binom{N}{y}\binom{y}{x}\binom{N-y}{n-x}. \tag{9}$$

In (8) and (9), Z is the set of n investors who contribute, Y is the set of investors who observe $\hat{G}$ and X is the intersection of Z and Y. In the summations, y indexes the size of Y and x indexes the size of X. The three counting functions provide the number of unique ways one can select y investors (out of N) who observe $\hat{G}$, x investors (out of y) who observe $\hat{G}$ and contribute, and $n-x$ investors (out of the $N-y$ investors who observe $\hat{B}$) who observe $\hat{B}$ and contribute. (8) and (9) only differ in the probabilities applied to investors' who observe $\hat{G}$ or $\hat{B}$.

Using (6), the principal will deploy capital to the risky project if, and only if,

$$\Delta \geq \mathcal{D}(n) \equiv \frac{1}{\rho(n, \vec{\pi})}. \tag{10}$$

It is straightforward to see that $\mathcal{D}(n)$ is weakly decreasing in n (so long as $g \geq b$, which occurs in equilibrium). If the principal observes a greater number of contributions, he believes that the project is good with a higher posterior probability and he is more likely to choose risky deployment. The return threshold $\mathcal{D}(n)$ can be mapped to a threshold number of contributing investors. Let $\mathcal{N} = \{n | \mathcal{D}(n) \geq \Delta\}$ and define $\underline{n}$ as,

$$\underline{n} = \min(\mathcal{N}). \tag{11}$$

The principal chooses risky deployment if, and only if, $n \geq \underline{n}$.

Investors internalize that the principal chooses how to deploy capital *after* observing their contribution decisions. Investors rationally anticipate the principal's ex post decision rule and internalize it as an ex ante hedge. All else equal, good projects attract more contributions and are likely to lead the principal to choose risky deployment. Bad projects, however, attract fewer contributions and are likely to lead the principal to choose safe deployment, shielding the smaller number of contributing investors from losses. The anticipated hedge, which we term a loser's blessing, distorts investors' incentives and their equilibrium contribution strategies.

Definition 1. *A **loser's blessing** is an asymmetric exposure to good projects.*

A loser's blessing is the converse to a winner's curse, which is an asymmetric exposure to bad projects. To see the relation between the loser's blessing and the winner's curse, consider a setting in which all projects are funded and capital is always deployed in the risky project. In this setting, when a project is good, it is likely that many investors will observe correct signals of $\hat{G}$, and, all else equal, many investors will contribute. When the good project's payoff is realized, it is shared among a large group of investors. If, however, the project is bad, it likely that only a few investors will observe incorrect signals of $\hat{G}$, and, all else equal, the project losses will be shared by a small group of misinformed investors. Thus, investors are less exposed to good projects and more exposed to bad projects, giving rise to the well-known winner's curse that is at the core of other financing models like Rock (1986). In our setting, the winner's curse is second-order because (i) capital is not sunk at the time of the investor's contribution decision, and (ii) the principal deploys capital based on his posterior beliefs to maximize the project's expected value.

Returning to the model, the principal's problem is to optimally choose the investors' fixed contribution amount κ to maximize the project's expected value. The principal's choice of κ has two effects. First, κ influences the total capital raised. Raising less than $\underline{K}$ makes risky deployment impossible and raising less than $\overline{K}$ leads to underinvestment (assuming risky deployment). Second, κ influences investors' strategies. For example, if κ is set equal to $\overline{K}$, then one investor can finance the project to full capacity and investor payoffs are rationed when more than one investor contributes. Investors rationally incorporate the effect of κ and the effect of the principal's ex post decision-making rule into their expected payoffs and their equilibrium contribution strategies.

Formally, the principal's problem is given by,

$$\max_{\kappa\in\mathbb{R}^+} E\left[V(K,F|d)\right] \tag{12}$$

$$\text{s.t.}\quad g_i \in \arg\max_{\hat{g}\in[0,1]} \Pi(\hat{G},\{\hat{g},\hat{b}\}|\vec{\pi}_{-i},\kappa,\underline{n}), \tag{12.1}$$

$$b_i \in \arg\max_{\hat{b}\in[0,1]} \Pi(\hat{B},\{\hat{g},\hat{b}\}|\vec{\pi}_{-i},\kappa,\underline{n}), \tag{12.2}$$

$$g_i = g_{-i} \text{ and } b_i = b_{-i}, \tag{12.3}$$

$$\Pi(\hat{G},\vec{\pi}_i|\vec{\pi}_{-i},\kappa,\underline{n}) \geq 0 \text{ and } \Pi(\hat{B},\vec{\pi}_i|\vec{\pi}_{-i},\kappa,\underline{n}) \geq 0, \tag{12.4}$$

$$d = \begin{cases} r & \text{if } \rho(n,\vec{\pi})\Delta - 1 \geq 0 \text{ and } K \geq \underline{K} \\ s & \text{otherwise,} \end{cases} \tag{12.5}$$

in which $\Pi(\hat{F},\vec{\pi}_i|\vec{\pi}_{-i},\kappa,\underline{n})$ is investor i's expected payoff as a function of her signal $\hat{F}$ and strategy $\vec{\pi}_i$, given all other investors use strategy $\vec{\pi}_{-i}$, the principal chooses contribution amount κ, and risky deployment occurs if $n \geq \underline{n}$.[9] Constraints 12.1-12.2 require that g_i and b_i are chosen as mutual best responses by the investors, Constraint 12.3 requires that the strategies are symmetric, and Constraint 12.4 ensures participation (which is trivially satisfied if the mechanism is incentive compatible). Furthermore, Constraint 12.5 requires that the principal chooses risky deployment of capital if the project is positive valued in expectation and raised capital exceeds $\underline{K}$. It is equivalent to interpret Constraint 12.5 as limited commitment by the principal. If, conditional on his posterior beliefs, the principal believes the project is positive valued, he will invest regardless of any ex ante promises.[10]

[9]Explicit forms of investor i's expected payoff $\Pi(\hat{F},\vec{\pi}_i|\vec{\pi}_{-i},\kappa,\underline{n})$ are located in (A5) and (A6) in Appendix A.

[10]Hakenes and Schlegel (2014) and Chang (2016) effectively assume the decision maker can commit to a threshold rule ex ante. By committing to invest in projects that are ex post negative valued, investors are less hedged against bad outcomes, which can attenuate the loser's blessing and improve financing efficiency. Within the feedback literature, Goldstein, Ozdenoren and Yuan (2011) and Bond and Goldstein (2015) both show that informational inefficiencies can be mitigated when decision makers are willing and able to commit to actions that may run against their best interest.

We define an equilibrium according to a quartet,

$$\Omega^* = \{\kappa^*, g^*, b^*, \underline{n}^*\}, \tag{13}$$

in which κ^* solves the principal's problem in (12), g^* and b^* are the equilibrium contribution strategies of investors who observe $\hat{G}$ and $\hat{B}$ respectively, and $\underline{n}^*$ is the minimum number of contributions the principal must observe to choose risky deployment (conditional on strategies g^* and b^*).

A first-best (denoted with superscript FB) solution Ω^{FB} to (12) requires that: (i) conditional on choosing risky deployment, underinvestment does not occur; and (ii) investors truthfully report their signals through their contribution decisions. Formally stated:

Definition 2. *First-best financing efficiency is characterized by,*

(i) $K \geq \overline{K}$ *if* $\Delta \geq \frac{1}{\rho(n,\vec{\pi})}$ *(equivalently,* $n \geq \underline{n}$*),*

(ii) and truthful reporting by each investor, $g = 1$ *and* $b = 0$*.*

2.1 First-Best Mechanism

Denote a first-best equilibrium quartet as $\Omega^{FB} = \{\kappa^{FB}, g^{FB}, b^{FB}, \underline{n}^{FB}\}$. In a first-best equilibrium, the principal's posterior beliefs outlined in (7) simplify to,

$$\rho(n, \Omega^{FB}) = \frac{\alpha^n(1-\alpha)^{N-n}}{\alpha^n(1-\alpha)^{N-n} + (1-\alpha)^n\alpha^{N-n}}, \tag{14}$$

because first-best requires $g^{FB} = 1$ and $b^{FB} = 0$ (we make ρ a function of all equilibrium parameters Ω for simplicity). Thus, the function $\mathcal{D}(n)$ outlined in (10) is given by,

$$\mathcal{D}(n) \equiv \frac{\alpha^n(1-\alpha)^{N-n} + (1-\alpha)^n\alpha^{N-n}}{\alpha^n(1-\alpha)^{N-n}}, \tag{15}$$

and the principal will choose risky deployment of capital if, and only if, $\Delta \geq \mathcal{D}(n)$.

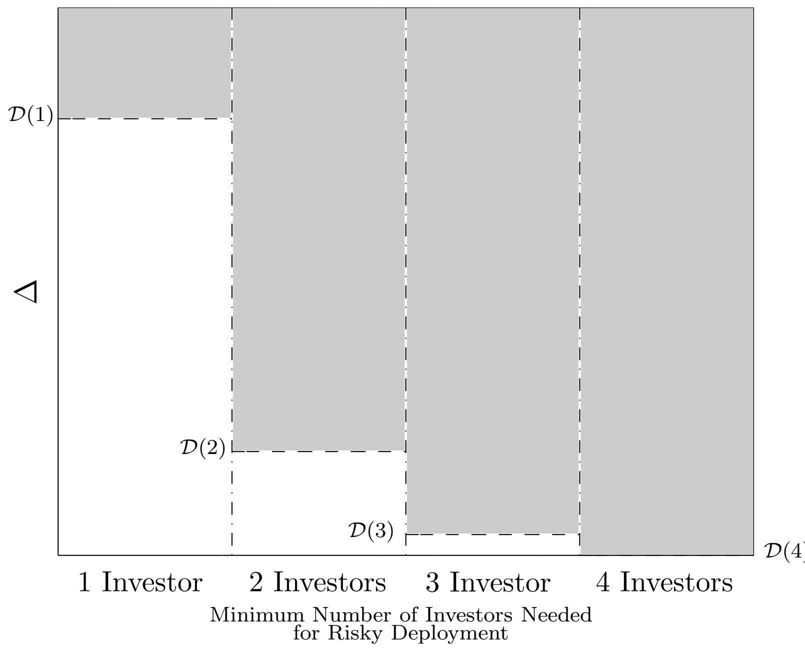


Figure 1: Risky deployment regions with $N = 4$ investors and $\alpha = \frac{2}{3}$.

To guarantee there is not underinvestment if $\Delta \geq \mathcal{D}(n)$, first-best requires that a contribution quantity κ^{FB} satisfies,

$$\kappa^{FB} \geq \frac{\overline{K}}{\underline{n}^{FB}}. \tag{16}$$

If κ is set to a value smaller than $\overline{K}/\underline{n}^{FB}$, the principal risks raising a capital quantity $K < \overline{K}$ despite observing $n \geq \underline{n}^{FB}$ good signals (via investors' contributions). Thus, $\kappa < \overline{K}/\underline{n}^{FB}$ risks not economizing on the project's total capacity and there may be a welfare loss due to underinvestment.

Figure 1 depicts the the principal's ex post decision rule for $N = 4$ investors. The horizontal axis depicts the four possible scenarios for non-zero investor participation: one investor contributes, two investors contribute, and so on. The vertical axis depicts the threshold values of Δ for which the principal will choose risky deployment if at least n investors contribute. For example, $\mathcal{D}(2)$, reflects the project return for which the entrepreneur will choose risky deployment if two or more investors contribute. Figure 1 shows that for lower return projects, the principal requires that more investors contribute to warrant risky deployment.

Given an arbitrary $\underline{n}$, investor contribution strategies $g = 1$ and $b = 0$ must be payoff maximizing (i.e., incentive compatible) to attain truth-telling. Formally, the following constraints must be satisfied for $g = 1$ and $b = 0$ to be payoff maximizing,

$$\sum_{n=\underline{n}-1}^{N-1} \left(\alpha \Pr(n|G, \vec{\pi}, N-1) \frac{(\Delta - 1)\underline{n}}{n+1} - (1-\alpha) \Pr(n|B, \vec{\pi}, N-1) \frac{\underline{n}}{n+1} \right) \geq 0, \tag{17}$$

$$\sum_{n=\underline{n}-1}^{N-1} \left((1-\alpha) \Pr(n|G, \vec{\pi}, N-1) \frac{(\Delta - 1)\underline{n}}{n+1} - \alpha \Pr(n|B, \vec{\pi}, N-1) \frac{\underline{n}}{n+1} \right) < 0, \tag{18}$$

in which, given $g = 1$ and $b = 0$, $\Pr(n|G, \vec{\pi}, N-1)$ and $\Pr(n|B, \vec{\pi}, N-1)$ from (8) and (9) simplify to,

$$\Pr(n|G, \vec{\pi}, N-1) = \binom{N-1}{n} \alpha^n (1-\alpha)^{N-1-n}, \tag{19}$$

$$\Pr(n|B, \vec{\pi}, N-1) = \binom{N-1}{n} (1-\alpha)^n \alpha^{N-1-n}. \tag{20}$$

(17) requires that an investor's payoff (in terms of expected return), conditional on observing $\hat{G}$ and choosing to contribute, is weakly positive. The summation begins at $n = \underline{n} - 1$, reflecting that the investor internalizes her contribution decision (i.e., by contributing, she increases the total number of contributing investors by one) and also reflecting that the principal chooses risky deployment only if at least $\underline{n}$ investors contribute. (18) requires an investor's payoff (in terms of expected return), conditional on observing $\hat{B}$ and choosing to contribute, to be strictly negative. Both inequalities must be satisfied for investors to truthfully report their signals via contributions.

(17) can be rearranged to give the lower-bound on Δ that makes it incentive compatible for an investor who observes $\hat{G}$ to truthfully report her signal via contributing,

$$\Delta_{\underline{n},\hat{G}} \equiv \frac{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1) \frac{1}{n+1} + (1-\alpha) \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1) \frac{1}{n+1}}{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1) \frac{1}{n+1}}. \tag{21}$$

Thus, an investor will contribute $\kappa \geq \frac{\overline{K}}{\underline{n}^{FB}}$ after receiving signal $\hat{G}$ if $\Delta \geq \Delta_{\underline{n},\hat{G}}$. Likewise, (18) can be rearranged to give the upper-bound on Δ that makes it incentive compatible for an investor who observes $\hat{B}$ to truthfully report their signal by not contributing,

$$\Delta_{\underline{n},\hat{B}} \equiv \frac{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1} + \alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}. \quad (22)$$

Thus, an investor will abstain from contributing after receiving signal $\hat{B}$ if $\Delta < \Delta_{\underline{n},\hat{B}}$. The following lemmas establish several relations among investors' incentive compatibility thresholds.

Lemma 1. $\Delta_{\underline{n},\hat{G}}$ *is strictly smaller than* $\Delta_{\underline{n},\hat{B}}$.

According to Lemma 1, for any given $\underline{n}$, there is a non-empty set of Δ that satisfy incentive compatibility with respect to truthfully revealing private signals via contribution decisions. The set of Δ is defined by the interval $\left[\Delta_{\underline{n},\hat{G}}, \Delta_{\underline{n},\hat{B}}\right)$.

Lemma 2. $\Delta_{\underline{n},\hat{G}}$ *and* $\Delta_{\underline{n},\hat{B}}$ *are both decreasing in* $\underline{n}$.

Lemma 2 shows that the upper and lower bounds, $\Delta_{\underline{n},\hat{G}}$ and $\Delta_{\underline{n},\hat{B}}$, are decreasing in $\underline{n}$. Using the result of the lemma, an upper bound on Δ may be established for which no incentive compatible first-best mechanism exists. Define

$$\overline{\Delta} \equiv \Delta_{1,\hat{B}} = \frac{(1-\alpha^N)\alpha^2 + (1-(1-\alpha)^N)(1-\alpha)^2}{(1-(1-\alpha)^N)(1-\alpha)^2}. \quad (23)$$

For any $\Delta \geq \overline{\Delta}$, it is not incentive compatible for an investor who observes $\hat{B}$ to choose $b = 0$. Instead, because the return is sufficiently large, investors who observe $\hat{B}$ will participate with strictly positive probability. Lemma 1 also implies that a lower bound on Δ may be established for which no incentive compatible first-best mechanism exists. Define

$$\underline{\Delta} \equiv \Delta_{N,\hat{G}} = \frac{\alpha^N + (1-\alpha)^N}{\alpha^N}. \quad (24)$$

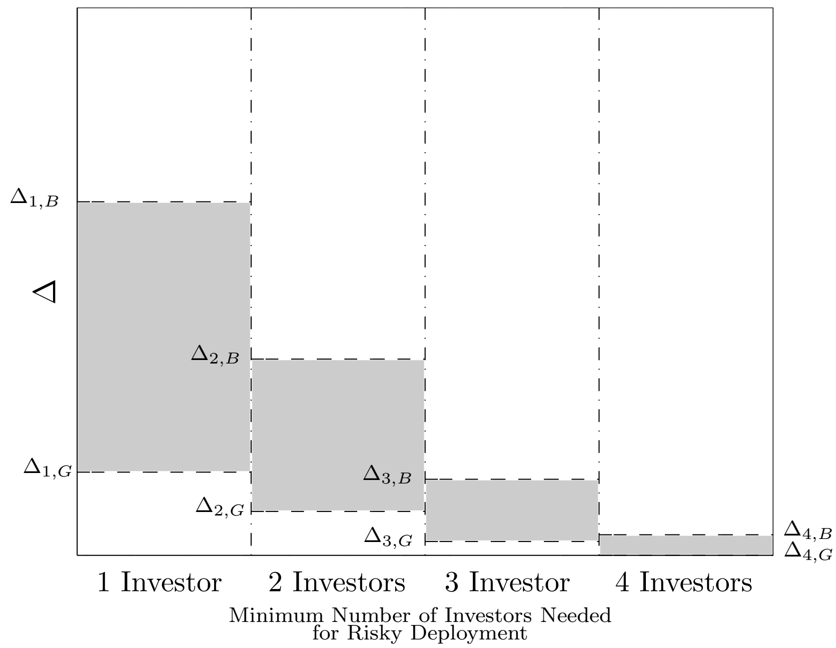


Figure 2: Incentive Compatibility regions with $N = 4$ investors and $\alpha = \frac{2}{3}$.

For any $\Delta < \underline{\Delta}$, it is not incentive compatible for an investor who observes $\hat{G}$ to choose $g = 1$. Without loss of generality, we assume $\Delta \in \left[\underline{\Delta}, \overline{\Delta}\right)$ to exclude highly profitable projects and highly unprofitable projects for which a first-best mechanism cannot exist.

Lemma 3. *For any* $\underline{n} \geq 2$, $\Delta_{\underline{n},\hat{B}} \geq \Delta_{\underline{n}-1,\hat{G}}$.

Lemma 3, together with Lemma 1 and Lemma 2, implies that adjacent incentive compatible regions overlap, i.e. $\left[\Delta_{\underline{n},\hat{G}}, \Delta_{\underline{n},\hat{B}}\right)$ overlaps with $\left[\Delta_{\underline{n}-1,\hat{G}}, \Delta_{\underline{n}-1,\hat{B}}\right)$ for at least one value of Δ.

Lemma 4. *For any value of* $\Delta \in \left[\underline{\Delta}, \overline{\Delta}\right)$*, there exists at least one* $\underline{n}$ *for which* $\Delta \in \left[\Delta_{\underline{n},\hat{G}}, \Delta_{\underline{n},\hat{B}}\right)$.

By Lemma 4, all values of $\Delta \in \left[\underline{\Delta}, \overline{\Delta}\right)$ correspond to at least one value of $\underline{n}$ at which it is incentive compatible for investors to truthfully report their private signals via their contributions.

Figure 2 depicts the truth-telling incentive compatibility regions for $N = 4$ investors. The horizontal axis depicts the principal's four possible ex post decision rules: the principal chooses risky deployment if at least one investor contributes, if at least two investors contribute, and so on. The vertical axis depicts the threshold bounds on the incentive compatible regions of Δ for which investors truthfully report their signals via their contribution strategies. For example, the shaded region between $\Delta_{2,G}$ and $\Delta_{2,B}$ reflects the project returns for which investors will truthfully report their signals if the principal's ex post decision is risky deployment if at least two investors contribute. The figure also illustrates the analytic results of Lemmas 1–4.

Our analysis shows that three features are required for first-best financing efficiency. First, assuming investors truthfully reveal their signals via their contributions, the principal will choose risky deployment if, and only if, $n \geq \underline{n}^{FB}$ investors contribute. This implies that the project's promised return Δ is weakly greater than $\mathcal{D}(\underline{n}^{FB})$. Second, to ensure that the project is always financed to full capacity when risky deployment is chosen, the principal chooses a contribution quantity $\kappa^{FB} \geq \overline{K}/\underline{n}^{FB}$. Third, truth-telling via contribution decisions is incentive compatible if, and only if, $\Delta \in \left[\Delta_{\underline{n}^{FB},\hat{G}}, \Delta_{\underline{n}^{FB},\hat{B}}\right)$. Proposition 1 formally states the necessary and sufficient conditions for first-best financing efficiency.

Proposition 1. *If, and only if,* $\Delta \geq \mathcal{D}(\underline{n}^{FB})$ *and* $\Delta \in \left[\Delta_{\underline{n}^{FB},\hat{G}}, \Delta_{\underline{n}^{FB},\hat{B}}\right)$*, there exists an equilibrium solution* $\Omega^{FB} = \{\kappa^{FB}, g^{FB}, b^{FB}, \underline{n}^{FB}\}$ *to the principal's problem outlined in (12) which corresponds to first-best financing efficiency as defined in Definition 2.*

In a first-best solution, the principal chooses risky deployment of capital if, and only if, sufficiently many investors participate. The first-best solution exhibits an all-or-nothing financing threshold in which risky deployment is only undertaken if the project attracts sufficient capital.

Remark 1. *All-or-nothing financing thresholds are a feature of the first-best solution to the principal's problem outlined in (12).*

In practice, all-or-nothing financing, in which entrepreneurs and/or intermediaries set thresholds below which capital is returned to investors, is prominent in securities-based, reward-based and donation-based crowdfunding.[11] Our analysis suggests that the prominence of such thresh-

[11] In the United States, all-or-nothing financing mechanisms are mandated by Regulation Crowdfunding (the SEC's

olds is consistent with an optimal mechanism when investors' only means to convey their private information is via contributions.

Corollary 1.1. *For any $N \geq 2$, there exists a non-empty set of Δ for which a first-best solution Ω^{FB} is possible and achieves first-best financing efficiency as defined in Definition 2.*

Corollary 1.1 shows that for any number of investors N, there is a range of projects whose promised returns Δ satisfy the conditions of Proposition 1. Importantly, Corollary 1.1 does not say that first-best is possible for *any* Δ. Figure 3 illustrates the overlapping regions from Figure 1 and Figure 2. The shaded areas represent values of Δ for which the principal's ex post decision making rule is congruent with the investors' incentive compatibility constraints, i.e., project returns for which first-best financing efficiency is possible. Figure 3 shows that there are many values of Δ for which first-best is not possible.

2.2 Financing Efficiency

In this section, we first show that for $N > 2$ investors there always exists a range of Δ for which a first-best equilibrium does not exist. Second, we show that the percentage of Δ which have first-best equilibria shrinks as N increases, indicating that financing efficiency worsens as N increases. Third, we show that multiple equilibria can exist, and the principal may find himself in an equilibrium featuring strategic misreporting by investors even if a first-best equilibrium is possible. Furthermore, under some parameter sets, there is a babbling equilibrium in which all investors report good signals and financing efficiency is at its minimum. Before proceeding with our analysis, we introduce a welfare metric to compare the financing efficiency of different equilibria. The welfare metric we use is the project's expected value,

$$E[V(K, F|s)] = \Pr(\text{Risky Deployment}) \left(\Pr(G|\text{Risky Deployment})\Delta - 1\right). \tag{25}$$

implementation of the CROWDFUND Act) for securities-based crowdfunding and have been used in Regulation D filings since 1982. The all-or-nothing financing mechanism is also known as a "provision point mechanism" (Bagnoli and Lipman 1989).

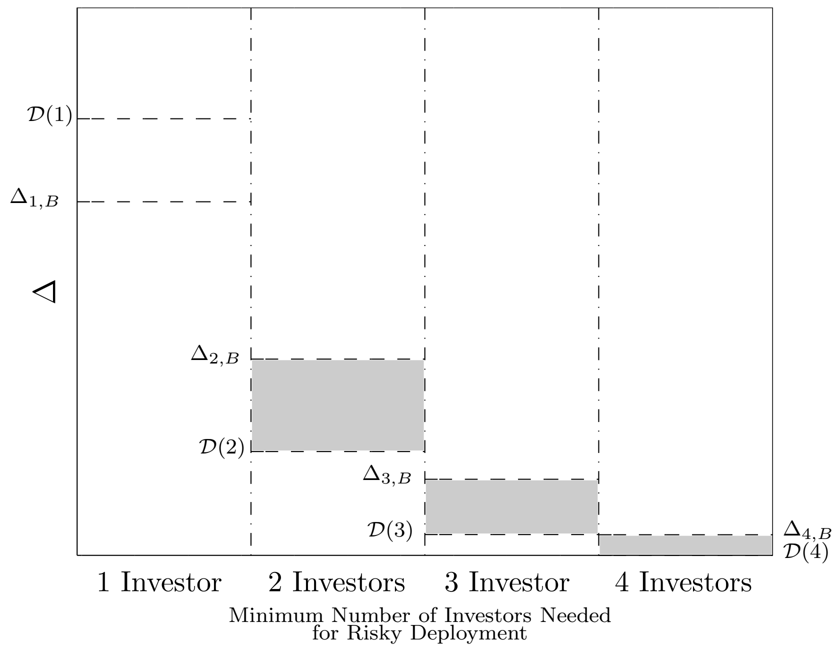


Figure 3: First-best regions with $N = 4$ investors and $\alpha = \frac{2}{3}$.

Using the ex ante expected value considers the implications of both choosing risky deployment and choosing safe deployment. The term within parenthesis in (25) corresponds to the project's expected value, conditional on choosing risky deployment. The term within parenthesis is then scaled by the probability of risky deployment.

2.2.1 Breakdown of First-Best Equilibria

Figure 3 provides a simple example of how first-best equilibria fail to exist for some ranges of Δ. In this example, when $\Delta \in [\Delta_{3,\hat{B}}, \mathcal{D}(2)]$ and $\Delta \in [\Delta_{2,\hat{B}}, \overline{\Delta}]$, first-best equilibria do not exist. In these regions, either the promised return is too low for the principal to choose risky deployment without more contributions, the promised return is too high for investors to abstain from contributing when they observe $\hat{B}$, or both. For example, when $\Delta \in [\mathcal{D}(3), \mathcal{D}(2))$, the principal chooses risky deployment only if at least three investors contribute. By choosing safe deployment with less than three investors, the principal creates a loser's blessing for investors who observe $\hat{B}$. To see this, consider a project with $\Delta \in \left[\Delta_{3,\hat{B}}, \mathcal{D}(2)\right)$ and consider a hypothetical equilibrium in which investors only contribute if they observe $\hat{G}$. Holding all other investors' strategies fixed, if investor i observes $\hat{B}$ and decides to deviate by contributing, and fewer than two other investors contribute, then the principal chooses safe deployment and no capital is at risk. However, if at least two other investors contribute, then the principal chooses risky deployment. With all other investors only contributing after observing $\hat{G}$, it is more likely that two investors contribute for a good project than a bad project and, thus, risky deployment is more likely when the project is good. As such, investor i's deviation is profitable, eliminating the hypothetical equilibrium. In general, asymmetric exposure to good projects leads investors to contribute after observing $\hat{B}$ in the hope of free-riding on other investors' information. Furthermore, higher returns make contributing after observing $\hat{B}$ more profitable.

Proposition 2. *For any $N > 2$, there exists at least one continuous range of Δ in the interval $\left[\underline{\Delta}, \overline{\Delta}\right]$ for which first-best financing efficiency, as defined in Definition 2, is not possible.*

Corollary 2.1. *For any $N > 2$, first-best financing efficiency, as defined in Definition 2, is not possible for some Δ in the interval $\left[\frac{\alpha^2+(1-\alpha)^2}{\alpha^2}, 2\right)$.*

Proposition 2 shows that there is always at least one region in which first-best is not possible. In particular, Corollary 2.1 shows that a region without first-best always exists around the threshold that requires half of the investors to contribute. That threshold corresponds to projects with $\Delta = 2$, which are ex ante zero valued and at the threshold of being economically valuable (in the absence of information). In the context of venture financing, low value projects are likely candidates for securities-based crowdfunding because more traditional types of financing, for example, venture capital, are likely to fund more valuable projects.

Proposition 2 and Corollary 2.1 show that first-best equilibria break down because the principal's threshold Δ is not within the range of investors' incentive compatible Δs. While it is possible that the misalignment is due to investors who observe $\hat{G}$, the following corollary shows that violations of the incentive compatibility constraints for investors who observe $\hat{B}$ prevent first-best.

Corollary 2.2. *Deviations from first-best financing inefficiency are due to a loser's blessing and never due to a winner's curse.*

Corollary 2.2 implies that the principal's ex post decision-making is congruent with the incentive compatibility constraints of investors who observe $\hat{G}$. In other words, the inability to attain first-best is never due to a winner's curse (i.e., abstaining with strictly positive probability after observing $\hat{G}$). Mathematically, the corollary implies that the Δ at which an investor who observes $\hat{G}$ is willing to contribute, given that the principal chooses risky deployment only if $\underline{n}$ investors contribute, is always smaller than the Δ for which the principal chooses risky deployment if $\underline{n}$ investors contribute.

Proposition 2 along with Corollaries 2.1 and 2.2, show that first-best equilibria break down because the principal's ex post decision-making introduces a loser's blessing for investors. To analyze the affect of N on first-best equilibria, we compute the principal's first-best decision thresholds and investors' incentive compatible regions for values of N ranging from two to forty. Figure 4 shows that as N increases, the percentage of first-best equilibria approaches zero. The figure shows the percentage of Δ that have first-best equilibria for all $\Delta \in [\underline{\Delta}, \overline{\Delta}]$ for three levels of information precision: $\alpha \in \{\frac{3}{5}, \frac{2}{3}, \frac{3}{4}\}$. In each case, as N increases, the percentage of first-best equilibria decrease. When investors have more precise information, the percentage of first-best equilibria drops more quickly. For $\alpha = \frac{3}{4}$, the percentage falls below 1% for $N \geq 13$, while for $\alpha = \frac{2}{3}$, the percentage falls below 1% for $N \geq 19$. By $N = 40$, the three lines are visually indistinguishable from each other and from zero. Figure 4 shows that first-best equilibria are effectively not feasible as N gets large.

2.2.2 Second-Best and Multiple Equilibria

When a first-best equilibrium is not feasible, financing efficiency of the second-best equilibrium can suffer either due to underinvestment (not maximizing the production technology) or noisy financing (funding more bad projects and/or fewer good projects due to less precise information). Recall that when a first-best equilibrium exists, $\kappa^{FB} \geq \overline{K}/\underline{n}^{FB}$, so that the production technology is fully used when at least $\underline{n}^{FB}$ investors contribute. Choosing $\kappa < \overline{K}/\underline{n}^{FB}$ leads to underinvestment when

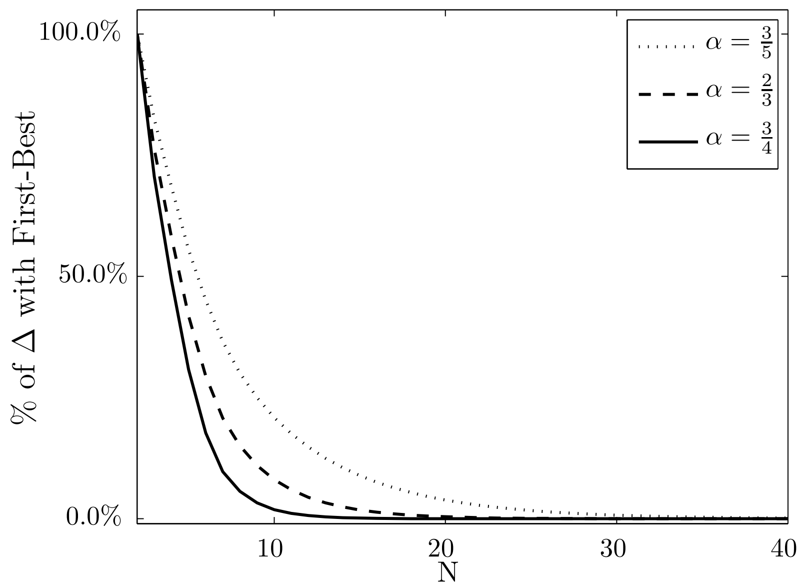


Figure 4: Percentage of $\Delta \in [\underline{\Delta}, \overline{\Delta})$ that support first-best equilibria as a function of N for $\alpha \in \left\{\frac{3}{5}, \frac{2}{3}, \frac{3}{4}\right\}$.

relatively fewer investors contribute. While underinvestment lowers financing efficiency, there is a possibility that choosing $\kappa < \overline{K}/\underline{n}^{FB}$ alters investors' incentives, leading to more truthful reporting. Lemma 5 shows that this is never the case and that lowering κ leads to less truthful reporting.[12] In equilibrium, underinvestment never occurs from not maximizing the production technology.

Lemma 5. *Choosing $\kappa < \overline{K}/\underline{n}$ exacerbates the loser's blessing.*

According to Lemma 5, lowering κ leads to less truth-telling. Lowering κ smooths the rationing faced by an investor who observes $\hat{B}$. To see this, consider a setting in which the principal requires at least $\underline{n}$ investors to choose risky deployment. If the principal chooses a contribution quantity $\kappa' = \overline{K}/\underline{n}$, an investor receives a full share of the risky payoff only if $\underline{n}$ investors contribute. If, however, $n > \underline{n}$ investors contribute, the investor receives a smaller share of $\underline{n}/n$. Now, suppose

[12] In Section 3, we consider the principal's ability to use price setting as an additional means to enhance financing efficiency. Furthermore, if the principal could perfectly commit to choosing risky deployment, even if his ex post beliefs were not sufficiently high, the principal could attenuate the loser's blessing and enhance financing efficiency.

the principal still requires at least $\underline{n}$ investors to choose risky deployment, but instead chooses contribution quantity $\hat{\kappa} = \overline{K}/(\underline{n}+1) < \overline{K}/\underline{n}$. An investor receives a full share of the risky payoff if $\underline{n}$ or $\underline{n}+1$ investors contribute. Moreover, if $n > \underline{n}+1$ investors contribute, the investor receives a share of $(\underline{n}+1)/n$ which is larger than what she received with κ'. Notably, with contribution quantity $\hat{\kappa}$, rationing is reduced which favors good projects because they attract more contributions, all else equal. Thus, if the principal lowers κ below $\overline{K}/\underline{n}$, rationing is reduced and contributing investors are relatively more exposed to good projects which encourages less truth-telling for investors who observe $\hat{B}$.

When considering second-best equilibria, it is important to remember that the principal's threshold rule for deploying capital to the risky project depends on the quality of the information received from investors. Our analysis of first-best equilibria assumes that all investors only contribute after observing $\hat{G}$. In second-best equilibria (denoted with superscript SB), investors do not always truthfully report, so $\mathcal{D}(n)$ is a function of g^{SB} and b^{SB}, as shown in (8) and (9). As b^{SB} increases, the principal's information becomes less precise, and he may require more contributions to deploy capital to the risky project.

Because of the dependence of the principal's threshold rule on investors' strategies, multiple equilibria may exist. Figure 5 depicts multiple equilibria across three panels: the first panel shows the lower bound of the principal's optimal κ^* as a percentage of $\overline{K}$ (since any $\kappa \geq \overline{K}/\underline{n}^*$ is also optimal), the second panel shows the equilibrium contribution strategy b^* of investors who observe $\hat{B}$, and the third panel shows the percentage of first-best expected value achieved by an equilibrium. The superscript $*$ denotes generic equilibrium values, some of which correspond to first-best or second-best.

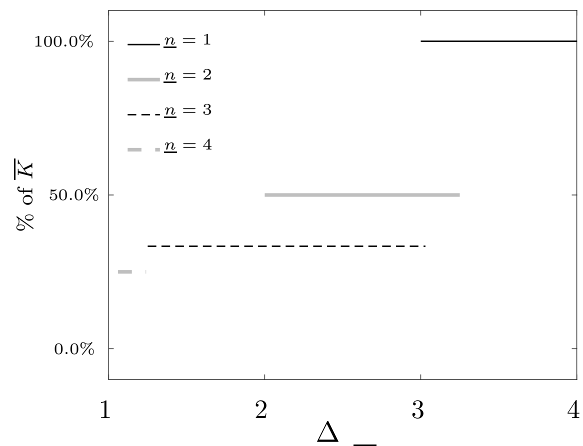
To see the potential for multiple equilibria, consider the two different equilibria that exist for $\Delta = 2.5$ in Figure 5. One equilibrium is first-best and corresponds to $\kappa^{FB} \geq \frac{\overline{K}}{2}$ in Panel A, $b^{FB} = 0$ in Panel B, and $E[V(K, F|d)] = 100\%$ in Panel C (100% represents the expected value from a first-best equilibrium, while 0% represents the expected value from a no-information equilibrium).[13] In the first-best equilibrium, the principal chooses risky deployment if at least two investors participate. In the other equilibrium, investors who receive bad signals participate 37% of the time ($b^{SB} = 0.37$ in Panel B) and the principal chooses risky deployment if at least three investors contribute (and $\kappa^{SB} \geq \frac{\overline{K}}{3}$ so that three investors will fully fund the project). The additional noise in the principal's information creates a value loss due to undertaking bad projects and forgoing good projects, as Panel C shows $E[V(K, F|d)] = 63\%$ in this equilibrium.

With limited commitment, the principal cannot influence which equilibrium will result through his choice of κ. By choosing a larger $\kappa \geq \frac{\overline{K}}{2}$, the principal does not commit to risky deployment in projects that attract at least 2 investors. Instead, the principal still has the option to deploy the capital in the safe project. As a result, if investors contribute after observing $\hat{B}$, the principal only chooses risky deployment when at least 3 investors contribute. In those cases, the principal has excess capital and rations investors. If, however, the principal could ex ante commit to risky deployment if two investors contribute, the principal can influence the equilibrium and implement first-best. By committing to risky deployment with two contributing investors, the loser's blessing is mitigated and the only incentive compatible equilibrium is first-best. In the context of securities-based crowdfunding, ex ante listed funding thresholds may be optimal as they provide a credible means to select more efficient equilibria.

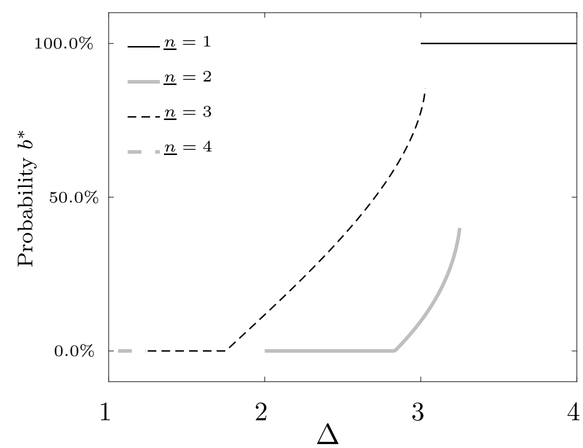
Finally, as Δ increases, more investors contribute after observing $\hat{B}$, and the principal requires more investors to contribute in order to deploy capital to the risky project. In particular, when $\Delta \geq \frac{\alpha+(1-\alpha)}{1-\alpha}$, an equilibria always exists in which all investors contribute, no information is conveyed, and all projects are funded (i.e., a babbling equilibrium). In these cases, the returns are high enough for the principal to invest without information, and are high enough for investors who observe $\hat{B}$ to invest despite that information. While this particular equilibrium with $\Delta = \frac{\alpha+(1-\alpha)}{1-\alpha}$ can be eliminated by a trembling-hand refinement, when Δ is sufficiently high, a babbling equilibrium

[13] All equilibrium values, other than first-best, are obtained through numerical methods.

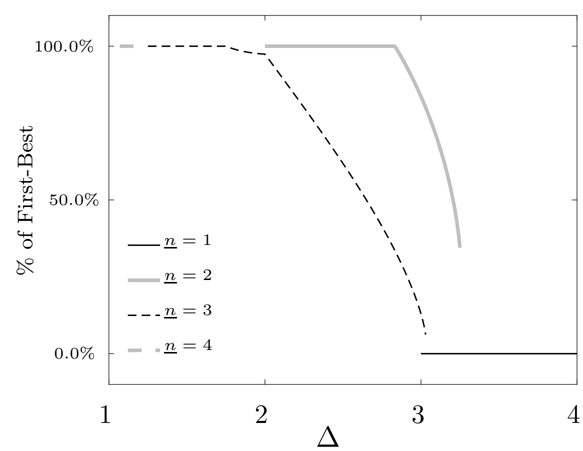
Figure 5: Equilibrium outcomes as a function of Δ for $\alpha = 3/4$ and $N = 4$.



(a) κ^*, as a percentage of $\overline{K}$, as a function of Δ.



(b) Equilibrium b^* as a function of Δ.



(c) Expected value, over first-best, as a function of Δ.

is the only possible equilibrium. In Figure 5, only the no-information equilibrium exists when $\Delta \geq 3.27$, which is therefore associated with $E[V(K, F|d)] = 0\%$.

3 Price Setting and Information Production Costs

In our base model, the loser's blessing prevents first-best financing efficiency when Δ is sufficiently high such that it is not incentive compatible for investors' observing $\hat{B}$ to abstain from contributing. To mitigate the loser's blessing, the principal must make contributing less attractive to investors who observe $\hat{B}$. The principal can disincent contributions by keeping some of the project's return for himself, effectively raising the price of the offering. We model this by assuming the principal offers a fraction $\beta \in [0, 1]$ of the project's risky return Δ to investors. Thus, $\beta\Delta$ replaces Δ in investor i's expected payoff function.

With the ability to price set, the principal's problem is given by,

$$\max_{\kappa \in \mathbb{R}^+, \beta \in [0,1]} E\left[V(K, F|d)\right] \tag{26}$$

$$\text{s.t. } g_i \in \arg\max_{\hat{g} \in [0,1]} \Pi(\hat{G}, \{\hat{g}, \hat{b}\} | \vec{\pi}_{-i}, \kappa, \beta, \underline{n}), \tag{26.1}$$

$$b_i \in \arg\max_{\hat{b} \in [0,1]} \Pi(\hat{B}, \{\hat{g}, \hat{b}\} | \vec{\pi}_{-i}, \kappa, \beta, \underline{n}), \tag{26.2}$$

$$g_i = g_{-i} \text{ and } b_i = b_{-i}, \tag{26.3}$$

$$\Pi(\hat{G}, \vec{\pi}_i | \vec{\pi}_{-i}, \kappa, \beta, \underline{n}) \geq 0 \text{ and } \Pi(\hat{B}, \vec{\pi}_i | \vec{\pi}_{-i}, \kappa, \beta, \underline{n}) \geq 0, \tag{26.4}$$

$$d = \begin{cases} r & \text{if } \rho(n, \vec{\pi})\Delta - 1 \geq 0 \text{ and } K \geq \underline{K} \\ s & \text{otherwise,} \end{cases} \tag{26.5}$$

in which $\Pi(\hat{F}, \vec{\pi}_i | \vec{\pi}_{-i}, \kappa, \beta, \underline{n})$ is investor i's expected payoff as a function of her signal $\hat{F}$ and strategy $\vec{\pi}_i$, given all other investors use strategy $\vec{\pi}_{-i}$, the principal chooses contribution amount κ and price setting level β, and risky deployment occurs if $n \geq \underline{n}$. As in the base model, Constraints 26.1-26.2

require that g_i and b_i are chosen as mutual best responses by the investors, Constraint 26.3 requires that the strategies are symmetric, Constraint 26.4 ensures participation (which is trivially satisfied if the mechanism is incentive compatible), and Constraint 26.5 requires that the principal chooses risky deployment of capital if the project is positive valued in expectation and raised capital exceeds $\underline{K}$.

With the ability to set prices, we define an equilibrium according to a quintet,

$$\Omega^{PS} = \{\kappa^{PS}, \beta^{PS}, g^{PS}, b^{PS}, \underline{n}^{PS}\}, \tag{27}$$

in which κ^{PS} and β^{PS} solve the principal's problem in (26), g^{PS} and b^{PS} are the equilibrium contribution strategies of investors who observe $\hat{G}$ and $\hat{B}$ respectively, and $\underline{n}^{PS}$ is the minimum number of contributions the principal must observe to choose risky deployment (conditional on strategies g^{PS} and b^{PS}). The superscript PS denotes an equilibrium with price setting.

To solve the problem, the principal first considers the optimal decision rule conditional on the project's actual Δ, establishing $\underline{n}^{PS}$. He then sets β such that the investor's payoff by contributing with a bad signal is strictly negative, that is,

$$\sum_{n=\underline{n}^{PS}-1}^{N-1} \left((1-\alpha)\Pr(n|G,\vec{\pi},N-1)\frac{(\beta\Delta-1)\underline{n}^{PS}}{n+1} - \alpha \Pr(n|B,\vec{\pi},N-1)\frac{\underline{n}^{PS}}{n+1} \right) < 0. \tag{28}$$

For the preceding inequality to hold, β must be strictly smaller than an upper bound $\overline{\beta}$. Noting that $\overline{\beta}$ is bound above by 1, (28) may be rearranged to provide an explicit formula for $\overline{\beta}$,

$$\overline{\beta} = \min\left\{ \frac{(1-\alpha)\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1} + \alpha\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\Delta(1-\alpha)\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}, 1 \right\}. \tag{29}$$

If the principal chooses too small of a β, however, investors who observe $\hat{G}$ will not find contributing optimal. As such, the principal must choose β such that an investor's payoff from contributing with a good signal is positive, that is,

$$\sum_{n=\underline{n}^{PS}-1}^{N-1}\left(\alpha\Pr(n|G,\vec{\pi},N-1)\frac{(\beta\Delta-1)\underline{n}^{PS}}{n+1}-(1-\alpha)\Pr(n|B,\vec{\pi},N-1)\frac{\underline{n}^{PS}}{n+1}\right)\geq 0. \tag{30}$$

For the preceding inequality to hold, β must be greater than a lower bound $\underline{\beta}$. Noting that $\underline{\beta}$ is also bound above by 1, (30) may be rearranged to provide an explicit formula for $\underline{\beta}$,

$$\underline{\beta}=\min\left\{\frac{\alpha\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}+(1-\alpha)\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\Delta\alpha\sum_{n=\underline{n}^{PS}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}},1\right\}. \tag{31}$$

Proposition 3. *For any $\Delta\geq\underline{\Delta}$, there exists an equilibrium solution $\Omega^{PS}=\{\kappa^{PS},\beta^{PS},g^{PS},b^{PS},\underline{n}^{PS}\}$ to the principal's problem outlined in (26) which corresponds to first-best financing efficiency as defined in Definition 2.*

Proposition 3 shows that first-best is always possible with price setting. First-best is obtained by shrinking investor returns to make abstaining with a bad signal incentive compatible. Said differently, *penalizing* investors with lower returns improves information aggregation. This result contrasts the IPO literature that shows *rewarding* investors improves efficiency (e.g., see Rock (1986) in which underpricing is used to mitigate the winner's curse and Benveniste and Spindt (1989) in which larger allocations reward more positive reports). The result suggests that efficient securities-based crowdfunding may be characterized by limited investor returns as price setting is used to curb the loser's blessing.

In Proposition 3, the principal sets the price such that investors who receive bad signals earn negative expected payoffs by participating. Furthermore, the principal sets the price such that an investor who receives a good signal expects a positive payoff. However, it is not clear that this profit could adequately compensate for information costs that might be incurred in acquiring the investors' signals. For example, suppose that investors must bear an information production cost

$c > 0$.[14] While we do not formally state the principal's problem, with costly information, we define an equilibrium according to the septet,

$$\Omega^C = \{\kappa^C, \beta^C, \vec{g}^C, \vec{b}^C, \vec{u}^C, \vec{\tau}^C, \underline{n}^C\}, \tag{32}$$

in which κ^C and β^C solve the principal's problem, $\vec{g}^C$ and $\vec{b}^C$ are the vectors of equilibrium contribution strategies of investors who observe $\hat{G}$ and $\hat{B}$ respectively, $\vec{u}^C$ is the vector of contribution strategies of uninformed investors that do not purchase a signal, $\vec{\tau}^C$ is the mixed strategies of investors with regards to whether or not they purchase a signal, and $\underline{n}^C$ is the minimum number of contributions the principal must observe to choose risky deployment (conditional on strategies $\vec{g}^C$, $\vec{b}^C$, and $\vec{u}^C$).[15] The superscript C denotes an equilibrium with costly information acquisition.

For investor i to be willing to pay the cost, the expected payoff (in dollars) for purchasing a signal must meet or exceed c:

$$\left(\frac{1}{2}\Pi_i(\hat{G}|\Omega^C) + \frac{1}{2}\Pi_i(\hat{B}|\Omega^C)\right) \min\left\{\kappa^C, \frac{\overline{K}}{\underline{n}^C}\right\} - c \geq 0, \tag{33}$$

in which $\Pi_i(\hat{F}|\Omega^C)$ is investor i's expected payoff as a function of her signal $\hat{F}$ in equilibrium Ω^C. If investor i does not purchase a signal, her expected payoff is given by,

$$u_i \Pi(\text{uninformed}|\Omega^C) + (1 - u_i)0, \tag{34}$$

in which $\Pi(\text{uninformed}|\Omega^C)$ is an investor's expected payoff when she submits an uninformed contribution with probability u_i and she abstains with probability $(1 - u_i)$.

The payoff in (33) is related to the difference between the payoffs for good and bad signals. As the net payoff to a bad signal is set to zero in a first-best equilibrium, the good signal payoff must

[14] Alternatively, we could consider an admission cost to participate on the platform. We model the information production cost, as it is a more natural interpretation, but a model with admission costs also gives rise to a loser's blessing.

[15] We acknowledge that assuming symmetric strategies by investors is too restrictive in this setting. Hence, we use the vector notation on g, b, u and τ to allow for nonsymmetric strategies.

be sufficiently large to compensate for the information cost c. For some projects, however, the good signal payoff goes to zero as N increases, in particular, projects that are ex ante negative valued (i.e., projects with $\Delta \leq 2$ which are also characterized by the principal's requiring at least half of the investors to contribute for risky deployment, i.e., $\underline{n}^C \geq \frac{N}{2}$ if all investors purchase signals and truthfully report them).

Corollary 3.1. *If $c > 0$ and N is sufficiently large, an ex ante negative valued project ($\Delta \leq 2$) cannot achieve first-best financing efficiency as defined in Definition 2.*

While Proposition 3 shows that first-best is always possible via price setting, Corollary 3.1 shows that first-best is not always attainable if information is costly and N is large.[16] To understand the result, consider a project with $\Delta = 2$ and a hypothetical equilibrium in which all investors purchase signals ($\vec{\tau} = 1$) and follow them ($\vec{g} = 1$ and $\vec{b} = 0$). In this hypothetical equilibrium, the principal chooses risky deployment if at least half the investors contribute, $\underline{n} = \frac{N}{2}$. If the project is good, the distribution of contributing investors is a binomial distribution centered at αN. Conversely, if the project is bad, the distribution is a binomial distribution centered at $(1 - \alpha)N$. As N gets large, the probability mass of the good distribution above $\frac{N}{2}$ increases and the probability mass of the bad distribution below $\frac{N}{2}$ increases. Consequently, the wedge between good and bad projects increases and a loser's blessing ensues. To dissuade investors from contributing after observing $\hat{B}$, the principal lowers β. However, as N gets sufficiently large, the crowd has the potential to almost perfectly screen projects and the principal lowers β to a level at which investors just breakeven (ignoring c), even with perfect screening. That is, β goes to $\frac{1}{\Delta}$ as $N \to \infty$ and price setting makes investors gross return 1 and net return 0 (ignoring c). While such price setting accomplishes first-best when signals are costless, first-best is not possible if information is costly as net payoffs are strictly negative with signal costs.

If c and N are sufficiently large such that first-best is not possible, some investors in equilibrium

[16] Hakenes and Schlegel (2014) also models investors' information costs, showing that information production and aggregation is not first-best when firms do not know their own types. While our analysis focuses on financing efficiency, the model in Hakenes and Schlegel (2014) maximizes the entrepreneur's payoff.

choose to not purchase signals and are uninformed. In such an equilibrium, it must be the case that investors who do purchase a signal adhere to a strategy of following their signal, i.e., $\vec{g}^C = 1$ and $\vec{b}^C = 0$. To see this, suppose that an investor purchases a signal, observes $\hat{B}$, and chooses to contribute despite the signal. Because the investor incurred the cost c, she would be better off if she had never purchased a signal and instead contributed without information. Thus, contributing after observing $\hat{B}$ cannot be an equilibrium strategy. Additionally, in such an equilibrium, if there are any informed investors, it must be the case that the payoff after observing $\hat{G}$ is strictly positive. If this were not the case, then no investor would purchase a signal in equilibrium. However, if the payoffs for purchasing a signal are sufficiently large, uninformed investors contribute despite not having information. Thus, uninformed contributions are the manifestation of the loser's blessing in a setting with costly information.[17]

4 Concluding Discussion

Crowdfunding campaigns are characterized by small contributions from many dispersed investors. When products are pre-sold to consumers, campaigns can aggregate the wisdom of the crowd to fund projects with sufficient demand. When securities are sold to investors, however, we show that information aggregation breaks down, even in an optimal mechanism.

Our analysis suggests that crowds of strictly-profit-motivated investors are unlikely to provide significant capital to early-stage ventures due to inefficient financing. Rewards-based and donations-based crowdfunding have grown successfully based on contributions from individuals who receive private benefits. To the extent that profit-motivated investors also enjoy private benefits from a project, e.g., access to a new local brewery, a portion of investors may be able to commit to following their signals, restoring some degree of financing efficiency. By attracting consumer-like investors, securities-based crowdfunding platforms may be able to efficiently finance projects by

[17] In such a setting, the principal rationally anticipates that some investors are uninformed and controls for it in his Bayesian updating. Thus, while informed investors play truth-telling strategies, $\vec{g}^C = 1$ and $\vec{b}^C = 0$, the principal faces two sources of noise: (i) fewer investors are informed and (ii) uninformed investors contribute.

enticing investment with the promise of future returns (having a common value), while relying on investors' preferences (their private values) to reveal their true beliefs.

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Appendix A

Derivation of Investors' Payoffs:

The investor's expected payoff (as a net return) from contributing is attained by using her subjective state probabilities for there being n contributions when the project's type is F, conditional on her private signal $\hat{F}$, that is, $\Pr(n \cap F|\hat{F})$. Using Bayes' Rule, the subjective state probability may be rewritten as,

$$\frac{\Pr(n \cap F \cap \hat{F})}{\Pr(\hat{F})} = \frac{\Pr(n \cap \hat{F}|F)\Pr(F)}{\Pr(\hat{F})}, \tag{A1}$$

and, using conditional independence, expands to,

$$= \frac{\Pr(n|F)\Pr(\hat{F}|F)\Pr(F)}{\Pr(\hat{F})}, \tag{A2}$$

and, because $\Pr(F) = \Pr(\hat{F}) = \frac{1}{2}$, simplifies to,

$$= \Pr(n|F)\Pr(\hat{F}|F), \tag{A3}$$

and, using again Bayes' Rule and $\Pr(F) = \Pr(\hat{F}) = \frac{1}{2}$, finally may be rewritten as,

$$= \Pr(n|F)\Pr(F|\hat{F}). \tag{A4}$$

Investor i's expected payoff, as a function of the signal $\hat{G}$ and mixing strategy g_i, is given by,

$$\begin{aligned}&\Pi(\hat{G}, g_i|\vec{\pi}_{-i}, \kappa, \underline{n})\\ =&g_i \sum_{n=\underline{n}-1}^{N-1} \left(\alpha \Pr(n|G, \vec{\pi}_{-i}, N-1) \frac{(\Delta - 1)\underline{n}}{n+1} - (1-\alpha)\Pr(n|B, \vec{\pi}_{-i}, N-1) \frac{\underline{n}}{n+1} \right),\end{aligned} \tag{A5}$$

Similarly, investor i's expected payoff, as a function of the signal $\hat{B}$ and mixing strategy b_i, is given by,

$$\Pi(\hat{B}, b_i|\vec{\pi}_{-i}, \kappa, \underline{n})$$
$$= b_i \sum_{n=\underline{n}-1}^{N-1} \left((1-\alpha) \Pr(n|G, \vec{\pi}_{-i}, N-1) \frac{(\Delta-1)\underline{n}}{n+1} - \alpha \Pr(n|B, \vec{\pi}_{-i}, N-1) \frac{n}{n+1} \right). \tag{A6}$$

Proof of Lemma 1: To perform the proof of the lemma, we show that $\Delta_{\underline{n},\hat{B}} - \Delta_{\underline{n},\hat{G}}$ is strictly positive valued. Using (21) and (22), the difference is given by,

$$\Delta_{\underline{n},\hat{B}} - \Delta_{\underline{n},\hat{G}} = \frac{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1} + \alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}}$$
$$- \frac{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1} + (1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}} \tag{A7}$$
$$= 1 + \frac{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}}$$
$$- 1 - \frac{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{\alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}} \tag{A8}$$
$$= \left(\frac{\alpha}{1-\alpha} - \frac{1-\alpha}{\alpha}\right) \frac{\sum_{n=\underline{n}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}}, \tag{A9}$$

which is strictly positive because $\alpha > \frac{1}{2}$.

■

Proof of Lemma 2: To perform the proof of the lemma, we begin by showing that $\Delta_{\underline{n},\hat{G}}$ is decreasing in $\underline{n}$. Consider $\underline{\dot{n}} < N$ and $\underline{\ddot{n}} = \underline{\dot{n}} + 1$, i.e., $\underline{\ddot{n}} > \underline{\dot{n}}$. Using (21), $\Delta_{\underline{\ddot{n}},\hat{G}}$ is given by,

$$\Delta_{\underline{\ddot{n}},\hat{G}} = \frac{\alpha \sum_{n=\underline{\ddot{n}}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1} + (1-\alpha)\sum_{n=\underline{\ddot{n}}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{\alpha \sum_{n=\underline{\ddot{n}}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}} \tag{A10}$$
$$= 1 + \frac{(1-\alpha)\sum_{n=\underline{\ddot{n}}-1}^{N-1} \Pr(n|B, \vec{\pi}, N-1)\frac{1}{n+1}}{\alpha \sum_{n=\underline{\ddot{n}}-1}^{N-1} \Pr(n|G, \vec{\pi}, N-1)\frac{1}{n+1}}, \tag{A11}$$

and, it is straightforward to show that $\Delta_{\underline{\dot{n}},\hat{G}}$ is given by,

$$\Delta_{\underline{\dot{n}},\hat{G}} = 1 + \frac{(1-\alpha)\Pr(\dot{n}-1|B,\vec{\pi},N-1)\frac{1}{\dot{n}} + (1-\alpha)\sum_{n=\underline{\ddot{n}}-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\alpha\Pr(\dot{n}-1|G,\vec{\pi},N-1)\frac{1}{\dot{n}} + \alpha\sum_{n=\underline{\ddot{n}}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}, \tag{A12}$$

The sign on difference in $\Delta_{\underline{\dot{n}},\hat{G}}$ and $\Delta_{\underline{\ddot{n}},\hat{G}}$ is determined by the sign on,

$$\begin{aligned} &\alpha(1-\alpha)\Pr(\dot{n}-1|B,\vec{\pi},N-1)\frac{1}{\dot{n}}\sum_{n=\underline{\ddot{n}}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1} \\ &-\alpha(1-\alpha)\Pr(\dot{n}-1|G,\vec{\pi},N-1)\frac{1}{\dot{n}}\sum_{n=\underline{\ddot{n}}-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}, \end{aligned} \tag{A13}$$

which is simplified using the explicit forms of $\Pr(n|G,\vec{\pi},N-1)$ and $\Pr(n|B,\vec{\pi},N-1)$ found in (19) and (20) respectively to,

$$\begin{aligned} &\binom{N-1}{\dot{n}-1}\frac{1}{\dot{n}}\sum_{n=\underline{\ddot{n}}-1}^{N-1}\binom{N-1}{n}\alpha^{N+n-\dot{n}+1}(1-\alpha)^{N-(n-\dot{n}+1)}\frac{1}{n+1} \\ &-\binom{N-1}{\dot{n}-1}\frac{1}{\dot{n}}\sum_{n=\underline{\ddot{n}}-1}^{N-1}\binom{N-1}{n}\alpha^{N-(n-\dot{n}+1)}(1-\alpha)^{N+n-\dot{n}+1}\frac{1}{n+1}, \end{aligned} \tag{A14}$$

and further simplifies to,

$$\binom{N-1}{\dot{n}-1}\frac{1}{\dot{n}}\sum_{n=\underline{\ddot{n}}-1}^{N-1}\binom{N-1}{n}\frac{1}{n+1}\left(\alpha^{N+n-\dot{n}+1}(1-\alpha)^{N-(n-\dot{n}+1)} - \alpha^{N-(n-\dot{n}+1)}(1-\alpha)^{N+n-\dot{n}+1}\right), \tag{A15}$$

which is strictly positive because $n-\dot{n}+1>0$ for all $n \geq \underline{\ddot{n}}-1$. A similar proof can be constructed to show that $\Delta_{\underline{n},\hat{B}}$ is decreasing in $\underline{n}$.

■

Proof of Lemma 3: To perform the proof of the lemma, consider $\underline{\dot{n}} < N$ and $\underline{\ddot{n}} = \underline{\dot{n}} + 1$, i.e.,

$\underline{\ddot{n}} > \underline{\dot{n}}$. We now define some shorthand notation for the sake of exposition. Define the following,

$$u \equiv \binom{N-1}{\underline{\dot{n}}-1} \frac{\alpha^{\underline{\dot{n}}-1}(1-\alpha)^{N-\underline{\dot{n}}}}{\underline{\dot{n}}}, \tag{A16}$$

$$v \equiv \binom{N-1}{\underline{\dot{n}}-1} \frac{(1-\alpha)^{\underline{\dot{n}}-1}\alpha^{N-\underline{\dot{n}}}}{\underline{\dot{n}}}, \tag{A17}$$

$$w \equiv \sum_{n=\underline{\ddot{n}}-1}^{N-1} \binom{N-1}{n} \frac{\alpha^{n}(1-\alpha)^{N-1-n}}{n+1}, \tag{A18}$$

$$x \equiv \sum_{n=\underline{\ddot{n}}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^{n}\alpha^{N-1-n}}{n+1}, \tag{A19}$$

$$\begin{aligned} y &\equiv u + w \\ &= \sum_{n=\underline{\dot{n}}-1}^{N-1} \binom{N-1}{n} \frac{\alpha^{n}(1-\alpha)^{N-1-n}}{n+1}, \end{aligned} \tag{A20}$$

$$\begin{aligned} z &\equiv v + x \\ &= \sum_{n=\underline{\dot{n}}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^{n}\alpha^{N-1-n}}{n+1}. \end{aligned} \tag{A21}$$

Using the preceding shorthand notation and (21) and (22), $\Delta_{\underline{\dot{n}},\hat{G}}$ and $\Delta_{\underline{\ddot{n}},\hat{B}}$ are given by,

$$\Delta_{\underline{\dot{n}},\hat{G}} = \frac{\alpha y + (1-\alpha) z}{\alpha y} \tag{A22}$$

$$= \frac{\alpha(u+w) + (1-\alpha)(v+x)}{\alpha(u+w)}, \tag{A23}$$

$$\Delta_{\underline{\ddot{n}},\hat{B}} = \frac{(1-\alpha)w + \alpha x}{(1-\alpha)w}. \tag{A24}$$

Thus, $\Delta_{\ddot{\underline{n}},\hat{B}} - \Delta_{\dot{\underline{n}},\hat{G}}$ is given by,

$$\Delta_{\ddot{\underline{n}},\hat{B}} - \Delta_{\dot{\underline{n}},\hat{G}} = \frac{(1-\alpha)w + \alpha x}{(1-\alpha)w} - \frac{\alpha(u+w) + (1-\alpha)(v+x)}{\alpha(u+w)} \tag{A25}$$

$$= \frac{w + \frac{\alpha}{(1-\alpha)}x}{w} - \frac{(u+w) + \frac{(1-\alpha)}{\alpha}(v+x)}{u+w} \tag{A26}$$

$$= \frac{w(u+w) + \frac{\alpha}{(1-\alpha)}x(u+w) - w(u+w) - \frac{(1-\alpha)}{\alpha}w(v+x)}{w(u+w)} \tag{A27}$$

$$= \frac{\frac{\alpha}{(1-\alpha)}x(u+w) - \frac{(1-\alpha)}{\alpha}w(v+x)}{w(u+w)}. \tag{A28}$$

The sign on the difference is determined by the sign on,

$$\frac{\alpha}{(1-\alpha)}x(u+w) - \frac{(1-\alpha)}{\alpha}w(v+x). \tag{A29}$$

We conjecture that the difference is positive valued, and we show that it is:

$$\frac{\alpha}{(1-\alpha)}x(u+w) - \frac{(1-\alpha)}{\alpha}w(v+x) \geq 0, \tag{A30}$$

which can be rearranged as,

$$\frac{\alpha^2}{(1-\alpha)^2} \geq \frac{w(v+x)}{x(u+w)}, \tag{A31}$$

and further rearranged as,

$$\frac{\alpha^2}{(1-\alpha)^2} \geq \frac{\frac{(v+x)}{(u+w)}}{\frac{x}{w}} \tag{A32}$$

$$= \frac{\sum_{n=\dot{\underline{n}}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n}}{n+1} \Big/ \sum_{n=\dot{\underline{n}}-1}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n}}{n+1}}{\sum_{n=\ddot{\underline{n}}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n}}{n+1} \Big/ \sum_{n=\ddot{\underline{n}}-1}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n}}{n+1}} \tag{A33}$$

$$= \frac{\frac{\alpha^{N-2(\dot{\underline{n}}-1)-1}}{(1-\alpha)^{N-2(\dot{\underline{n}}-1)-1}} \left({}_2F_1\left(1, \dot{\underline{n}} - N; \dot{\underline{n}} + 1; \frac{-(1-\alpha)}{\alpha}\right) \Big/ {}_2F_1\left(1, \dot{\underline{n}} - N; \dot{\underline{n}} + 1; \frac{-\alpha}{(1-\alpha)}\right)\right)}{\frac{\alpha^{N-2(\ddot{\underline{n}}-1)-1}}{(1-\alpha)^{N-2(\ddot{\underline{n}}-1)-1}} \left({}_2F_1\left(1, \ddot{\underline{n}} - N; \ddot{\underline{n}} + 1; \frac{-(1-\alpha)}{\alpha}\right) \Big/ {}_2F_1\left(1, \ddot{\underline{n}} - N; \ddot{\underline{n}} + 1; \frac{-\alpha}{(1-\alpha)}\right)\right)}, \tag{A34}$$

in which ${}_2F_1(a,b;c;z)$ is the ordinary hypergeometric function.[18] Noting that $\underline{\ddot{n}} = \underline{\dot{n}} + 1$, the expression in (A34) may be rewritten as,

$$= \frac{\alpha^2}{(1-\alpha)^2} \frac{\left({}_2F_1\left(1, \underline{\dot{n}} - N; \underline{\dot{n}} + 1; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, \underline{\dot{n}} - N; \underline{\dot{n}} + 1; -\frac{\alpha}{1-\alpha}\right)\right)}{\left({}_2F_1\left(1, \underline{\dot{n}} + 1 - N; \underline{\dot{n}} + 2; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, \underline{\dot{n}} + 1 - N; \underline{\dot{n}} + 2; -\frac{\alpha}{1-\alpha}\right)\right)}. \tag{A35}$$

A proof by George Gasper in Addendum 1 of Appendix B shows that the ratio

$$\frac{\left({}_2F_1\left(1, \underline{\dot{n}} - N; \underline{\dot{n}} + 1; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, \underline{\dot{n}} - N; \underline{\dot{n}} + 1; -\frac{\alpha}{1-\alpha}\right)\right)}{\left({}_2F_1\left(1, \underline{\dot{n}} + 1 - N; \underline{\dot{n}} + 2; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, \underline{\dot{n}} + 1 - N; \underline{\dot{n}} + 2; -\frac{\alpha}{1-\alpha}\right)\right)}, \tag{A36}$$

is bounded above by one. Therefore, (A36) is smaller than $\frac{\alpha^2}{(1-\alpha)^2}$, proving that the difference $\Delta_{\underline{\ddot{n}},\hat{B}} - \Delta_{\underline{\dot{n}},\hat{G}}$ is positive valued.

■

Proof of Lemma 4: The proof follows directly from Lemma 1, Lemma 2, and Lemma 3.

■

Proof of Proposition 1: The result follows directly from the analysis.

■

Proof of Corollary 1.1: To prove the corollary, we consider one non-empty subset of Δ that satisfies the conditions in Proposition 1. While the proof considers a single non-empty subset, the analysis does not fully characterize the entire set of Δ that satisfies the conditions in Proposition 1.

Suppose there are N investors and consider a project with $\Delta \in [\mathcal{D}(N), \mathcal{D}(N-1))$, in which, using (15),

$$\mathcal{D}(N) = \frac{\alpha^N + (1-\alpha)^N}{\alpha^N}, \tag{A37}$$

$$\mathcal{D}(N-1) = \frac{\alpha^{N-1}(1-\alpha) + (1-\alpha)^{N-1}\alpha}{\alpha^{N-1}(1-\alpha)}. \tag{A38}$$

The project's promised return is sufficiently low and the principal chooses risky deployment if, and only if, all investors participate, i.e., $\underline{n}^{FB} = N$. The principal chooses $\kappa^{FB} \geq \frac{\overline{K}}{\underline{n}^{FB}}$. Using (21) and

[18] See Lemma B2, Lemma B3, Lemma B4, Lemma B5, and Lemma B6 in Appendix B for the analytic steps showing how (A34) is obtained.

(22) respectively, the threshold values $\Delta_{\underline{n}^{FB},\hat{G}}$ and $\Delta_{\underline{n}^{FB},\hat{B}}$ at $\underline{n}^{FB} = N$ with $\kappa^{FB} \geq \frac{\overline{K}}{\underline{n}^{FB}}$ are given by,

$$\Delta_{N,\hat{G}} = \frac{\alpha\binom{N-1}{N-1}\alpha^{N-1}\frac{1}{N} + (1-\alpha)\binom{N-1}{N-1}(1-\alpha)^{N-1}\frac{1}{N}}{\alpha\binom{N-1}{N-1}\alpha^{N-1}\frac{1}{N}} \tag{A39}$$

$$= \frac{\alpha^N + (1-\alpha)^N}{\alpha^N}, \tag{A40}$$

$$\Delta_{N,\hat{B}} = \frac{(1-\alpha)\binom{N-1}{N-1}\alpha^{N-1}\frac{1}{N} + \alpha\binom{N-1}{N-1}(1-\alpha)^{N-1}\frac{1}{N}}{(1-\alpha)\binom{N-1}{N-1}\alpha^{N-1}\frac{1}{N}} \tag{A41}$$

$$= \frac{\alpha^{N-1}(1-\alpha) + (1-\alpha)^{N-1}\alpha}{\alpha^{N-1}(1-\alpha)}. \tag{A42}$$

As such, when all N investors are required for risky deployment, the range of Δ for which it is incentive compatible for investors to truthfully report their signals via contribution is the same range of Δ for which the principal chooses risky deployment if, and only if, N investors contribute with contribution strategies of $g = 1$ and $b = 0$. That is, $\mathcal{D}(N) = \Delta_{N,\hat{G}}$ and $\mathcal{D}(N-1) = \Delta_{N,\hat{B}}$. Furthermore, if N investors participate the principal receives at least $\overline{K}$ units of capital which fully economizes the production technology. Therefore, since $\mathcal{D}(N-1) > \mathcal{D}(N)\ \forall\ N$, there is a non-empty set of Δ for any $N \geq 2$ for which the principal chooses κ^{FB} and achieves first-best financing efficiency as defined in Definition 2.

■

Proof of Proposition 2: To prove the proposition, consider a setting with $N > 2$. If $\Delta_{\hat{n},\hat{B}} < \mathcal{D}(\hat{n}-1)$ for some $\hat{n} \leq N$ then there exists a region of Δ between $\left[\Delta_{\hat{n},\hat{B}}, \mathcal{D}(\hat{n}-1)\right)$ for which incentive compatibility and optimal deployment of contributed capital are not mutually possible. In the preceding range of Δ, the principal would choose risky-deployment if at least $\hat{n}$ investors participate, however, it is not incentive compatible for an investor who observes $\hat{B}$ to truthfully report the signal via abstaining from contributing. We now show that such a range of Δ exists if $\hat{n} = N-1$, but note that there are also additional regions at different values of $\hat{n}$ for $N > 3$. Using (15) and (22), the difference $\mathcal{D}(N-2) - \Delta_{N-1,\hat{B}}$ is given by,

$$\frac{\alpha^{N-2}(1-\alpha)^2 + (1-\alpha)^{N-2}\alpha^2}{\alpha^{N-2}(1-\alpha)^2} - \frac{(1-\alpha)\sum\limits_{n=N-2}^{N-1}\frac{\Pr(n|G)}{n+1} + \alpha\sum\limits_{n=N-2}^{N-1}\frac{\Pr(n|B)}{n+1}}{(1-\alpha)\sum\limits_{n=N-2}^{N-1}\frac{\Pr(n|G)}{n+1}}, \tag{A43}$$

and is simplified to,

$$=\frac{(1-\alpha)^{N-1}\alpha^2\left(\binom{N-1}{N-2}\frac{\alpha^{N-2}(1-\alpha)}{N-1}+\binom{N-1}{N-1}\frac{\alpha^{N-1}}{N}\right)}{\alpha^{N-2}(1-\alpha)^3\left(\sum_{n=N-2}^{N-1}\frac{\Pr(n|G)}{n+1}\right)}-$$
$$-\frac{\alpha^{N-1}(1-\alpha)^2\left(\binom{N-1}{N-2}\frac{(1-\alpha)^{N-2}\alpha}{N-1}+\binom{N-1}{N-1}\frac{(1-\alpha)^{N-1}}{N}\right)}{\alpha^{N-2}(1-\alpha)^3\left(\sum_{n=N-2}^{N-1}\frac{\Pr(n|G)}{n+1}\right)}, \tag{A44}$$

which further reduces to,

$$=\frac{\frac{(1-\alpha)^{N-1}\alpha^{N-1}}{N}\left(\alpha^2-(1-\alpha)^2\right)}{\alpha^{N-2}(1-\alpha)^3\left(\sum_{n=N-2}^{N-1}\frac{\Pr(n|G)}{n+1}\right)} \tag{A45}$$

$$>0, \tag{A46}$$

because N is finite and $\alpha > \frac{1}{2}$.

■

Proof of Corollary 2.1: To prove the corollary, without loss of generality, consider the case in which N is divisible by 2 and $N > 2$. Conditional on investors truthfully reporting their signals, the principal chooses risky deployment of capital for ex ante zero valued projects, i.e., $\Delta = 2$, if at least $N/2$ investors participate. That is,

$$\mathcal{D}(N/2)=\frac{\alpha^{N/2}(1-\alpha)^{N/2}+\alpha^{N/2}(1-\alpha)^{N/2}}{\alpha^{N/2}(1-\alpha)^{N/2}}=2. \tag{A47}$$

Furthermore, if $\Delta \in \left[\frac{\alpha^2+(1-\alpha)^2}{\alpha^2}, 2\right)$, the principal requires at least $N/2+1$ investors, because

$$\mathcal{D}(N/2+1)=\frac{\alpha^{N/2+1}(1-\alpha)^{N/2-1}+\alpha^{N/2-1}(1-\alpha)^{N/2+1}}{\alpha^{N/2+1}(1-\alpha)^{N/2-1}}=\frac{\alpha^2+(1-\alpha)^2}{\alpha^2}. \tag{A48}$$

The preceding values of $\mathcal{D}(N/2)$ and $\mathcal{D}(N/2+1)$ hold for any N. Therefore, for any N, there exists a region of Δ given by $\left[\frac{\alpha^2+(1-\alpha)^2}{\alpha^2}, 2\right)$, for which the principal chooses risky deployment if $N/2+1$ investors participate.

Now, consider the upper bound on investors' incentive compatibility region of Δ for $\underline{n} = N/2+1$,

$\Delta_{N/2+1,\hat{B}}$. The upper bound is given by,

$$\Delta_{N/2+1,\hat{B}} \equiv \frac{(1-\alpha)\sum\limits_{n=N/2}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1} + \alpha \sum\limits_{n=N/2}^{N-1} \frac{\Pr(n|B,\vec{\pi},N-1)}{n+1}}{(1-\alpha)\sum\limits_{n=N/2}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}}, \tag{A49}$$

which is rewritten as,

$$1 + \frac{\sum\limits_{n=N/2}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-n}}{n+1}}{\sum\limits_{n=N/2}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-n}}{n+1}}. \tag{A50}$$

Note that because the summation begins at $N/2$ in the preceding equation, each term in the summation in the numerator is weakly smaller than its analog term in the denominator. For example, for the term in the summations in which $n = N/2+1$, the term in the numerator is given by $\binom{N-1}{N/2+1} \frac{(1-\alpha)^{N/2+1}\alpha^{N/2-1}}{N/2+2}$ and the term in the denominator is given by $\binom{N-1}{N/2+1} \frac{\alpha^{N/2+1}(1-\alpha)^{N/2-1}}{N/2+2}$, which is larger because $\alpha > 1/2$. Thus, for any $N > 2$, the ratio of sums in (A50) is always smaller than one and the total expression in (A50) is strictly less than two.

Therefore, because $\Delta_{N/2+1,\hat{B}} < \mathcal{D}(N/2)$ and $\mathcal{D}(N/2) - \mathcal{D}(N/2+1) > 0$, for any $N > 2$, there exists at least one continuous range of Δ in the interval $\left[\frac{\alpha^2+(1-\alpha)^2}{\alpha^2}, 2\right) \subseteq \left[\underline{\Delta}, \overline{\Delta}\right)$ for which first-best financing efficiency, as defined in Definition 2, is not possible.

■

Proof of Corollary 2.2: To perform the proof of the corollary, we show that $\mathcal{D}(\underline{n}) - \Delta_{\underline{n},\hat{G}}$ is positive valued which implies that the truth-telling incentive compatibility constraint of investors observing $\hat{G}$ is satisfied. Using (15) and (21), the difference is given by,

$$\mathcal{D}(\underline{n}) - \Delta_{\underline{n},\hat{G}} = \frac{\alpha^{\underline{n}}(1-\alpha)^{N-\underline{n}} + (1-\alpha)^{\underline{n}}\alpha^{N-\underline{n}}}{\alpha^{\underline{n}}(1-\alpha)^{N-\underline{n}}} \tag{A51}$$

$$- \frac{\alpha \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1} + (1-\alpha) \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|B,\vec{\pi},N-1)}{n+1}}{\alpha \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}} \tag{A52}$$

$$= \frac{(1-\alpha)^{\underline{n}}\alpha^{N-\underline{n}}}{\alpha^{\underline{n}}(1-\alpha)^{N-\underline{n}}} - \frac{(1-\alpha) \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|B,\vec{\pi},N-1)}{n+1}}{\alpha \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}} \tag{A53}$$

$$= \frac{(1-\alpha)^{\underline{n}}\alpha^{N-\underline{n}+1} \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1} - \alpha^{\underline{n}}(1-\alpha)^{N-\underline{n}+1} \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|B,\vec{\pi},N-1)}{n+1}}{\alpha^{\underline{n}+1}(1-\alpha)^{N-\underline{n}} \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}} \tag{A54}$$

$$= \frac{\sum_{n=\underline{n}-1}^{N-1} \left(\binom{N-1}{n} \frac{1}{n+1} \left((1-\alpha)^{\underline{n}+N-1-n}\alpha^{N-\underline{n}+1+n} - \alpha^{\underline{n}+N-1-n}(1-\alpha)^{N-\underline{n}+1+n} \right) \right)}{\alpha^{\underline{n}+1}(1-\alpha)^{N-\underline{n}} \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}} \tag{A55}$$

$$= \frac{\alpha^N(1-\alpha)^N \sum_{n=\underline{n}-1}^{N-1} \left(\binom{N-1}{n} \frac{1}{n+1} \left(\left(\frac{\alpha}{1-\alpha}\right)^{n+1-\underline{n}} - \left(\frac{1-\alpha}{\alpha}\right)^{n+1-\underline{n}} \right) \right)}{\alpha^{\underline{n}+1}(1-\alpha)^{N-\underline{n}} \sum_{n=\underline{n}-1}^{N-1} \frac{\Pr(n|G,\vec{\pi},N-1)}{n+1}}. \tag{A56}$$

The sign on the difference is determined by the sign on,

$$\sum_{n=\underline{n}-1}^{N-1} \left(\binom{N-1}{n} \frac{1}{n+1} \left(\left(\frac{\alpha}{1-\alpha}\right)^{n+1-\underline{n}} - \left(\frac{1-\alpha}{\alpha}\right)^{n+1-\underline{n}} \right) \right). \tag{A57}$$

Note that every term in the summation is weakly positive valued because the summation begins at $n = \underline{n} - 1$ and because $\alpha > \frac{1}{2}$. Thus, $\mathcal{D}(\underline{n}) - \Delta_{\underline{n},\hat{G}}$ is positive valued and the truth-telling incentive compatibility constraint of investors observing $\hat{G}$ is satisfied.

■

Proof of Lemma 5: To prove the lemma, first consider the case in which $\kappa = \frac{\overline{K}}{\underline{n}}$. In such a setting, if at least $\underline{n}$ investors contribute, the principal chooses risky deployment and does not risk underinvestment. Each investor that contributes earns the risky return on a pro rata share of her contribution. The pro rata share is given by $\frac{\underline{n}}{n}$ in which n is the number of contributing investors. If only $\underline{n}$ investors participate, each investor receives a full share and if all N investors participate, each investor receives a $\frac{\underline{n}}{N}$ share.

Next, consider the case in which $\kappa < \frac{\overline{K}}{\underline{n}}$, and, without loss of generality, suppose $\kappa = \frac{\overline{K}}{\underline{n}+1}$. In such a setting, the principal will chose risky deployment if at least $\underline{n}$ investors participate, but he faces underinvestment when exactly $\underline{n}$ investors participate. Each investor that contributes earns the risky return on a pro rata share of her contribution, however, in this case, the share is given by $\min\left(\frac{\underline{n}+1}{n}, 1\right)$, because if either $n = \underline{n}$ or $n = \underline{n} + 1$ investors contribute, returns are not rationed and investors receive a full share. Notably, with $\kappa < \frac{\overline{K}}{\underline{n}}$, the pro rata share schedule becomes flatter.

Now, we formally show that a flatter pro rata share schedule exacerbates the loser's blessing. Suppose that κ is set such that $\underline{n} + \hat{n}$ investors must participate to economizing on the project's total capacity, in which $\hat{n} \geq 0$ and $\hat{n} \leq N - \underline{n} - 1$. Without loss of generality, assume $\kappa = \frac{\overline{K}}{\underline{n}+\hat{n}}$ and if the number of contributing investors n is weakly smaller than $\underline{n} + \hat{n}$, they each receive a full share. Only if $n > \underline{n} + \hat{n}$ are returns rationed. Under that proposed setting, the upper bound on the region of Δ for which it is incentive compatible for investors to truthfully report their signals is given by $\Delta^{\hat{n}}_{\underline{n},\hat{B}}$, in which the superscript $\hat{n}$ denotes the number of investors greater than $\underline{n}$ that are required to economize on the project's total capacity. Now, consider a setting in which $\underline{n} + \hat{n} + 1$ investors must participate to economize on the project's total capacity and denote the upper bound on the region of Δ for which it is incentive compatible for investors to truthfully report there signals is given by $\Delta^{\hat{n}+1}_{\underline{n},\hat{B}}$. The difference in $\Delta^{\hat{n}}_{\underline{n},\hat{B}}$ and $\Delta^{\hat{n}+1}_{\underline{n},\hat{B}}$ is given by,

$$\frac{(1-\alpha)A + \alpha B}{(1-\alpha)A} - \frac{(1-\alpha)Y + \alpha Z}{(1-\alpha)Y}, \tag{A58}$$

in which,

$$A = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} \alpha^n (1-\alpha)^{N-1-n} + \sum_{n=\underline{n}+\hat{n}}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n} (\underline{n}+\hat{n})}{n+1}, \tag{A59}$$

$$B = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} (1-\alpha)^n \alpha^{N-1-n} + \sum_{n=\underline{n}+\hat{n}}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n} (\underline{n}+\hat{n})}{n+1}, \tag{A60}$$

$$Y = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}} \binom{N-1}{n} \alpha^n (1-\alpha)^{N-1-n} + \sum_{n=\underline{n}+\hat{n}+1}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n} (\underline{n}+\hat{n}+1)}{n+1}, \tag{A61}$$

$$Z = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}} \binom{N-1}{n} (1-\alpha)^n \alpha^{N-1-n} + \sum_{n=\underline{n}+\hat{n}+1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n} (\underline{n}+\hat{n}+1)}{n+1}. \tag{A62}$$

Furthermore, note that,

$$A = \omega_G + \gamma_G + \theta_G \frac{\underline{n}+\hat{n}}{\underline{n}+\hat{n}+1}, \tag{A63}$$

$$B = \omega_B + \gamma_B + \theta_B \frac{\underline{n}+\hat{n}}{\underline{n}+\hat{n}+1}, \tag{A64}$$

$$Y = \omega_G + \gamma_G \frac{\underline{n}+\hat{n}+1}{\underline{n}+\hat{n}} + \theta_G, \tag{A65}$$

$$Z = \omega_B + \gamma_B \frac{\underline{n}+\hat{n}+1}{\underline{n}+\hat{n}} + \theta_B, \tag{A66}$$

in which,

$$\omega_G = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} \alpha^n (1-\alpha)^{N-1-n}, \tag{A67}$$

$$\omega_B = \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} (1-\alpha)^n \alpha^{N-1-n}, \tag{A68}$$

$$\gamma_G = \sum_{n=\underline{n}+\hat{n}+1}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n} (\underline{n}+\hat{n})}{n+1}, \tag{A69}$$

$$\gamma_B = \sum_{n=\underline{n}+\hat{n}+1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n} (\underline{n}+\hat{n})}{n+1}, \tag{A70}$$

$$\theta_G = \binom{N-1}{\underline{n}+\hat{n}} \alpha^{\underline{n}+\hat{n}} (1-\alpha)^{N-1-\underline{n}-\hat{n}}, \tag{A71}$$

$$\theta_B = \binom{N-1}{\underline{n}+\hat{n}} (1-\alpha)^{\underline{n}+\hat{n}} \alpha^{N-1-\underline{n}-\hat{n}}. \tag{A72}$$

Using the preceding shorthand notation, $\frac{(1-\alpha)A+\alpha B}{(1-\alpha)A} - \frac{(1-\alpha)Y+\alpha Z}{(1-\alpha)Y}$ is given by,

$$\frac{\alpha\left(\omega_B(\gamma_G(1+\underline{n}+\hat{n}) + \theta_G(\underline{n}+\hat{n})) - \omega_G(\gamma_B(1+\underline{n}+\hat{n}) + \theta_B(\underline{n}+\hat{n}))\right)}{(1-\alpha)(\underline{n}+\hat{n})(1+\underline{n}+\hat{n})\left(\omega_G + \gamma_G + \theta_G \frac{\underline{n}+\hat{n}}{\underline{n}+\hat{n}+1}\right)\left(\omega_G + \gamma_G \frac{\underline{n}+\hat{n}+1}{\underline{n}+\hat{n}} + \theta_G\right)}, \tag{A73}$$

and the sign on the expression is determined by the sign on,

$$\omega_B(\gamma_G(1+\underline{n}+\hat{n}) + \theta_G(\underline{n}+\hat{n})) - \omega_G(\gamma_B(1+\underline{n}+\hat{n}) + \theta_B(\underline{n}+\hat{n})), \tag{A74}$$

which is given by,

$$\begin{aligned}(\underline{n}+\hat{n})(\underline{n}+\hat{n}+1)&\left(\sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} (1-\alpha)^n \alpha^{N-1-n} \left(\sum_{n=\underline{n}+\hat{n}}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n}}{n+1}\right)\right.\\ &\left. - \sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1} \binom{N-1}{n} \alpha^n (1-\alpha)^{N-1-n} \left(\sum_{n=\underline{n}+\hat{n}}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n}}{n+1}\right)\right),\end{aligned} \tag{A75}$$

and simplifies further to,

$$
\begin{aligned}
&(\underline{n}+\hat{n})(\underline{n}+\hat{n}+1)\alpha^{N-1}(1-\alpha)^{N-1}\left(\sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1}\binom{N-1}{n}\left(\frac{1-\alpha}{\alpha}\right)^{n}\sum_{n=\underline{n}+\hat{n}}^{N-1}\binom{N-1}{n}\frac{\left(\frac{\alpha}{1-\alpha}\right)^{n}}{n+1}\right. \\
&\left.-\sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1}\binom{N-1}{n}\left(\frac{\alpha}{1-\alpha}\right)^{n}\sum_{n=\underline{n}+\hat{n}}^{N-1}\binom{N-1}{n}\frac{\left(\frac{1-\alpha}{\alpha}\right)^{n}}{n+1}\right). \qquad \text{(A76)}
\end{aligned}
$$

The previous expression can be seen to be positive valued by noting that the product,

$$
\sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1}\binom{N-1}{n}\left(\frac{1-\alpha}{\alpha}\right)^{n}\sum_{n=\underline{n}+\hat{n}}^{N-1}\binom{N-1}{n}\frac{\left(\frac{\alpha}{1-\alpha}\right)^{n}}{n+1}, \qquad \text{(A77)}
$$

yields a summation of values and each value in the summation contains $\left(\frac{\alpha}{1-\alpha}\right)$ raised to some positive integer. The product,

$$
\sum_{n=\underline{n}-1}^{\underline{n}+\hat{n}-1}\binom{N-1}{n}\left(\frac{\alpha}{1-\alpha}\right)^{n}\sum_{n=\underline{n}+\hat{n}}^{N-1}\binom{N-1}{n}\frac{\left(\frac{1-\alpha}{\alpha}\right)^{n}}{n+1}, \qquad \text{(A78)}
$$

yields a similar summation of values and each value in the summation contains $\left(\frac{\alpha}{1-\alpha}\right)$ raised to some negative integer. That is, for any term in the first summation which is generically given by $\chi\left(\frac{\alpha}{1-\alpha}\right)^{\zeta}$ in which $\chi>0$ and ζ is some positive integer, there is a corresponding term in the second summation given by $\chi\left(\frac{\alpha}{1-\alpha}\right)^{-\zeta}$. Because the two summations are indexed identically, the first summation is always greater in value than the second summation.

■

Proof of Proposition 3: The analysis in Subsections 2.1 and 2.2 show that, if first-best is not possible, it is because $\Delta \geq \Delta_{\underline{n},\hat{B}}$ (see Corollary 2.2). Therefore, since decreasing β below 1 shifts $\Delta_{\underline{n},\hat{G}}$ and $\Delta_{\underline{n},\hat{B}}$ proportionally by a factor of $\frac{1}{\beta}$, first-best is possible by choosing a value of β such that $\Delta \in \left[\Delta_{\underline{n},\hat{G},\beta}, \Delta_{\underline{n},\hat{B},\beta}\right)$ in which $\Delta_{\underline{n},\hat{G},\beta}$ and $\Delta_{\underline{n},\hat{B},\beta}$ are defined as,

$$
\Delta_{\underline{n},\hat{G},\beta} \equiv \frac{\alpha\sum_{n=\underline{n}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}+(1-\alpha)\sum_{n=\underline{n}-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\beta\alpha\sum_{n=\underline{n}-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}, \qquad \text{(A79)}
$$

and

$$\Delta_{\underline{n},\hat{B},\beta} \equiv \frac{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1} + \alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\beta(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}. \tag{A80}$$

■

Proof of Corollary 3.1: The proof of the corollary is constructed as follows: First, we consider projects with $\Delta \leq 2$ and, without loss of generality, focus on the special case in which $\Delta = 2$. We begin by showing that $\overline{\beta}$ is decreasing in N and goes to $\frac{1}{\Delta}$ in the limit, all else equal. We then show that, without an information cost, an investor who observes $\hat{G}$ earns an expected profit of zero in the limit, that is, $\lim_{N\to\infty} \Pi(\hat{G}|\vec{\pi}^*,\beta^*) = 0$. To complete the proof, we argue that a strictly positive information cost implies that the inequality in (33) cannot be satisfied for a large number of investors N.

Without loss of generality, consider a $\Delta \leq 2$ in a region in which first-best is not possible with $\beta = 1$ (see Section 2 for analysis of these regions). $\overline{\beta}$ is internally valued and given by,

$$\overline{\beta} = \frac{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1} + \alpha \sum_{n=\underline{n}-1}^{N-1} \Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{\Delta(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}, \tag{A81}$$

and the right-hand side simplifies to,

$$\frac{1}{\Delta}\left(1 + \frac{\alpha\sum_{n=\underline{n}-1}^{N-1} \Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=\underline{n}-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}\right). \tag{A82}$$

Showing that the preceding expression is decreasing in N is most easily shown by focusing on a project with return $\Delta = 2$ and by assuming N is divisible by 2 (the analysis can be extended to values of $\Delta < 2$ using numerical methods). Under first-best, $\underline{n} = N/2$ and $\overline{\beta}$ is given by,

$$\frac{1}{2}\left(1 + \frac{\alpha\sum_{n=N/2-1}^{N-1} \Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=N/2-1}^{N-1} \Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}\right). \tag{A83}$$

Now, suppose that one more investor is added so that there are $\hat{N} = N+1$ total investors. $\overline{\beta}$ with $\hat{N}$ investors is given by,

$$\frac{1}{2}\left(1 + \frac{\alpha\sum_{n=\hat{N}/2+\frac{1}{2}-1}^{\hat{N}-1} \Pr(n|B,\vec{\pi},\hat{N}-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=\hat{N}/2+\frac{1}{2}-1}^{\hat{N}-1} \Pr(n|G,\vec{\pi},\hat{N}-1)\frac{1}{n+1}}\right), \tag{A84}$$

which simplifies, after substituting $N+1$ for $\hat{N}$, to,

$$\frac{1}{2}\left(1+\frac{\alpha\sum_{n=N/2}^{N}\Pr(n|B,\vec{\pi},N)\frac{1}{n+1}}{(1-\alpha)\sum_{n=N/2}^{N}\Pr(n|G,\vec{\pi},N)\frac{1}{n+1}}\right). \tag{A85}$$

The difference in the two values of $\overline{\beta}$ is given by,

$$\begin{aligned}&\overline{\beta}\big|_{N}-\overline{\beta}\big|_{N+1}=\\&\frac{1}{2}\left(\frac{\alpha\sum_{n=N/2-1}^{N-1}\Pr(n|B,\vec{\pi},N-1)\frac{1}{n+1}}{(1-\alpha)\sum_{n=N/2-1}^{N-1}\Pr(n|G,\vec{\pi},N-1)\frac{1}{n+1}}-\frac{\alpha\sum_{n=N/2}^{N}\Pr(n|B,\vec{\pi},N)\frac{1}{n+1}}{(1-\alpha)\sum_{n=N/2}^{N}\Pr(n|G,\vec{\pi},N)\frac{1}{n+1}}\right),\end{aligned} \tag{A86}$$

which, using Lemmas B2-B6 in Appendix B, may be rewritten as,

$$\frac{\alpha}{2(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\frac{{}_2F_1\left(1,N/2-N,N/2+1,-\frac{1-\alpha}{\alpha}\right)}{{}_2F_1\left(1,N/2-N,N/2+1,-\frac{\alpha}{1-\alpha}\right)}-\frac{{}_2F_1\left(1,N/2-N,N/2+2,-\frac{1-\alpha}{\alpha}\right)}{{}_2F_1\left(1,N/2-N,N/2+2,-\frac{\alpha}{1-\alpha}\right)}\right), \tag{A87}$$

in which ${}_2F_1\left(a,b;c;z\right)$ is the ordinary hypergeometric function. It is straightforward to show numerically that the preceding expression is positive valued for all $\alpha>\frac{1}{2}$ and $N>2$, implying that $\overline{\beta}$ is decreasing in N for $\Delta=2$.

Next, we show that $\overline{\beta}$ goes to $\frac{1}{2}$ in the limit for $\Delta=2$. Note that, using Lemmas B2-B6 in Appendix B, $\overline{\beta}$ may be rewritten as,

$$\frac{1}{2}\left(1+\frac{\alpha^2}{(1-\alpha)^2}\frac{{}_2F_1\left(1,N/2-N,N/2+1,-\frac{1-\alpha}{\alpha}\right)}{{}_2F_1\left(1,N/2-N,N/2+1,-\frac{\alpha}{1-\alpha}\right)}\right). \tag{A88}$$

It is straightforward to show numerically that the preceding expression goes to $\frac{1}{2}$ for all $\alpha>\frac{1}{2}$ as $N\to\infty$.

We now consider the case in which the information cost is zero. If $\overline{\beta}$ goes to $\frac{1}{2}$ for $\Delta=2$ in the limit as $N\to\infty$, then $\Pi(\hat{G}|\vec{\pi}^*,\beta^*)$ goes to zero. To see this, consider $\Pi(\hat{G}|\vec{\pi}^*,\beta^*)$ evaluated at $\beta^*=\overline{\beta}$.[19] $\Pi(\hat{G}|\vec{\pi}^*,\overline{\beta})$ is given by,

$$\sum_{n=\underline{n}-1}^{N-1}\left(\alpha\Pr(n|G,\vec{\pi},N-1)\frac{(2\overline{\beta}-1)\,\underline{n}}{n+1}-(1-\alpha)\Pr(n|B,\vec{\pi},N-1)\frac{\underline{n}}{n+1}\right), \tag{A89}$$

[19]Technically, $\beta^*<\overline{\beta}$, thus, assume β^* is smaller than $\overline{\beta}$ by some arbitrarily small value of ϵ.

and, in the limit as $N \to \infty$, goes to zero (because $\lim_{N\to\infty} \Pr(n|B, \vec{\pi}, N-1) = 0$ for all $n \geq \frac{N}{2}$ and $\overline{\beta} \to \frac{1}{2}$).

To complete the proof, we argue that a strictly positive information cost implies that the inequality in (33) cannot be satisfied for a large number of investors N. If both $\Pi(\hat{G}|\vec{\pi}^*, \overline{\beta})$ and $\Pi(\hat{B}|\vec{\pi}^*, \overline{\beta})$ go to zero, then the information acquisition constraint in (33) cannot be satisfied for a strictly positive c and arbitrarily large number of investors N.

■

Appendix B

In this auxiliary appendix, we discuss an extension to our base model in which investors are atomistic. Furthermore, we provide additional analytic results that are used in proving the paper's main propositions, corollaries, and lemmas.

B.1 Non-Pivotal Investors

Our base model assumes that investors make their contribution decisions based partially on the probability that they may be pivotal in affecting the principal's decision of how to deploy capital. For example, the upper and lower bounds on the range of incentive compatible Δ values for which investors truthfully report signals outlined in (21) and (22) are computed from the perspective that at least $\underline{n}-1$ other investors must contribute for the principal to choose risky deployment of capital. The assumption places an incredible onus on the investors, particularly when N is large. In this subsection, we consider a setting in which investors do not see themselves as being potentially pivotal, which may better represent large crowds. We show that, with non-pivotal investors, the loser's blessing is exacerbated.

Consider a setting in which investors are atomistic. We maintain all other assumptions regarding the project from Section 2. The principal solicits contributions from a unit continuum of risk-neutral investors. Each investor i can contribute κ dollars to the project so that the total available capital equals $\kappa = \int_0^1 \kappa \, \mathrm{d}i$. After the principal raises capital, if the total level of capital exceeds $\underline{K}$, the principal chooses between risky and safe deployment. The principal's problem is to choose κ to maximize project value.

As in the base model, investors receive conditionally i.i.d. private signals and then simultaneously choose whether or not to contribute. Investors who observe $\hat{G}$ contribute with probability $g^{NP} \in [0,1]$ and investors who observe $\hat{B}$ contribute with probability $b^{NP} \in [0,1]$ (NP denotes non-pivotal crowdfunding). While we restrict our analysis to equilibria in which investors play identical strategies, we denote the strategy of investor i as, $\vec{\pi}_i^{NP} = \{g_i^{NP}, b_i^{NP}\}$, and we denote the strategy of all other investors, not including investor i, as, $\vec{\pi}_{-i}^{NP} = \{g_{-i}^{NP}, b_{-i}^{NP}\}$.

Whether an investor contributes is determined by the sign on her expected payoff (expressed as a return on invested capital),

$$\Pi(\hat{F}|\vec{\pi}_i^{NP}, \vec{\pi}_{-i}^{NP}) = \Pr(G|\hat{F})\sigma(G)(\Delta - 1) - \Pr(B|\hat{F})\sigma(B), \tag{B1}$$

in which the share of her committed capital deployed to the project is,

$$\sigma(F) \equiv \frac{1}{\ell(F)} \mathbb{1}_F. \tag{B2}$$

$\ell(F)$ is the measure of contributing investors, which depends on α and investors' strategies. $\mathbb{1}_F$ is an indicator function that equals one if (i) total capital exceeds $\underline{K}$ and (ii) the principal believes based on his posterior beliefs that the project is positive valued. $\sigma(G)$ therefore represents each investor's expected contribution in good projects and $\sigma(B)$ represents expected contribution in bad projects.

As in our base model, a loser's blessing may exist. If a loser's blessing exists, by the strong law of large numbers the principal receives a perfect signal regarding the project's unobservable quality. When total capital reflects a bad project, the principal never chooses risky deployment and $\sigma(B) = 0$.

Because investor i is atomistic, she is not pivotal and she does not internalize the impact of her contribution decision on whether or not the principal chooses risky deployment. Therefore, her contribution decision is dictated by the sign on the expression in (B1). Taking all other investors' equilibrium contribution strategies as given, investor i will contribute with probability one if (B1) is strictly positive. If the expression equals zero, investor i is indifferent between contributing and not contributing and she may choose to mix between her options. Investor i abstains from contributing if (B1) is strictly negative.

If investor i observes $\hat{G}$, (B1) is explicitly given by,

$$\underbrace{\alpha}_{\Pr(G|\hat{G})} \underbrace{\left(\frac{\mathbb{1}_G}{(\alpha g_{-i}^{NP} + (1-\alpha) b_{-i}^{NP})} \right)}_{\sigma(G)} (\Delta - 1) - \underbrace{(1-\alpha)}_{\Pr(B|\hat{G})} \underbrace{\left(\frac{\mathbb{1}_B}{((1-\alpha) g_{-i}^{NP} + \alpha b_{-i}^{NP})} \right)}_{\sigma(B)}, \tag{B3}$$

in which $\mathbb{1}_G$ equals one if both $\kappa\left(\alpha g_{-i}^{NP} + (1-\alpha) b_{-i}^{NP}\right) \geq \underline{K}$ and the principal's posterior beliefs $\rho(n, \vec{\pi})$ are sufficiently high that he believes the project is positive valued. $\mathbb{1}_B$ equals one if both $\kappa\left((1-\alpha) g_{-i}^{NP} + \alpha b_{-i}^{NP}\right) \geq \underline{K}$ and the principal's posterior beliefs $\rho(n, \vec{\pi})$ are sufficiently high that he believes the project is positive valued. Similarly, if investor i observes $\hat{B}$ instead, (B1) is explicitly given by,

$$(1-\alpha) \left(\frac{\mathbb{1}_G}{(\alpha g_{-i}^{NP} + (1-\alpha) b_{-i}^{NP})} \right) (\Delta - 1) - \alpha \left(\frac{\mathbb{1}_B}{((1-\alpha) g_{-i}^{NP} + \alpha b_{-i}^{NP})} \right). \tag{B4}$$

If investor contribution strategies vary depending on the project's unobservable type, by the strong law of large numbers there are only two capital contribution levels: (i) a good project raises $\kappa\left(\alpha g_{-i}^{NP} + (1-\alpha)b_{-i}^{NP}\right)$ and (ii) a bad project raises $\kappa\left((1-\alpha)g_{-i}^{NP} + \alpha b_{-i}^{NP}\right)$. Thus, if investor contribution strategies vary (i.e., $g \neq b$), the principal receives a perfect signal regarding the project's unobservable quality and $\mathbb{1}_B = 0$ meaning there is a perfect loser's blessing since the principal never chooses risky deployment for a bad project.

Lemma B1. *A loser's blessing never exists in equilibrium.*

Proof of Lemma B1: A loser's blessing cannot exist in equilibrium. Suppose not: in equilibrium, $\mathbb{1}_G = 1$ and $\mathbb{1}_B = 0$, implying that type G projects raise strictly more dollars than type B projects, allowing the principal to perfectly screen projects. Investor i's payoff in (B1) based on signal $\hat{G}$ or signal $\hat{B}$ is positive valued with $\mathbb{1}_G = 1$ and $\mathbb{1}_B = 0$. As such, investor i chooses to invest under both signals. This deviation is profitable for all investors. By the strong law of large numbers, both type G projects and type B projects raise κ dollars from the unit continuum of investors. Therefore, there is a contradiction as type G projects do not raise more dollars than type B projects.

■

For a loser's blessing to exist, there must exist a wedge between the quantities of capital raised by good and bad projects. If, however, a loser's blessing exists in equilibrium, all investors will find contributing optimal. Thus, the equilibrium will unravel and investors will contribute for both types of projects, eliminating the financing wedge.

Remark 2. *With non-pivotal investors, the principal receives no information from the fundraising process and first-best financing efficiency, as defined in Definition 2, is never possible.*

The preceding remark highlights the inefficiencies introduced by the loser's blessing when investors are atomistic. Despite the crowd collectively possessing a perfect signal regarding the project's quality, the principal is unable to extract the information due to the loser's blessing.

B.2 Additional Proofs

In this auxiliary subsection, we include additional analytic results that are used in the proofs of the model's main results.

Lemma B2. *The expression*

$$\sum_{n=\underline{n}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n}}{n+1}, \tag{B5}$$

may be rewritten as,

$$\alpha^{N-1}\left(\frac{1-\alpha}{\alpha}\right)^{\underline{n}-1}\Gamma(\underline{n})\binom{N-1}{\underline{n}-1}\sum_{n=\underline{n}-1}^{N-1}\frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)}\left(\frac{1-\alpha}{\alpha}\right)^{n-(\underline{n}-1)}. \tag{B6}$$

Proof of Lemma B2:

Beginning with (B5), the expression $\sum_{n=\underline{n}-1}^{N-1}\binom{N-1}{n}\frac{(1-\alpha)^n\alpha^{N-1-n}}{n+1}$ may be simplified to,

$$=\alpha^{N-1}\sum_{n=\underline{n}-1}^{N-1}\binom{N-1}{n}\frac{(1-\alpha)^n\alpha^{-n}}{n+1} \tag{B7}$$

$$=\alpha^{N-1}\sum_{n=\underline{n}-1}^{N-1}\binom{N-1}{n}\frac{(1-\alpha)^n\alpha^{-n}}{n+1}\frac{(\underline{n}-1)!}{(\underline{n}-1)!} \tag{B8}$$

$$=\alpha^{N-1}\Gamma(\underline{n})\sum_{n=\underline{n}-1}^{N-1}\frac{(N-1)!}{n!(N-1-n)!(\underline{n}-1)!(n+1)}\left(\frac{1-\alpha}{\alpha}\right)^{n} \tag{B9}$$

$$=\alpha^{N-1}\Gamma(\underline{n})\sum_{n=\underline{n}-1}^{N-1}\frac{(N-1)!}{n!(N-1-n)!(\underline{n}-1)!(n+1)}\left(\frac{1-\alpha}{\alpha}\right)^{n}\frac{(N-\underline{n})!}{(N-\underline{n})!} \tag{B10}$$

$$=\alpha^{N-1}\Gamma(\underline{n})\binom{N-1}{\underline{n}-1}\sum_{n=\underline{n}-1}^{N-1}\frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)}\left(\frac{1-\alpha}{\alpha}\right)^{n} \tag{B11}$$

$$=\alpha^{N-1}\left(\frac{1-\alpha}{\alpha}\right)^{\underline{n}-1}\Gamma(\underline{n})\binom{N-1}{\underline{n}-1}\sum_{n=\underline{n}-1}^{N-1}\frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)}\left(\frac{1-\alpha}{\alpha}\right)^{n-(\underline{n}-1)}. \tag{B12}$$

■

Lemma B3. *The expression*

$$\sum_{n=\underline{n}-1}^{N-1}\binom{N-1}{n}\frac{\alpha^n(1-\alpha)^{N-1-n}}{n+1}, \tag{B13}$$

may be rewritten as,

$$(1-\alpha)^{N-1}\left(\frac{\alpha}{1-\alpha}\right)^{\underline{n}-1}\Gamma(\underline{n})\binom{N-1}{\underline{n}-1}\sum_{n=\underline{n}-1}^{N-1}\frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)}\left(\frac{\alpha}{1-\alpha}\right)^{n-(\underline{n}-1)}. \tag{B14}$$

Proof of Lemma B3:

The proof follows from the proof of Lemma B2.

■

Lemma B4. *The summation,*

$$\sum_{n=\underline{n}-1}^{N-1} \frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)} \left(\frac{1-\alpha}{\alpha}\right)^{n-(\underline{n}-1)}, \tag{B15}$$

is equivalent to the ordinary hypergeometric function,

$${}_2F_1\left(1, \underline{n}-N; \underline{n}+1; -\frac{1-\alpha}{\alpha}\right). \tag{B16}$$

Proof of Lemma B4:

The proof follows directly from the definition of the ordinary hypergeometric function.

■

Lemma B5. *The summation,*

$$\sum_{n=\underline{n}-1}^{N-1} \frac{(N-\underline{n})!}{n!(N-1-n)!(n+1)} \left(\frac{\alpha}{1-\alpha}\right)^{n-(\underline{n}-1)}, \tag{B17}$$

is equivalent to the ordinary hypergeometric function,

$${}_2F_1\left(1, \underline{n}-N; \underline{n}+1; -\frac{\alpha}{1-\alpha}\right). \tag{B18}$$

Proof of Lemma B5:

The proof follows directly from the definition of the ordinary hypergeometric function.

■

Lemma B6. *The ratio*

$$\frac{\sum\limits_{n=\underline{n}-1}^{N-1} \binom{N-1}{n} \frac{(1-\alpha)^n \alpha^{N-1-n}}{n+1}}{\sum\limits_{n=\underline{n}-1}^{N-1} \binom{N-1}{n} \frac{\alpha^n (1-\alpha)^{N-1-n}}{n+1}}, \tag{B19}$$

may be rewritten as,

$$\left(\frac{\alpha}{1-\alpha}\right)^{N-1} \left(\frac{1-\alpha}{\alpha}\right)^{2(\underline{n}-1)} \frac{{}_2F_1\left(1, \underline{n}-N; \underline{n}+1; -\frac{1-\alpha}{\alpha}\right)}{{}_2F_1\left(1, \underline{n}-N; \underline{n}+1; -\frac{\alpha}{1-\alpha}\right)}. \tag{B20}$$

Proof of Lemma B6:

The proof follows from the proofs of Lemma B2, Lemma B4, Lemma B3, and Lemma B5.

■

Addendum 1. *For $\alpha \in \left(\frac{1}{2}, 1\right]$, $n > 1$, and $N > n$, the expression,*

$$\frac{\left({}_2F_1\left(1, n-N; n+1; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, n-N; n+1; -\frac{\alpha}{1-\alpha}\right)\right)}{\left({}_2F_1\left(1, n+1-N; n+2; -\frac{1-\alpha}{\alpha}\right) \Big/ {}_2F_1\left(1, n+1-N; n+2; -\frac{\alpha}{1-\alpha}\right)\right)}, \tag{B21}$$

is bounded above by one.

We conjectured this Addendum based on numerical attempts and posted it as an open problem on March 28, 2018 in the Mathematics StackExchange (https://math.stackexchange.com/questions/2712056/bounding-a-ratio-of-gaussian-hypergeometric-functions). We further communicated with some mathematicians that were familiar with hypergeometric functions. Robert S. Maier suggested that we write to Richard Askey to see if he had any ideas, or could suggest one of his collaborators. Fortunately, Askey forwarded it to George Gasper who was able to discover the following proof (presented with his permission for the convenience of the reader). In a separate paper, George Gasper will extend the following formulas to power series and more general formulas.

Proof of Addendum 1:

The proceeding analytic proof was provided to us by George Gasper, Emeritus Professor of Mathematics at Northwestern University, Evanston, Illinois, United States. Recommended citation: ***Omitted to maintain anonymity during journal review process.***

Let $n > -1$, $x = \frac{1-\alpha}{\alpha}$, $\alpha \in \left(\frac{1}{2}, 1\right)$, and $m = N - n = 1, 2, \ldots$. Then $0 < x < 1$, and the four ${}_2F_1$ series are positive, terminating series. Using the preceding notation of n, x, α, and m, our conjectured inequality in Addendum 1 may be rewritten as,

$$\frac{{}_2F_1\left(1, -m; 1+N-m; -x\right) {}_2F_1\left(1, 1-m; 2+N-m; -x^{-1}\right)}{{}_2F_1\left(1, -m; 1+N-m; -x^{-1}\right) {}_2F_1\left(1, 1-m; 2+N-m; -x\right)} < 1, \tag{B22}$$

which, by cross-multiplication, is equivalent to the inequality,

$$\begin{aligned} G_m(x; N) \equiv & {}_2F_1\left(1, -m; 1+N-m; -x^{-1}\right) {}_2F_1\left(1, 1-m; 2+N-m; -x\right) \\ & - {}_2F_1\left(1, -m; 1+N-m; -x\right) {}_2F_1\left(1, 1-m; 2+N-m; -x^{-1}\right) > 0. \end{aligned} \tag{B23}$$

In terms of the power series,

$$G_m(x;N) = \sum_{r=0}^{m} \frac{(-m)_r(-x)^{-r}}{(1+N-m)_r} \sum_{s=0}^{m-1} \frac{(1-m)_s(-x)^s}{(2+N-m)_s} - \sum_{r=0}^{m} \frac{(-m)_r(-x)^{r}}{(1+N-m)_r} \sum_{s=0}^{m-1} \frac{(1-m)_s(-x)^{-s}}{(2+N-m)_s}, \tag{B24}$$

in which $(a)_r$ is the shifted factorial (also called the (rising) Pochhammer symbol), which is defined by:

$$(a)_r = \begin{cases} 1 & r = 0 \\ a(a+1)\dots(a+r-1) & r > 0. \end{cases} \tag{B25}$$

By series manipulations, the expression in (B24) further simplifies to,

$$G_m(x;N) = \sum_{r=0}^{m} \sum_{s=0}^{m-1} \frac{(-m)_r(1-m)_s(-1)^{r+s}}{(1+N-m)_r(2+N-m)_s} \left(x^{s-r} - x^{r-s}\right) \tag{B26}$$

$$= \sum_{k=1}^{m} A_k(m,N)(x^{-k} - x^k), \tag{B27}$$

with

$$A_k(m,N) = \sum_{r=k}^{m} \frac{(-m)_r(1-m)_{r-k}(-1)^k}{(1+N-m)_r(2+N-m)_{r-k}} - \sum_{r=0}^{m-1-k} \frac{(-m)_r(1-m)_{r+k}(-1)^k}{(1+N-m)_r(2+N-m)_{r+k}}. \tag{B28}$$

It is easy to check for small values of m that $A_k(m,N) > 0$, $k = 1, 2, \dots, m$, and hence that $G_m(x;N) > 0$ for these values of m since $x^{-k} - x^k > 0$ when $0 < x < 1$ and k is a positive integer. In order to prove that all of the coefficients $A_k(m,N)$ in (B27) are positive, and hence that $G_m(x;N) > 0$ (given $0 < x < 1$, m is a positive integer and $N+1 > m$), proving the conjectured inequalities in (B22) and (B23), we proceed as follows. Via a shift of the index of summation in the first series on the right-hand side of (B28), we obtain that,

$$\sum_{r=k}^{m} \frac{(-m)_r(1-m)_{r-k}(-1)^k}{(1+N-m)_r(2+N-m)_{r-k}} = \sum_{r=0}^{m-k} \frac{(-m)_{r+k}(1-m)_r(-1)^k}{(1+N-m)_{r+k}(2+N-m)_r} \tag{B29}$$

$$= \frac{(-m)_m(1-m)_{m-k}(-1)^k}{(1+N-m)_m(2+N-m)_{m-k}} + \sum_{r=0}^{m-k-1} \frac{(-m)_{r+k}(1-m)_r(-1)^k}{(1+N-m)_{r+k}(2+N-m)_r}. \tag{B30}$$

Hence, using (B28) and identities involving shifted factorials $(a)_r$ such as in Appendix II of Slater (1966),

$$A_k(m,N) = \frac{(-m)_m(1-m)_{m-k}(-1)^k}{(1+N-m)_m(2+N-m)_{m-k}} + \sum_{r=0}^{m-k-1} \left[\frac{(-m)_{r+k}(1-m)_r(-1)^k}{(1+N-m)_{r+k}(2+N-m)_r} - \frac{(-m)_r(1-m)_{r+k}(-1)^k}{(1+N-m)_r(2+N-m)_{r+k}} \right], \tag{B31}$$

in which the difference of the two functions inside the square brackets equals,

$$\frac{(-m)_r(1-m)_{r+k-1}(-1)^{k+1}\left((r+k-m)(1+N-m+r)-(r-m)(1+N-m+r+k)\right)}{(1+N-m)_{r+k+1}(2+N-m)_r}, \tag{B32}$$

which simplifies to,

$$\frac{k(N+1)(-m)_r(1-m)_{r+k-1}(-1)^{k+1}}{(1+N-m)_{r+k+1}(2+N-m)_r}. \tag{B33}$$

Thus, from (B31),

$$A_k(m,N) = \frac{(-m)_m(1-m)_{m-k}(-1)^k}{(1+N-m)_m(2+N-m)_{m-k}} + \sum_{r=0}^{m-k-1} \frac{k(N+1)(-m)_r(1-m)_{r+k-1}(-1)^{k+1}}{(1+N-m)_{r+k+1}(2+N-m)_r} \tag{B34}$$

$$= \sum_{r=0}^{m} \frac{k(N+1)(-m)_r(1-m)_{r+k-1}(-1)^{k+1}}{(1+N-m)_{r+k+1}(2+N-m)_r} \tag{B35}$$

$$= \frac{k(N+1)(m-1)!}{(m-k)!(1+N-m)_{k+1}} {}_3F_2\left[\left. {}^{1,-m,k-m}_{2+N-m,2+N+k-m} \right| 1 \right], \tag{B36}$$

which is clearly positive when $m = 1, 2, \ldots,$, $k = 1, 2, \ldots, m$, and $N + 1 > m$. This completes the proof of the inequalities in (B22) and (B23). It is obvious that (B22) and (B23) hold with equality when $x = 1$, i.e., $\alpha = \frac{1}{2}$.[20]

■

[20] It is also obvious that the inequalities (B22) and (B23) are reversed when $x > 1$.